Optimal growth when environmental quality is a research asset

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Abstract

We advance an original assumption according to which a good state of the environment positively affects the productivity of labor in R&D, so that a deteriorating environmental quality negatively impacts R&D. We study the implications of this assumption for the optimal solution in a model of growth based on R&D, where the use of a non-renewable resource generates pollution. It is shown that it is socially optimal to postpone extraction, as compared to the case with no effect of the environment on productivity in R&D. Moreover, to the extent that environmental quality first declines and later recovers, we find it is optimal to re-allocate employment to R&D as its productivity changes. If environmental quality recovers only partially from pollution, R&D effort optimally starts above its long run level, then progressively declines to a trough, and eventually increases to its steady state level.

Key words: Endogenous growth; Non-renewable resources; Environmental quality; Environmental hysteresis.
JEL codes: O40, Q30, Q50

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1 Introduction

We allow environmental quality to exert a positive effect on the productivity of labor in research and development (R&D) and study the implications of this assumption for the properties of the socially optimal dynamic path of the economy.

Our hypothesis is plausible since a clean and life-supporting environment is an essential factor for human activity in general. In this perspective the environment is an essential input for most creative economic activities and R&D in particular. In a number of sectors, ecosystems provide valuable services not only to production processes but also at the stage of design and conception. In the pharmaceutical industry, for instance, biodiversity is a crucial asset, source of inspiration, and provider of test opportunities (Craft and Simpson, 2001). In general, an environment in a stable state provides potential access to a wealth of information and of possibilities to test theories and improve both fundamental and applied research. Environmental degradation may limit this function of ecosystems. [2]

Despite its plausibility the assumption of a link from environmental quality to research productivity constitutes an original way of introducing environmental externalities in models of growth. Aside from the obvious externality on agents utility, most authors have considered the case when environmental quality (i.e. a measure of the stock of pollution) or polluting emissions affect total factor productivity in the aggregate production function (e.g. Bovenberg and Smulders, 1995, Groth and Schou, 2007). Some have explored alternative linkages, such as pollution being harmful to human capital accumulation

[2] This argument is based on the view according to which scientific understanding is most often based on repeated observation, thorough inspection and reflection, formulation of competing potential explanations and repeated testing. Most of these phases require substantial time, more so given that they are built up in a cumulative process.

(Gradus and Smulders, 1993, van Ewijk and van Wijnbergen, 1995).

With our approach we are able to contribute to the literature that studies models of growth with environmental constraints to emphasize the crucial role of R&D for allowing the economy to overcome the limits imposed by these constraints. For the case where natural inputs (polluting emissions or non renewable resources) are not essential inputs to R&D Aghion and Howitt (1998, ch.5) showed that trajectories where natural inputs are maintained constant or even decline are compatible with sustained growth. This is so because technological progress resulting from R&D can compensate for the ever ongoing dematerialization of production.[3] Such trajectories may emerge at equilibrium, with public intervention, in the form of environmental policy, necessary to control polluting emissions. In the context of mounting pressure for environmental protection, R&D experiences a boom for two reasons. First, the value of innovations increases to the extent that these are relatively clean (a demand pull effect) (e.g. Hart 2004, Ricci 2007). Second, the (relative) production costs fall as factors of production exit relatively dirty sectors to the benefit of R&D (a favorable cost shift effect) (e.g. Elbasha and Roe, 1996).

In our opinion an additional aspect should be considered: Environmental degradation may increase R&D costs (an unfavorable cost shift). According to the hypothesis that we advance, a worsening state of ecosystems will call for a re-allocation of R&D effort. Taking into account this additional cost shift effect on the R&D sector is a step toward a more comprehensive understanding of the complex consequences of environmental policy on innovation and technical change.

Our aim here is to clarify the role of the original assumption in a framework which is well understood for its other features (see Groth, 2007). For the sake

[3] In other terms, the rate of growth of total factor productivity must be higher than the growth rate of output and capital, and much higher than that of natural inputs (which is nil or negative).

of clarity, we prefer to consider this environmental externality in isolation. Thus we abstract from any direct effect of environmental quality on social welfare, since in this case there is additional room for intertemporal substitution (e.g. Michel and Rotillon, 1995). For the same reason, we also abstract from any direct effect of environmental quality on total factor productivity in manufacturing. Considering several environmental externalities at once is useful for the purpose of studying their interactions, a task left for future research.

The environment plays two distinct roles in the economic system of our model. First it provides material inputs to production. Accordingly we assume that a non-renewable natural resource is a necessary input in manufacturing. Second, environmental quality is supposed to be a necessary input in R&D. There is a trade-off between these two functions of the environment. The use of the natural resource adds to production and thus consumption. However it implies polluting emissions that stock up and worsen environmental quality. This impacts R&D negatively and thus potentially decreases economic growth and, in the end, future consumption.[4]

Given that the polluting natural resource is non-renewable, its use must ultimately decline and the flow of polluting emissions shrink. Environmental quality will eventually recover and approach some upper bound. Such an environmental Kuznets-curve suggests that there is scope for intertemporal substitution of R&D effort, leading to richer dynamics than in related literature (e.g. Schou 2000). In fact, as the polluting-exhaustible resource becomes increasingly scarce, limiting the pace of growth, the implied improvement in

[4] Closest to our study is the paper by van Ewijk and van Wijnbergen (1995). But whereas they consider human capital accumulation as the engine of growth and assume that pollution, as a flow, reduces the productivity of time devoted to education, we focus on non-rival knowledge as the growth engine and consider the damage from the stock of pollution. We also characterize the global dynamics, while they study comparative dynamics at steady state.

environmental quality can foster the growth rate, via its beneficial effect on the productivity of R&D.

In Section 2 we first set up the model and derive the necessary conditions for optimality. Next we examine the local and global dynamics of the implied dynamic system. We find that, as compared to the case where R&D is not directly affected by environmental quality, it is optimal to postpone extraction of the resource and that the optimal time path of R&D is non-monotonic.

In this first model it is assumed that the environment ultimately recovers all damages from pollution. We think that this assumption of asymptotic full recovery of environmental quality for its function as a research asset is defendable in general. [5] Admittedly, however, assuming full recovery is a particularly strong case. Concerning biodiversity, for instance, the loss of genes due to the extinction of an organism is not perfectly compensated by the emergence of new genes from surviving spices. Hence it would be preferable to model pollution as a definite loss of biodiversity (e.g. Goeschel and Swanson, 2002). In Section 3 we consider an extension of the basic framework, where we allow for only partial recovery in the long run of the damage from pollution on environmental quality. As a consequence hysteresis in environmental quality occurs. Apart from this, the results are qualitatively similar to those of the first model.

In the last section we summarize results, comment them and put them in perspective with respect to related literature.

[5] Consider for instance global warming and the services provided by ecosystems to researchers. This phenomenon is largely imputable to fossil use, a non renewable natural resource. For ecosystems it implies widespread migration phenomena of flora and fauna. Yet ecosystems should stabilize at some point in a different state. Probably during this transition, scientists will find it relatively more difficult to understand evolving ecosystems and use them to test theories. Hence carbon emissions provide productive services, but their accumulation implies a loss for the research sector, a loss which is temporary to the extent that ecosystems will eventually stabilize and provide as much informational services it used to do.

2 The model

Let L denote the constant size of population (and labor force). Consider the social planner's problem: choose $(L_{Yt}, R_t)_{t=0}^{\infty}$ so as to

$$\max U_0 = \int_0^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} L e^{-\rho t} dt \quad \text{s.t.} \tag{1}$$

$$c_t = Y_t/L = A_t^{\sigma} L_{Yt}^{\beta} R_t^{1-\beta}/L, \quad 0 \leq L_{Yt} \leq L, \quad R_t \geq 0, \tag{2}$$

$$\dot{A}_t = \gamma A_t E_t^{\varepsilon} (L - L_{Yt}), \quad A_t \geq 0, \quad A_0 > 0, \text{ given}, \tag{3}$$

$$\dot{S}_t = -R_t, \quad S_t \geq 0, \quad S_0 > 0 \text{ given}, \tag{4}$$

$$\dot{E}_t = b\left(\bar{E} - E_t\right) - aR_t, \quad 0 < E_t \leq \bar{E}, \quad E_0 \text{ given}, \tag{5}$$

where $L, \theta, \rho, \sigma, \gamma, \varepsilon, a, b,\ \bar{E} > 0$ and $\beta \in (0,1)$. The criterion function, (1), discounts future utility from per-capita consumption, c, by the rate of time preference, ρ. Production of a homogeneous manufacturing good, Y, employs two inputs: labor, L_Y, and a flow of an extracted resource, R, under constant returns to scale. Total factor productivity, A^{σ}, is increasing in the stock of technical knowledge, A, which grows through R&D according to (3).

The productivity of R&D is affected by two public goods: the stock of knowledge, proxied by cumulative R&D output, A, (see Romer, 1990); and the state of environmental quality, E. The latter formalizes our original assumption and the focus of our analysis.

The stock of the non-renewable resource is denoted by S and decreases over time, due to resource extraction, according to (4). Together with $S_t \geq 0$ this implies the restriction

$$\int_0^{\infty} R_t dt \leq S_0, . \tag{6}$$

Environmental quality evolves according to (5): it falls with extraction, R, and regenerates spontaneously at rate b. An ecological threshold, $E = 0$, exists which, if transgressed, implies disaster The maximum environmental quality is a given positive constant, $\bar{E}$. In the next section we study the case where this variable is negatively affected by pollution.

2.1 Dynamic system

Until further notice, all variables (but not growth rates) are assumed positive. We suppress explicit dating of the variables. Let $g_x \equiv \dot{x}/x$ denote the growth rate of any variable x.

The current-value Hamiltonian for problem (1)-(5) is

$$H = \frac{c^{1-\theta}-1}{1-\theta}L + \lambda_1 \gamma A E^{\varepsilon}(L - L_Y) - \lambda_2 R + \lambda_3 \left[b\left(\bar{E} - E\right) - aR\right],$$

where λ_1, λ_2, and λ_3 are the shadow prices of the state variables, A, S, and E, respectively. Necessary first-order conditions for an interior optimal solution are:

$$\frac{\partial H}{\partial L_Y} = c^{-\theta}\beta\frac{Y}{L_Y} - \lambda_1 \gamma A E^{\varepsilon} = 0, \tag{7}$$

$$\frac{\partial H}{\partial R} = c^{-\theta}(1-\beta)\frac{Y}{R} - \lambda_2 - a\lambda_3 = 0, \tag{8}$$

$$\frac{\partial H}{\partial A} = c^{-\theta}\sigma\frac{Y}{A} + \lambda_1 \gamma E^{\varepsilon}(L - L_Y) = \rho\lambda_1 - \dot{\lambda}_1, \tag{9}$$

$$\frac{\partial H}{\partial S} = 0 = \rho\lambda_2 - \dot{\lambda}_2, \tag{10}$$

$$\frac{\partial H}{\partial E} = \lambda_1 \varepsilon \frac{\dot{A}}{E} - \lambda_3 b = \rho\lambda_3 - \dot{\lambda}_3. \tag{11}$$

Defining $h \equiv \lambda_3/\lambda_2$ (the shadow price of environmental quality in terms of the resource) and $u \equiv R/S$ (the depletion rate), we can derive the following dynamic system from the optimality conditions (7)-(11) and equations (2)-(5):[6]

$$\dot{S} = -uS, \tag{12}$$

[6] For the detailed derivations see Appendix A.

$$\dot{h} = bh - \varepsilon \frac{\beta u S}{(1-\beta) L_Y E}(1+ah)(L - L_Y), \tag{13}$$

$$\dot{E} = b(\bar{E} - E) - auS, \tag{14}$$

$$\dot{u} = \Big\{ \theta u - (1-\theta)\,\beta\varepsilon b(\frac{\bar{E}}{E} - 1) + \left[\frac{1-\beta(1-\theta)}{1-\beta}\frac{L}{L_Y} - \frac{\theta}{1-\beta}\right]\beta\varepsilon a \frac{uS}{E} + (1-\theta)\,\sigma\gamma E^{\varepsilon} L - [1-\beta(1-\theta)]\,b\frac{ah}{1+ah} - \rho \Big\} \frac{u}{\theta}, \tag{15}$$

$$\dot{L}_Y = \Big\{ \theta \frac{\sigma\gamma}{\beta} E^{\varepsilon} L_Y - [\beta + (1-\beta)\theta]\,\varepsilon b(\frac{\bar{E}}{E} - 1) + \left[(1-\theta)\,\beta\frac{L}{L_Y} + \theta\right]\varepsilon a \frac{uS}{E} + (1-\theta)\,\sigma\gamma E^{\varepsilon} L - (1-\theta)\,(1-\beta) b \frac{ah}{1+ah} - \rho \Big\} \frac{L_Y}{\theta}. \tag{16}$$

Equations (12)-(16) constitute a five-dimensional dynamic system in S, h, E, u, and L_Y. There are two pre-determined variables, S and E, and three jump variables, L_Y, u, and h.

2.2 Optimal dynamics

A viable path (ensuring that $Y > 0$ for all t) is incompatible with a steady state. In fact constancy of E requires, by (5), $R = b(\bar{E} - E)/a$ constant, which contradicts (6) unless $R = 0$, thus $Y = 0$. We study instead a viable path that converges towards an *asymptotic steady state* (S^*, h^*, E^*, u^*, L_Y^*) for $t \to \infty$.

If the following parametric restriction is satisfied

$$(1-\theta)\sigma\gamma \bar{E}^{\varepsilon} L < \rho < \sigma\gamma \bar{E}^{\varepsilon} L, \tag{A}$$

the system admits an asymptotic steady state where

$$S^* = h^* = 0, \quad E^* = \bar{E}, \quad u^* = \frac{1}{\theta}\left[(\theta - 1)\sigma\gamma\bar{E}^{\varepsilon}L + \rho\right] > 0; \tag{17}$$

$$g_R^* = g_S^* = -u^* < 0, \tag{18}$$

$$L_Y^* = \frac{\beta}{\theta\sigma\gamma\bar{E}^{\varepsilon}}\left[(\theta - 1)\sigma\gamma\bar{E}^{\varepsilon}L + \rho\right] \in (0, L), \tag{19}$$

$$g_A^* = \frac{1}{\theta}\left\{[\beta + (1-\beta)\theta]\,\gamma\bar{E}^{\varepsilon}L - \frac{\beta}{\sigma}\rho\right\} > 0, \tag{20}$$

$$g_c^* = g_Y^* = \frac{1}{\theta}(\sigma\gamma\bar{E}^{\varepsilon}L - \rho) > 0. \tag{21}$$

Linearizing the system we find that the Jacobian matrix evaluated at the asymptotic steady state has two negative and three positive eigenvalues (see Appendix A). Hence, there exists a neighborhood of (S^*, E^*) such that when (S_0, E_0) belongs to this neighborhood, there is a unique path $(S_t, h_t, E_t, u_t, L_{Yt})$ converging towards the steady state.

To study the qualitative features of the global dynamics, we have run simulations for system (12)-(16) using the relaxation algorithm (Trimborn et al., 2008). Figure 1 shows results from a simulation, based on the following parameter values: $\theta = 2.5$, $L = 1.5$, $\sigma = 1$, $\beta = .8$, $\gamma = 1$, $S_0 = 4$, $E_0 = \bar{E}$ $= 1$, $a = .01$, $b = .01$, and $\rho = .02$. The qualitative features of the results hold for alternative values of parameters. The case with a productive role of E in R&D ($\varepsilon = .5$, on the right-hand panels) is compared with the case where labor productivity in R&D is independent of environmental quality ($\varepsilon = 0$, on the left-hand panels). The trajectories are represented in terms of percentage variation with respect to the steady state value of each variable.

As expected, resource depletion implies an environmental Kuznets curve, with an initial degradation of environmental quality followed by a recovery phase. Similar dynamics for environmental quality hold in the case $\varepsilon = 0$ (see top panels). But as indicated by the middle and bottom left-hand panels of Figure

1, this non-monotone evolution of E does not affect the optimal dynamics of control variables if $\varepsilon = 0$. When instead E is a productive asset in R&D, its non-monotone optimal path has implications for the optimal dynamics of the control variables u and L_Y as it can be seen from the middle and bottom right-hand panels of Figure 1.

First, notice from the middle panels of Figure 1 that with $\varepsilon > 0$, the resource depletion rate is persistently lower than in the case with $\varepsilon = 0$. This is due to extraction having a greater social cost when $\varepsilon > 0$. Not only does extraction now imply less resource availability in the future. It also lowers labor productivity in R&D, for given A. This optimal policy allows the economy to maintain environmental quality above the optimal level prevailing in the case with $\varepsilon = 0$.

Second, comparing the bottom panels of Figure 1 we see that the optimal R&D effort evolves non-monotonically over time if $\varepsilon > 0$, while it is constant when $\varepsilon = 0$. There are two contrasting forces behind the adjustment of R&D effort over time: on the one hand, there is an incentive to take advantage of research opportunities when they are favorable, and, on the other hand, there is a desire for consumption smoothing. In the case presented in the bottom right-hand panel of Figure 1, R&D employment falls when environmental quality falls, that is when the productivity of labor in R&D, for given A, is decreasing. Vice versa, when the environment recovers and the productivity of labor in R&D increases, the optimal policy progressively allocates more labor to R&D. This case where R&D effort and environmental quality are synchronized holds only for some patterns of parameters. Instead we find that the non-monotonicity of R&D effort is a general property of the optimal solution.

3 Permanent loss of environmental quality

In this section we consider the case where the damage from pollution on environmental quality is not fully recoverable. This case is interesting on its own sake and allows us to inspect the reasons for the non-monotonic dynamics of optimal employment in R&D.

In the previous analysis we assumed that the environment can recover from any damage of pollution, all the way to its pristine state, if polluting emissions indefinitely decline. This may be considered an unrealistic assumption, since pollution can exert permanent impacts on ecosystems. For instance in the case of biodiversity, if pollution causes a loss of genes as a consequence of the extinction of an organism, this loss can be considered as a definite loss. In the introduction we have argued in favor of our assumption since we should expect, even in this case, that new genetic variations of surviving organisms will eventually renew biodiversity. In this section we reconsider this assumption.

Let us first notice that assuming full recovery simplifies the analysis because the steady state level of environmental quality (and therefore of labor productivity in R&D) is exogenous. If instead the entire path of resource extraction, and therefore of pollution, determines the steady state level of environmental quality (and hence of research productivity for given level of technical knowledge, A), the system in our framework is characterized by hysteresis. We say that *hysteresis* in environmental quality occurs if environmental quality in the long run depends on environmental quality in the short run.

Here we extend the model by allowing pollution to exert a permanent negative effect on environmental quality. We find that there are cases in which the qualitative features of optimal paths of resource extraction and R&D employment are similar to those obtained in the simpler model. In particular R&D employment is adjusted intertemporally non-monotonically.

We introduce the concept of rest point for environmental quality, $\bar{E}$, defined as the level of environmental quality that prevails once the stock of pollution $(\bar{E} - E)$ vanishes. Although we use the same notation as in the model of Section 2, the rest point is not any longer the pristine state of environmental quality. To problem (1)-(5) we add the law of motion:

$$\dot{\bar{E}}_t = -m\left(\bar{E}_t - E_t\right) \quad \text{where } 0 < E_0 \leq \bar{E}_0 \text{ given} \tag{22}$$

and $m > 0$ is a parameter. With this specification the meaning of pollution stock is modified. When the rest point was exogenous and constant, the pollution stock measured simultaneously the cumulative damage imposed on the environment and the potential for recovery once polluting emissions stop. But now the dynamics of the ecosystem are given by (5) and (22), so that the evolution of the pollution stock is given by

$$\frac{d\left(\bar{E} - E\right)}{dt} = -\left(m + b\right)\left(\bar{E}_t - E_t\right) + aR_t \tag{23}$$

In the absence of emissions the pollution stock $\left(\bar{E} - E\right)$ falls both because environmental quality E recovers and because the rest point $\bar{E}$ worsens.

The top panels of Figure 2 depict a possible trajectory showing non-monotone dynamics of environmental quality (dubbed Environmental Kuznets Curve, EKC). The trajectory is drawn in the $\left(\bar{E} - E, R\right)$ plane on the top left panel and in the $\left(E, \bar{E}\right)$ plane on the top right panel. The EKC trajectory prevails in a system characterized by a high regeneration-damage ratio (b/a) and a small permanent impact of pollution on the rest point (m).

We have simulated the global dynamics for the system obtained from the first order and Euler conditions of the extended model.[7] It is easy to find paths along which the EKC emerges. In all the cases with EKC we have identified,

[7] See Appendix B for details.

the optimal intertemporal allocation of labor to R&D varies non monotonically. Figure 3 shows results obtained for parameter values: $\varepsilon = .5$, $\theta = 2.5$, $L = 1.5$, $\sigma = 1$, $\beta = .8$, $\gamma = 1$, $S_0 = .25$, $E_0 = \bar{E}_0 = 1$, $a = .01$, $b = .01$, $m = .01$ and $\rho = .02$. The trajectories are drawn in terms of percentage variation with respect to the steady state value of each variable. On the left-hand panel the continuous line depicts the trajectory of environmental quality E, and the dashed one that of its rest point $\bar{E}$. The right-hand panel illustrates optimal R&D employment $(L - L_Y)$. The qualitative similarity with the trajectories that emerge when $m = 0$ is striking. In particular R&D effort evolves non-monotonically and reflects the evolution of environmental quality, which follows an EKC. Interestingly, however, the optimal R&D effort is initially higher than in the steady state. This comes at no surprise since labor productivity in R&D (for given A) is greater at the initial state than in the long run, precisely because the pollution indefinitely reduces the rest point of environmental quality.

When $m > 0$, environmental quality (and hence labor productivity in R&D for given A) is permanently lower than in the baseline case with $m = 0$. To the purpose of comparing these two cases, let us first notice that in our model there isn't any direct disutility of pollution and deteriorating environmental quality. Hence, the relevant change in going from an economy where $m = 0$ to one where $m > 0$ is that the production possibility set shrinks, in the sense that the economy is confronted to an increasingly stringent constraint on the productivity of its factors of production.

Finally, in the case of large permanent consequences of pollution on environmental quality (i.e. high m) and of a small regeneration-damage ratio (b/a) the optimal trajectories can be characterized by monotonically declining environmental quality. The bottom panels of Figure 2 depict this case. Figure 4 shows simulations obtained for three different values of parameter m: .001, .1 and 1 (holding other parameters as those used for Figure 3). Inspecting the left-hand panel of Figure 4 one sees that in the case of a strong permanent impact ($m = 1$) the rest point of environmental quality follows closely the deterioration of environmental quality (on printed scale the two schedules seem to coincide). In practice there is no scope for the environment to recover once polluting emissions dwindle away. This is not the case instead for more moderate values of the permanent impact factor (m), when the rest point approaches environmental quality with a lag, and therefore the latter can recover, giving rise to an EKC.

Turning to the right-hand panel of Figure 4, we find that, as the permanent effect of pollution increases, the intertemporal adjustment in R&D employment is tilted toward earlier dates, when the labor productivity in R&D is highest, and away from the doomed distant future. We conclude that the non-monotonicity of the optimal intertemporal adjustment of R&D employment reflects the renewable nature of environmental quality as a production input (in our case as a determinant of labor productivity in R&D).

4 Conclusion

The objective of our analysis is understanding the consequences of the original externality that is introduced by this paper: the positive effect that environmental quality may exert on the productivity of factors of production in the R&D sector. We find that sustained economic growth is feasible and optimal, in the case where the services from the environment to R&D are modeled as a renewable resource. The presence of environmental quality as a research asset affects the optimal policy. First, the rate of extraction of the polluting resource should be relatively low during the entire adjustment period. In fact, the social cost of extraction is ceteris paribus greater, since the implied pollution persistently lowers the productivity of R&D. Second, R&D effort should evolve non-monotonically. As resource exploitation implies first a deterioration and then a recovery of environmental quality, R&D effort adapts to the changes in labor productivity in the R&D sector. If there is only partial recovery in the long run of the damage from pollution on environmental quality, environmental hysteresis occurs. In this case, the permanent degradation of environmental quality constitutes a drag on the productivity of labor in R&D, and therefore on growth, in the long run.

We do not present equilibrium analysis for a decentralized economy, whose aggregate representation coincides with the one considered in Section 2 (or 3). However we can make here a simple point concerning the dynamics of the optimal rate of taxation on extraction (or polluting emissions). Let us assume that the tax rate $(\tau - 1)$ is levied on each unit sold in a competitive sector extracting the non-renewable resources out of privately owned stocks. The competitive final sector purchases resources according to its inverse demand function $p_R = Y_R/\tau$, where $Y_R \equiv \partial Y/\partial R$. Each extracting firm maximizes the present value of revenues net of taxes (extraction costs are nil), so that the revenue net of taxes per unit extracted satisfies the *Hotelling rule* $g_{p_R} = r$, where r is the interest rate. Together these two equilibrium conditions give: $r = g_{Y_R} - g_\tau$. We can obtain the corresponding condition from the social optimum problem (1)-(5). Compute first the optimum interest rate as $r = \rho + \theta g_c$ where g_c is optimal. Use next conditions (8), (10)-(11), and notation Y_R, to get $r = g_{Y_R} - a\left[b\lambda_3 - \lambda_1 \varepsilon \gamma A E^{\varepsilon-1}\left(L - L_Y\right)\right] / \left(\lambda_2 + a\lambda_3\right)$. If public intervention takes care of other distortions through appropriate policy instruments,[8] the tax rate on extraction is targeted in such a way that the time path of environmental quality at equilibrium coincides with the optimal one (first best policy). From direct comparison of the two conditions obtained, we see that the optimal tax

[8] In models of decentralized economies resulting in the aggregate representation that we analyze, it is usually assumed that there is a monopolistic market for intermediate goods, and a positive intertemporal externality in knowledge accumulation in R&D (see Romer, 1990, for the case without natural resources). Two policy instruments can be used in order to correct these two market imperfections: a subsidy for purchasing intermediate inputs, and a subsidy for employing labor in R&D.

rate on extraction (or polluting emissions) should satisfy:

$$g_{\tau} = \frac{ah}{1+ah}\left[b - \varepsilon\gamma A E^{\varepsilon-1}\left(L - L_Y\right)\frac{\lambda_1}{\lambda_3}\right]$$

where the endogenous variables $h \equiv \lambda_3/\lambda_2$, λ_1/λ_3, A, E and L_Y evolve along the optimal solution. Our original assumption materializes in the second term in brackets on the right-hand side of the expression, which reflects the marginal contribution of environmental quality to the accumulation of technical knowledge. We can see the direct impact on the optimal tax of the externality, as measured by parameter ε. The greater is the externality parameter ε, the faster should the tax rate fall everything else equal. Faced with a falling tax rate the firms tend to delay extraction.

Our assumption can therefore bring new insights into the ongoing debate on the timing of an optimal environmental policy, when the polluting resource is non renewable. Since the early contribution of Ulph and Ulph (1994), this debate has shifted its focus on the implications of environmental policy on technological change. Although our model takes this into account, since technological progress is endogenous, the topical issues debated hinge on how environmental policy may alter the direction of technical change. Typically the optimal policy is characterized in terms of a portfolio of investments in R&D, each targeted to improve the productivity of a specific natural resource, be it renewable or nonrenewable (Di Maria and Valente, 2008, Grimaud and Rougé 2009, Acemoglu et al. 2009). In particular, Sinn (2008), Hoel (2008) and Chakravorty et al. (2010) underscore the potential unintentional consequences of the timing of environmental policy: even though its aim is to initially reduce polluting emissions, it may cause more resources to be extracted earlier on because of the endogenous response by resource owners. Our results indicate that by taking the indirect harmful impact on R&D productivity into account, the costs of such an outcome may be higher than those pointed to by previous analysis. Admittedly this is a tentative conclusion, since our model differs crucially from those developed in this latter strand of literature, notably because the non-renewable resource is essential for production and the direction of technological progress in exogenous.

References

Acemoglu, D., P. Aghion, L. Burstyn and D. Hemous, 2009,The Environment and Directed Technical Change. NBER working paper n.15451, Washington DC.

Aghion, P. and P. Howitt, 1998, *Endogenous Growth Theory* (The MIT Press, Cambridge MA).

d'Albis, H., P. Gourdel and C. Le Van (2008), Existence of Solutions in Continuous-time Optimal Growth Models, *Economic Theory* 37, 321-333.

Bovenberg, A.L., and S. Smulders, 1995, Environmental Quality and Pollution-Augmenting Technological Change in a Two-Sector Endogenous Growth Model, *Journal of Public Economics* 57, 369–391.

Chakravorty, U., A. Leach and M. Moreaux, 2010, Would Hotelling Kill the Electric Car?, LERNA working paper 10.08.314, Toulouse.

Craft, A. and D. Simpson, 2001, The Value of Biodiversity in Pharmaceutical Research with Differentiated Products, *Environmental and Resource Economics* 18, 1-17.

Di Maria, C. and S. Valente, 2008, The Direction of Technical Change in Capital-Resource Economies. *Environment and Development Economics* 13, .

Elbasha E, and T. Roe, 1996, On Endogenous Growth: The Implications of Environmental Externalities, *Journal of Environmental Economics and Management* 31, 240-268.,

Goeschl, T. and T. Swanson, 2002, The Social Value of Biodiversity for R&D, *Environmental and Resource Economics* 22, 477-504.

Gradus, R. and S. Smulders, 1993, The Trade-off Between Environmental Care and Long-term Growth: Pollution in Three Prototype Growth Models, *Journal of Economics* 58, 25-51.

Grimaud, A. and L. Rougé, 2008, Environment, Directed Technical Change and Economic Policy, *Environmental and Resource Economics* 41, 439-463.

Groth, C., 2007, A New-Growth Perspective on Non-renewable Resources. In L. Bretschger and S. Smulders (Ed.s) *Sustainable Resource Use and Economic Dynamics*, Springer (Dordrecht).

Groth, C. and P. Schou, 2007, Growth and non-renewable resources: The different roles of capital and resource taxes, *Journal of Environmental Economics and Management* 53, 80-98.

Hart, R., 2004, Growth, Environment and Innovation: A Model with Production Vintages and Environmentally Oriented Research, *Journal of Environmental Economics and Management* 48, 1078-1098.

Hoeal, M. 2008, Bush Meets Hotelling: Effects of Improved Renewable Energy Technology on Greenhouse Gas Emissions, Memorandum n.29, Department of Economics, University of Oslo.

Michel, P. and G. Rotillon, 1995, Disutility of Pollution and Endogenous Growth, *Environmental and Resource Economics* 6, 279-300.

Romer, P., 1990, Endogenous Technological Change, *Journal of Political Economy*, 98, S71-102.

Ricci, F., 2007, Environmental Policy and Growth when Inputs Are Differentiated in Pollution Intensity, *Environmental and Resource Economics* 38, 285-310.

Schou, P., 2000, Polluting Non-Renewable Resources and Growth, *Environmental and Resource Economics*, 16, 211-227.

Seierstad, A. and K. Sydsaeter, 1987, *Optimal Control Theory with Economic Applications* (North-Holland, Amsterdam).

Sinn, H-W, 2008, Public policies against global warming: a supply side approach, *International Tax and Public Finance* 15, 360–394

Trimborn, T., K-J. Koch and T. Steger, 2008, Multidimensional transitional dynamics: A simple numerical procedure, *Macroeconomic Dynamics* 12, 301-319.

Ulph, A. and D. Ulph, 1994, The Optimal Time Path of a Carbon Tax, *Oxford Economic Papers* 46, 857-868.

van Ewijk, C. and S. van Wijnbergen, 1995, Can Abatement Overcome the Conflict Between Environment and Economic Growth?, *De Economist* 143, 197-216.

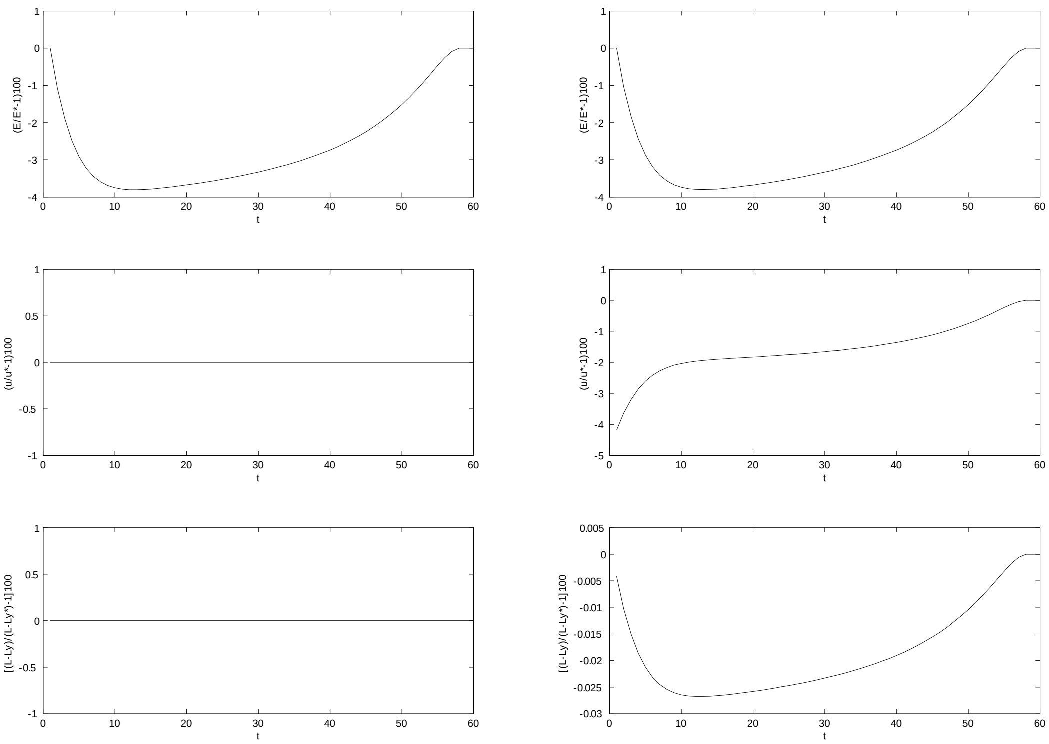

Fig. 1: Optimal time paths of environmental quality (E), extraction (u), and R&D ($L - L_Y$) (in % variation from steady state): case $\varepsilon = 0$ left-hand panels; case $\varepsilon = .5$ right-hand panels.

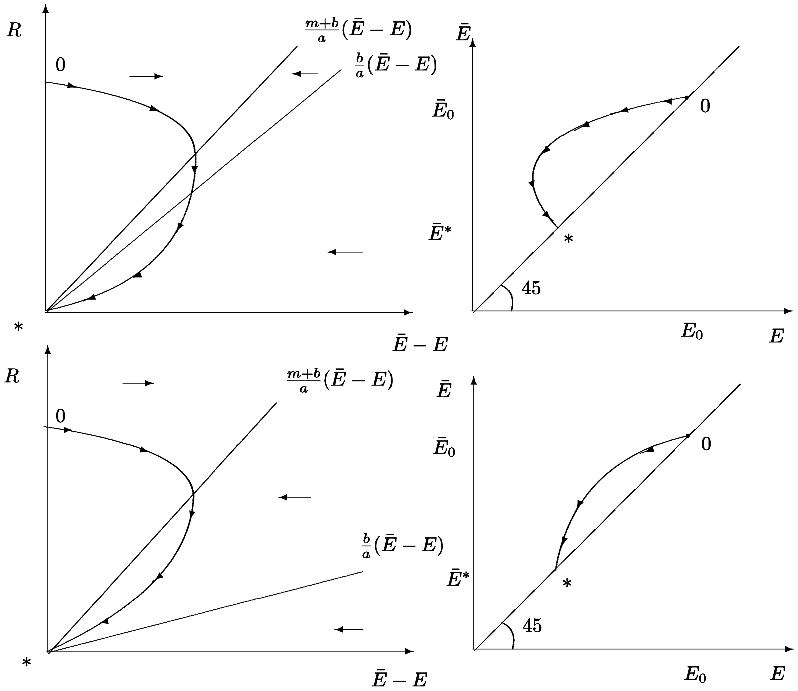

Fig. 2: Permanent loss in environmental quality: trajectories in phase diagrams.

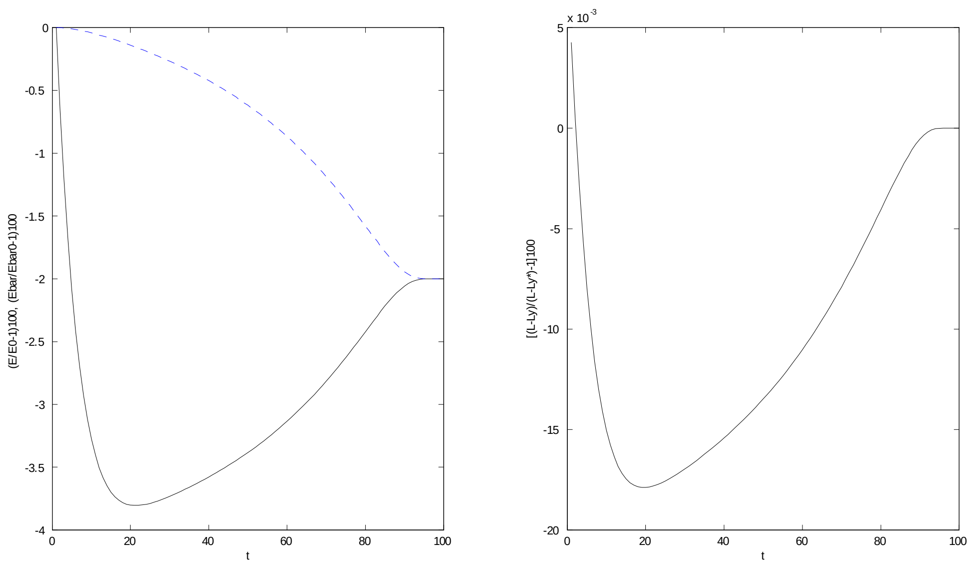

Fig. 3: Permanent loss : optimal paths of E (continuous curve), $\bar{E}$ (dashed curve) and $L - L_Y$ (in % variations from steady state).

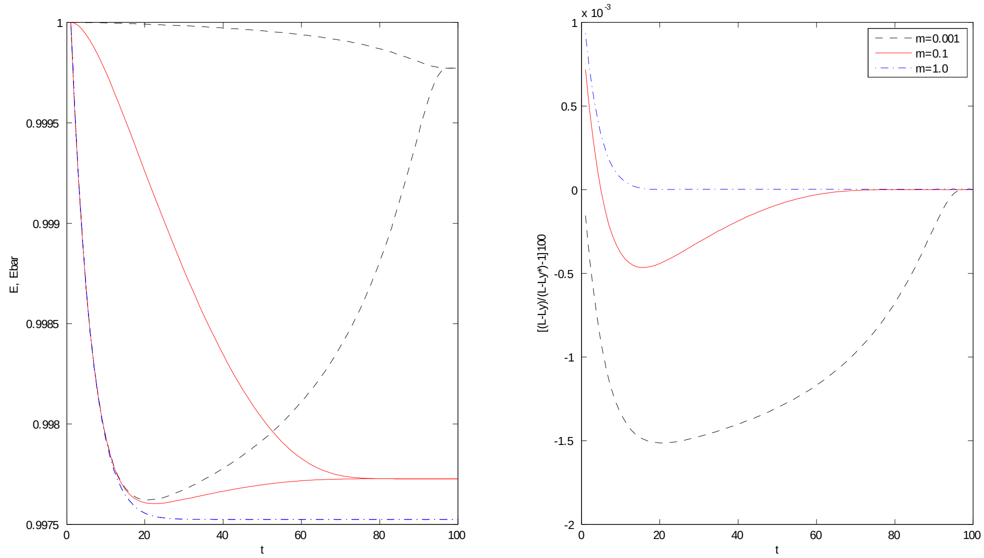

Fig. 4: Role of m: optimal paths of E, $\bar{E}$ (in absolute levels on the left-hand panel, $\bar{E}$ schedule above or merged with the corresponding E schedule) and $L - L_Y$ (in % variations from steady state on the right-hand panel) for three different values of parameter m: $m = .001$ (dashed curve), $m = .1$ (continuous curve), $m = 1$ (dashed-dotted curve).

Appendix A

This appendix contains the detailed derivation of the results presented in Section 2. We first show how the dynamic system (12)-(16) is derived, next we consider the asymptotic steady state, and then the linearization of the system around the steady state in order to study the local dynamics. Finally we address the question of how to establish that our candidate for an optimal solution, the unique converging path, is in fact optimal.

Dynamic system. Two growth accounting conditions obtained from the model are useful. First, (2) implies

$$g_c = g_Y = \sigma g_A + \beta g_{L_Y} + (1-\beta)\, g_R. \tag{24}$$

Second, (3) gives

$$g_A = \gamma E^{\varepsilon}(L - L_Y). \tag{25}$$

Ordering (7) and log-differentiating wrt. time, using $g_c = g_Y$, gives

$$(1-\theta) g_Y - g_{L_Y} = g_{\lambda_1} + \varepsilon g_E + g_A, \tag{26}$$

Ordering (9) yields

$$g_{\lambda_1} = \rho - c^{-\theta}\sigma \frac{Y}{\lambda_1 A} - \gamma E^{\varepsilon}(L - L_Y) = \rho - \frac{\sigma\gamma E^{\varepsilon} L_Y}{\beta} - g_A, \tag{27}$$

by (7) and (25). Now substitute (27) into (26) to get

$$g_{L_Y} = (1-\theta) g_Y - \rho + \frac{\sigma\gamma E^{\varepsilon} L_Y}{\beta} - \varepsilon g_E. \tag{28}$$

Combining (7) and (8) gives

$$\frac{(1-\beta)\, L_Y}{\beta R} = \frac{\lambda_2 + a\lambda_3}{\lambda_1 \gamma A E^{\varepsilon}} = \frac{1 + ah}{\frac{\lambda_1}{\lambda_2}\gamma A E^{\varepsilon}}. \tag{29}$$

Log-differentiating (29) wrt. time and ordering, using (27) and (10), leads to

$$g_R = g_{L_Y} - \frac{\sigma\gamma E^{\varepsilon} L_Y}{\beta} + \varepsilon g_E - \frac{a}{1+ah}\dot{h}. \tag{30}$$

Considering the stock value ratio $\lambda_1 A/(\lambda_3 E)$, we have

$$\frac{\lambda_1 A}{\lambda_3 E} \equiv \frac{\frac{\lambda_1}{\lambda_2} A}{hE} = \frac{\beta R(1+ah)}{(1-\beta) L_Y h E \gamma E^{\varepsilon}}, \tag{31}$$

in view of (29). Using $R \equiv uS$, (4), and (5) immediately yield (12) and (14), respectively.

By (10) and (11),

$$g_h = g_{\lambda_3} - g_{\lambda_2} = b - \varepsilon \frac{\lambda_1 A}{\lambda_3 E} g_A = b - \varepsilon \frac{\beta R(1+ah)}{(1-\beta)\, L_Y h E}(L - L_Y), \tag{32}$$

in view of (31) and (25). This explains (13). From (24) and (30),

$$g_Y = \sigma g_A + g_{L_Y} + (1-\beta)\left(-\frac{\sigma\gamma E^{\varepsilon} L_Y}{\beta} + \varepsilon g_E - \frac{a}{1+ah}\dot{h}\right). \tag{33}$$

Substituting this into (28) yields

$$\begin{aligned}
g_{L_Y} =& (1-\theta)\left[\sigma g_A + g_{L_Y} + (1-\beta)\left(-\frac{\sigma\gamma}{\beta}E^{\varepsilon}L_Y + \varepsilon g_E - \frac{a}{1+ah}\dot{h}\right)\right] \\
&-\rho + \frac{\sigma\gamma}{\beta}E^{\varepsilon}L_Y - \varepsilon g_E \\
=& (1-\theta)\left[\sigma\gamma E^{\varepsilon}(L-L_Y) + g_{L_Y} - \frac{\sigma}{\beta}\gamma E^{\varepsilon}L_Y + \sigma\gamma E^{\varepsilon}L_Y\right. \\
&\left. + (1-\beta)\left(\varepsilon g_E - \frac{a}{1+ah}\dot{h}\right)\right] - \rho + \frac{\sigma\gamma E^{\varepsilon}L_Y}{\beta} - \varepsilon g_E \qquad \text{(by (25))} \\
=& (1-\theta)\left[\sigma\gamma E^{\varepsilon}L + g_{L_Y} - (1-\beta)\frac{a}{1+ah}\dot{h}\right] - \rho + \frac{\theta\sigma\gamma}{\beta}E^{\varepsilon}L_Y \\
&+ \left[(1-\theta)(1-\beta) - 1\right]\varepsilon g_E.
\end{aligned}$$

Solving for g_{L_Y} gives

$$\begin{aligned}
g_{L_Y} =& \frac{1}{\theta}\left\{(1-\theta)\left[\sigma\gamma E^{\varepsilon}L - (1-\beta)\frac{a}{1+ah}\dot{h}\right]\right. \\
&\left. -\rho + \frac{\theta\sigma\gamma}{\beta}E^{\varepsilon}L_Y - \left[\beta + \theta(1-\beta)\right]\varepsilon g_E\right\}.
\end{aligned} \tag{34}$$

Log-differentiating $u \equiv R/S$ wrt. t gives

$$\begin{aligned}
g_u =& g_R - g_S = g_R + u = g_{L_Y} - \frac{\sigma\gamma E^{\varepsilon}L_Y}{\beta} + \varepsilon g_E - \frac{a}{1+ah}\dot{h} + u \qquad \text{(from (30))} \\
=& \frac{\sigma\gamma}{\beta}E^{\varepsilon}L_Y - (\beta/\theta + 1 - \beta)\varepsilon g_E + \frac{1-\theta}{\theta}\sigma\gamma E^{\varepsilon}L - \frac{1-\theta}{\theta}(1-\beta)\frac{a}{1+ah}\dot{h} - \frac{\rho}{\theta} \\
&-\frac{\sigma\gamma}{\beta}E^{\varepsilon}L_Y + \varepsilon g_E - \frac{a}{1+ah}\dot{h} + u \qquad \text{(from (34))} \\
=& u - (\beta/\theta - \beta)\varepsilon g_E + \frac{1-\theta}{\theta}\sigma\gamma E^{\varepsilon}L - (\frac{1-\theta}{\theta}(1-\beta) + 1)\frac{a}{1+ah}\dot{h} - \frac{\rho}{\theta},
\end{aligned}$$

from which follows

$$\dot{u} = \left(u - \frac{1-\theta}{\theta}\beta\varepsilon g_E + \frac{1-\theta}{\theta}\sigma\gamma E^{\varepsilon} L - \frac{1-\beta(1-\theta)}{\theta}\frac{a}{1+ah}\dot{h} - \frac{\rho}{\theta} \right) u.$$

Taking into account (13) and (14) this can be written as (15). Finally, (34) can be written

$$\dot{L}_Y = \left[\frac{\sigma\gamma}{\beta} E^{\varepsilon} L_Y - (\frac{\beta}{\theta} + 1 - \beta)\varepsilon g_E + \frac{1-\theta}{\theta}\sigma\gamma E^{\varepsilon} L \right. \\ \left. - \frac{1-\theta}{\theta}(1-\beta)\frac{a}{1+ah}\dot{h} - \frac{\rho}{\theta} \right] L_Y.$$

Taking into account (13) and (14) one obtains (16).

Asymptotic steady state. By the parameter restriction (A) follows $u^* > 0$, and so the asymptotic steady state has $S^* = 0$, in view of (12). Since $S^* = 0$, $\dot{h} = 0$ requires $h^* = 0$, in view of (13), and $\dot{E} = 0$ requires $E^* = \bar{E}$ according to (14). The remainder of (17) follows from (15). Further, by (16), L_Y^* must satisfy

$$\frac{\sigma\gamma}{\beta}\bar{E}^{\varepsilon} L_Y^* = \frac{1}{\theta}\left[(\theta - 1)\sigma\gamma\bar{E}^{\varepsilon} L + \rho \right] = u^*. \tag{35}$$

This can be rearranged, using (17), to obtain (19). Given that u^* is constant, (18) follows from (12). Then, by (24), (17), (19), and (18) we get

$$\begin{aligned} g_c^* = g_Y^* &= \sigma\gamma\bar{E}^{\varepsilon}(L - L_Y^*) + (1-\beta)\, g_R^* = \sigma\gamma\bar{E}^{\varepsilon}(L - L_Y^*) - (1-\beta)\, u^* \\ &= \sigma\gamma\bar{E}^{\varepsilon} L - \sigma\gamma\bar{E}^{\varepsilon} L_Y^* - (1-\beta)\, u^* = \sigma\gamma\bar{E}^{\varepsilon} L - u^* \\ &= \sigma\gamma\bar{E}^{\varepsilon} L - \frac{1}{\theta}\left[(\theta - 1)\sigma\gamma\bar{E}^{\varepsilon} L + \rho \right], \end{aligned}$$

which can be reduced to (21). Finally, (20) is obtained using (19) in (25).

Linearization. The system can be approximated around the asymptotic steady state by a linearized system. The Jacobian matrix of the system (12)-(16), evaluated at the asymptotic steady state, is given by

$$\begin{array}{cccccc} & S & h & E & u & L_Y \\ \dot{S} & -u^* & 0 & 0 & 0 & 0 \\ \dot{h} & -(L - L_Y^*)\frac{\varepsilon\beta u^*}{(1-\beta)L_Y^*\bar{E}} & b & 0 & 0 & 0 \\ \dot{E} & -au^* & 0 & -b & 0 & 0 \\ \dot{u} & \{[1-\beta(1-\theta)]\, L - \theta L_Y^*\}\, \frac{\varepsilon\beta a u^{*2}}{(1-\beta)\theta\bar{E}L_Y^*} & -\left[1-\beta(1-\theta)\right]\frac{bau^*}{\theta} & j_{43} & u^* & 0 \\ \dot{L}_Y & \left[\beta\,(1-\theta)\, L + \theta L_Y^*\right]\frac{\varepsilon a u^*}{\bar{E}\theta} & -\frac{1-\theta}{\theta}(1-\beta)baL_Y^* & j_{53} & 0 & u^* \end{array}$$

where $j_{43} = \frac{1-\theta}{\theta}\left(\beta b + \sigma\gamma\bar{E}^{\varepsilon}L\right)\frac{\varepsilon u^*}{\bar{E}}$ and $j_{53} = \{[\beta + (1-\beta)\theta]\,\beta b + [\beta\,(1-\theta)\,L$ $+\,\theta L_Y^*]\,\sigma\gamma\bar{E}^{\varepsilon}\}\,\frac{\varepsilon L_Y^*}{\beta\theta\bar{E}}$.

We see the Jacobian matrix is triangular so that the eigenvalues are the entries in the main diagonal. Two eigenvalues are negative and three are positive. This corresponds to the number of pre-determined variables (S and E) and jump variables (h, u, and L_Y), respectively.[9] Yet, since the linearized system is recursive, one should check whether also each of the subsystems in the causal ordering has a number of negative eigenvalues equal to the number of predetermined variables in that subsystem. Inspection of the Jacobian shows this to be the case. Thus, there exists a neighborhood of (S^*, E^*) such that when (S_0, E_0) belongs to this neighborhood, there is a unique path $(S_t, h_t, E_t, u_t, L_{Yt})$ converging towards the steady state.

Checking sufficient conditions. The transversality conditions of problem (1)-(5) are given by

$$\lim_{t\to\infty} \lambda_{1t}A_t e^{-\rho t} = 0, \qquad \text{(TVC1)}$$

$$\lim_{t\to\infty} \lambda_{2t}S_t e^{-\rho t} = 0, \qquad \text{(TVC2)}$$

$$\lim_{t\to\infty} \lambda_{3t}(\bar{E} - E_t)e^{-\rho t} \geq 0. \qquad \text{(TVC3)}$$

Indeed, along the converging path, $\lambda_1 A e^{-\rho t}$ grows ultimately at the rate

$$g_{\lambda_1} + g_A^* - \rho = -\frac{\sigma\gamma}{\beta}\bar{E}^{\varepsilon}L_Y^* < 0,$$

by (27). Thus, the first transversality condition is satisfied. Along the converging path the second transversality condition also holds since $\lambda_2 S e^{-\rho t}$ grows ultimately at the rate

$$g_{\lambda_2} + g_S^* - \rho = -u^* < 0,$$

by (10), (12) and (A). The third transversality condition is stated in a more general (and less common) form than the two others. This is because, seemingly, we cannot be sure that our candidate solution satisfies the more demanding condition $\lim_{t\to\infty} \lambda_{3t}E_t e^{-\rho t} = 0$. On the other hand, (TVC3) definitely holds, since $E_t \leq \bar{E}$ and $\lambda_{3t} > 0$ (and this is sufficient for our present purpose).

If only the maximized Hamiltonian were jointly concave in (A, E), our candidate solution would now satisfy a set of sufficient conditions for optimality according to Arrow's sufficiency theorem (Seierstad and Sydsaeter, 1987 pp. 235-36). Unfortunately, however, the maximized Hamiltonian is not jointly concave in (A, E). Indeed, the maximized Hamiltonian is

[9] Interestingly, the eigenvalues appear in a symmetric way. In a pairwise manner they are of the same absolute size, but with opposite signs.

$$\hat{H}(A,S,E,\lambda_1,\lambda_2,\lambda_3,t) = \max_{L_Y,R} H(A,S,E,L_Y,R,\lambda_1,\lambda_2,\lambda_3,t)$$
$$= C_1 A^{-\frac{\beta(1-\theta)}{\theta}} E^{-\varepsilon\frac{\beta(1-\theta)}{\theta}} + \lambda_1\gamma L A E^{\varepsilon}$$
$$-C_2 A^{-\left[\frac{\beta(1-\theta)}{\theta}+\sigma\right]} E^{-\varepsilon\frac{\beta(1-\theta)}{\theta}} - C_3,$$

where C_1, C_2, and C_3 are positive coefficients not depending on A or E. We know the function $f(x,y) = x^{\alpha}y^{\beta}$ is concave if and only if

$$0 \leq \alpha \leq 1, \tag{36}$$
$$0 \leq \beta \leq 1, \quad \text{and} \tag{37}$$
$$\alpha + \beta \leq 1. \tag{38}$$

Thus, we come closest to concavity if $\theta = 1$. But even then, the term $\lambda_1 LAE^{\varepsilon}$ implies lack of joint concavity in (A,E). We therefore need to go via *existence* of an optimal solution.

Existence of an optimal solution. Given the parametric restriction (A), we can establish existence of an optimal solution by appealing to the existence theorem of d'Albis et al. (2008). To apply this theorem, consider c and R as control variables and substitute $L_Y = A^{-\sigma/\beta}c^{1/\beta}R^{-(1-\beta)/\beta}$ into (3). Then the required joint concavity in the control variables in the integrand of the integral in (1) as well as the right-hand sides of (3), (4), and (5) is satisfied. And given (A), $\rho > (1-\theta)g_c^*$ holds and so the utility integral U_0 is bounded from above. As an implication, an optimal solution exists. Above we found that among the dynamic paths satisfying the necessary first-order conditions, there is only one converging path, all other paths being divergent. This leaves us with the converging path as the unique optimal solution.

Appendix B

In this appendix we derive the dynamic system that is simulated in Figure 3-4. Taking into account (22) the current-value Hamiltonian for the extended problem is

$$H = \frac{c^{1-\theta}-1}{1-\theta}L + \lambda_1\gamma A E^{\varepsilon}(L-L_Y) - \lambda_2 R + \lambda_3\left[b\left(\bar{E}-E\right) - aR\right] + \lambda_4 m\left(E-\bar{E}\right),$$

which implies (7)-(10) and the following:

$$\frac{\partial H}{\partial E} = \lambda_1\varepsilon\frac{\dot{A}}{E} - \lambda_3 b + \lambda_4 m = \rho\lambda_3 - \dot{\lambda}_3, \tag{39}$$

$$\frac{\partial H}{\partial \bar{E}} = \lambda_3 b - \lambda_4 m = \rho\lambda_4 - \dot{\lambda}_4. \tag{40}$$

Defining $\eta \equiv \lambda_3/\lambda_4$, we can derive the following dynamic system from the optimality conditions (7)-(10), (39)-(40) and equations (2)-(5), (22):

$$\dot{S} = -uS$$

$$\dot{E} = b(\bar{E} - E) - auS$$

$$\dot{\bar{E}} = m\left(E - \bar{E}\right)$$

$$\dot{h} = bh - \varepsilon\frac{\beta uS}{(1-\beta)L_Y E}(1+ah)(L - L_Y) - m\frac{h}{\eta}$$

$$\dot{\eta} = (1+\eta)(b\eta - m) - \varepsilon\frac{\beta}{1-\beta}\frac{uS}{EL_Y}(L - L_Y)(1+ah)\frac{\eta}{h}$$

$$\dot{u} = \left\{\theta u - (1-\theta)\beta\varepsilon b(\frac{\bar{E}}{E} - 1) + \left[\frac{1-\beta(1-\theta)}{1-\beta}\frac{L}{L_Y} - \frac{\theta}{1-\beta}\right]\beta\varepsilon a\frac{uS}{E} + (1-\theta)\sigma\gamma E^{\varepsilon}L - [1-\beta(1-\theta)]\left(b - m\frac{1}{\eta}\right)\frac{ah}{1+ah} - \rho\right\}\frac{u}{\theta}$$

$$\dot{L}_Y = \left\{\theta\frac{\sigma\gamma}{\beta}E^{\varepsilon}L_Y - [\beta + (1-\beta)\theta]\varepsilon b(\frac{\bar{E}}{E} - 1) + \left[(1-\theta)\beta\frac{L}{L_Y} + \theta\right]\varepsilon a\frac{uS}{E} + (1-\theta)\sigma\gamma E^{\varepsilon}L - (1-\theta)(1-\beta)\left(b - m\frac{1}{\eta}\right)\frac{ah}{1+ah} - \rho\right\}\frac{L_Y}{\theta}.$$

This is a seven-dimensional dynamic system in S, E, $\bar{E}, h$, η, u, and L_Y, with three pre-determined variables, S, $\bar{E}$, and E, and four jump variables, L_Y, u, η, and h. If $\bar{E}^*$ allows (A) to hold, it admits an asymptotic steady state with

$$S^* = h^* = 0, \quad 0 < E^* = \bar{E}^* < \bar{E}_0, \quad u^* = \frac{1}{\theta}\left[(\theta-1)\sigma\gamma\bar{E}^{*\varepsilon}L + \rho\right] > 0,$$

$$g_R^* = g_S^* = -u^* < 0,$$
$$\eta^* = \frac{m}{b},$$
$$L_Y^* = \frac{\beta}{\theta\sigma\gamma\bar{E}^{\varepsilon}}\left[(\theta-1)\sigma\gamma\bar{E}^{*\varepsilon}L + \rho\right] \in (0, L),$$
$$g_A^* = \frac{1}{\theta}\left\{[\beta + (1-\beta)\theta]\gamma\bar{E}^{*\varepsilon}L - \frac{\beta}{\sigma}\rho\right\} > 0,$$
$$g_c^* = g_Y^* = \frac{1}{\theta}(\sigma\gamma\bar{E}^{*\varepsilon}L - \rho) > 0.$$

The long-run result thus looks similar to that of the simple model with exogenous rest point, except that now hysteresis is present. The rest point of environmental quality $\bar{E}^*$ depends on the historically given initial values of S, $\bar{E}$, and E, as indicated in the right panels of Figure 2. Given S_0, for $\bar{E}_0$ large and E_0 not too far below $\bar{E}_0$, the condition (A), with $\bar{E}$ replaced by $\bar{E}$, needed for sustained economic growth, remains valid.