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revealed preference analysis  
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**DISCUSSION  
PAPER**

# Commitment in intertemporal household consumption: a revealed preference analysis\*

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## Abstract

We present a revealed preference methodology for analyzing intertemporal household consumption behavior. In doing so, we follow a collective approach, which explicitly recognizes that multi-member households consist of multiple decision makers with their own rational preferences. Following original work of Mazzocco (2007), we develop tests that can empirically verify whether observed consumption behavior is consistent with (varying degrees of) intrahousehold commitment. In

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our set-up, commitment means that households choose consumption allocations on the ex ante Pareto frontier. The distinguishing feature of our tests is that they are entirely nonparametric, i.e. their implementation does not require an a priori (typically non-verifiable) specification of the intrahousehold decision process (e.g. individual utilities). We demonstrate the practical usefulness of our methodology by means of an empirical application. For the data at hand, our results suggest using a so-called limited commitment model that allows for household-specific commitment patterns. Importantly, our application also shows that bringing intertemporal dynamics in the empirical analysis can substantially increase the discriminatory power of the revealed preference methodology.

**JEL Classification:** D11, D12, D13, C14.

**Keywords:** collective models of household consumption, intertemporal consumption, commitment, revealed preferences.

## 1 Introduction

There is a growing consensus that a realistic modeling of household consumption behavior must take into account preference heterogeneity within the household: in many cases, the household decision-making process cannot be explained by the restrictive unitary framework, which models the household as if it were a single decision maker.<sup>1</sup> In addition, many household decisions are inherently intertemporal. For example, such intertemporal interdependence typically applies to decisions on family savings, investment in human capital, housing purchase, fertility decisions, etc. This directly extends to a consumption setting. In many cases, consumption today impacts on consumption tomorrow and vice versa (e.g. because of (dis)saving possibilities).

This paper presents a revealed preference methodology for analyzing intertemporal consumption decisions of multi-person households. Specifically, we develop a methodology for revealed preference analysis in terms of the intertemporal ‘collective’ framework set out by Mazzocco (2007). In this introductory section, we will briefly recapture Mazzocco’s main ideas and point

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<sup>1</sup>Many studies reject the empirical validity of the unitary model for multi-person household behavior. See, for example, Lundberg (1988), Thomas (1990), Fortin and Lacroix (1997), Browning and Chiappori (1998), Chiappori, Fortin and Lacroix (2002), Duflo (2003) and Cherchye and Vermeulen (2008).

out our own methodological contribution. In addition, we will indicate an important empirical motivation for our following analysis. Specifically, as we will demonstrate, bringing intertemporal dynamics in the empirical analysis can substantially enhance the ‘discriminatory power’ of the revealed preference methodology. In our opinion, this is a most interesting observation, as low power is a frequently cited concern for static (or atemporal) revealed preference analysis.

**Intertemporal consumption and commitment.** Mazzocco (2007) introduced a household consumption model that simultaneously accounts for the non-unitary and intertemporal nature of household consumption decisions. He adopts a collective approach, which means that he explicitly recognizes that a multi-person household consists of multiple members (decision makers) with their own preferences. Mazzocco’s model considers the observed household consumption behavior as the result of a within-household bargaining process between these members. It (only) assumes that this process yields a Pareto efficient within-household allocation.<sup>2</sup> The household’s location on the Pareto frontier is then defined by the distribution of the within-household bargaining power over the individual household members; and this power distribution may vary depending on the specific situation at hand. In particular, shifts in the bargaining power can be induced by changes of so-called distribution factors (which influence the bargaining power distribution but not the household members’ preferences). In Mazzocco’s original model these distribution factors are explicitly taken up in the intertemporal decision model (which will constitute an important difference with the approach we follow in this paper; see below). In his analysis, Mazzocco adopts the life cycle model for describing intertemporal consumption decisions; and, thus, he obtains a collective extension of the widely applied unitary life cycle consumption model.<sup>3</sup>

More specifically, Mazzocco develops empirical tests for two versions of his intertemporal collective consumption model: a ‘full commitment’ model and a ‘no commitment’ model. In the full commitment model, household con-

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<sup>2</sup>Chiappori (1988, 1992) originally suggested this collective approach for analyzing a household’s labor supply behavior (in a static setting); see also Apps and Rees (1988) for a similar analysis. Browning and Chiappori (1998) extended Chiappori’s original analysis to apply to the more general setting of household consumption behavior (again in a static setting); see also Chiappori and Ekeland (2006, 2009) for closely related contributions.

<sup>3</sup>See, for example, Browning and Crossley (2001) for an extensive overview of the literature on the unitary life cycle model.

sumption allocations are assumed to be situated on the ex ante Pareto frontier: household members commit themselves not to respond to unexpected shocks of the variables affecting the bargaining power. In the no commitment model, the ex ante Pareto frontier is replaced by the ex post Pareto frontier: household allocations no longer satisfy the stated commitment criterion but they still meet Pareto efficiency. More precisely, it is assumed that, if some unexpected shock makes an individual participation constraint binding, the household members renegotiate to decide on the intrahousehold allocations. We will briefly recapture Mazzocco’s full and no commitment models in Section 2, where we will also indicate how these models relate to the different models (with varying commitment) considered in the present study.

**Revealed preference analysis.** In Sections 3 and 4, we will consider the revealed preference analysis of models characterized by varying degrees of commitment. This will extend Mazzocco’s analysis in two ways. Firstly, we present a revealed preference characterization (and corresponding tests) of intertemporal collective consumption behavior. In the tradition of Afriat (1967) and Varian (1982),<sup>4</sup> we derive necessary and sufficient conditions for household consumption data to be consistent with a particular behavioral model. These conditions then enable checking consistency of a given data set with the model; in the spirit of Varian (1982), we refer to this as ‘testing’ data consistency with the model under study.<sup>5</sup> Attractively, our revealed preference approach enables a full nonparametric verification of these conditions, i.e. the associated tests do not require an a priori (typically non-verifiable) parametric specification of the intrahousehold decision process (e.g. individual preferences). This contrasts with the tests that were originally proposed by Mazzocco (2007), which do require such a parametric specification in practice.<sup>6</sup>

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<sup>4</sup>See also Samuelson (1948), Houthakker (1950) and Diewert (1973) for seminal contributions on the revealed preference approach to analyzing consumption behavior.

<sup>5</sup>As is standard in the revealed preference literature, the type of tests that we consider here are ‘sharp’ tests; either a data set satisfies the data consistency conditions or it does not.

<sup>6</sup>In particular, Mazzocco originally developed a so-called ‘differential’ characterization of intertemporal consumption behavior in a collective framework. The specific feature of this differential approach is that it focuses on properties of a function representing household consumption behavior (e.g. cost, indirect utility and demand functions). Practical applications of this approach typically require a prior parametric specification of the function subject to study. By contrast, the revealed preference approach that we follow here (only) uses a finite set of household consumption observations.

Our second extension pertains to the modeling of commitment. In contrast to Mazzocco’s original work, we no longer need that deviations from commitment are modeled explicitly in terms of (observed) distribution factors. As we will argue in Section 2, the requirement that all relevant distribution factors are effectively observed is quite demanding. To account for this, our revealed preference tests solely use information on the household’s observed consumption quantities and the corresponding prices. As such, their practical implementation does not need any explicit information on distribution factors.

Three preliminary remarks are in order with respect to our following revealed preference analysis. Firstly, this analysis is directly related to earlier work of Browning (1989) and Cherchye, De Rock and Vermeulen (2007, 2011). Browning provided a revealed preference characterization of the unitary life cycle model.<sup>7</sup> We extend this characterization to a collective setting. Next, Cherchye, De Rock and Vermeulen established the revealed preference characterization of the static collective consumption model. We add to this work by accounting for intertemporal relations between household consumption decisions.

Secondly, we will follow Browning (1989) by assuming perfect capital markets and perfect foresight with regard to all relevant economic variables, and by using intertemporally separable utility functions to represent the individual preferences. Admittedly, these are very strong assumptions. However, as pointed out by Browning (in *casu* in a unitary context), the flip side of the coin is that our method allows for testing the collective model of intertemporal behavior without requiring an *ad hoc* functional specification or error process. Thus, if we can accept our strong assumptions for some given data set, then any test of weaker assumptions on the same data can be interpreted as a test of the functional form that is adopted rather than the behavioral assumptions as such.

Finally, our following analysis will concentrate on the characterization of intertemporal consumption models, and testing consistency of observed behavior with alternative model specifications. If observed behavior is consistent with a particular model, then a natural next question pertains to

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<sup>7</sup>Crawford (2010) provides another, more recent contribution on revealed preference analysis of the unitary life cycle model. Specifically, he extended Browning’s original characterization by accounting for habit formation in the household consumption. We see the integration of Crawford’s analysis with the analysis developed in this paper as an interesting avenue for future research.

recovering/identifying the decision model that underlies the observed consumption behavior (e.g. individual preferences). To focus our discussion, such recovery will not be studied here. However, it is worth emphasizing that our revealed preference characterizations do allow for subsequent recovery analysis. For example, Varian (1982) and, more recently, Blundell, Browning and Crawford (2003, 2008) considered such recovery (based on revealed preferences) for the unitary consumption model, and Cherchye, De Rock and Vermeulen (2011) studied similar recovery issues for the static collective model. The analyses of these authors can be extended to the current setting when starting from the revealed preference characterizations established below.

**Empirical performance: goodness-of-fit versus power.** In Section 5, we will demonstrate the practical usefulness of our revealed preference methodology by an empirical application to panel data drawn from the Russian Longitudinal Monitoring Survey (RLMS). A main conclusion will be that adding intertemporal dynamics to the collective model substantially increases the discriminatory power of the revealed preference analysis. This is an important point to make, as lack of power is often mentioned as an important weakness of revealed preference tests. Bronars (1987) and, more recently, Andreoni and Harbaugh (2008) and Beatty and Crawford (2011) -rather convincingly- argue the need to complement the basic revealed preference test results (indicating pass or fail of the data for some behavioral model) with measures of discriminatory power if one wants a fair empirical assessment of the model under evaluation.<sup>8</sup> Indeed, favorable test results (i.e. a high pass rate for some given data), which prima facie suggest a good empirical fit of the model under study, have little value if the test has little discriminatory power (i.e. the model is hard to reject for the data at hand).

This argument pro conducting power assessments in addition to goodness-of-fit evaluations is particularly valid in the context of revealed preference analysis of collective consumption models. The few existing empirical studies that make use of revealed preference methods to test the static collective model suggest a need for information on assignable goods (i.e. goods for

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<sup>8</sup>In fact, Beatty and Crawford (2010) also propose a measure for the ‘predictive’ success of a model, which is computed by combining power and pass rates for the corresponding revealed preference tests. For compactness, we will not explicitly use this predictive success measure in our empirical analysis in Section 5. But, in principle, the measure can easily be computed from the results that are reported.

which the consumption of individual household members is known) to obtain reasonable power results; see, for example, Cherchye, De Rock and Vermeulen (2009, 2011). To some extent, this is problematic as such assignable information is often not available in household budget surveys.<sup>9</sup> In our empirical application, we will show that an intertemporal collective model does have powerful empirical restrictions even if no assignable quantity information is used. We then evaluate alternative intertemporal collective consumption models (with varying commitment) by simultaneously considering their goodness-of-fit and power. Our findings suggest that a model with ‘limited’ commitment (i.e. situated between full and no commitment) provides the best description of the particular data at hand.

The concluding Section 6 summarizes our main findings and suggests some avenues for further research. Appendix A contains the proofs of our theoretical results.

## 2 Intertemporal collective consumption

We consider a household with  $M$  members. The household has to decide over the consumption of a bundle of  $N$  private goods and a bundle of  $K$  public goods. Given that there is private and public consumption in the household, the utility of each member  $m$  is given by the function  $u^m(\mathbf{q}^m, \mathbf{Q})$ , with  $\mathbf{q}^m \in \mathbb{R}_+^N$  the private consumption bundle of  $m$  and  $\mathbf{Q} \in \mathbb{R}_+^K$  the public consumption bundle.<sup>10</sup> Throughout, we will assume that each utility function  $u^m$  is continuous, concave, non-satiated and non-decreasing in its arguments.

The empirical analysis of household consumption starts from a time series of consumption choices. Let  $T = \{1, \dots, |T|\}$  represent such a time series, which pertains to a given number of periods  $|T|$ . We thus have a set of observations  $S = \{\mathbf{p}_t, \mathbf{P}_t, \mathbf{q}_t, \mathbf{Q}_t | t \in T\}$ . For each period  $t \in T$ , the vectors  $\mathbf{q}_t (= \sum_{m=1}^M \mathbf{q}_t^m)$  and  $\mathbf{Q}_t$  represent the chosen household bundles of private and public goods; we use  $\mathbf{p}_t \in \mathbb{R}_{++}^N$  for the discounted price vector of the

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<sup>9</sup>However, data sets with detailed assignable information are increasingly available in the literature. See, for example, Browning and Gørtz (2006) and Cherchye, De Rock and Vermeulen (2010).

<sup>10</sup>We will abstract from externalities associated with privately consumed quantities. Importantly, however, our setting can actually account for such externalities. Specifically, if an individual is the exclusive consumer of a particular private good, then we can account for externalities for this good by formally treating it as a public good.



private commodities and  $\mathbf{P}_t \in \mathbb{R}_{++}^K$  for the discounted price vector of the public commodities. Note that we make the standard assumption that we only observe  $\mathbf{q}_t$  and not the intrahousehold allocation  $\mathbf{q}_t^m$ ; i.e. we assume the individual consumption of private goods is not observed and, thus, no goods are assignable to individual household members. In what follows, we will use  $y_t (= \mathbf{p}'_t \mathbf{q}_t + \mathbf{P}'_t \mathbf{Q}_t)$  for the household's income at period  $t$  and  $Y (= \sum_{t=1}^{|T|} y_t)$  for the household's total income.

## 2.1 Intertemporal collective model

As indicated in the Introduction, we adopt a life cycle approach for modeling intertemporal collective consumption behavior. Specifically, we make the same basic assumptions as Browning (1989) used for his revealed preference analysis of intertemporal consumption behavior in a unitary setting: we assume household members have perfect foresight, face perfect capital markets, and are endowed with preferences that can be represented by an intertemporally (additively) separable utility function. Next, following Mazzocco (2007), for each member  $m$  we assume exponential utility discounting on the basis of a time-invariant discount factor  $\beta^m \in ]0, 1]$ . Given all this, a notable difference between our set-up and the one of Mazzocco is that we assume perfect foresight. We will return to this difference below.

A specific feature of the collective model is that it assumes Pareto optimal intrahousehold allocations. Formally, using intertemporal separability, this implies for each period  $t$  that there exist bargaining (or Pareto) weights  $\mu_t^m$  and quantities  $\mathbf{q}_t^m$  (with  $\sum_{m=1}^M \mathbf{q}_t^m = \mathbf{q}_t$ ) such that

$$\{\mathbf{q}_t^1, \dots, \mathbf{q}_t^M, \mathbf{Q}_t\} \in \arg \max_{\mathbf{q}^1, \dots, \mathbf{q}^M, \mathbf{Q}} \sum_{m=1}^M \mu_t^m (\beta^m)^{t-1} u^m(\mathbf{q}^m, \mathbf{Q})$$

$$s.t. \mathbf{p}'_t \left( \sum_{m=1}^M \mathbf{q}_t^m \right) + \mathbf{P}'_t \mathbf{Q} \leq y_t.$$

In our life cycle approach, we simultaneously consider all periods  $t$  in  $T$  and account for the household's total income  $Y$ . The intertemporal intrahousehold allocation must then satisfy the following condition: for each period  $t$  there exist weights  $\mu_t^m$  and quantities  $\mathbf{q}_t^m$  (with  $\sum_{m=1}^M \mathbf{q}_t^m = \mathbf{q}_t$ ) such

that

$$\begin{aligned} \{\mathbf{q}_t^1, \dots, \mathbf{q}_t^M, \mathbf{Q}_t\}_{t \in T} \in \arg \max_{\mathbf{q}^1, \dots, \mathbf{q}^M, \mathbf{Q}} \sum_{t=1}^{|T|} \sum_{m=1}^M \mu_t^m (\beta^m)^{t-1} u^m(\mathbf{q}^m, \mathbf{Q}) \\ s.t. \sum_{t=1}^{|T|} \mathbf{p}'_t \left( \sum_{m=1}^M \mathbf{q}^m \right) + \mathbf{P}'_t \mathbf{Q} \leq Y. \end{aligned}$$

## 2.2 Commitment

In Mazzocco's (2007) model of intertemporal collective consumption behavior, commitment is the key notion. To introduce this concept in our present set-up, we partition the set  $T$  (representing the full period) in  $\Upsilon$  mutually exclusive subsets  $T_\tau$  (representing sub-periods). More formally, we consider partitions  $\mathbf{T}$  of the following form:

$$\begin{aligned} \mathbf{T} = \{T_1, \dots, T_\Upsilon\} \text{ (with } T = \bigcup_{\tau=1}^\Upsilon T_\tau \text{ and } T_{\tau_1} \cap T_{\tau_2} = \emptyset \text{ if } \tau_1 \neq \tau_2) \text{ such that} \\ \tau_1 < \tau_2 \text{ implies } t_1 < t_2 \text{ for all } t_1 \in T_{\tau_1} \text{ and } t_2 \in T_{\tau_2}. \end{aligned}$$

Using this notation, we assume commitment over each subset  $T_\tau$ . This means that the bargaining weights  $\mu^m$  are held constant over all periods  $t$  included in the subset (or sub-period)  $T_\tau$ . For a given partition  $T$ , commitment then requires the following restriction for the intrahousehold allocation: there must exist quantities  $\mathbf{q}_t^m$  (with  $\sum_{m=1}^M \mathbf{q}_t^m = \mathbf{q}_t$ ) for each period  $t$  and bargaining weights  $\mu_\tau^m$  for each subset  $T_\tau$  such that

$$\begin{aligned} \{\mathbf{q}_t^1, \dots, \mathbf{q}_t^M, \mathbf{Q}_t\}_{t \in T} \in \arg \max_{\mathbf{q}^1, \dots, \mathbf{q}^M, \mathbf{Q}} \sum_{m=1}^M \sum_{\tau=1}^\Upsilon \mu_\tau^m \sum_{t \in T_\tau} (\beta^m)^{t-1} u^m(\mathbf{q}^m, \mathbf{Q}) \quad (\text{LC}) \\ s.t. \sum_{t=1}^{|T|} \mathbf{p}'_t \left( \sum_{m=1}^M \mathbf{q}^m \right) + \mathbf{P}'_t \mathbf{Q} \leq Y. \end{aligned}$$

In the collective model, the concept of commitment is intrinsically related to so-called distribution factors, which influence the bargaining weights (represented by  $\mu^m$ ) but not the member's preferences (represented by  $u^m$ ). These factors include, for example, exogenous income of individual house-

hold members, sex ratio within the relevant region, the country’s divorce legislation, etc. (see, for example, Browning, Bourguignon, Chiappori and Lechene (1994) and Bourguignon, Browning and Chiappori (2009) for more discussion). By assuming commitment for a subset of periods  $T_\tau$ , we keep the bargaining weight constant for all periods  $t$  in  $T_\tau$ . The interpretation is that household members commit themselves to defining their bargaining weights for consumption decisions in  $T_\tau$  (only) on the basis of distribution factors that realize in this sub-period.

In this respect, it is worth to point out that Mazzocco (2007) distinguishes between a ‘full commitment’ model and a ‘no commitment’ model. We will adopt a similar distinction below. In doing so, we must account for the fact that Mazzocco did not assume perfect foresight, while we do use perfect foresight as a maintained hypothesis. In Mazzocco’s full commitment model, household members account for all expected shocks over the full period, and commitment then means that household members cannot respond to unexpected shocks in the distribution factors. In our set-up (with perfect foresight), this model complies with a fixed bargaining weight over all periods  $t$  in  $T$  (or  $\mathbf{T} = \{T\}$  and  $\Upsilon = 1$ ). Next, in Mazzocco’s no commitment model, the members can renegotiate the decisions after an unexpected shock in the distribution factors but only if some participation constraint becomes binding. This complies with the general version of our limited commitment model (with  $\Upsilon \geq 1$ ) as it is presented above. Below, we will introduce a different ‘no commitment’ model as a limiting case of this general model.

At this point, we want to stress that our revealed preference characterizations will make no explicit use of distribution factors. Our motivation is both empirical and theoretical. From an empirical point of view, this minimizes the data requirement: we no longer need to observe these distribution factors, and it is well known that useful distribution factors are often hard to find in empirical studies. From a theoretical point of view, abstracting from distribution factors implies that we do not explicitly model how a partition  $\mathbf{T}$  is obtained. Our following revealed preference characterization will allow us to verify whether or not some given partition  $\mathbf{T}$  obtains consistency with the intertemporal collective model. (Interestingly, as we will illustrate in our empirical application, our characterization will also allow us to endogenously define a partition  $\mathbf{T}$  consistent with the model.) An attractive by-product is that this allows the bargaining power to depend not only on observed distribution factors (such as the ones listed above) but also on distribution factors that are typically not observed (such as love, health status, ...).

To end this section, we introduce two limiting cases of the general limited commitment model, i.e. a full commitment model and a no commitment model (which is actually a limiting case of Mazzocco's no commitment model mentioned above). These cases will be useful to structure our following discussion. As discussed before, full commitment means that the bargaining weight is constant over the full period, which obtains  $\mathbf{T} = \{T\}$  and  $\Upsilon = 1$ . Formally, this means that there exist bargaining weights  $\mu^m$  such that for each period  $t$  the household consumes private quantities  $\mathbf{q}_t^m$  and public quantities  $\mathbf{Q}_t$  that satisfy

$$\begin{aligned} \{\mathbf{q}_t^1, \dots, \mathbf{q}_t^M, \mathbf{Q}_t\}_{t \in T} \in \arg \max_{\mathbf{q}^1, \dots, \mathbf{q}^M, \mathbf{Q}} \sum_{m=1}^M \mu^m \sum_{t=1}^{|T|} (\beta^m)^{t-1} u^m(\mathbf{q}^m, \mathbf{Q}) \quad (\text{FC}) \\ s.t. \sum_{t=1}^{|T|} \mathbf{p}'_t \left( \sum_{m=1}^M \mathbf{q}^m \right) + \mathbf{P}'_t \mathbf{Q} \leq Y. \end{aligned}$$

By contrast, in our no commitment model the bargaining weights can change in each different decision period  $t$ , which corresponds to  $\mathbf{T} = \{\{1\}, \dots, \{|T|\}\}$  and  $\Upsilon = |T|$ . In this case, there exist bargaining weights  $\mu_t^m$  such that in each period  $t$  the household chooses private quantities  $\mathbf{q}_t^m$  and public quantities  $\mathbf{Q}_t$  that satisfy

$$\begin{aligned} \{\mathbf{q}_t^1, \dots, \mathbf{q}_t^M, \mathbf{Q}_t\}_{t \in T} \in \arg \max_{\mathbf{q}^1, \dots, \mathbf{q}^M, \mathbf{Q}} \sum_{m=1}^M \sum_{t=1}^{|T|} \mu_t^m (\beta^m)^{t-1} u^m(\mathbf{q}^m, \mathbf{Q}) \quad (\text{NC}) \\ s.t. \sum_{t=1}^{|T|} \mathbf{p}'_t \left( \sum_{m=1}^M \mathbf{q}^m \right) + \mathbf{P}'_t \mathbf{Q} \leq Y. \end{aligned}$$

### 3 Revealed preference characterization

This section develops the revealed preference characterization of our intertemporal collective model with limited commitment. This characterization implies testable conditions for data consistency with the model while avoiding an a priori (typically parametric) specification of the utility functions  $u^m$  and the bargaining weights  $\mu^m$ . In our analysis below, we consider the discount factors  $\beta^m$  and the partition  $\mathbf{T}$  as given. In our empirical application

in Section 5, we will consider several scenarios for the partition  $\mathbf{T}$ , which basically imply alternative specifications of the general limited commitment model. In this empirical exercise, we will also consider a number of different values for  $\beta^m$ , so to identify the values that best fit the data at hand.<sup>11</sup>

Before introducing the revealed preference characterization of the intertemporal collective model, we first define the basic optimality criterion for an intertemporal intrahousehold allocation in our set-up. Following the arguments of Bewley (1977) and Hall (1978), the life-cycle hypothesis implies that an optimal allocation smooths the marginal utility of income in every time period. The following Strong Rational Expectations Hypothesis (SREH) defines this criterion for our specific setting (using  $(\mathbf{x})_z$  for the  $z$ -th entry of a vector  $\mathbf{x}$ ).

**Definition 1** *Consider a set of observations  $S$ , a partition  $\mathbf{T}$  and discount factors  $\beta^m$ . Suppose further that each member  $m$  is endowed with a utility function  $u^m$ , and that each subset  $T_\tau$  is associated with bargaining weights  $\mu_\tau^m$ .*

*Then, the Strong Rational Expectations Hypothesis (SREH) is satisfied if, for each subset  $T_\tau$ , there exists  $\lambda_\tau \in \mathbb{R}_+$  such that, for  $t \in T_\tau$ ,*

$$\begin{aligned} \frac{\partial}{\partial (q_t)_n} \left( \sum_{m=1}^M \mu_\tau^m (\beta^m)^{t-1} u^m(\mathbf{q}_t^m, \mathbf{Q}_t) \right) &= \lambda_\tau (p_t)_n, \text{ and} \\ \frac{\partial}{\partial (Q_t)_k} \left( \sum_{m=1}^M \mu_\tau^m (\beta^m)^{t-1} u^m(\mathbf{q}_t^m, \mathbf{Q}_t) \right) &= \lambda_\tau (P_t)_k \end{aligned}$$

*for all private goods  $n \in \{1, \dots, N\}$  and public goods  $k \in \{1, \dots, K\}$ .*

Clearly, direct verification of SREH requires a specification of the utility functions  $u^m$  and the bargaining weights  $\mu^m$ . As indicated above, the revealed preference approach that we follow here avoids such a specification. Essentially, it defines conditions that must be satisfied such that there exists a possible specification of the functions  $u^m$  and weights  $\mu^m$  that obtains consistency with SREH. These conditions will be given in the following Theorem 1. Before stating this theorem, we first define the concept of limited commitment (LC) rationalizability.

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<sup>11</sup>Crawford (2010) followed a similar approach to deal with utility discounting in his revealed preference analysis of unitary intertemporal consumption behavior.

**Definition 2** Consider a set of observations  $S$ , a partition  $\mathbf{T}$  and discount factors  $\beta^m$ . The set  $S$  is LC-rationalizable if there exist utility functions  $u^m$ , bargaining weights  $\mu_\tau^m \in \mathbb{R}_{++}$ , and private consumption bundles  $\mathbf{q}_t^m \in \mathbb{R}_+^N$  (with  $\sum_{m=1}^M \mathbf{q}_t^m = \mathbf{q}_t$ ) such that the set  $\{\mathbf{q}_t^1, \dots, \mathbf{q}_t^M, \mathbf{Q}_t\}_{t \in T}$  satisfies (LC).

We can now formulate the characterization of LC-rationalizable consumption behavior. Below, we will explain the relation between this characterization and the SREH condition in Definition 1.

**Theorem 1** Consider a set of observations  $S$ , a partition  $\mathbf{T}$  and discount factors  $\beta^m$ . The following conditions are equivalent:

- (i) The set  $S$  is LC-rationalizable.
- (ii) There exist  $\mathbf{P}_t^m \in \mathbb{R}_+^K$ ,  $\mathbf{q}_t^m \in \mathbb{R}_+^N$  and  $\lambda_\tau^m, u_t^m \in \mathbb{R}_{++}$  such that the following conditions hold for all  $m \in \{1, \dots, M\}$ ,  $\tau \in \{1, \dots, \Upsilon\}$ ,  $t \in T_\tau$  and  $s \in \{1, \dots, |T|\}$ :

$$\begin{aligned}
(a) \quad & \mathbf{P}_t = \sum_{m=1}^M \mathbf{P}_t^m; \\
(b) \quad & \mathbf{q}_t = \sum_{m=1}^M \mathbf{q}_t^m; \\
(c) \quad & u_s^m - u_t^m \leq \frac{\lambda_\tau^m}{(\beta^m)^{t-1}} [(\mathbf{p}'_t \mathbf{q}_s^m + (\mathbf{P}_t^m)' \mathbf{Q}_s) - (\mathbf{p}'_t \mathbf{q}_t^m + (\mathbf{P}_t^m)' \mathbf{Q}_t)].
\end{aligned}$$

The interpretation of conditions (a), (b) and (c) in statement (ii) is as follows. First, the prices  $\mathbf{P}_t^m$  in condition (a) express the marginal willingness to pay of each member  $m$  for the publicly consumed quantities  $\mathbf{Q}_t$ . They can be interpreted as Lindahl prices since they must add up to the observed prices. Cherchye, De Rock and Vermeulen (2007, 2011) use a similar concept of Lindahl prices for the public goods in their revealed preference characterization of the static collective consumption model. Next, condition (b) follows from the fact that we do not observe the intrahousehold allocation of the private goods. As such, we need that there exists at least one feasible allocation. Finally, condition (c) states so-called Afriat inequalities (after Varian, 1982; based on Afriat, 1967) for our intertemporal collective model with limited commitment. These inequalities provide an explicit construction of the utility levels ( $u_t^m$ ) associated with each member  $m$  and observation  $t$  for the given data set  $S$ . Condition (c) introduces SREH in our characterization of the limited commitment model (see in particular the variables  $\lambda_\tau^m$  that apply to each subset  $T_\tau$ ). In the next section, we will provide further intuition for these Afriat inequalities.

## 4 Practical tests

Statement (ii) in Theorem 1 implies conditions for data consistency with the limited commitment model that do not require a prior specification of the utility functions  $u^m$  and the bargaining weights  $\mu^m$ . This is attractive from a conceptual point of view, as such a specification is typically nonverifiable in practical applications. However, from an empirical point of view, the characterization in Theorem 1 is of limited use. The reason is that condition (c) is nonlinear in the unknown variables  $\lambda_\tau^m$ ,  $\mathbf{P}_t^m$  and  $\mathbf{q}_s^m$ , which makes it difficult to verify this condition for a given data set. In this section, we will consider reformulations of the conditions in Theorem 1 that do have practical usefulness because they no longer involve nonlinearities. Specifically, we first consider the two limiting cases that we also discussed before, i.e. the full commitment model and the no commitment model. Subsequently, we introduce linear conditions for the general version of the limited commitment model.

### 4.1 Full commitment

In the full commitment model the bargaining weights  $\mu^m$  are constant over all decision periods (i.e.  $\mathbf{T} = \{T\}$  and  $\Upsilon = 1$ ). Building on Theorem 1, we obtain the following characterization.

**Proposition 1** *Consider a set of observations  $S$ , a partition  $\mathbf{T}$  and discount factors  $\beta^m$ . The following conditions are equivalent:*

- (i) *The set  $S$  is FC-rationalizable.*
- (ii) *There exist  $\mathbf{P}_t^m \in \mathbb{R}_+^K$ ,  $\mathbf{q}_t^m \in \mathbb{R}_+^N$  and  $u_t^m \in \mathbb{R}_{++}$  such that the following conditions hold for all  $m \in \{1, \dots, M\}$  and  $t, s \in \{1, \dots, |T|\}$ :*

$$\begin{aligned}
 (a) \quad & \mathbf{P}_t = \sum_{m=1}^M \mathbf{P}_t^m; \\
 (b) \quad & \mathbf{q}_t = \sum_{m=1}^M \mathbf{q}_t^m; \\
 (c) \quad & u_s^m - u_t^m \leq \frac{1}{(\beta^m)^{t-1}} [(\mathbf{p}'_t \mathbf{q}_s^m + (\mathbf{P}_t^m)' \mathbf{Q}_s) - (\mathbf{p}'_t \mathbf{q}_t^m + (\mathbf{P}_t^m)' \mathbf{Q}_t)].
 \end{aligned}$$

The main difference with the characterization in Theorem 1 pertains to condition (c) in statement (ii). Under full commitment, this condition no longer involves a specification of  $\lambda_\tau^m$ . The underlying argument is that, under full commitment, SREH implies that each member's marginal utility of

income must be constant over all time periods  $t \in T$ . In terms of condition (c) in Theorem 1, this implies a single value  $\lambda^m$  for each member  $m$ . As shown in the proof of Proposition 1, we can then drop  $\lambda^m$  altogether in the Afriat inequalities, so to obtain condition (c) in Proposition 1.

The characterization in statement (ii) of Proposition 1 is particularly attractive from a computational point of view. For a given specification of  $\beta^m$ , it implies a set of Afriat inequalities that are linear in terms of the unknowns  $\mathbf{P}_t^m$ ,  $\mathbf{q}_t^m$  and  $u_t^m$ . Thus, a practical test of the full commitment model can use standard linear programming techniques to verify the inequalities.

As a final remark, we indicate that the revealed preference characterization in Proposition 1 is formally close to Browning's (1989) revealed preference characterization of rational intertemporal behavior in a unitary context. In fact, Browning also provided an equivalent characterization in terms of so-called cyclical monotonicity conditions. It is easy to verify that the characterization in statement (ii) of Proposition 1 can equally be expressed in terms of such cyclical monotonicity conditions; in this case, we will get such a condition for each  $m$ . Interestingly, these cyclical monotonicity conditions will also imply a set of linear inequalities, which can thus be checked through linear programming. However, the verification of these conditions is computationally more complex than checking the Afriat inequalities in Proposition 1. Basically, for a given data set the number of cyclical monotonicity inequalities will generally be (often substantially) higher than the number of Afriat inequalities. Therefore, we choose to focus on the Afriat inequalities in this paper.

## 4.2 No commitment

In the no commitment model the bargaining weights can be different in each decision period (i.e  $\mathbf{T} = \{\{1\}, \dots, \{|T|\}\}$  and  $\Upsilon = |T|$ ). We now get the following characterization.

**Proposition 2** *Consider a set of observations  $S$ , a partition  $\mathbf{T}$  and discount factors  $\beta^m$ . The following conditions are equivalent:*

- (i) *The set  $S$  is NC-rationalizable.*
- (ii) *There exist  $\mathbf{P}_t^m \in \mathbb{R}_+^K$ ,  $\mathbf{q}_t^m \in \mathbb{R}_+^N$  and  $\lambda_t^m, u_t^m \in \mathbb{R}_{++}$  such that the*



following conditions hold for all  $m \in \{1, \dots, M\}$  and  $t, s \in \{1, \dots, T\}$ :

$$\begin{aligned}
 (a) \quad & \mathbf{P}_t = \sum_{m=1}^M \mathbf{P}_t^m; \\
 (b) \quad & \mathbf{q}_t = \sum_{m=1}^M \mathbf{q}_t^m; \\
 (c) \quad & u_s^m - u_t^m \leq \lambda_t^m [(\mathbf{p}_t' \mathbf{q}_s^m + (\mathbf{P}_t^m)' \mathbf{Q}_s) - (\mathbf{p}_t' \mathbf{q}_t^m + (\mathbf{P}_t^m)' \mathbf{Q}_t)].
 \end{aligned}$$

As compared to the characterization in Theorem 1, the Afriat inequalities in Proposition 2 no longer include discount factors. The reason is that we now have a different value for  $\lambda_t^m$  associated with each observation  $t$ . As shown in the proof of Proposition 2, these (observation specific) variables  $\lambda_t^m$  also incorporate the (observation specific) discount factors.

In fact, statement (ii) of Proposition 2 implies a characterization for the no commitment model that is formally equivalent to the one obtained by Cherchye, De Rock and Vermeulen (2011) for the static collective model. As such, we conclude that the static collective model and the no commitment model are empirically indistinguishable. This may seem paradoxical at first sight, because the structural decision problems that underlie the two models are intrinsically different. The explanation is that SREH has no testable implications if the bargaining weights can alter in each different period, which is the case under no commitment. Indeed, given that the empirical researcher cannot observe the bargaining weights, (s)he cannot conclude for the data at hand whether the observed consumption patterns are either explained by consumption smoothing or changing bargaining weights.

Finally, as for empirical verification, we note that the Afriat inequalities in Proposition 2 are nonlinear in the unknown variables  $\lambda_t^m$ ,  $\mathbf{P}_t^m$  and  $\mathbf{q}_t^m$ . Interestingly, however, in their analysis of the (empirically equivalent) static collective model, Cherchye, De Rock and Vermeulen (2011) have shown that the conditions in statement (ii) of Proposition 2 can equivalently be formulated as mixed integer programming (MIP) constraints, which can be checked through standard MIP techniques. Given this, we conclude that the nonlinear Afriat inequalities in Proposition 2 can equivalently be expressed in MIP terms. In turn, this implies a practical test for data consistency with the no commitment model.

### 4.3 Limited commitment

For the general case of limited commitment, verifying the revealed preference conditions in Theorem 1 implies solving a system of nonlinear inequalities; and this system cannot be reformulated in MIP terms. To see this last point, we note that the Afriat inequalities contain multiple values of  $\lambda_\tau^m$ , and each value  $\lambda_\tau^m$  simultaneously applies to a subset of observations (i.e. all observations  $t \in T_\tau$ ). This has two implications. Firstly, the fact that the Afriat inequalities contain multiple  $\lambda_\tau^m$  makes that we cannot just drop these variables as in the full commitment case. Secondly, because each  $\lambda_\tau^m$  simultaneously applies to different observations, it is impossible to apply the insights of Cherchye, De Rock and Vermeulen (2011) to equivalently reformulate the Afriat inequalities in MIP terms.

All this makes verifying the necessary and sufficient conditions in Theorem 1 a difficult matter. Therefore, in our empirical application we will focus on linear conditions that are necessary (but not sufficient) for data consistency with the limited commitment model. These conditions no longer account for the fact that the Afriat inequalities pertaining to different subsets of observations  $T_\tau$  are interrelated. More precisely, we solve the Afriat inequalities for each subset separately: for each  $t \in T_\tau$  we (only) consider the Afriat inequalities pertaining to other observations  $s$  that belong to the same subset  $T_\tau$  (rather than to all other observations  $s \in \{1, \dots, T\}$ ). As such, our necessary condition essentially requires that observed household behavior is consistent with the full commitment model (and the associated SREH condition) only for each separate subset of observations  $T_\tau$ . In fact, in contrast to the necessary and sufficient condition in Theorem 1, this necessary condition allows the individual utility functions ( $u^m$ ) to be different in each other sub-period.<sup>12</sup>

**Proposition 3** *Consider a set of observations  $S$ , a partition  $\mathbf{T}$  and discount factors  $\beta^m$ . The set  $S$  is LC-rationalizable only if there exist  $\mathbf{P}_t^m \in \mathbb{R}_+^K$ ,  $\mathbf{q}_t^m \in \mathbb{R}_+^N$ , and  $u_t^m \in \mathbb{R}_{++}$  such that the following conditions hold for all*

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<sup>12</sup>In a unitary setting, Browning (1989) followed a closely similar reasoning when introducing his so-called SREH-2 model.

$m \in \{1, \dots, M\}$ ,  $\tau \in \{1, \dots, \Upsilon\}$  and  $t, s \in T_\tau$ :

$$\begin{aligned}
 (a) \quad & \mathbf{P}_t = \sum_{m=1}^M \mathbf{P}_t^m; \\
 (b) \quad & \mathbf{q}_t = \sum_{m=1}^M \mathbf{q}_t^m; \\
 (c) \quad & u_s^m - u_t^m \leq \frac{1}{(\beta^m)^{t-1}} [(\mathbf{p}'_t \mathbf{q}_s^m + (\mathbf{P}_t^m)' \mathbf{Q}_s) - (\mathbf{p}'_t \mathbf{q}_t^m + (\mathbf{P}_t^m)' \mathbf{Q}_t)].
 \end{aligned}$$

## 5 Empirical application

In this section we present an empirical application of the practical tests discussed above. As discussed in the Introduction, we will evaluate the empirical performance of alternative specifications of the intertemporal collective model in terms of both goodness-of-fit and discriminatory power. This application complements the earlier study of Mazzocco (2007) by providing revealed preference tests of intertemporal collective models (with varying commitment). It also complements empirical studies that focused on revealed preference tests of the static collective model; see Cherchye, De Rock and Vermeulen (2009, 2011). We start by discussing the data. Next, we consider some methodological extensions of the basic tests presented in the previous section, which will be useful for our empirical analysis. Finally, we present our empirical results.

### 5.1 Data

We use data drawn from the Russia Longitudinal Monitoring Survey (RLMS). The RLMS is an extensive panel data set containing detailed consumption expenditure data of various categories of commodities for a large sample of households. For each household, we have consumption data for eight years (i.e. the years 1994 until 2003 except from 1997 and 1999). We focus on couples without children (i.e.  $M = 2$ ), so as to homogenize the sample. Furthermore, in order to abate the concern of non-separability between consumption and leisure (see, for example, Browning and Meghir (1991)), we only select couples with both members employed. In the end, this obtains a sample with 148 couples, each observed for eight periods. Cherchye, De Rock, Sabbe and Vermeulen (2008) and Cherchye, De Rock and Vermeulen (2009, 2011) provided revealed preference analyses of the same sample in

terms of the static collective consumption model; these authors also provide additional details on the data.

The commodity bundle under consideration consists of 21 nondurables: (1) food outside the home, (2) clothing, (3) car fuel, (4) wood fuel, (5) gas fuel, (6) luxury goods, (7) services, (8) housing rent, (9) bread, (10) potatoes, (11) vegetables, (12) fruit, (13) meat, (14) dairy products, (15) fat, (16) sugar, (17) eggs, (18) fish, (19) other food items, (20) alcohol and (21) tobacco. Throughout, we will assume that wood fuel, gas fuel and housing rent represent public consumption (i.e.  $K = 3$ ), while all other goods are assumed to be private (i.e.  $N = 18$ ). For all goods, we have discounted the real prices ( $\mathbf{p}_t, \mathbf{P}_t$ ) by using compound real interest rates in Russia as obtained from Thomson Reuters Datastream.

For the given selection of publicly and privately consumed goods, we will evaluate intertemporal collective models at the level of the individual households. This implies that each household's quantity and price observations form a separate set  $S$  with  $T = \{1, \dots, 8\}$ . The fact that we test the models for each household separately avoids possibly controversial preference homogeneity assumptions across different households.

As a final remark, we recall from the Introduction that our following analysis will not make use of information on assignable goods. The reason is that the RLMS does not directly provide this information. Interestingly, our application will demonstrate that the intertemporal collective consumption model can have strong empirical restrictions even in the absence of such assignable information. We believe this is an interesting empirical result for, as we also indicated in the Introduction, existing revealed preference analyses of the static collective model typically needed assignable information to obtain powerful tests. See, for example, the above mentioned studies that analyzed the same RLMS data in terms of the static collective consumption model; these studies imputed assignable information (based on observed expenditures for singles in the RLMS) to obtain reasonable power results.

## 5.2 Methodological extensions

As indicated before, an important focus in our following analysis will be on the evaluation of the discriminatory power of the different models subject to evaluation. Next, our empirical analysis will also account for optimization error, i.e. behavior might (slightly) violate the exact rationalizability conditions defined above. We here briefly present the corresponding methodolog-

ical extensions of the tests presented in Section 4.

**Power measurement.** For a given data set, power quantifies the probability of detecting (simulated) behavior that is not consistent with the behavioral model subject to testing; we will refer to such inconsistent behavior as ‘random’ behavior. Given the general importance of power for the type of revealed preference tests that we consider here, we will measure power in two different ways. The difference between the two power measures (which we label Power-1 and Power-2) pertains to the procedure that is used for simulating random behavior. As we will explain, each measure captures a different aspect of the discriminatory power of revealed preference tests.

Our Power-1 measure uses a notion of randomness that is based on Becker’s (1962) definition of irrational behavior, which states that households randomly choose consumption bundles that exhaust the available budget. Bronars (1987) was the first one to use this notion in the context of evaluating the power of revealed preference tests (in casu in a unitary setting). Specifically, the Power-1 measure is constructed in two steps. In the first step, we construct household-specific power measures, as follows. For each household, we simulate 1000 random series of eight consumption choices by constructing, for each of the eight observed household budgets, a random quantity bundle exhausting the given budget (for the corresponding prices); we construct these random quantity bundles by drawing budget shares for the 21 goods from a uniform distribution. The household-specific power measure is then calculated as one minus the proportion of the randomly generated consumption series that are consistent with the model under evaluation. In the second step, the aggregate Power-1 is measure is defined as the sample average of these household-specific power measures. This Power-1 measure is independent of the observed quantities and, thus, it quantifies discriminatory power as it directly follows from the given prices and budgets.

Next, our Power-2 measure uses a bootstrap procedure to simulate random behavior, and is based on a proposal of Andreoni and Harbaugh (2006). Specifically, instead of using the uniform distribution, we now draw from the empirical distribution to simulate random behavior: we define random quantity bundles by drawing budget shares (for the 21 goods) from the set of 1184 (= 148 x 8) observed household choices in the original data set. The motivation for using this alternative randomization procedure is that it incorporates more information on the actual choices made by households, while preserving a notion of randomness. For the given randomization, we

calculate the Power-2 measure by using a similar two-step procedure as for our Power-1 measure.

We consider our Power-2 measure in addition to our Power-1 measure as a robustness check. More precisely, in practice it may well be that some data are associated with a high Power-1 measure but a low Power-2 measure because of specific consumption patterns revealed by the quantity data. For example, it can be verified that the Power-2 measure will always be zero if only one good is effectively consumed by the households in the sample at hand (for example, because this is the only good that is deemed ‘desirable’ by these households). In such a case, the Power-1 measure will be artificially high because, by its very construction, it assigns positive quantities to all goods (including the goods that are never consumed in practice).

**Optimization error.** The tests defined in Section 4 are ‘sharp’ tests: they only tell us whether households are exact optimizers in terms of the behavioral model that is under evaluation. This is a demanding premise, especially in our intertemporal setting. In fact, one may argue that exact optimization is not a very interesting hypothesis, but that we rather want to know whether the behavioral model under study provides a reasonable way to describe observed behavior. Therefore, in our empirical analysis we will also consider extended versions of the basic (sharp) tests that account for optimization error; these extended tests focus on nearly optimizing behavior rather than exactly optimizing behavior. See also Varian (1990) for a general discussion on the usefulness of considering such nearly optimizing behavior in empirical revealed preference analysis.

To deal with optimization error, we adapt an original proposal of Afriat (1973) for revealed preference tests in a unitary setting. Specifically, optimization error is captured by some ‘Afriat index’  $e \in [0, 1]$ ; to obtain the extended tests, we replace  $(\mathbf{p}'_t \mathbf{q}_t^m + (\mathbf{P}_t^m)' \mathbf{Q}_t)$  by  $e * (\mathbf{p}'_t \mathbf{q}_t^m + (\mathbf{P}_t^m)' \mathbf{Q}_t)$  in the Afriat inequalities in Propositions 1-3 (see condition (c) in each statement (ii)). Clearly, if the Afriat index  $e = 1.00$ , then the extended tests coincide with the original sharp tests, while lower values for  $e$  account for optimization error. In our empirical exercise, we allow for small optimization error by considering  $e = 0.95$  and  $e = 0.90$ . For every behavioral model subject to study, we will evaluate goodness-of-fit and power for each value of  $e$  that we consider.

### 5.3 Results

One remark is in order before presenting our empirical results. As indicated above, we need to specify the discount factors  $\beta^m$  (in casu for  $m = 1, 2$ ) prior to the actual testing exercise. To facilitate our further exposition, we only present test results for a limited number of possible specifications of  $\beta_1$  and  $\beta_2$ . Specifically, we will report results for scenarios corresponding to  $\beta_1$  and/or  $\beta_2$  equal to 0.80, 0.90 and 1.00.<sup>13</sup> Importantly, this includes several scenarios in which household members have a different time preference (i.e.  $\beta_1 \neq \beta_2$ ). In this respect, we note that the identity of the individual household members (1 and 2) is irrelevant in our following analysis because we do not use assignable quantity information.

Table 1 gives pass rates and Tables 2a and 2b present power results (for Power-1 and Power-2, respectively) for the different intertemporal collective models that we consider. To structure our following discussion, we will first consider the test results for the extreme cases of our general limited commitment model, i.e. the full commitment model (FC, with  $\mathbf{T} = \{T\}$  and  $\Upsilon = 1$ ) and the no commitment model (NC, with  $\mathbf{T} = \{\{1\}, \dots, \{|T|\}\}$  and  $\Upsilon = |T|$ ). Subsequently, we will analyze four alternative model specifications that account for limited commitment in the household (LC-1 to LC-4, with  $1 < \Upsilon < |T|$ ).

**Full commitment and no commitment.** Let us first evaluate the full commitment (FC) model. We start by considering the ‘sharp’ tests, which -to recall- correspond to Afriat index  $e = 1.00$ . From Tables 1, 2a and 2b, we learn that exactly the same results apply to any combination of  $\beta_1$  and  $\beta_2$  that we consider: pass rates are everywhere 0% and power measures always amount to (approximately) 100%.

These results indicate that the sharp tests are overly demanding for the full commitment model. Therefore, we next consider Afriat index  $e = 0.95$  and  $e = 0.90$ , which accounts for some (but not much) optimization error. The overall picture remains the same as before: pass rates are everywhere (close to) 0% and power measures are always (nearly) 100%.

All this suggests that the full commitment model is strongly rejected for the data at hand. This is visualized in Figure 1, which gives pass rates for our different versions of the full commitment model as a function of optimization

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<sup>13</sup>Test results corresponding to other values of  $\beta_1$  and  $\beta_2$  are available from the authors upon request.

error (captured by the Afriat index  $e$ ). Consistent with the results in Table 1, we only obtain reasonable pass rates when we allow for an implausible degree of optimization error.

Figure 2 presents the corresponding power results. For compactness, we only consider Power-1; results for Power-2 are qualitatively similar. Expectedly, power decreases rather rapidly with optimization error. When simultaneously considering pass rates and power result, we do not find strong empirical support for any value of  $e$ : reasonably high pass rates are generally associated with very low power and vice versa.

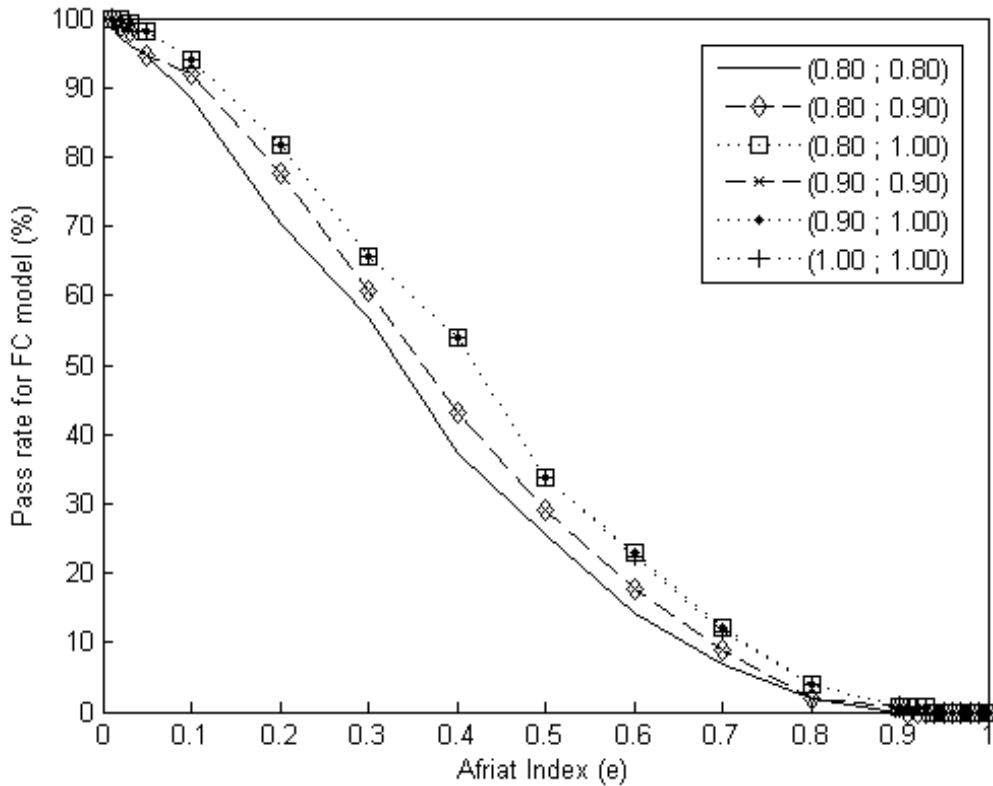


Figure 1: Full Commitment pass rate for varying optimization error



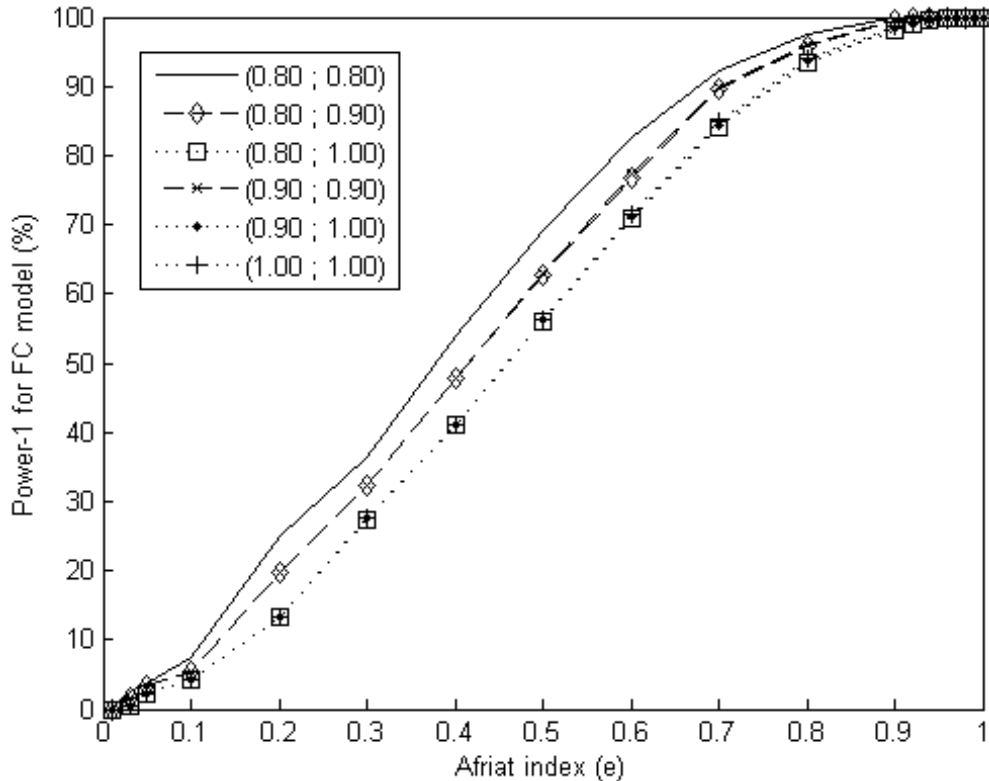


Figure 2: Full commitment Power-1 for varying optimization error

Given these negative results for the full commitment model, we next turn to the other limiting model, i.e. the no commitment (NC) model. As indicated in Section 4, the values of the discount factors  $\beta_1$  and  $\beta_2$  are irrelevant for the model tests in this case. Therefore, it comes as no surprise that Tables 1, 2a and 2b report the same pass rates and power results for every different combination of  $\beta_1$  and  $\beta_2$ . In general, we now get exactly the opposite picture to what we found before: pass rates are everywhere 100% and power measures are always 0%.

Overall, we can conclude that neither the full commitment model nor the no commitment model is well supported by the data: the first model implies too stringent rationalizability conditions, while the conditions for the second model have too little discriminatory power. At a more general level, our results indicate the following relation between, on the one hand, the degree of intrahousehold commitment in the intertemporal consumption model and, on the other hand, the apparent trade-off between goodness-fit and power results for the corresponding revealed preference tests: including

commitment restrictions in the model contributes to the power of the tests; however, too stringent commitment assumptions has an adverse effect on the pass rates for the model. In our opinion, this pleads for limited commitment models, which are situated between the extreme full and no commitment models.

**Limited commitment.** We evaluate four limited commitment models situated between the full and no commitment models. For each model specification, we use the practical test based on the necessary condition for LC-rationalizability in Proposition 3. We begin by considering three specifications that use an a priori specified partition  $\mathbf{T}$  that is common to each of the 148 households in our sample. By using a common partition, these specifications implicitly assume the same commitment ‘breaks’ for each household. Using our terminology of Section 2, such a commitment break then corresponds to a change in the bargaining weights (due to changing distribution factors) between the corresponding sub-periods.

Subsequently, we also evaluate a fourth specification that allows the partition  $\mathbf{T}$  to vary depending on the specific household at hand. In this exercise, each household-specific partition is (endogenously) defined such that it rationalizes the household behavior in terms of the corresponding LC-rationalizability test. In particular, for each household we select the partition that corresponds to the smallest number of commitment breaks. Contrary to the first three specifications, this specification accounts for the possibility that commitment breaks are idiosyncratic at the household level. Essentially, this means that we allow for distribution factors (causing these breaks) to be particular to individual households, which effectively does seem to be a plausible prior. In fact, by accounting for inter-household heterogeneity, this fourth model specification better exploits the panel structure of our data set.

We refer to our three model specifications with common commitment breaks as the LC-1, LC-2 and LC-3 models. The respective partitions are:

$$\begin{aligned} \text{LC-1} & : \quad \mathbf{T} = \{T_1 = \{1, 2\}, T_2 = \{3, 4\}, T_3 = \{5, 6\}, T_4 = \{7, 8\}\}, \\ \text{LC-2} & : \quad \mathbf{T} = \{T_1 = \{1\}, T_2 = \{2, 3\}, T_3 = \{4, 5\}, T_4 = \{6, 7\}, T_5 = \{8\}\}, \\ \text{LC-3} & : \quad \mathbf{T} = \{T_1 = \{1, 2, 3, 4\}, T_2 = \{5, 6, 7, 8\}\}. \end{aligned}$$

The interpretation is as follows. Firstly, our LC-1 and LC-2 models implicitly assume that bargaining weights only depend on the distribution factors of two consecutive periods, which implies subsets containing (at most) 2 years.

The two models differ from each other in terms of their starting year (i.e. year 1 for the LC-1 model and year 2 for the LC-2 model). Our motivation for including these two models is that, in his revealed preference analysis of the unitary life cycle model, Browning (1989) used a closely similar SREH-2 model; in a certain sense, our LC-1 and LC-2 models extend Browning’s intuition for his SREH-2 model to the collective setting we study. Next, our LC-3 model is inspired on a recent finding of Lacroix and Radtchenko (2011) for the RLMS data set that we study: these authors found empirical evidence that several important distribution factors shifted around the period of the financial crisis in Russia, which appeared in 1998. Because 1998 coincides with the fourth observation in our time series data, this effectively suggests the partition given above.

Like before, Tables 1, 2a and 2b present the empirical performance results for the three models under consideration. As for the pass rates, we find that especially the models LC-1 and LC-2 substantially outperform the full commitment model. For example, for an Afriat index  $e = 0.90$  we get a pass rate above 30% for the LC-1 model and above 40% for the LC-2 model (each time for  $\beta_1 = 0.80$  and  $\beta_2 = 1.00$ ). However, pass rates are still fairly low in general. This is all the more true for the LC-3 model, for which the pass rates nowhere exceed 13%.

On the other hand, we observe that both our Power-1 and Power-2 measures are quite high for any values of  $\beta_1$  and  $\beta_2$ . As we can expect a priori, lower values of the Afriat index  $e$  generally imply lower power; but even for  $e = 0.90$  our power measures are everywhere (and often substantially) above 50%. In general, we get the same trade-off as before: models that are associated with high pass rates typically have low discriminatory power and vice versa. Overall, the partitions underlying our LC-1, LC-2 and LC-3 models seem to be only weakly supported by the data. We may conclude that the assumption of common commitment breaks for all households is too strong for our sample.

This weak evidence pro common commitment breaks motivates considering our fourth limited commitment model, which -to recall- accounts for household-specific commitment breaks. We refer to this model as the LC-4 model. Specifically, we endogenously define household-specific partitions  $\mathbf{T}$  as follows: for each household, the full period  $T$  is subdivided in continuous sub-periods that each separately satisfy the condition for FC-rationalizability in Proposition 1. By the very construction of these (household-specific) partitions  $\mathbf{T}$ , each household will pass the corresponding condition for LC-

rationalizability in Proposition 3. As such, we get by construction a pass rate of 100% for our LC-4 model (for any value of the Afriat index  $e$ ). As an illustration, Figure B.1 in Appendix B gives an overview of the obtained partitions used for rationalizing the data in terms of our LC-4 model.<sup>14</sup>

Let us then focus on the discriminatory power of the LC-4 model. In this case, power is defined as the probability that (simulated) random behavior fails the condition in Proposition 3 when using the endogenously defined partitions as described above (and illustrated in Figure B.1). From Tables 2a and 2b we find that, even though the Power-1 and Power-2 values are substantially below the ones for the full commitment model and the LC-1 to LC-3 models, the LC-4 model turns out to be considerably more powerful than the no commitment model. Overall, power does not seem to vary too much for the different values of  $\beta_1$  and  $\beta_2$  that we study. The ‘most powerful’ combination of  $\beta_1$  and  $\beta_2$  generally depends on the value of  $e$  and the specific power measure (Power-1 or Power-2) under consideration.

As an additional exercise, Figure 3 presents power result for the LC-4 model as a function of optimization error (measured by  $e$ ). The figure has a similar interpretation as Figure 2; for brevity, we again only consider Power-1. The figure confirms that the model has reasonable power for values of the Afriat index  $e$  that are generally considered plausible (e.g. above 0.90).

As a general conclusion, we can argue that our LC-4 model strikes a rather good balance between goodness-of-fit and discriminatory power: it combines a 100% pass rate with reasonable power results (for  $e$  sufficiently high). In turn, this leads us to conclude that the limited commitment model with household-specific commitment breaks provides a rather good description of the intertemporal consumption behavior under study.

In our opinion, an interesting following step can relate these findings on household-dependent commitment breaks to observable distribution factors, to investigate which household (member) characteristics and/or other exogenous variables effectively drive the commitment breaks. Such an exercise falls beyond the scope of the current study (also because of limited data availability). But our empirical analysis does indicate that the revealed preference tests presented in this paper can be useful for addressing this type of questions.

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<sup>14</sup>The figure refers to the scenario with  $\beta_1 = 0.80$ ,  $\beta_2 = 1.00$  and  $e = 0.95$ . Similar figures for alternative values of  $\beta_1$ ,  $\beta_2$  and  $e$  are available upon request.

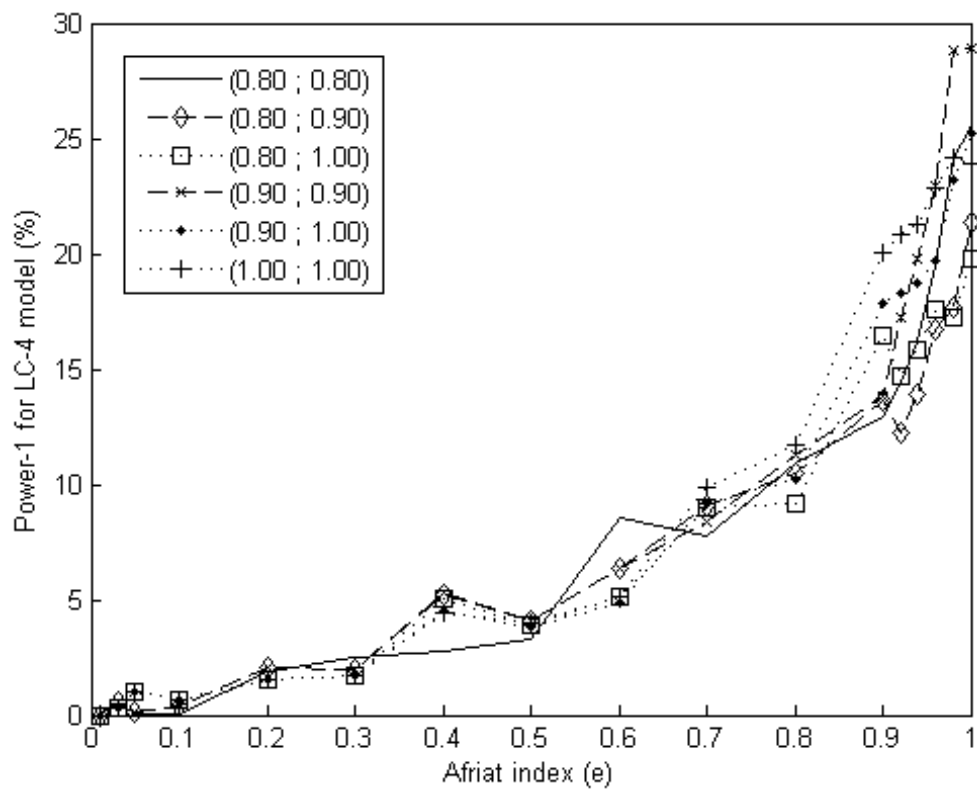


Figure 3: LC-4 Power-1 for varying optimization error

Table 1: Collective commitment models - Pass rate  
 (% of households passing the test)

	FC	NC	LC-1	LC-2	LC-3	LC-4
Afriat index $e = 1.00$						
$(\beta_1 ; \beta_2) = (0.80 ; 0.80)$	0.00	100.00	6.08	10.81	0.00	100.00
$(\beta_1 ; \beta_2) = (0.80 ; 0.90)$	0.00	100.00	8.78	16.89	0.68	100.00
$(\beta_1 ; \beta_2) = (0.80 ; 1.00)$	0.00	100.00	10.14	22.30	0.68	100.00
$(\beta_1 ; \beta_2) = (0.90 ; 0.90)$	0.00	100.00	6.08	8.78	0.00	100.00
$(\beta_1 ; \beta_2) = (0.90 ; 1.00)$	0.00	100.00	8.78	16.89	0.68	100.00
$(\beta_1 ; \beta_2) = (1.00 ; 1.00)$	0.00	100.00	6.76	8.78	0.00	100.00
Afriat index $e = 0.95$						
$(\beta_1 ; \beta_2) = (0.80 ; 0.80)$	0.00	100.00	10.14	18.92	0.68	100.00
$(\beta_1 ; \beta_2) = (0.80 ; 0.90)$	0.00	100.00	14.19	25.00	3.78	100.00
$(\beta_1 ; \beta_2) = (0.80 ; 1.00)$	0.00	100.00	21.62	31.76	6.76	100.00
$(\beta_1 ; \beta_2) = (0.90 ; 0.90)$	0.00	100.00	12.84	20.95	2.03	100.00
$(\beta_1 ; \beta_2) = (0.90 ; 1.00)$	0.00	100.00	16.89	27.03	4.73	100.00
$(\beta_1 ; \beta_2) = (1.00 ; 1.00)$	0.00	100.00	15.54	19.59	4.05	100.00
Afriat index $e = 0.90$						
$(\beta_1 ; \beta_2) = (0.80 ; 0.80)$	0.00	100.00	18.92	29.73	4.73	100.00
$(\beta_1 ; \beta_2) = (0.80 ; 0.90)$	0.68	100.00	24.32	35.14	8.11	100.00
$(\beta_1 ; \beta_2) = (0.80 ; 1.00)$	0.68	100.00	33.78	43.92	12.16	100.00
$(\beta_1 ; \beta_2) = (0.90 ; 0.90)$	0.00	100.00	19.59	32.43	5.41	100.00
$(\beta_1 ; \beta_2) = (0.90 ; 1.00)$	0.68	100.00	31.08	40.54	9.46	100.00
$(\beta_1 ; \beta_2) = (1.00 ; 1.00)$	0.68	100.00	29.73	35.81	9.46	100.00

Table 2a: Estimated statistical power (%) - Power-1

Power - Bronars	FC	NC	LC-1	LC-2	LC-3	LC-4
Afriat index $e = 1.00$						
$(\beta_1 ; \beta_2) = (0.80 ; 0.80)$	100.00	0.00	92.20	88.75	99.81	25.54
$(\beta_1 ; \beta_2) = (0.80 ; 0.90)$	100.00	0.00	90.33	83.60	99.43	21.38
$(\beta_1 ; \beta_2) = (0.80 ; 1.00)$	100.00	0.00	86.68	78.12	98.49	20.03
$(\beta_1 ; \beta_2) = (0.90 ; 0.90)$	100.00	0.00	91.90	89.66	99.73	28.83
$(\beta_1 ; \beta_2) = (0.90 ; 1.00)$	100.00	0.00	88.88	84.23	99.13	25.09
$(\beta_1 ; \beta_2) = (1.00 ; 1.00)$	100.00	0.00	91.48	89.96	99.68	23.93
Afriat index $e = 0.95$						
$(\beta_1 ; \beta_2) = (0.80 ; 0.80)$	100.00	0.00	87.28	79.49	98.31	16.77
$(\beta_1 ; \beta_2) = (0.80 ; 0.90)$	100.00	0.00	82.42	73.19	96.48	15.74
$(\beta_1 ; \beta_2) = (0.80 ; 1.00)$	99.67	0.00	73.88	66.33	93.71	16.59
$(\beta_1 ; \beta_2) = (0.90 ; 0.90)$	100.00	0.00	84.67	78.37	97.41	21.58
$(\beta_1 ; \beta_2) = (0.90 ; 1.00)$	99.82	0.00	77.26	70.91	94.92	17.92
$(\beta_1 ; \beta_2) = (1.00 ; 1.00)$	99.95	0.00	80.65	78.00	96.42	21.37
Afriat index $e = 0.90$						
$(\beta_1 ; \beta_2) = (0.80 ; 0.80)$	99.97	0.00	78.04	69.49	95.11	13.04
$(\beta_1 ; \beta_2) = (0.80 ; 0.90)$	99.42	0.00	70.40	63.58	92.26	13.36
$(\beta_1 ; \beta_2) = (0.80 ; 1.00)$	98.17	0.00	57.33	57.87	86.37	16.55
$(\beta_1 ; \beta_2) = (0.90 ; 0.90)$	99.63	0.00	73.36	66.79	93.48	13.85
$(\beta_1 ; \beta_2) = (0.90 ; 1.00)$	98.33	0.00	60.21	60.31	87.64	17.90
$(\beta_1 ; \beta_2) = (1.00 ; 1.00)$	98.62	0.00	63.95	64.95	89.76	20.07

Table 2b: Estimated statistical power (%) - Power-2

Power - Bootstrap	FC	NC	LC-1	LC-2	LC-3	LC-4
Afriat index $e = 1.00$						
$(\beta_1 ; \beta_2) = (0.80 ; 0.80)$	100.00	0.00	92.70	89.04	99.87	27.22
$(\beta_1 ; \beta_2) = (0.80 ; 0.90)$	100.00	0.00	90.48	81.76	99.42	20.74
$(\beta_1 ; \beta_2) = (0.80 ; 1.00)$	99.99	0.00	85.41	75.46	98.24	19.07
$(\beta_1 ; \beta_2) = (0.90 ; 0.90)$	100.00	0.00	92.61	89.44	99.80	31.97
$(\beta_1 ; \beta_2) = (0.90 ; 1.00)$	100.00	0.00	89.03	82.44	99.13	26.05
$(\beta_1 ; \beta_2) = (1.00 ; 1.00)$	100.00	0.00	92.45	90.35	99.76	27.49
Afriat index $e = 0.95$						
$(\beta_1 ; \beta_2) = (0.80 ; 0.80)$	100.00	0.00	88.50	80.69	98.76	19.39
$(\beta_1 ; \beta_2) = (0.80 ; 0.90)$	99.98	0.00	82.42	72.40	96.53	16.76
$(\beta_1 ; \beta_2) = (0.80 ; 1.00)$	99.67	0.00	74.06	65.65	93.61	17.61
$(\beta_1 ; \beta_2) = (0.90 ; 0.90)$	100.00	0.00	86.28	78.46	97.86	23.91
$(\beta_1 ; \beta_2) = (0.90 ; 1.00)$	99.81	0.00	78.20	70.29	95.16	19.55
$(\beta_1 ; \beta_2) = (1.00 ; 1.00)$	99.97	0.00	82.74	78.96	96.91	25.36
Afriat index $e = 0.90$						
$(\beta_1 ; \beta_2) = (0.80 ; 0.80)$	99.95	0.00	79.11	69.53	95.31	15.49
$(\beta_1 ; \beta_2) = (0.80 ; 0.90)$	99.53	0.00	71.42	63.17	92.50	15.10
$(\beta_1 ; \beta_2) = (0.80 ; 1.00)$	98.75	0.00	59.03	57.03	87.22	17.04
$(\beta_1 ; \beta_2) = (0.90 ; 0.90)$	99.79	0.00	75.16	66.60	94.02	15.75
$(\beta_1 ; \beta_2) = (0.90 ; 1.00)$	98.90	0.00	62.25	59.77	88.88	18.34
$(\beta_1 ; \beta_2) = (1.00 ; 1.00)$	99.19	0.00	66.75	64.85	91.60	22.48

## 6 Summary and conclusions

We have presented a methodology for revealed preference analysis of intertemporal collective consumption behavior. Following Mazzocco (2007), we have defined tests that allow for empirically verifying consistency of observed behavior with intertemporal consumption models characterized by varying degrees of commitment. The particular feature of these tests is that they are entirely nonparametric, which means that their practical application does not require an a priori (typically nonverifiable) specification of the individual utilities and the intrahousehold bargaining process. An empirical application to RLMS data has demonstrated the practical usefulness of the methodology. Specifically, we have evaluated alternative behavioral models (with varying commitment) in terms of their goodness-of-fit as well as their



discriminatory power. This application provided empirical evidence pro using a limited commitment model that allows for household-specific commitment patterns. Importantly, our results also show that accounting for intertemporal relations between household consumption decisions may substantially increase the power of the revealed preference analysis.

We see multiple avenues for future research. Firstly, we have adopted a basic life cycle model for describing intertemporal consumption; as we have indicated, this follows Browning's (1989) original work on revealed preference analysis in a unitary setting. A more realistic modeling of intertemporal household consumption behavior may weaken the strong underlying assumptions, i.e. perfect capital markets, perfect foresight and intertemporally separable utility functions. This can obtain consumption models that achieve an even better empirical performance (in terms of goodness-of-fit and power) than the models we considered in this paper. In this respect, follow-up research may fruitfully build on recent methodological advances for revealed preference analysis of unitary consumption behavior. For example, relaxing the assumption of perfect capital markets can use the insights of Demuyneck and Verriest (2010). Similarly, intertemporal separability may be weakened by integrating Crawford's (2010) characterization of intertemporal behavior under habit formation.

Secondly, we have adopted Mazzocco's (2007) original starting point that households achieve Pareto efficient intrahousehold allocations, which basically means that household members interact in a 'cooperative' way. We believe it will be interesting to extend our framework to account for possibly 'noncooperative' behavior of individual household members and, thus, to model household allocations as Nash equilibria rather than Pareto efficient outcomes. This may also imply a more subtle modeling of the notion of intrahousehold commitment. In this respect, the recent study of Cherchye, Demuyneck and De Rock (2011) can serve as a useful starting point; these authors considered revealed preference analysis of noncooperative household consumption in a static setting.

## Appendix A: proofs

### Proof of Theorem 1

(i)  $\Rightarrow$  (ii). Suppose condition (i) of Theorem 1 holds for the allocation  $\{\mathbf{q}_t^1, \dots, \mathbf{q}_t^M, \mathbf{Q}_t\}$ , with  $t \in T$ , for a partition  $T = \{T_1, \dots, T_\Upsilon\}$ . Let  $\eta$  denote the Lagrange multiplier associated with the household budget constraint. We get the following first order constraints for the household optimization problem under LC-rationalizability (with  $\frac{\partial u^m(\mathbf{q}_t^m, \mathbf{Q}_t)}{\partial \mathbf{q}_t^m}$  and  $\frac{\partial u^m(\mathbf{q}_t^m, \mathbf{Q}_t)}{\partial \mathbf{Q}_t}$  ( $m = 1, \dots, M$ ) the subgradients for the function  $u^m$  at bundle  $(\mathbf{q}_t^m, \mathbf{Q}_t)$ ), for all  $t \in T_\tau$  ( $\tau = 1, \dots, \Upsilon$ ):

$$\begin{aligned} \frac{\partial u^m(\mathbf{q}_t^m, \mathbf{Q}_t)}{\partial \mathbf{q}_t^m} &\leq \frac{1}{(\beta^m)^{t-1}} \frac{\eta}{\mu_\tau^m} \mathbf{P}_t, \text{ and} \\ \sum_{m=1}^M \mu_\tau^m (\beta^m)^{t-1} \frac{\partial u^m(\mathbf{q}_t^m, \mathbf{Q}_t)}{\partial \mathbf{Q}_t} &\leq \eta \mathbf{P}_t. \end{aligned}$$

Under concavity of the individual utility functions  $u^m$ , we have

$$u^m(\mathbf{q}_s^m, \mathbf{Q}_s) - u^m(\mathbf{q}_t^m, \mathbf{Q}_t) \leq \left( \frac{\partial u^m(\mathbf{q}_t^m, \mathbf{Q}_t)}{\partial \mathbf{q}_t^m} \right)' (\mathbf{q}_s^m - \mathbf{q}_t^m) + \left( \frac{\partial u^m(\mathbf{q}_t^m, \mathbf{Q}_t)}{\partial \mathbf{Q}_t} \right)' (\mathbf{Q}_s - \mathbf{Q}_t)$$

Now define for each  $t$

$$\begin{aligned} \mathbf{P}_t^m &= \frac{\mu_\tau^m}{\eta} (\beta^m)^{t-1} \frac{\partial u^m(\mathbf{q}_t^m, \mathbf{Q}_t)}{\partial \mathbf{Q}_t}, \quad m = 1, \dots, M-1, \quad t \in T_\tau, \\ \mathbf{P}_t^M &= \mathbf{P}_t - \sum_{m=1}^{M-1} \mathbf{P}_t^m, \text{ and} \\ \lambda_\tau^m &= \frac{\eta}{\mu_\tau^m}, \quad m = 1, \dots, M, \quad t \in T_\tau. \end{aligned}$$

Then we obtain

$$\begin{aligned}
u^m(\mathbf{q}_s^m, \mathbf{Q}_s) - u^m(\mathbf{q}_t^m, \mathbf{Q}_t) &\leq \frac{\lambda_\tau^m}{(\beta^m)^{t-1}} \mathbf{p}'_t(\mathbf{q}_s^m - \mathbf{q}_t^m) + \frac{\lambda_\tau^m}{(\beta^m)^{t-1}} (\mathbf{P}_t^m)'(\mathbf{Q}_s - \mathbf{Q}_t) \\
&\iff \\
u^m(\mathbf{q}_s^m, \mathbf{Q}_s) - u^m(\mathbf{q}_t^m, \mathbf{Q}_t) &\leq \frac{\lambda_\tau^m}{(\beta^m)^{t-1}} [(\mathbf{p}'_t \mathbf{q}_s^m + (\mathbf{P}_t^m)' \mathbf{Q}_s) - (\mathbf{p}'_t \mathbf{q}_t^m + (\mathbf{P}_t^m)' \mathbf{Q}_t)]
\end{aligned}$$

Using  $u^m(\mathbf{q}_s^m, \mathbf{Q}_s) = u_s^m$  and imposing the sum conditions on the personalized quantities  $\mathbf{q}_t^m$  and Lindahl prices  $\mathbf{P}_t^m$  gives condition (ii) of Theorem 1.

(ii)  $\Rightarrow$  (i). Suppose condition (ii) of Theorem 1 holds. Define the following utility function, using the appropriate  $\lambda_\tau^m$  for the period in which the bundle  $(\mathbf{x}^m, \mathbf{X})$  is consumed:

$$u^m(\mathbf{x}^m, \mathbf{X}) = \min_s \left( u_s^m + \frac{\lambda_\tau^m}{(\beta^m)^{s-1}} [\mathbf{p}'_s(\mathbf{x}^m - \mathbf{q}_s^m) + \mathbf{P}'_s(\mathbf{X} - \mathbf{Q}_s)] \right)$$

Using a straightforwardly similar argument as Varian (1982), we can derive  $u^m(\mathbf{q}_t^m, \mathbf{Q}_t) = u_t^m$ .

Consider any consumption path  $(\mathbf{x}_t^m, \mathbf{X}_t)$  such that

$$\sum_{m=1}^M \sum_{t=1}^{|T|} [\mathbf{p}'_t(\mathbf{x}_t^m - \mathbf{q}_t^m) + (\mathbf{P}_t^m)'(\mathbf{X}_t - \mathbf{Q}_t)] \leq 0,$$

i.e. the consumption path  $(\mathbf{x}_t^m, \mathbf{X}_t)$  is not more expensive than the path  $(\mathbf{q}_t^m, \mathbf{Q}_t)$ . Then we need to show that

$$\sum_{m=1}^M \sum_{\tau=1}^{\Upsilon} \mu_\tau^m \sum_{t \in T_\tau} (\beta^m)^{t-1} u^m(\mathbf{x}_t^m, \mathbf{X}_t) \leq \sum_{m=1}^M \sum_{\tau=1}^{\Upsilon} \mu_\tau^m \sum_{t \in T_\tau} (\beta^m)^{t-1} u^m(\mathbf{q}_t^m, \mathbf{Q}_t)$$

Without losing generality, we can assume that  $\mu_\tau = \frac{1}{\lambda_\tau}$ . As such, we

obtain

$$\begin{aligned}
& \sum_{m=1}^M \sum_{\tau=1}^{\Upsilon} \mu_{\tau}^m \sum_{t \in T_{\tau}} (\beta^m)^{t-1} u^m(\mathbf{x}_t^m, \mathbf{X}_t) \\
\leq & \sum_{m=1}^M \sum_{\tau=1}^{\Upsilon} \mu_{\tau}^m \sum_{t \in T_{\tau}} (\beta^m)^{t-1} \left( u_t^m + \frac{\lambda_{\tau}^m}{(\beta^m)^{t-1}} [\mathbf{p}'_t(\mathbf{x}_t^m - \mathbf{q}_t^m) + \mathbf{P}'_t(\mathbf{X}_t - \mathbf{Q}_t)] \right) \\
= & \sum_{m=1}^M \sum_{\tau=1}^{\Upsilon} \mu_{\tau}^m \sum_{t \in T_{\tau}} (\beta^m)^{t-1} u_t^m + \sum_{m=1}^M \sum_{\tau=1}^{\Upsilon} \mu_{\tau}^m \lambda_{\tau}^m \sum_{t \in T_{\tau}} [\mathbf{p}'_t(\mathbf{x}_t^m - \mathbf{q}_t^m) + \mathbf{P}'_t(\mathbf{X}_t - \mathbf{Q}_t)] \\
\leq & \sum_{m=1}^M \sum_{\tau=1}^{\Upsilon} \mu_{\tau}^m \sum_{t \in T_{\tau}} (\beta^m)^{t-1} u_t^m + \sum_{m=1}^M \sum_{t=1}^{|T|} [\mathbf{p}'_t(\mathbf{x}_t^m - \mathbf{q}_t^m) + \mathbf{P}'_t(\mathbf{X}_t - \mathbf{Q}_t)] \\
\leq & \sum_{m=1}^M \sum_{\tau=1}^{\Upsilon} \mu_{\tau}^m \sum_{t \in T_{\tau}} (\beta^m)^{t-1} u_t^m
\end{aligned}$$

Since  $u^m(\mathbf{q}_t^m, \mathbf{Q}_t) = u_t^m$ , we obtain that condition (i) of Theorem 1 is satisfied.

## Proof of Proposition 1

We have the partition  $\mathbf{T} = \{T\}$  and  $\Upsilon = 1$ . Because this partition involves only a single subset, there is only a single bargaining weight ( $\mu^m$ ) that is relevant for each member  $m$  ( $= 1, \dots, M$ ). Given this, we can use an analogous argument as in our proof of Theorem 1 to show (i)  $\Rightarrow$  (ii). Specifically, as compared to the proof of Theorem 1, we now define  $\lambda^m = \eta/\mu^m$  and, subsequently, we use  $u^m(\mathbf{q}_s^m, \mathbf{Q}_s) = u_s^m/\lambda^m$ .

To prove (ii)  $\Rightarrow$  (i), we define the following utility function:

$$u^m(\mathbf{x}^m, \mathbf{X}) = \min_s \left( u_s^m + \frac{\lambda^m}{(\beta^m)^{s-1}} [\mathbf{p}'_s(\mathbf{x}^m - \mathbf{q}_s^m) + \mathbf{P}'_s(\mathbf{X} - \mathbf{Q}_s)] \right)$$

The remainder of the argument is analogous as for the general limited commitment model (see our proof of Theorem 1).

## Proof of Proposition 2

We have the partition  $\mathbf{T} = \{\{1\}, \dots, \{|T|\}\}$  and  $\Upsilon = |T|$ . This specification of the LC-model implies time-dependent bargaining power  $\mu_t^m$  ( $m = 1, \dots, M$ ;  $t \in T$ ). Given this, we can use an analogous argument as in our proof of Theorem 1 to show (i)  $\Rightarrow$  (ii). Specifically, as compared to the proof of Theorem 1, we now define

$$\lambda_t^m = \frac{1}{(\beta^m)^{t-1}} \frac{\eta}{\mu_t^m}$$

for all  $m = 1, \dots, M$ ,  $t \in T$ , and we use  $u^m(\mathbf{q}_s^m, \mathbf{Q}_s) = u_s^m$ .

To prove (ii)  $\Rightarrow$  (i), we define the following utility function:

$$u^m(\mathbf{x}^m, \mathbf{X}) = \min_s (u_s^m + \lambda_t^m [\mathbf{p}'_s(\mathbf{x}^m - \mathbf{q}_s^m) + \mathbf{P}'_s(\mathbf{X} - \mathbf{Q}_s)])$$

The remainder of the argument is again analogous as for the general limited commitment model (see our proof of Theorem 1).

## Proof of Proposition 3

Consider a set of observations  $S$ , a partition  $\mathbf{T}$  and discount factors  $\beta^m$ . By Proposition 1, LC-rationalizability implies there exist Lindahl prices and personalized quantities  $\mathbf{P}_t^m \in \mathbb{R}_+^K$ ,  $\mathbf{q}_t^m \in \mathbb{R}_+^N$  that are feasible (i.e. sum up to the observed prices  $\mathbf{P}_t$  and quantities  $\mathbf{q}_t$ , respectively), and numbers  $\lambda_\tau^m$ ,  $u_t^m \in \mathbb{R}_{++}$  such that the following conditions hold for all  $m \in \{1, \dots, M\}$ ,  $\tau \in \{1, \dots, \Upsilon\}$ ,  $t \in T_\tau$  and  $s \in \{1, \dots, T\}$ :

$$u_s^m - u_t^m \leq \frac{\lambda_\tau^m}{(\beta^m)^{t-1}} [(\mathbf{p}'_t \mathbf{q}_s^m + (\mathbf{P}_t^m)' \mathbf{Q}_s) - (\mathbf{p}'_t \mathbf{q}_t^m + (\mathbf{P}_t^m)' \mathbf{Q}_t)].$$

Let us now consider these inequalities for two observations  $s, t$  that belong to the same subset  $T_\tau$ . By simply rescaling the numbers  $u_s^m$  and  $u_t^m$  by the common factor  $\lambda_\tau^m$ , we obtain the set of conditions stated in Proposition 3, which are linear for a given discount factor  $\beta^m$ .



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