B A D A N I A O P E R A C Y J N E I D E C Y Z J E

Nr 4 2009

Jacek W. MERCIK*,[1]

A PRIORI VETO POWER OF THE PRESIDENT OF POLAND

The *a priori* power of the president of Poland, lower chamber of parliament (*Sejm*) and upper chamber of parliament (*Senate*) in the process of legislation are considered in this paper. The evaluation of power is made using the Johnston power index.

Keywords: *a priori power, Johnston power index, president of Poland*

1. Introduction

In Poland, in the process of legislation, any bill accepted by the *Sejm* (the lower chamber of the Polish parliament) is considered by the *Senate*, which may accept, amend or reject a bill. If a bill is amended or rejected by the *Senate*, then it goes back to the *Sejm*. The *Sejm* may, by absolute majority, reject the *Senate*'s objection. After that, a bill accepted by the *Sejm* goes to the president of Poland who can within 21 days accept and sign a bill or may declare his veto and send a bill back to the *Sejm*.

The presidential veto is considered as a cognizable attribute of the president regarding any bill resolved by parliament. According to the Constitutional Act, the president signs and declares a bill in the official monitor (gazette). In the case of important state interests or poor quality of constituted law, the president may reject a bill. Presidential rejection of a bill (veto) has a conditional character: the *Sejm* may accept a bill once more by a majority of 3/5 of votes in the presence of at least half of the members of the *Sejm* (representatives). In this case, the president has to sign a bill within seven days and publish the bill in the official monitor. The real effectiveness of

* Institute of Organization and Management, Wrocław University of Technology, ul. Smoluchowskiego 25, 50-372 Wrocław, e-mail: Jacek.Mercik@pwr.wroc.pl

[1] We would like to thank two anonymus referees for their enlightening comments. We also have to thank David Ramsey who contributed substantially to improve the paper.

the president's veto is therefore strongly subordinated to the present structure of parties in the *Sejm*.

Another way for the president to stop the legislation process is by sending an unsigned bill to the Constitutional Court asking it to establish a bill's conformity to the Constitutional Act. If the Constitutional Court declares the bill's conformity to the Constitutional Act, the president must sign it and may not declare his/her veto against it.

2. Analysis of the power of the members of a legislative process

The analysis of the power of members of a legislative process will be conducted via so called power indices. There are many different power indices in the literature. Among them, the most popular are the SHAPLEY–SHUBIK power index [5], [6] and the BANZHAF power index [1]. These two indices came from game theory[2] and are well tailored to an *a priori* evaluation of the ability to form a winning coalition (the Shapley–Shubik index of power) or of the permanency of a coalition[3] (the Banzhaf index of power). However, something different needs to be used when analysing the legislation process. A winning coalition must be formed, but the way to do so is a sequence of decisions whose summary results in a bill at the end of it. In the literature (for example [2]) it is assumed that the Johnston power index [3] is the best suited to reflect and evaluate this process.

For the Johnston power index, it is crucial to define a so called vulnerable coalition.

Definition: a winning coalition is vulnerable if, among its members, there is at least one whose defection would cause the coalition to lose[4]. Such a member is called critical. If only one player is critical, then this player is uniquely powerful in the coalition.

For example, a president in coalition with 232 representatives (simple majority +1) is uniquely powerful. If, however, a coalition comprises of exactly the president and 231 representatives, then the president, as well as any other member of the coalition, shares

[2] One may find a different approach to defining power indices for example in Turnovec et al. [8].

[3] For more on this see for example [7], [4].

[4] Historically, the word "defection" has been used, but it is also possible to use the concept of "swing", which is probably more often in use right now (see for example Turnovec et al. [9]). Therefore, the first sentence of the definition is as follows: a winning coalition is vulnerable if, among its members, there is at least one in a swing position, whose swing would cause the coalition to lose.

power equally with 231 other players[5]. In such a coalition there are 232 critical members and each has 1/232 of the power.

Defining the Johnston power index, first we count the number of players that are critical in each vulnerable coalition c (critical defections). The inverse of the number of critical defections is called the fractional defections for the coalition, $f(c)$. For example, if there are only two such players in the coalition c, then $f(c) = 1/2$.

The Johnston power of player i is the sum of the fractional critical defections over all the vulnerable coalitions in which i is critical, divided by the total number of fractional critical defections of all players, in other words, i's proportion of fractional critical defections.

Let V be the set of all vulnerable coalitions. Formally, for each vulnerable coalition $c \in V$, we define the set $f_i(c)$

$$f_i(c) = \begin{cases} f(c) & \text{i is critical in c} \\ 0 & \text{otherwise} \end{cases},$$

and the Johnston power index:

$$J(i) = \frac{\sum_{c \in V} f_i(c)}{\sum_{j=1}^{n} \sum_{c \in V} f_j(c)}$$

Let us consider the following example: the game [4; 3, 2, 1], i.e. voting where there are three voters with 3, 2 and 1 votes each. The majority needed for a decision is 4. The following are vulnerable coalitions in this game: (3, 2), (3, 1) and (3, 2, 1) (vulnerable coalitions must be winning coalitions).

Table 1. The Johnston power indexes for the game [4; 3, 2, 1]

Vulnerable coalitions	Number of vulnerable coalitions	Critical defections			Fractional critical defections		
		3 votes player	2 votes player	1 vote player	3 votes player	2 votes player	1 vote player
(3, 2)	1	1	1	0	1/2	1/2	0
(3, 1)	1	1	0	1	1/2	0	1/2
(3, 2, 1)	1	1	0	0	1	0	0
Total	3	3	1	1	2	1/2	1/2
$J(i)$					4/6	1/6	1/6

[5] We note that when 231 representatives in the *Sejm* decide to enact a bill, no one decision of the *Senate* can stop it. The *Sejm* may overrule any decision of the *Senate* by a simple majority. This means that the *a priori* power of the *Senate* equals 0, independently of which power index is in use.

For example, in the USA system of enacting bills, overruling of the president's veto needs at least a 2/3 majority in both chambers. It follows that the USA *Senate* may be crucial in the legislative process. This is not the case for the Polish *Senate*.

It is easy to notice that the vector (4/6, 1/6, 1/6) of Johnston power indexes in this example differs from the vector of Banzhaf power indexes (3/5, 1/5, 1/5) and is equal to the vector of Shapley–Shubik power indexes (4/6, 1/6, 1/6).

3. Analysis of the enactment of bills

In Poland all bills are resolved if:
– an absolute majority of representatives (p) and the president (z) are for[6], or
– in the case of a veto by the president, at least 3/5 of the representatives are for[7].
The legislative procedure in the Polish parliamentary system is presented in Fig. 1.

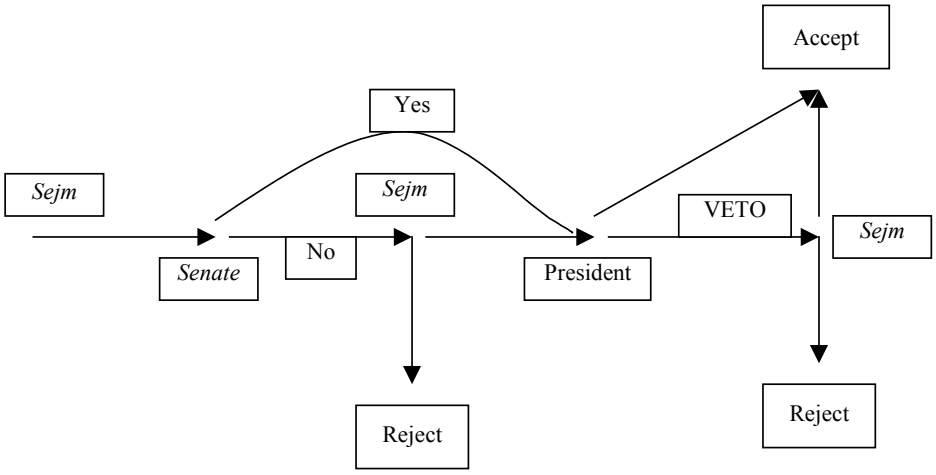

Fig. 1. Legislative procedure in the Polish parliamentary system
(*Sejm* stands for the House of Representatives)

Therefore, we have coalitions: $(z, p_{j_1}, p_{j_2}, ..., p_{j_{460}})$ where z denotes the president and p denotes a representative. Some of these coalitions are winning, some are not. Below, one can find derivations of vulnerable (winning) coalitions in the three following cases:

[6] Once again we would like to confirm that the Polish *Senate* has no effective influence during the legislative process. The *Sejm* may reject the objections of the *Senate* at any moment by a simple majority, i.e. 231 deputies when all of them are present (460). In the *a priori* analysis we only consider simple majority winning coalitions.

[7] This is a slightly simplified model, because the Supreme Court may also by simple majority recognize the bill as contradicting the Constitutional Act (or both chambers may change the Constitutional Act itself).

- case #1: we assume that there are no party structures in the parliament,
- case #2: just the *Sejm* has party structures, and
- case #3: the president favours one of the opposition parties.

We would like to find out how the above assumptions influence the *a priori* estimate of the power of each member of the legislative process, with special emphasis given to the position of the president.

Case #1.

In case #1 we assume that each member (including the representatives) of the legislative process is autonomous and acts independently. This is equivalent to the situation in which party affiliation, both of the president and representatives, is no longer valid. There is only one criterion for supporting or opposing a bill: an individual's personal attitude for or against the bill, not party discipline or belonging to the governing coalition. This means that we are analyzing the case of a hypothetical 3-level system of legislation.

In case #1 the winning coalitions are as follows[8]:

- $(z, p_{j_1}, p_{j_2}, ..., p_{j_n})$, where $n \geq 231$ (we also assume that all representatives participate in each vote).

Among the winning coalitions, the following coalitions are vulnerable:

- for $n = 231$ all the players, i.e. the president and 231 representatives, are critical,
- for $232 \leq n < 276$ only the president is critical.

Note, that for $n \geq 276$ all coalitions are winning, but no one member of such a coalition is critical.

Case #2.

In case #2 we assume that the representatives are members of parties and they vote according to the party leaders. The structure of the sixth Polish *Sejm* is presented in table 2.

Table 2. Structure of the *Sejm*[9]

Description	Party	Number of seats
a	Civic Platform	208
b	Law and Justice	157
c	Leftwing (Lewica)	42
d	Polish Popular Party (PSL)	31
e	SDPL – New Leftwing	5
f	Poland XXI	5
g	Democratic Faction of Representatives	3
h	Non-affiliated representatives	9
	In total	460

[8] The Polish *Sejm* consists of 460 representatives, and the *Senate* of 100 senators.

[9] As of January 4, 2009 http://www.*Sejm*.gov.pl/poslowie/kluby.htm

In case #2 there are two types of winning coalitions[10]:

- $(z, \{a, d\}, S)$,

where:

S can be formed from any factions of the parliament, except Civil Platform and PSL. S can be any of the following sets: $\{\varnothing\}$, $\{b\}$, $\{c\}$, $\{e\}$, $\{f\}$, $\{g\}$, $\{h\}$, $\{b, c\}$, $\{b, e\}$, $\{b, f\}$, $\{b, g\}$, $\{b, h\}$, $\{c, e\}$, $\{c, f\}$, $\{c, g\}$, $\{c, h\}$, $\{e, f\}$, $\{e, g\}$, $\{e, h\}$, $\{f, g\}$, $\{f, h\}$, $\{g, h\}$, $\{b, c, e\}$, $\{b, c, f\}$, $\{b, c, g\}$, $\{b, c, h\}$, $\{b, e, f\}$, $\{b, e, g\}$, $\{b, e, h\}$, $\{b, f, g\}$, $\{b, f, h\}$, $\{b, g, h\}$, $\{c, e, f\}$, $\{c, e, g\}$, $\{c, e, h\}$, $\{c, f, g\}$, $\{c, f, h\}$, $\{c, g, h\}$, $\{e, f, g\}$, $\{e, f, h\}$, $\{f, g, h\}$, $\{b, c, e, f\}$, $\{b, c, e, g\}$, $\{b, c, e, h\}$, $\{b, c, f, g\}$, $\{b, c, f, h\}$, $\{b, c, g, h\}$, $\{b, e, f, g\}$, $\{b, e, f, h\}$, $\{b, e, g, h\}$, $\{b, f, g, h\}$, $\{c, e, f, g\}$, $\{c, e, f, h\}$, $\{c, e, g, h\}$, $\{c, f, g, h\}$, $\{e, f, g, h\}$, $\{b, c, e, f, g\}$, $\{b, c, e, f, h\}$, $\{b, c, e, g, h\}$, $\{b, c, f, g, h\}$, $\{b, e, f, g, h\}$, $\{c, e, f, g, h\}$, $\{b, c, e, f, g, h\}$, and

- $(\{a, d\}, S)$,

where:

S could be any of the following sets: $\{b\}$, $\{c\}$, $\{b, c\}$, $\{b, e\}$, $\{b, f\}$, $\{b, g\}$, $\{b, h\}$, $\{c, e\}$, $\{c, f\}$, $\{c, g\}$, $\{c, h\}$, $\{b, c, e\}$, $\{b, c, f\}$, $\{b, c, g\}$, $\{b, c, h\}$, $\{b, e, f\}$, $\{b, e, g\}$, $\{b, e, h\}$, $\{b, f, g\}$, $\{b, f, h\}$, $b, g, h\}$, $\{c, e, f\}$, $\{c, e, g\}$, $\{c, e, h\}$, $\{c, f, g\}$, $\{c, f, h\}$, $\{c, g, h\}$, $\{b, c, e, f\}$, $\{b, c, e, g\}$, $\{b, c, e, h\}$, $\{b, c, f, g\}$, $\{b, c, f, h\}$, $\{b, c, g, h\}$, $\{b, e, f, g\}$, $\{b, e, f, h\}$, $\{b, e, g, h\}$, $\{b, f, g, h\}$, $\{c, e, f, g\}$, $\{c, e, f, h\}$, $\{c, e, g, h\}$, $\{c, f, g, h\}$, $\{b, c, e, f, g\}$, $\{b, c, e, f, h\}$, $\{b, c, e, g, h\}$, $\{b, c, f, g, h\}$, $\{b, e, f, g, h\}$, $\{c, e, f, g, h\}$, $\{b, c, e, f, g, h\}$.

Searching for vulnerable coalitions, we find that:

If card(S) $\leq$ 37, then[11] the coalition $(z, \{a, d\}, S)$ is vulnerable; the critical players are the president and the "governmental" coalition {a, d}.

If card(S) > 37, then the coalition $(z, \{a, d\}, S)$ is vulnerable, but only the "governmental" set $\{a, d\}$ is critical.

None of e, f, g, h can be critical as a member of coalition $(z, \{a, d\}, S)$ or $(\{a, d\}, S)$.

If 37 < card(S) < 199, then the coalition $(\{a, d\}, S)$ is vulnerable; the critical players are the "governmental" coalition $\{a, d\}$ and one element of the set $\{b, c\}$, depending on which one is included in S.

If card(S) $\geq$ 199, then the coalition $(\{a, d\}, S)$ is vulnerable; the only critical player is the "governmental" coalition $\{a, d\}$.

Case #3.

In case #3 we assume that the president acts in the same way as one of the opposition parties, namely Law and Justice (PiS). Therefore, case #3 is similar to case #2, since the winning coalitions are of the form $(\{z, b\}, \{a, d\}, S)$ or $(\{a, d\}, S)$.

[10] Description of representatives according to tab. 2. Governing coalition consists of Civic Platform (a) and PSL (d).

[11] card(S) denotes the number of seats at the disposal of the parties forming the coalition S.

For $(\{z,b\},\{a,d\},S)$, S could be any of the following subsets: $\{\varnothing\}$, $\{c\}$, $\{e\}$, $\{f\}$, $\{g\}$, $\{h\}$, $\{c, e\}$, $\{c,f\}$, $\{c, g\}$, $\{c, h\}$, $\{e,f\}$, $\{e, g\}$, $\{e, h\}$, $\{f, g\}$, $\{f, h\}$, $\{g, h\}$, $\{c, e,f\}$, $\{c, e, g\}$, $\{c, e, h\}$, $\{c,f, g\}$, $\{c,f, h\}$, $\{c, g, h\}$, $\{c, g, h\}$, $\{e,f, g\}$, $\{e,f, h\}$, $\{f, g, h\}$, $\{c, e,f, g\}$, $\{c, e,f, h\}$, $\{c,f, g, h\}$, $\{e,f, g, h\}$, $\{c, e,f, g, h\}$.

For $(\{a, d\}, S)$, S could be any of the following subsets: $\{c\}$, $\{c, e\}$, $\{c,f\}$, $\{c, h\}$, $\{e, g\}$, $\{c, e,f\}$, $\{c, e, g\}$, $\{c, e, h\}$, $\{c,f, g\}$, $\{c,f, h\}$, $\{c, g, h\}$, $\{c, g, h\}$, $\{c, e,f, g\}$, $\{c, e,f, h\}$, $\{c,f, g, h\}$, $\{c, e,f, g, h\}$.

Again, searching for a vulnerable coalition, one obtains the following conditions:

If card(S) ≤ 37, then the coalition $(\{z,b\},\{a,d\},S)$ is vulnerable; the critical players are the president together with the major opposition party PiS[12] and the "governmental" coalition $\{a, d\}$.

If card(S) > 37, then the coalition $(\{z,b\},\{a,d\},S)$ is vulnerable but only the "governmental" set $\{a, d\}$ is critical.

None of e, f, g, h can be critical as a member of coalition $(\{z,b\},\{a,d\},S)$ or $(\{a, d\}, S)$.

If card(S) > 37, then the coalition $(\{a, d\}, S)$ is vulnerable and the "governmental" set $\{a, d\}$ and player $\{c\}$ are critical.

4. Power analysis

Having the list of all the winning coalitions at one's disposal, one can calculate the *a priori* Johnston index of power. All three cases are presented below.

Case #1.

Among the winning coalitions there are the following vulnerable coalitions:

- for n = 231 all players, i.e. the president and representatives are critical. Therefore, the Johnston fraction of critical defections for each of them equals $f(\bullet) = 1/232$.

There are $\binom{460}{231} = 1.10\text{E}+137$ coalitions with the president and $\binom{459}{230} = 5.5347\text{E}+136$ with a given representative.

- for $232 \le n < 276$ only the president is critical and the Johnston fraction of critical defections for him equals $f(\bullet) = 1$. There are $\left[\binom{460}{232}+\binom{460}{233}+\ldots+\binom{460}{275}\right] = 1.323\text{E}+138$ such coalitions.

[12] It is not clear how to share the power between these two players.

The above results give the following values of the Johnston index of power:

- for the president: 0.92342978817
- for a representative: 0.0001664570
- for the *Sejm* as a whole: 0.0765702118

Case #2.

Let us recall that in case #2 we take into account the party affiliation of representatives by assuming that they vote according to their party leaders. We also know the present governing coalition (Civic Platform and PSL).

The results of the calculations for coalitions $(z,\{a,d\},S)$ and $(\{a, d\}, S)$ are presented in tab. 3 and tab. 4, respectively.

Table 3. Calculation of Johnston power index for case #2 and coalition $(z, \{a, d\}, S)$

Vulnerable coalitions $(z, \{a, d\}, S)$ for $S =$	card(S)	Fraction of critical defections	
		President z	Government $\{a, d\}$
1	**2**	**3**	**4**
$\{\varnothing\}$	0	$^1/_2$	$^1/_2$
$\{g\}$	3	$^1/_2$	$^1/_2$
$\{e\}$	5	$^1/_2$	$^1/_2$
$\{f\}$	5	$^1/_2$	$^1/_2$
$\{h\}$	9	$^1/_2$	$^1/_2$
$\{e, g\}$	8	$^1/_2$	$^1/_2$
$\{f, g\}$	8	$^1/_2$	$^1/_2$
$\{e, f\}$	10	$^1/_2$	$^1/_2$
$\{g, h\}$	12	$^1/_2$	$^1/_2$
$\{e, f, g\}$	13	$^1/_2$	$^1/_2$
$\{e, h\}$	14	$^1/_2$	$^1/_2$
$\{f, h\}$	14	$^1/_2$	$^1/_2$
$\{f, g, h\}$	17	$^1/_2$	$^1/_2$
$\{e, f, h\}$	19	$^1/_2$	$^1/_2$
$\{e, f, g, h\}$	22	$^1/_2$	$^1/_2$
$\{c\}$	42		1
$\{c, g\}$	45		1
$\{c, e\}$	47		1
$\{c, f\}$	47		1
$\{c, e, g\}$	50		1
$\{c, f, g\}$	50		1
$\{c, h\}$	51		1
$\{c, e, f\}$	52		1
$\{c, g, h\}$	54		1

1	2	3	4
{*c*, *e*, *f*, *g*}	55		1
{*c*, *e*, *h*}	56		1
{*c*, *f*, *h*}	56		1
{*c*, *e*, *g*, *h*}	59		1
{*c*, *f*, *g*, *h*}	59		1
{*c*, *e*, *f*, *h*}	61		1
{*c*, *e*, *f*, *g*, *h*}	64		1
{*b*}	157		1
{*b*, *g*}	160		1
{*b*, *e*}	162		1
{*b*, *f*}	162		1
{*b*, *e*, *g*}	165		1
{*b*, *f*, *g*}	165		1
{*b*, *h*}	166		1
{*b*, *e*, *f*, }	167		1
{*b*, *g*, *h*}	169		1
{*b*, *e*, *f*, *g*}	170		1
{*b*, *e*, *h*}	171		1
{*b*, *f*, *h*}	171		1
{*b*, *e*, *g*, *h*}	174		1
{*b*, *f*, *g*, *h*}	174		1
{*b*, *e*, *f*, *h*}	176		1
{*b*, *e*, *f*, *g*, *h*}	179		1
{*b*, *c*}	199		1
{*b*, *c*, *g*}	202		1
{*b*, *c*, *e*}	204		1
{*b*, *c*, *f*}	204		1
{*b*, *c*, *e*, *g*}	207		1
{*b*, *c*, *f*, *g*}	207		1
{*b*, *c*, *h*}	208		1
{*b*, *c*, *e*, *f*}	209		1
{*b*, *c*, *g*, *h*}	211		1
{*b*, *c*, *e*, *f*, *g*}	212		1
{*b*, *c*, *e*, *h*}	213		1
{*b*, *c*, *f*, *h*}	213		1
{*b*, *c*, *e*, *g*, *h*}	216		1
{*b*, *c*, *f*, *g*, *h*}	216		1
{*b*, *c*, *e*, *f*, *h*}	218		1
{*b*, *c*, *e*, *f*, *g*, *h*}	221		1
	In total:	7.5	55.5

Table 4. Calculation of the Johnston power index for case #2 and coalition ($\{a, d\}, S$)

Vulnerable coalitions ($\{a, d\}, S$) for S =	card(S)	Fraction of critical defections			
		President z	Government $\{a, d\}$	Party $\{b\}$	Party $\{c\}$
1	**2**	**3**	**4**	**5**	**6**
$\{c\}$	42		$^1/_2$		$^1/_2$
$\{c, g\}$	45		$^1/_2$		$^1/_2$
$\{c, e\}$	47		$^1/_2$		$^1/_2$
$\{c, f\}$	47		$^1/_2$		$^1/_2$
$\{c, e, g\}$	50		$^1/_2$		$^1/_2$
$\{c, f, g\}$	50		$^1/_2$		$^1/_2$
$\{c, h\}$	51		$^1/_2$		$^1/_2$
$\{c, e, f\}$	52		$^1/_2$		$^1/_2$
$\{c, g, h\}$	54		$^1/_2$		$^1/_2$
$\{c, e, f, g\}$	55		$^1/_2$		$^1/_2$
$\{c, e, h\}$	56		$^1/_2$		$^1/_2$
$\{c, f, h\}$	56		$^1/_2$		$^1/_2$
$\{c, e, g, h\}$	59		$^1/_2$		$^1/_2$
$\{c, f, g, h\}$	59		$^1/_2$		$^1/_2$
$\{c, e, f, h\}$	61		$^1/_2$		$^1/_2$
$\{c, e, f, g, h\}$	64		$^1/_2$		$^1/_2$
$\{b\}$	157		$^1/_2$	$^1/_2$	
$\{b, g\}$	160		$^1/_2$	$^1/_2$	
$\{b, e\}$	162		$^1/_2$	$^1/_2$	
$\{b, f\}$	162		$^1/_2$	$^1/_2$	
$\{b, e, g\}$	165		$^1/_2$	$^1/_2$	
$\{b, f, g\}$	165		$^1/_2$	$^1/_2$	
$\{b, h\}$	166		$^1/_2$	$^1/_2$	
$\{b, e, f, \}$	167		$^1/_2$	$^1/_2$	
$\{b, g, h\}$	169		$^1/_2$	$^1/_2$	
$\{b, e, f, g\}$	170		$^1/_2$	$^1/_2$	
$\{b, e, h\}$	171		$^1/_2$	$^1/_2$	
$\{b, f, h\}$	171		$^1/_2$	$^1/_2$	
$\{b, e, g, h\}$	174		$^1/_2$	$^1/_2$	
$\{b, f, g, h\}$	174		$^1/_2$	$^1/_2$	
$\{b, e, f, h\}$	176		$^1/_2$	$^1/_2$	
$\{b, e, f, g, h\}$	179		$^1/_2$	$^1/_2$	
$\{b, c\}$	199		1		
$\{b, c, g\}$	202		1		
$\{b, c, e\}$	204		1		
$\{b, c, f\}$	204		1		
$\{b, c, e, g\}$	207		1		
$\{b, c, f, g\}$	207		1		
$\{b, c, h\}$	208		1		
$\{b, c, e, f\}$	209		1		

1	2	3	4	5	6
{*b*, *c*, *g*, *h*}	211		1		
{*b*, *c*, *e*, *f*, *g*}	212		1		
{*b*, *c*, *e*, *h*}	213		1		
{*b*, *c*, *f*, *h*}	213		1		
{*b*, *c*, *e*, *g*, *h*}	216		1		
{*b*, *c*, *f*, *g*, *h*}	216		1		
{*b*, *c*, *e*, *f*, *h*}	218		1		
{*b*, *c*, *e*, *f*, *g*, *h*}	221		1		
	In total:	0	32	8	8

For case #2 the *a priori* Johnston power indexes are as follows:

- for the *president*: 0.067568,
- for the *government*: 0.788288,
- for party {*b*} (PiS) and {*c*} (Leftwing): 0.072072 each,

which means that in the Polish parliamentary system for the duration of the sixth Parliament, the *government* is 11.66 times stronger than the *president*.

Indirectly we also obtain an answer to the question as to which factions of parliament the president and the governmental coalition, respectively, should form coalitions with: for the president it is better to form a coalition with an "*S*", for which *card*(*S*) < 39 (upper part of tab. 3). For the governmental coalition it is better to act in quite the reverse way (lower part of tab.3), which is obvious.

Case #3.

In this case we assume that the president conducts his voting together with the biggest opposition party, namely PiS (denoted in tab. 2 by *b*). One can find the results obtained under this assumption in tabs. 5 and 6.

Table 5. Johnston power index for case #3 with coalition ({*z*, *b*},{*a*, *d*}, *S*)

Vulnerable coalitions ({*z*, *b*},{*a*, *d*},*S*) for *S* =	card(*S*)	Fraction of critical defections	
		President *z*	Government {*a*, *d*}
1	**2**	**3**	**4**
{Ø}	0	$^1/_2$	$^1/_2$
{*g*}	3	$^1/_2$	$^1/_2$
{*e*}	5	$^1/_2$	$^1/_2$
{*f*}	5	$^1/_2$	$^1/_2$
{*h*}	9	$^1/_2$	$^1/_2$
{*e*, *g*}	8	$^1/_2$	$^1/_2$
{*f*, *g*}	8	$^1/_2$	$^1/_2$
{*e*, *f*}	10	$^1/_2$	$^1/_2$
{*g*, *h*}	12	$^1/_2$	$^1/_2$
{*e*, *f*, *g*}	13	$^1/_2$	$^1/_2$

1	2	3	4
{*e*, *h*}	14	1/2	1/2
{*f*, *h*}	14	1/2	1/2
{*f*, *g*, *h*}	17	1/2	1/2
{*e*, *f*, *h*}	19	1/2	1/2
{*e*, *f*, *g*, *h*}	22	1/2	1/2
{*c*}	42		1
{*c*, *g*}	45		1
{*c*, *e*}	47		1
{*c*, *f*}	47		1
{*c*, *e*, *g*}	50		1
{*c*, *f*, *g*}	50		1
{*c*, *h*}	51		1
{*c*, *e*, *f*}	52		1
{*c*, *g*, *h*}	54		1
{*c*, *e*, *f*, *g*}	55		1
{*c*, *e*, *h*}	56		1
{*c*, *f*, *h*}	56		1
{*c*, *f*, *g*, *h*}	59		1
{*c*, *e*, *f*, *h*}	61		1
{*c*, *e*, *f*, *g*, *h*}	64		1
	In total:	7.5	22.5

Table 6. Johnston power index for case #3 with coalition ({*a*, *d*}, *S*)

Vulnerable coalitions ({*a*, *d*}, *S*) for *S* =	card(*S*)	Fraction of critical defections		
		President *z*	Government {*a*, *d*}	Party {*c*}
{*c*}	42		1/2	1/2
{*c*, *g*}	45		1/2	1/2
{*c*, *e*}	47		1/2	1/2
{*c*, *f*}	47		1/2	1/2
{*c*, *e*, *g*}	50		1/2	1/2
{*c*, *f*, *g*}	50		1/2	1/2
{*c*, *h*}	51		1/2	1/2
{*c*, *e*, *f*}	52		1/2	1/2
{*c*, *g*, *h*}	54		1/2	1/2
{*c*, *e*, *f*, *g*}	55		1/2	1/2
{*c*, *e*, *h*}	56		1/2	1/2
{*c*, *f*, *h*}	56		1/2	1/2
{*c*, *f*, *g*, *h*}	59		1/2	1/2
{*c*, *e*, *f*, *h*}	61		1/2	1/2
{*c*, *e*, *f*, *g*, *h*}	64		1/2	1/2
	In total:	0	7.5	7.5

For case #3, the changes compared to case #2 can be seen in the possibilities for forming the coalition *S*: the number of such coalitions is less than in case #3. All the remaining conditions are unchanged. Therefore, the Johnston power indexes are as follows:

- for the *president*: 0.166667
- for the *government*: 0.666667,
- for *party* {*c*} (Leftwing): 0.166667,

which means that in the Polish legislative system and under the conditions of the sixth *Sejm*, the governmental coalition is still stronger than the president, but only 4 times stronger. This results directly from the coalition of the president with the biggest opposition party[13].

The suggested coalition partners for the president and the government, respectively, are the same as in case #2.

5. Conclusions

A summary of all the calculations of the Johnston power index under the different assumptions used are presented in tab. 7.

Table 7. Summary of the calculations of the Johnston power index under the different assumptions used (the values for the USA are taken from [2])

	Johnston power index			
	Case #1	Case #2	Case #3	USA
President	0.9234	0.0676	0.1668	0.7700
Sejm (government for case #2 and #3)	0.0766	0.9324	0.8332	0.0736
Senate	0	0	0	0.1560

On the basis of the Johnston power indexes obtained one may give the following conclusions:

1) The legislative structure (*president–Sejm–Senate* or the equivalent in the USA) without an inside party structure (case #1) are similar in Poland and the USA. One may suppose that introducing a veto overruling condition for the *Senate* in Poland would make these results even more similar.

[13] His situation is symmetric with respect to parties *b* and *c* and a coalition of the president with party c would result in the same values of the power indexes.

2) The multi-party system in the Polish parliament radically affects the values of the Johnston power index in Poland and the USA. The *a priori* power of the governmental coalition is much higher than the power of the president. This is a direct result of the fact that the governing coalition is formed by a majority parliamentary coalition.

3) An alliance of the president with one of the major opposition parties increases his power as measured by the Johnston power index. Evidently, this is important only in the situation of so called cohabitation, i.e. when the president and the governmental coalition are from opposite factions of the parliament. The Johnston power index is not suited for the case in which the president and the government represent the same political faction (party). In this case, the president would not veto a bill supported by the government.

4) It seems that the *Senate* should be empowered by enabling it to overrule vetoes by the president. The present power of the *Senate*, measured by the Johnston power index, is a good argument for its eventual liquidation.

In most cases, the power of the right to veto cannot be measured directly, because this right is only part of the characteristics of players. However, we can indirectly estimate the influence of the right to veto on the power of a player by comparing her power both with and without this right. Quite intuitively, the right of veto will increase the power of a player in most cases. It is not so obvious how large this increase will be and in some cases power is associated only with the right to veto (as is the case of the president of Poland and parliament with a party structure). This example calculates the measure of power of the right to veto in absolute terms. In most cases it is not possible to measure it so directly.

Bibliography

[1] BANZHAF J.F., *Weighted voting doesn't work: a mathematical analysis*, Rutgers Law Review, 1965, 19, No. 2 (winter), pp. 317–343.

[2] BRAMS S.J., *Negotiation games*, Routledge, New York–London, 1990.

[3] JOHNSTON R.J., *On the measurement of power: some reactions to Laver*, Environment and Planning, 1978, A10, No. 8, pp. 907–914.

[4] MERCIK J.W., *Power and expectations* (in Polish), PWN, Warszawa–Wrocław, 1999.

[5] SHAPLEY L.S., *A value for n-person games*, [in:] *Contributions to the Theory of Games*, Vol. II, by H.W. Kuhn and A.W. Tucker (editors), Annals of Mathematical Studies, 1953, 28, pp. 307–317.

[6] SHAPLEY L.S., SHUBIK M., *A method of evaluating the distribution of power in a committee system*, American Political Science Review, 1954, 48, No. 3 (September), pp. 787–792.

[7] STRAFFIN Ph.D., *Topics in the theory of voting*, Birkhauser, Boston, 1980,

[8] TURNOVEC F., MERCIK J., MAZURKIEWICZ M., *Power indices: Shapley-Shubik or Penrose-Banzhaf*? Operational Research and Systems 2004, Decision making, Methodological base and applications. (R. Kulikowski, J. Kacprzyk, R. Słowiński – eds.). Warszawa: "Exit" 2004, pp. 121–127.

[9] TURNOVEC F., MERCIK J., MAZURKIEWICZ M., *Power indices methodology: decisiveness, pivots and swings*, [in:] *Power, freedom, and voting*, Matthew Braham, Frank Steffen (eds.). Springer, Berlin–Heidelberg, 2008.

Aprioryczna siła weta prezydenta RP

W Polsce w procesie ustawodawczym ustawę przyjętą przez *Sejm* rozpatruje *Senat*, który ma prawo zgłosić do niej poprawki lub ją odrzucić (po odrzuceniu ustawa wraca do *Sejm*u, gdzie bezwzględną większością głosów można odrzucić sprzeciw *Senatu*). Uchwalona przez *Sejm* i *Senat* ustawa trafia następnie do Prezydenta RP, który może w ciągu 21 dni przyjąć i podpisać ją, bądź zgłosić weto i przekazać do ponownego rozpatrzenia *Sejmowi*.

Weto Prezydenta RP to jego kompetencja do sprzeciwu wobec ustawy uchwalonej przez *Sejm*. Zgodnie z Konstytucją RP prezydent podpisuje i zarządza ogłoszenie ustawy w Dzienniku Ustaw RP. Może on jednak, kierując się ważnym interesem państwa i dbałością o jakość stanowionego prawa, odmówić podpisania ustawy. Taka odmowa ma charakter weta zawieszającego, ponieważ *Sejm* może powtórnie uchwalić ustawę większością 3/5 głosów w obecności co najmniej połowy ustawowej liczby posłów. W tym wypadku prezydent musi w ciągu 7 dni podpisać ustawę i zarządzić jej ogłoszenie w Dzienniku Ustaw RP. Rzeczywista skuteczność sprzeciwu prezydenta jest więc uzależniona od układu sił w *Sejmie*. W pracy przedstawiono ocenę aprioryczną siły prezydenta, *Sejmu* i *Senatu* w procesie ustawodawczym w Polsce. Ocenę przeprowadzono z użyciem indeksu siły Johnstona.

Słowa kluczowe: *siła aprioryczna, indeks Johnsona, prezydent RP*