Metz, Christina E.

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Christina E. Metz

Editor:
Christof Kreuter
+49 69 910-31748
christof.kreuter@db.com

Deutsche Bank Research
Frankfurt am Main
Germany
E-mail: marketing.dbr@db.com
Fax: +49 69 910-31877

DB Research Management
Axel Siedenberg
Norbert Walter

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Private and Public Information in Self-Fulfilling Currency Crises

Christina E. Metz*

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Abstract

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* University of Kassel, Nora-Platiel-Str. 4, 34127 Kassel, metz@wirtschaft.uni-kassel.de

1 Introduction

Following the currency crises of the last decade in Europe, Mexico and Asia, a growing literature has appeared trying to explain and formalize financial turmoil. Due to the growing importance of expectations and beliefs, multiple equilibria models of currency attacks, proposed by Obstfeld (1996), have gained momentum in explaining currency crises. A major drawback of these models, however, is the ambiguity of possible outcomes for a rather large range of economic fundamentals. As actions of economic agents are motivated by their beliefs, economic outcomes are influenced by expectations as well, so that in multiple equilibria models *anything* can happen, as long as agents believe it to happen. Moreover, the multiple equilibria approach does not explain the shift in beliefs, which incites the economy to move from one equilibrium to the next. Consequently, there is no way of generating comparative statics and therefore policy devices are hardly derivable.

The procedure of eliminating equilibria in a currency-market setting was firstly taken up by Morris and Shin (1998a). Proceeding from a simple model with multiple equilibria, they show that the introduction of noisy private information leads to a unique equilibrium as long as the noise value is relatively small.

In this paper we will apply their approach to an information structure with normally distributed parameters. We allow for public as well as private information/signals. Public information is disseminated by the government, for instance by publishing economic data and other statistics, and might be interpreted as common priors about the fundamental state of the economy. The government can determine the precision of its signal ex-ante, whereas from then on this parameter stays constant. Private information, in contrast, might be seen as speculators' slightly different interpretations of the unknown true fundamental state. In this setting the uniqueness result is restored if the precision of private information is large relative to the precision of public information. If there is a unique equilibrium we are then able to analyze comparative statics, giving particular attention to the influence of private and public information on the probability of a currency attack. We can infer from our model that, for a government that tries to prevent an attack on the fixed exchange rate, in case of bad prior fundamentals it is better to have established a system with low precision of public information. However, if the a priori expected fundamental state of the economy is good, there is an incentive for the government to increase the precision of its signal. In contrast, the precision of the private signal influences the probability of a speculative attack exactly in the opposite direction.

In case that it is not ex ante certain if there will be a unique equilibrium in the game between government and speculators, our model delivers another interesting finding. For a rather bad expected state of economic fundamentals there are two ways for the government to prevent a speculative attack: by either disseminating very precise signals about the true value of the fundamentals or by passing on extremely imprecise public information. As such, our model gives a link between the implications of the multiple equilibria approach and the unique equilibrium case.

The remainder of the paper is organized as follows. Section two presents the basic form of a general model of currency crises. Section three gives a brief review of a multiple equilibria model due to complete information of the underlying fundamentals. Section four concentrates on a model of incomplete information and shows that the lack of higher order knowledge under certain conditions generates a unique equilibrium. Subsequently, comparative statics are analyzed with emphasis on the influence of the precision of private and public signals. Section five analyzes the importance of private and public information for equilibrium selection. Section six concludes.

2 The Basic Model

We examine a model with a continuum of speculators, indexed by the unit interval $[0, 1]$. Each speculator disposes of one unit of the currency and can decide whether to short-sell this unit, i.e., attack the currency peg, or not to do so. If the attack is successful he gets a fixed profit of D, $D > 0$. Attacking the currency, however, also leads to cost t, $t > 0$, which can be interpreted as either actual transaction costs or as the interest rate differential between the considered currencies. If a speculator refrains from selling the currency he is not exposed to any cost, but he does not gain anything either.

We assume that the government defends the currency if the costs of this action are not too high. The costs of defending the peg are increasing in the proportion of speculators short-selling the domestic currency and decreasing in the underlying fundamental state of the economy. Let the proportion of speculators attacking the currency peg be denoted by l. An index of the fundamental state is given by θ. If θ is sufficiently high, the government is able to always defend the peg irrespective of the number of attacking speculators. However, if θ is sufficiently low, the government abandons the peg in favour of a devaluation even if none of the speculators sell the currency. More precisely,

- if $l < \theta$, the attack is unsuccessful
- if $l \geq \theta$, the attack leads to success

The game between speculators and government is then structured as follows: Nature chooses the value of the fundamental index according to a normal distribution with mean $\bar{\theta}$ and variance $\frac{1}{\alpha}$. The mean $\bar{\theta}$ of this distribution is exogenous to the government, whereas the variance $\frac{1}{\alpha}$ can be influenced. This might be interpreted in terms of transparency of fiscal or monetary measures. Thus, by making its policy measures and targets transparent, i.e., by increasing α, the government can stabilize speculators' expectations concerning the development of the fundamental index around the a priori expected value $\bar{\theta}$ and vice versa. As we assume that the government intends to prevent devaluations of the currency peg, it will choose variance $\frac{1}{\alpha}$ so as to minimize the probability of a currency attack given $\bar{\theta}$. In accordance with Morris and Shin (1999), these two distributional parameters of the fundamental state θ are called *public signal* with α also denoted as the precision of the public signal. This public signal is disseminated by the government to all speculators. The truly realized value of the fundamental index θ, however, is only known to the government. Additionally to the public signal, speculators individually receive noisy *private signals* $x_i = \theta + \varepsilon_i$, with ε_i being normally distributed with mean zero and variance $\frac{1}{\beta}$ and independent of the realized value of θ. Mean and variance of the noise parameter in the private signal are assumed to be known to all speculators, but as long as the variance $\frac{1}{\beta}$ is higher than zero, i.e., as long as the precision of the private signals is finite, speculators cannot accurately establish the signals of their opponents. On the basis of their private signals and the common public signal, speculators simultaneously have to decide whether to attack the currency or to stay with the peg. The government observes the proportion l of attacking agents and decides on maintaining the peg or abandoning it.

Concerning the different kinds of signals we would like to emphasize that there are two sources of uncertainty in the market about the truly realized value of the fundamental index. Firstly, there is a governmental-based uncertainty or prior uncertainty due to the public signal, which only tells speculators *how close* the true value of θ is to the prior mean $\bar{\theta}$. By determining α, the government can influence speculators' prior beliefs about the fundamental state of the economy. The second layer of uncertainty arises from the different private signals. These can be seen as speculators' individual interpretations of the fundamental state of the economy, which might be due to different channels of information selection.

It is crucial for the distinction between the models of complete and incomplete information to correctly define which elements of the game are common knowledge. Something is said to be common knowledge if everybody knows it, everybody knows that everybody knows it and so on to an infinite order. In both models, the full *structure* of the game is common knowledge, i.e. D,t, mean $\bar{\theta}$ and precision α of the public signal, as well as mean and precision β of the private signals. The difference between the two models, however, lies in the knowledge of θ, as speculators are not able to observe θ directly but only receive individual signals about it. Moreover, they cannot precisely establish the informational content of their opponents' private signals, for they only observe their own signal. Thus, the degree of lack of common knowledge of θ depends on the value of the noise term ε, respectively on the finiteness of its precision, β.

3 The Complete Information Case - Multiple Equilibria

If $\varepsilon_i = 0$ for all i due to zero variance ($\beta = \infty$), so that each speculator's signal is completely precise and equal to the true value of θ, the fundamental index becomes common knowledge, and we get the typical tripartition of fundamentals as in the original multiple equilibria model by Obstfeld:

- If $\theta > 1$, it is optimal for speculators not to attack the currency, as, even if all of them sell the domestic currency (i.e. $l = 1$), the economy is sound enough so that the government can defend the currency peg. Thus, the *critical mass condition* can never be satisfied. For this range of fundamentals the currency peg is said to be *stable.*

- If, however, $\theta \leq 0$, the government always abandons the peg, irrespective of the actions of speculators and the currency peg is *unstable.*

- In between these two boundaries, i.e. for $0 < \theta \leq 1$, there can be attacks as well as financial stability. This interval is called the *ripe for attack* region. If speculators expect an attack to be successful they will all attack the currency peg which in turn leads to success and as such vindicates their initial beliefs, whereas they will (all) not engage in selling the domestic currency if they are not convinced of a positive payoff. We therefore speak of self-fulfilling expectations. The outcome of the model may then depend on so-called sunspots, i.e., arbitrary correlating devices which coordinate the expectations of speculators concerning the behaviour of others.

Thus, if θ is known to lie in the range from zero to one, even incidences that seem to be absolutely unrelated to economic fundamentals can induce speculators to sell the currency and consequently lead to a speculative attack. Although in this interval the state of fundamentals can coordinate speculators' actions as well, the shift in beliefs, which leads to a shift from an attack-equilibrium to a no-attack-equilibrium and vice versa, does not depend on the fundamental index. Obviously, this feature of multiple equilibria models runs counter to the intuition that, above all, countries in economic distress should be vulnerable to speculative attacks.

4 The Incomplete Information Case - Unique Equilibrium

In the model of incomplete information we assume that β takes on a finite, strictly positive value, so that θ is no longer common knowledge. Hence, speculators do not know the true value of θ but obtain a signal that is more or less close to the realized value of θ.

In accordance with Morris and Shin (1999) we can state the following condition for a unique equilibrium:
If the private signal x is sufficiently precise, i.e. for $\beta > \frac{\alpha^2}{2\pi}$, there exists a unique equilibrium consisting of a unique value of the fundamental index θ^* up to which the government always abandons the peg, and a unique value of the signal x^*, such that every speculator who receives a signal lower than x^* attacks the currency peg.

The general intuition behind this proposition is the following: A fundamental value of θ^* generates a distribution of signals, such that there is exactly one signal x^*, which would make a speculator receiving that signal indifferent between attacking and not-attacking and which - if all speculators with signals smaller than x^* would decide to attack - would just generate a proportion $l = \theta^*$ of attackers that were sufficient to force a devaluation of the currency peg.

Before we turn to the influence of the informational parameters on the outcome of the game between speculators and government, we will derive the equilibrium and show that it is indeed unique if the above condition is satisfied.

4.1 Derivation of the Unique Equilibrium

The two equilibrium values θ^* and x^* belong to two situations of indifference: for $\theta = \theta^*$ the government is indifferent between defending the currency peg and abandoning it, whereas speculators receiving a signal of x^* are indifferent between attacking the peg and refraining from doing so[1].

θ^* and x^* can be obtained as follows: speculator i observes signal x_i and forms expectations of the unknown value of θ on the grounds of his signal:

$$E(\theta|x_i) = \frac{1}{\alpha+\beta}(\alpha \cdot \bar{\theta} + \beta \cdot x_i) = \theta^e(x_i) \tag{1}$$

with variance[2]

$$\text{Var}(\theta|x_i) = \frac{1}{\alpha+\beta} \tag{2}$$

As can be seen, the posterior expectation of θ is a weighted average of the information the speculator possesses. The higher the precision of the public information, α, the more important $\bar{\theta}$ gets, whereas the private signal gains importance the higher the precision β of this signal.

Moreover each speculator forms expectations about the signals x_j of his opponents:

$$E(x_j|x_i) = \frac{1}{\alpha+\beta}(\alpha \cdot \bar{\theta} + \beta \cdot x_i) = \theta^e(x_i) \tag{3}$$

and

$$\text{Var}(x_j|x_i) = \frac{\alpha+2\beta}{\beta(\alpha+\beta)} \tag{4}$$

Conditional on his signal, each speculator assumes that the other agents receive signals with mean equal to his posterior expectation of the fundamental index, $\theta^e(x_i)$. However, he assigns a higher variability to their signals than the variance he ascribes to the posterior expected value of θ. Consequently, even if a speculator receives a signal which rules out some states of the world, he cannot neglect these states in his decision-making process, as his payoff not only depends on the true value of the fundamentals but also on the actions of the other speculators which might have got different signals that do not rule out the same states. Furthermore,

[1]For reasons of mathematical tractability we assume that after receiving the signal x^* a speculator decides to attack rather than not-attack.

[2]For the calculation of these distributional parameters see appendix I.

even if all the other speculators neglected the same states of the world, due to their signals, they might not know that he did, too, so that again, he cannot rule out these fundamental states. This lack of common knowledge is the essential feature of the model which renders unambiguity of the equilibrium.

After receiving the private and the public signal, each speculator has to decide whether to attack the currency, which leads to costs of t and an uncertain payoff of D, or not to sell the currency which is associated with a net profit of zero with certainty. As can be shown, there exists exactly one value of the signal, namely x^*, which makes each speculator indifferent between these two possibilities. Indifference means, that the two actions lead to the same expected utility[3], with utility defined as net profit. For indifference it is thus required that:

$$t = D \cdot \text{Prob}(\text{attack successful}|x^*) \tag{5}$$

The government will abandon the peg if θ is smaller than or equal to θ^*, so that the probability of a successful attack equals the probability that the realized value of θ is smaller than or equal to θ^*, given x^*. Thus, with Φ being the cumulated normal density:

$$\begin{aligned} t &= D \cdot \text{Prob}(\theta \leq \theta^* | x^*) \\ &= D \cdot \Phi\left(\frac{\theta^* - E(\theta|x^*)}{\sqrt{\text{Var}(\theta|x^*)}}\right) \\ &= D \cdot \Phi(\sqrt{\alpha+\beta}(\theta^* - \theta^e(x^*))) \\ &= D \cdot \Phi(\sqrt{\alpha+\beta}(\theta^* - \frac{\alpha}{\alpha+\beta}\bar{\theta} - \frac{\beta}{\alpha+\beta}x^*)) \end{aligned} \tag{6}$$

The government on the other hand is indifferent between defending the currency peg and abandoning it, if the proportion of speculators attacking the peg $l = \theta^*$. The proportion of attacking speculators, however, equals the proportion of speculators who get a signal smaller than or equal to x^*. As ε is assumed to be independent of the true value of θ, this proportion corresponds to the probability with which one single speculator observes a signal smaller than or equal to x^*, given θ. l can thus be calculated as:

$$l \;=\; \text{Prob}(x \leq x^* | \theta)$$

[3]Assuming risk-neutrality of speculators' behaviour.

$$
\begin{aligned}
&= \Phi\left(\frac{x^* - E(x|\theta)}{\sqrt{\mathrm{Var}(x|\theta)}}\right) \\
&= \Phi\left(\frac{x^* - \theta}{\sqrt{\frac{1}{\beta}}}\right) \\
&= \Phi(\sqrt{\beta}(x^* - \theta))
\end{aligned} \tag{7}
$$

Hence, the government is indifferent between defending the peg and abandoning it if:

$$\theta^* = \Phi(\sqrt{\beta}(x^* - \theta^*)) \tag{8}$$

From equations (6) and (8) we can thus derive the indifference curve for the speculators, denoted by $x^{*SP}(\theta^*)$, and the government, denoted by $x^{*G}(\theta^*)$:

$$x^{*SP}(\theta^*) = \frac{\alpha+\beta}{\beta}\theta^* - \frac{\alpha}{\beta}\bar{\theta} - \frac{\sqrt{\alpha+\beta}}{\beta} \quad \Phi^{-1}\left(\frac{t}{D}\right) \tag{9}$$

and

$$x^{*G}(\theta^*) = \frac{1}{\sqrt{\beta}}\Phi^{-1}(\theta^*) + \theta^* \tag{10}$$

The equilibrium is then given as the intersection point of the two indifference curves, so that the equilibrium value of θ is determined as

$$\theta^* = \Phi\left(\frac{\alpha}{\sqrt{\beta}}\left(\theta^* - \bar{\theta} - \frac{\sqrt{\alpha+\beta}}{\alpha} \quad \Phi^{-1}\left(\frac{t}{D}\right)\right)\right) \tag{11}$$

while x^* can be obtained from equation (9).

In the present model, (θ^*, x^*) forms a trigger-point equilibrium, which means that exactly at that point speculators switch their strategy from *attack* to *not-attack* and the government switches its strategy from *abandon the peg* to *hold the peg*.

4.2 Sufficient Condition for Uniqueness

To show that the equilibrium is unique, we have to prove that there can be only one value of the fundamental index and one signal which make both the government and the speculators indifferent, i.e., there is only one intersection point of $x^{*SP}(\theta^*)$ and $x^{*G}(\theta^*)$. This condition for a unique equilibrium is satisfied if one of the curves runs steeper than the other throughout the whole range of possible values. As neither of the two indifference functions is limited to any range, the unique equilibrium then exists with certainty.

The slopes of the two indifference curves are equal to:

$$\frac{\partial x^{*SP}}{\partial \theta^*} = \frac{\alpha + \beta}{\beta} \tag{12}$$

and

$$\frac{\partial x^{*G}}{\partial \theta^*} = \frac{1}{\sqrt{\beta}} \cdot \frac{\partial \Phi^{-1}(\theta^*)}{\partial \theta^*} + 1 \tag{13}$$

respectively.

Thus, the (sufficient but not necessary) condition for a unique equilibrium is satisfied, if

$$\frac{\alpha + \beta}{\beta} < \frac{1}{\sqrt{\beta}} \quad {}_{min}\left(\frac{\partial \Phi^{-1}(\theta^*)}{\partial \theta^*}\right) \quad + \quad 1 \tag{14}$$

For the following derivation of the uniqueness condition note that the smallest value of $\frac{\partial \Phi^{-1}(\theta^*)}{\partial \theta^*}$ is equal to the reciprocal of the maximum value of the partial derivative of $\Phi(\theta^*)$ with respect to θ^*. This maximum value is given by the normal density $\phi(\theta^*)$ at its mean μ with $\phi(\mu) = \frac{1}{\sqrt{2\pi}}$. Thus, the above sufficient condition of uniqueness is fulfilled, if

$$\begin{aligned} \frac{\alpha + \beta}{\beta} &< 1 + \frac{1}{\sqrt{\beta}} \cdot \frac{1}{\frac{1}{\sqrt{2\pi}}} \\ \beta &> \frac{\alpha^2}{2\pi} \end{aligned} \tag{15}$$

Hence, for a given precision of the public signal, α, the depicted equilibrium is unique as long as the precision of the private signal, β, is high enough. If θ^* is unique, then x^* must be unique as well.

4.3 Comparative Statics

In the following, we will assume that uniqueness of equilibrium is guaranteed, i.e., $\beta > \frac{\alpha^2}{2\pi}$, and examine the influence which the different parameters exert on the unique switching point (θ^*, x^*). From equation (11), which gives the equilibrium value of the fundamental index and as such the range in which an attack will take place and be successful, we can infer the following propositions:

Proposition 1 The probability of an attack *rises* with *decreasing* t and *increasing* D.

Proof:
The partial derivatives of θ^* with respect to t and D are:

$$\frac{\partial \theta^*}{\partial t} = \phi(\cdot)\left(\frac{\alpha}{\sqrt{\beta}}\frac{\partial \theta^*}{\partial t} - \sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial \Phi^{-1}(\frac{t}{D})}{\partial t}\right) = \frac{-\sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial \Phi^{-1}(\frac{t}{D})}{\partial t}}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}} < 0$$

$$\frac{\partial \theta^*}{\partial D} = \phi(\cdot)\left(\frac{\alpha}{\sqrt{\beta}}\frac{\partial \theta^*}{\partial D} - (-1)\sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial \Phi^{-1}(\frac{t}{D})}{\partial D}\right) = \frac{\sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial \Phi^{-1}(\frac{t}{D})}{\partial D}}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}} > 0$$

The partial derivative of θ^* with respect to t (D) is always negative (positive), as due to the condition of uniqueness the denominator stays positive and nonzero. A rising t (D) thus decreases (increases) the switching value θ^* and thereby the probability of an attack. Consequently, as rising t reduces the expected net profit of an attack for every probability of success, controlling for the costs of international capital transactions might be a possibility to prevent speculative attacks on currency pegs.

Proposition 2 The ex-ante mean $\bar{\theta}$ influences the probability of an attack *negatively.*

Proof:

$$\frac{\partial \theta^*}{\partial \bar{\theta}} = \phi(\cdot)\left(\frac{\alpha}{\sqrt{\beta}}\frac{\partial \theta^*}{\partial \bar{\theta}} - \frac{\alpha}{\sqrt{\beta}}\right) = \frac{-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}} < 0$$

Consequently, the higher the publicly known ex-ante mean value of θ, the lower

the switching point θ^* turns out to be, and thus the narrower gets the range of fundamentals for which an attack would be successful and vice versa. This is a desirable feature of our model which is not contained in the models of multiple equilibria. Whereas under common knowledge of θ, the switch in beliefs about the outcome of the game does not depend on the fundamentals, in the model of incomplete knowledge the probability of a (successful) attack rises when the a priori expected value of the fundamentals deteriorates.

What remains to be analyzed are the effects of transparency, i.e. the preciseness of signals, on the unique switching point. We can show that for equilibrium values θ^* sufficiently high above $\bar{\theta}$ the preciseness of the private signal exerts a negative influence on the equilibrium value θ^*, whereas the precision of the public signal influences θ^* positively.

Proposition 3 If $\theta^* > \bar{\theta} + \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})$, the precision of the private signal β exerts a *negative* influence on the probability of a currency crisis.

Proof:

$$\begin{aligned}\frac{\partial \theta^*}{\partial \beta} &= \phi(\cdot)\left(-\frac{\alpha}{2\sqrt{\beta^3}}\theta^* + \frac{\alpha}{\sqrt{\beta}}\frac{\partial \theta^*}{\partial \beta} + \frac{\alpha}{2\beta^2}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})\right) \\ &= \frac{\phi(\cdot)\left(-\frac{\alpha}{2\sqrt{\beta^3}}\theta^* + \frac{\alpha}{2\sqrt{\beta^3}}\bar{\theta} + \frac{\alpha}{2\beta^2}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})\right)}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}}\end{aligned}$$

In the unique equilibrium, $\frac{\partial \theta^*}{\partial \beta}$ is negative if θ^* is larger than $\bar{\theta} + \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})$, so that the nominator becomes negative.

A changing value of β has three individual effects on the equilibrium value θ^*, with proposition 3 giving the net effect. The individual effects can be illustrated using figure one below, which describes the interaction between signals and fundamentals in the process of determining the unique equilibrium.

Suppose nature chooses a fundamental index equal to θ_0. This generates a distribution of signals corresponding to a normal density with mean $E(x|\theta_0) = x_0$, depicted below the horizontal axis. For a fundamental value of θ_0 at least a fraction $l = \theta_0$ of all agents have to attack the peg to make the attack successful. If all speculators follow a switching strategy around a certain signal, this proportion would be given for a switching signal of x_0^*, which is defined by the condition that for the given fundamental index θ_0, the probability of receiving a signal smaller than or equal to x_0^* is equal to θ_0. A speculator receiving exactly the switching signal x_0^*, however, calculates a posterior mean value of the fundamental index of

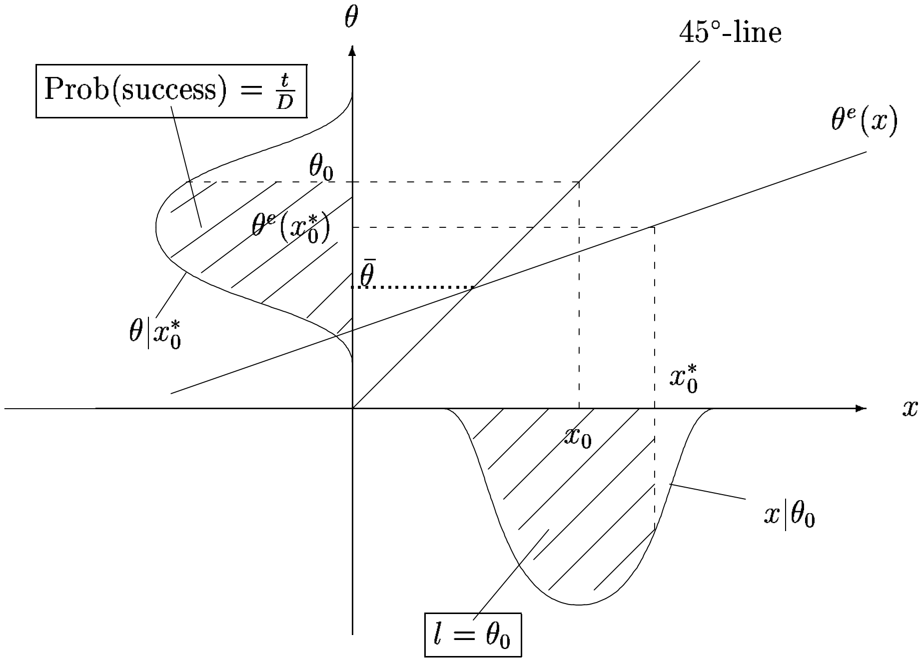

Figure 1: The unique equilibrium

$E(\theta|x_0^*) = \theta^e(x_0^*)$, which is depicted on the vertical axis. As can be seen from the normal density of $\theta|x_0^*$, the probability that θ is smaller than θ_0, which in that case is the switching-value of the fundamental index ($\theta_0 = \theta_0^*$) corresponding to the switching signal x_0^*, is exactly equal to $\frac{t}{D}$. Therefore, (θ_0^*, x_0^*) is the switching equilibrium which makes the government indifferent, as $l = \text{Prob}(x \leq x_0^*|\theta_0^*) = \theta_0^*$, and the speculators are indifferent due to $\text{Prob}(\theta \leq \theta_0^*|x_0^*) = \frac{t}{D}$.

A first effect of a growing preciseness of the private signal β is to make the probability function of the signals conditional on the fundamental index, drawn on the horizontal axis, more dense. Consequently, the necessary speculative mass l is already reached at a lower signal x for every θ. From this it follows that the probability distribution of θ conditional on the potential switching signal $x^*(\theta)$, which is depicted on the vertical axis, shifts downwards. Therefore, $\text{Prob}(\theta \leq \theta^*|x^*)$ rises for every potential switching signal $x^*(\theta)$, so that for the former equilibrium value θ_0^* the probability of success now is too high (higher than $\frac{t}{D}$) and consequently, θ^* has to decline.

However, there is a second effect due to a rising β that increases θ^*. This second effect is the consequence of the line $\theta^e(x)$ getting steeper when β increases, i.e. it rotates around the intersection point with the 45°-line. Due to a rising precision of the private signal, speculators attach more weight to their signal in calculating the most probable value of the fundamental index and put less weight on the public information $\bar{\theta}$. Therefore, in the case of a sufficiently high θ, the conditional distribution $\theta|x$ is shifted upwards so that the probability of success decreases for the given fundamental index. Consequently, the unique equilibrium value θ^* has to increase in order to satisfy the conditions of indifference again and the range of fundamental values in which an attack will take place broadens. Thus, this second effect leads to an increase in θ^* following a rising precision of the private signal.

Yet, there is still a third effect which again might lead to a declining value of θ^* in β. This last influence takes into account that a rising β lowers the variance of the distribution of θ conditional on x, as this variance is given by $\frac{1}{\alpha+\beta}$. Thus, the distribution of $\theta|x$ becomes more dense around the mean θ^e, which might intensify the first effect of β and again lowers the unique value θ^*.

On a whole, however, for θ^* being sufficiently high, a rising β leads to a declining probability of a currency attack, since θ^* will decrease. As a high value of θ^* can be caused by a low ex-ante mean value $\bar{\theta}$, we can infer that in the case of bad a priori expected fundamentals it is advantageous for the government if the speculators can establish each other's information very precisely. If in such a situation β is increased, speculators will put more weight on their signals than on the ex-ante fundamental mean $\bar{\theta}$ and will therefore partly disregard that public information and refrain from attacking[4]. However, if the a priori expected fundamental state of the economy is good so that θ^* turns out to be very low, increasing β leads to a higher probability of attack. This might be explained by the fact that with a higher β, speculators can establish the signals of their colleagues more easily and can try to force a devaluation through their sheer mass despite the rather good economic state.

Proposition 4 The precision of the public signal α exerts a *positive* influence on the probability of an attack if $\theta^* > \bar{\theta} + \frac{1}{2\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})$.

[4]This is not in contrast to the explanation of the second effect of a rising β, as there we described speculator's considerations in equilibrium, whereas here we have already moved away from equilibrium.

Proof:

$$\begin{aligned}\frac{\partial\theta^*}{\partial\alpha} &= \phi(\cdot)\left(\frac{1}{\sqrt{\beta}}\theta^* + \frac{\alpha}{\sqrt{\beta}}\frac{\partial\theta^*}{\partial\alpha} - \frac{1}{\sqrt{\beta}}\bar{\theta} - \frac{1}{2\beta}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})\right) \\ &= \frac{\phi(\cdot)\left(\frac{1}{\sqrt{\beta}}\theta^* - \frac{1}{\sqrt{\beta}}\bar{\theta} - \frac{1}{2\beta}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})\right)}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}}\end{aligned}$$

In the unique equilibrium, the partial derivative of θ^* with respect to the precision of the public signal α is positive, if $\theta^* > \bar{\theta} + \frac{1}{2}\frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})$.

The effect of the public signal's precision is threefold as well. First of all, α influences the range of possible values of the fundamental index, as the ex-ante variance of θ is given by $\frac{1}{\alpha}$. A low value of α makes extreme values of θ possible. If the realized value of θ is very high compared to the ex ante mean $\bar{\theta}$, then the proportion of attacking speculators has to be very high as well, as it has to exceed θ. Therefore, the relative difference between the conditional mean signal and the corresponding switching signal increases with higher fundamental indices, i.e., with decreasing α. However, as the slope of the function $\theta^e(x)$ is less than one, the most probable value of θ, calculated by a speculator receiving the switching signal, slowly falls behind the original value of θ that generated the distribution of signals. As a consequence, the probability of success inherently increases and the unique equilibrium value θ^*, that guarantees a success-probability of $\frac{t}{D}$, has to decrease. In other words, the first effect of a change in the precision of the public signal shows that θ^* decreases with a declining α, respectively increases with a growing α, given the depicted range of parameters.

The second effect is similar to the second effect of a change in β, except for the sign. If α increases, speculators put more weight on the public information in calculating the most probable value of θ. Therefore, the slope of $\theta^e(x)$ becomes smaller, so that as a consequence θ^* will decrease.

However, there is still a third effect, showing that a rising α decreases the variance of the distribution of θ conditional on the signal x, equal to the third effect of a change in β.

As a whole, in the considered case with θ^* being sufficiently high, α has a positive influence on θ^*, i.e., a high precision of the public signal is detrimental as it increases the probability of a successful attack additionally. Again, a high value of θ^* might be the consequence of bad a priori expected fundamentals, so that the government will only be willing to choose a large α, if $\bar{\theta}$ is high.

What can be seen from the discussion of comparative statics in this section is that *better* information, in the sense of *more precise* information, does not always lead to the desired outcome of the game. Moreover, the mechanisms of a changing informativeness are very complex. However, the general result is that if parameters like the ex-ante fundamental mean $\bar{\theta}$, cost t or payoff D lead to a rather high value of the switching fundamental index θ^*, an increased precision of private signals β is likely to narrow the attack-interval whereas a higher precision of the public information α will broaden it.

Consequently, if the a priori expected fundamentals are bad, the government is better off with a low precision of the public signal and a high precision of the private signal. In contrast, if the prior fundamentals are good, the government benefits from disseminating very precise public information and from a low precision of the private signals, as this tends to reduce the probability of a speculative attack. These results are explainable by recognizing that the precision of the public signal affects the question of how close the a priori expected fundamental index is to the truly realized value, whereas the precision of the private signals matters for the degree of coordination between speculators. As the maintenance of the currency peg is susceptible to both factors, i.e. to the perceived economic deterioration as well as speculative mass, they both affect the probability of an attack. If the fundamental state of the economy is good, it is intuitive that the government is interested in disseminating this information very precisely, i.e., in the public having very precise priors. In such a case, the currency peg might only come down due to sufficiently speculative mass, but if speculators are not able to establish the other agents' signals precisely, i.e., if the precision of the private signals is low, so that speculators are not able to coordinate on the attack-strategy, this danger is warded off as well. What becomes very clear is that actions in this model are still based on self-fulfilling expectations. An individual agent only attacks the currency peg if she is convinced that others will do so as well and vice versa.

Although we examined a very simple model with a successful attack giving a constant payoff D, we can show that if we assume the payoff to decrease in the fundamental index, i.e., $\frac{\partial D(\theta)}{\partial \theta} < 0$, our results remain valid. Thus, for θ^* being high enough above $\bar{\theta}$, α exerts a positive and β a negative influence on the probability of a speculative attack.[5] For uniformly distributed private signals

[5]For the calculation of the respective partial derivatives and the sufficient conditions see appendix II

Morris/Shin (1998a) and Heinemann/Illing (1999) come to similar results concerning β, whereas they do not take into account a public signal. In our model, however, by allowing for a second layer of uncertainty, the so-called governmental-based uncertainty, we are able to show that if θ^* exceeds a certain threshold, the precision of the governmental information increases the probability of an attack.

5 Unique and Multiple Equilibria and the Importance of Private and Public Information

As we know that a unique equilibrium can only be sustained for rather precise private signals relative to the public signal, a declining β, respectively a rising α, will eventually lead to multiple equilibria. This can easily be seen from figure two below, which depicts the sufficient condition for uniqueness of equilibrium.

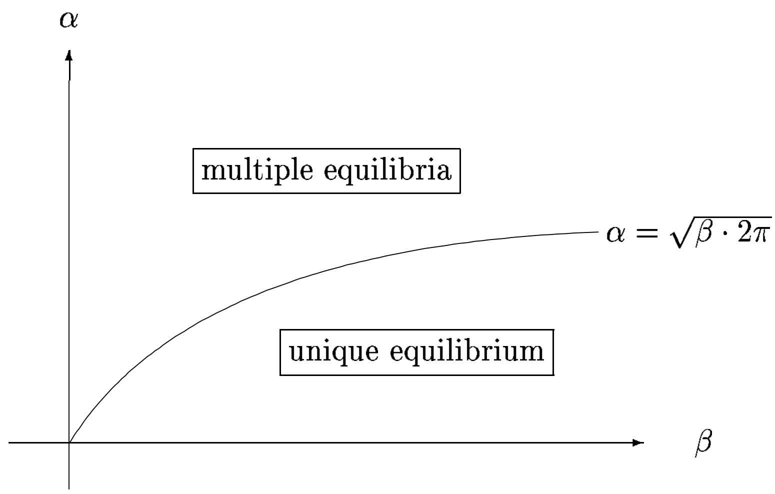

Figure 2: Regions of Unique and Multiple Equilibria

If β declines, while α stays constant, speculators are not able to precisely establish the information received by the other agents, so that they are not able to coordinate on a specific action for a given θ. As they cannot be sure that the necessary amount of coordination, i.e., the proportion of attacking agents, will be achieved, the optimal action for them is again to attack if everyone else attacks and to refrain from short-selling if that is what they expect everyone else to do. However, if the precision of public information α increases for a given β, the informational content of the private signal falls more and more behind so that it is eventually neglected and again, multiple equilibria arise.

Hence, in case that it is not ex-ante clear if the condition for a unique equilibrium will be satisfied and if the a priori expected fundamentals are rather bad, there is another interesting finding of our model: On the one hand the government benefits from a low precision of the public signal according to what was said in section 4.3. Decreasing α moreover ascertains the effect as through this action the uniqueness of equilibrium is restored. On the other hand, if the public information's precision is extraordinarily high, the condition for uniqueness will most probably be violated. In that case, there is a certain probability that in the revived multiplicity of equilibria speculators will coordinate on the no-attack equilibrium, as this coordination will not necessarily depend on the fundamental state of the economy any longer. Consequently, in the case of bad expected fundamentals, a government that tries to maintain the fixed exchange rate can either increase or decrease α. Both are reasonable actions in preventing currency attacks. Decreasing α lowers the probability of an attack in a unique equilibrium setting, if the a priori expected fundamentals are bad enough. Increasing α does not affect the attack-probability directly but influences the possibility of multiple equilibria in the considered range of fundamentals, so that the coordination result does not necessarily depend on the realized fundamental state but also on expectations, which might lead to the no-attack equilibrium.

6 Conclusion

From the delineated model we are able to see that the introduction of noise to the information gathering process of speculators brings a further crucial condition (of a net expected profit equal to zero) into the model so that all equilibria but one are eliminated, if the variance of the noisy information is small enough. It is therefore possible for an outside observer who knows θ, to specify for each realized value of θ whether there will be a (successful) speculative attack or not.

In contrast to the earlier multiple equilibria models of currency crises we are now able to give directions for policy devices. Firstly, the increase of transaction costs certainly reduces speculators' incentives to intervene on international financial markets and therefore reduces the probability of a speculative currency attack. Secondly, the sheer increase of the amount of information in the market is obviously not enough to prevent currency crises. In particular, if the risk of a speculative attack is quite high, for instance due to a low value of the a priori expected fundamentals, disseminating very precise public information is crucial as it might increase the probability of an attack further, whereas a high precision of the private signals will be beneficial, and vice versa. However, if the prior fundamental state of the economy is bad and it is not ex-ante certain that private information will be precise enough to ensure a unique equilibrium, the government might also be better-off with an extremely high precision of the public signal. For this increases the probability of moving into the multiple equilibria region in which speculators might as well coordinate on the no-attack equilibrium, independent of the underlying fundamentals.

During the last months and years, a lot of economists tried to tackle some of the still open questions. One inquiry of research (see for instance Sbracia and Zaghini (2000)) analyzes further aspects of information variability in a slightly different setting. Morris and Shin (1998b) include a more sophisticated reaction function of the government. Another interesting aspect is to depart from simultaneous move games, in which speculators have to decide on their strategies at the same point in time, and to look at sequential move games. In these models, agents not only get a private signal but they can also observe the actions that earlier speculators decided on. As an example of such a model see for instance Dasgupta (1999). An again different aspect in current research is to allow for speculators of different sizes (Corsetti/Dasgupta/Morris/Shin (2000)).

Appendix I

As θ and ε are both normally distributed, the expected value of the unknown θ, conditional on the observation of signal x, as well as the conditional variance of θ are determined as follows:

$$
\begin{aligned}
E(\theta|x) &= E(\theta) + \mathrm{Corr}(\theta, x) \cdot \sqrt{\frac{\mathrm{Var}(\theta)}{\mathrm{Var}(x)}} \cdot (x - E(x)) \qquad (16)\\
&= E(\theta) + \frac{\mathrm{Cov}(\theta, x)}{\sqrt{\mathrm{Var}(\theta)\mathrm{Var}(x)}} \cdot \sqrt{\frac{\mathrm{Var}(\theta)}{\mathrm{Var}(x)}} \cdot (x - E(x))\\
&= \bar{\theta} + \frac{\mathrm{Cov}(x, \theta)}{\mathrm{Var}(x)} \cdot (x - \bar{\theta})\\
&= \bar{\theta} + \frac{E(\theta \cdot x) - E(\theta) \cdot E(x)}{\mathrm{Var}(x)} \cdot (x - \bar{\theta})\\
&= \bar{\theta} + \frac{\mathrm{Var}(\theta)}{\mathrm{Var}(x)} \cdot (x - \bar{\theta})\\
&= \bar{\theta} + \frac{\frac{1}{\alpha}}{\frac{1}{\alpha} + \frac{1}{\beta}} \cdot (x - \bar{\theta})\\
&= \frac{1}{\alpha + \beta}(\alpha \cdot \bar{\theta} + \beta \cdot x)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathrm{Var}(\theta|x) &= \mathrm{Var}(\theta) \cdot (1 - \mathrm{Corr}(x, \theta)^2)\\
&= \frac{1}{\alpha} \cdot (1 - \frac{\mathrm{Cov}(x, \theta)^2}{\mathrm{Var}(x) \cdot \mathrm{Var}(\theta)})\\
&= \frac{1}{\alpha} \cdot \left(1 - \frac{\mathrm{Var}(\theta)^2}{\mathrm{Var}(x) \cdot \mathrm{Var}(\theta)}\right)\\
&= \frac{1}{\alpha} \cdot \left(1 - \frac{\mathrm{Var}(\theta)}{\mathrm{Var}(x)}\right)\\
&= \frac{1}{\alpha} \cdot \left(1 - \frac{\frac{1}{\alpha}}{\frac{1}{\alpha} + \frac{1}{\beta}}\right)\\
&= \frac{1}{\alpha + \beta}
\end{aligned}
$$

Appendix II

If we assume that payoff D is negatively influenced by θ, equation (11) changes to

$$\theta^* = \Phi\Big(\frac{\alpha}{\sqrt{\beta}}\big(\theta^* - \bar{\theta} - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}(\frac{t}{D(\theta^*)})\big)\Big) \tag{17}$$

The partial derivative of θ^* with respect to α is then calculated as

$$\begin{aligned}\frac{\partial\theta^*}{\partial\alpha} &= \phi(\cdot)\Big(\frac{1}{\sqrt{\beta}}\theta^* + \frac{\alpha}{\sqrt{\beta}}\frac{\partial\theta^*}{\partial\alpha} - \frac{1}{\sqrt{\beta}}\bar{\theta} - \frac{1}{2\beta}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}\big(\frac{t}{D(\theta^*)}\big) \\ &\quad -\sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial\Phi^{-1}(\frac{t}{D(\theta^*)})}{\partial\frac{t}{D(\theta^*)}}\frac{\partial\frac{t}{D(\theta^*)}}{\partial\theta^*}\frac{\partial\theta^*}{\alpha}\Big) \\ &= \frac{\phi(\cdot)\Big(\frac{1}{\sqrt{\beta}}\theta^* - \frac{1}{\sqrt{\beta}}\bar{\theta} - \frac{1}{2\beta}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}(\frac{t}{D(\theta^*)})\Big)}{1-\phi(\cdot)\Big(\frac{\alpha}{\sqrt{\beta}} - \sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial\Phi^{-1}(\frac{t}{D(\theta^*)})}{\partial\frac{t}{D(\theta^*)}}\frac{\partial\frac{t}{D(\theta^*)}}{\partial\theta^*}\Big)}\end{aligned}$$

This partial derivative of θ^* with respect to α is positive if, again, $\theta^* > \bar{\theta} + \frac{1}{2}\frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D(\theta^*)})$ and if

$$1 > \phi(\cdot)\Big(\frac{\alpha}{\sqrt{\beta}} - \sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial\Phi^{-1}(\frac{t}{D(\theta^*)})}{\partial\frac{t}{D(\theta^*)}}\frac{\partial\frac{t}{D(\theta^*)}}{\partial\theta^*}\Big)$$

In case that the private signal is precise enough relative to the public signal to ensure uniqueness of equilibrium, this latter condition is always satisfied.

For the partial derivative of θ^* with respect to β we get:

$$\begin{aligned}\frac{\partial\theta^*}{\partial\beta} &= \phi(\cdot)\Big(-\frac{\alpha}{2\sqrt{\beta^3}}\theta^* + \frac{\alpha}{\sqrt{\beta}}\frac{\partial\theta^*}{\partial\beta} + \frac{\alpha}{2\sqrt{\beta^3}}\bar{\theta} + \frac{\alpha}{2\beta^2}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}\big(\frac{t}{D(\theta^*)}\big) \\ &\quad -\sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial\Phi^{-1}(\frac{t}{D(\theta^*)})}{\partial\frac{t}{D(\theta^*)}}\frac{\partial\frac{t}{D(\theta^*)}}{\partial\theta^*}\frac{\partial\theta^*}{\partial\beta}\Big) \\ &= \frac{\phi(\cdot)\Big(-\frac{\alpha}{2\sqrt{\beta^3}}\theta^* + \frac{\alpha}{2\sqrt{\beta^3}}\bar{\theta} + \frac{\alpha}{2\beta^2}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}(\frac{t}{D(\theta^*)})\Big)}{1-\phi(\cdot)\Big(\frac{\alpha}{\sqrt{\beta}} - \sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial\Phi^{-1}(\frac{t}{D(\theta^*)})}{\partial\frac{t}{D(\theta^*)}}\frac{\partial\frac{t}{D(\theta^*)}}{\partial\theta^*}\Big)}\end{aligned}$$

This partial derivative is negative, so that an increasing β reduces the probability of a speculative attack, if $\theta^* > \bar{\theta} + \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D(\theta^*)})$ and if a unique equilibrium is guaranteed.

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