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# SINGLE EQUIVALENT OF
# DOUBLE MAJORITY VOTING SYSTEM

In the paper, the consequences of using double majority voting in the European Council of Ministers are analysed. An equivalent single majority distribution of seats is proposed and evaluated.

Keywords: *majority voting, single equivalent*

## 1. Principle of proportionality

At the beginning of every democratic representative system lies down the proportionality.

Let a given assembly consist of $m$ seats, which have to be distributed among $N$ subgroups $s_1, s_2, ..., s_N$, $s_i \geq 1$, $i = 1, 2, ..., N$, $\sum_{i=1}^{N} s_i = m$. In the case of European Union institutions a subgroup usually denotes a country.

Democratic apportionment of seats has to be proportionally correlated with population of a country. Let $p_i$ denotes cardinality of the $i$-th subgroup, $i = 1, 2, ..., N$ and $p = \sum_{i=1}^{N} p_i$. Hence $\lambda = p / m$ is a coefficient of proportionality, which enables us to determine the number of seats for every subgroup in $m$-seats assembly.

Such an apportionment must produce integer values so a kind of rounding has to be used: $\delta(s_i)$, $s_i = [p_i / \lambda]$. It gives non-deterministic result: $s_i$ or $s_i + 1$ seats. In Balinski and Young, 1975, we have:

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$$s_i^* = \begin{cases} s_i & \text{for } \dfrac{p_i}{\lambda} < \delta(s_i), \\ s_i + 1 & \text{dor } \dfrac{p_i}{\lambda} > \delta(s_i). \end{cases}$$

The function $\delta$ is a monotonic increasing function of non-negative integers $s$ and only for $p/\lambda = \delta(s)$ gives a unique solution.

Proportional distribution of seats is the core idea of democracy but is not in use, at least in European Union! For example, Widgren (1994) described the number of seats in the EU Council of Ministers as a function of population:

$$\log v = 0.0063 \log^{2.465} p$$

where $p$ denotes population and $v$ denotes number of seats.

Paterson (1997) modifies this approximation by

$$v_i = \begin{cases} 2 & \text{for } p_i < 712000 \\ [1.731 p_i^{0.42}] & \text{for } 712000 \le p_i \le 73000000 \,. \\ 10 & \text{for } p_i > 73000000 \end{cases}$$

In both cases exponential function of approximation contradicts proportionality.

The entering of new members changes this situation very much (entering countries have very different population: from Poland with nearly 39 millions of citizens to 0.36 million of Malta). Standard regression attempt produces the following approximation (square root-$p$ model):

$$v_i = 0.9498 + 1.1317 \sqrt{p_i}$$

with $R^2 = 97.5092$, while the next two models: linear and multiplicative ones gave $R^2 = 91.1542$ and $R^2 = 92.4794$, respectively. The result is not very far from linear, i.e., it is close to proportional[1].

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[1] In most cases, the Council takes decisions by a "qualified majority". The authors of the Treaty of Rome determined the qualified majority using a system of weighted votes reflecting the population of the Member States, with a correction in favour of States with smaller populations. This system has been adapted to take account of successive enlargements, without changing the relative weight of the Member States as laid down at the outset. Under the present arrangement (EU-15), the number of votes attributed to Member States varies from 10 (for those with the largest populations) to 2, making a grand total of 87 votes. Under Article 205 EC, acts are adopted by qualified majority if there are at least 62 votes in favour – excerption from official www site of EC.

**Table 1**

Proportional apportionment of seats for EU-25 European Council
(before the Treaty of Nice)

| Country | Votes (V) | Population (P) | (3)/lambda | Seats I round | Seats II round | Residuals | Seats III round |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Belgium | 5 | 10.06 | 2.757217 | 2 | 2 | 0.757217 | 2 |
| Denmark | 3 | 5.19 | 1.4224609 | 1 | 2 | –0.5775391 | 2 |
| Germany | 10 | 80.77 | 22.137218 | 22 | 22 | 0.1372183 | 21 |
| Greece | 5 | 10.38 | 2.8449217 | 2 | 2 | 0.8449217 | 2 |
| Spain | 8 | 39.13 | 10.724642 | 10 | 10 | 0.7246422 | 9 |
| France | 10 | 57.65 | 15.800553 | 15 | 15 | 0.8005526 | 14 |
| Ireland | 3 | 3.57 | 0.9784557 | 0 | 2 | –1.0215443 | 2 |
| Italy | 10 | 57.84 | 15.852627 | 15 | 15 | 0.8526273 | 15 |
| Luxemburg | 2 | 0.39 | 0.1068901 | 0 | 2 | –1.8931099 | 2 |
| Holland | 5 | 15.28 | 4.1879002 | 4 | 4 | 0.1879002 | 3 |
| Austria | 4 | 7.94 | 2.1761733 | 2 | 2 | 0.1761733 | 2 |
| Portugal | 5 | 9.85 | 2.6996608 | 2 | 2 | 0.6996608 | 2 |
| Finland | 3 | 5.07 | 1.3895716 | 1 | 2 | –0.6104284 | 2 |
| Sweden | 4 | 8.71 | 2.3872127 | 2 | 2 | 0.3872127 | 2 |
| United Kingdom | 10 | 58.04 | 15.907443 | 15 | 15 | 0.9074428 | 15 |
| Malta | 2 | 0.38 | 0.1041493 | 0 | 2 | –1.8958507 | 2 |
| Cyprus | 2 | 0.71 | 0.1945948 | 0 | 2 | –1.8054052 | 2 |
| Estonia | 2 | 1.48 | 0.4056343 | 0 | 2 | –1.5943657 | 2 |
| Slovenia | 2 | 1.99 | 0.5454137 | 0 | 2 | –1.4545863 | 2 |
| Latvia | 3 | 2.42 | 0.6632669 | 0 | 2 | –1.3367331 | 2 |
| Lithuania | 3 | 3.69 | 1.011345 | 1 | 2 | –0.988655 | 2 |
| Slovakia | 3 | 5.38 | 1.4745355 | 1 | 2 | –0.5254645 | 2 |
| Hungary | 5 | 10.18 | 2.7901063 | 2 | 2 | 0.7901063 | 2 |
| Czech Republic | 5 | 10.32 | 2.8284771 | 2 | 2 | 0.8284771 | 2 |
| Poland | 8 | 38.71 | 10.60953 | 10 | 10 | 0.6095298 | 9 |
| Total: | 122 | 445.13 | 122 | 109 | 127 | –5 | 122 |

As is seen from table 1 the proportional apportionment (truncated to 2) of seats for $\lambda = 3.6486$ (according to the least remainders method) is also approximation. Linear model for $a = 0.650508$ and $b = 0.236575$ gives $R^2 = 0.973703$, which is comparable with linear model of square $p$ (both models have high determination) and also gives some privileges for small countries.

**Table 2**

Proportional apportionment of seats for EU-25 European Council
(after the Treaty of Nice)

| Country | Votes (V) Nice Treaty | Population (P) | (3)/lambda lamb = 1.3866 | Seats I round | Seats II round (>=3) | Residuals | Seats III round |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Belgium | 12 | 10.06 | 7.2546447 | 7 | 7 | 0.2546447 | 6 |
| Denmark | 7 | 5.19 | 3.7427044 | 3 | 3 | 0.7427044 | 3 |
| Germany | 29 | 80.77 | 58.246288 | 58 | 58 | 0.2462876 | 57 |
| Greece | 12 | 10.38 | 7.4854088 | 7 | 7 | 0.4854088 | 7 |
| Spain | 27 | 39.13 | 28.218116 | 28 | 28 | 0.2181161 | 27 |
| France | 29 | 57.65 | 41.573585 | 41 | 41 | 0.5735852 | 41 |
| Ireland | 7 | 3.57 | 2.5744614 | 2 | 3 | –0.4255386 | 3 |
| Italy | 29 | 57.84 | 41.710601 | 41 | 41 | 0.7106014 | 41 |
| Luxemburg | 4 | 0.39 | 0.2812437 | 0 | 3 | –2.7187563 | 3 |
| Holland | 13 | 15.28 | 11.018983 | 11 | 11 | 0.0189832 | 10 |
| Austria | 10 | 7.94 | 5.7258329 | 5 | 5 | 0.7258329 | 5 |
| Portugal | 12 | 9.85 | 7.1032058 | 7 | 7 | 0.1032058 | 6 |
| Finland | 7 | 5.07 | 3.6561679 | 3 | 3 | 0.6561679 | 3 |
| Sweden | 10 | 8.71 | 6.2811089 | 6 | 6 | 0.2811089 | 6 |
| United Kingdom | 29 | 58.04 | 41.854829 | 41 | 41 | 0.8548289 | 41 |
| Malta | 3 | 0.38 | 0.2740323 | 0 | 3 | –2.7259677 | 3 |
| Cyprus | 4 | 0.71 | 0.5120077 | 0 | 3 | –2.4879923 | 3 |
| Estonia | 4 | 1.48 | 1.0672837 | 1 | 3 | –1.9327163 | 3 |
| Slovenia | 4 | 1.99 | 1.4350639 | 1 | 3 | –1.5649361 | 3 |
| Latvia | 4 | 2.42 | 1.7451531 | 1 | 3 | –1.2548469 | 3 |
| Lithuania | 7 | 3.69 | 2.6609979 | 2 | 3 | –0.3390021 | 3 |
| Slovakia | 7 | 5.38 | 3.8797205 | 3 | 3 | 0.8797205 | 3 |
| Hungary | 12 | 10.18 | 7.3411812 | 7 | 7 | 0.3411812 | 7 |
| Czech Republic | 12 | 10.32 | 7.4421405 | 7 | 7 | 0.4421405 | 7 |
| Poland | 27 | 38.71 | 27.915238 | 27 | 27 | 0.9152382 | 27 |
| Total: | 321 | 445.13 | 321 | 309 | 326 | –5 | 321 |

The Nice Treaty solves the number of seats in European Council for each of 25 countries, as presented in table 2. A qualified majority will be secured, where[2]:

• the number of votes in favour of a decision is close to the present threshold (71.26% of votes) in a Union of fifteen Member States. At first, the threshold will evolve in step with the accessions, up to a maximum of 73.4% of votes. Afterwards, once the twelve applicant countries with which the Union has already started accession

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[2] Excerption from official www site of EC.

negotiations have joined, the qualified majority threshold will be set at 255 votes out of 345; a majority of Member States votes in favour of a decision,

 • in addition, a Member State may ask for verification that the qualified majority comprises at least 62% of the total population of the EU. Should this not be the case, the decision will not be adopted.

The proportional method of apportionment truncated to 3 gives results presented in table 2. As we see, actual number of seats (after the Nice Treaty) allocated to given countries differs from the proportional one very much, so the actual distribution is not proportional.

Having a template (proportional apportionment of seats) one can evaluate how far from ideal distribution the actual one is. For certain distribution of seats we can calculate the mean distance from the template as:

$$d = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(v_i - v_i^{\text{ideal}})^2}$$

where:

 − $v_i$ denotes the actual number of seats for a given country *i*,

 − $v_i^{\text{ideal}}$ denotes the ideal (proportional) number of seats for a given country *i*.

For distribution of seats from the Treaty of Nice we obtain the mean distance from the template as equal to 7.7408. For EU-25 with distribution of seats according to the pre-Nice Treaty conditions this distance is 3.1749. As we see, the actual distance is almost twice that noted before, so the Treaty did not improve the proportionality.

The above inconsistency of seats distribution was probably the reason for introducing the second criterion into the process of decision-making: countries "for" must represent at least 62% of population of EU. This leads us to the problem of double (in general multi-) dimensional evaluation of apportionment of seats. It is not obvious that it should be the population of the countries. For example, Turnovec (1995, 1997), Widgren (1995) suggest that the second criterion could be GDP: popular public opinion says that net payers to EU budget should have more "to say" than "consumers" of the budget.

## 2. Multi-dimensional power index

Let us assume that the same set of decision-makers $s_1, s_2, ..., s_N$ is differently regarded according to *k* different criterions and decision is made via voting. Such differentiation produces the following structures:

 • structure 1: $(d_1; s_1^1, s_2^1, ..., s_N^1)$,

 • structure 2: $(d_2; s_1^2, s_2^2, ..., s_N^2)$,

• ...
• structure $k$: $(d_k; s_1^k, s_2^k, ..., s_N^k)$,

where $d_i$ is the threshold of a decision.

For the above structures one can define a winning coalition as a coalition, which is winning in every structure, and subsequently using definition of a winning coalition one can modify power indices.

Following the idea of Turnovec (1995, 1997) we calculate the double power index for EU Council of Ministers for two criterions: population and GDP. As can be seen in table 3, this leads us to different and not obvious results. We may notice that introducing population changes the power of certain country significantly: bigger power of countries is corrected up, making the seats distribution more close to proportional one.

**Table 3**

EU-25 Council of Ministers (before Nice Treaty)

| Country | Votes (V) | Votes % | Population (P) | Population % | GDP | GDP % | Shapley–Shubik power index | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | V | V + P | V+GDP | V + GDP + P |
| Belgium | 5 | 3.79 | 10.06 | 2.11 | 213.4 | 2.84 | 4.04 | 2.20 | 3.40 | 2.20 |
| Denmark | 3 | 2.27 | 5.19 | 1.09 | 137.6 | 1.83 | 2.39 | 1.15 | 2.07 | 1.15 |
| Germany | 10 | 7.58 | 80.77 | 16.95 | 1903 | 25.33 | 8.45 | 19.27 | 18.51 | 19.30 |
| Greece | 5 | 3.79 | 10.38 | 2.18 | 76.7 | 1.02 | 4.04 | 2.26 | 2.53 | 2.25 |
| Spain | 8 | 6.06 | 39.13 | 8.21 | 533.9 | 7.11 | 6.64 | 8.53 | 6.97 | 8.52 |
| France | 10 | 7.58 | 57.65 | 12.10 | 1289 | 17.16 | 8.45 | 12.85 | 12.57 | 12.88 |
| Ireland | 3 | 2.27 | 3.57 | 0.75 | 44.9 | 0.60 | 2.39 | 0.82 | 1.50 | 0.82 |
| Italy | 10 | 7.58 | 57.84 | 12.14 | 1135 | 15.10 | 8.45 | 12.89 | 11.26 | 12.88 |
| Luxemburg | 2 | 1.52 | 0.39 | 0.08 | 14.2 | 0.19 | 1.58 | 0.17 | 0.89 | 0.17 |
| Holland | 5 | 3.79 | 15.28 | 3.21 | 316.4 | 4.21 | 4.04 | 3.24 | 4.13 | 3.25 |
| Austria | 4 | 3.03 | 7.94 | 1.67 | 183.5 | 2.44 | 3.22 | 1.72 | 2.79 | 1.73 |
| Portugal | 5 | 3.79 | 9.85 | 2.07 | 77.7 | 1.03 | 4.04 | 2.16 | 2.53 | 2.15 |
| Finland | 3 | 2.27 | 5.07 | 1.06 | 96.2 | 1.28 | 2.39 | 1.12 | 1.80 | 1.12 |
| Sweden | 4 | 3.03 | 8.71 | 1.83 | 216.3 | 2.88 | 3.22 | 1.87 | 3.00 | 1.88 |
| United Kingdom | 10 | 7.58 | 58.04 | 12.18 | 1042 | 13.88 | 8.45 | 12.94 | 10.46 | 12.93 |
| Malta | 2 | 1.52 | 0.38 | 0.08 | 3.1 | 0.04 | 1.58 | 0.16 | 0.83 | 0.16 |
| Cyprus | 2 | 1.52 | 0.71 | 0.15 | 7.5 | 0.10 | 1.58 | 0.23 | 0.85 | 0.24 |
| Estonia | 2 | 1.52 | 1.48 | 0.32 | 4.7 | 0.06 | 1.58 | 0.39 | 0.83 | 0.39 |
| Slovenia | 2 | 1.52 | 1.99 | 0.42 | 12.6 | 0.17 | 1.58 | 0.47 | 0.88 | 0.47 |
| Latvia | 3 | 2.27 | 2.42 | 0.54 | 5.2 | 0.07 | 2.39 | 0.63 | 1.25 | 0.62 |
| Lithuania | 3 | 2.27 | 3.69 | 0.79 | 4.9 | 0.07 | 2.39 | 0.85 | 1.25 | 0.85 |
| Slovakia | 3 | 2.27 | 5.38 | 1.12 | 10.1 | 0.14 | 2.39 | 1.17 | 1.28 | 1.17 |
| Hungary | 5 | 3.79 | 10.18 | 2.16 | 34.2 | 0.46 | 4.04 | 2.24 | 2.27 | 2.23 |
| Czech Republic | 5 | 3.79 | 10.32 | 2.17 | 28.2 | 0.38 | 4.04 | 2.25 | 2.24 | 2.24 |
| Poland | 8 | 6.06 | 38.71 | 8.07 | 87.3 | 1.16 | 6.64 | 8.41 | 3.92 | 8.40 |
| Total: | 122 | 100 | 445.13 | 100 | 7477.6 | 100 | 100 | 100 | 100 | 100 |

Data for new enterers are extracted from official www pages of their Statistical Offices.

Interpreting the results from tab. 4 one can find that declaration about supporting small countries by great countries by special arrangement of seats is true when we look at the Shapley–Shubik power index value and it is not true when we add the criterion of 62% of population being "for" for the decision to be made. This second criterion reverses the possibility of countries to influence the final voting result: bigger countries are even stronger in the sense of Shapley–Shubik measure of power than it emanates from their population.

One of the new proposals for the time after 2009 is to use double voting where the first criterion for a decision to be made is majority of countries and the second criterion for those countries is to represent at least 60% of EU population. This criterion is equivalent to uniform seats distribution where each country has 1 vote. The results of Shapley–Shubik power index calculation are presented in table 5 and they confirm that this way of decision-making is in favour of small countries.

**Table 4**

EU-25 Council of Ministers (after Nice Treaty)

| Country | Votes (V) | Votes % | Population (P) | Population % | Shapley–Shubik power index | |
|---|---|---|---|---|---|---|
| | | | | | V | V + P |
| Belgium | 12 | 3.74 | 10.06 | 2.11 | 3.64 | 2.13 |
| Denmark | 7 | 2.18 | 5.19 | 1.09 | 2.09 | 1.10 |
| Germany | 29 | 9.03 | 80.77 | 16.95 | 9.32 | 19.56 |
| Greece | 12 | 3.74 | 10.38 | 2.18 | 3.64 | 2.20 |
| Spain | 27 | 8.41 | 39.13 | 8.21 | 8.61 | 8.65 |
| France | 29 | 9.03 | 57.65 | 12.10 | 9.32 | 13.04 |
| Ireland | 7 | 2.18 | 3.57 | 0.75 | 2.09 | 0.75 |
| Italy | 29 | 9.03 | 57.84 | 12.14 | 9.32 | 13.09 |
| Luxemburg | 4 | 1.25 | 0.39 | 0.08 | 1.19 | 0.10 |
| Holland | 13 | 4.05 | 15.28 | 3.21 | 3.96 | 3.27 |
| Austria | 10 | 3.12 | 7.94 | 1.67 | 3.02 | 1.67 |
| Portugal | 12 | 3.74 | 9.85 | 2.07 | 3.64 | 2.09 |
| Finland | 7 | 2.18 | 5.07 | 1.06 | 2.09 | 1.07 |
| Sweden | 10 | 3.12 | 8.71 | 1.83 | 3.02 | 1.82 |
| United Kingdom | 29 | 9.03 | 58.04 | 12.18 | 9.32 | 13.14 |
| Malta | 3 | 0.93 | 0.38 | 0.08 | 0.89 | 0.09 |
| Cyprus | 4 | 1.25 | 0.71 | 0.15 | 1.19 | 0.17 |
| Estonia | 4 | 1.25 | 1.48 | 0.32 | 1.19 | 0.34 |
| Slovenia | 4 | 1.25 | 1.99 | 0.42 | 1.19 | 0.42 |
| Latvia | 4 | 1.25 | 2.42 | 0.54 | 1.19 | 0.53 |
| Lithuania | 7 | 2.18 | 3.69 | 0.79 | 2.09 | 0.79 |
| Slovakia | 7 | 2.18 | 5.38 | 1.12 | 2.09 | 1.13 |
| Hungary | 12 | 3.74 | 10.18 | 2.16 | 3.64 | 2.18 |
| Czech Republic | 12 | 3.74 | 10.32 | 2.17 | 3.64 | 2.19 |
| Poland | 27 | 8.41 | 38.71 | 8.07 | 8.61 | 8.51 |
| Total: | 321 | 100 | 445.13 | 100 | 99.99 | 100.3 |

**Table 5**

EU-25 probable Council of Ministers (after year 2009)

| Country | Votes (V) | Votes % | Population (P) | Population % | Shapley–Shubik power index | |
|---|---|---|---|---|---|---|
| | | | | | V | V + P |
| Belgium | 1 | 4.00 | 10.06 | 2.11 | 4.00 | 2.46 |
| Denmark | 1 | 4.00 | 5.19 | 1.09 | 4.00 | 1.71 |
| Germany | 1 | 4.00 | 80.77 | 16.95 | 4.00 | 16.41 |
| Greece | 1 | 4.00 | 10.38 | 2.18 | 4.00 | 2.51 |
| Spain | 1 | 4.00 | 39.13 | 8.21 | 4.00 | 7.76 |
| France | 1 | 4.00 | 57.65 | 12.10 | 4.00 | 11.09 |
| Ireland | 1 | 4.00 | 3.57 | 0.75 | 4.00 | 1.46 |
| Italy | 1 | 4.00 | 57.84 | 12.14 | 4.00 | 11.13 |
| Luxemburg | 1 | 4.00 | 0.39 | 0.08 | 4.00 | 0.97 |
| Holland | 1 | 4.00 | 15.28 | 3.21 | 4.00 | 3.31 |
| Austria | 1 | 4.00 | 7.94 | 1.67 | 4.00 | 2.12 |
| Portugal | 1 | 4.00 | 9.85 | 2.07 | 4.00 | 2.43 |
| Finland | 1 | 4.00 | 5.07 | 1.06 | 4.00 | 1.69 |
| Sweden | 1 | 4.00 | 8.71 | 1.83 | 4.00 | 2.25 |
| United Kingdom | 1 | 4.00 | 58.04 | 12.18 | 4.00 | 11.17 |
| Malta | 1 | 4.00 | 0.38 | 0.08 | 4.00 | 0.96 |
| Cyprus | 1 | 4.00 | 0.71 | 0.15 | 4.00 | 1.02 |
| Estonia | 1 | 4.00 | 1.48 | 0.32 | 4.00 | 1.15 |
| Slovenia | 1 | 4.00 | 1.99 | 0.42 | 4.00 | 1.21 |
| Latvia | 1 | 4.00 | 2.42 | 0.54 | 4.00 | 1.30 |
| Lithuania | 1 | 4.00 | 3.69 | 0.79 | 4.00 | 1.49 |
| Slovakia | 1 | 4.00 | 5.38 | 1.12 | 4.00 | 1.73 |
| Hungary | 1 | 4.00 | 10.18 | 2.16 | 4.00 | 2.50 |
| Czech Republic | 1 | 4.00 | 10.32 | 2.17 | 4.00 | 2.51 |
| Poland | 1 | 4.00 | 38.71 | 8.07 | 4.00 | 7.66 |
| Total: | 25 | 100 | 445.13 | 100 | 100 | 100 |

## 3. Power index and distribution of seats

Multi-power index is a right measure of power of certain group relatively to multi--criterions. Nevertheless, it is Shapley–Shubik power index or Banzhaf power index (both are mostly accepted as a measure of power) that obviously do not linearly depend on the number of seats.

Let us look at the following example of two decision structures: (*51*; 49, 49, 2) and (*51*; 33, 33, 34). We shall assume that in both situations decisions are made via 51% majority; the number of subgroups is equal to 3, distributions of seats are different, but Shapley–Shubik power index is the same: (1/3, 1/3, 1/3). Moreover, it is easy

to show that for the above structures any distribution of seats with the lowest number of seats not less than 2 and the highest not exceeding 50 gives the same result of power calculation.

Let us introduce the idea of right power distribution. Quite intuitively we expect direct proportionality between the number of seats and the power. In this sense, the structure (*51*; 33, 33, 34) is more correct than the structure (*51*; 49, 49, 2). More formally (Bruckner 1996), a structure has (*a*, *b*) right seats distribution $\mu$ in the sense of power distribution if:

$$\varepsilon_i < | \mu_i - \Psi_i(d;w) | < \varepsilon^i$$

where $\Psi_i(d;w)$ denotes value of power index in structure $(d;w)$.

Values $\varepsilon_i, \varepsilon^i$ are determined as follows:

$$\varepsilon_i = \max\left[ \min\left[ \mu_i\left(1 - \frac{a}{100}\right), \mu_i - \frac{b}{100} \right], 0 \right],$$

$$\varepsilon^i = \min\left[ \max\left[ \mu_i\left(1 + \frac{a}{100}\right), \mu_i + \frac{b}{100} \right], 1 \right].$$

It is obvious that such a distribution of seats is right only in the above sense and can be determined only very roughly and fuzzy. We propose the following algorithm to obtain the right distribution of seats, where the stop rule is the distance from initial Shapley–Shubik value[3].

**The algorithm**

Step I. Establish a set of criterions distinguishing every subgroup $(k \geq 1)$.

Step II. Calculate multi power index (if $k = 1$ then it is a regular power index).

Step III. Calculate the number of seats adequate to power distribution (it is suggested to use methods of least remainders).

Step IV. Calculate again multi-power index for distribution of seats from step III.

Step V. Calculate the difference between power indices from step II and step IV. If the sum of absolute differences is not decreasing then stop.

Step VI. If the differences are not acceptable then correct every group's representation in such a way that for the greatest difference add 1 seat if the difference is

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[3] It is a slightly modified algorithm presented in paper of Mercik and Mazurkiewicz (2002).

Table 6

Table 6 cd.

positive and extract 1 seat if the difference is negative. Otherwise, do the same with the group with the greatest opposite difference[4]. Go to step V.

# 4. Example

Let us calculate right distribution of seats for EU-25 after the Treaty of Nice. In table 6, we present the result of application of the above algorithm. We assume that every country has the number of seat declared in Nice and decision is made if more than 50% of votes are for and those countries represent more than 62% of EU population (as presented in table 4). Using the algorithm we find single equivalent of double criterion decision-making, i.e., such a distribution of seats for which the value of Shapley–Shubik power index will be almost the same as for double voting.

In table 6, column 9, the light grey (originally green) marks the country (here Germany) for which the number of seats should be increased by 1, the dark grey (originally red) marks the country (Austria) for which the number of seats should be decreased by 1. The last position in column 9 represents the distance between Shapley–Shubik value for double voting and single equivalent for all countries.

As we may see from table 6, step by step, the apportionment of seats being an equivalent of double voting in European Council is achieved. The last value (7.56) of the measure of differences (sum of residuals) is lower than the last but one (7.61), which means that the algorithm should be continued, but the truncation to the minimum number of seats equalled 3 stopped it. The last right distribution shown in column 25 is therefore the final apportionment.

Table 7 shows distribution of seats after the Treaty of Nice compared with the distribution of seats being a single equivalent of actual distribution.

As we may see in table 7, the differences between power indices for double voting and single equivalent are not significant as a whole: the value of $t$ test is 0.010438 for paired test and is much smaller than any one or two tail critical values. What is more, this single equivalent distribution really supports small countries: in the new distribution it is more effective (in a sense of influence on the final result of voting) to have 3 seats than even 7 seats in the previous one. The only inconsequence is inequality between Poland and Spain: the algorithm was artificially stopped[5].

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[4] To some extent it is an adoption of the method used in classical transportation algorithm.

[5] What is surprising is that at one of the pre final Treaty stages such an inequality was proposed by the president of France.

**Table 7**

Distribution of seats after the Treaty of Nice
with double voting and its single equivalent

| Country | Votes (V) | S–S power index V + P | Seats single equivalent | S–S single equivalent |
|---|---|---|---|---|
| Belgium | 12 | 2.13 | 7 | 2.06 |
| Denmark | 7 | 1.1 | 3 | 0.87 |
| Germany | 29 | 19.56 | 56 | 19.11 |
| Greece | 12 | 2.2 | 7 | 2.06 |
| Spain | 27 | 8.65 | 28 | 8.42 |
| France | 29 | 13.04 | 40 | 12.69 |
| Ireland | 7 | 0.75 | 3 | 0.87 |
| Italy | 29 | 13.09 | 40 | 12.69 |
| Luxemburg | 4 | 0.1 | 3 | 0.87 |
| Holland | 13 | 3.27 | 11 | 3.26 |
| Austria | 10 | 1.67 | 5 | 1.47 |
| Portugal | 12 | 2.09 | 7 | 2.06 |
| Finland | 7 | 1.07 | 3 | 0.87 |
| Sweden | 10 | 1.82 | 6 | 1.76 |
| United Kingdom | 29 | 13.14 | 40 | 12.69 |
| Malta | 3 | 0.09 | 3 | 0.87 |
| Cyprus | 4 | 0.17 | 3 | 0.87 |
| Estonia | 4 | 0.34 | 3 | 0.87 |
| Slovenia | 4 | 0.42 | 3 | 0.87 |
| Latvia | 4 | 0.53 | 3 | 0.87 |
| Lithuania | 7 | 0.79 | 3 | 0.87 |
| Slovakia | 7 | 1.13 | 3 | 0.87 |
| Hungary | 12 | 2.18 | 7 | 2.06 |
| Czech Republic | 12 | 2.19 | 7 | 2.06 |
| Poland | 27 | 8.51 | 27 | 0.05 |
| Total: | 321 | 100.3 | 321 | 100.01 |

# 5. Summary

The results presented can be summarised as follows:
• after the Treaty of Nice the distribution of seats in the EU Council is not proportional,
• improving the voting system by adding criterion about 62% of the population "for" makes this system more proportional,
• the proposed single equivalent (new distribution) of seats is clearer and fulfils declaration about being in favour of small countries.

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### Jednowymiarowy ekwiwalent podwójnego większościowego systemu głosowania

W artykule rozważano konsekwencje zastosowania zasady podwójnego głosowania w odniesieniu do decyzji podejmowanych w Radzie Ministrów Unii Europejskiej. Przyjmuje się, że dana koalicja jest wygrywającą w systemie głosowania podwójnego, jeżeli jest koalicją wygrywającą w każdym z tych systemów oddzielnie. Każdy z tych systemów głosowania oddzielnie może mieć inną wartość kworum oraz inny rozkład wag głosów. W konsekwencji, rola i siła poszczególnych głosujących może być inna w każdym z systemów głosowania oddzielnie. Przedstawiono także ekwiwalent głosowania podwójnego w postaci pojedynczego systemu głosowania z inną dystrybucją wag głosów.

Słowa kluczowe: *głosowanie większościowe, jednowymiarowy ekwiwalent*