The number and size of nations revisited: Endogenous border formation with non-uniform population distributions

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Abstract

The endogenous border formation model of Alesina and Spolaore (1997) has received a lot of attention in the economics community. One of its central messages is that in a democratic world in equilibrium there is an inefficiently large number of nation states. However, this result is obtained under very specific assumptions like a uniform population distribution and no population mobility. In this paper, I generalize the model of Alesina and Spolaore allowing for population distributions other than the uniform distribution. Since this generalization is accompanied by the loss of tractability in closed form, I calculate the equilibria by means of numerical computation. It turns out that the above-mentioned central result is highly sensitive to the choice of population distribution and that the model shows four different regimes depending on the chosen distribution. Furthermore, the behaviour implied by the Alesina and Spolaore model with uniform population distribution is the exception, not the rule.

JEL-Classification: F50, H40
Keywords: Size of Nations, Endogenous Border Formation, Computational Economics

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1 Introduction

In the past century national borders have been redrawn to an enormous extent. More than 140 of the currently existing 193 nation states[1] have emerged by gaining sovereignty since 1900. Obviously, dramatic political events like the two World Wars, the collapse of the USSR and the decline of colonialism were major driving forces in this process. Still, there is desire for independence in many regions all around the world where people raise their voice and aim for more independence either peacefully or by means of force. Considering these developments, the assumption of a fixed nation state as basic unit of (macro) economic analysis seems problematic.

Although these issues traditionally belong to the realm of disciplines like (political) philosophy, history and political science, a recent economic literature addresses questions regarding the process of political disintegration as well. The most prevalent of these approaches models national border formation dependent on a trade-off in the provision of public goods [Alesina and Spolaore, 1997]. In this model, Alesina and Spolaore are able to derive a number of very general statements concerning the formation of nation states. However, these results are obtained under very specific assumptions like a uniform population distribution, perfect correlation between an individual's location and her preferences, and the exclusion of population mobility. In the present paper I build on the work of the Alesina and Spolaore model and try to generalize their work with regard to the population distribution.

In the following section I present a short overview of the existing economic literature on secession and integration with a special focus on the basic model of Alesina and Spolaore (AS henceforth) and some of its extensions. In Section 3 I introduce some generalizations into the framework regarding the population distributions; the consequences of this modification will be investigated in Sections 4 and 5. The last two sections of the paper focus on the possible implications of the obtained results on the existing literature and some speculations on promising future research directions.

2 Existing Literature

In the last twenty years political economists became more and more interested in exploring and modelling analytically several aspects of nation state

[1] I am aware of the conceptual shortcomings resulting from the lack of adequate definitions of terms like jurisdiction, nation, state and nation-state and their interchangeable use in most of the relevant literature in political economics. In the context of international relations, this point was made forcefully by Lars-Erik Cederman [Cederman, 1997, p.17]. Further elaboration on this topic would justify an article on its own, but for the purpose of this paper I will stick with the blurry use of these concepts and use these terms interchangeably.

formation. Bolton et al. [Bolton et al., 1996] provide a survey on the existing economic literature until the mid-1990s, Michele Ruta [Ruta, 2005] and Enrico Spolaore [Spolaore, 2006] summarize recent advances in the field. David Friedman [Friedman, 1977] was one of the first economists providing an analytical answer to the question of the size of nations. In his model he assumes a world of dictators that leads to a configuration of countries such that the dictators' joint revenue received from taxes on trade is maximized. Bolton and Roland [Bolton and Roland, 1997] provide a model that allows individuals to vote for unification and secession of nations, thus representing an idealized democratic world. Their study concentrates on the influence of regional income disparities and preferences over fiscal policy (i.e. redistribution policies) on the break-up of nations. However, the model assumes two regions facing the possibility to unify as exogenously given.

A number of studies in the late 1990s are loosely based on Hotelling's location model [Hotelling, 1929]. In these models agents are uniformly distributed on a line and their position on this line represents their location and/or preferences. Casella and Feinstein tackle the problem of political (dis-)integration by employing a variation of Hotelling's location model to represent the voters' heterogeneity of preferences over the provision of public goods. Then they go on and investigate the relationship between the formation of markets and the formation of jurisdictions [Casella and Feinstein, 2002]. Donald Wittman presents a similar but comparably richer approach and models the preferential and the spatial dimensions separately [Wittman, 2000]. His framework also includes a simple production function, which allows to analyze the interactions between the polity and the market, as well as an explicit formulation of coercion by means of an extortion function. These additional mechanisms, however, force him to compare only the relative sizes of two states.

The seminal work of Alesina and Spolaore [Alesina and Spolaore, 1997] has probably received the most attention in this area so far. Also starting from a location model loosely based on Hotelling's approach they describe the equilibrium number and sizes of jurisdictions endogenously. In the AS model the size of a nation state is determined by a basic trade-off in the provision of tax financed public goods. Larger nation states are able to provide the public good (e.g. governmental services) at a lower per capita cost. However, the larger the nation state the more difficult it is to satisfy the demands of the more heterogeneous electorate by majority voting on the location/type of the public good. In the AS model, individuals are uniformly distributed on the segment $[0, 1]$ representing the world. The world consists of at least one country and each country has to provide a single public good, its government, which is financed by taxes on its residents. In return, these residents can take advantage of their country's and only their country's public good. The resulting utility from the public good is decreasing linearly with the distance between the individual and the government. For the sake

of simplicity, it is assumed that in a given country every agent has to pay the same amount of taxes.

A benevolent social planner who wishes to maximize the sum of all individuals' utilities divides the world into N* countries, where the socially optimal number of countries N* is given by

$$N^* = \sqrt{ga/4k}. \tag{1}$$

The socially optimal number of nation states is rising in the utility g generated by the public good and in the parameter a, which weights the utility loss from distance to government. At the same time, N^* is declining in the cost k of the public good. Because of the assumption of a uniform population distribution, all N^* nations are of the same size and the social planner locates each government in the middle of its country.[2]

However, the socially optimal number of nations N^* does not coincide with the stable number of nations in a democratic model world. In a world without a benevolent social planner, individuals are allowed to decide on national borders according to a number of straight forward behavioural rules. Furthermore, it is assumed that the location of a country's government is decided by majority rule after the national borders are established. These assumptions then lead to a democratic equilibrium number of nation states $\tilde{N}$ given by

$$\tilde{N} = \sqrt{ga/2k}. \tag{2}$$

As in the social planner solution, in this equilibrium solution all nation states are of the same size and each government is located in the middle of its country (once again as a consequence of the uniform distribution).

To sum up, given a uniform population distribution the stable number of nations in democratic equilibrium $\tilde{N}$ is always larger than the social optimum N^*, since

$$\sqrt{ga/2k} > \sqrt{ga/4k}. \tag{3}$$

For any given value of $g > 0$, there are more jurisdictions in the democratic equilibrium than in the social planner case.

Regarding the three central results of the AS model[3], the proposition that "in equilibrium one generally observers an inefficiently large number of countries" is of immediate relevance for my purpose and the only proposition

[2]For details regarding the derivation of this and the following results, the reader is referred to [Alesina and Spolaore, 1997] or [Alesina and Spolaore, 2003].

[3]The three central results of the AS model are:

- "democratization leads to secessions"
- "in equilibrium one generally observes an inefficiently large number of countries"
- "the equilibrium number of countries is increasing in the amount of economic integration." [Alesina and Spolaore, 1997]

that follows directly from the calculations above. The remaining two propositions will not be treated in this paper, although it cannot be ruled out that they are affected as well by the consequences of the proposed extension to the model.

The model presented above clearly is just a first crude attempt to grasp the phenomenon under investigation. Several authors have tried to refine the basic framework in one or another way. Klaas Staal [Staal, 2004] focuses on making public spending and taxation[4] exogenously dependent on the size of the countries and confirms the results of Alesina and Spolaore. Federico Etro [Etro, 2006] endogenously determines the size of public spending and derives optimal solutions for the size of nations and their respective public spending together. His conclusions are not unambiguously in favour of the original AS results. Alesina and Spolaore [Alesina and Spolaore, 2006] extend the model to investigate the role of international conflicts and defence spending on the size and number of nations.

2.1 Shortcomings

The list of objections to the AS model in this section is by no means complete but rather tries to emphasize some of the more pressing drawbacks. Its originators are well aware of many of these shortcomings and give an extensive roadmap for model improvement [Alesina and Spolaore, 2003, pp. 221-223].

The AS Model has been criticised for some of its rather strong assumptions which have been justified for reasons of mathematical tractability in closed form. First, there is perfect correlation between the geographical location and the individuals' preferences, which in turn implies perfect sorting of the population in terms of preferences. This simplification, however, rules out the investigation of isolated minority groups. Empirical studies by Easterly and Levine [Easterly and Levine, 2001] show a rather fractal income distribution while Easterly and Levine [Easterly and Levine, 1997] as well as Alesina et al. [Alesina et al., 2003] point out how ethnic, linguistic and religious diversity affects economic growth of a country. So consideration of the population distribution regarding attributes like income, ethnicity, and language may well yield valuable insights.

Considerable objection was also raised against the idea of marginal border adjustment. The AS model lets an individual living at the border between two countries decide which jurisdiction it belongs to. Because there is no population mobility, in such situations borders adjust marginally around

[4]In fact, the amount of tax each individual has to pay is already defined exogenously dependent on the size of the respective country in the original AS model, since it is given by the quotient of public good cost k over population size s. In the case of uniform population distribution (as applied in the AS model), the population size coincides with the territorial size of the country.

the border individuals. In fact, howeer, border changes impose very high fixed cost which in turn might be the most important reason why they occur only in the way of discrete non-linear steps rather than infinitesimally small ones. As Allan Drazen correctly notes,

> [...] the marginalist approach may be sometimes relevant for localities like cities or counties. They will make marginal adjustments in their borders for purely economic reasons by absorbing an unincorporated area or (more rarely) giving up a small area to another city or county. At the national level, making marginal adjustments for economic reasons, as opposed to political or socio-ethnic reasons, does not seem especially descriptive of reality. [Drazen, 2000, p. 727]

Additionally, the static approach to equilibrium analysis inherent in this model poses questions on the methodological adequacy of the problem at hand. Since state formation and secession are clearly dynamic long run phenomena contingent on historical events, this theory is able to derive conclusions only within a comparably stable and stylised context.

Another major point of criticism concerns the distribution of individuals. In the AS model the population is distributed uniformly. A direct consequence of this assumption is that in any given solution all countries are of the same size - no matter which institutional setting is investigated (dictatorship, democracy, benevolent social planner). Obviously this result doesn't live up with a comparison against real world data. In fact, country size in terms of population is distributed extremely asymmetrically with relatively few large countries and a large number of very small nation states. Various extensions to the basic AS model deal with openness and international trade, conflict situations and possible power constellations in economic and political supranational institutions. The assumption of a uniform population distribution, and therefore countries of equal size, seems hard to justify considering these extensions. This paper is a first step in analyzing the consequences of introducing jurisdictions of different size by means of non-uniform population distributions.

3 Non-Uniform Population Distributions

In the basic AS model, the utility function of individual i residing in country x is given by

$$U_{ix} = g(1 - al_{ix}) + y - t_{ix}, \tag{4}$$

y denoting exogenous private income, l_{ix} representing the distance between individual i's location and the location of the government of country x and t_{ix} measuring the amount of tax individual i has to pay to finance the

government of country x. Since every inhabitant of country x has to pay the same amount of tax, t_{ix} can be expressed by

$$t_{ix} = k/p_x, \tag{5}$$

where p_x, the population size of country x, is given by

$$p_x = F(b^x) - F(b_x). \tag{6}$$

$F(.)$ is the cumulative distribution function of the population distribution and b^x (b_x) denotes the upper (lower) border of country x on the $[0,1]$-segment. Note that in the case of the uniform distribution equation (6) simplifies to $p_x = b^x - b_x$, which is the formal representation of the fact that territorial size and population size of country x coincide in the basic AS model.

Rewriting equation (4) in a more general way yields

$$U_{ix} = g(1 - a|i - m_x|) + y - k/p_{ix}, \tag{7}$$

where the distance l_{ix} is expressed explicitly as the absolute value of the difference between the location of individual i (denoted simply as i) and the location m_x of the government of country x. While in democratic equilibrium m_x is given by the median position of country x[5], in the social planner case m_x is chosen to maximize total population utility. It can be shown, however, that in this case the location of the government is given by the median of country x (irrespective of the chosen population distribution) as well. The median of country x is determined by

$$m_x \equiv m(b_x, b^x) = F^{-1}\left(\frac{1}{2}(F(b_x) + F(b^x))\right). \tag{8}$$

$F^{-1}(.)$ is the inverse function of $F(x)$, so that $F^{-1}(F(x)) = x$. Assuming a uniform population distribution, equation (8) simplifies to $m_x = \frac{1}{2}(b_x + b^x)$.

As the above elaborations already indicate, the introduction of a non-uniform population distribution into the AS framework results in the loss of tractability in closed form, mostly since this modification leads to the dissolution of identity between territorial size and population size of a country.

Because my method of investigation is numerical computation, any alternative distribution has to offer comparably simple functional forms for density function, distribution function, and median. Another desirable property of the candidate distribution is versatility in terms of being capable of representing a lot of different kinds of population density. In the ideal case, these scenarios also contain the uniform distribution employed in the original model in order to validate the obtained results against the existing

[5]This result is a direct consequence of the application of majority voting and makes use of the Median Voter Theorem.

work. These requirements limit the available range of functions seriously. The Beta-distribution, which is a generalization of the uniform distribution, is out of the question due to its rather complicated form. So I turned to a distribution suggested by Indian hydrologist Poondi Kumaraswamy [Kumaraswamy, 1980] which is very similar to the Beta-distribution. This double-bounded distribution takes the following form:

$$f(i; a, b) = abx^{a-1}(1 - x^a)^{b-1} \tag{9}$$

$$F(i; a, b) = 1 - (1 - x^a)^b \tag{10}$$

The parameters a and b are shape parameters and $0 \leq x \leq 1$. Setting $a = 1$ and $b = 1$ results in the uniform distribution. As can be seen in figure 1, this distribution allows for a variety of different shapes apart from the uniform distribution depending on the chosen shape parameters a and b. Choosing similar values for a and b results in rather symmetric distributions; with increasing difference between the values of the shape parameters, however, asymmetric distributions arise.

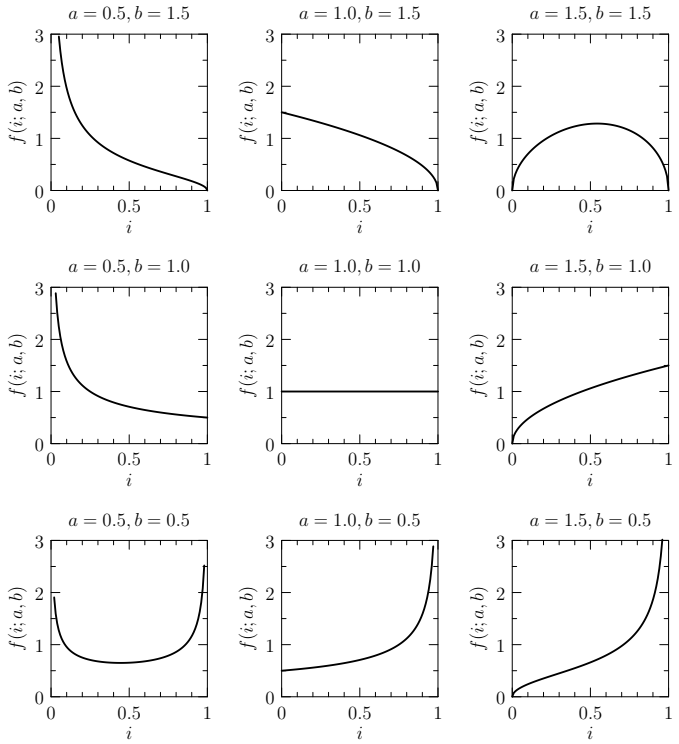

Figure 1: Density functions of Kumaraswamy's double bounded distribution for various sets of shape parameters

3.1 The Social Planner Solution

Analytically, the problem of the social planner remains the same as in the case with uniform population distribution. However, as already mentioned, the introduction of non-uniformity results in the loss of a number of simplifications and the problem is no longer solvable in closed form. Particularly, the social planner problem, which is originally given by

$$\max \int_0^1 U_i di$$
$$s.t. \int_0^1 t_i di = Nk$$

turns to

$$\max \int_0^1 U_i f(i) di \tag{11}$$
$$s.t. \int_0^1 t_i f(i) di = Nk. \tag{12}$$

Since the population is no longer distributed uniformly, utility as well as collected taxes in every location on the $[0, 1]$-segment have to be weighted with the population size in this particular location, which is given by $f(.)$, the density function of the population distribution. Substitution of equations (7) and (12) into (11) and reformulation of the problem yields

$$\min_{N,b_1,...,b_N,b^1,b^N} Nk - y + g + ag \sum_{x=1}^{N} \int_{b_x}^{b^x} |i - m(b_x, b^x)| f(i) di \tag{13}$$

subject to $b^x = b_{x+1}$, $b_1 = 0$ and $b^N = 1$. Thus, the social planner problem is an optimization problem in the N dimensions $N \times b^1 \times ... \times b^{N-1}$. I bypass the loss of tractability in closed form by exhaustively searching the space of possible configurations of nations (and locations of the public good given by the resulting medians of the nations). This is done by varying parameter g for a given set of the model parameters y, k, a and l and evaluating the configuration that maximizes total population utility. Because this task is computationally very demanding, I am able to present results for only a comparatively small range of parameter values. Nevertheless, this is sufficient for detecting qualitative changes in the model behaviour.

Figure 2 shows the results of the calculations for parameter values $y = 50$, $a = 0.8$, $k = 8.0$ and different values of g as a function of g.[6] The solid

[6]The choice of parameter values may seem arbitrary, but since the high level of abstraction in this model renders empirical validation nearly impossible, this choice was guided only by the requirement to generate solutions up to $N = 5$ in a comparatively low range of parameter g. This, in turn, is a concession to run time issues resulting from the computational complexity of the social planner solution. Variation of the parameters y, a and k does not lead to qualitatively different behavior.

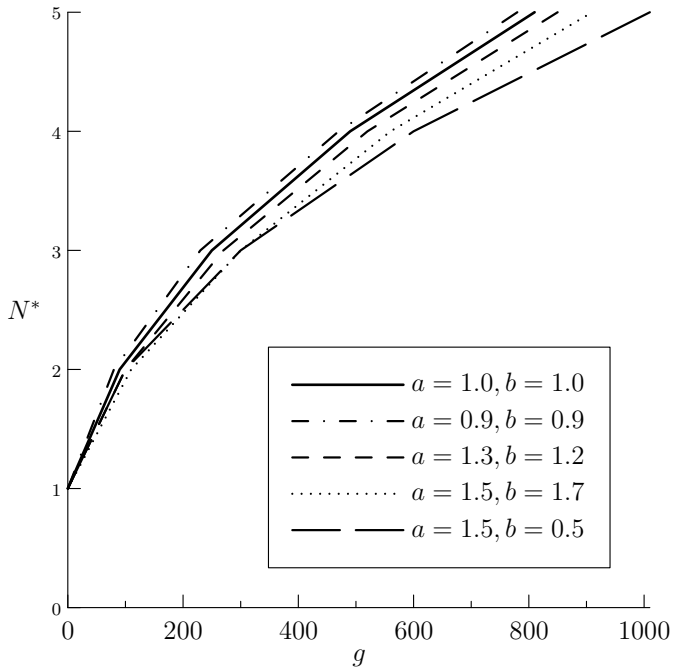

Figure 2: The social planner solution N^* dependent on public good utility g for various population distributions

line represents the corresponding values of the uniform distribution, i.e. $K(1.0, 1.0)$[7]. As can be seen, given a value of g the optimum solution leads to more countries the smaller are the values of the shape parameters and vice versa to a smaller number of countries for higher values of the shape parameters a and b.

3.2 The Democratic Equilibrium

For the evaluation of democratic equilibria I calculate the stability conditions for all possible configurations.[8] A democratic equilibrium under RULE A requires all individuals living at the border between two countries x and y to be indifferent about which country to belong to or formally

$$l_{iy} - l_{ix} = \frac{k}{ag} \frac{p_y - p_x}{p_x p_y} \tag{14}$$

[7]Henceforth, I will use the notation $K(a, b)$ for Kumaraswamy's double bounded distribution with shape parameters a and b.

[8]For this purpose, I employ the stability conditions of RULE A from the original AS model. One might object that omitting calculation of the equilibria regarding RULE B (internationally coordinated secessions and unifications) and RULE C (unilateral secessions) weakens my argument. Since both the sets of equilibria under RULE B and under RULE C, however, are subsets of all possible equilibria under RULE A by definition, all results that follow are valid for equilibria under all three rules.

for all $N-1$ interior borders. In order to be a stable equilibrium as well, small perturbations of this equilibrium (i.e. small border changes) must not lead to a different outcome but instead the system has to converge to the pre-shock equilibrium. Since the location of the public good is decided by majority voting in each country, I can make use of the median voter theorem and assume that each such public good is located at the position of the median voter in each country.

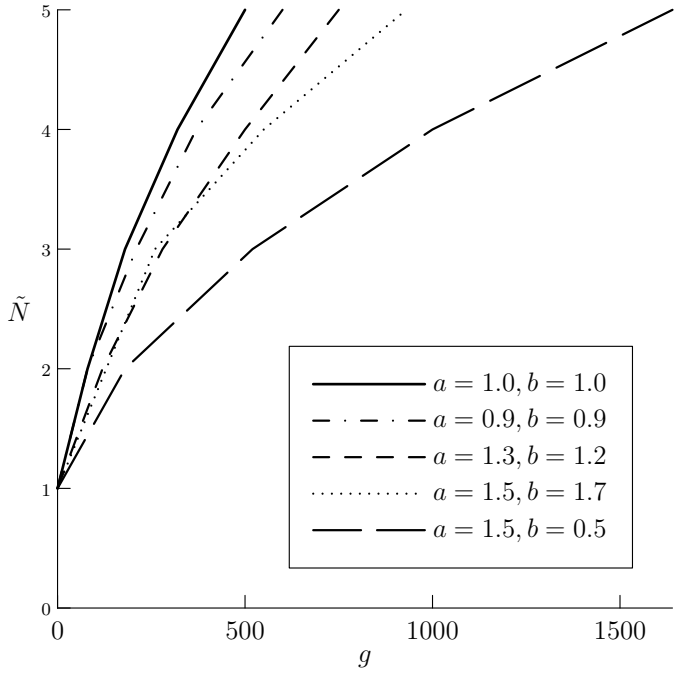

Figure 3: The democratic equilibrium solution $\tilde{N}$ for various population distributions dependent on public good utility g

Figure 3 shows the results of the calculations with the same parameter values as in Figure 2. No matter what values the shape parameters take, it can be said that in general the stable number of nations is smaller than in the uniform case (represented by the bold line once again). While the deviations from the benchmark case with uniform distribution was rather small regarding the optimum number of nations, the border stable equilibrium tends to react much stronger to changes in the underlying population distribution.

4 Comparison Of Results

Given the high level of abstraction of the model, it would not be sensible to derive quantitative conclusion. Nevertheless, the results allow for deci-

sive qualitative conclusions when compared with another. In the following graphs, I therefore compare the social planner solution and the democratic equilibrium solution for the various population distributions investigated above with the results of the original AS model. As will be seen, four different model behaviours can be sorted out.

4.1 Case 1: Confirmation of the AS model

Comparison of the case of $(a, b) = (0.9, 0.9)$ yields a confirmation of the results of the original AS model with uniform population distribution. This rather minor deviation from the uniform case $(a, b) = (1.0, 1.0)$ changes the stable solution and the efficient solution only slightly, thereby not altering the qualitative behaviour of the model. Still, the stable number of nations is larger than the efficient number of nations for all values $g > 0$. A look at figure 4 reveals the details.

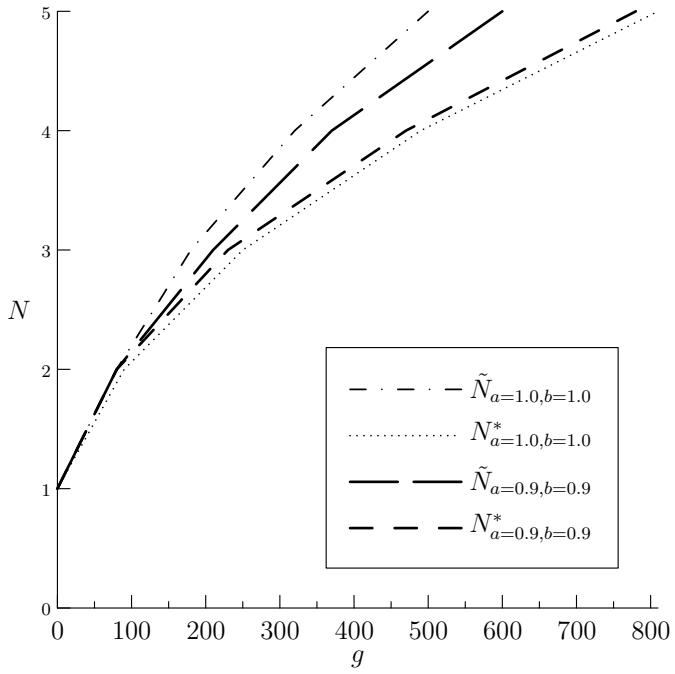

Figure 4: Results of the uniform distribution compared with K(0.9, 0.9)

The general result is very similar to the case of the uniform distribution, namely, that in equilibrium the stable number of nations is larger than the socially optimal number of nations, i.e. there are too many nations in equilibrium. But the extent of the difference between social planner solution and border stable equilibrium is smaller. Nevertheless, this finding confirms the stability of the results obtained by Alesina and Spolaore regarding minor deviations from the uniform population distribution.

4.2 Case 2: Ambiguous outcome with 1 intersection

The picture changes, however, when the shape parameters are altered a little bit more. For the values $(a, b) = (1.3, 1.2)$, the outcome is neither clearly in favour nor strictly against the results derived from the original AS model. As Figure 5 shows, the social planner solution intersects with the equilibrium solution at about $g = 350$ and it depends on the exact value of parameter g, whether the stable number of nations is larger than the efficient number of nations or vice versa.

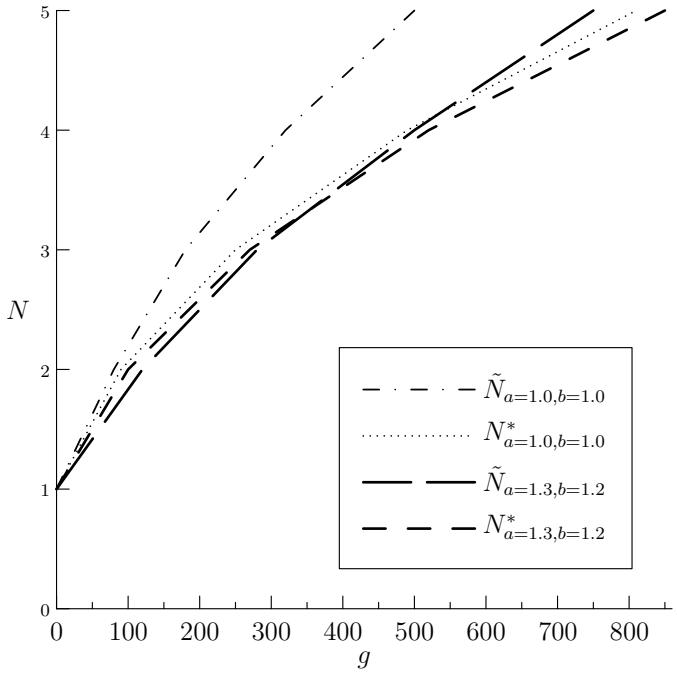

Figure 5: Results of the uniform distribution compared with K(1.3, 1.2)

4.3 Case 3: Ambiguous outcome with 2 intersections

An even more puzzling outcome results for instance with shape parameters $(a, b) = (1.5, 1.7)$. As figure 6 shows, this time there is not only one intersection point, but there are two of them. Now, the basic result that the stable number of nations is larger than the efficient number of nations is valid only within a certain value range of parameter g. If g lies outside of this range, then the results are reversed again.

4.4 Case 4: Complete rejection of the AS model

The final case I want to illustrate is the complete reversal of the AS results. For certain values of the shape parameters, the efficient number of nations is

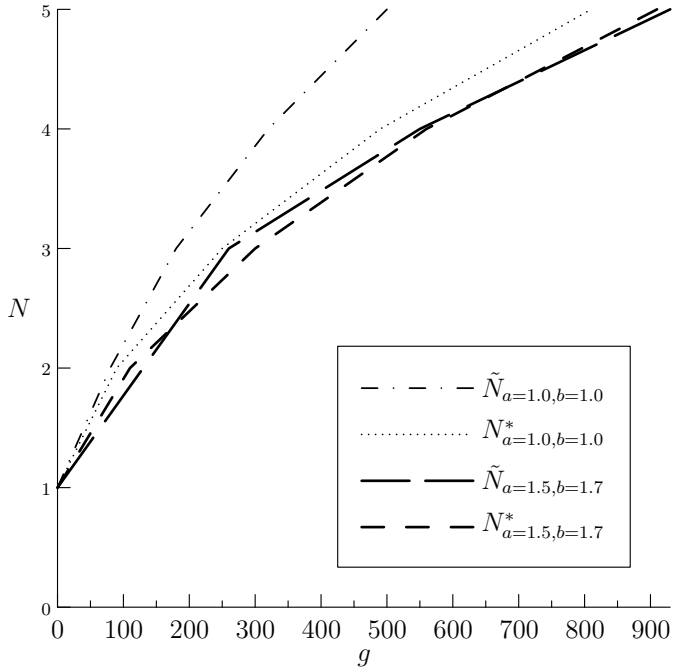

Figure 6: Results of the uniform distribution compared with K(1.5, 1.7)

larger than the stable number of nations, thus contradicting the statement of the AS model with uniform population distribution. As can be seen in figure 7, in the case of the $K(1.5, 0.5)$-distribution, the socially optimal number of nations is larger than the stable number of nations, which contradicts the basic message stated by the uniform case of the AS model.

These examples have clearly shown that the qualitative behaviour of this framework crucially depends on the choice of the population distribution. For shape parameter values close to 1, the same propositions obtain as in the uniform case. However, by further deviating from the benchmark uniform case, the model changes its qualitative behaviour. We have seen that in the cases of $K(1.3, 1.2)$ and $K(1.5, 1.7)$, no globally valid conclusions on the relationship between the efficient solution and the stable solution can be drawn. Finally, at shape parameter values $a = 1.5$ and $b = 0.5$ the proposition derived from the uniform model is even reversed and in equilibrium there are "too few" nation states compared to the socially optimal case.

Until here, I have only shown a comparably small number of cases yielding different results. To get the bigger picture of the dependence of model behaviour on the choice of the population distribution, figure 8 shows the outcomes for a great number of population distributions with shape parameters a and b ranging from 0.1 to 2.0. The point in parameter space corresponding to the uniform population distribution (and therefore also corresponding to the results of Alesina and Spolaore) is marked by the X

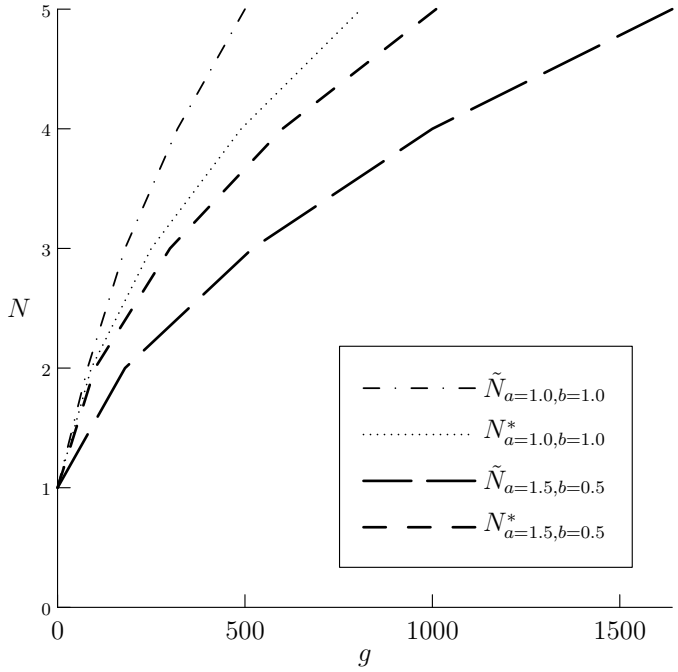

Figure 7: Results of the uniform distribution compared with K(1.5, 0.5)

at $(1, 1)$. As can be seen by the hollow circles, the predictions of the AS model only hold in a very small surrounding area of the uniform population distribution (case 1). For slightly larger deviations from the uniform case, the model adapts a different behaviour characterized by the intersection of the social planner solution and the stable solution (case 2). Going even farther away from the center of the diagram, the model finally reverts its behaviour in most of the investigated cases (marked by the hollow squares) (case 4). An additional type of model behaviour can be found in the lower left and the upper right region of the parameter space. The cases marked by the triangle-signs characterize situations where the social planner solution and the stable solution have two intersections (case 3). The general pattern indicates that for asymmetrical distributions the results of the AS model are negated, while in the case of rather symmetrical distributions the results are conformed or - more oftenly - ambiguous outcomes occur.

While these results still represent a very selective view on possible parameter constellations, they suffice to show that the model behaviour is highly sensitive to the chosen population distribution and serve as existence proof for outcomes completely different from the ones proposed by the original AS-model with uniform population distribution. Especially the relative frequency of the four different regimes represents a strong result. The overwhelming majority of population distributions points to model behavior contrary to the insights of the AS model. Furthermore, the results of the AS

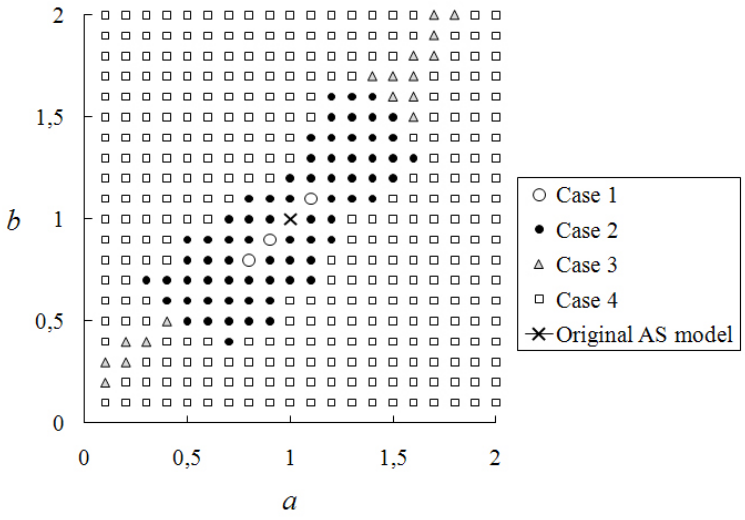

Figure 8: Model behavior for the state space of shape parameters a, b

model hold only within a narrow range of distributions close to the uniform distribution.

5 Interpretation

What drives these results? Figures 2 and 3 have already shown that the democratic equilibrium reacts much stronger to changes in the population distribution than the social planner solution. As a means of illustration I would like to give a specific example. Figure 9 presents the social planner solution and the democratic solution for $K(1.5, 0.5)$ and parameters $y = 50$, $a = 0.8$, $k = 8.0$ and $g = 400$. As stated above, $K(1.5, 0.5)$ belongs to the behavioral regime of case 4, i.e. in equilibrium there are too few nations. The borders which the social planner would decree are represented by the dashed lines; the single interior border in the democratic case is represented by the solid line. The diagram depicts the distribution function, thus on the horizontal axis the borders indicate the territorial size of a country, on the vertical axis they represent the population size. Given the parameters stated above, the social planner would divide the world into three countries: the leftmost country is populated sparsely, i.e. population size is much smaller then territorial size. For the middle country population size and territorial size are nearly equal; the rightmost encompasses nearly half of the world population while occupying less then 20% of territorial space. This solution, however, is not stable in a democratic world. In such a world the individuals would create only two countries, a sparsely populated one taking up two thirds of territorial space but encompassing only one third of

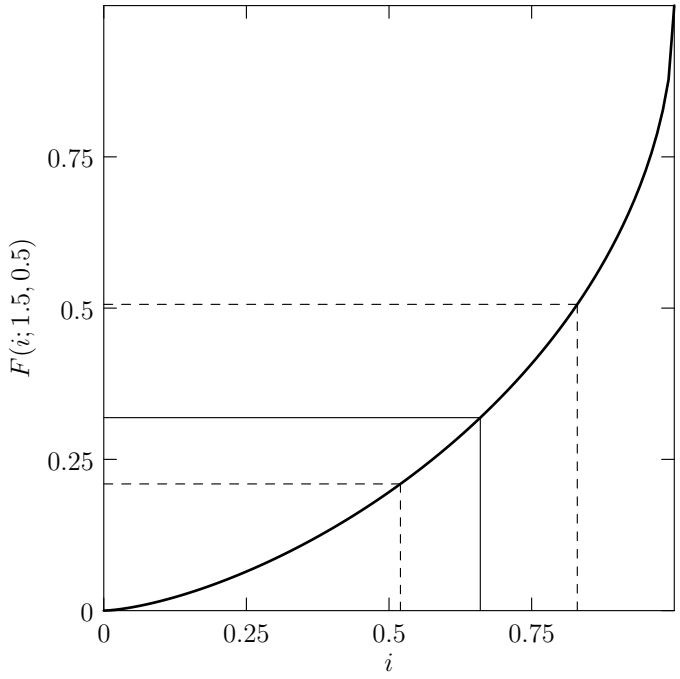

Figure 9: An example of the democratic solution and the social planner solution in the case of $K(1.5, 0.5)$

the world population and a densely populated one accounting for one third of territorial space and two thirds of world population.

The logic behind the individuals' desire to create fewer countries than the social planner would decree, however, is the same as in the original AS model, only the direction of effect is reversed. Starting at the social planner solution, the individual next to the left border of the densely populated country to the right would prefer to be part of this country rather than the country in the middle since distance to the government is much smaller in the rightmost country. Additionally, the per capita cost of public good provision is much lower there because of the high population density. The individual's choice for the right country even increases the benefits of being a citizen of that country because per capita costs are reduced by a higher amount than the utility losses from distance to government. For the same reasons the next individuals would choose the right rather than the middle country as well until the gain from decreased per capita costs equals the utility losses from distance to government. Similar reasoning applies to the border between the left and the middle country. Making public good utility g endogenously dependent on taxes collected - instead of treating it as constant - as proposed by Klaas Staal [Staal, 2004] even amplifies this process.

In the remaining part of this article I will elaborate on the implications

of the lack of robustness of the AS model and propose a different approach to the issue.

6 Implications

As has been shown above, the results derived from the AS framework are strictly dependent on the choice of the distribution. So it is fair to say that this model is not robust to variations of some of its assumptions.

Clearly, the results derived from the uniform distribution case make some sense. If, for instance, you take a look at the many small nation states with a population size less than one million people, it seems plausible that they are operating at an inefficiently low scale. Furthermore, if you consider that some of these small nations are still subject to secession ambitions, therefore potentially leading to even smaller states - as documented by the efforts of Curacao or St. Martin to become independent[9] - the prediction of the AS model that in equilibrium there are more nation states than in the social optimum seems like an accurate description of some real world processes.

These results are derived, however, under very strict and artificial assumptions. Especially the uniform size distribution of nation states seems troublesome because it leads to results with nation states of equal size which obviously contradicts the highly asymmetrical size distribution of countries - whether measured in population size or in territorial size. Though, by relaxing the assumptions regarding the population distribution and therefore allowing for states of different sizes, the proposition holds only within a very limited range of distributions for shape parameters close to the uniform case. In the vast majority of cases, the model behaviour is ambiguous or even opposed to the results of the original model.

Considering the extensions of the AS model that are concerned with international conflict [Alesina and Spolaore, 2006] or the formation of international federations [Alesina et al., 2005], the question at hand is, whether not incorporating heterogeneous state sizes is a valid simplification for capturing the driving forces of these phenomena. I doubt the validity of this abstraction for both issues are determined by the interaction patterns of a few very large actors and a majority of comparably small ones. Negotiations within international federations and conflict situations rarely take place on equal terms with regard to bargaining power. The neglect of this asymmetry clearly limits the explanatory power of the AS model regarding the description and understanding of international political dynamics. Most if not all models in the tradition of the AS approach assume a uniform population distribution for mathematical convenience. It seems likely that the obtained results are dependent on this simplifying assumption as well, although these models haven't been tested yet.

[9] The Economist, May 26th 2007, p. 58

Additionally, empirical investigation is of high importance in order to single out those forms of distributions that prevail in reality. Thereafter, validation or falsification of the obtained results would be possible. Given the high abstraction level of this framework, however, it seems hard to come up with a sound empirical strategy. Above all it is not clear how empirical data could be transformed to one-dimensional distance measures as used in the AS model.

7 Conclusion

Without a doubt, the AS model serves as a starting point for a better understanding of the complex issues at hand, but as in the present paper I tried to show, there is still much research to do in order to grasp them in a more adequate way. While the treatment at hand represents a first generalization of the underlying distribution assumptions that allows for nation states of different sizes, more realistic multimodal population distributions are still missing in the picture. Non-linear modelling approaches like agent based modelling (ABM) seem to be the method of choice in this case. Not only would ABM allow for more flexibility regarding population distributions, but other drawbacks of the AS model (e.g. absence of population mobility, perfect correlation between preferences and location) could be tackled as well.

In my opinion a comparative static approach with marginal border adjustments cannot do justice to the issue of secession and unification. Border changes happen in discrete non-linear steps and are subject to a number of dynamic influences, path dependencies, and the like. It is to be doubted if a static mathematical model driven by the self-interested decisions of single individuals is the right method to tackle this collective problem. ABM, on the other hand, allows for the formulation of the issues by means of complex adaptive systems. These systems are able to exhibit non-linearities as well as path-dependent behaviour which I believe to be relevant elements in the context of this topic. Lars-Erik Cederman's agent based models, for instance, seem particularly promising [Cederman, 1997]. He develops a number of non-equilibrium models on the emergence of states and nations by employing this method which resulted in a number of valuable insights. However, like most work originating in the political sciences, he neglects the interplay between politics and economics in the process of secession and unification and reduces the agents' motivation to conquest. On the other hand, existing work in the economics literature focuses primarily on economic issues like public good provision and tax collection.

It is my belief that neither of these approaches is fully capable of handling the issues tackled in this paper. In fact, I believe that interdisciplinary social science research is necessary to gain deeper insights in many hot issues

related to the problem. The role of the nation state in a unified Europe and a globalized world, the ongoing trend for secessions in many parts of the world, and the challenges of so-called failed states show impressively that there is a pressing need for theories which can substantially improve our knowledge about the role of the nation state in the international political economy.

Clearly, the presented results are just a first step. Nevertheless, I think that the work on this topic so far deserves further investigation in many directions, since the central insights of Alesina and Spolaore have turned into common sense among many economists. Yet, as the treatment at hand has shown, their results are heavily dependent on the appropriate choice of distribution function and valid only in the minority of cases. In the face of the many problems related to the role of the nation state it would be urgent to put forward a new and improved theory that manages to integrate political, economic and cultural factors that accompany the issue of secession and unification.

References

[Alesina et al., 2005] Alesina, A., Angeloni, I., and Etro, F. (2005). International unions. *The American Economic Review*, 95:602–615.

[Alesina et al., 2003] Alesina, A., Devleeschauwer, A., Easterly, W., Kurlat, S., and Wacziarg, R. (2003). Fractionalization. *Journal Of Economic Growth*, 8:155–194.

[Alesina and Spolaore, 1997] Alesina, A. and Spolaore, E. (1997). On the number and size of nations. *Quarterly Journal Of Economics*, 112:1027–1056.

[Alesina and Spolaore, 2003] Alesina, A. and Spolaore, E. (2003). *The Size Of Nations*. MIT Press.

[Alesina and Spolaore, 2006] Alesina, A. and Spolaore, E. (2006). Conflict, defense spending and the number of nations. *European Economic Review*, 50:91–120.

[Bolton and Roland, 1997] Bolton, P. and Roland, G. (1997). The breakup of nations: A political economy analysis. *Quarterly Journal Of Economics*, 112:1057–1090.

[Bolton et al., 1996] Bolton, P., Roland, G., and Spolaore, E. (1996). Economic theories of the break-up and integration of nations. *European Economic Review*, 40:697–705.

[Casella and Feinstein, 2002] Casella, A. and Feinstein, J. S. (2002). Public goods in trade: On the formation of markets and political jurisdictions. *International Economic Review*, 43:437–462.

[Cederman, 1997] Cederman, L.-E. (1997). *Emergent Actors In World Politics: How States & Nations Develop & Dissolve*. Princeton University Press.

[Drazen, 2000] Drazen, A. (2000). *Political Economy In Macroeconomics*. Princeton University Press.

[Easterly and Levine, 1997] Easterly, W. and Levine, R. (1997). Africa's growth tragedy: Policies and ethnic divisions. *Quarterly Journal Of Economics*, 112:1203–1250.

[Easterly and Levine, 2001] Easterly, W. and Levine, R. (2001). What have we learned from a decade of empirical research on growth? it's not factor accumulation: Stylized facts and growth models. *World Bank Economic Review*, 15:177–219.

[Etro, 2006] Etro, F. (2006). Political geography. *Public Choice*, 127:329–351.

[Friedman, 1977] Friedman, D. (1977). A theory of the size and shape of nations. *Journal Of Political Economy*, 85:59–77.

[Hotelling, 1929] Hotelling, H. (1929). Stability in competition. *Economic Journal*, 39:41–57.

[Kumaraswamy, 1980] Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. *Journal of Hydrology*, 39:79–88.

[Ruta, 2005] Ruta, M. (2005). Economic theories of political (dis)integration. *Journal Of Economic Surveys*, 19:1–21.

[Spolaore, 2006] Spolaore, E. (2006). National borders and the size of nations. In Weingast, B. R. and Wittman, D. A., editors, *The Oxford Handbook of Political Economy*, pages 778–798. Oxford University Press.

[Staal, 2004] Staal, K. (2004). Country size and public good provision. *Tinbergen Institute Discussion Paper*, 26.

[Wittman, 2000] Wittman, D. (2000). The wealth and size of nations. *Journal Of Conflict Resolution*, 44:868–884.