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Christoph Braun[1]

Taxing Human Capital: A Good Idea

Abstract

This paper studies a Ramsey optimal taxation model with human capital in an infinite-horizon setting. Contrary to Jones, Manuelli, and Rossi (1997), the human capital production function does not include the current stock of human capital as a production factor. As a result, the return to human capital, namely labor income, does not vanish in equilibrium. In a stationary state, the household underinvests in human capital relative to the first best, i.e., education is distorted. Human capital is effectively taxed. The optimal tax scheme prescribes making the cost of education not fully tax-deductible.

JEL Classification: H21, I28, J24

Keywords: Optimal taxation; human capital; Ramsey approach

September 2010

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1 Introduction

"Is physical capital special?" Jones, Manuelli, and Rossi (1997) ask. Using the Ramsey approach (Ramsey, 1927), they add human capital to an optimal taxation model with physical capital similar to that of Chamley (1986) and Judd (1985). By modeling human capital almost symmetrically to physical capital they show that in a stationary state all taxes are zero. The Chamely and Judd's result is thus shown to extend to human capital. What drives this zero-tax result is that the human capital production function features constant returns to scale with respect to the stock of human capital. Jones, Manuelli, and Rossi (1997) call this specification a zero-profit condition. As a consequence, human capital disappears as an object of taxation in a competitive equilibrium. But they acknowledge that if the human capital production function violates the assumption of constant returns to scale, the stationary-state labor tax will not be zero. Jones, Manuelli, and Rossi (1997) raise an intriguing question and provide useful insights into the nature of optimal taxation, but in the end, unfortunately, no answer is evident. The difference between physical and human capital still is not clear, because they have made it disappear by means of zero-profit conditions.[1]

This paper takes up the issue of modeling human capital almost symmetrically to physical capital. I drop the constant-returns-to-scale assumption. The human capital production function does not include the current stock of human capital, which therefore is not self-productive. It does not raise the productivity of human capital investments, or interchangeably, education. The increasing and concave production function only includes the household's time devoted to education. Time spent on education can-

[1]See also Ljungqvist and Sargent (2004, pp. 534) for this line of argument.

not be substituted by physical goods.[2] Instead, the household has to pay for verifiable[3] direct costs, e.g.,s tuition fees, that depend on the amount of education. The government may choose to subsidize this cost. It therefore has two instruments at its disposal to guide education. Labor taxes and the subsidy both affect the opportunity cost of education. The next periods' labor tax rates affect the discounted stream of marginal earnings from education.

I derive two results: The first one is not surprising but nonetheless important, as it helps to clarify the role of zero capital taxation when the model features human capital. The other is new and shows how to deal with profits coming from education. First, optimal taxation in the stationary state prescribes not taxing capital income, as Chamley (1986) and Judd (1985) show. The zero-capital-tax result holds despite the presence of human capital. Lucas (1990), Jones, Manuelli, and Rossi (1997) and Chari and Kehoe (1999) also derive this result. The education decision depends only on how the labor tax and the education subsidy interact with each other. This relates to the second result, stating that in the optimum the marginal social return to education is larger than the marginal social cost. The so-called *Education Efficiency Theorem* (Richter, 2009), which states that the education decision is undistorted given certain assumptions, does not hold. From the inequality between the marginal social return and the marginal social cost it follows that education is effectively taxed, i.e., the private rate of return to education is smaller than the social rate of return. Turning to the underlying tax rates, it results that the cost of education is not fully tax-deductible, the labor income tax rate is higher than the rate

[2]Allowing for physical goods as an additional production factor does not affect the results obtained by Jones, Manuelli, and Rossi (1997), as Chari and Kehoe (1999) show.

[3]Reis (2007, chapter 4) assumes that the government cannot distinguish between consumption and expenditures on education and finds that it is optimal to tax human capital.

of subsidization. As a consequence, the household underinvests in human capital relative to the first best.

The second result is striking. Since the household is endowed with perfect foresight and therefore must be able to internalize the effects of its actions, one would have expected to derive an equality between the private and social rates of return to education, and the *Education Efficiency Theorem* to hold - a result that Jones, Manuelli, and Rossi (1997), among others,[4] also obtain. Their zero-tax results imply that all private and social rates of return from investments in physical and human capital are equal in the stationary state. The difference in results is due to how I model the accumulation of human capital. The specification used gives rise to profits in equilibrium. Profits from education are not pure in the strict sense, because they still depend on raw labor supply. The government taxes away part of the return to education, thereby accepting the distortion of education.

To derive clear-cut results, the analysis is confined to an examination of the stationary state. In the stationary state, the household's decision variables remain constant. As usual, it is assumed that a unique stationary state exists and that the economy converges to it. It would be straightforward to introduce exogenous growth. To allow for a setting in which the economy grows endogenously is however not possible. The reason for this limitation is the specification of the human capital production function. Lucas (1988) and Caballe and Santos (1993) provide a discussion of the existence and properties of a balanced growth path. They show that the human capital production function must feature constant returns to scale with respect to the stock of human capital.

[4]For further reference, see Lucas (1990), Bull (1993), Milesi-Ferretti and Roubini (1998), Chari and Kehoe (1999), Barbie and Hermeling (2006), and Richter (2009).

2 The Model

2.1 Household's Problem

The household solves the following maximization problem:

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ & u(c_t, 1 - n_t - e_t) \\
& - \lambda_t \Big(c_t + k_{t+1} + b_{t+1} + (1 - \tau_t^e) f e_t - (1 - \tau_t^n) w_t n_t h_t \\
& - R_t^k k_t - R_t^b b_t \Big) \qquad (1) \\
& - \mu_t \Big(h_{t+1} - (1 - \delta_h) h_t - G(e_t) \Big) \Big\} \qquad (2)
\end{aligned}
$$

The household's utility function u is strictly increasing and concave in both arguments and continuously differentiable everywhere. The Inada conditions apply to ensure interior solutions. In each period t the household faces a consumption-labor-leisure choice. It consumes c_t, which is not taxed,[5] and devotes n_t time units to work in the labor market and e_t time units to investment in human capital. The total time endowment is normalized to one, i.e., $n_t + e_t + \ell_t = 1$, where ℓ_t is the amount of leisure. The household combines its raw labor supply n_t with the current stock of human capital h_t. The product $z_t \equiv n_t h_t$ is called the effective labor supply,[6] it earns the after-tax wage rate $(1 - \tau_t^n) w_t$ where w_t is the real wage rate. The household must spend resources $(1 - \tau_t^e) f$ per time unit invested in human capital. One may think of f as tuition, books, and other related

[5] Taxing consumption only complicates the analysis without yielding further insights in the present context.

[6] This specification is a special case of Jones, Manuelli, and Rossi (1997), who use the more general function $z = M(x, h, n)$ and assume that it exhibits constant returns to scale with respect to h and market goods x. Judd (1999) works out that this specification is not innocuous, as any deviation gives rise to positive taxation of human capital. This point Jones, Manuelli, and Rossi (1997, p. 103) acknowledge.

expenses. The government subsidizes this cost at rate τ_t^e. The household lends capital k_{t+1} to the firm. The rate of return net of taxes and depreciation is $R_{t+1}^k \equiv (1-\tau_t^k)r_{t+1} + 1 - \delta_k$, where r_t is the real interest rate and δ_k is the rate at which capital depreciates. The household may lend b_{t+1} to the government which offers a rate of return of R_{t+1}^b in the next period. In period 0, the household earns income from capital $R_0^k k_0$ and government debt $R_0^b b_0$. (1) is the household's budget constraint in period t, which is associated with the Lagrange multipliers λ_t.

Associated with the Lagrange multiplier μ_t, the law of motion (2) describes the accumulation of human capital. Investments e_t enter the human capital production function G, which is strictly increasing and concave, i.e., $G'' < 0 < G'$. The crucial assumption is that the current stock of human capital does not enter the production function; it does not increase productivity. The output of G is added to the depreciated stock of human capital, the rate of depreciation being $0 < \delta_h \leq 1$.[7] Furthermore, the human capital production function G is assumed to be isoelastic, that is, $\gamma \equiv G'e/G < 1$.[8] Finally, β is the household's discount factor, which, for simplicity, stays constant over time.

The first-order conditions are

$$c_t : \frac{\partial u}{\partial c_t} \equiv u_{c_t} = \lambda_t \tag{3}$$

$$n_t : \frac{\partial u}{\partial \ell_t} \equiv u_{\ell_t} = (1-\tau_t^n) w_t h_t \lambda_t \tag{4}$$

$$e_t : u_{\ell_t} + \lambda_t (1-\tau_t^e) f = \mu_t G'(e_t) \tag{5}$$

$$h_{t+1} : \lambda_{t+1}\beta(1-\tau_{t+1}^n) w_{t+1} n_{t+1} + \mu_{t+1}\beta(1-\delta_h) = \mu_t \tag{6}$$

[7] $\delta_h = 1$ means the household cannot use the stock of human capital accumulated so far in the next period.

[8] This is an assumption that features prominently in the literature. See Jacobs and Bovenberg (2008) for a discussion and their footnote 3 for more references.

$$k_{t+1} : \lambda_t = \beta \lambda_{t+1} R^k_{t+1} \tag{7}$$

$$b_{t+1} : \lambda_t = \beta \lambda_{t+1} R^b_{t+1} \tag{8}$$

Combine (7) and (8) to derive

$$R^b_{t+1} = R^k_{t+1}. \tag{9}$$

(9) is a familiar condition that states that there is arbitrage-freeness between investments in physical capital and government bonds. Both investments promise the same rate of return in equilibrium.

Human capital can be regarded as an asset, similar to physical capital, that yields a rate of return, which in equilibrium must be equal to the other assets' rates of return. To see this, recursively eliminate μ_{t+1} in (6), and use (5) and (7)[9]:

$$R^k_{t+1} = \frac{\sum_{i=0}^{\infty} \left(\prod_{j=1}^{i} \left(R^k_{t+1+j} \right)^{-1} \right) (1 - \tau^n_{t+1+i}) w_{t+1+i} n_{t+1+i} G'(e_t)(1-\delta_h)^i}{(1-\tau^e_t) f + (1-\tau^n_t) w_t h_t} \tag{10}$$

The numerator in (10) summarizes the discounted sum of returns due to a marginal investment e_t, henceforth referred to as the marginal (private) return to education. The investment in period t not only increases

[9]Then the transversality condition

$$\lim_{i \to \infty} \left(\prod_{j=0}^{i} \left(R^k_{t+j} \right)^{-1} \right) (1-\delta_h)^i \frac{G'(e_t)}{G'(e_{t+i})} \left[(1-\tau^n_{t+i}) w_{t+i} h_{t+i} + (1-\tau^h_{t+i}) f \right] = 0$$

also emerges, which holds as long as $0 < \delta_h \leq 1$.

tomorrow's stock of human capital and thereby the wage earned, but also the stock afterwards at the decreasing rate $1 - \delta_h$.[10] The denominator summarizes the marginal (private) cost of education in period t, comprising direct cost and foregone earnings. The optimality condition (10) reveals arbitrage-freeness between investments in human and physical capital.

(10) also shows that the depreciation rate δ_h and the after-tax rate of return to physical capital investments R^k affect the discounted present value of a time unit e_t invested in human capital similarly. An increasing capital tax rate, which reduces R^k, and an increasing rate of depreciation both raise the marginal return to education (Davies and Whalley, 1991).

For further reference, the stationary state version of (10) is [11]

$$\frac{\beta}{1 - \beta(1 - \delta_h)}(1 - \tau^n)wnG' = (1 - \tau^n)wh + (1 - \tau^e)f. \tag{11}$$

(11) can be interpreted in the same way as (10). The household devotes time to education up to the point where the marginal cost equals the marginal return to education. One can also see that if the direct cost of education were 100% tax-deductible, i.e., $\tau^n = \tau^e$, the choice of education would be undistorted. Boskin (1975) was the first to state this insight.

(11) also reveals that capital taxation does not affect the marginal return to education, because only the household's discount factor β matters. This means that only the labor tax rate τ^n and the rate of subsidization τ^e affect the wedge between the marginal return to and the marginal cost of education.

[10] To allow for $\delta_h = 1$, $0^0 = 1$ must hold.

[11] Use $1 = \beta R^k$, which is the stationary state version of (7).

2.2 Firm's problem

The representative firm produces the single consumption good using a neoclassical constant-returns-to-scale production function. It maximizes profits

$$F(k_t, n_t h_t) - r_t k_t - w_t n_t h_t$$

in capital k_t and effective labor $z_t \equiv n_t h_t$, taking the capital rental rate r_t and the wage rate w_t as given . As a result,

$$F_{k_t} \equiv \frac{\partial F(k_t, z_t)}{\partial k_t} = r_t \tag{12}$$

$$F_{z_t} \equiv \frac{\partial F(k_t, z_t)}{\partial z_t} = w_t \tag{13}$$

The constant-returns-to-scale production technology implies that the firm makes zero profit in equilibrium.

2.3 Government's problem

The government finances an exogenously given stream of government expenditures $\{g_t\}_{t=0}^{\infty}$. Its per-period budget constraint is

$$g_t + R_t^b b_t = \tau_t^k r_t k_t + \tau_t^n w_t h_t n_t - \tau_t^e f e_t + b_{t+1}. \tag{14}$$

2.4 Competitive Equilibrium

A competitive equilibrium consists of a feasible allocation

$$\{c_t, n_t, e_t, k_t, h_t, g_t\}_{t=0}^{\infty},$$

a price system

$$\{w_t, r_t, R_t^b\}_{t=0}^{\infty},$$

and a government policy

$$\{\tau_t^n, \tau_t^k, \tau_t^e, b_t, g_t\}_{t=0}^{\infty},$$

such that, given the price system and the government policy, the allocation solves the household's and firm's problems, and the government balances its budget. $\mathcal{C}$ is the set of the competitive equilibria that result from different government policies. Put formally:

$$\begin{aligned}\mathcal{C} = \Big\{ &\{c_t, n_t, e_t, k_t, h_t, g_t\}_{t=0}^{\infty} : \exists \{\tau_t^n, \tau_t^k, \tau_t^e, b_t, g_t\}_{t=0}^{\infty}, \{w_t, r_t, R_t^b\}_{t=0}^{\infty} \\ &\text{s.t. } (3)-(8), (12)-(13), (1), (2) \text{ and } (14) \text{ hold for all } t = 0, 1, \ldots, \\ &\qquad k_0, b_0 \text{ and } h_0 \text{ are given.} \Big\}\end{aligned}$$

2.5 Social Planner's Problem – First-Best Analysis

The social planner maximizes the household's utility subject to the resource constraint and the law of motion for human capital. The Lagrangian reads

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ &u(c_t, 1 - n_t - e_t) \\ &+ \theta_t \Big(F(k_t, z_t) + (1 - \delta_k) k_t - c_t - k_{t+1} - f e_t - g_t \Big) \\ &- \mu_t \Big(h_{t+1} - (1 - \delta_h) h_t - G(e_t) \Big) \Big\}.\end{aligned}$$

The first-order conditions for c_t, e_t, n_t, h_{t+1}, and k_{t+1} are

$$c_t : u_{c_t} = \theta_t \tag{15}$$

$$e_t : \mu_t G'(e_t) = u_{\ell_t} + \theta_t f \tag{16}$$

$$n_t : \theta_t F_{z_t} h_t = u_{\ell_t} \tag{17}$$

$$h_{t+1} : \theta_{t+1} \beta F_{z_{t+1}} n_{t+1} - \mu_t + \beta \mu_{t+1}(1 - \delta_h) = 0 \tag{18}$$

$$k_{t+1} : \theta_t = \theta_{t+1} \beta (F_{k_t} + 1 - \delta_k) \tag{19}$$

Analogously to the household's problem, the following condition shows how the social planner optimally chooses education[12]:

$$\begin{aligned} & F_{k_{t+1}} + 1 - \delta_k \\ & = \frac{\sum_{i=0}^{\infty} \left(\prod_{j=1}^{i} (F_{k_{t+1+j}} + 1 - \delta_k)^{-1} \right) F_{z_{t+1+i}} n_{t+1+i} G'(e_t)(1 - \delta_h)^i}{f + F_{z_t} h_t} \end{aligned} \tag{20}$$

The numerator in (20) is the discounted sum of marginal returns to investment e_t, henceforth called marginal (social) return to education. The investment in period t increases not only tomorrow's stock of human capital and thereby the productivity but also the stock afterwards at the decreasing rate $1 - \delta_h$. The denominator captures the marginal (social) cost in period t, comprising the direct cost of education and the loss of labor income. The optimality condition (20) reveals that the rates of return to physical and human capital accumulation are equal.

For further reference, the stationary-state version of (20) reads

$$\frac{\beta}{1 - \beta(1 - \delta_h)} F_z n G' = F_z h + f. \tag{21}$$

[12] Recursively eliminate μ_{t+1} in (18), and use (16) and (19).

The efficiency condition (21) will serve as a benchmark when analyzing below how the education decision is affected by the use of distortionary taxation. The preceding discussion therefore suggests the following

Definition 1. *Education efficiency is achieved if the marginal social return to education equals the marginal social cost of education. In the first best, there is no wedge between the marginal social return to and the marginal social cost of education.*

2.6 Ramsey Problem – Second-Best Analysis

Linear taxes are chosen to finance a given stream of government expenditures. The choice of taxes should maximize social welfare subject to resource and budget constraints and taking the household's and firm's competitive equilibrium behavior into account. Each government policy gives rise to a different competitive equilibrium. The Ramsey problem is to choose the competitive equilibrium that yields the highest utility. To solve the problem, the primal approach (Lucas and Stokey (1983), Atkinson and Stiglitz (1980), Chari and Kehoe (1999)) is adopted.

This approach is one way to take into account the competitive equilibrium behavior. Instead of choosing the optimal policy directly, which yields the optimal allocation and prices, one chooses the optimal allocation that is consistent with competitive equilibrium behavior and then solves for the government policy and prices that support this outcome. The key to solving this problem is to use the so-called implementability constraint that summarizes the household's competitive equilibrium behavior.

In the present model, three conditions on the Ramsey problem must hold. The first one, the implementability constraint, is the household's budget constraint after having substituted for after-tax prices by means of the household's first-order conditions.

Combining the per-period budget constraints (1) leads to the intertemporal budget constraint (using (8)):

$$R_0^k k_0 + R_0^b b_0 + \sum_{t=0}^{\infty} \left(\prod_{j=1}^{t} \left(R_j^k \right)^{-1} \right) (1 - \tau_t^n) w_t n_t h_t = \sum_{t=0}^{\infty} \left(\prod_{j=1}^{t} \left(R_j^k \right)^{-1} \right) \left(c_t + (1 - \tau_t^e) f e_t \right) \quad (22)$$

The transversality conditions

$$\lim_{t \to \infty} \left(\prod_{j=0}^{t} \left(R_j^k \right)^{-1} \right) k_{t+1} = 0 \quad (23)$$

and

$$\lim_{t \to \infty} \left(\prod_{j=0}^{t} \left(R_j^b \right)^{-1} \right) b_{t+1} = 0 \quad (24)$$

must hold. If (23) and (24) were positive, then the household could find an alternative allocation yielding a higher utility by simply consuming more in finite time. The reverse cannot hold either, because some other household has to be on the lending side and could increase utility for the reason just explained.

Then, using the household's first-order conditions (3) and (4) and thereby substituting out $(1 - \tau_t^n) w_t h_t$, the intertemporal budget constraint (22) can be written as

$$W_0 + \sum_{t=0}^{\infty} \beta^t u_{\ell_t} n_t = \sum_{t=0}^{\infty} \beta^t u_{c_t} \left(c_t + (1 - \tau_t^e) f e_t \right) \quad (25)$$

with $W_0 \equiv u_{c_0}\big(R_0^k k_0 + R_0^b b_0\big)$, which is the value of the initial endowment of physical capital and government bonds. (25) is the first constraint in the planner's problem.

The first-order conditions for e_t and h_{t+1}, (5) and (6), which yield (10) and determine the dynamic choice of h_{t+1}, have not been used. Therefore, they give rise to a second constraint, which Jones, Manuelli, and Rossi (1997) and Atkeson, Chari, and Kehoe (1999) call an Euler equation for the accumulation of human capital:

$$\beta n_{t+1} u_{\ell_{t+1}} h_{t+1}^{-1} + \beta(1-\delta_h)\frac{u_{\ell_{t+1}} + u_{c_{t+1}}(1-\tau_{t+1}^e)f}{G'(e_{t+1})} = \frac{u_{\ell_t} + u_{c_t}(1-\tau_t^e)f}{G'(e_t)} \tag{26}$$

$(1-\tau_t^e)f$ could also be eliminated in (25) using (26). But dealing with the resulting double sum is cumbersome, which is why it is more convenient to work with two implementability constraints. In any case, either approach must yield the same solution. Pursuing the present way mixes the primal and the dual approach, as the planner has to optimize over the allocation and over the tax rate τ_t^e.

Third, the economy's resource constraint is

$$F(k_t, z_t) + (1-\delta_k)k_t - c_t - k_{t+1} - f e_t - g_t = 0. \tag{27}$$

The set $\mathcal{R}$ consists of all allocations that satisfy the three constraints above and the law of motion (2) for human capital. Put formally,

$$\mathcal{R} = \Big\{ \{c_t, n_t, e_t, k_t, h_t, g_t\}_{t=0}^{\infty} : \text{(2), (25), (26) and (27) hold for all } t = 0, 1, \ldots \Big\}$$

The Ramsey problem is to choose a member belonging to the set $\mathcal{R}$ that yields the highest utility.

The key result to solving the Ramsey problem is the following

Proposition 1 (see Chari and Kehoe (1999), Proposition 1)**.** *The competitive equilibrium allocations satisfy the resource constraints and the implementability constraint. Furthermore, given allocations that satisfy these constraints, one can construct policies and prices that, together with the given allocations, constitute a competitive equilibrium. Put formally,* $\mathcal{C} = \mathcal{R}$*.*

Proof. See appendix A. □

The Ramsey problem therefore reads

$$\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ & W(c_t, n_t, e_t, \tau_t^e, \phi) \\
& + \theta_t \Big(F(k_t, z_t) + (1-\delta_k)k_t - c_t - k_{t+1} - fe_t - g_t \Big) \\
& - \mu_t \Big(h_{t+1} - (1-\delta_h)h_t - G(e_t) \Big) \\
& - \eta_t \Bigg(\beta n_{t+1} u_{\ell_{t+1}} h_{t+1}^{-1} - \frac{u_{\ell_t} + u_{c_t}(1-\tau_t^e)f}{G'(e_t)} \\
& \qquad + \beta(1-\delta_h) \frac{u_{\ell_{t+1}} + u_{c_{t+1}}(1-\tau_{t+1}^e)f}{G'(e_{t+1})} \Bigg) \Bigg\} - \phi W_0,
\end{aligned}$$

with

$$W(c_t, n_t, e_t, \tau_t^e, \phi) = u(c_t, 1 - n_t - e_t) + \phi \Big(u_{c_t} \big(c_t + (1-\tau_t^e) fe_t \big) - u_{\ell_t} n_t \Big)$$

defining the so-called pseudo-welfare function, which includes the implementability constraint and also depends on the endogenous Lagrange multiplier ϕ. Jones, Manuelli, and Rossi (1997) follow the same approach.

But their and the present setup differ substantially. First, due to the special assumptions made regarding the human capital production function, they show that human capital does not appear in the implementability constraint. Second, when solving the Ramsey problem they neglect the Euler equation for the accumulation of human capital (26). After having found the solution to this relaxed problem, they show that this equation is satisfied anyway. Similarly, they derive a stationary-state arbitrage-freeness condition for human capital and a corresponding Ramsey problem's condition. Because in their setup time devoted to education only gives rise to some cost in the form of forgone earnings and because the labor income tax is proportional, the tax cannot have an effect on education in a stationary state. Both the return and the cost are taxed at the same rate, and both are reduced in the same proportion.[13] It is the implementability constraint (26) that captures the transitional dynamics of the accumulation of human capital. Setting up the problem in a way that allows one to put this constraint aside and then to show that it is satisfied anyway does not, however, help to explore the special nature of human capital.

As k_0 is exogenous, τ_0^k works like a lump-sum tax.[14] To rule out this trivial form of taxation, it is common to assume $\tau_0^k = 0$.

[13]Even more obviously, this is the case in Chari and Kehoe (1999), too.

[14]To see this point, maximize the Lagrangian over τ_0^k:

$$\frac{\partial \mathcal{L}}{\partial \tau_0^k} = \phi u_{c_0} F_{k_0} k_0$$

ϕ measures the costs of using distortionary taxation. Optimally, τ_0^k should be chosen such that all government expenditures could be financed by taxing away the return to the initial stock of physical capital and thereby abstaining from levying distorting taxes on capital and labor. The other three factors are positive. Therefore, $\phi > 0$. It is then possible to increase τ_0^k until $\phi = 0$ and the present problem coincides with the first-best problem. This renders the whole analysis uninteresting. See also Jones, Manuelli, and Rossi (1997, p. 111).

Under the assumption that a unique stationary state exists,[15] the first-order conditions for c_t, e_t, n_t, h_{t+1}, k_{t+1} and τ^e_t, evaluated at the stationary state, are

$$c : W_c - \theta - \eta \left(n u_{c\ell} h^{-1} - \delta_h \frac{u_{\ell c} + u_{cc}(1-\tau^e) f}{G'} \right) = 0 \tag{28}$$

$$\begin{aligned} e : W_e - \theta f + \mu G' - \eta \Bigg(& -n u_{\ell\ell} h^{-1} \\ & - \delta_h \frac{(-u_{\ell\ell} - u_{c\ell}(1-\tau^e) f) G' - (u_\ell + u_c(1-\tau^e) f) G''}{G'^2} \Bigg) = 0 \end{aligned} \tag{29}$$

$$n : W_n + \theta F_z h - \eta \left(h^{-1}(-u_{\ell\ell} n + u_\ell) - \delta_h \frac{-u_{\ell\ell} - u_{c\ell}(1-\tau^e) f}{G'} \right) = 0 \tag{30}$$

$$h : \mu = \theta \frac{\beta}{1 - \beta(1-\delta_h)} F_z n + \eta \frac{\beta}{1 - \beta(1-\delta_h)} n u_\ell h^{-2} \tag{31}$$

$$k : 1 = \beta(F_k + 1 - \delta_k) \tag{32}$$

$$\tau^e : \eta \frac{\delta_h}{G'} = -\phi e \tag{33}$$

The first-order conditions (28)-(33), the resource constraint (27), the implementability constraint (25), and the Euler equation (26) for the accumulation of human capital determine the Ramsey allocation $\{c, e, n, h, k, \tau^e\}$ along with the Lagrange multipliers θ, η, and ϕ.[16]

The following analysis is devoted to studying the tax rates τ^k, τ^n, and

[15] This is a common assumption frequently found in the literature. Judd (1999) uses a compactness assumption on the marginal social value of government wealth instead of the convergence assumption adopted here and shows that the average capital tax rate is zero for any long interval.

[16] Chari, Christiano, and Kehoe (1994) explain how to compute ϕ. First, fix ϕ and solve for the entire allocation, using all first-order conditions and resource constraints. Then, check whether this allocation satisfies the implementability constraint. If not, iterate on ϕ until the constraint holds.

τ^e that implement the Ramsey allocation as a competitive equilibrium, $\mathcal{R} \subseteq \mathcal{C}$.[17]

Proposition 2. *Capital income is not taxed in the stationary state, i.e.,* $\tau^k = 0$.

Proof. Combine (32) with (7) evaluated at the stationary state. □

This is the seminal result by Chamley (1986) and Judd (1985). Evidently, the private and social rates of return to capital investments are equal. The zero-capital-tax result is independent of whether the model features human capital or not. From this follows that there is no trade-off between efficiency in physical and human capital formation.

Proposition 3.

1. *Labor income is taxed in the stationary state if the human capital production function's elasticity is sufficiently small.*
2. *The labor income tax rate is not higher than* 100% *if preferences satisfy the following condition:*

$$-\frac{u_{cc}c}{u_c} + \frac{u_{c\ell}}{u_c}\big((1-\gamma)n + e\big) < 1 + 1/\phi \tag{34}$$

Proof. Combining (3), (4) and (28), (30) and rearranging yields

$$1 - \tau^n = \frac{1 + \phi\left(1 + \frac{u_{cc}}{u_c}c - \frac{u_{c\ell}}{u_c}\big((1-\gamma)n + e\big)\right)}{1 + \phi\left(1 - \gamma - \frac{u_{\ell\ell}}{u_\ell}\big((1-\gamma)n + e\big) + \frac{u_{c\ell}}{u_c}c\right)}.$$

1. $\tau^n > 0$ amounts to requiring

$$\gamma < \frac{\left(-\frac{u_{\ell\ell}}{u_\ell} + \frac{u_{c\ell}}{u_c}\right)(n+e) + \left(-\frac{u_{cc}}{u_c} + \frac{u_{c\ell}}{u_\ell}\right)c}{1 + \left(-\frac{u_{\ell\ell}}{u_\ell} + \frac{u_{c\ell}}{u_c}\right)n}.$$

[17] See Proposition 1 for the central argument.

2. $\tau^n < 1$ amounts to requiring the condition (34) to hold.

□

Restrictions are imposed on the household's preferences and the properties of the human capital production function. Suppose the utility function reads $u(c, \ell) = \ln c + \kappa \ln \ell$. In this special case, conditions 1 and 2 then reduce to $\gamma < 1/(1 + n/\ell)$ and $1 < 1 + 1/\phi$. Condition 1 says that the elasticity parameter has to be below unity, as has been assumed above on page 8. Condition 2 is always satisfied as long as distortionary taxes are used.

Proposition 4. *Even if the human capital production function is isoelastic, the education decision is distorted. In the second best, there is underinvestment in human capital relative to the first best.*

Proof. Combine (26), (29), (30), (31), and (33) to obtain[18]

$$\frac{\beta}{1 - \beta(1 - \delta_h)} F_z n G' = \frac{\phi}{\theta} \gamma u_\ell + F_z h + f. \tag{35}$$

Equation (35) states that the discounted flow of marginal returns to education equals the marginal cost plus some distortion term,

$$\frac{\phi}{\theta} \gamma u_\ell. \tag{36}$$

The distortion term is positive, as each Lagrange multiplier is positive. Therefore, the marginal social return, which is decreasing in e, is larger than the marginal social cost, which is constant in e. The household is required to underinvest in human capital relative to the first best. □

[18]See appendix B for the details.

Given that the education decision is distorted, the next question is what this means for the tax rates.

Corollary 1. *In the stationary state, the direct cost of education is not fully tax-deductible, that is,* $\tau^e < \tau^n$*.*

Proof. Multiply (35) by $1 - \tau^n$, and combine the result with (11) using (13):

$$
\begin{aligned}
&(1-\tau^n)\frac{\phi}{\theta}\gamma u_\ell + (1-\tau^n)(F_z h + f) = (1-\tau^n)F_z h + (1-\tau^e)f \\
\Leftrightarrow\ &(1-\tau^n)\frac{\phi}{\theta}\gamma u_\ell = (\tau^n - \tau^e)f
\end{aligned}
$$

All the Lagrange multipliers are positive. Given that $\tau^n < 1$, the desired result follows. □

The preceding results allow one to study how the social and the private return to education are related to each other.

Corollary 2.

1. *The private rate of return to capital investments is equal to the social rate of return.*

2. *The private rate of return to capital investments is equal to the private rate of return to education.*

3. *The private rate of return to education is smaller than the social rate of return. Education is effectively taxed.*

Proof. One has to show that

$$
(1-\tau^k)r + 1 - \delta_k = F_k + 1 - \delta_k = \frac{\frac{F_z(1-\tau^n)nG'}{1-\beta(1-\delta_h)}}{F_z(1-\tau^n)h + f(1-\tau^e)} < \frac{\frac{F_z nG'}{1-\beta(1-\delta_h)}}{F_z h + f}.
$$

The first and second equalities follow from Proposition 2 and (11). (32) and Proposition 4 imply the inequality. □

The marginal social return is taxed at a higher rate than the marginal social cost. The result is that this tax scheme negatively distorts education incentives, as Proposition 4 clarifies.

To shed more light on the above results, consider the government's stationary-state budget constraint (14), which can be written as follows:

$$g + (R^B - 1)b = \tau^n(whn - fe) + (\tau^n - \tau^e)fe$$

The direct cost of education is taxed at the rate $(\tau^n - \tau^e)$ as long as $\tau^n > \tau^e$. Suppose that the converse were true, and consider a marginal decrease of τ^e. Then τ^n has to decline as well if the government's budget constraint is to continue to hold. $\tau^n \leq \tau^e$ implies the private rate of return to education to be larger than the social rate of return. The considered tax reform has the effect that the marginal cost of education, consisting of the direct cost and forgone earnings, increases less than the marginal return. As a result, the private rate of return to education increases. Ceteris paribus, the household earns more income and hence consumption rises which increases utility. An efficiency gain would result, which is not possible, given that the planner maximizes efficiency.

3 Conclusion and Disussion

This paper explores the special nature of human capital compared to physical capital in an optimal taxation model. Capital income remains untaxed in the stationary state. The presence of human capital does not interfere with this result. This means taxing capital and human capital are two distinct issues and capital taxation is not a means to guide efficient education policy. This leaves labor taxation and the subsidization of the direct cost of education as the only instruments to set efficient education incentives.

As the human capital production function includes time as the only production factor and not the current stock of human capital, the analysis calls for effective taxation of education, thereby partly extracting the ability rent. To achieve this end, the cost of education is not fully tax-deductible. As a consequence, the subsidy is insufficient in encouraging education and to offset the distortions caused by the tax on labor.

Critical is the assumption that the cost of education is fully observable. This allows the government to use this piece of information to set an effective tax on education. Otherwise it has to resort to the labor tax alone to achieve this end, which would imply higher welfare costs. In reality it is not that easy to get exact data on the time spent on education. Likewise it is not possible to precisely estimate the stock of human capital.

References

ATKESON, A., V. V. CHARI, AND P. J. KEHOE (1999): “Taxing capital income: a bad idea,” *Quarterly Review*, (Sum), 3–17.

ATKINSON, A. B., AND J. E. STIGLITZ (1980): *Lectures on Public Economics*. McGraw-Hill Inc.

BARBIE, M., AND C. HERMELING (2006): “Optimal Taxation in a Simple Model of Human Capital Accumulation,” *Economics Bulletin*, 5(5), 1–8.

BOSKIN, M. J. (1975): “Notes on the Tax Treatment of Human Capital,” NBER Working Papers 0116, National Bureau of Economic Research, Inc.

BULL, N. (1993): “When all the optimal dynamic taxes are zero,” Working Paper Series / Economic Activity Section 137, Board of Governors of the Federal Reserve System (U.S.).

CABALLE, J., AND M. S. SANTOS (1993): "On Endogenous Growth with Physical and Human Capital," *Journal of Political Economy*, 101(6), 1042–67.

CHAMLEY, C. (1986): "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54(3), 607–22.

CHARI, V. V., L. J. CHRISTIANO, AND P. J. KEHOE (1994): "Optimal Fiscal Policy in a Business Cycle Model," *Journal of Political Economy*, 102(4), 617–52.

CHARI, V. V., AND P. J. KEHOE (1999): "Optimal fiscal and monetary policy," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, vol. 1 of *Handbook of Macroeconomics*, chap. 26, pp. 1671–1745. Elsevier.

DAVIES, J., AND J. WHALLEY (1991): "Taxes and Capital Formation: How Important is Human Capital?," in *National Saving and Economic Performance*, NBER Chapters, pp. 163–200. National Bureau of Economic Research, Inc.

JACOBS, B., AND A. L. BOVENBERG (2008): "Optimal Taxation of Human Capital and the Earnings Function," CESifo Working Paper Series CESifo Working Paper No. 2250, CESifo GmbH.

JONES, L. E., R. E. MANUELLI, AND P. E. ROSSI (1997): "On the Optimal Taxation of Capital Income," *Journal of Economic Theory*, 73(1), 93–117.

JUDD, K. L. (1985): "Redistributive taxation in a simple perfect foresight model," *Journal of Public Economics*, 28(1), 59–83.

——— (1999): "Optimal taxation and spending in general competitive growth models," *Journal of Public Economics*, 71(1), 1–26.

LJUNGQVIST, L., AND T. J. SARGENT (2004): *Recursive Marcoeconomic Theory*. MIT Press, 2nd edn.

LUCAS, R. E. J. (1988): "On the mechanics of economic development," *Journal of Monetary Economics*, 22(1), 3–42.

——— (1990): "Supply-Side Economics: An Analytical Review," *Oxford Economic Papers*, 42(2), 293–316.

LUCAS, R. E. J., AND N. L. STOKEY (1983): "Optimal fiscal and monetary policy in an economy without capital," *Journal of Monetary Economics*, 12(1), 55–93.

MILESI-FERRETTI, G. M., AND N. ROUBINI (1998): "On the taxation of human and physical capital in models of endogenous growth," *Journal of Public Economics*, 70(2), 237–254.

RAMSEY, F. P. (1927): "A Contribution to the Theory of Taxation," *The Economic Journal*, 37(145), 47–61.

REIS, C. (2007): "Essays on Optimal Taxation," Ph.D. thesis, Massachusetts Institute of Technology, Department of Economics.

RICHTER, W. F. (2009): "Taxing Education in Ramsey's Tradition," *Journal of Public Economics*, 93(11-12), 1254–1260.

A Proof of Proposition 1

1. $\mathcal{C} \subseteq \mathcal{R}$:

   The statement is true because the implementability constraint is the intertemporal budget constraint after having substituted out prices using the household's first-order conditions. Derive (26) by combining (5) and (6) and substituting out prices again. Because the household's and government's budget constraints are satisfied, the resource constraint is satisfied by Walras's law. This proves the first inclusion.

2. $\mathcal{R} \subseteq \mathcal{C}$:

   The converse, that any allocation satisfying the implementability and resource constraints satisfies competitive equilibrium behavior, is also true. This amounts to finding prices and a government policy, namely tax rates, such that the allocation that is in $\mathcal{R}$ is also in $\mathcal{C}$. To derive R^b_{t+1} use (3) and (8). Obtain r_t and w_t from (12) and (13). (3) and (4) yield τ^n_t. (3) and (7) determine τ^k_t. τ^e_t is defined recursively by (5) and (6).

By construction, the Ramsey allocation satisfies the household's budget constraint and the economy's resource constraint. By Walras' law, the government's budget constraint is satisfied as well.

B Derivation of (35)

(33) serves to eliminate η:

$$\eta = -\phi \frac{G'e}{\delta_h} \tag{37}$$

Equalize (29) and (30), and plug in (37):

$$\begin{aligned} &-\theta f + \mu G' - \phi \frac{G'e}{\delta_h} \delta_h \frac{-u_{\ell\ell} G' - \big(u_\ell + u_c(1-\tau^e) f\big) G''}{G'^2} \\ &- \phi \frac{G'e}{\delta_h h} n u_{\ell\ell} + \phi\big(u_c(1-\tau^e) + u_\ell\big) \\ &\qquad = \theta F_z h + \phi \frac{G'e}{\delta_h h}(-u_\ell n + u_\ell) + \phi \frac{G'e}{\delta_h} \delta_h \frac{u_\ell \ell}{G'} + \phi u_{\ell\ell} n - \phi u_\ell \end{aligned} \tag{38}$$

(2) yields $h = G + (1-\delta_h)h$, which is equivalent to $1/G = 1/(\delta_h h)$. Using the constant elasticity γ of G and substituting for μ by means of (31), one can manipulate (38) as follows:

$$\begin{aligned} &\theta \frac{\beta}{1-\beta(1-\delta_h)} F_z n G' - \phi\gamma \left(\frac{\beta}{1-\beta(1-\delta_h)} G' n u_\ell h^{-1} - u_c(1-\tau^e) f \right) \\ &\qquad = \theta(F_z h + f) \end{aligned} \tag{39}$$

The stationary-state version of (26) reads

$$\frac{\beta}{1-\beta(1-\delta_h)} G' n u_\ell h^{-1} = u_\ell + u_c(1-\tau^e) f. \tag{40}$$

Plug (40) into (39) to finally derive (35):

$$\frac{\beta}{1-\beta(1-\delta_h)} F_z n G' = \frac{\phi}{\theta} \gamma u_\ell + F_z h + f \tag{35}$$