Eliciting Beliefs: Proper Scoring Rules, Incentives, Stakes and Hedging*

Olivier Armantier† Nicolas Treich‡

December 2, 2010

Abstract

Accurate measurements of probabilistic beliefs have become increasingly important both in practice and in academia. Introduced by statisticians in the 1950s to promote truthful reports in simple environments, Proper Scoring Rules (PSR) are now arguably the most popular incentivized mechanisms to elicit an agent's beliefs. This paper generalizes the analysis of PSR to richer environments relevant to economists. More specifically, we combine theory and experiment to study how beliefs reported with a PSR may be biased when i) the PSR payments are increased, ii) the agent has a financial stake in the event she is predicting, and iii) the agent can hedge her prediction by taking an additional action. Our results reveal complex distortions of reported beliefs, thereby raising concerns about the ability of PSR to recover truthful beliefs in general economic environments.

*We would especially like to thank Amadou Boly and Alexander Wagner for their help and insights during the course of this research project. We also thank Monique Chyba, Edi Karni, Alexander Koch, Peter Wakker, and Bob Winkler as well as seminar participants at the NHH of Bergen, the university of Canterbury at Christchurch and at the ESA conferences in Tucson (2009) and Melbourne (2010) for helpful discussions. All remaining errors are ours. The views expressed in this paper are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of New York or the Federal Reserve System. Nicolas Treich acknowledges the chair "Marché des risques et création de valeurs" Fondation du risque/SCOR and the ANR project "NanoInnov" for financial support.

†Federal Reserve Bank of New York, U.S. E-mail: olast1@gmail.com.

‡Toulouse School of Economics (LERNA, INRA), France. E-mail: ntreich@toulouse.inra.fr

1. Introduction

Introduced by statisticians in the 1950s, Proper Scoring Rules (PSR hereafter) are belief elicitation techniques designed to provide an agent the incentives to report her subjective beliefs in a thoughtful and truthful manner.[1] Although the analysis of PSR has recently been generalized to modern theories of risk and ambiguity (Offerman et al. 2009), the properties of PSR have yet to be characterized in general economic environments. This paper combines theory and experiment in an attempt at partially filling this void. More precisely, we characterize the possible PSR biases (i.e. the systematic differences between an agent's subjective and reported beliefs) for all risk averse agents and all PSR under three effects: i) a change in the PSR payments, ii) the introduction of a financial stake in the event the agent is predicting, and iii) the possibility for the agent to hedge her prediction by taking an additional action. The empirical significance of the biases identified are then tested in an experiment.

Accurate measurements of probabilistic beliefs have become increasingly important both in practice and in academia. In practice, numerous websites now offer public opinions (about e.g. consumer products, movies, or restaurants) and predictions (about e.g. sporting or political events).[2] To be meaningful, these opinions and predictions must be informative. When soliciting advice from experts (about e.g. health, environmental, or financial issues) individuals typically expect unbiased recommendations. Recent suspicions of conflict of interest suggest that this may not always be the case.[3] Finally, in an effort to better manage risk, firms are increasingly turning to their employees to forecast (e.g.) sales, completion dates, or industry trends.[4] Precise belief assessments are also important in academia. In particular, Manski (2002, 2004) argues that separate measures of choices and beliefs are useful to estimate decision models properly. Modern macroeconomic theory considers that monetary policy consists in large part in managing expectations (about e.g. inflation), which requires correct measures of probabilistic beliefs.[5] Finally, experimental economists are increasingly eliciting their subjects' beliefs to understand observed behavior better.[6]

Because they are incentive compatible under expected payoff maximization, PSR have been one of the most popular belief elicitation techniques, with applications to numerous

[1] Early references in statistics include Brier (1950), Good (1952) and McCarthy (1956). Interestingly, it seems that De Finetti first introduced PSR (De Finetti 1981). A broad and influential paper in the statistical literature is Savage (1971). Early references in economics include Friedman (1979) and Holt (1979).

[2] Such opinion websites include *ePinion, Ebay, Zagat,* or Amazon. Prediction websites include the *Iowa Electronic Market*, the *Hollywood Stock Exchange*, or *Intrade*.

[3] See e.g. the *New York Times* article "Questions Grow About a Top CNBC Anchor" (February 12, 2007) about TV anchors for the financial news channel CNBC. Likewise, the pharmaceutical industry has long been suspected to influence doctors' prescription behavior through various "marketing" campaigns.

[4] Such firms include *Yahoo!, Microsoft, Google, Chevron, General Electric,* and *General Motors*. The economic value of accurate forecasts may be illustrated with the case of the *Dreamliner*'s delays which are expected to cost the *Boeing Corporation* up to $10 billion.

[5] See e.g. Woodford (2005), or Blinder et al. (2008) for a review of this literature. Note also that efforts are currently underway at the Federal Reserve Bank of New York to develop better instruments to measure individuals' inflation expectations (Bruine de Bruin et al. 2009).

[6] Wagner (2009) identifies more than 40 economic experiments using incentivized beliefs elicitation techniques.

fields such as meteorology, business, education, psychology, finance, and economics.[7] Recently, with the rapid development of prediction markets, there has been an upsurge of interest in PSR. In particular, Market Scoring Rules have been proposed as a way to overcome the liquidity problems that have affected some prediction markets.[8] In short, Market Scoring Rules may be described as follows. A group of agents is sequentially asked to make a prediction about a particular event. Each agent is paid for her prediction according to a PSR, but she also agrees to pay the previous agent for his prediction according to the same PSR. Because of their attractive properties, Market Scoring Rules have been rapidly adopted by several firms operating prediction markets.[9]

It has long been recognized, however, that PSR can generate biases when agents are not risk neutral (Winkler and Murphy 1970). The nature of these biases has typically been analyzed in simple and specific contexts. Namely, a particular PSR and a given utility function are selected, and the agent's income is generally assumed to depend only on the PSR payments. This paper contributes to the literature by extending the analysis i) to all PSR and ii) to richer economic environments. More precisely, we characterize the possible biases PSR generate in response to three effects.

First, we consider how varying the PSR payments affects reported probabilities. Although experimental economists have long debated how incentives affect choices (Camerer and Hogarth 1999, Hertwig and Ortmann 2001), to the best of our knowledge, the problem has not been explicitly addressed for PSR.[10]

Second, we consider an environment in which the agent has a financial stake in the event, a situation common in practice. For instance, an agent may be asked to make a prediction about an economic indicator (e.g. the stock market, the inflation level) or an event (e.g. a flood, a favorable jury verdict). Similarly, an agent facing a Market Scoring Rule always has a stake in the event she predicts, as her payment to the previous predictor depends on the outcome of the event. Finally, subjects in (e.g.) public good experiments are often asked to predict the contributions of others.[11] In all those cases, independently of the PSR payments, the agent's income depends on the outcome of the random variable she is predicting. As we shall see, such violations of the "no stake" condition (Kadane and Winkler 1988) may induce further distortions.

Third, we offer the agent the possibility of hedging her prediction by taking an additional action whose payoff also depends on the event. For instance, in the previous examples, the agent predicting an economic indicator might also have to choose how to diversify her portfolio, while the agent predicting a catastrophic event may also have to decide on her insurance coverage. Likewise, subjects in public good experiments have to choose their own contributions.[12] As we shall see, because they are not independent, the prediction and the

[7] See Camerer (1995), Offerman et al. (2009), or Palfrey and Wang (2009) for references.

[8] See, e.g., Hanson (2003), Ledyard (2006) or Abramowicz (2007).

[9] E.g. *Inkling Markets*, *Consensus Point*, *Yoopick*, *Crowdcast*, and *Microsoft Corporation*.

[10] Some have compared financially versus non-financially incentivized belief elicitation techniques (Beach and Philips 1967, Rutström and Wilcox 2009). We extend this analysis by considering variations of strictly positive financial incentives.

[11] See e.g. Croson (2000), Gachter and Renner (2010), or Fischbacher and Gachter (2010).

[12] Although experimental economists have recently been aware of stakes and hedging opportunities (Palfrey and Wang 2009, Andersen et al. 2009, Fehr et al. 2010), these issues have typically been ignored (Costa-Gomez and Weizsacker 2008, Fischbacher and Gachter 2010). Concerns have also been raised that eliciting

additional action are in general different from what each decision would be if made separately. In such cases, we show that hedging creates an additional source of distortions in the reported probabilities.

Our analysis is divided in three parts. In the theory part, we assume expected utility and consider the class of all PSR for binary events (Gneiting and Raftery 2007). We first generalize previous results (Winkler and Murphy 1970, Offerman et al. 2009, Andersen et al. 2009) by showing that risk averters report more uniform probabilities, i.e. probabilities skewed toward one half. In contrast to popular belief, we find that changing the rewards of the PSR has an ambiguous impact on reported probabilities. In particular, smaller PSR payments can either reduce or reinforce the PSR biases depending on whether the utility function displays increasing or decreasing relative risk aversion. We then show that the presence of a bonus (i.e. a positive stake) when the event occurs, lowers reported probabilities under risk aversion. Finally, we show that the possibility of hedging by betting on the event can severely alter predictions. In particular, we identify a region in our model where the reported probabilities remain unchanged and are therefore completely independent of the agent's subjective probabilities.

In the second part, we report on an experiment aimed at testing whether the biases induced by PSR are empirically relevant. The basic design is similar to Offerman et al. (2009) (OSKW hereafter). Subjects are presented with a list of events describing the possible outcome of the roll of two 10-sided dice. The probabilities are elicited with a quadratic scoring rule. In addition to a control treatment, six treatments are conducted by varying i) the PSR payments (the "High Incentives" and "Hypothetical Incentives" treatments), ii) the stake in the event (the "Low Stakes" and "High Stakes" treatments), and iii) the returns on the amount bet on the event (the "Low Hedging" and "High Hedging" treatments). Although not perfectly consistent in magnitude, the experimental results are generally in line with the directions of the theoretical predictions made under risk aversion. In particular, we find significantly larger biases when the PSR pays higher amounts, which suggests that subjects exhibit increasing relative risk aversion. In contrast, the absence of incentives produces less biased yet noisier elicited probabilities. Consistent with the theory, the presence of a stake leads subjects to report significantly lower probabilities. Furthermore, we find a positive correlation between the amount bet on an event and the bias in the reported probability for that event, thereby providing evidence of hedging. Finally, we observe larger biases when the underlying objective probabilities are compound or complex, which is inconsistent with expected utility.

In the third part, we discuss some implications of our results for the elicitation of beliefs with PSR in general economic environments. We conclude that accurate measures of subjective probabilities may be difficult to obtain in the field. For lab experiments, we discuss possible remedial measures which (in theory) guarantee accurate elicitation of subjective probabilities when subjects have a stake or a hedging opportunity.

---

beliefs of subjects engaged in a game, even absent any stakes and hedging considerations, may lead them to think more strategically, therefore affecting their behavior (Croson 2000, Rutström and Wilcox 2009, Gachter and Renner 2010).

2. Theory

In this section, we study the properties of the "response function", that is the function that gives the optimal reported probabilities of an agent who is rewarded according to a PSR. The properties derived hold for all PSR, and are therefore not restricted to the quadratic scoring rule (QSR hereafter) we use in the experiment.

2.1. Preliminary Assumptions

We consider a binary random variable, that is, an event and its complement. We assume probabilistic sophistication, and thus restrict our attention to the subjective probability of this event p held by the agent. We assume that p is exogenous: it is fixed and cannot be affected by the agent (i.e. there is no learning and no moral hazard).[13]

Let $q \in [0, 1]$ be the agent's reported probability. A scoring rule gives the agent a monetary reward $S_1(q)$ if the event occurs and $S_0(q)$ if the event does not occur. We assume that the scoring rule is differentiable and real-valued, except possibly that $S_1(0) = -\infty$ and $S_0(1) = -\infty$.

In this section, we consider a standard expected utility framework. We assume that the agent's utility function over income, denoted $u(.)$, is thrice differentiable, strictly increasing, state-independent, and satisfies the von Neumann Morgenstern axioms. The expected utility assumption is then relaxed in Appendix B where we study the effect of ambiguity aversion. We start by assuming that the agent's income depends only on the scoring rule payments. We will relax this "no-stake" condition in sections 2.5 and 2.6 where we assume that the agent's income may also depend on the outcome of the random variable.

2.2. Proper Scoring Rules

A scoring rule is said to be proper if and only if a risk neutral agent truthfully reveals her subjective probability.

Definition 2.1. Proper Scoring Rule. *A scoring rule $S = (S_1(q), S_0(q))$ is proper if and only if:*

$$p = \arg\max_{q \in [0,1]} pS_1(q) + (1 - p)S_0(q) \tag{2.1}$$

In the remainder of this section, we will illustrate some of our results using the popular QSR, defined by

$$\begin{aligned} S_1(q) &= 1 - (1 - q)^2 \\ S_0(q) &= 1 - q^2 \end{aligned} \tag{2.2}$$

The QSR is represented in Figure 1a. It is straightforward to show that a QSR satisfies (2.1) and is therefore proper.

[13] See Osband (1989), Ottaviani and Sorensen (2007) and Wagner (2009) for somewhat related theoretical results obtained when p is endogenous.

We now provide a simple characterization of all PSR for binary random variables. This characterization has been recently proposed in the statistics literature by Gneiting and Raftery (2007) in the multi-event situation, building on the pioneering work of McCarthy (1956), Savage (1971), and Schervish (1989).

Proposition 2.1. *A scoring rule S is proper if and only if there exists a function $g(.)$ with $g''(q) > 0$ for all $q \in [0,1]$ such that*

$$\begin{aligned} S_1(q) &= g(q) + (1-q)g'(q) \\ S_0(q) &= g(q) - qg'(q) \end{aligned} \tag{2.3}$$

The sufficiency of Proposition 2.1 is easy to prove.[14] Indeed, under (2.3), the agent's expected payoff equals

$$\pi(q) \equiv pS_1(q) + (1-p)S_0(q) = g(q) + (p-q)g'(q)$$

which reaches a maximum at $q = p$ since g' is strictly increasing.

Proposition 2.1 indicates that a PSR can be fully characterized by a single function $g(.)$ and a simple property on this function, namely, its convexity. In particular, it is easy to show that $g(q) = 0.5(q^2 + (1-q)^2 + 1)$ yields the traditional QSR in (2.2).[15] Observe also that

$$\begin{aligned} S_1'(q) &= (1-q)g''(q) > 0 \\ S_0'(q) &= -qg''(q) < 0 \end{aligned}$$

Hence, when a scoring rule is proper, the convexity of $g(.)$ implies the intuitive property that $S_1(q)$ must be increasing, and that $S_0(q)$ must be decreasing (see Figure 1a). This implies that $S_1(q)$ and $S_0(q)$ cross at most once. Note also that PSR are invariant to any transformation of the form $aS + b$ where $a \in \mathbb{R}^{*+}$ and $b \in \mathbb{R}^2$. To simplify the presentation, we often consider in what follows the set of "standard" PSR satisfying

$$g'(1/2) = 0 \tag{2.4}$$

so that $S_1(.)$ and $S_0(.)$ cross at 1/2. Observe that this condition is implicit in the literature, as it is satisfied for the most common PSR. In particular, the traditional quadratic, logarithmic and spherical scoring rules all verify (2.4).[16]

2.3. Risk Aversion

From now on, we relax the assumption of risk neutrality and allow for risk aversion. Winkler and Murphy (1970), Kadane and Winkler (1988) and OSKW have examined the response

[14] The proof of the necessity of this Proposition, as well as the proofs of all the Propositions derived in this section may be found in Appendix A.

[15] Likewise, the traditional logarithmic scoring rule $S = (\log q, \log(1-q))$ and spherical scoring rule $S = (q^{\eta-1}, (1-q)^{\eta-1})(q^\eta + (1-q)^\eta)^{(1-\eta)/\eta}$ with $\eta > 1$, are obtained with respectively $g(q) = q \log q + (1-q)\log(1-q)$ and $g(q) = (q^\eta + (1-q)^\eta)^{1/\eta}$.

[16] These three PSR also verify a stronger symmetry condition whereby $g(q) = g(1-q)$ which implies $S_1(q) = S_0(1-q)$.

function of a risk averse agent facing a QSR. They show that risk aversion leads the agent to report probabilities skewed toward one half in the case of binary events. This makes a risk-averter better off since this reduces the difference across terminal payoffs. We first generalize this result to the class of all standard PSR satisfying (2.3) and (2.4), and then to the class of all PSR in subsection 2.5.

We define the response function $R(p)$ as follows

$$R(p) = \arg\max_{q\in[0,1]} pu(S_1(q)) + (1-p)u(S_0(q)) \tag{2.5}$$

where S is a PSR as defined in (2.3). Our objective in the remainder of this section is to analyze the properties of this response function. In particular, we want to characterize the response function's "bias", $|R(p) - p|$.

The first order condition of the program above can be written as follows

$$f(p,q) \equiv p(1-q)u'(S_1(q)) - (1-p)qu'(S_0(q)) = 0. \tag{2.6}$$

It is easy to see that $\frac{\partial f(p,q)}{\partial q} < 0$, so that the program is concave and $R(p)$ is unique. It is also easy to check that it is optimal to report 0 when $p = 0$ and to report 1 when $p = 1$. Observe, moreover, that $\frac{\partial f(p,q)}{\partial p} > 0$, so that the response function $R(p)$ is strictly increasing in p, as stated in the following Lemma.

Lemma 2.1 *For all PSR defined by (2.3), $R'(p) > 0$ together with $R(0) = 0$ and $R(1) = 1$.*

Next, we show that truthful revelation of subjective probabilities is in general not optimal under risk aversion. Moreover, we show that the deviation from truth telling is systematic and depends on p. To see this, observe that $f(p,q)$ in (2.6) evaluated at $q = p$ has the sign of $u'(S_1(p)) - u'(S_0(p))$, which captures the marginal benefit of increasing q at $q = p$. This means that under risk aversion the response function $R(p)$ is larger (respectively lower) than p when $S_1(p)$ is lower (respectively larger) than $S_0(p)$. In particular, for a standard PSR defined by (2.3) and (2.4), we have $S_1(p) \leq S_0(p)$ if and only if $p \leq 1/2$. This implies that the agent reports more uniform probabilities in the following sense: the response function is higher than p when $p < 1/2$ and lower than p when $p > 1/2$, as stated in the following corollary.[17]

Corollary 2.1 *For all standard PSR defined by (2.3) and (2.4), $R(p) \geq p$ if and only if $p \leq 1/2$.*

The response function is therefore "regressive" (i.e., it crosses the diagonal from above), with a fixed point equal to one half. Figure 1b displays such a regressive response function for the QSR in (2.2) together with a quadratic utility function $u(x) = -(2-x)^2$ with $x \leq 2$.

Note that Corollary 2.1 implies $R(1/2) = 1/2$, so that the agent truthfully reveals her subjective probability at $p = 1/2$. This result is due to the condition in (2.4). However,

[17] In contrast, risk lovers facing a PSR always report more extreme probabilities. In particular, when an interior solution exists, it is easy to show that the response function is lower than p when $p < 1/2$ and higher than p when $p > 1/2$.

if one applies a positive affine transformation to one of the two PSR payoffs, $S_1(1/2)$ and $S_0(1/2)$ would differ, while the scoring rule remains proper. In that case, the result that risk aversion leads to reporting more uniform probabilities does not hold anymore. This effect is studied in more details in subsection 2.5.

Finally, we derive a Proposition that generalizes Corollary 2.1. This Proposition states that more risk averse agents (in the classical sense of Pratt 1964) always report more uniform probabilities. We denote $R_u(p)$ and $R_v(p)$, the response functions associated with utility functions $u(.)$ and $v(.)$.

Proposition 2.2. *Let $v(w) = \Phi(u(w))$ with $\Phi' > 0$ and $\Phi'' \leq 0$. For all standard PSR defined by (2.3) and (2.4),*

$$R_v(p) \geq R_u(p) \text{ if and only if } p \leq 1/2$$

The intuition is that a more risk averse agent is willing to sacrifice more in terms of expected payoff to reduce the difference across terminal payoffs. This can be achieved by reporting more uniform probabilities, i.e. probabilities closer to one half. An increase in risk aversion therefore leads the response function to increase before the fixed point, and to decrease afterwards. The response function thus moves further away from p, which can naturally be interpreted as an increase in the response function's bias.

2.4. Incentives

We now study the effect of changing the incentives provided by the PSR. More precisely, we study the effect of changing $a > 0$ on the response function

$$R(p, a) = \arg\max_{q \in [0,1]} pu(aS_1(q)) + (1-p)u(aS_0(q)) \tag{2.7}$$

We show that this effect depends on the relative risk aversion coefficient $\gamma(x) = \frac{-xu''(x)}{u'(x)}$.

Proposition 2.3. *For all standard PSR defined by (2.3) and (2.4), and for all $a > 0$, under $\gamma'(x) \geq (\leq) 0$,*

$$\frac{\partial R(p, a)}{\partial a} \geq (\leq) 0 \text{ if and only if } p \leq 1/2.$$

In other words, when the relative risk aversion with respect to income is increasing (decreasing), raising the PSR payments leads the agent to report more (less) uniform probabilities. One can present the intuition as follows. There are two effects when the PSR payments increase: i) a wealth effect, as the agent gets a higher reward for any given reported probability, and ii) a risk effect, as the difference between the rewards in the two states becomes more important. The sign of the derivative of the relative risk aversion $\gamma(x)$ ensures that one effect always dominates the other. In particular, when relative risk aversion is increasing, the risk effect dominates the wealth effect so that the agent reports more uniform probabilities to reduce the variability of her payoff.

An implication of this result is that it may not be possible to mitigate the bias of the response function by adjusting the incentives of the PSR. In particular, changing the PSR payments has no effect on the response function when the utility exhibits constant relative risk aversion (CRRA) with respect to income.[18] Furthermore, a reduction of the PSR payments may in fact exacerbate the PSR bias as long as the utility function displays decreasing relative risk aversion (DRRA) with respect to income. Hence, the common belief that paying agents smaller amounts necessarily induces more truthful reports is misleading in general.[19]

The result of Proposition 2.3 is illustrated in Figure 1c. The added response function compared to Figure 1b is calculated for $a = 2$. Observe that both response functions are regressive with a fixed point at 1/2. Yet, the increased incentives lead to reporting more uniform probabilities. This is because the quadratic utility function used for the numerical example displays increasing relative risk aversion.

2.5. Stakes

Kadane and Winkler (1988) define the presence of a stake as a situation in which, absent any PSR payments, the agent's final wealth varies depending on whether or not the event occurs. Consistent with that definition, we introduce a stake by assuming that income increases by an exogenous amount $\Delta \in \mathbb{R}$ when the event occurs. Observe, however, that this is formally equivalent to adding a constant Δ to $S_1(q)$ in (2.3). As a result, it is immediate that any PSR remains proper in the presence of a stake. Note also that the PSR defined by $(S_0(q), S_1(q) + \Delta)$ does not necessarily satisfy condition (2.4). Consequently, the results in this section generalize the analysis under risk aversion to the class of all PSR.

The response function is defined by

$$R(p, \triangle) = \arg \max_{q \in [0,1]} pu(\triangle + S_1(q)) + (1-p)u(S_0(q)) \tag{2.8}$$

in which the added reward $\triangle$ is a finite (positive or negative) "stake". The first order condition can be written

$$f(\triangle, q) \equiv p(1-q)u'(\triangle + S_1(q)) - (1-p)qu'(S_0(q)) = 0 \tag{2.9}$$

As before, $\frac{\partial f(\triangle,q)}{\partial q} < 0$ under risk aversion, so that the program is concave. In addition, it is immediate to see that the properties of Lemma 2.1 are still satisfied. Finally, observe that $\frac{\partial f(\triangle,q)}{\partial \triangle} < 0$ so that the response function is decreasing in $\triangle$ under risk aversion, as stated in

[18] To illustrate, consider a CRRA utility function $u(x) = (1-\gamma)^{-1}x^{1-\gamma}$ with $\gamma > 0$ and a spherical scoring rule $S = (q, (1-q))(q^2 + (1-q)^2)^{-1/2}$. This combination yields a closed-form solution $R(p, a) = p^{1/(1+\gamma)}(p^{1/(1+\gamma)} + (1-p)^{1/(1+\gamma)})^{-1}$, which is indeed independent from a.

[19] This is not incompatible with a well-known result in the literature showing that the agent reveals her beliefs truthfully when a tends toward 0, i.e. $\lim_{a \to 0} R(p, a) = p$ (Kadane and Winkler 1988, Jaffray and Karni 1999, Karni 1999). Indeed, these authors consider a utility function of the form $u(x) = U(w + x)$, where w is the agent's initial wealth, and with $U'(w) < \infty$. This implies that $\gamma'(0) = -U''(w)/U'(w)$ is strictly positive under risk aversion. Therefore, the agent necessarily displays increasing relative risk aversion with respect to income as x tends toward 0. However, the utility function may still be DRRA locally for some $x > 0$. An example of such a utility function is $u(x) = -\exp(1/(w + x))$. Consistent with Proposition (2.3), a reduction of the income through a could then initially move the response function away from p for this utility function, but, once a gets sufficiently close to 0, $R(p, a)$ would start converging toward p. In other words, although it is correct that $\lim_{a \to 0} R(p, a) = p$ when $U'(w)$ is defined, a reduction in a does not guarantee more truthful responses.

the following Lemma.

Lemma 2.2 *For all PSR defined by (2.3),* $\frac{\partial R(p,\triangle)}{\partial \triangle} \leq 0$

The intuition for this result is straightforward. Under risk aversion, an increase in $\triangle$ reduces the marginal utility when the event occurs. Therefore, to compensate for the difference in marginal utility across states, the agent wants to increase the rewards of the PSR when the event does not occur. This can be done by reducing the reported probability that the event occurs. We can now state a Proposition that characterizes the response function when the agent has a stake.

Proposition 2.4. *For all PSR defined by (2.3), the response function* $R(p, \triangle)$ *is characterized as follows:*
i) if there exists a $\widehat{p}$ *such that* $\triangle + S_1(\widehat{p}) = S_0(\widehat{p})$*, then we have*

$$R(p, \triangle) \geq p \text{ if and only if } p \leq \widehat{p}$$

ii) if $\triangle + S_1(p) \geq (\leq) S_0(p)$ *for all* p*, then we have*

$$R(p, \triangle) \leq (\geq) p$$

Proposition 2.4 is illustrated in Figure 1d. Compared to Figure 1b, there are two additional response functions in Figure 1d, one for $\triangle = 1/2$ and the other (the High Stake) for $\triangle = 1$. Both response functions are regressive, but the first has a fixed point at $1/4$ and the second is below the diagonal everywhere. Note also that the presence of a stake does not necessarily increase the PSR bias. In particular, for any $p \leq 1/4$, $|R(p) - p|$ is smaller when $\triangle = 1/2$ than when $\triangle = 0$.

2.6. Hedging

In addition to her prediction, we now assume that the agent can make another decision whose payoff depends on the outcome of the event. This hedging opportunity can create a stake in the event, and it may therefore be interpreted as an endogenous stake. Suppose the agent receives an endowment $\overline{\alpha}$ and can invest an amount α in $[0, \overline{\alpha}]$ in a risky asset. This investment returns k if the event occurs, and -1 (i.e., the investment is lost) if the event does not occur. We assume that k and $\overline{\alpha}$ are strictly positive and finite. The problem becomes

$$\max_{q\in[0,1],\alpha\in[0,\overline{\alpha}]} pu(S_1(q) + k\alpha + \overline{\alpha}) + (1-p)u(S_0(q) - \alpha + \overline{\alpha}) \tag{2.10}$$

The following Proposition presents properties of the response function when this particular form of hedging is available.

Proposition 2.5. *For all standard PSR defined by (2.3) and (2.4), the solutions $R(p)$ and $\alpha(p)$ to program (2.10) satisfy the following properties:*

> *i) for $p \leq \underline{p}(k)$, we have $\alpha(p) = 0$ and $R(p) \in [0, \frac{1}{1+k}]$ with $R'(p) > 0$,*
> *ii) for p in $[\underline{p}(k), \overline{p}(k)]$, we have $\alpha(p) \in [0, \overline{\alpha}]$ with $\alpha'(p) > 0$ and $R(p) = \frac{1}{1+k}$,*
> *iii) for $p \geq \overline{p}(k)$, we have $\alpha(p) = \overline{\alpha}$ and $R(p) \in [\frac{1}{1+k}, 1]$ with $R'(p) > 0$,*

together with

$$
\begin{aligned}
\underline{p}(k) &= \frac{u'(S_0(\frac{1}{1+k}) + \overline{\alpha})}{ku'(S_1(\frac{1}{1+k}) + \overline{\alpha}) + u'(S_0(\frac{1}{1+k}) + \overline{\alpha})} \text{ and} \\
\overline{p}(k) &= \frac{u'(S_0(\frac{1}{1+k}))}{ku'(S_1(\frac{1}{1+k}) + (k+1)\overline{\alpha}) + u'(S_0(\frac{1}{1+k}))}
\end{aligned}
$$

This Proposition tells us that there are two critical threshold values for subjective probabilities that shape the optimal investment rule: when $p \leq \underline{p}(k)$, the agent does not invest at all; and when $p \geq \overline{p}(k)$, the agent invests the maximum amount $\overline{\alpha}$. According with intuition, the investment opportunity is not attractive when p is low and becomes more attractive as p increases. Note that when p is high enough so that the agent invests the maximum amount $\overline{\alpha}$, then the investment opportunity has an effect similar to a stake equal to $(k+1)\overline{\alpha}$. In the intermediate range where p belongs to $]\underline{p}(k), \overline{p}(k)[$, the agent invests some strictly positive amount in $]0, \overline{\alpha}[$. Perhaps most interestingly, the agent reports probabilities that are constant in this interval, and are therefore independent from p. The intuition is that the PSR is used as a transfer scheme across states, while the investment opportunity is used to adjust risk exposure to changes in p. Note finally, that when $k > 1$, we have $\underline{p}(k) < 1/(1+k)$, so that it can be optimal to invest in the risky asset even when its expected return is negative. This shows that the presence of the PSR can also alter the investment decision. We plot in Figures 1e and 1f the optimal investment share $\alpha(p)/\overline{\alpha}$ and the response function under $k = 1$ and $\overline{\alpha} = 0.5$.

3. Experimental Treatments

The main features of the experimental design are similar to OSKW's (2009) calibration experiment without explicit reference to beliefs or probability. The subjects are presented with a list of 30 events (e.g. "the two dice sum up to 4"), each describing the possible outcome of the roll of two 10-sided dice. Observe that, as in OSKW, each event has an objective probability (i.e. it can be calculated using standard probability theory). Unlike OSKW, however, the events in our experiment are not homogenous and consist of three different series of 10 events. A precise description of the events and series is postponed to subsection 3.5.

For each of the 30 events, a subject is asked to make a choice consisting of selecting 1 out of 149 possible options called "choice numbers". To each choice number correspond two payments generated with a QSR. As further explained below, the first is the payment to the subject when the event occurs, while the second is the payment to the subject when the event does not occur. A subject's set of possible choice numbers, as well as their corresponding payments, was presented in the form of a "Choice Table" (see Appendix C).[20] Observe that the Choice Table is ordered such that, as the choice number increases, the payment when the event occurs increases, while the payment when the event does not occur decreases. Note also that the choice number 75 guarantees the same payment to a subject regardless of the roll of the dice.

3.1. The Control Treatment

The subjects' payments in the control treatment (T_0) are generated with a QSR of the form $S_1(q) = a \cdot [1 - (1-q)^2]$ and $S_0(q) = a \cdot [1 - q^2]$, where $a = 4,000FCFA$ in our experiment.[21] Each entry in the choice table, and in particular the link between choices and payments, was explained in details and illustrated through several examples (see Appendix C). After reading the instructions, the subjects' understanding of the table was submitted to a test, which was then solved by the experimenter. The subjects were then presented with the list of 30 events, one series at a time. No time limit was imposed, and the subjects could modify any of their previous choices at any time.

Once all subjects had completed their task, the experimenter randomly selected one of the 30 events and rolled the two dice once to determine whether this event occurred or not. All subjects in a session was then paid according to their choice number for the event randomly drawn. For instance, if a subject selects the choice number 30 for the event randomly selected, she receives either 1,440 FCFA if the event obtains or 3,840 FCFA if the event does not obtain (see Appendix C). This amount constitutes the entirety of a subject's payments, as no show-up fee was provided in the control treatment.

Based on the theoretical analysis conducted in the previous section, we can frame the experimental hypotheses for each treatment in terms of the properties of the response function $R(p)$. In particular, assuming subjects in our experiment are risk averse, we can use Corollary 2.1 to formulate our first hypothesis.

$\mathbf{H_0}$: *The response function in* T_0 *is i) regressive (i.e. it crosses the diagonal from above) and ii) has a fixed point at* 1/2.

3.2. The "High Incentives" and "Hypothetical Incentives" Treatments

Two treatments were conducted to study the effect of incentives. As indicated in Table 1, where the differences between treatments are summarized, the "High Incentives" treatment (T_1) is identical to the control treatment except that every payment in the choice table is now multiplied by 10. For instance, if a subject chose row 30, she received either 14,400 FCFA (instead of 1,440 FCFA in T_0) if the event obtains or 38,400 FCFA (instead of 3,840 FCFA in T_0) if the event does not obtain. The "Hypothetical Incentives" treatment (T_2) is identical to the "High Incentives" treatment except that payments are now hypothetical.

[20] How best to present PSR to subjects remains an open question. Tables, although not ideal, have been often adopted in part because they are simple to implement (see e.g. McKelvey and Page 1990, Sonnemans and Offerman 2004, Rutström and Wilcox 2009, Blanco et al. 2009).

[21] The Franc CFA is the currency used in Burkina Faso where the experiment was conducted (see Section 3.6 for details). The conversion rate at the time was roughly \$1 for 455 FCFA.

More specifically, subjects in T_2 were asked to make their choices as if they would be paid the amounts in the choice table. Yet, they knew they would receive only a flat fee of 3,000 FCFA for completing the task, regardless of their choices.

Proposition 2.3 shows that higher incentives affect the response function only when relative risk aversion is non-constant. To derive a hypothesis, we follow most of the experimental economics literature (including recent papers on scoring rules such as OSKW or Andersen et al. 2009) and assume constant relative risk aversion with respect to income.

$\mathbf{H_1}$: *The response function in* T_1 *is identical to the response function in* T_0.

Since subjects' choices are not incentivized in T_2, no theoretical prediction can be derived. However, Holt and Laury (2002) suggest that treatments with high hypothetical payments and treatment with low real payoffs yield similar results. This leads to our next hypothesis.

$\mathbf{H_2}$: *The response function in* T_2 *is identical to the response function in* T_0.

3.3. The "Low Stake" and "High Stake" Treatments

Two treatments were conducted to study the effect of stakes. These treatments are identical to the control treatment except that subjects receive a bonus when the event occurs. The bonus is 2,000 FCFA in the "Low Stake" treatment (T_3) and 8,000 FCFA in the "High Stake" treatment (T_4).

Lemma 2.2 demonstrates that adding a positive stake when the event occurs lowers the response function. Therefore, the response functions in T_3 and T_4 are predicted to be less elevated everywhere than the response function in T_0. Moreover, Proposition 2.4 identifies two special cases, one in which the response function is regressive with an interior fixed point and one in which the response function is below the diagonal everywhere (and therefore has no interior fixed point). It is easy to show, given the size of the stakes and the specific QSR we used, that T_3 corresponds to the first case with a fixed point at 1/4, while T_4 corresponds to the second case. This leads to the following hypotheses.

$\mathbf{H_3}$: *The response function in* T_3 *i) is lower than in* T_0 *and ii) has a fixed point at* $1/4$.

$\mathbf{H_4}$: *The response function in* T_4 *i) is lower than in* T_3 *and ii) is lower than* p.

3.4. The "Low Hedging" and "High Hedging" Treatments

Two treatments were conducted to study the effect of hedging. These treatments are identical to the control treatment except that subjects are asked to make an additional decision for each event. Namely, subjects were given $2,000$ FCFA and offered the opportunity to bet a share of this endowment. If the event does not occur, the bet is lost. If the event occurs, the bet multiplied by 2 (respectively, 4) is paid to the subjects in the "Low Hedging" treatment T_5 (respectively, the "High Hedging" treatment T_6). Finally, in both states of the world, the subjects retain the part of the $2,000$ FCFA they did not bet. In other words, in addition to selecting a choice number, subjects in the hedging treatments are asked to make a simple portfolio decision with two assets, a riskless and a risky asset.

Proposition 2.5 shows that, with the possibility for hedging, the response function is regressive, yet constant when the share of the endowment invested is in]0,1[. Proposition 2.5 also shows that a subject invests all her endowment when p is high enough. In this case, the hedging opportunity operates as a stake of $4,000$ FCFA (respectively, $8,000$ FCFA) in treatment T_5 (respectively, treatment T_6), and the response functions are reduced accordingly compared to T_0. Observe finally that T_5 corresponds to the case $k = 1$ and T_6 to the case $k = 3$ in section 2.6. It is then easy to show that the fixed points in T_5 and T_6 are respectively $1/2$ and $1/4$. This leads to the following hypotheses:

$\mathbf{H_5}$: *The response function in T_5 is i) regressive with a fixed point at $1/2$, ii) equal to $1/2$ when the share invested is in]0,1[, and iii) lower than in T_0 when p is close to 1.*

$\mathbf{H_6}$: *The response function in T_6 is i) regressive with a fixed point at $1/4$, ii) equal to $1/4$ when the share invested is in]0,1[, and iii) lower than in T_5 when p is close to 1.*

3.5. Comparison of the Three Series

As mentioned previously, the 30 events presented to a subject had been split into 3 series of 10 events. In each series, the 10 events describe the possible outcome resulting from the roll of two 10-sided dice (one black, the other red). To better compare a subject's choices across series, the 10 events in each series have the same objective probabilities 3%, 5%, 15%, 25%, 35%, 45%, 61%, 70%, 80%, and 90%. The events in Series 1 are similar to those in OSKW's calibration exercise. Namely, we told subjects that the red die determines the first digit and the black die determines the second digit of a number between 1 and 100. For instance, we described the event with an objective probability of 25% as "the number drawn is between 1 (included) and 25 (included)". A complete description of the events and the order in which they were presented to subjects may be found in Appendix C.

In Series 2, we consider events whose probabilities, although still objective, are arguably more difficult to calculate than those in Series 1. Namely, we told subjects that the two dice would be added to form a number between 0 and 18. For instance, we described the event with 25% probability as "the sum obtained is between 2 (included) and 6 (included)".

The object of Series 3 is to test how subjects respond when faced with (objective) compound probabilities. To do so, we asked subjects to select a single choice number not for one but for two possible events. The subjects were told that the experimenter would throw a fair coin to determine which of the two possible events would be taken into consideration for payments. The events used in Series 3 are similar to those in Series 1, i.e. the roll of the red and black dice produces a number between 1 and 100 and the events give a possible range for that number. For instance, we described the event with a probability of 25% as "If the coin falls on the **Heads** side, then the event is : "the number drawn is between 82 (included) and 89 (included)" ; otherwise, if the coin falls on the **Tails** side, then the event is : "the number drawn is between 25 (included) and 66 (included)"".

Since the objective probabilities are the same in each series, the response functions should not differ across the three series if subjects behave in a way consistent with expected utility based on von Neumann Morgenstern axioms.[22] Nevertheless, experimental evidence suggests that subjects' responses may be affected by the complexity in calculating objective probabilities. In particular Halevy (2007) finds that most subjects in his experiments do not reduce

[22] This statement remains true if one assumes that subjects are only able to identify the objective probability P_0 with some noise in Series 2 or 3. Indeed, if a subject's subjective belief may be written $P = P_0 + \varepsilon$ with $\mathrm{E}[\varepsilon] = 0$, then the response function is still characterized by (2.5) under expected utility.

compound probabilities. From a theoretical perspective, there are different ways to relax the standard reduction axiom. We opted for a simple model recently introduced by Klibanoff, Marinacci and Mukerji (2005) (KMM hereafter), in which ambiguity aversion may induce violations of the reduction axiom under objective probabilities.[23]

Under this model, we show in Proposition B.1 (see Appendix B) that, in the context of our experiment, ambiguity aversion reinforces the effect of risk aversion. As a result, the response functions obtained with Series 2 and 3 may be more biased than the response function obtained with Series 1, as shown in Proposition 2.2. To derive a hypothesis, however, we consider the standard expected utility approach.

$\mathbf{H_7}$: *The response functions obtained in Series 1, 2, and 3 are identical.*

3.6. Implementation of the Experiment

The experiment took place in Ouagadougou, Burkina Faso, in June 2009.[24] The choice of location was motivated by two factors. First, we wanted to take advantage of a favorable exchange rate i) to create salient financial differences between treatments (e.g. between the reference and the "Hypothetical Incentives" treatments), and ii) to provide subjects with substantial incentives so that risk aversion had a fair chance to play a role.[25] Second, one of the authors had conducted several experiments in Ouagadougou over the past three years (see e.g. Armantier and Boly 2009, 2010). Building on our experience, we followed a well established protocol to hire subjects and rent a lab to conduct the experiment.

More specifically, we used a local recruiting firm (Opty-RH) to place fliers around Ouagadougou stating that we were looking for subjects for a paid economic experiment. The subjects had to be at least 18 years old and be current or former university students. People interested had to come to the recruiting firm location with a proof of identification and either a valid university diploma or a proof of university enrollment. After their credentials were validated, subjects were randomly assigned to a session and told when and where to show-up for the experiment.

The sessions were conducted in a centrally located high school we had already rented in the past to conduct other experiments. Upon arrival, the subjects were gathered in a large room. The instructions were read aloud, followed by questions and a comprehension test. The subjects were then presented with the 30 events and asked to make their choices using pen and paper. Finally, the subjects filled out a short survey, after which the experimenter made the random draws and the subjects were paid in cash. Two sessions were conducted for each treatment, with each session taking on average 90 minutes to complete.

[23] See the discussion in KMM (2005: 1863-64). We realize that the term "ambiguity aversion" is not appropriate to characterize the behavior of an agent facing objective probabilities. We only use this term in the paper for simplicity. Note, however, that Halevy (2007) finds a tight association between ambiguity aversion and the failure to reduce compound objective lotteries. Finally, our choice of KMM preferences is partially motivated by their ability to distinguish ambiguity aversion from risk aversion, a property that we will use in the proof in Appendix B.

[24] Burkina Faso is a Francophone country in West Africa with over 13 million inhabitants, among which around 1.4 million live in the capital city Ouagadougou.

[25] The maximum payment of 40,000 FCFA in the "High Incentives" treatment slightly exceeds the monthly average entry salary for a university graduate.

As indicated in Table 2, a total of 301 subjects participated in the experiment, with a minimum of 41 subjects per treatment. The subjects were composed mostly of men (74%) and students currently enrolled at the university (68%), ranging in age between 19 and 38 (with a median age of 25). In the post-experiment survey, slightly more than half the subjects reported having taken a probability or a statistics class at the university. Finally, most of the subjects (86%) reported not having participated in a similar economic or psychology experiment. Excluding the "Hypothetical Incentives" treatment (where earnings were fixed at $3,000$ FCFA), the average earnings of a subject were $8,861$ FCFA. As indicated in Table 2, however, earnings varied greatly across subjects and treatments (the smallest amount paid was 100 FCFA and the maximum was $40,000$ FCFA).

4. Experimental Results

4.1. The Control Treatment (T_0)

Figure 2 shows the subjects' average responses to the events in the three series for each of the 7 treatments conducted. According with hypothesis H_0, the three response functions in the control treatment are regressive. Moreover, they exhibit the traditional inverse S-shape with a fixed point around 1/2. This observation is consistent with the literature, as similar shapes have been previously identified when eliciting beliefs (Huck and Weizsaker 2002, Sonnemans and Offerman 2004, Hurley and Shogren 2005, OSKW). Figure 2 also reveals that the three series can be ordered with respect to their respective biases. Indeed, it appears that Series 1 (the simple probabilities) yields the smallest biases for virtually all objective probabilities (i.e. Series 1's response function is consistently the closest to the diagonal), while Series 2 (the complex probabilities) generates the largest biases. This result appears to contradict hypothesis H_7 which states that under expected utility there should be no systematic differences across the three series.

To test our hypotheses more formally, we compare statistically the subjects' choices across series and treatments with a parametric model of the form:

$$\widehat{P}_{it} = \varphi\left(P_t\right) + \eta_i + u_{it} \tag{4.1}$$

where $\widehat{P}_{it}$ is the reported probability corresponding to the choice number N_{it} selected by subject i for event $t = 1, ..., 30$ (i.e. $\widehat{P}_{it} = 2/3 \cdot N_{it}$), P_t is the objective probability of occurrence of event t, η_i is a zero-mean normally distributed individual specific error term, u_{it} follows a normal distribution truncated such that $\widehat{P}_{it} \in [0, 1]$, and $\varphi(.)$ is a continuous function satisfying $\varphi(0) = 0$, $\varphi(1) = 1$ and $\varphi'(.) > 0$. Consistent with previous literature, we consider a function that may exhibit an inverse S-shape:

$$\varphi\left(P_t\right) = \exp\left(\left[\ln P_t\right]^b \cdot \left[\ln(a)\right]^{1-b}\right) \tag{4.2}$$

where $a \in]0, 1]$ and $b > 0$.

Observe that under the reparametrization $\left\{b = \alpha; a = \exp\left(-\beta^{\frac{1}{1-\alpha}}\right)\right\}$, (4.2) is in fact the probability weighting function $w\left(P_t\right) = \exp\left(-\beta\left[-\ln P_t\right]^{\alpha}\right)$ proposed in a different context by Prelec (1998). The specification in (4.2) was preferred to Prelec's because the parameters are easier to interpret with our experimental data. Indeed, observe that $\varphi(a) = a$ and $\varphi'(a) = b$. In other words, a captures where the function φ crosses the diagonal, while b captures the slope of φ at this fixed point.[26] Finally, we control for treatment and series effects by modeling the parameters in (4.2) as follows:

$$a = a_0 + a_1 \cdot (S_2 + S_3) + a_2 \cdot S_3 + a_3 \cdot T_0 + a_4 \cdot T_0 \cdot (S_2 + S_3) + a_5 \cdot T_0 \cdot S_3$$

where T_0 is a dummy variable equal to 1 when the observation was collected in the control treatment, while S_2 and S_3 are dummy variables equal to 1 when the event belongs respectively to Series 2 and Series 3. The parameter b is modeled in an analog fashion. To estimate the model with the data collected only in the control treatment, the parameters a_3 to a_5, as well as b_3 to b_5, are all set equal to zero. The parameters, estimated by Maximum Simulated Likelihood, are reported in Table 4.

First, observe that in the control treatment a_0 is not significantly different from 1/2, while b_0 is significantly smaller than 1. This therefore confirms that the subjects' response function for the events in Series 1 exhibits an inverse S-shape and crosses the diagonal near 1/2. Note also that a_1 and a_2 are not significant in the control treatment. In other words, the response functions' fixed points do not appear to vary significantly for the three series in the control treatment. In contrast, b_1 is significantly smaller than 0, while b_2 is positive and significant. This result confirms that the curvature of the inverse S-Shape is the most pronounced for the events in Series 2 and the least pronounced for the events in Series 1. Observe also in Table 4 that the sign and significance of (a_1, a_2), as well as (b_1, b_2), are generally consistent across treatments. This therefore implies that the ranking of the three series in terms of the biases they generate is generally preserved regardless of treatment.[27]

To gain a different perspective on the data, we calculate four statistics in Table 3. The first is the average number of "extreme predictions", that is, the average number of times a subject selects a choice number below 10 (which corresponds to a reported probability below 6.66%) or above 140 (which corresponds to a reported probability above 93.33%). We also calculate two measures of the errors made by subjects when ranking the objective probabilities. The first one (Error 1) consists of the average number of times a subject incorrectly ranks two consecutive choice numbers with respect to their objective probabilities (e.g. the choice number selected by a subject for the objective probability 5% is higher than the choice number he selected for the objective probability 15%). The second measure captures the number of reported probabilities on the incorrect side of 1/2. More specifically, "Error 2" consists of the average number of times a subject selects a choice number above (below) 75 for an objective probability below (above) 50%. Finally, the choice numbers a subject selects for a given objective probability are ordered across the three series from least

[26] More generally, a captures the "elevation" of $\varphi(.)$ and b captures its "curvature" (Gonzalez and Wu, 1999). Indeed, assuming $b < 1$ so that $\varphi(.)$ is inverse S-shaped, then when a increases $\varphi(.)$ increases, while when b increases $\varphi(.)$ increases if and only if $p > a$.

[27] Similar conclusions can be reached nonparametrically using Friedman tests for each objective probability and series. Table 6 shows that in most treatments Series 1 (Series 2) generally has the lowest (highest) ranking of reported probabilities for objective probabilities below 50%, and the lowest (highest) ranking of reported probabilities for objective probabilities above 50%.

to most biased. We can see in Table 3 that these four criteria paint a consistent picture: Series 1 (respectively, Series 2) has the most (least) extreme predictions, the fewest (most) errors, and the best (worst) ranking in terms of bias. These results therefore provide further support against the hypothesis that subjects respond similarly to the events in the three series.

To summarize, we find statistical evidence that the response functions in the control treatment exhibit the traditional inverse S-shape with a fixed point around 1/2. This result is consistent with subjects being risk averse expected utility maximizers, and therefore supports hypothesis H_0. Moreover, we find that the responses to the events in Series 1 (Series 2) are statistically the most (least) biased. This result cannot be explained under expected utility, and therefore it contradicts hypothesis H_7. Instead, the systematic differences between the three series could be explained by a form of ambiguity aversion under the additional assumption that Series 1 (the simple probabilities), Series 3 (the compound probabilities) and Series 2 (the complex probabilities) are perceived as ambiguous, and ranked in increasing order of ambiguity. This assumption could perhaps find support in the fact that, although all objective, these three types of probabilities (simple, compound, and complex) require different levels of computational sophistication to calculate. In a recent paper, Halevy (2007) concludes that attitudes toward ambiguity and compound objective lotteries are tightly associated. Our experimental results support Halevy's conclusion, but also extend it by suggesting that complex objective probabilities may also be perceived "as if" ambiguous.[28]

4.2. The Incentives Treatments (T_1 and T_2)

Figure 2 indicates that the response functions for the three series in the "High Incentives" treatment are flatter than in the control treatment, although they still cut the diagonal around 1/2. This observation is confirmed statistically in Table 4. Indeed, a_3 to a_5 are not significantly different from 0 in T_1, thereby suggesting that the fixed points of the different response functions cannot be distinguished statistically across the two treatments. In contrast, we find the parameter b_3 to be positive and significant in T_1. This confirms that, compared to the control treatment, the response functions are generally flatter in the "High Incentive" treatment. Similar conclusions are reached nonparametrically by using Mann-Whitney tests for each objective probability and each series. Indeed, Table 5 shows that for most series and objective probabilities (except some objective probabilities around 1/2) the distributions of responses in T_1 are closer to the diagonal than in T_0. Note also that the the signs and magnitudes of b_4 and b_5 in Table 4 suggest that subjects' choices are more homogenous across the three series in T_1. This observation finds additional support in the criteria reported in Table 3. Indeed, responses to the events in Series 2 (Series 1) remain the most (least) biased and the most (least) prone to errors, but the differences across series are not as severe as in T_0.

[28] Our experiment also points out a potential problem with OSKW approach to correct for risk aversion when eliciting beliefs with a PSR: for the same agent, different correction functions could emerge in their calibration exercise depending on the type of objective probabilities considered. An argument could be made for using the "simplest" objective probabilities possible (such as those in Series 1), but our experience suggests that simplicity is a relative concept: while obvious for anyone with basic knowledge of probability, the events in Series 1 seemed challenging for some of our subjects.

To sum up, responses in T_1 are significantly different from those in T_0, which refutes hypothesis H_1 derived under the assumption of constant relative risk aversion. Instead, as explained in Section 2.4, choices in T_1 are consistent with subjects exhibiting increasing relative risk aversion. In a recent paper, Andersen et al. (2009) also conclude that their subjects' behavior in a similar belief elicitation experiment may be best described under increasing relative risk aversion. It is also interesting to note that our results imply that paying more does not necessarily yield "better" answers. Instead, we find that, because of our subjects' specific form of relative risk aversion, using a PSR that provides higher incentives generates more biases.

As for the "Hypothetical Incentives" treatment T_2, Figure 2 reveals that subjects' responses, although still exhibiting the inverse S-shape, are on average closer to the diagonal than in the control treatment. This observation is confirmed statistically in Table 4 as b_3 is found to be negative and significant in T_2. Note also that σ_u, the standard deviation of the error term u_{it} in (4.1), is significantly larger for T_2 than for T_0. In addition, observe in Table 3 that the number of extreme predictions and errors (of both forms) is systematically greater in T_2 than in T_0. In other words, it appears that, although not as biased, subjects' responses are noisier in the "Hypothetical Incentives" treatment. These results therefore do not support hypothesis H_2. Instead, we find that, when eliciting beliefs with a QSR, subjects behave differently when provided with real or hypothetical incentives. Our conclusions are only partially consistent with the literature. Like Gachter and Renner (2010), we find that financial incentives reduce the noise in the beliefs elicited. In contrast with our experiment, however, Sonnemans and Offerman (2004) find no difference between rewarding predictors with a QSR or with a flat fee.

4.3. The Treatments with Stakes (T_3 and T_4)

For the smallest objective probabilities, no obvious difference is visible in Figure 2 between the response functions in the control treatment and those in the low and high stakes treatments. In contrast, Figure 2 clearly shows that, compared to T_0, responses for the highest objective probabilities are lower in T_3 and lowest in T_4. These observations are confirmed by the nonparametric tests in Table 5. There, we can see that the samples of responses for each of the three series are stochastically lower in T_0 for most objective probabilities above 25%. In addition, the comparison of the low and high stakes treatments in Table 5 indicates that in general there exists a significant difference between the two treatments, whereby the probabilities stated by subjects are generally lower in T_4 than in T_3. The parametric estimations in Table 4 confirm these results. Indeed, a_3 and b_3 are positive and significant for both treatments T_3 and T_4 but significantly larger for treatment T_4. This implies that, compared to T_0, the response functions become lower and flatter in the "Low Stake" treatment and that the magnitude of this effect is stronger in the "High Stake" treatment. In other words, part i) of predictions H_3 and H_4 (i.e. the response functions are less elevated in T_3 and T_4) is verified. Behavior in the experiment, however, is not fully consistent with our predictions. Indeed, observe in Table 4 that the parameter a_3 is significantly different from 1/4 in T_3, and from 0 in T_4, thereby contradicting part ii) of H_3 and H_4.

To sum up, we find that when they have a stake in the event, subjects in our experiment tend to smooth their payoffs across the two states, especially when the event is likely to occur. This treatment effect is only partially in agreement with the theory: the direction is correct, but the magnitude is insufficient.

4.4. The Treatments with Hedging (T_5 and T_6)

The response functions for the two hedging treatments plotted in Figure 2 reveal several differences from those in the control treatment. First, for the highest objective probabilities, the response functions become lower in T_5 and lowest in T_6. Second, although the fixed point of the response function is also around 1/2 in T_5, it is slightly above 40% in T_6. Third, the response functions appear slightly flatter (but not perfectly flat) around the diagonal in both hedging treatments. Most of these observations are confirmed statistically by the parametric and nonparametric tests in Tables 4 and 5. In particular, observe in Table 4 that the estimate of a_3 is insignificant in T_5, while it is positive and significant in T_6. The former is consistent with part i) of hypothesis H_5, as we cannot exclude that, as in the control treatment, the response functions in the "Low Hedging" treatment cut the diagonal at 1/2. In the "High Hedging" treatment, however, the parameter a_0 is found to be significantly greater than 1/4, which contradicts part i) of hypothesis H_6. Observe also that b_3 is significant and positive in both T_5 and T_6, thereby indicating flatter responses around the diagonal in the two hedging treatments compared to T_0. The nonparametric tests in Table 5 also confirm that, compared to T_0, responses for most objective probabilities above 60% are statistically lower in T_5, and lowest in T_6. This result therefore supports part iii) of hypotheses H_5 and H_6.

Turning now to the subjects' betting behavior in the two hedging treatments, we can see in Figure 2 that subjects in the "High Hedging" treatment invest more in the risky asset for any objective probability than in the "Low Hedging" treatment. This observation is confirmed statistically by the nonparametric tests reported in the last column of Table 5. The subjects' betting behavior, however, is not fully consistent with the theory. In particular, we can see in Figure 2 that, on average, subjects invest strictly positive amounts even for low probabilities, while they do not invest all of their endowments even for high probabilities.

To summarize, although not fully consistent with the theory, subjects in our experiment appear to take partial advantage of their hedging opportunities. In particular, we find that subjects tend to bet high on the most likely events, while simultaneously making lower predictions than in the control treatment. In other words, it seems that subjects are willing to take some risk on the bet, while using the scoring rule as an insurance in case the event does not occur.

5. Discussion and Conclusion

Introduced in the 1950s by statisticians, Proper Scoring Rules (PSR) have arguably become the most popular incentivized belief elicitation mechanism. In the simplest environment, a well known result is that, under the most common PSR, risk averters are better off misreporting their beliefs by stating more uniform probabilities (i.e. closer to one half in the case of a binary event). Combining theory and experiment, we find that this result does not generalize to richer environments of particular interest to economists. Instead, we have shown that higher incentives, stakes, and hedging may lead to severe distortions in reported probabilities.

We believe our results have implications for the elicitation of subjective probabilities with PSR in general field settings. Indeed, as argued in the introduction, eliciting beliefs in the field typically involves some form of a stake or the possibility to hedge one's prediction. In particular, agents who participate in Prediction Markets based on PSR (the so called Market Scoring Rule) always have a stake in the event they predict. In addition, as in our experiment, the stakes in field environments are likely to be far larger than the prediction's reward. For instance, most prediction payments are likely to pale in comparison with the stakes the agent may have in the stock market, a natural catastrophe, or the future of her industry. Our results therefore suggest that in general field settings stakes and hedging are likely to distort substantially the beliefs reported with a PSR.

Moreover, the stakes and hedging opportunities an agent may have in the field are typically unobserved by the analyst. In such cases, theory cannot be used to predict and correct the distortions generated by PSR. Suppose, nevertheless, that an agent has no stake in the event she is predicting. Then, two additional issues arise. First, one may be concerned that the beliefs elicited are not informative since the agent had no incentives ex-ante to acquire information about the event. Second, one cannot rule out the possibility that the agent will look for hedging opportunities ex-post. The presence of such opportunities may lead the agent to bias her reported beliefs even though she has no stake when making her prediction. In other words, (unobserved) stakes and hedging may lead to unpredictable distortions in reported beliefs, thereby rendering PSR ineffective in recovering subjective probabilities in general field environments.

Our results may also be relevant for traditional lab experiments. Indeed, as argued in the introduction, belief elicitation in many lab experiments involve a stake or a hedging opportunity. This may therefore explain why i) some subjects fail to best-respond to their stated beliefs (e.g. Costa-Gomez and Weizsaker 2008), and ii) observers make different predictions about the play of a game than the subjects actually playing the game (e.g. Palfrey and Wang 2009). Observe, however, that our subjects were paid more than in most lab experiments.[29] Consequently, our results may not generalize to traditional lab settings where risk aversion may play a lesser role. Nevertheless, there is evidence that stakes and hedging can play a role in lab experiments. Indeed, consistent with our experimental outcomes, Blanco et al. (2010) find that, when faced with transparent and strong hedging opportunities, many subjects in their lab experiment distort both their beliefs and their actions. In contrast with the field, however, the analyst has more control in the lab and possible remedial measures may be devised to recover subjective beliefs when using a PSR. We now discuss the effectiveness of some of these remedial measures in the presence of stakes and hedging.

The method most frequently implemented in the lab to mitigate the effect of risk aversion on elicited beliefs consists in using a PSR that pays smaller amounts (e.g. Nyarko and Schotter 2002, Rutström and Wilcox 2009). This approach, however, is at odds with one of the basic principle of experimental economics whereby thoughtful decisions by subjects should be incentivized with salient monetary rewards. More importantly, there is no guarantee that this approach produces more truthful reports. In fact, we have shown theoretically

[29] A notable exception is Andersen et al. (2009) whose subjects were paid up to $100 for a similar belief elicitation task.

that smaller PSR payments can actually exacerbate the PSR biases if the agent exhibits decreasing relative risk aversion.

Recently, OSKW and Andersen et al. (2009) have independently proposed a different approach to correct PSR biases in simple environments (i.e. without stakes or hedging). Following (e.g.) Kadane and Winkler (1988), these "truth serums" are built on the premise that if an agent's primitives (e.g. utility, wealth) are known, then her optimal reported probability can be calculated for any subjective probability. This function could then be inverted to recover the agent's unobserved beliefs from her stated probabilities. In OSKW, relevant information is first collected in a calibration exercise to estimate this correction function. This approach, however, tends to make belief elicitation with a PSR (an already intrusive method) even more cumbersome. It is also unclear how this approach can be generalized to economic environments with stakes and hedging.

Another well known remedial measure is to induce risk neutrality by paying agents in lottery tickets that give them a chance to win a prize (Roth and Malouf 1979, Allen 1987, Schlag and van der Weele 2009). In theory, it is easy to show that this approach is effective under expected utility in eliciting truthful beliefs as long as all payments (including the stakes and hedging revenues) are made in lottery tickets. In practice, however, doubts have been expressed about the ability of this approach to control for risk attitude (Davis and Holt 1993, Selten, Sadrieh and Abbink 1999). In addition, if the stake or hedging opportunity arises from a task involving a risky choice (e.g. an investment or insurance decision), then the analyst may not necessarily want to induce risk neutrality for that task.

A related approach when eliciting beliefs while playing a game consists in using random draws to make the prediction and the game decision independent. For instance, Blanco et al. (2010) propose to pay subjects randomly either their game or their prediction payoffs. Likewise, a subject in Armantier and Treich (2009) is randomly matched with two different partners. The subject plays the game against the first partner, and her prediction is scored against the play of the second partner. Observe, however, that, although promising, the practical effectiveness of these methods remains mostly unproven.

To conclude, note that the adverse effects of stakes and hedging are not specific to PSR. It is easy to show that other incentivized belief elicitation techniques, although they may offer some protection against risk aversion in the simplest environments, are not immune to stakes and hedging. This is the case, for instance, for the standard lottery mechanism (Kadane and Winkler 1988) and for the direct revelation mechanism recently proposed by Karni (2009). In fact, we are not aware of any incentivized belief elicitation method that would directly address these issues.[30] In this context, one may want to consider the merits of eliciting beliefs without offering any financial reward for accuracy. Although not incentive compatible, this approach is simple, transparent, and commonly used in statistics, psychology, and field surveys. In his review of the survey literature in economics, Manski (2004) concludes that the beliefs elicited in such a way are informative. Our results support this view and suggest that hypothetical payoffs may be preferred if one is willing to trade noise for unbiasedness.

[30] Importantly, Karni and Safra (1995) show that unbiased belief elicitation based on marginal rates of substitution is impossible when stakes are not observed by the experimenter. This impossibility result holds even if the utility function is observable and even if several experiments can be implemented.

6. References

Abramowicz, A., 2007, The hidden beauty of the quadratic scoring market rule: A uniform liquidity market maker, with variations, The Journal of Prediction Markets, 2, 111-125.

Allen, F., 1987, Discovering personal probabilities when utility functions are unknown, Management Science, 33, 542-44.

Andersen, S., Fountain, J., Harrison G.W. and E.E. Rutstrom, 2009, Estimating subjective probabilities, mimeo.

Armantier, O. and A. Boly, 2009, A controlled field experiment on corruption, working paper, université de Montréal.

Armantier, O. and A. Boly, 2010, Can corruption be studied in the lab?, working paper, université de Montréal.

Armantier, O. and N. Treich, 2009, Subjective probabilities in games: An application to the overbidding puzzle, International Economic Review, 50, 1079-1102.

Beach, L.R. and L.D. Philips, 1967, Subjective probabilities inferred from estimates and bets, Journal of Experimental Psychology, 75, 354-9.

Blanco, M.D., Engelmann, D., Koch A.K. and H.-T. Normann, 2010, Belief elicitation in experiments: Is there a hedging problem?, Experimental Economics, forthcoming.

Blinder A., Ehrmann M., Fratzscher M., De Haan J., and D. Jansen, 2008, Central bank communication and monetary policy: A survey of theory and evidence, Journal of Economic Literature, 46, 910–945.

Brier, G.W., 1950, Verification of forecasts expressed in terms of probability, Monthly Weather Review, 78, 1-3.

Bruine de Bruin W., van der Klauw W., Downs J., Fischoff B., Topa G. and O. Armantier, 2009, Expectations of inflation: The role of financial literacy and demographic variables, Journal of Consumer Affairs, forthcoming.

Camerer, C.F., 1995, Individual decision making, In: Kagel, J.H. & Roth, A.E. (eds.): Handbook of Experimental Economics, Princeton, NJ: Princeton University Press, 587-703.

Camerer, C.F. and R.M. Hogarth, 1999, The effects of financial incentives in experiments: A review and capital-labor-production framework, Journal of Risk and Uncertainty, 19, 7-42.

Costa-Gomez, M.A. and G. Weizsacker, 2008, Stated beliefs and play in normal-form games, Review of Economic Studies, 75, 729-762.

Croson R., 2000, Thinking like a game theorist: Factors affecting the frequency of equilibrium play, Journal of Economic Behavior and Organization, 41, 299-314.

Davis, D. and C. Holt, 1993, Experimental Economics. Princeton, NJ: Princeton University Press.

De Finetti, B., 1981, The role of "dutch books" and "proper scoring rules", British Journal for the Philosophical of Science, 32, 55-56.

Fehr, D., Kübler, D. and D.N. Danz, 2009, Information and beliefs in a repeated normal-form game, mimeo, Humboldt University, Berlin.

Fischbacher, U. and S. Gachter, 2008, Social preferences, beliefs, and the dynamics of free riding in public good experiments, American Economic Review, 100(1): 541–56..

Friedman, D., 1979, An effective scoring rule for probability distributions, UCLA Economics Working Papers 164.

Gachter, S. and E. Renner, 2010, The effects of (incentivized) belief elicitation in public good experiments, Experimental Economics, 13, 364-377.

Gneiting, T. and A.E. Raftery, 2007, Strictly proper scoring rules, prediction and estimation, Journal of the American Statistical Association, 102, 359- 78.

Gonzalez, R. and G. Wu, 1999, On the shape of the probability weighting function, Cognitive Psychology, 38, 129-66.

Good, I.J., 1952, Rational decisions, Journal of the Royal Statistical Society, Ser. B, 14, 107-14.

Halevy, Y., 2007, Ellsberg revisited: An experimental study, Econometrica, 75, 503-36.

Hanson, R., 2003, Combinatorial information market design, Information Systems Frontiers, 5, 107-119.

Hertwig, R. and A. Ortmann, 2001, Experimental practices in economics: A methodological challenge for psychologists?, Behavioral and Brain Sciences, 24, 383–451.

Holt, C, 1979, Elicitation of subjective probability distributions and von Neumann Morgenstern utility functions, University of Minnesota, Discussion Paper No. 79-128.

Holt, C.A. and S.K. Laury, 2002, Risk aversion and incentive effects, American Economic Review, 92, 1644-55.

Huck, S. and G. Weizsäcker, 2002, Do players correctly estimate what others do? Evidence of conservatism in beliefs, Journal of Economic Behavior & Organization, 47, 71–85.

Hurley, T. and J. Shogren, 2005, An experimental comparison of induced and elicited beliefs, The Journal of Risk and Uncertainty, 30, 169–188.

Jaffray, J.Y and E. Karni, 1999, Elicitation of subjective probabilities when initial endowment is unobservable, Journal of Risk and Uncertainty, 8, 5-20.

Kadane, J.B. and R.L. Winkler, 1988, Separating probability elicitation from utilities, Journal of the American Statistical Association, 83, 357-63.

Karni, E., 1999, Elicitation of subjective probabilities when preference are state-dependent, International Economic Review, 40, 479-486.

Karni, E., 2009, A mechanism for eliciting probabilities, Econometrica, 77, 603-06.

Karni, E. and Z. Safra, 1995, The impossibility of experimental elicitation of subjective probabilities, Theory and Decision, 38, 313-320.

Kimball, A. W., 1951, On dependent tests of significance in the analysis of variance, The Annals of Mathematical Statistics, 22, 600-602.

Klibanoff, P., Marinacci M. and S. Mukerji, 2005, A smooth model of decision making under ambiguity, Econometrica, 73, 1849-1892.

Ledyard, J.O., 2006, Designing information markets for policy analysis, in: Information Markets: A New Way of Making Decisions, R.W. Hahn and P.C. Tetlock (editors), AEI-Brookings Joint Center for Regulatory Studies.

Manski, C. F., 2002, Identification of decision rules in experiments on simple games of proposal and response, European Economic Review, 46, 880–891.

Manski, C. F., 2004, Measuring expectations, Econometrica, 72, 1329–1376.

McCarthy, J., 1956, Measures of the value of information, Proceedings of the National Academy of Sciences, 654-5.

McKelvey, R.D. and T. Page, 1990, Public and private information: An experimental study of information pooling, Econometrica, 58, 1321-39.

Nyarko, Y. and A. Schotter, 2002, An experimental study of belief learning using elicited beliefs, Econometrica, 70, 971-1005.

Offerman, T., Sonnemans, J., van de Kuilen, G. and P.P. Wakker, 2009, A truth-serum for non-bayesians: Correcting proper scoring rules for risk attitudes, Review of Economics Studies, 76, 1461-1489.

Osband, K., 1989, Optimal forecasting incentives, Journal of Political Economy, 97, 1091-1112.

Ottaviani, M. and P.N. Sorensen, 2007, Outcome manipulation in corporate prediction markets, Journal of the European Economic Association, 5, 554-63.

Palfrey, T.R. and S.W. Wang, 2009, On eliciting beliefs in strategic games, Journal of Economic Behavior and Organization, 71, 2, 98-109.

Pratt, J. W., 1964, Risk aversion in the small and in the large, Econometrica, 32, 122-36.

Prelec, D., 1998, The probability weighting function, Econometrica, 66, 497-527.

Roth, A. E. and M. W. Malouf, 1979, Game-theoretic models and the role of information in bargaining, Psychological Review, 86, 574–594

Rutström E. and N. Wilcox, 2009, Stated beliefs versus inferred beliefs: A methodological inquiry and experimental test, 67, 2, 616-632.

Savage, L.J., 1971, Elicitation of personal probabilities and expectation, Journal of the American Statistical Association, 66, 783-801.

Schervish, M.J., 1989, A general method for comparing probability assessors, The Annals of Statistics, 17, 1856-1879.

Schlag K. and J. van der Weele, 2009, Eliciting probabilities, means, medians, variances and covariances without assuming risk neutrality, mimeo.

Selten R., Sadrieh A. and K. Abbink, 1999, Money does not induce risk neutral behavior, but binary lotteries do even worse, Theory and Decision, 46, 211-249.

Sonnemans J. and T. Offerman, 2004, Is the quadratic scoring rule really incentive compatible?, mimeo, University of Amsterdam.

Wagner, A., 2009, Essays on the role of beliefs in economic decisions and interactions, Ph. Dissertation, Toulouse School of Economics.

Winkler, R.L. and A.H. Murphy, 1970, Nonlinear utility and the probability score, Journal of Applied Meteorology, 9, 143-48.

Woodford M., 2005, Central bank communication and policy effectiveness, Proceedings, Federal Reserve Bank of Kansas City, August issue, 399-474.

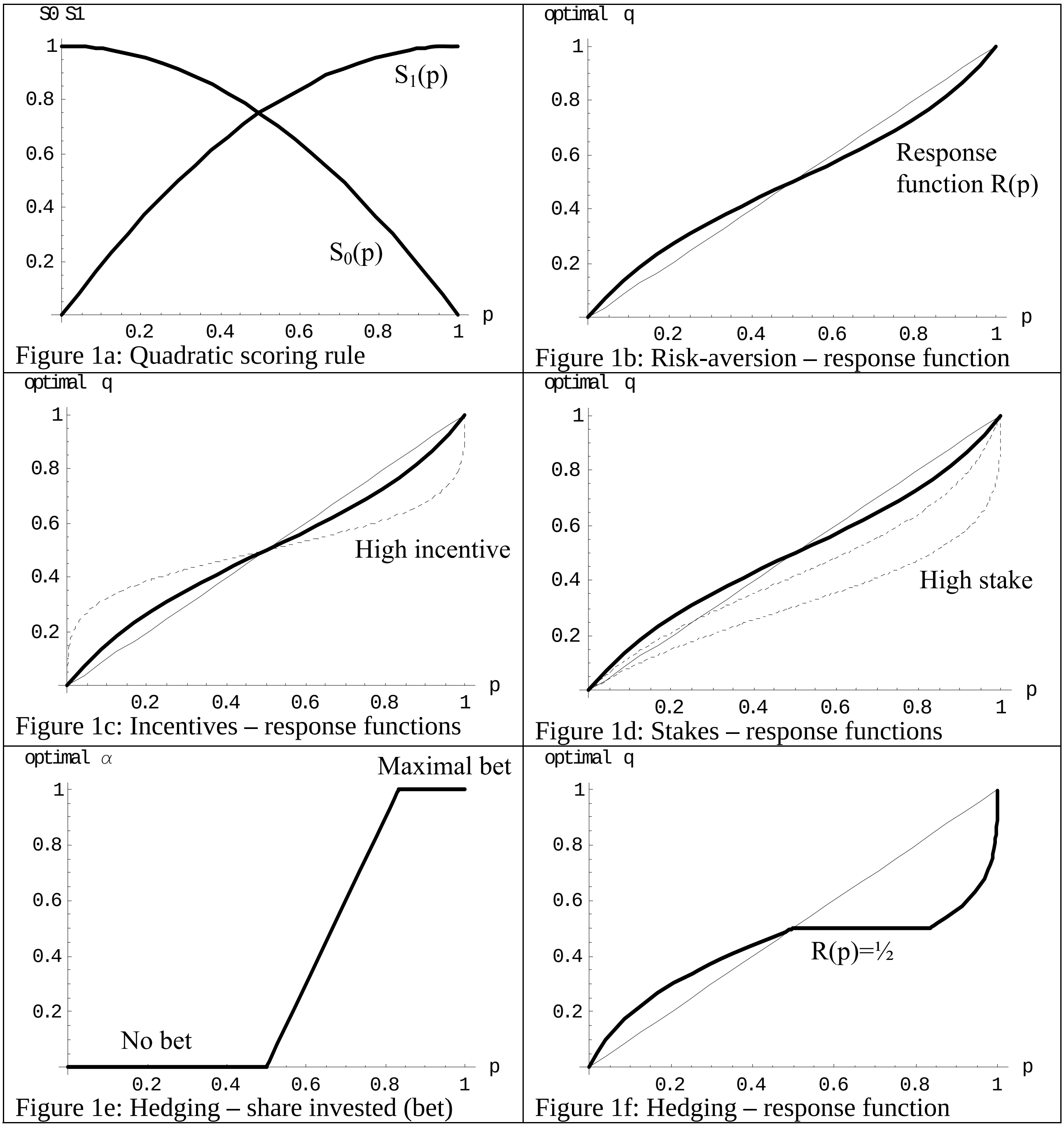

Figure 1a: Quadratic scoring rule
Figure 1b: Risk-aversion – response function
Figure 1c: Incentives – response functions
Figure 1d: Stakes – response functions
Figure 1e: Hedging – share invested (bet)
Figure 1f: Hedging – response function

Simulations were done with Mathematica.
Figure 1a represents a quadratic scoring rule (QSR): $S_1(q)=1-(1-q)^2$ and $S_0(q)=1-q^2$.
Figures 1b, 1c and 1d represent the response functions under a QSR and a quadratic utility function: $u(x)=-(2-x)^2$ with $x\leq 2$. Figure 1c represents the response functions for respective incentives $a=1$(plain curve) and $a=2$ (dashed curve). Figure 1d represents the response function for respective stakes $\Delta=0$ (plain curve), $\Delta=½$ (dashed curve) and $\Delta=1$ (dashed curve, below the diagonal).
Figures 1e and 1f represent respectively the optimal share invested (amount bet divided by maximal possible amount to bet) and the response function under a QSR, a quadratic utility function, a double-or-nothing investment opportunity ($k=1$) and a maximal possible amount to bet equal to 0.5.

Figure 1

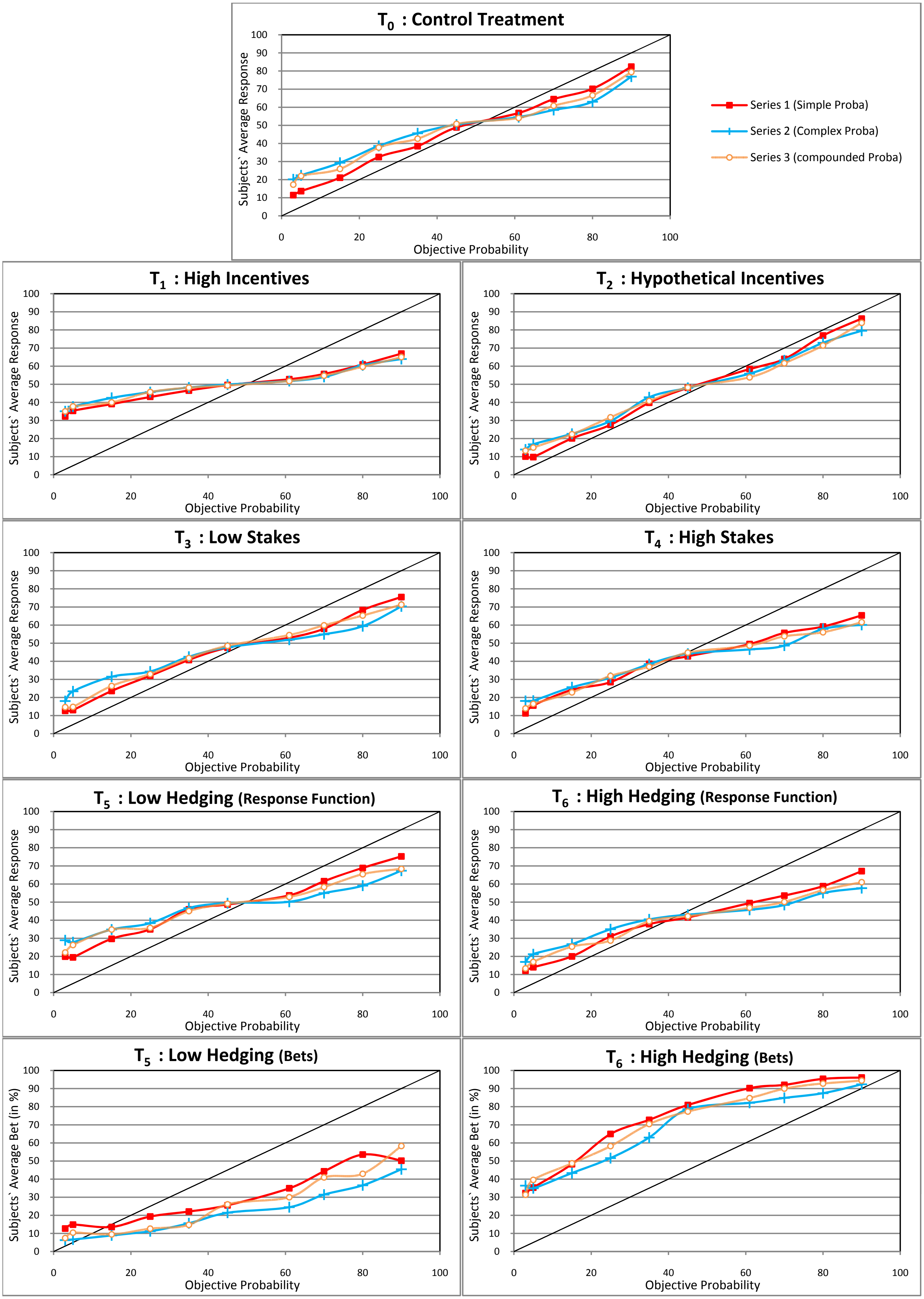
T0 : Control Treatment
Series 1 (Simple Proba)
Series 2 (Complex Proba)
Series 3 (compounded Proba)
T1 : High Incentives
T2 : Hypothetical Incentives
T3 : Low Stakes
T4 : High Stakes
T5 : Low Hedging (Response Function)
T6 : High Hedging (Response Function)
T5 : Low Hedging (Bets)
T6 : High Hedging (Bets)
Subjects` Average Response
Subjects` Average Bet (in %)
Objective Probability
0
10
20
30
40
50
60
70
80
90
100

Figure 2

Table 1 : Financial Differences between Treatments (in FCFA)							
	T0 Control	T1 High Incentives	T2 Hypothetical Incentives	T3 Low Stakes	T4 High Stakes	T5 Low Hedging	T6 High Hedging
Show-up-fee	0	0	3,000	0	0	0	0
Maximum Scoring Rule Payment	4,000	40,000	0	4,000	4,000	4,000	4,000
Stakes	0	0	0	2,000	8,000	0	0
Maximum Return on Investment	0	0	0	0	0	4,000	8,000

Table 2 : Characteristics of the Subject Pool							
	T0 Control	T1 High Incentives	T2 Hypothetical Incentives	T3 Low Stakes	T4 High Stakes	T5 Low Hedging	T6 High Hedging
Number of Subjects	43	43	48	41	44	41	41
Age	24.581 (2.373)	25.119 (2.350)	24.208 (2.042)	24.350 (2.315)	25.605 (2.977)	24.850 (3.286)	25.475 (2.792)
% of Female	27.9%	23.8%	25.0%	29.3%	23.3%	25.0%	29.3%
% Currently Enrolled at the University	67.2%	64.5%	71.6%	70.5%	60.8%	68.0%	61.5%
% with University Course in Probability	60.0%	52.8%	54.2%	61.0%	57.5%	50.0%	53.7%
% with Previous Participation in Experiment	14.6%	15.0%	16.7%	9.8%	20.9%	15.4%	12.2%
Subjects' average Earnings	3,055.2 (1,565.5)	28,341.8 (8,121.7)	3,000 (__)	5,277.3 (1,173.2)	7,864.9 (4,189.5)	4,992.8 (1,226.3)	6,155.8 (1,583.9)

Table 3 : Features of Subjects' Responses								
	Series	T0 Control	T1 High Incentives	T2 Hypothetical Incentives	T3 Low Stakes	T4 High Stakes	T5 Low Hedging	T6 High Hedging
Extreme Prediction[1]	S1	1.070 (0.985)	0.023 (0.152)	2.116 (1.531)	1.148 (1.236)	0.977 (1.000)	1.122 (1.166)	1.244 (1.985)
	S2	0.558 (0.796)	0.093 (0.366)	1.465 (1.120)	0.415 (0.774)	0.864 (1.268)	0.439 (0.808)	0.780 (2.019)
	S3	0.721 (0.959)	0.070 (0.258)	1.488 (1.470)	0.902 (0.944)	0.932 (1.208)	0.683 (1.059)	1.000 (1.987)
Error 1[2]	S1	1.093 (0.947)	0.953 (1.045)	1.698 (1.301)	1.049 (0.973)	0.955 (0.861)	1.171 (1.093)	1.049 (0.973)
	S2	1.744 (1.157)	1.465 (1.437)	2.349 (1.066)	1.683 (0.934)	1.727 (1.169)	1.878 (1.364)	1.439 (1.285)
	S3	1.395 (1.158)	1.209 (1.186)	2,372 (1.047)	1.537 (1.247)	1.409 (1.085)	1.488 (0.952)	1.293 (1.123)
Error 2[3]	S1	0.419 (0.626)	0.372 (0.655)	0.605 (0.760)	0.634 (0.733)	0.932 (1.087)	0.537 (0.636)	0.878 (1.345)
	S2	0.767 (0.947)	0.674 (0.919)	1.163 (0.998)	0.878 (0.872)	1.523 (1.191)	0.683 (0.960)	1.220 (1.475)
	S3	0.674 (0.680)	0.372 (0.725)	1.163 (0.924)	0.805 (0.782)	1.227 (1.309)	0.390 (0.737)	1.073 (1.403)
Series Ranking[4]	S1	1.515 (0.669)	1.828 (0.629)	1.795 (0.721)	1.732 (0.254)	1.748 (0.694)	1.793 (0.251)	1.741 (0.263)
	S2	2.398 (0.724)	2.112 (0.712)	2.091 (0.792)	2.318 (0.250)	2.255 (0.731)	2.220 (0.242)	2.310 (0.241)
	S3	2.087 (0.708)	2.060 (0.676)	2.114 (0.783)	1.950 (0.292)	1.998 (0.717)	1.988 (0.257)	1.949 (0.228)

In each cell, the first number is the average per subject, while the number in parenthesis is the standard deviation.

[1] For each subject and each Series, "Extreme Prediction" captures the number of time his choice number is below 10 or above 140.

[2] For each subject and each Series, "Error 1" captures the number of time his choice numbers are incorrectly ordered (e.g. In Series 1, the choice number associated with the 5% probability event is greater than the choice number associated with the 15% probability event).

[3] For each subject and each Series, "Error 2" captures the number of time a choice number above 75 is selected for an event with probability below 50%, plus the number of time a choice number below 75 is selected for an event with probability above 50%.

[4] For each subject and each of the 10 objective probabilities, the three series are ranked from least to most biased. "Series Ranking" therefore equals to 1, 2 or 3, depending on that ranking.

Table 4 : Estimation of the Response Function							
	T0 Control	T1 High Incentives	T2 Hypothetical Incentives	T3 Low Stakes	T4 High Stakes	T5 Low Hedging	T6 High Hedging
a_0	0.480*** (0.024)	0.501*** (0.006)	0.499*** (0.156)	0.425*** (0.016)	0.393*** (0.032)	0.493*** (0.012)	0.402*** (0.029)
a_1 $(S_2 + S_3)$	0.026 (0.021)	0.004 (0.005)	-0.018 (0.016)	0.019 (0.013)	0.006 (0.014)	-0.024 (0.016)	0.013 (0.013)
a_2 (S_3)	0.004 (0.011)	-0.002 (0.006)	-0.020 (0.043)	2.851E-4 (0.014)	-0.007 (0.012)	0.009 (0.013)	-0.010 (0.008)
a_3 (T_0)	—	-0.034 (0.023)	-0.038 (0.059)	0.048** (0.023)	0.109*** (0.040)	-0.031 (0.029)	0.101*** (0.035)
a_4 $(T_0.[S_2 + S_3])$	—	0.020 (0.020)	0.047 (0.048)	0.005 (0.024)	0.003 (0.031)	0.052* (0.028)	-0.009 (0.023)
a_5 $(T_0.S_3)$	—	0.008 (0.013)	0.024 (0.045)	0.004 (0.017)	0.015 (0.016)	-0.005 (0.017)	0.025* (0.014)
b_0	0.736*** (0.054)	0.289*** (0.033)	0.901*** (0.060)	0.622*** (0.037)	0.518*** (0.038)	0.543*** (0.057)	0.499*** (0.040)
b_1 $(S_2 + S_3)$	-0.242*** (0.029)	-0.053*** (0.015)	-0.195*** (0.043)	-0.189*** (0.025)	-0.119*** (0.024)	-0.210*** (0.032)	-0.157*** (0.017)
b_2 (S_3)	0.067*** (0.021)	0.009 (0.013)	0.015 (0.035)	0.129*** (0.024)	0.056** (0.026)	0.086*** (0.024)	0.072*** (0.013)
b_3 (T_0)	—	0.434*** (0.061)	-0.147** (0.068)	0.107** (0.051)	0.222*** (0.071)	0.207** (0.083)	0.197*** (0.062)
b_4 $(T_0.[S_2 + S_3])$	—	-0.179*** (0.032)	-0.058 (0.055)	-0.048 (0.039)	-0.103** (0.042)	-0.042 (0.046)	-0.049 (0.031)
b_5 $(T_0.S_3)$	—	0.055 (0.034)	0.052 (0.042)	-0.063 (0.042)	0.012 (0.036)	-0.017 (0.035)	-0.010 (0.024)
σ_η	0.020*** (0.004)	0.024*** (0.003)	0.029*** (0.003)	0.026*** (0.005)	0.093*** (0.017)	0.025*** (0.003)	0.114*** (0.015)
σ_u	0.107*** (0.006)	0.097*** (0.003)	0.133*** (0.004)	0.101*** (0.003)	0.106*** (0.004)	0.117*** (0.004)	0.103*** (0.004)
ln (L)	-2602.37	-5362.18	-5532.78	-5165.09	-5220.24	-4832.89	-4937.02

In each cell, the first number corresponds to the point estimate, while the number in parenthesis is the estimated standard deviation of the parameter. ***, **, and * respectively indicate parameters significant at the 1%, 5% and 10% levels.

Table 5 : Non-Parametric Comparison of Treatments

Objective Probability	Series	T_0 vs. T_1	T_0 vs. T_2	T_0 vs. T_3	T_0 vs. T_4	T_3 vs. T_4	T_0 vs. T_5	T_0 vs. T_6	T_5 vs. T_6	
									Prediction	Bet
3%	S1	2.60E+02 8.64E-09	1.20E+03 1.74E-01	8.29E+02 6.37E-01	9.48E+02 9.90E-01	9.60E+02 6.08E-01	7.02E+02 1.07E-01	8.78E+02 9.71E-01	1.02E+03 9.10E-02	5.92E+02 1.60E-02
	S2	3.98E+02 5.19E-06	1.37E+03 7.00E-03	9.65E+02 4.55E-01	1.07E+03 2.90E-01	9.33E+02 7.88E-01	5.98E+02 1.10E-02	1.01E+03 2.40E-01	1.19E+03 1.00E-03	5.00E+02 1.00E-03
	S3	3.50E+02 6.55E-07	1.22E+03 1.37E-01	9.88E+02 3.42E-01	1.08E+03 2.39E-01	9.42E+02 7.28E-01	7.90E+02 4.09E-01	1.05E+03 1.23E-01	1.06E+03 3.90E-02	5.63E+02 6.00E-03
5%	S1	2.23E+02 1.30E-09	1.26E+03 7.10E-02	8.97E+02 8.93E-01	8.49E+02 4.09E-01	7.97E+02 3.52E-01	7.18E+02 1.43E-01	8.38E+02 6.93E-01	9.55E+02 2.87E-01	5.05E+02 1.00E-03
	S2	3.83E+02 2.70E-06	1.28E+03 4.50E-02	8.22E+02 5.94E-01	1.13E+03 1.26E-01	1.13E+03 4.80E-02	7.15E+02 1.34E-01	9.12E+02 7.85E-01	1.04E+03 6.50E-02	4.41E+02 6.44E-05
	S3	3.60E+02 1.04E-06	1.35E+03 1.10E-02	1.16E+03 1.20E-02	1.19E+03 4.10E-02	8.56E+02 6.82E-01	7.78E+02 3.54E-01	1.07E+03 9.20E-02	1.08E+03 2.40E-02	3.96E+02 1.89E-05
15%	S1	2.29E+02 1.73E-09	1.14E+03 3.75E-01	6.91E+02 8.80E-02	7.38E+02 7.70E-02	8.98E+02 9.72E-01	5.36E+02 2.00E-03	9.34E+02 6.41E-01	1.17E+03 2.00E-03	3.37E+02 1.96E-06
	S2	3.31E+02 2.71E-07	1.36E+03 9.00E-03	7.65E+02 2.95E-01	1.09E+03 2.26E-01	1.15E+03 3.20E-02	6.38E+02 2.90E-02	9.68E+02 4.38E-01	1.13E+03 7.00E-03	3.38E+02 1.80E-06
	S3	3.77E+02 2.04E-06	1.16E+03 2.92E-01	8.09E+02 5.13E-01	1.06E+03 3.46E-01	1.07E+03 1.35E-01	5.22E+02 1.00E-03	8.79E+02 9.79E-01	1.15E+03 4.00E-03	2.94E+02 1.89E-07
25%	S1	3.54E+02 7.59E-07	1.33E+03 1.80E-02	8.77E+02 9.68E-01	1.14E+03 9.90E-02	1.09E+03 9.50E-02	7.44E+02 2.18E-01	9.06E+02 8.26E-01	9.76E+02 2.09E-01	2.58E+02 4.34E-08
	S2	4.84E+02 1.24E-04	1.52E+03 9.87E-05	1.11E+03 4.50E-02	1.25E+03 1.00E-02	9.83E+02 4.76E-01	8.65E+02 8.82E-01	9.60E+02 4.84E-01	9.39E+02 3.60E-01	2.84E+02 1.29E-07
	S3	4.48E+02 3.36E-05	1.35E+03 1.10E-02	1.10E+03 4.80E-02	1.20E+03 3.00E-02	9.16E+02 9.02E-01	9.51E+02 5.32E-01	1.22E+03 3.00E-03	1.08E+03 2.90E-02	2.51E+02 1.83E-08
35%	S1	3.54E+02 7.02E-07	9.24E+02 3.88E-01	7.29E+02 1.72E-01	9.38E+02 9.42E-01	1.02E+03 2.88E-01	3.99E+02 1.46E-05	8.02E+02 4.76E-01	1.22E+03 3.98E-04	2.25E+02 5.29E-09
	S2	7.46E+02 1.13E-01	1.16E+03 3.13E-01	1.10E+03 4.50E-02	1.36E+03 3.70E-04	1.09E+03 9.80E-02	7.73E+02 3.18E-01	1.04E+03 1.42E-01	1.08E+03 2.10E-02	2.14E+02 3.14E-09
	S3	4.95E+02 1.36E-04	1.16E+03 3.13E-01	9.35E+02 6.34E-01	1.25E+03 1.00E-02	1.16E+03 2.40E-02	7.11E+02 1.22E-01	9.29E+02 6.73E-01	1.01E+03 1.22E-01	1.47E+02 5.35E-11
45%	S1	7.81E+02 1.95E-01	1.03E+03 9.90E-01	9.90E+02 3.26E-01	1.25E+03 1.00E-02	1.09E+03 8.70E-02	8.59E+02 8.34E-01	1.11E+03 4.20E-02	1.08E+03 2.50E-02	1.63E+02 1.21E-10
	S2	9.98E+02 5.08E-01	1.11E+03 5.37E-01	1.12E+03 2.80E-02	1.24E+03 1.10E-02	9.73E+02 5.28E-01	1.00E+03 2.47E-01	1.17E+03 7.00E-03	1.02E+03 7.00E-02	1.46E+02 3.90E-11
	S3	1.12E+03 7.50E-02	1.21E+03 1.61E-01	1.07E+03 8.40E-02	1.41E+03 7.18E-05	1.16E+03 2.30E-02	1.02E+03 2.05E-01	1.38E+03 5.68E-06	1.22E+03 2.48E-04	2.40E+02 1.01E-08
61%	S1	1.28E+03 2.00E-03	9.79E+02 6.73E-01	1.18E+03 7.00E-03	1.33E+03 1.00E-03	1.03E+03 2.43E-01	1.11E+03 3.80E-02	1.14E+03 2.10E-02	8.86E+02 6.69E-01	1.43E+02 1.48E-11
	S2	1.27E+03 3.00E-03	1.13E+03 4.31E-01	1.08E+03 7.80E-02	1.44E+03 2.63E-05	1.16E+03 2.10E-02	1.29E+03 2.52E-04	1.35E+03 1.91E-05	1.05E+03 4.50E-02	1.42E+02 2.68E-11
	S3	1.11E+03 1.04E-01	1.08E+03 7.04E-01	8.59E+02 8.40E-01	1.22E+03 2.00E-02	1.18E+03 1.40E-02	9.82E+02 3.51E-01	1.16E+03 1.30E-02	1.02E+03 8.20E-02	1.59E+02 7.82E-11
70%	S1	1.48E+03 1.60E-06	1.11E+03 5.21E-01	1.23E+03 2.00E-03	1.38E+03 2.45E-04	9.51E+02 6.69E-01	1.07E+03 9.10E-02	1.29E+03 2.71E-04	1.05E+03 4.50E-02	2.51E+02 6.00E-09
	S2	1.23E+03 7.00E-03	7.81E+02 4.50E-02	1.10E+03 4.80E-02	1.45E+03 1.47E-05	1.20E+03 9.00E-03	1.10E+03 4.50E-02	1.22E+03 3.00E-03	9.78E+02 1.98E-01	1.63E+02 8.45E-11
	S3	1.32E+03 1.00E-03	1.08E+03 7.26E-01	9.48E+02 5.51E-01	1.25E+03 9.00E-03	1.14E+03 4.00E-02	1.07E+03 9.70E-02	1.27E+03 1.00E-03	1.05E+03 4.60E-02	2.65E+02 1.65E-08
80%	S1	1.36E+03 1.62E-04	7.57E+02 2.90E-02	9.89E+02 3.38E-01	1.38E+03 2.57E-04	1.22E+03 5.00E-03	9.35E+02 6.34E-01	1.24E+03 1.00E-03	1.10E+03 1.40E-02	3.30E+02 8.78E-08
	S2	1.07E+03 2.19E-01	6.33E+02 1.00E-03	1.10E+03 4.60E-02	1.16E+03 7.50E-02	9.05E+02 9.79E-01	1.07E+03 9.50E-02	1.09E+03 5.90E-02	8.73E+02 7.65E-01	2.16E+02 1.91E-09
	S3	1.31E+03 1.00E-03	8.81E+02 2.28E-01	9.66E+02 4.51E-01	1.38E+03 2.34E-04	1.25E+03 2.00E-03	9.92E+02 3.21E-01	1.18E+03 7.00E-03	1.03E+03 8.40E-02	2.35E+02 1.91E-09
90%	S1	1.50E+03 5.68E-07	8.06E+02 7.10E-02	1.17E+03 9.00E-03	1.53E+03 6.37E-07	1.24E+03 3.00E-03	1.10E+03 5.20E-02	1.27E+03 4.60E-04	9.82E+02 1.88E-01	3.15E+02 2.66E-08
	S2	1.42E+03 1.92E-05	9.48E+02 5.04E-01	1.12E+03 3.50E-02	1.51E+03 1.87E-06	1.24E+03 3.00E-03	1.18E+03 7.00E-03	1.39E+03 5.68E-06	1.05E+03 4.70E-02	2.55E+02 5.56E-09
	S3	1.43E+03 1.45E-05	8.10E+02 7.70E-02	1.20E+03 5.00E-03	1.50E+03 2.69E-06	1.24E+03 3.00E-03	1.24E+03 1.00E-03	1.36E+03 1.67E-05	9.47E+02 3.20E-01	3.26E+02 9.87E-08

In each cell, the first number is the Mann-Whitney U Test Statistic, while the second number is the p-value.
A cell shaded in dark (light) gray indicates a difference between the two treatments significant at the 5% (10% level).

Table 6 : Non-Parametric Comparison of Series

Objective Probability		Series	T_0	T_1	T_2	T_3	T_4	T_5 Prediction	T_5 Bet	T_6 Prediction	T_6 Bet
3%	Rank Sum	S1	53.5	76	78.5	68.5	77.5	67.5	88.5	71.5	82.5
		S2	108	92.5	102.5	100	103.5	104	75.5	97.5	83
		S3	96.5	89.5	107	77.5	83	74.5	82	77	80.5
	Friedman Statistic		40.256	5.024	11.313	14.327	10.652	23.638	5.93	11.827	0.230
	p-value		1.81E-9	0.081	0.003	0.001	0.005	7.36E-6	0.052	0.003	0.892
5%	Rank Sum	S1	58	76.5	78.5	65	79	68.5	86	67	84.5
		S2	103	90	108.5	111	101	95	75	100.5	76
		S3	97	91.5	101	70	84	82.5	85	78.5	85.5
	Friedman Statistic		28.429	4.299	11.538	32.253	7.189	11.071	4.625	16.797	2.627
	p-value		6.71E-7	0.117	0.003	9.92E-5	0.027	0.004	0.099	2.25E-4	0.269
15%	Rank Sum	S1	60	78.5	82	65	88	67.5	90.5	65	84
		S2	102.5	90	104	103	93	86.5	76.5	96	75
		S3	95.5	89.5	102	78	83	92	79	85	87
	Friedman Statistic		25.024	2.793	6.265	19.128	1.307	10.836	6.559	16.197	4.727
	p-value		3.68E-6	0.247	0.044	7.02E-5	0.520	0.004	0.038	3.04E-4	0.094
25%	Rank Sum	S1	67	74.5	89	76	75	78.5	92	74.5	96.5
		S2	97.5	89.5	91	91	92	92	76.5	98	69.5
		S3	93.5	94	108	79	97	75.5	77.5	73.5	80
	Friedman Statistic		13.485	7.068	4.589	3.294	6.734	4.828	7.167	12.607	17.435
	p-value		0.001	0.029	0.101	0.193	0.034	0.089	0.028	0.002	1.64E-4
35%	Rank Sum	S1	59	79.5	88.5	73.5	84.5	80	90.5	72.5	90
		S2	112.5	89	106.5	88.5	97.5	89.5	80.5	92	69.5
		S3	86.5	89.5	93	84	82	76.5	75	81.5	86.5
	Friedman Statistic		35.565	2.209	3.795	3.058	3.693	2.919	6.024	5.953	12.998
	p-value		1.89E-8	0.331	0.150	0.217	0.158	0.232	0.049	0.051	0.002
45%	Rank Sum	S1	69.5	86.5	93.5	80.5	81.5	76.5	88	80.5	88.5
		S2	95	91	97.5	82	96.5	85.5	72.5	88.5	77.5
		S3	93.5	80.5	97	83.5	86	84	82.5	77	80
	Friedman Statistic		10.920	2.056	0.22	0.123	3.038	1.706	5.957	2.482	3.800
	p-value		0.004	0.358	0.896	0.940	0.219	0.426	0.051	0.289	0.150
61%	Rank Sum	S1	102	92	106	87.5	96.5	86	94	98	89
		S2	77.5	81.5	95	74.5	81.5	72.5	69	70	73.5
		S3	78.5	84.5	87	84	86	87.5	83	78	83.5
	Friedman Statistic		9.859	1.814	3.978	2.277	3.203	4.707	13.362	14.726	8.157
	p-value		0.007	0.404	0.137	0.320	0.202	0.095	0.001	0.001	0.014
70%	Rank Sum	S1	109	95	105	86	108.5	97	91.5	94	88.5
		S2	65.5	77	96	63.5	63	65.5	67	72	73.5
		S3	83.5	86	87	96.5	92.5	83.5	87.5	80	84
	Friedman Statistic		23.448	5.184	3.447	14.392	27.856	16.65	14.702	8.198	11.023
	p-value		8.10E-6	0.075	0.178	0.001	8.93E-7	2.42E-4	0.001	0.017	0.004
80%	Rank Sum	S1	109.5	89.5	110	96.5	98.5	93.5	98.5	92	88
		S2	59.5	86	92.5	61	88.5	68	66	71.5	73.5
		S3	89	82.5	85.5	88.5	77	84.5	81.5	82.5	84.5
	Friedman Statistic		32.191	0.778	6.777	17.337	6.431	11.34	19.757	7.322	13.086
	p-value		1.02E-7	0.678	0.034	1.72E-4	0.040	0.003	5.13E-5	0.026	0.001
90%	Rank Sum	S1	105	95	103	96	101	98	86.5	103	85.5
		S2	69.5	77	83	71.5	75.5	73.5	73	64.5	78.5
		S3	83.5	86	102	78.5	87.5	74.5	86.5	78.5	82
	Friedman Statistic		15.988	5.184	5.676	8.327	8.857	12.924	6.000	28.13	6.125
	p-value		1.74E-3	0.075	0.059	0.016	0.012	0.002	0.050	7.79E-7	0.047

Friedman tests are conducted to compare a subject's individual responses across Series. Under the null hypothesis, the distributions of a subject's responses are the same across series. Cells shaded in dark (light) gray indicate that the subject's responses are significantly different across Series at the 5% (10% level).

Appendix A: Demonstration of the Propositions in Section 2

Proposition 2.1 *A scoring rule S is proper if and only if there exists a function $g(.)$ with $g''(q) > 0$ for all $q \in [0,1]$ such that*

$$\begin{aligned} S_1(q) &= g(q) + (1-q)g'(q) \\ S_0(q) &= g(q) - qg'(q) \end{aligned}$$

Proof: We proved the sufficiency in the text. We now prove the necessity. Define

$$g(p) \equiv \max_q pS_1(q) + (1-p)S_0(q) = pS_1(p) + (1-p)S_0(p) \tag{6.1}$$

by the definition of a proper scoring rule. By the envelope theorem, we have $g'(p) = S_1(p) - S_0(p)$. Replacing $S_1(p)$ by $g'(p) + S_0(p)$ and $S_0(p)$ by $S_1(p) - g'(p)$ in (6.1) directly gives the result.■

Proposition 2.2 *Let $v(w) = \Phi(u(w))$ with $\Phi' > 0$ and $\Phi'' \leq 0$. For all standard PSR defined by (2.3) and (2.4),*

$$R_v(p) \geq R_u(p) \textit{ if and only if } p \leq 1/2$$

Proof: The function $R_u(p) \equiv R_u$ is defined by the first order condition:

$$p(1-R_u)u'(S_1(R_u)) - (1-p)R_u u'(S_0(R_u)) = 0 \tag{6.2}$$

We want to examine the sign of the similar first order condition for $v(.)$ evaluated at $R_v(p) = R_u$:

$$\begin{aligned} L(p) &\equiv p(1-R_u)v'(S_1(R_u)) - (1-p)R_u v'(S_0(R_u)) \\ &= p(1-R_u)u'(S_1(R_u))\Phi'(u(S_1(R_u))) - (1-p)R_u u'(S_0(R_u))\Phi'(u(S_0(R_u))) \\ &= (1-p)R_u u'(S_0(R_u))[\Phi'(u(S_1(R_u))) - \Phi'(u(S_0(R_u)))] \end{aligned}$$

where the second inequality uses $v(w) = \Phi(u(w))$ and the last inequality uses (6.2). Observe that $L(p)$ has the sign of the term in brackets, and thus of $[S_0(R_u))) - S_1(R_u))]$ since Φ' is decreasing. Consequently, $L(p)$ is positive if and only if $R_u \leq 1/2$. By the properties of R_u exhibited in Corollary 2.1, this holds true if and only if $p \leq 1/2$.■

Proposition 2.3 *For all standard PSR defined by (2.3) and (2.4), and for all $a > 0$, under $\gamma'(x) \geq (\leq) 0$,*

$$\frac{\partial R(p,a)}{\partial a} \geq (\leq) 0 \textit{ if and only if } p \leq 1/2.$$

Proof: The response function $R(p,a) \equiv R$ is defined by the first order condition

$$M(a) \equiv p(1-R)au'(aS_1(R)) - (1-p)Rau'(aS_0(R))) = 0 \tag{6.3}$$

Since the objective function is concave, the sign of $\frac{\partial R(p,a)}{\partial a}$ is the same as that of $M'(a)$. We obtain

$$
\begin{aligned}
M'(a) &= p(1-R)aS_1(R)u''(aS_1(R)) - (1-p)RaS_0(R)u''(aS_0(R)) \\
&= p(1-R)u'(aS_1(R))\frac{aS_1(R)u''(aS_1(R))}{u'(aS_1(R))} - \\
&\quad (1-p)Ru'(aS_0(R))\frac{aS_0(R)u''(aS_0(R))}{u'(aS_0(R))} \\
&= p(1-R)u'(aS_1(R))[\gamma(aS_0(R)) - \gamma(aS_1(R))]
\end{aligned}
$$

where the last equality uses (6.3) and the definition of $\gamma(x)$. Notice that $M'(a)$ has the sign of the term in brackets. Using the properties of S_0 and S_1 in (2.3) and (2.4), and those of R, we conclude that, under $\gamma(x)$ increasing (respectively decreasing), $M'(a)$ is positive if and only if p is lower (respectively larger) than 1/2. ■

Proposition 2.4 *For all PSR defined by (2.3), the response function $R(p, \triangle)$ is characterized as follows:*
i) if there exists a $\widehat{p}$ such that $\triangle + S_1(\widehat{p}) = S_0(\widehat{p})$, then we have

$$R(p, \triangle) \geq p \text{ if and only if } p \leq \widehat{p}$$

ii) if $\triangle + S_1(p) \geq (\leq) S_0(p)$ for all p, then we have

$$R(p, \triangle) \leq (\geq) p$$

Proof: The response function $R(p, a) \equiv R$ is characterized by the following first order condition

$$g(\triangle, R) \equiv p(1-R)u'(\triangle + S_1(R)) - (1-p)Ru'(S_0(R)) = 0$$

We compute the marginal benefit of increasing R at p :

$$g(\triangle, p) = p(1-p)[u'(\triangle + S_1(p)) - u'(S_0(p))]$$

which is positive if and only if

$$N(p) \equiv \triangle + S_1(p) - S_0(p) \leq 0$$

We thus have $R \geq p$ if and only if $N(p) \leq 0$. Since, by our assumptions on the PSR, $N(p)$ is strictly increasing, there is at most one $\widehat{p}$ satisfying $N(\widehat{p}) = 0$. Therefore either $\widehat{p}$ exists and we are in case i), or $\widehat{p}$ does not exist and we are in case ii). It is then direct to conclude the proof for each case i) and ii).■

Proposition 2.5 *For all standard PSR defined by (2.3) and (2.4), the solutions $R(p)$* and

$\alpha(p)$ to program (2.10) satisfy the following properties:

i) for $p \leq \underline{p}(k)$, we have $\alpha(p) = 0$ and $R(p) \in [0, \frac{1}{1+k}]$ with $R'(p) > 0$,
ii) *for p in $[\underline{p}(k), \overline{p}(k)]$, we have $\alpha(p) \in [0, \overline{\alpha}]$ with $\alpha'(p) > 0$ and $R(p) = \frac{1}{1+k}$,*
iii) for $p \geq \overline{p}(k)$, we have $\alpha(p) = \overline{\alpha}$ and $R(p) \in [\frac{1}{1+k}, 1]$ with $R'(p) > 0$,

together with

$$\begin{aligned}\underline{p}(k) &= \frac{u'(S_0(\frac{1}{1+k}) + \overline{\alpha})}{ku'(S_1(\frac{1}{1+k}) + \overline{\alpha}) + u'(S_0(\frac{1}{1+k}) + \overline{\alpha})} \text{ and} \\ \overline{p}(k) &= \frac{u'(S_0(\frac{1}{1+k}))}{ku'(S_1(\frac{1}{1+k}) + (k+1)\overline{\alpha}) + u'(S_0(\frac{1}{1+k}))}\end{aligned}$$

Proof: The conditions which characterize the interior solutions (α^*, q^*) of the program (2.10) are

$$pku'(S_1(q^*) + k\alpha^* + \overline{\alpha}) - (1-p)u'(S_0(q^*) - \alpha^* + \overline{\alpha}) = 0 \tag{6.4}$$
$$p(1-q^*)u'(S_1(q^*) + k\alpha^* + \overline{\alpha}) - (1-p)q^*u'(S_0(q^*) - \alpha^* + \overline{\alpha}) = 0 \tag{6.5}$$

which imply $q^* = \frac{1}{1+k}$.

Therefore the condition (6.4) writes

$$pku'(S_1(\frac{1}{1+k}) + k\alpha^* + \overline{\alpha}) - (1-p)u'(S_0(\frac{1}{1+k}) - \alpha^* + \overline{\alpha}) = 0 \tag{6.6}$$

Differentiating with respect to p and rearranging yields

$$\frac{\partial \alpha^*}{\partial p} = -\frac{ku'(S_1(\frac{1}{1+k}) + k\alpha^* + \overline{\alpha}) + u'(S_0(\frac{1}{1+k}) - \alpha^* + \overline{\alpha})}{pk^2u''(S_1(\frac{1}{1+k}) + k\alpha^* + \overline{\alpha}) + (1-p)u''(S_0(\frac{1}{1+k}) - \alpha^* + \overline{\alpha})} > 0$$

Therefore α^* can only increase in p; moreover, the condition (6.4) cannot be satisfied at $p = 0$ or at $p = 1$. Indeed it is strictly negative at $p = 0$ and strictly positive at $p = 1$. Consequently, $\alpha(p)$ is first equal to zero, then equal to $\alpha^* > 0$ and strictly increasing in p, and finally constant and equal to $\overline{\alpha}$. There are thus two critical values of subjective probability denoted $\underline{p}(k)$ and $\overline{p}(k)$ with $0 < \underline{p}(k) < \overline{p}(k) < 1$ such that the optimal α is equal to zero for $p \leq \underline{p}(k)$ and is equal to $\overline{\alpha}$ for $p \geq \overline{p}(k)$. Moreover, the response function $R(p) = \frac{1}{1+k}$ when p is in $[\underline{p}(k), \overline{p}(k)]$.

We now study more specifically the response function when $p \leq \underline{p}(k)$. Since $\alpha^* = 0$, the response function $R(p)$ is equal to the reported probability without hedging effects, as characterized by q solving

$$p(1-q)u'(S_1(q) + \overline{\alpha}) - (1-p)qu'(S_0(q) + \overline{\alpha}) = 0$$

The threshold probability $\underline{p}(k)$ is defined by the p solving

$$pku'(S_1(\frac{1}{1+k})+\overline{\alpha})-(1-p)u'(S_0(\frac{1}{1+k})+\overline{\alpha})=0$$

We thus have

$$\underline{p}(k)=\frac{u'(S_0(\frac{1}{1+k})+\overline{\alpha})}{ku'(S_1(\frac{1}{1+k})+\overline{\alpha})+u'(S_0(\frac{1}{1+k})+\overline{\alpha})}$$

as stated in the Proposition. Moreover, the properties of the standard PSR (2.3) and (2.4) imply $\underline{p}(k)<\frac{1}{1+k}$ if and only if $k>1$. Notice also that an increase in k decreases $\underline{p}(k)$. Finally, it is easy to check that $R(\underline{p}(k))=\frac{1}{1+k}$.

We finally study the response function when $p\geq\overline{p}(k)$. Since $\alpha^*=\overline{\alpha}$, the optimal reported probability q is defined by

$$p(1-q)u'(S_1(q)+(k+1)\overline{\alpha})-(1-p)qu'(S_0(q))=0$$

The threshold probability $\overline{p}(k)$ is defined by the p solving

$$pku'(S_1(\frac{1}{1+k})+(k+1)\overline{\alpha})-(1-p)u'(S_0(\frac{1}{1+k}))=0,$$

that is by

$$\overline{p}(k)=\frac{u'(S_0(\frac{1}{1+k}))}{ku'(S_1(\frac{1}{1+k})+(k+1)\overline{\alpha})+u'(S_0(\frac{1}{1+k}))}$$

as stated in the Proposition.■

Appendix B: The Effect of Ambiguity Aversion

We consider the theory of ambiguity introduced by KMM (2005). We assume that the agent's subjective probabilities are represented by a random variable $\widetilde{p}$, to which the possible realizations belong to $[0,1]$. Consistent with KMM preferences, an ambiguity averse agent facing a PSR solves

$$\max_q (E\phi(\widetilde{p}u(S_1(q)) + (1-\widetilde{p})u(S_0(q))))$$

in which ϕ is assumed to be continuously differentiable, strictly increasing, and concave. Under KMM preferences, the concavity of ϕ corresponds to ambiguity aversion. The first order condition is

$$K(q) \equiv E\{(\widetilde{p}(1-q)u'(S_1(q)) - (1-\widetilde{p})qu(S_0(q)))\phi'(\widetilde{p}u(S_1(q)) + (1-\widetilde{p})u(S_0(q)))\} = 0 \quad (6.7)$$

It can be easily checked that the second order condition is satisfied under u and ϕ concave.

Our objective is to compare the response function of an ambiguity averse agent to that of an ambiguity neutral agent. Under ϕ linear, the agent is ambiguity neutral and essentially behaves as an expected utility maximizer, reporting q^* defined by

$$E(\widetilde{p}(1-q^*)u'(S_1(q^*)) - (1-\widetilde{p})q^*u(S_0(q^*))) = 0$$

Observe that this last condition is equivalent to (2.6) with $p = E\widetilde{p}$. We are done if we can compute the sign of $K(q^*)$, which expresses the marginal benefit under ambiguity aversion of increasing q at the optimal reported probability q^* under ambiguity neutrality. Denoting as before $f(p,q) \equiv p(1-q)u'(S_1(q)) - (1-p)qu'(S_0(q))$ and using the previous equality, we have

$$K(q^*) = Cov_{\widetilde{p}}[f(\widetilde{p},q^*), \phi'(\widetilde{p}u(S_1(q^*)) + (1-\widetilde{p})u(S_0(q^*)))]$$

We now use the following Lemma stating the well-known covariance rule (e.g., Kimball 1951).

Lemma B.1 *If $X(p)$ is increasing in p, then $Cov_{\widetilde{p}}(X(\widetilde{p}), Y(\widetilde{p})) \leq 0$ if and only if $Y(p)$ is decreasing in p.*

Observe that $f(p,q)$ is increasing in p. Moreover, observe that the derivative of $\phi'(pu(S_1(q^*)) + (1-p)u(S_0(q^*)))$ with respect to p has the sign of $S_0(q^*) - S_1(q^*)$ under $\phi'' < 0$. Consequently, under (2.3) and (2.4) $K(q^*)$ is positive if and only if q^* is lower than $1/2$. We then use Corollary 2.1 that characterizes q^* to obtain the next Proposition. In this Proposition, we denote $R_a(\widetilde{p})$ the optimal reported probability under ambiguity aversion (ϕ concave) and $R(\widetilde{p})$ the optimal reported probability under expected utility (ϕ linear) with a subjective probability $E\widetilde{p}$.

Proposition B.1 *For all standard PSR defined by (2.3) and (2.4), $R_a(\widetilde{p}) \geq R(\widetilde{p})$ if and only if $E\widetilde{p} \leq 1/2$.*

Essentially, this result indicates that ambiguity aversion leads to reporting more uniform probabilities compared to ambiguity neutrality (i.e., expected utility). In other words, ambiguity aversion reinforces the bias induced by risk aversion. It is also possible to show that ambiguity aversion reinforces the effect of risk aversion when there is a stake. That is, ambiguity aversion leads to increasing the response function before the fixed point and to decreasing the response function after the fixed point.

A particular case is $u(x) = x$. In that case, the agent is ambiguity averse but risk neutral. This implies $q^* = E\widetilde{p}$: when the agent is both ambiguity neutral and risk neutral, she always reports the mean of her subjective beliefs $E\widetilde{p}$ by the definition of a PSR. Proposition B.1 then reduces to the following Corollary.

Corollary B.1 *For all standard PSR defined by (2.3) and (2.4), $R_a(\widetilde{p}) \geq E\widetilde{p}$ if and only if $E\widetilde{p} \leq 1/2$.*

This Corollary, similar to Corollary 2.1, thus further suggests that the effect of ambiguity aversion is similar to that of risk aversion.

Appendix C

Identification Code :______

INSTRUCTIONS
(translated from French)

You are about to take part in an experiment aimed at better understanding decisions made under uncertainty. In the experiment you will earn an amount of money. This amount of money will be paid to you at the end of the experiment, outside the lab, in private, and in cash. The amount of money you will earn may be larger if :

1. You read the instructions below carefully.
2. You follow these instructions precisely.
3. You make thoughtful decisions during the experiment.

If you have any questions while we read the instructions or during the experiment, then call us **by raising your hand.** Any form of communication between participants is absolutely forbidden. If you do not follow this rule, then we will have to exclude you from the experiment without any payment.

The Task

You will be given 30 different «events», divided into 3 series of 10. Each of these events describes the possible outcome produced by the roll of 2 dice. One of the die is red, the other die is black. Each die has 10 sides numbered from 0 to 9. Each die is fair, which means that any of the 10 sides has an equal chance to come up when the die is rolled. Consider now two examples of events we could give you:

- Event 1: ***«The red die equals 5 and the black die equals 3»***.
- Event 2: ***«The red die produces a number strictly greater than the black die»***.

As explained below, 1 out of the 30 events will be randomly selected for payment at the end of the experiment. We will then roll the 2 dice once in order to determine whether the event occurs or whether the event does not occur. For instance, if Event 1 above is randomly selected for payment, then we will say that Event 1 occurred when the outcome of the roll of the 2 die is such that the red die produces a 5 and the black die produces a 3. For any other number produced by either the black or the red die, we will say that Event 1 did not occur. Likewise, if Event 2 is randomly selected for payment, then we will say that Event 2 occurred when the outcome of the roll of the 2 dice is such that the red die produces a number strictly greater than the black die. Otherwise, we will say that Event 2 did not occur.

Your Choices:

For each of the 30 events, you will be asked to make a choice. One of these choices will determine the amount of money you will earn both when the event randomly selected for payment occurs and when it does not occur. Each of your choices consists in selecting a number between 1 and 149 in the table we gave you separately. We will now explain how your choice for the event randomly selected for payment affects the amount of money you will earn.

If you look at the table, you can see that there are two amounts associated with each of the 149 possible choice numbers. The first is the amount of money you receive if the event occurs. The second is the amount of money you receive if the event does not occur. For instance, you can see in the table that the amounts associated with the choice number "1" are 53 and 4,000. This means that the amount of money you earn would be 53FCFA if the event occurs or 4,000FCFA if the event does not occur. As you can see, when the choice number increases from 1 to 149, the amounts in the first columns increase, while the amounts in the second column decrease. For instance, the amounts associated with the choice number "90" are 3,360FCA and 2,560FCFA. In other words, if you choose the number "90" instead of the number "1" then you would earn more if the event occurs (3,360FCFA instead of 53FCFA), but you would earn less if the event does not occur (2,560FCFA instead of 4,000FCFA). Note also, that the highest choice numbers (those closer to 149) produce the largest amounts of money when the event occurs, but the smallest amounts of money when the event does not occur. For instance, the choice number "140" produces 3,982FCFA if the event occurs, but only 516FCFA when the event does not occur.

For each of the 30 events, you are free to select any choice number you want. Note that there is no correct or incorrect choice. The choice numbers selected may differ from one individual to the next. In general however, you may find it profitable to choose a higher choice number when you think the chances that the event occurs are higher. Indeed, as we just explained, such a choice number will produce a larger amount if the event occurs. Conversely, you may find it profitable to choose a smaller number when you think the chances that the event occurs are lower.

Your Payment

The amount of money you receive today will be determined in 3 steps. In a first step, we will randomly select one of the 30 events for payment. In a second step, we will roll the 2 dice once to determine whether the event selected for payment occurs or does not occur. Finally, in a third step, we will look at the choice number you chose for the event selected for payment in order to determine the amount of money you will receive.

We will proceed as follows to select one of the 30 events for payment. At the beginning of the experiment, we will ask you to write your identification code on a piece of paper that you will then fold. Your identification code is located on the top right hand corner on the first page of the instructions. At the end of the experiment, we will draw at random one of the pieces of paper. The person whose identification number has been drawn will randomly choose 1 out of 30 numbered tokens from a bag. The number written on the token selected indicates the event that will be considered for the payment of each person in the room.

We will then draw at random a second piece of paper. The person whose identification code has been drawn will roll the 2 dice once to determine whether the event selected occurs or not. This single roll will be used to determine the payment of each person in the room.

If you do not wish to be one of the persons rolling the dice or drawing the token, then simply leave your piece of paper blank. Just fold it without writing your identification code.

<u>Comprehension Test:</u>

Understanding the instructions well is important if you want to improve your chances to earn a larger amount of money during the experiment. In order to make sure you understand the instructions well, we will now conduct a quick test without monetary consequences. Imagine first that Event 1: ***«The red die equals 5 and the black die equals 3»*** has been selected for payment. In addition, imagine that an individual selected the choice number **98** for this event, while a different individual selected the choice number **139**. Please, write in the table below the amount of money each of these 2 individuals would receive if the roll of the dice produces the following outcomes:

Outcome produced by the roll of the 2 dice	Payment to the individual with	
	A choice number of **98**	A choice number of **139**
The red die equals 6 and the black die equals 4	________FCFA	________FCFA
The red die equals 5 and the black die equals 4	________FCFA	________FCFA
The red die equals 5 and the black die equals 3	________FCFA	________FCFA

Imagine now that Event 2: **«*The red die produces a number strictly greater than the black die*»** has been selected for payment. In addition, imagine that an individual selected the choice number **6** for this event, while a different individual selected the choice number **71**. Please, write in the table below the amount of money each of these 2 individuals would receive if the roll of the dice produce the following outcomes:

Outcome produced by the roll of the 2 dice	Payment to the individual with	
	A choice number of **6**	A choice number of **71**
The red die equals 3 and the black die equals 9	________FCFA	________FCFA
The red die equals 5 and the black die equals 2	________FCFA	________FCFA
The red die equals 0 and the black die equals 5	________FCFA	________FCFA

Please, do not hesitate to raise your hand now if the instructions we just read were not perfectly clear. Once the experiment starts you can still call us to answer any question <u>by raising your hand</u>.

Note that the amount of money you will receive today may be larger or smaller depending on your choices <u>and</u> on the outcome produced by the roll of the 2 dice. By accepting to participate in the experiment, you accept the consequences associated with your choices and with the roll of the dice. If you do not wish to participate in the experiment you are free to leave now, in which case you will receive a flat fee of 500FCFA.

Series 1 :

For the first series of 10 events, we will consider that the red die determines the first digit (meaning 0, 10, 20, 30, 40, 50, 60, 70, 80, or 90) and the black die determines the second digit (meaning 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9) of a number between 1 and 100 (both dice equal to zero corresponds to the number 100). As a result, every number between 1 and 100 has an equal chance to come out from the roll of the 2 dice.

Event	Description	Your Choice Number
1	***« the number is between 1 (included) and 25 (included) »***	
2	***« the number is between 62 (included) and 66 (included) »***	
3	***« the number is between 16 (included) and 76 (included) »***	
4	***« the number is between 3 (included) and 92 (included) »***	
5	***« the number is between 52 (included) and 96 (included) »***	
6	***« the number is between 9 (included) and 88 (included) »***	
7	***« the number is between 44 (included) and 58 (included) »***	
8	***« the number is between 23 (included) and 25 (included) »***	
9	***« the number is between 37 (included) and 71 (included) »***	
10	***« the number is between 28 (included) and 97 (included) »***	

Series 2 :

For the next series of 10 events, we will sum the outcome of the red die to the outcome of the black die. Since each die can only produce a number between 0 and 9, the sum obtained can only be a number between 0 and 18. Observe that some of these sums (for instance 0) can only be obtained from a unique combination of the 2 dice, while other sums (for instance 6) can be obtained from multiple combinations of the 2 dice. As a result, some of the 19 possible sums have more chances to come out than other sums.

Event	Description	Your Choice Number
11	***« The sum is between 0 (included) and 4 (included) »***	
12	***« The sum is between 2 (included) and 10 (included) »***	
13	***« The sum is equal to 16 »***	
14	***« The sum is between 4 (included) and 14 (included) »***	
15	***« The sum is between 5 (included) and 13 (included) »***	
16	***« The sum is between 0 (included) and 14 (included) »***	
17	***« The sum is between 10 (included) and 18 (included) »***	
18	***« The sum is equal to 4 »***	
19	***« The sum is between 11 (included) and 17 (included) »***	
20	***« The sum is between 2 (included) and 6 (included) »***	

<u>Series 3 :</u>

The last series of 10 events is similar to the first series. The red die determines the first digit and the black die determines the second digit of a number between 1 and 100. The difference with the first series is that, when you select your choice number, you are not facing 1, but 2 possible events. For instance, the 1st of the 2 possible events could be ***«the number is between 1 (included) and 25 (included)»*** and the 2nd of the 2 possible events could be ***«the number is between 55 (included) and 59 (included)»***. You are asked to select a single choice number without knowing which of the 2 possible events will be used to determine your payment. It is only at the end of the experiment that we will toss a coin to identify which of the 2 possible events will be used for payment. If the coin lands on <u>**Heads**</u>, then your payment will be determined using the 1st event. If the coin lands on <u>**Tails**</u>, then your payment will be determined using the 2nd event. As with Series 1, we will then roll the 2 dice to determine whether the event identified by the coin toss occurs or not. Here is an example :

♦ If the coin lands on <u>**Heads**</u>, then the event is :

«the number is between 1 (included) and 25 (included)».

♦ <u>Or</u>, if the coin lands on <u>**Tails**</u>, then the event is :

«the number is between 55 (included) and 59 (included)».

You must select a unique choice number before you know which of the possible 2 events will be used for payment. Imagine for instance that an individual selects the choice number 70. We have to distinguish between different 2 situations to determine how much the individual will be paid:

♦ <u>Either</u> the coin tossed at the end of the experiment lands on <u>**Heads**</u>. In this case, the event used for payment is ***«the number is between 1 (included) and 25 (included)»***. Then, the event occurs if the 2 dice produce a number that is indeed between 1 (included) and 25 (included), and the individual in our example is paid 2,862FCFA. On the other hand, if the 2 dice produce a number that is not between 1 (included) and 25 (included), then the event does not occur and the individual in our example is paid 3,129FCFA.

♦ <u>Or</u> the coin tossed at the end of the experiment lands on <u>**Tails**</u>. In this case, the event identified is ***«the number is between 55 (included) and 59 (included)»***. Then, the event occurs if the 2 dice produce a number that is indeed between 55 (included) and 59 (included), and the individual in our example is paid 2,862FCFA. On the other hand, if the 2 dice produce a number that is between 55 (included) and 59 (included), then the event does not occur and the individual in our example is paid 3,129FCFA.

To summarize, there are only 2 cases under which the event occurs : 1) The coin lands on <u>**Heads**</u> and the 2 dice produce a number between 1 (included) and 25 (included), <u>or</u> 2) the coin lands on <u>**Tails**</u> and the 2 dice produce a number between 55 (included) and 59 (included). In all other cases, the event does not occur. Thus, when you select your choice number, you might want to imagine the different cases under which the event occurs and does not occur.

If these explanations are not sufficiently clear, please call us by <u>raising your hand</u>. We will then come to your desk to answer any questions you may have. We would like to remind you that it is important for you to understand the instructions well so that you can make the decisions that suit you the best.

Event	Description	Your Choice Number
21	♦ If the coin lands on **Heads**, then the event is : ***«the number is between 48 (included) and 82 (included)»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 14 (included) and 48 (included)».***	
22	♦ If the coin lands on **Heads**, then the event is : ***«the number is between 21 (included) and 35 (included)»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 30 (included) and 44 (included)».***	
23	♦ If the coin lands on **Heads**, then the event is : ***«the number is between 25 (included) and 89 (included)»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 2 (included) and 96 (included)».***	
24	♦ If the coin lands on **Heads**, then the event is : ***«the number is between 66 (included) and 97 (included)»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 13 (included) and 70 (included)».***	
25	♦ If the coin lands on **Heads**, then the event is : ***«the number is between 56 (included) and 58 (included)»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 78 (included) and 80 (included)».***	
26	♦ If the coin lands on **Heads**, then the event is : ***«the number is between 82 (included) and 89 (included)»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 25 (included) and 66 (included)».***	
27	♦ If the coin lands on **Heads**, then the event is : ***«the number is between 7 (included) and 88 (included)»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 3 (included) and 100 (included)».***	
28	♦ If the coin lands on **Heads**, then the event is : ***«the number is equal to 12»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 49 (included) and 57 (included)».***	
29	♦ If the coin lands on **Heads**, then the event is : ***«the number is between 26 (included) and 86 (included)»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 14 (included) and 74 (included)».***	
30	♦ If the coin lands on **Heads**, then the event is : ***«the number is between 1 (included) and 83 (included)»***. ♦ Or, if the coin lands on **Tails**, then the event is : ***«the number is between 36 (included) and 91 (included)».***	

Choice Number	Your Payment (in FCFA) when the Event		Choice Number	Your Payment (in FCFA) when the Event		Choice Number	Your Payment (in FCFA) when the Event	
	Occurs	**Does not Occur**		**Occurs**	**Does not Occur**		**Occurs**	**Does not Occur**
1	53	4000	51	2258	3538	101	3573	2186
2	106	3999	52	2293	3519	102	3590	2150
3	158	3998	53	2327	3501	103	3607	2114
4	210	3997	54	2362	3482	104	3624	2077
5	262	3996	55	2396	3462	105	3640	2040
6	314	3994	56	2429	3442	106	3656	2002
7	365	3991	57	2462	3422	107	3671	1965
8	415	3989	58	2495	3402	108	3686	1926
9	466	3986	59	2528	3381	109	3701	1888
10	516	3982	60	2560	3360	110	3716	1849
11	565	3978	61	2592	3338	111	3730	1810
12	614	3974	62	2623	3317	112	3743	1770
13	663	3970	63	2654	3294	113	3757	1730
14	712	3965	64	2685	3272	114	3770	1690
15	760	3960	65	2716	3249	115	3782	1649
16	808	3954	66	2746	3226	116	3794	1608
17	855	3949	67	2775	3202	117	3806	1566
18	902	3942	68	2805	3178	118	3818	1525
19	949	3936	69	2834	3154	119	3829	1482
20	996	3929	70	2862	3129	120	3840	1440
21	1042	3922	71	2890	3104	121	3850	1397
22	1087	3914	72	2918	3078	122	3861	1354
23	1133	3906	73	2946	3053	123	3870	1310
24	1178	3898	74	2973	3026	124	3880	1266
25	1222	3889	75	3000	3000	125	3889	1222
26	1266	3880	76	3026	2973	126	3898	1178
27	1310	3870	77	3053	2946	127	3906	1133
28	1354	3861	78	3078	2918	128	3914	1087
29	1397	3850	79	3104	2890	129	3922	1042
30	1440	3840	80	3129	2862	130	3929	996
31	1482	3829	81	3154	2834	131	3936	949
32	1525	3818	82	3178	2805	132	3942	902
33	1566	3806	83	3202	2775	133	3949	855
34	1608	3794	84	3226	2746	134	3954	808
35	1649	3782	85	3249	2716	135	3960	760
36	1690	3770	86	3272	2685	136	3965	712
37	1730	3757	87	3294	2654	137	3970	663
38	1770	3743	88	3317	2623	138	3974	614
39	1810	3730	89	3338	2592	139	3978	565
40	1849	3716	90	3360	2560	140	3982	516
41	1888	3701	91	3381	2528	141	3986	466
42	1926	3686	92	3402	2495	142	3989	415
43	1965	3671	93	3422	2462	143	3991	365
44	2002	3656	94	3442	2429	144	3994	314
45	2040	3640	95	3462	2396	145	3996	262
46	2077	3624	96	3482	2362	146	3997	210
47	2114	3607	97	3501	2327	147	3998	158
48	2150	3590	98	3519	2293	148	3999	106
49	2186	3573	99	3538	2258	149	4000	53
50	2222	3556	100	3556	2222			