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## Another look at yield spreads: Monetary policy and the term structure of interest rates

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**Another Look at Yield Spreads\* :**  
**Monetary Policy and the Term Structure**  
**of Interest Rates**

Dong Heon Kim

October 1998

**Abstract**

Liquidity plays an important role in explaining how banks determine their allocation of funds. This paper analyses whether this fact can explain the term structure of interest rates and yield spreads. The paper models banks' demand for liquidity in a manner similar to that used to study household need for liquidity, namely, by using a cash-in-advance type model. The paper finds that the shadow price of the cash-in-advance constraint plays an important role in determining yield spreads. The model predicts that short-term rates respond more to monetary policy than long-term rates, consistent with earlier empirical findings. The empirical part of the paper shows that the expectations hypothesis might be salvaged under the maintained hypothesis concerning the liquidity premium and default risk premium. This paper confirms the finding that monetary contractions raise nominal interest rates.

**JEL:** E43; E44; E52

**Keywords:** Term structure of interest rates, Expectations hypothesis, Yield Spreads, Liquidity, Cash-in-advance constraint, Monetary policy

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## I. Introduction

An attractive theory of the term structure of interest rates is the expectations hypothesis, which holds that fluctuations in the slope of the yield curve reflect expected future interest rate changes. Many empirical studies such as Shiller, Campbell and Schoenholtz (1983), Fama (1984), Mankiw and Miron (1986), Fama and Bliss (1987), Mishkin (1988), Hardouvelis (1988), Froot (1989), Simon (1989, 1990), Cook and Hahn (1990), Campbell and Shiller (1991), and Roberds et al. (1996) find that the estimated coefficients in a regression of the change in the expected future short-term interest rates on the yield spread are significantly less than the value of unity predicted by the expectations hypothesis and differ as the forecast horizon varies<sup>1</sup>. Even though Fama (1984), Mishkin (1988), Hardouvelis (1988), and Simon (1990) have found yield spreads do help to predict future rates, the coefficient appears inconsistent with the expectations hypothesis.

Various previous studies have focused on the possibility of a the time-varying risk premium and concluded that a time-varying risk premium can help explain the failures of the expectations theory. Examples include Engle, Lilien and Robins (1987), Simon (1989, 1990), Friedman and Kuttner (1992) and Lee (1995).

On the other hand, several studies such as Mankiw and Miron (1986), McCallum (1994), and Rudebusch (1995) have shown that even if the expectations theory holds, supporting empirical evidence cannot be obtained from the forecasting ability of the slope of the yield curve due to interest rate smoothing by the Fed. Mankiw and Miron (1986) argued that the negligible predictive power of the spread after the founding of the Fed did not reflect a failure of the expectations theory. Instead, they suggested that the Fed ‘stabilized’ short-term rates, such as the three-month rate, by inducing a random-walk behavior, which eliminated any predictable variation. McCallum (1994) proposes that the empirical failure of the rational expectations theory of the term structure of interest rates can be rationalized with the

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<sup>1</sup> Rudebusch (1995) refers to the ‘U-shaped’ pattern of the predictability of the yield curve. Roberds and Whiteman (1997) state the existence of a “predictability smile” in the term structure of interest rates: spreads between long maturity rates and short rates predict subsequent movements in interest

expectations theory by recognition of an exogenous random term premium plus the assumption that monetary policy involves smoothing of an interest rate instrument -- the short-term rate -- together with the responses to the prevailing level of the spread. Rudebusch (1995) states that the Federal Reserve's interest rate targeting behavior accompanied by the maintained rational expectations hypothesis explains the varying predictive ability of the yield curve.

However, these reasons are not sufficient to address why the result of the empirical test of the expectations hypothesis is not in accordance with the prediction of the expectations hypothesis. Evans and Lewis (1994) point out that a time-varying risk premium alone is not sufficient to explain the time-varying term premium in the Treasury bill. Dotsey and Otrok (1995) say that a deeper understanding of interest rate behavior will be produced by jointly taking into account the behavior of the monetary authority along with a more detailed understanding of what determines term premia.

Recently, Bansal and Coleman (1996) argue that some assets other than money play a special role in facilitating transactions, which affects the rate of return that they offer. In their model, they show that securities that back checkable deposits provide a transaction service return in addition to their nominal return. Since short-term government bonds facilitate transactions by backing checkable deposits, this results in equilibrium in a lower nominal return for these bonds. In fact, the transaction service return of short-term bonds comes from the liquidity of short-term bonds. Such a view implies that liquidity plays an important role in determining the returns of various securities.

In general, liquidity refers to the ease with which an asset can be bought or sold. Asset purchases or sales are subject to transaction costs and the degree of liquidity of an asset decreases as the costs incurred in buying and selling it increase. So, if liquidity is an important factor determining the returns of financial assets among investors in financial markets, we will also have to incorporate liquidity into the term structure of interest rates.

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rates provided the long horizon is three months or less or if the long horizon is two years or more, but not for immediate maturities.

But how do investors consider liquidity in allocating their funds between securities of different terms? Since commercial banks are principal investors and primary dealers in instruments such as federal funds, commercial papers, and Eurodollar CDs, a study of liquidity demand by commercial banks may provide the key to answering this question. Stigum (1990) states that “in the money market, in particular, banks are players of such major importance that any serious discussion of the various markets that comprise the money market must be prefaced with a careful look at banking.” Cook and La Roche (1993) emphasize that commercial banks play an important role in the money market.

In terms of banks, liquidity means having ready cash (reserves) in all currencies to pay the bills, to fund the drawdowns of loan commitments, to meet depositor withdrawals, to honor cash calls on foreign exchange contracts and guarantees, and to meet reserve requirements (Abboud (1987)). If a bank might at some point be unable to turn its assets into ready cash, the bank faces a liquidity risk. Liquidity is a crucial fact of life for banks, and for this reason may have an implication for the term structure of interest rates. For example, since banks’ loans are relatively long-term assets and illiquid, they are not appropriate in terms of bank’s liquidity management. By contrast, government short-term securities such as Treasury bills are very liquid. Stigum (1990) emphasizes that all banks hold government securities for liquidity and profit. .

In addition, since banks’ liquidity can be time-varying in accordance with the policy of the Fed, the situation of the financial markets, the individual bank’s specific demand for reserves and so on, banks’ liquidity might play an important role in explaining time-varying term premium. Most previous studies, however, have not focused on banks as the main investors in financial markets and thus, banks’ liquidity.

This paper attempts to answer the following question. Can the fact that liquidity plays an important role in explaining how banks determine their allocation of funds explain the term structure of interest rates?

The paper begins by developing a model of a bank's optimal behavior. This model incorporates the cash-in-advance constraint (liquidity constraint) of Clower (1967), Lucas (1982), Svensson (1985), Lucas and Stokey (1987) and Bansal and Coleman (1996) into a model of bank decision-making similar to Cosimano (1987), Cosimano and Van Huyck (1989), Elyasiani, Kopecky and Van Hoose (1995), and Kang (1997). In addition, this model models a time-varying default risk premium. The paper then studies the implications resulting from banks' liquidity needs. The paper finds that the shadow price of the cash-in-advance constraint has an important role in determining the term structure of interest rates. The empirical part of the paper shows that the expectations hypothesis might be salvaged under the maintained hypothesis concerning the liquidity premium and the default risk premium. The paper also predicts the effect of a monetary policy shock on the short-term rate is bigger than on the long-term rate, consistent with earlier empirical findings of Cook and Hahn (1989). This paper also confirms the finding of Hamilton (1996, 1997a, 1997b) that the liquidity effect is real, i.e., a monetary contraction raises short-term interest rate.

The plan of this paper is as follows. Section II develops a model which incorporates liquidity into banks' optimal behavior and explains determination of the term structure of interest rates. Section III provides a brief empirical test of the simple expectations hypothesis and shows the empirical results for theoretical model developed in Section II. A brief summary and concluding remark are given on Section VI.

## **II. Banks' Optimal Behavior Subject to a Cash-in-advance Constraint (Liquidity Constraint)**

This section develops a model that incorporates liquidity into banks' optimal behavior. The first part presents a simplified model in which bank loans are two period assets and federal funds lent are one-period assets. The second part generalizes to the case when bank

loans are n-period assets and federal funds are one-period assets. The key results are the same in both cases.

## II.1 Basic Model

There are many banks in the banking system. Banks have an infinite-period horizon. Each period consists of two sessions, the beginning of period  $t$  and the end of period  $t$ . We assume that the public prefers demand deposits to cash and so all the cash is deposited in the bank at the end of the period. The reserve supply in the banking system does not change unless the Fed changes it. Banks face default risks on loans and this default risk increases as the quantity of loans or the maturity of loans increases. In addition, it is assumed that there is an unobserved shock to default risk. Suppose that a representative bank has following profit function

$$\Pi_t = r_t^L L_t - (\delta_0 L_t + \frac{\delta_1}{2} L_t^2 + q_t L_t) + r_{t-1}^L L_{t-1} - (\delta_0 L_{t-1} + \frac{\delta_1}{2} L_{t-1}^2 + q_{t-1} L_{t-1}) + r_t^F F_t$$

where  $L_t$  is the quantity of new two-period loans made in period  $t$ ,  $r_t^L$  is the yield on two-period loan made at time  $t$ ,  $r_t^F$  is the yield on federal funds lent or borrowed at time  $t$ ,  $F_t$  is the federal funds lent or borrowed at time  $t$ , and  $q_t$  represents an unobserved shocks to default risk of loan supply. The  $\delta$  s' are non-negative constants. The two expressions in parentheses represent the default risk on loans each period.

The bank chooses the level of new loans  $L_t$  at the beginning of period  $t$ . At the end of period  $t$ , it chooses the quantity of federal funds to lend  $F_t$ . These choices, along with some other exogenous or predetermined factors, determine the level of reserves  $R_t$  with which the bank will end the period. These other factors are (1) the bank's level of demand deposits,  $\bar{D}_t$ , which is taken to be exogenous, with positive value for  $\bar{D}_t - \bar{D}_{t-1}$  increasing the bank's end-of-period reserve position; (2) the repayment of the Fed funds the bank lent the previous



period ( $F_{t-1}$ ); and (3) the repayment of the loans the bank made 2 periods previously ( $L_{t-2}$ ).

The bank's end-of-period reserves thus evolve according to

$$R_t = R_{t-1} + L_{t-2} - L_t + F_{t-1} + \bar{D}_t - \bar{D}_{t-1} - F_t$$

In addition, the bank must satisfy the reserve requirement

$$R_t \geq \theta \bar{D}_t$$

where  $\theta$  is the ratio of the required reserves.

The state variables that are relevant for the bank's decision about the quantity of Fed funds to lend at the end of period  $t$  are  $L_t^*, L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \bar{D}_t$ . Consider following the value function formulation of the decision for the end-of-period at time  $t$ :

$$\begin{aligned} & U_t(L_t, L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \bar{D}_t) \\ &= \max_{\{F_t\}} \{r_t^F F_t + \beta E_t V_{t+1}(L_t, L_{t-1}, \bar{D}_t, F_t, R_{t-1} + L_{t-2} + F_{t-1} + \bar{D}_t - \bar{D}_{t-1} - L_t - F_t) \\ & \quad + \lambda_t (R_{t-1} + L_{t-2} + F_{t-1} + \bar{D}_t - \bar{D}_{t-1} - L_t - F_t - \theta \bar{D}_t)\} \end{aligned}$$

where  $U_t(\cdot)$  denotes the lifetime value of the bank's optimal program as of the second session of period  $t$  whereas  $V_t(L_t, L_{t-1}, F_t, R_t)$  is the value as of the first session of period  $t+1$  and  $\lambda_t$  is the multiplier of the required reserve constraint. The first-order conditions are as follows;

$$r_t^F + \beta E_t \frac{\partial V_{t+1}}{\partial F_t} = \beta E_t \frac{\partial V_{t+1}}{\partial R_t} + \lambda_t \quad (1)$$

$$R_t \geq \theta \bar{D}_t, \text{ with equality if } \lambda_t > 0 \quad (2)$$

The left-hand side in equation (1) is the marginal benefit of lending federal funds and the right is the marginal cost.

In the beginning of a period  $t$ , a representative bank starts with reserve balances given by  $R_{t-1}$ . Given the loan rate and the federal funds rate, a representative bank must choose its loan supply,  $L_t$ , before knowing the deposit,  $\bar{D}_t$ . We model the bank's need for liquidity in the same way that the cash-in-advance (hereafter CIA) literature has modeled demand for liquidity by private households. In the conventional cash-in-advance formulation, goods must

be purchased with cash, and a consumer can only obtain goods if he has cash on hand sufficient to pay for them. In our model, a borrower from the bank wants cash, and the bank can only extend a loan to such a customer if it has cash on hand equal to the amount of the loan. In either application, the strict cash-in-advance requirement ignores such real-life institutions as credit cards available to consumers or within-day overdraft privileges available to banks on their accounts with the Fed. Even so, the requirement of needing actual cash on hand for certain transactions seems to capture the key idea of what is meant by liquidity and has proven a useful framework for thinking about liquidity demand by private households. We propose that it may also be fruitful for seeing how the need for liquidity may make a difference for understanding the rates of return on assets of different maturities held by banks. Thus, we propose that a representative bank faces a CIA or liquidity constraint such as the following:

$$L_t \leq R_{t-1} \quad (3)$$

where  $R_{t-1}$  is the reserve balances transferred from the previous period to the start of period  $t$ . The state variables for the decision at the beginning of the period  $t$  are

$$L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \bar{D}_{t-1}.$$

The value function of the beginning-of-period at time  $t$  is

$$\begin{aligned} V_t(L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \bar{D}_{t-1}) = \max_{\{L_t\}} & r_t^L L_t - (\delta_0 L_t + \frac{\delta_1}{2} L_t^2 + q_t L_t) + r_{t-1}^L L_{t-1} \\ & - (\delta_0 L_{t-1} + \frac{\delta_1}{2} L_{t-1}^2 + q_{t-1} L_{t-1}) + E_t U_t(L_t, L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \bar{D}_t) \\ \text{subject to} & \quad L_t \leq R_{t-1} \end{aligned}$$

The first-order conditions are as follows;

$$r_t^L + E_t \frac{\partial U_t}{\partial L_t} - (\delta_0 + \delta_1 L_t + q_t) = \eta_t \quad (4)$$

$$R_{t-1} \geq L_t, \text{ with equality if } \eta_t > 0 \quad (5)$$

where  $\eta_t$  denotes the multiplier of CIA constraint and represents the shadow price of the CIA constraint. The left-hand side of equation (4) is the marginal benefit of increasing loans and the right is the marginal cost.

By using the envelope condition from the value function of the end-of-period t, we get

$$\frac{\partial V_{t+1}}{\partial F_t} = \beta E_{t+1} \frac{\partial V_{t+2}}{\partial R_{t+1}} + \lambda_{t+1}, \text{ and } \frac{\partial V_{t+1}}{\partial R_t} = \beta E_{t+1} \frac{\partial V_{t+2}}{\partial R_{t+1}} + \eta_{t+1} + \lambda_{t+1} \quad (6)$$

Substituting equation (6) into equation (1), we get following condition

$$r_t^F + \beta^2 E_t \frac{\partial V_{t+2}}{\partial R_{t+1}} + \beta E_t \lambda_{t+1} - \beta^2 E_t \frac{\partial V_{t+2}}{\partial R_{t+1}} - \beta E_t \lambda_{t+1} - \beta E_t \eta_{t+1} - \lambda_t = 0, \text{ and so}$$

$$r_t^F = \beta E_t \eta_{t+1} + \lambda_t \quad (7)$$

Equation (7) is the equilibrium condition to determine the federal funds rate at time t since at this point, a representative bank is indifferent to choosing  $F_t = 0$ . Equation (7) means that the current federal funds rate is the discounted expected value of the next-period shadow price of the CIA constraint plus the shadow price of the required reserve constraint at time t and thus, the federal funds rate reflects liquidity. In addition, we get the following result by using the envelope condition from the value function of the beginning-of-period t,

$$\begin{aligned} \frac{\partial U_t}{\partial L_t} &= \beta E_t r_t^L - \beta(\delta_0 + \delta_1 E_t L_t + E_t q_t) + \beta^3 E_t \frac{\partial V_{t+3}}{\partial R_{t+2}} + \beta^2 E_t \lambda_{t+2} \\ &\quad - \beta E_t \eta_{t+1} - \beta E_t \lambda_{t+1} - \beta^3 E_t \frac{\partial V_{t+3}}{\partial R_{t+2}} - \beta^2 E_t \eta_{t+2} - \beta^2 E_t \lambda_{t+2} - \lambda_t \\ &= \beta E_t r_t^L - \beta(\delta_0 + \delta_1 E_t L_t + E_t q_t) - (\beta^2 E_t \eta_{t+2} - \beta E_t \lambda_{t+1}) - (\beta E_t \eta_{t+1} - \lambda_t) \\ &= \beta E_t r_t^L - \beta(\delta_0 + \delta_1 E_t L_t + E_t q_t) - r_t^F - \beta E_t r_{t+1}^F \end{aligned} \quad (8)$$

since  $\beta^2 E_t \eta_{t+2} + \beta E_t \lambda_{t+1} = \beta E_t r_{t+1}^F$ ,  $\beta E_t \eta_{t+1} + \lambda_t = r_t^F$

Substituting equation (8) into equation (4) results in

$$r_t^L + \beta r_t^L - (\delta_0 + \delta_1 L_t + q_t) - \beta(\delta_0 + \delta_1 L_t + q_t) = r_t^F + \beta E_t r_{t+1}^F + \eta_t \quad (9)$$

Equation (9) can be rewritten as follows;

$$r_t^L = \frac{1}{1+\beta}[r_t^F + \beta E_t r_{t+1}^F] + (\delta_0 + \delta_1 L_t + q_t) + \frac{1}{1+\beta} \eta_t \quad (10)$$

Equation (10) means that the loan rate is the weighted average of the current federal funds rate and future federal funds rate plus the cost resulting from default risk (or transaction cost) on loans and the cost of loss of the liquidity benefit. The second term of the right-hand side is the default risk premium while the third term of the right-hand side is the liquidity premium.

Multiplying equation (9) by  $\beta$  and taking expectation at time t-1, we get

$$\begin{aligned} \beta(1+\beta)E_{t-1}r_t^L - \beta(1+\beta)[\delta_0 + \delta_1 E_{t-1}L_t] - \beta(1+\beta)E_{t-1}q_t \\ = \beta E_{t-1}r_t^F + \beta^2 E_{t-1}r_{t+1}^F + \beta E_{t-1}\eta_t \end{aligned} \quad (11)$$

Equation (11) is an Euler equation and an optimality condition between the loan market and the federal funds market. The left hand side of the equation (11) is the marginal benefit of increasing loans and the right is the marginal benefit of lending federal funds. In our model, a representative bank has two options at time t-1. One option is for the bank to hold reserves  $R_{t-1}$  at the end of period t-1 in order to lend them at the beginning of period t. The other option is that the bank rolls over the reserves as federal funds for two periods. For two cases, the streams of bank's return are shown in Table 1.

Table 1. The streams of bank's returns for two options of allocating funds

time	t-1	t	t+1	t+2
Return (loan)	0	$r_t^L$	$r_t^L$	get back the loan
Return (F. funds)	$r_{t-1}^F$	$r_t^F$	$r_{t+1}^F$	get back the federal funds

Equation (11) can be rewritten as follows;

$$E_{t-1}r_t^L = \frac{1}{\beta(1+\beta)}(\beta E_{t-1}r_t^F + \beta^2 E_{t-1}r_{t+1}^F) + (\delta_0 + \delta_1 E_{t-1}L_t + E_{t-1}q_t) + \frac{1}{\beta(1+\beta)}\beta E_{t-1}\eta_t \quad (12)$$

By using equation (7), equation (12) can be rewritten as follows;

$$E_{t-1}r_t^L = \frac{1}{\beta(1+\beta)}(r_{t-1}^F + \beta E_{t-1}r_t^F + \beta^2 E_{t-1}r_{t+1}^F) + (\delta_0 + \delta_1 E_{t-1}L_t + E_{t-1}q_t) - \frac{1}{\beta(1+\beta)}\lambda_{t-1} \quad (13)$$

Equation (13) implies that the loan rate is a weighted average of the previous federal funds rate, the current federal funds rate and the future federal funds rate plus the default risk premium and the shadow price of the required reserve constraint at time t-1. Comparing equation (10) with equation (13) gives some intuition. Since the current shadow price of the cash-in-advance constraint is related to the previous federal funds rate, the previous federal funds rate impacts the loan rate - and thus - the term structure of interest rates.

Since in practice, observed excess reserves are always greater than zero during this sample period, we assume that the required reserve constraint is always non-binding<sup>2</sup>. Then, equation (13) can be rewritten as follows;

$$E_{t-1}r_t^L = \frac{1}{\beta(1+\beta)}(r_{t-1}^F + \beta E_{t-1}r_t^F + \beta^2 E_{t-1}r_{t+1}^F) + (\delta_0 + \delta_1 E_{t-1}L_t + E_{t-1}q_t) \quad (14)$$

Moreover, because the federal funds rate is never observed to be zero, from equations (2), (5) and (7), the shadow price of the CIA constraint is always greater than zero and hence the constraint is always binding. Therefore, equation (5) becomes

$$L_t = R_{t-1} \quad (15)$$

Substituting equation (15) into equation (14) gives us

$$E_{t-1}r_t^L = \frac{1}{\beta(1+\beta)}(r_{t-1}^F + \beta E_{t-1}r_t^F + \beta^2 E_{t-1}r_{t+1}^F) + (\delta_0 + \delta_1 R_{t-1} + E_{t-1}q_t) \quad (16)$$

Equation (16) implies that the reserve balances (i.e., banks' liquidity) have impact not only on two interest rates but also on the term premium and thus, monetary policy can affect the term structure of interest rates.

So far, we considered a representative bank's optimal behavior given the loan rate and federal funds rate. However, since the loan rate and the federal funds rate are determined on the loan market and federal funds market, we need to incorporate the general equilibrium into our model. We assume that the public's demand for new loans is as follows;

$$\bar{L}_t = a_t - br_t^L, \quad a_t > 0, b > 0 \quad (17)$$

Equation (17) implies that the new total demand for loans is an inverse function of the loan rate. The term  $a_t$  reflects the loan demand shock. The equilibrium condition for new loans is

$$\bar{L}_t = L_t \quad (18)$$

where  $L_t$  is the new total supply of loans at time t. The total supply of loans is the sum of the individual bank's supply of loans at time t. Since the CIA constraint is always binding, equation (15) becomes the new total loan supply of the banking system. Hence, equation (15), (17) and (18) determine the loan rate as follows

$$r_t^L = \frac{1}{b}a_t - \frac{1}{b}R_{t-1} \quad (19)$$

Equation (19) means that the loan rate is a function of the shock to the demand for loan and the level of reserve balances. Combining equation (16) with equation (19) and rewriting it results in

$$\begin{aligned} r_{t-1} + \beta E_{t-1}r_t + \beta^2 E_{t-1}r_{t+1} \\ = \frac{\beta(1+\beta)}{b} E_{t-1}a_t - \frac{\beta(1+\beta)}{b} [1 + b\delta_1]R_{t-1} - \beta(1+\beta)\delta_0 - \beta(1+\beta)E_{t-1}q_t \end{aligned} \quad (20)$$

Equation (20) is the second-order stochastic difference equation with respect to the federal funds rate. In order to examine the effect of the exogenous change of the state variables on the interest rate, we need to know the process of the state variable. Suppose for example that

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<sup>2</sup> The data for the excess reserves of depository institution which are taken from Statistical Release provided by the Federal Reserve Board of Governor show that excess reserves are always positive

the reserve supply, the shock to loan demand, and the unobserved shock to the default risk of the loan supply follow a random walk without drift,

$$R_t = R_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (21)$$

$$a_t = a_{t-1} + e_t, \quad e_t \sim N(0, \sigma_e^2) \quad (22)$$

$$q_t = q_{t-1} + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2) \quad (23)$$

We also assume that the innovations,  $\varepsilon_t, e_t, \xi_t$  are serially uncorrelated and mutually independent. We conjecture the following solution function for the federal funds rate

$$r_t^F = \alpha_1 + \alpha_2 a_t + \alpha_3 R_t + \alpha_4 q_t \quad (24)$$

From (20) – (24), we get the following equation

$$r_t^F = \frac{(\beta + \beta^2)}{b(1 + \beta + \beta^2)} a_t - \frac{(\beta + \beta^2)(1 + b\delta_1)}{b(1 + \beta + \beta^2)} R_t - \frac{(\beta + \beta^2)\delta_0}{(1 + \beta + \beta^2)} - \frac{(\beta + \beta^2)}{(1 + \beta + \beta^2)} q_t \quad (25)$$

Equation (25) shows that the federal funds rate depends on the shock to loan demand, the level of reserve balances, and the unobserved shock to the default risk of loan supply. From equations (19) and (25), we can examine the effect of the exogenous shock to the loan rate and the federal funds rate, and thus, to the term structure of interest rates. Suppose that there occurs a positive loan demand shock. How do the loan rate and the federal funds rate respond to the loan demand shock? We can examine the effects by taking the differential of the loan rate and the federal funds rate with respect to  $a_t$ , obtaining

$$\frac{\partial r_t^L}{\partial a_t} = \frac{1}{b} > 0, \quad \frac{\partial r_t^F}{\partial a_t} = \frac{1}{b} \frac{(\beta + \beta^2)}{(1 + \beta + \beta^2)} > 0, \quad \text{and} \quad \frac{\partial r_t^L}{\partial a_t} > \frac{\partial r_t^F}{\partial a_t} \quad (26)$$

Equation (26) implies that the positive loan demand shock raises the loan rate and the federal funds rate. The response of the federal funds rate to a loan demand shock is less than that of the loan rate to the loan demand shock. The reason is that the change of the loan rate reflects the change of the shadow price of the CIA constraint while the change of the federal funds rate doesn't. In other words, since the positive loan demand shock raises the shadow price of

the CIA constraint, from equation (10) the increase in the loan rate is bigger than the increase in the federal funds rate. The response of the loan rate to the positive loan demand shock spreads over 2 returns while the response of the federal funds rate to the positive loan demand shock spreads over 3 returns.

To find the effect of a monetary shock to the loan rate and the federal funds rate in this model, we take the differential of the loan rate and the federal funds rate with respect to the reserve balance;

$$\frac{\partial r_t^L}{\partial R_{t-1}} = -\frac{1}{b} < 0, \frac{\partial r_t^F}{\partial R_t} = -\frac{1}{b} \frac{(\beta + \beta^2)[1 + b\delta_1]}{(1 + \beta + \beta^2)} < 0, \text{ and } \frac{\partial r_t^L}{\partial R_{t-1}} = \frac{\partial r_t^F}{\partial R_t} \quad (27)$$

Equation (27) means that an expansionary (contractionary) monetary shock decreases (increases) the loan rate and the federal funds rate. We can not determine whether the response of the federal funds rate to the monetary shock is bigger than that of the loan rate to the monetary shock. It depends on the parameter,  $\delta_1$ , which reflects the cost incurred from default risk on loans. If this parameter is big enough, the response of the federal funds rate to the monetary shock is bigger than that of the loan rate to the monetary shock. The reason is that the change in the default risk premium is opposite to the change in loan rate. If the change in the default risk premium is greater than the change in the shadow price of the CIA constraint, the change in the federal funds rate is bigger than the change in the loan rate in a no arbitrage equilibrium. For example, suppose that there occurs a contractionary monetary shock. On the one hand, because of the increase in the shadow price of the CIA constraint banks want to hold reserves and thus, transfer reserves from the loan market to the federal funds market. This force prevents the federal funds rate from increasing strongly. Loan rate, federal funds rate and the shadow price of the CIA constraint will increase. On the other hand, from equation (16), the default risk premium will decrease. If the decrease in the default risk premium is bigger than the increase in the shadow price of the CIA constraint, then the response of the federal funds rate to the contractionary monetary shock will be bigger than that of the loan rate in a no-arbitrage equilibrium.



## II.2 General Model

So far, we focused on the simple case in which loan was a two-period asset and federal funds lent a one-period asset. In this section, we will extend our model to the case in which loans are the n-period assets and federal funds lent one-period assets. The basic set-up is the same. A representative bank's profit function is as follows;

$$\begin{aligned} \Pi_t = & r_{n,t}^L L_t - (\delta_0 L_t + \frac{\delta_1}{2} L_t^2 + q_t L_t) + r_{n,t-1}^L L_{t-1} - (\delta_0 L_{t-1} + \frac{\delta_1}{2} L_{t-1}^2 + q_{t-1} L_{t-1}) \\ & + \dots + r_{n,t-n+1}^L L_{t-n+1} - (\delta_0 L_{t-n+1} + \frac{\delta_1}{2} L_{t-n+1}^2 + q_{t-n+1} L_{t-n+1}) + r_t^F F_t \end{aligned}$$

where  $r_{n,t}^L$  is the yield on n-period loan made at time t.

In this case, a representative bank has two options at time t-1. One option is for the bank to hold reserves  $R_{t-1}$  at the end of period t-1 in order to lend them over n period at the beginning of period t. The other option is that the bank rolls over reserves as federal funds for n periods. If a representative bank lends the public loans over n periods, the bank faces default risks on loans and this default risk increases as the quantity of loans or the maturity of loans increases. The expressions in parentheses represent the default risk on loans each period. Let us consider more specific behavior of a representative bank for two sessions of one period.

At the end of period t, the bank chooses the quantity of federal funds to lend  $F_t$ . These choices, along with some other exogenous or predetermined factors, determine the level of reserves  $R_t$  with which the bank will end the period. These other factors are the same for (1) and (2) in the basic model and (3) the repayment of the loans the bank made n periods previously ( $L_{t-n}$ ). The bank's end-of-period reserves thus evolve according to;

$$R_t = R_{t-1} + L_{t-n} + F_{t-1} + \bar{D}_t - \bar{D}_{t-1} - L_t - F_t$$

In addition, the bank must satisfy the reserve requirement

$$R_t \geq \theta \bar{D}_t$$

The state variables that are relevant for the bank's decision about the quantity of Fed funds to lend at the end of period  $t$  are  $L_t, L_{t-1}, \dots, L_{t-n}, F_{t-1}, R_{t-1}, \bar{D}_t$ . The value function formulation of the decision for the end-of-period at time  $t$  is as follows;

$$\begin{aligned} & U_t(L_t, L_{t-1}, \dots, L_{t-n}, F_{t-1}, R_{t-1}, \bar{D}_t) \\ &= \max_{\{F_t\}} r_t^F F_t + \beta E_t V_{t+1}(L_t, L_{t-1}, \dots, L_{t-n+1}, \bar{D}_t, F_t, R_{t-1} + L_{t-n} + F_{t-1} + \bar{D}_t - \bar{D}_{t-1} - L_t - F_t) \\ & \quad + \lambda_t (R_{t-1} + L_{t-n} + F_{t-1} + \bar{D}_t - \bar{D}_{t-1} - L_t - F_t - \theta \bar{D}_t) \end{aligned}$$

The first-order condition is

$$r_t^F + \beta E_t \frac{\partial V_{t+1}}{\partial F_t} = \beta E_t \frac{\partial V_{t+1}}{\partial R_t} + \lambda_t \quad (28)$$

$$R_t \geq \theta \bar{D}_t, \text{ with equality if } \lambda_t > 0 \quad (29)$$

In the beginning of the period  $t$ , a representative bank starts with reserve balances given by  $R_{t-1}$ . Given the loan rate and federal funds rate, a representative bank must choose its loan supply,  $L_t$ , before knowing the deposit,  $\bar{D}_t$ . The bank faces a CIA constraint as was the case in the basic model. The state variables for the decision at the beginning of the period  $t$  are  $L_{t-1}, L_{t-2}, \dots, L_{t-n}, R_{t-1}, F_{t-1}, \bar{D}_{t-1}$ . The value function of the beginning of period  $t$  is as follows;

$$\begin{aligned} & V_t(L_{t-1}, L_{t-2}, \dots, L_{t-n}, F_{t-1}, R_{t-1}, \bar{D}_{t-1}) \\ &= \max_{\{L_t\}} r_{n,t}^L L_t - (\delta_0 L_t + \frac{\delta_1}{2} L_t^2 + q_t L_t) + r_{n,t-1}^L L_{t-1} - (\delta_0 L_{t-1} + \frac{\delta_1}{2} L_{t-1}^2 + q_{t-1} L_{t-1}) + \dots \\ & \quad + r_{n,t-n+1}^L L_{t-n+1} - (\delta_0 L_{t-n+1} + \frac{\delta_1}{2} L_{t-n+1}^2 + q_{t-n+1} L_{t-n+1}) + E_t U_t(L_t, L_{t-1}, \dots, L_{t-n}, F_{t-1}, R_{t-1}, \bar{D}_t) \end{aligned}$$

$$\text{subject to } L_t \leq R_{t-1}$$

The first-order conditions are as follows;

$$r_{n,t}^L + \frac{\partial U_t}{\partial L_t} - (\delta_0 + \delta L_t + q_t) = \eta_t \quad (30)$$

$$R_{t-1} \geq L_t, \text{ with equality if } \eta_t > 0 \quad (31)$$

The left-hand side of equation (30) is the marginal benefit of increasing loans and the right is the marginal cost. By using the envelope condition, we get the same equilibrium condition for federal funds market as in the basic model;

$$r_t^F + \beta^2 E_t \frac{\partial V_{t+2}}{\partial R_{t+1}} + \beta E_t \lambda_{t+1} - \beta^2 E_t \frac{\partial V_{t+2}}{\partial R_{t+1}} - \beta E_t \lambda_{t+1} - \beta E_t \eta_{t+1} - \lambda_t = 0, \text{ and so}$$

$$r_t^F = \beta E_t \eta_{t+1} + \lambda_t \quad (7)$$

In addition, we get following result by using the envelope condition

$$\begin{aligned} \frac{\partial U_t}{\partial L_t} &= \beta E_t r_{n,t}^L - \beta(\delta_0 + \delta_1 L_t + q_t) + \beta^2 E_t r_{n,t}^L - \beta^2(\delta_0 + \delta_1 L_t + q_t) + \dots + \beta^{n-1} E_t r_{n,t}^L \\ &\quad - \beta^{n-1}(\delta_0 + \delta_1 L_t + q_t) + \beta^{n+1} E_t \frac{\partial V_{t+n+1}}{\partial R_{t+n}} + \beta^n E_t \lambda_{t+n} - \lambda_t - \beta E_t \lambda_{t+1} - \dots - \beta^n E_t \lambda_{t+n} \\ &\quad - \beta E_t \eta_{t+1} - \beta^2 E_t \eta_{t+2} - \dots - \beta^n E_t \eta_{t+n} - \beta^{n+1} E_t \frac{\partial V_{t+n+1}}{\partial R_{t+n}} \\ &= (\beta + \beta^2 + \dots + \beta^{n-1}) E_t r_{n,t}^L - (\beta + \beta^2 + \dots + \beta^{n-1})(\delta_0 + \delta_1 L_t + q_t) \\ &\quad - \beta E_t \eta_{t+1} - \dots - \beta^n E_t \eta_{t+n} - \lambda_t - \beta E_t \lambda_{t+1} - \dots - \beta^{n-1} E_t \lambda_{t+n-1} \\ &= (\beta + \beta^2 + \dots + \beta^{n-1}) E_t r_{n,t}^L - (\beta + \beta^2 + \dots + \beta^{n-1})(\delta_0 + \delta_1 L_t + q_t) \\ &\quad - (r_t^F + \beta E_t r_{t+1}^F + \dots + \beta^{n-1} E_t r_{t+n-1}^F) \end{aligned} \quad (32)$$

since  $\beta E_t \eta_{t+1} + \lambda_t = r_t^F, \dots, \beta^n E_t \eta_{t+n} + \beta^{n-1} E_t \lambda_{t+n-1} = \beta^{n-1} E_t r_{t+n-1}^F$

Substituting equation (32) into equation (30) results in

$$\begin{aligned} (1 + \beta + \dots + \beta^{n-1}) E_t r_{n,t}^L - (1 + \beta + \dots + \beta^{n-1})(\delta_0 + \delta_1 L_t + q_t) \\ = (r_t^F + \beta E_t r_{t+1}^F + \dots + \beta^{n-1} E_t r_{t+n-1}^F) + \eta_t \end{aligned} \quad (33)$$

Equation (33) can be rewritten as follows;

$$\begin{aligned} r_{n,t}^L &= \frac{1}{(1 + \beta + \dots + \beta^{n-1})} (r_t^F + \beta E_t r_{t+1}^F + \dots + \beta^{n-1} E_t r_{t+n-1}^F) \\ &\quad + (\delta_0 + \delta_1 L_t + q_t) + \frac{1}{(1 + \beta + \dots + \beta^{n-1})} \eta_t \end{aligned} \quad (34)$$

Equation (34) means that the n-period loan rate is the weighted average of the one-period current federal funds rate and the expected future federal funds rates over n periods plus the

cost resulting from the default risk on loans and the cost from the loss of the liquidity benefit.

Multiplying equation (33) by  $\beta$  and taking expectation at time t-1, we get

$$\begin{aligned} \beta(1 + \beta + \dots + \beta^{n-1})E_{t-1}r_{n,t}^L - \beta(1 + \beta + \dots + \beta^{n-1})(\delta_0 + \delta_1 E_{t-1}L_t + E_{t-1}q_t) \\ = \beta E_{t-1}r_t^F + \dots + \beta^n E_{t-1}r_{t+n-1}^F + \beta E_{t-1}\eta_t \end{aligned} \quad (35)$$

Equation (35) is an Euler equation and an optimality condition between the loan market and the federal funds market. This equation determines the term structure of the interest rates between the n-period loan rate and the one-period federal funds rate. Equation (35) can be rewritten

$$\begin{aligned} E_{t-1}r_{n,t}^L = \frac{1}{\beta(1 + \beta + \dots + \beta^{n-1})}(\beta E_{t-1}r_t^F + \dots + \beta^n E_{t-1}r_{t+n-1}^F) \\ + (\delta_0 + \delta_1 E_{t-1}L_t + E_{t-1}q_t) + \frac{1}{\beta(1 + \beta + \dots + \beta^{n-1})}\beta E_{t-1}\eta_t \end{aligned} \quad (36)$$

By using equation (7), Equation (36) can be rewritten

$$\begin{aligned} E_{t-1}r_{n,t}^L = \frac{1}{\beta(1 + \beta + \dots + \beta^{n-1})}(r_{t-1}^F + \beta E_{t-1}r_t^F + \dots + \beta^n E_{t-1}r_{t+n-1}^F) \\ + (\delta_0 + \delta_1 E_{t-1}L_t + E_{t-1}q_t) - \frac{1}{\beta(1 + \beta + \dots + \beta^{n-1})}\lambda_{t-1} \end{aligned} \quad (37)$$

Equation (37) implies that the loan rate is a weighted average of the previous federal funds rate, the current federal funds rate and the expected future federal funds rate over n periods plus the default risk premium plus the previous shadow price of the required reserve constraint. Since the current shadow price of the CIA constraint at time t is related to the previous federal funds rate, the previous federal funds rate has an impact on the loan rate and thus, on the term structure of interest rates. As in the basic model, we assume that the required reserve constraint is not binding. Since from equation (7), the shadow price of the CIA constraint is always greater than zero and so the CIA constraint is always binding, equation (37) can then be

$$\begin{aligned} E_{t-1}r_{n,t}^L = \frac{1}{\beta(1 + \beta + \dots + \beta^{n-1})}(r_{t-1}^F + \beta E_{t-1}r_t^F + \dots + \beta^n E_{t-1}r_{t+n-1}^F) \\ + (\delta_0 + \delta_1 E_{t-1}L_t + E_{t-1}q_t) \end{aligned} \quad (38)$$

Combining equation (15) and (38) gives us

$$E_{t-1}r_{n,t}^L = \frac{1}{\beta(1 + \beta + \dots + \beta^{n-1})} (r_{t-1}^F + \beta E_{t-1}r_t^F + \dots + \beta^n E_{t-1}r_{t+n-1}^F) + (\delta_0 + \delta_1 R_{t-1} + E_{t-1}q_t) \quad (39)$$

Comparing equation (16) with equation (39) gives us an implication. The default risk premium doesn't change while the maturity of the loan rate (long-term rate) increases. The reason is that for the n-period loan, the cost resulting from default risk on loans is incurred in every period.

Since the loan rate and the federal funds rate are determined on the loan market and the federal funds market, respectively, we need to incorporate the general equilibrium into our model. We assume that the public demand function for new n-period loans is similar to that of the basic model as follows;

$$\tilde{L}_t = \tilde{\alpha}_t - \tilde{b} r_{n,t}^L, \quad \tilde{\alpha}_t > 0, \quad \tilde{b} > 0 \quad (40)$$

where  $\tilde{L}_t$  is new total demand for n-period loans.

The equilibrium condition for new loans is

$$\tilde{L}_t = L_t \quad (41)$$

The new total supply of loans is the sum of the individual bank's supply of loans at time t. Since the CIA constraint is always binding, equation (15) becomes the new total loan supply of the banking system. Equations (15), (40) and (41) determine the loan rate as follows;

$$r_{n,t}^L = \frac{1}{\tilde{b}} \tilde{\alpha}_t - \frac{1}{\tilde{b}} R_{t-1} \quad (42)$$

Equation (42) means that the n-period loan rate is a function of the shock to demand for loans and the level of reserve balances. Combining equations (15), (39) with (42) results in

$$\begin{aligned} & r_{t-1}^F + \beta E_{t-1}r_t^F + \dots + \beta^n E_{t-1}r_{t+n-1}^F \\ &= \frac{(\beta + \beta^2 + \dots + \beta^n)}{\tilde{b}} E_{t-1}\tilde{\alpha}_t - \frac{(\beta + \beta^2 + \dots + \beta^n)}{\tilde{b}} (1 + \tilde{b} \delta_1) R_{t-1} \\ & \quad - (\beta + \beta^2 + \dots + \beta^n) \delta_0 - (\beta + \beta^2 + \dots + \beta^n) E_{t-1}q_t \end{aligned} \quad (43)$$

Equation (43) is the nth-order stochastic difference equation with respect to the federal funds rate. We conjecture the following solution function for the federal funds rate

$$r_t^F = \gamma_0 + \gamma_1 \tilde{a}_t + \gamma_2 R_t + \gamma_3 q_t \quad (44)$$

From (21) – (23) and (44), we get the following equation

$$r_t^F = -\frac{(\beta - \beta^{n+1})}{(1 - \beta^{n+1})} \delta_0 + \frac{(\beta - \beta^{n+1})}{\tilde{b}(1 - \beta^{n+1})} \tilde{a}_t - \frac{(\beta - \beta^{n+1})}{\tilde{b}(1 - \beta^{n+1})} (1 + \tilde{b} \delta_1) R_t - \frac{(\beta - \beta^{n+1})}{(1 - \beta^{n+1})} q_t \quad (45)$$

Equation (45) shows that the federal funds rate depends on the shock to loan demand, the level of reserve balances and unobserved shock to the default risk of loan supply. From equations (42) and (45), we can examine the effect of the exogenous shock to the loan rate and the federal funds rate, and thus, to the term structure of interest rates. Suppose there occurs a positive loan demand shock. We can examine the effect by taking the differential of the loan rate and the federal funds rate with respect to  $a_t$ , obtaining

$$\frac{\partial r_{n,t}^L}{\partial \tilde{a}_t} = \frac{1}{\tilde{b}} > 0, \frac{\partial r_t^F}{\partial \tilde{a}_t} = \frac{1}{\tilde{b}} \frac{(\beta - \beta^{n+1})}{(1 - \beta^{n+1})} > 0, \text{ and } \frac{\partial r_{n,t}^L}{\partial \tilde{a}_t} > \frac{\partial r_t^F}{\partial \tilde{a}_t} \quad (46)$$

We get similar results as the basic model, i.e., the positive loan demand shock raises the loan rate and the federal funds rate. The difference is that as the maturity of the loan rate increases, the response of the loan rate to the positive loan demand shock decreases because  $\tilde{b} > b$  as will be shown in the following result.

In addition, we can examine the monetary shock to the n-period loan rate and the federal funds rate. Taking the differential of the n-period loan rate and the federal funds rate with respect to the reserve balance, we get

$$\frac{\partial r_{n,t}^L}{\partial R_{t-1}} = -\frac{1}{\tilde{b}} < 0, \frac{\partial r_t^F}{\partial R_t} = -\frac{1}{\tilde{b}} \frac{(\beta - \beta^{n+1})}{(1 - \beta^{n+1})} (1 + \tilde{b} \delta_1) < 0, \text{ and } \frac{\partial r_{n,t}^L}{\partial R_{t-1}} \begin{matrix} < \\ > \end{matrix} \frac{\partial r_t^F}{\partial R_t} \quad (47)$$

The results are similar to the basic model. That is, the expansionary (contractionary) monetary shock causes the loan rate and the federal funds rate to decrease (increase). The response of the federal funds rate to the monetary shock might be less or bigger than that of the n-period loan rate to the monetary shock. It depends on the parameter,  $\delta_1$ , which reflects the cost incurred from the default risk of loans. If this parameter is big enough, the response

of the federal funds rate to the monetary shock is bigger than that of the loan rate to the monetary shock. The reason is the same as explained in the basic model.

Cook and Hahn (1989) find empirically that the effect of a monetary policy shock on the short-term rate is bigger than on the long-term rate and as the maturity of the long-term rate increases, the monetary policy shock effect to the long-term rate decreases. Is our result consistent with their finding? From equation (27) and (47), the response of the loan rates and the federal funds rate to a monetary shock are as follows;

$$\frac{\partial r_t^L}{\partial R_{t-1}} = -\frac{1}{b}, \quad \frac{\partial r_t^F}{\partial R_t} = -\frac{1}{b} \frac{(\beta + \beta^2)}{(1 + \beta + \beta^2)} [1 + b\delta_1] \quad (48-a)$$

$$\frac{\partial r_{n,t}^L}{\partial R_{t-1}} = -\frac{1}{\tilde{b}}, \quad \frac{\partial r_t^F}{\partial R_t} = -\frac{1}{\tilde{b}} \frac{(\beta - \beta^{n+1})}{(1 - \beta^{n+1})} [1 + \tilde{b}\delta_1] \quad (48-b)$$

Since the federal funds rate is a one-period rate, the responses of the federal funds rate should be the same in equation (48-a) and (48-b), and so we get

$$\frac{1}{b} \frac{(\beta + \beta^2)}{(1 + \beta + \beta^2)} [1 + b\delta_1] = -\frac{1}{\tilde{b}} \frac{(\beta - \beta^{n+1})}{(1 - \beta^{n+1})} [1 + \tilde{b}\delta_1] \quad (49)$$

Solving equation (49) with respect to  $\tilde{b}$  results in

$$\tilde{b} = b \frac{(1 + \beta + \beta^2)(\beta - \beta^{n+1})}{[(1 - \beta^{n+1})(\beta + \beta^2) - b\delta_1(\beta^3 - \beta^{n+1})]} \quad (50)$$

Equation (50) implies that

$$\tilde{b} > b \quad (51)$$

since  $(1 + \beta + \beta^2)(\beta - \beta^{n+1}) > (1 - \beta^{n+1})(\beta + \beta^2)$  and

$$\frac{(1 + \beta + \beta^2)(\beta - \beta^{n+1})}{[(1 - \beta^{n+1})(\beta + \beta^2) - b\delta_1(\beta^3 - \beta^{n+1})]} > 1$$

Equation (51) has two main implications. First of all, monetary shocks have less impact on

the n-period loan rate than on the two-period loan rate. Since  $\frac{\partial \tilde{b}}{\partial n} > 0$ , the response of the n-

period loan rate to monetary shocks decreases as the maturity of the loan rate increases. The

reason in our model is that as the maturity of the loan rate increases, the effect of the shadow

price of the CIA constraint on the loan rate decreases by spreading the liquidity effect (the shadow price of CIA constraint) over many periods<sup>3</sup>. That is, the change in the shadow price of the CIA constraint spreads over many period (n-period). So our result is consistent with the empirical study.

Another implication is that as the maturity of the loan rate increases, the demand for loans is more sensitive to the loan rate. This is consistent with the general theory of demand for goods. In general, long-term demand for goods is more sensitive to the price of goods than the short-term demand for goods. Similarly, long-term borrowing is more sensitive to the price of borrowing funds than short-term borrowing in financial instruments. The reason is that borrowers have more possibility to borrow various funds for long-term borrowing than for short-term borrowing. This is the same for the borrowers of bank loans.

In sum, banks' liquidity plays an important role in explaining the time-varying term premium and thus provides implications for the explanation of the rejection of expectations hypothesis. The term structure model into which we incorporate bank's liquidity explains the empirical phenomena that the monetary shock to the short-term rate is bigger than to the long-term rate. This model also shows that the response of long-term rates to the monetary shock decreases as the maturity of the long-term rate increases. Furthermore, these results confirm the finding of Hamilton (1996, 1997a, 1997b) that the liquidity effect is real, i.e., a monetary contraction raises short-term interest rate.

### **III. Estimation of the Term Structure Model**

#### **III.1 The Data**

The weekly data set we use runs from February, 1, 1984 to August, 27, 1997, which gives us 709 observations<sup>4</sup>. The interest rates and reserves figures are taken from the

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<sup>3</sup> In our model, a representative bank considers the liquidity effect once when it determines its funds' allocation between loans and Federal funds.

<sup>4</sup> Since lagged reserve accounting procedure changed to contemporaneous reserve accounting procedure from February 1984, we use only data after February 1984.



Statistical Release provided by the Federal Reserve Board of Governors. We also get quantities of loans from item 21 of the *Federal Reserve Bulletin*. Since this series refers to outstanding loans, we use the change of this series as a proxy for the volume of new loans extended. Federal funds rates (FFR) are averages of 7 calendar days ending on Wednesday and annualized using a 360-day year or bank interest. One-month commercial paper (CP) rates are also averages of 7 calendar days ending on Wednesday and annualized using a 360-day year or bank interest on a discount basis.<sup>5</sup> The Commercial paper rate is viewed as a substitute for the rate on banks' loans to financial and industrial companies. Our data on reserves represent aggregate reserves of depository institutions.

Figure 1 shows movements of the federal funds rate and one-month commercial paper rate during this sample. One interesting feature is that the federal funds rate fluctuated around the one-month commercial paper rate before the 1990 U.S. recession but after the recession the one-month commercial paper rate was higher than the federal funds rate. During the sample periods the mean of the federal funds rate and that of the one-month commercial paper rate are 6.41%, 6.46% respectively and thus the one-month commercial paper rate is on average higher by 5 basis points than the federal funds rate. However, the standard deviation of the federal funds rate is 2.11% and hence higher than that of the one-month commercial paper rate at 2%, which implies that the volatility of the federal funds rate is higher than that of the commercial paper rate.

### **III.2** The test for simple expectations hypothesis

We start from the test of the simple expectations hypothesis which implies that the long-term rate is the weighted average of the current short-term rate and the expected future short-term rates and that the current spread between the long-term rate and the short-term rate

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<sup>5</sup> The original commercial paper rates are business-daily averages of offer rates on commercial paper placed by several leading dealers for firms whose bond rating is AA or the equivalent. We construct the 7-days series per week by using the value of the previous business day for the holidays and we take weekly averages of 7-days series.

predicts the change in the future short-term rates. Assuming  $\beta = 1$ , Equation (34) can be rewritten as follows;

$$r_{n,t}^L = \frac{1}{n} E_t \left[ \sum_{i=0}^{n-1} r_{t+i}^F \right] + \delta_0 + \delta_1 L_t + q_t + \frac{1}{n} \eta_t \quad (52)$$

The simple expectations hypothesis implies that there is no default risk premium or liquidity premium and so  $\delta_0 = \delta_1 = q_t = \eta_t = 0$ . The regression to test the simple expectations hypothesis is as follows;

Model I:  $\delta_0 = \delta_1 = q_t = \eta_t = 0$

$$\frac{1}{n} \left[ \sum_{i=0}^{n-1} r_{t+i}^F \right] - r_t^F = \alpha + \phi (r_{n,t}^L - r_t^F) + v_t \quad (53)$$

where  $r_{n,t}^L, r_t^F$  are the n-period commercial paper rate (substitute for n-period loan rate) and the one-period federal funds rate respectively; here n corresponds to 4 or 12 weeks for one- and three-month commercial paper, respectively. Model I can be estimated by OLS with autocorrelation-heteroskedasticity consistent errors. According to the simple expectations hypothesis,  $\alpha = 0, \phi = 1$ . Table 2 shows the result for estimation of Model I (equation (53)).

The estimated coefficient on the spread is significantly less than unity and different from zero at conventional significance levels. In addition, the estimated constant is significantly less than zero. These results are very similar to previous empirical studies<sup>6</sup>.

### III.3 The test for the expectations hypothesis with default risk premium and liquidity premium

Our model developed in Section II implies that the simple expectations hypothesis doesn't hold because of the liquidity premium and the default risk premium. Since banks' optimal behavior is subject to a CIA constraint, bank's liquidity causes the shadow price of the CIA constraint to play an important role in the term structure of interest rates. Thus, our

<sup>6</sup> Rudebusch (1995) provides an excellent survey of previous empirical results.

model suggests that we need to incorporate a liquidity premium and a default risk premium into the simple expectations hypothesis.

How can we get a series for the shadow price of the CIA constraint reflecting the liquidity premium? Recall from Equation (7) that in equilibrium, the current federal funds rate is the expected value of next period's discounted shadow price of the CIA constraint by assuming the required reserve constraint not to be binding. Thus, we can take conditional expectations of both sides of equation (52) based on information available at t-1 as in (35) to use the previous federal funds rate for the expected value of the discounted shadow price of the CIA constraint. In Model II we assume that there is no unobserved shock to default risk. By using equations, (15) and (39) and setting  $\beta = 1$ , the regression for the expectations hypothesis with a liquidity premium and an observable default risk premium is as follows:

Model II:  $\delta_0 \neq 0, \delta_1 \neq 0, E_{t-1}q_t = 0, E_{t-1}\eta_t = r_{t-1}^F$

$$\frac{1}{n} \left[ \sum_{i=0}^{n-1} r_{t+i}^F \right] - r_t^F = \alpha + \phi(r_{n,t}^L - r_t^F) + \gamma_1 r_{t-1}^F + \gamma_2 R_{t-1} + v_t \quad (54)$$

Equation (54) cannot be estimated by OLS because  $v_t$  is correlated with the regressors,  $r_{n,t}^L$  and  $r_t^F$ . Rational expectations requires  $v_t$  to be uncorrelated with anything known to banks at time t-1 but  $r_t^F$  is not in the date t-1 information set. Equation (54) can be estimated by instrumental variables using a constant,  $r_{t-1}^L, r_{t-1}^F$ , and  $R_{t-1}$  as instruments. According to the Model II specification,  $\alpha = -\delta_0, \phi = 1, \gamma_1 = -n^{-1}, \gamma_2 = -\delta_1$ .

Table 3 reports the instrumental variable estimation of Model II. The model is totally unsuccessful. None of the coefficients are statistically significant or close to the predicted magnitudes.

**III.4** The test for expectations hypothesis with an unobserved default risk premium and liquidity premium : Unobserved shock to default risk,  $q_t$  and various instruments

In Model II, it was assumed that the expected unobserved shock to default risk at time  $t-1$  was zero and we considered only limited instruments. In Model III we allow an unobserved shock to default risk. If  $E_{t-1}q_t \neq 0$ , then it is not enough for the instruments to be known at time  $t-1$ ; They must also be uncorrelated with  $E_{t-1}q_t$ . If for example the lagged loan rate  $r_{t-1}^L$  is correlated with the probability of default risk  $E_{t-1}q_t$ , then  $r_{t-1}^L$  is not a valid instrument and the results for Model II are based upon misspecification. If the Fed adjusts the supply of reserves  $R_{t-1}$  to attempt to insulate  $r_{t-1}^F$  from fluctuations in the default risk premium, then  $R_{t-1}$  would also not be a valid instrument.

We re-estimated equation (54) using as instruments the federal funds rates and lagged changes in outstanding loans and replacing  $R_{t-1}$  with  $L_t$ . We employ Hansen's (1982) method in order to check the overidentifying restrictions and thus, test these conjectures about what is the correct set of instruments. Hansen's test statistic has an asymptotic  $\chi^2$  distribution with  $(r-k)$  degrees of freedom if the model is correctly specified, where  $r$  is the number of instruments and  $k$  is the number of estimated coefficients. The regression to test the expectations hypothesis with liquidity premium and default risk premium is as follows;

Model III:  $\delta_0 \neq 0, \delta_1 \neq 0, E_{t-1}\eta_t = r_{t-1}$

$$\frac{1}{n} \left[ \sum_{i=0}^{n-1} r_{t+i}^F \right] - r_t^F = \alpha + \phi(r_{n,t}^L - r_t^F) + \gamma_1 r_{t-1}^F + \gamma_2 L_t + v_t \quad (55)$$

where  $L_t$  is the quantity of the new loans at time  $t$ <sup>7</sup>.

Equation (55) can be estimated by two-stage least squares with a constant, lagged federal funds rates, and the lagged changes in outstanding loans as instruments. According to Model III,  $\alpha = -\delta_0, \phi = 1, \gamma_1 = -n^{-1}, \gamma_2 = -\delta_1$ . Table 4 shows the results.

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<sup>7</sup> The original data measure outstanding loans. Our measure of the new loan extended,  $L_t$ , is the change in outstanding loans.

The estimated coefficient on the spread is significantly different from zero and not significantly different from unity in contrast with the estimated coefficient on the spread in Model I. The t-test for the estimated coefficient of spread shows that the null hypothesis that the coefficient of spread is unity can not be rejected at the 5% level. These results imply that the expectations hypothesis might be salvaged under the maintained hypothesis concerning the liquidity premium and the default risk premium. In addition, the null hypothesis that Model III is correctly specified can not be rejected at the 5% level. In other words, Model III with these instruments is not overidentified. Table 5 reports Hansen's test statistic for various instruments. When we include the lagged commercial paper rates or the lagged reserve balances as instruments, the null hypothesis that the model with these instruments is correctly specified is rejected in most cases. These results imply that these instruments are invalid.

All the estimated coefficients (except the coefficient on the previous federal funds rate reflecting the liquidity premium) have the sign predicted by the theoretical model developed in Section II and all estimated coefficients are statistically significant at conventional level. The sign of estimated coefficient on the previous federal funds rate is opposite to the prediction of the theoretical model<sup>8</sup>. The value of the estimated coefficient on new loans extended is relatively high. This result suggests that a reserve supply shock has a significant impact on the loan rate and thus the term structure of interest rates.

As explained in the model of Section II, the relatively high value of the estimated coefficient on previous reserve balances implies that the response of the federal funds rate (short-term rate) to a monetary shock is bigger than the response of the loan rate (long-term rate) to a monetary shock. These results are consistent with previous studies. Consequently, monetary policy has an impact on the liquidity premium and the default risk premium as well as the interest rates of different terms.

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<sup>8</sup> This wrong sign might come from strong autocorrelation of the Federal funds rate and so we checked the sample autocorrelation of the Federal funds rate.  $\rho_1, \rho_2, \rho_3, \rho_5, \rho_{10}, \rho_{15}$  are 0.991, 0.986, 0.981,

#### IV. Conclusion

Even though this paper focuses only on commercial banks among various investors in the financial markets and employs a simplified model for thinking about liquidity demand by commercial banks, the results of this paper have important implications. Just like most households' transaction activities are subject to their liquidity conditions, banks' transaction activities are too. When banks allocate their funds into financial securities of different maturities, it is necessary for banks to incorporate liquidity consideration. Therefore, liquidity plays an important role in determining yield spreads. This might be a reason why previous studies did not produce a consensus about the empirical failure of the expectations hypothesis. When we incorporate the liquidity premium and the default risk premium resulting from transaction activities into the term structure of interest rates, the expectations hypothesis of the term structure of interest rates might be salvaged.

These results imply that investors think that long-term assets are relatively less liquid than short-term assets, and the difference in liquidity among financial assets of different terms needs to be priced into the returns of these assets. Thus, the compensation for holding illiquid assets is reflected in the return of illiquid assets as a liquidity premium.

In addition, the paper provides an explanation for the empirical finding that the effect of a monetary policy shock on the short-term rates is bigger than on the long-term rates. This empirical finding can be explained by the model presented here. In addition, a monetary policy has an impact on the term structure of interest rates through not only interest rates, but also the liquidity premium and the default risk premium on securities of different maturities. These results also confirm that the liquidity effect is real, i.e., a monetary contraction raises the short-term rates.

This paper has several limitations. The model developed in this paper doesn't consider highly liquid government securities such as Treasury bills. A more general models would encompass various investors and Treasury securities in the financial market. Moreover,

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0.973, 0.942, 0.905 respectively where  $n$  of  $\rho_n$  is the order of autocorrelation. However, since we have

empirical studies do not use various interest rates for different terms. The extension to various interest rates such as Treasury bills and Treasury bonds will be important future work for generalizing the findings of this paper.

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Table 2. The Expectation Hypothesis Test without liquidity premium and default risk premium

maturity of long-term rate	$\hat{\alpha}$	$\hat{\phi}$	$R^2$	
4-weeks	-0.022 (0.010)	0.304 (0.098)	0.171	Weekly Federal funds rate and one-month CP rate
12-weeks	-0.060 (0.039)	0.474 (0.090)	0.200	Weekly Federal funds rate and three-month CP rate

Note: a. The numbers in parenthesis are Hansen(1982)'s autocorrelation-heteroskedasticity consistent standard error.

b. 4-weeks are one-month CP rate (n=4) and 12-weeks are three-month CP rate (n=12).

Table 3. The Expectations hypothesis test with liquidity premium and default risk premium : Instrumental variable estimation

maturity of long-term rate	$\hat{\alpha}$	$\hat{\phi}$	$\gamma_1$	$\gamma_2$	Instrument
4-weeks (NBR)	-0.068 (0.091)	0.014 (0.100)	7.44e-05 (0.0067)	1.34e-06 (1.51e-06)	Constant, $r_{t-1}^L, r_{t-1}^F, R_{t-1}$
4-weeks (TR)	-0.056 (0.092)	0.016 (0.099)	-0.0001 (0.0072)	1.20e-06 (1.45e-06)	Constant, $r_{t-1}^L, r_{t-1}^F, R_{t-1}$

Note: a. The numbers in parenthesis are Hansen(1982)'s autocorrelation-heteroskedasticity consistent standard error.

b. NBR is non-borrowed reserves of depository institution and TR is total reserves of depository institution.

Table 4. The expectation hypothesis test with default risk and liquidity premium

: Two-stage least square with instruments, constant, lagged Federal funds rate and lagged changes in outstanding loans.

maturity of long-term rate	$\hat{\alpha}$	$\hat{\phi}$	$\gamma_1$	$\gamma_2$	Instrument
a. Estimation					
4-weeks (Loan)	-0.289 (0.056)	0.879 (0.119)	0.038 (0.009)	-0.00379 (0.00183)	constant, $r_{t-i}^F, i = 1, \dots, 14$ & $\Delta L_t$

b. Testing the overidentifying restriction and the correctness of model specification

$\chi_{12}^2 = 9.87$  and the critical value at 5% level  $\chi_{12,0.05}^2 = 21.03$ .  $H_0$  is accepted at 5% level.

c. Testing the coefficient of spread

$H_0: \phi = 1$ ,  $H_1: \phi \neq 1$

$t_{T-k} = -1.017$  and at 5% level  $t_{T-k,0.05} = 1.96$ .  $H_0$  is accepted at 5% level.

Note: a. The numbers in parenthesis are Hansen(1982)'s autocorrelation-heteroskedasticity consistent standard error.

b.  $\chi_n^2$  is the statistic of variable which has chi-square distribution with n degree of freedom and  $t_{T-k}$  is the statistic of variable which has t-distribution with T-k degree of freedom.

Table 5 The Hansen's test statistic for various instruments

Instruments	Statistic: $\chi^2_{r-k}$	Results
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $r_{t-1}^L$	31.22 (22.36)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $r_{t-1}^L, r_{t-2}^L$	31.21 (23.69)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $r_{t-i}^L, i = 1, 2, 3$	32.02 (25.00)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $r_{t-i}^L, i = 1, \dots, 4$	32.00 (26.30)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $r_{t-i}^L, i = 1, \dots, 5$	32.18 (27.59)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $r_{t-i}^L, i = 1, \dots, 6$	33.48 (28.87)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $r_{t-i}^L, i = 1, \dots, 7$	33.47 (30.14)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ , and $r_{t-i}^L, i = 1, \dots, 14$	37.18 (38.89)	Accepted
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $R_{t-1}$	31.83 (22.36)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $R_{t-i}, i = 1, 2$	32.40 (23.69)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $R_{t-i}, i = 1, 2, 3$	34.45 (25.00)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $R_{t-i}, i = 1, \dots, 4$	39.19 (26.30)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $R_{t-i}, i = 1, \dots, 5$	41.23 (27.59)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14, \Delta L_t$ and $R_{t-i}, i = 1, \dots, 14$	48.48 (38.89)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14$ , and $R_{t-1}$	30.94 (21.03)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14$ , and $R_{t-i}, i = 1, 2$	31.37 (22.36)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14$ , and $R_{t-i}, i = 1, 2, 3$	32.39 (23.69)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14$ , and $R_{t-i}, i = 1, \dots, 4$	36.65 (25.00)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14$ , and $R_{t-i}, i = 1, \dots, 5$	39.76 (26.30)	Rejected
$r_{t-i}^F, i = 1, 2, \dots, 14$ , and $R_{t-i}, i = 1, \dots, 14$	46.04 (37.65)	Rejected

Note: The values in parentheses are the critical values with the degree of freedom, r-k, where r is the number of instruments and k is the number of estimated coefficients.

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