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ABSTRACT

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This paper establishes stylized facts about the cyclicality of real consumer wages and real producer wages in Germany. As detrending methods we apply the deterministic trend model, the Beveridge-Nelson decomposition, the Hodrick-Prescott filter, the Baxter-King filter and the structural time series model. The detrended data are analyzed both in the time domain and in the frequency domain. The great advantage of an analysis in the frequency domain is that it allows to assess the relative importance of particular frequencies for the behavior of real wages. In the time domain we find that both real wages display a procyclical pattern and lag behind the business cycle. In the frequency domain the consumer real wage lags behind the business cycle and shows an anticyclical behavior for shorter time periods, whereas for longer time spans a procyclical behavior can be observed. However, for the producer real wage the results in the frequency domain remain inconclusive.

JEL Classification: E32, C22, C32, J30

Keywords: real wages, business cycle, frequency domain, time domain, Germany, trend-cycle decomposition, structural time series model, phase angle

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1 Introduction

At least since Keynes (1936) claimed in his General Theory that an increase in employment can only occur with a simultaneous decline in real wages, macroeconomists are debating about whether real wages are anticyclical, procyclical or do not exhibit any systematic relationship with the business cycle. A clarification of this issue could shed some light on the main sources of macroeconomic shocks and thereby be of some use in judgements about the empirical relevance of conflicting macroeconomic theories. A clearer empirical picture about the adjustment of real wages over the business cycle also helps in identifying the sources and features of wages and labor cost dynamics and therefore is of great relevance for monetary policy.

This paper contributes to the literature on the adjustment of aggregate real wages over the business cycle in several ways. First, we analyze the comovements between real wages and the cycle not only in the time domain, but also in the frequency domain. So far, most studies have focussed on the time domain approach and described the comovements between variables by traditional cross–correlations measures. However, as has been pointed out by Hart et al. (2009), in the time domain the observed cyclical behavior of the real wage hides a range of economic influences that give rise to cycles of different length and strength, thereby producing a distorted picture of real wage cyclicality. The great advantage of an analysis in the frequency domain is that it allows to assess the relative importance of particular frequencies for the behavior of real wages.

Second, it is analyzed whether the empirical results are robust to the method used to extract the cycle from the data. More specifically, we apply the deterministic trend model, the Beveridge–Nelson decomposition, the Hodrick–Prescott filter, the Baxter–King filter and the structural time series model of Harvey (1989) to the time series of aggregate real wages and gross domestic product. Since it is well known from the literature that the results may also be influenced by the price deflator used to compute real wages, we take this into account by considering both producer real wages and consumer real wages.

Third, we analyze the real wage behavior for the economy as a whole whereas many studies only consider real wages in the manufacturing sector, as for example in the recent study of the wage dynamics network of the ECB on real wage behavior in the OECD, see Messina et al. (2009).[1] Because of the much larger shares of the non-manufacturing sector in total output and employment, empirical results for the economy as a whole are certainly preferable.

Forth, whereas the question of the cyclicality of real wages in the US has been analyzed in a host of studies (see the surveys of Abraham and Haltiwanger, 1995, and Brandolini, 1995), surprisingly little systematic empirical evidence exists for Germany. This paper tries to fill this gap and provides a detailed picture of the wage dynamics in an economy in which labor unions (still) affect the majority of employment contracts.

The remainder of the paper is organized as follows. In Section 2 we describe our data and then analyze the stochastic properties of the time series. In Section 3 different trend–cycle decompositions are applied to consumer real wages, producer real wages and real GDP. In Section 4 we analyze the comovements between the particular real GDP cycle and the corresponding real wage cycles in the time and frequency domain. Section 5 summarizes and concludes.

2 Data and stochastic properties of the series

We use quarterly data for real GDP, consumer real wages and producer real wages in Germany from 1970.Q1 to 2009.Q1 (157 observations). All series that served to generate the project data were seasonally adjusted with the Census-X12-ARIMA

[1] In Messina et al. (2009) also time domain and frequency domain methods are used.

procedure. The data prior to 1991.Q1 refers to West Germany and has been linked to the data of unified Germany using annual averages for 1991. The data selection is described in more detail in Appendix A. All generated data are represented in natural logarithms.

Before we undertake the trend-cycle decompositions we study the stochastic properties of the data. We test for unit roots in real GDP and both real wage series applying the augmented Dickey-Fuller (ADF) test and the Phillips-Perron test. In both tests the alternative hypothesis is based on the assumption that the particular series follows a trend–stationary process since all series exhibit a clear upward course. The lag length for the ADF test is determined by the Akaike information criterion (AIC) and Schwartz information criterion (SIC) and its accuracy is then verified with the Ljung-Box test and the Breusch-Godfrey test. The results of both unit root tests show that for all series the unit root hypothesis cannot be rejected using conventional significance levels. Therefore, the underlying stochastic processes are not covariance stationary. We then apply both unit root tests to the first differences of the series. Since the null hypothesis can now be rejected, we conclude that all series are generated by I(1) processes.

3 Identification of the cyclical component

The general framework for the decomposition of each time series into trend and cycle is provided by the following model:

$$y_t = y_t^g + y_t^c + \varepsilon_t, \qquad t = 1, 2, ..., T \tag{1}$$

where t is a time index and y_t represents the natural logarithm of the series under consideration, i.e. real GDP, consumer real wages or producer real wages. The series y_t is decomposed into trend y_t^g, cycle y_t^c and (possibly) an irregular component ε_t. The latter is only relevant in the structural time series model (STSM), whereas it is neglected in the other decomposition methods applied in this paper, namely the linear trend model with broken trend (LBT), the Beveridge–Nelson decomposition (BN), the Hodrick–Prescott filter (HP) and the Baxter–King filter (BK). The latter methods assume the variance of ε_t to be zero, thereby attributing any disturbance left in the data after the removal of the trend to the cyclical component.

As a first decomposition method we consider the linear trend model. To check whether the time series under consideration is subject to structural breaks we apply the Quandt-Andrews test.[2] For all time series the test clearly rejects the hypothesis of no structural break. The estimated break point is 2002.Q4 for real GDP and 2003.Q1 for both real wages. Based on this result we estimate the following model:

$$y_t = \alpha_0 + \alpha_1 t + \beta_1 S_{t,k} + \nu_t,$$
$$S_{t,k} = \begin{cases} t-k, & \text{if } t > k \\ 0, & \text{if } t \leq k, \end{cases} \tag{2}$$

where ν_t is generated by a covariance stationary process which is uncorrelated with $\{y_t\}$ and $S_{t,k}$ reflects the change in the slope of the trend starting with period k. The results of the OLS estimation of model (2) are reported in Table 1. According to these findings real GDP grows with a (quarterly) rate of 0.57% until 2002.Q4. From 2002.Q4 on its growth slows down by 0.47 percentage points. The growth rate of the consumer real wage equals 0.35% before and -0.33% after the break point. The growth rate of the producer real wage exceeds that of the consumer real wage by 0.17 percentage points over the first period. However, after 2003.Q1 it falls more steeply than in the case of the consumer real wage. The deviations from the growth path, i.e. the residuals of the estimated model, represent the cyclical component, hence $y_t^c = \nu_t$. The examination of the residuals with correlograms and the Ljung-

[2]See Andrews (1993). This test overcomes the shortcomings of the commonly used Chow test in that it does not require any previous knowledge about the occurrence of a possible break point. We choose standard 15% as "trimming" level for this test.

Table 1: Estimation of segmented linear trend models for real GDP and real wages

regressor	**GDP**	**consumer wage**	**producer wage**
	coefficients [a)]		
t	0.0057	0.0035	0.0052
	(104.21)	(59.19)	(46.87)
$S_{k,t}$	−0.0047	−0.0068	−0.0112
	(−10.49)	(−13.4)	(−11.77)
constant	5.544	2.385	2.173
	(1278.237)	(510.194)	(246.321)

[a)] t-values in parentheses. Number of observations: 157. Break point $k = 132$ for real GDP and $k = 133$ for real wages.

Box test indicate that the obtained cycles of real GDP and real wages follow a persistent AR(1) process. This is confirmed by the results of the ADF test applied to each of these cycles. The hypothesis of a unit root cannot be rejected at the 5% significance level in each case. Since the LBT cycles do not satisfy the stationarity condition which is needed for the comovement analysis, we exclude the LBT cycles from further analysis.

As has been shown in Section 2, both real GDP and real wages are difference–stationary processes. For this case, a suitable decomposition method has been suggested by Beveridge and Nelson (1981). The BN decomposition assumes a I(1) process for the examined series and regards the trend as a prediction of future values of the series. The decomposition leads to a trend component which is a random walk with drift and to a covariance stationary cyclical component which are correlated with each other. In this respect, the BN decomposition differs from the

LBT model with its strong assumption of zero correlation between trend and cycle. However, the BN decomposition also bears some problems. For example, it requires an ARMA specification for the examined series but since distinct ARMA models can suit the data, different forecasts can result from these models. That in turn implies different trends and cycles. Furthermore, the a priori assumption about the trend being a random walk is somewhat controversial. Another problematic issue concerns the variance of the trend that could even exceed that of the series.

The procedure determining the trend and cycle requires truncation of infinite sums and is associated with heavy computational burden. In order to reduce these costs, different resolution methods have been proposed in the literature.[3] In this paper, we take the approach of Newbold (1990) which is based on the ARIMA$(p,1,q)$ representation of the series.[4] The BN cycle can be described as:

$$y_t^c = \sum_{j=1}^{q}[\hat{z}_t(j)-\mu] + (1-\phi_1-...-\phi_p)^{-1}\sum_{j=1}^{p}\sum_{i=j}^{p}\phi_i[\hat{z}_t(q-j+1)-\mu], \qquad (3)$$

where $\hat{z}_t(k)$ denotes the k-periods ahead forecast of $z=\Delta y$ made in period t. ϕ_j is the AR coefficient at lag j and μ is the mean of the process $\{z_t\}$. To isolate the cycle using (3) we have to identify the best ARMA specification for the first differences of real GDP and real wages. In doing so, we rely on the information criteria (AIC and SIC) beginning with an AR(4) model for each of the series in first differences. Since ARMA modeling technique aims at a parsimonious representation we reduce the number of AR terms and then optionally add some MA terms and compare all models with regard to the values of AIC and SIC. The initially considered AR(4) specification turns out to be the most suitable one. Next, we examine the residuals

[3] See, for example, Cuddington and Winters (1987), Miller (1988) and the more recent work of Morley (2002).

[4] According to Wold's theorem, each covariance stationary process has a MA(∞) representation which is also consistent with an ARMA(p,q) representation. Therefore, each I(1) process in first differences has an ARMA(p,q) representation.

from this model with the Ljung-Box test and the Breusch-Godfrey test. We find no evidence for serial correlation of the residuals in the case of real GDP as well as the real producer wage in first differences, respectively. Hence, for these series we choose an AR(4) model. As for the first differences of the real consumer wage, the residual autocorrelation vanishes after including an additional lag, so we finally end up with an AR(5) specification. Inserting the forecasts based on the selected models in (3) yields the cyclical components of real GDP and real wages.

As the next trend-cycle decompositions we use linear filters, the HP filter and the BK filter, which have proven popular in macroeconomic applications.[5] A great advantage of these methods can be seen in the fact that they are able to render the data stationary. They also avoid modeling of the series in contrast to, e.g. , the BN decomposition. However, the results of both filters are not without problems if they are applied to series which are generated by nonstationary processes. It has been shown in the literature that in this case the HP filter induces spurious cycles.[6] This is due to the fact that the frequency components of the resulting series have business cycle periodicity even though there are no important transitory fluctuations in the original data. As regarding the critique of the BK filter application for nonstationary series, Murray (2003) demonstrates that the first difference of an integrated trend enters the filtered series. As a result, the spectral properties of the filtered series depend on the trend in the unfiltered series. Because of the nonstationarity of the analyzed series the cycles obtained with the HP and the BK filter should be interpreted with some caution.

Finally, we consider structural time series models, which are defined in terms of unobserved components that have a direct economic interpretation.[7] The initial

[5]As suggested by Hodrick and Prescott (1980), for the HP filter we use the value 1600 for the smoothing parameter.

[6]See, for example, Cogley and Nason (1995) and Harvey and Jaeger (1993).

[7]See Harvey (1989, pp. 44–49).

specification of the model structure is left to the researcher. Within this framework, the data decide on the characteristics of the particular component. In contrast to the ad hoc filtering approaches, such as the HP and the BK filter, structural time series models rely on the stochastic properties of the data. Moreover, as opposed to ARMA modeling they do not aim at a parsimonious specification. It is quite probable that a parsimonious ARMA model identified by means of standard techniques (e.g. correlograms) does not exhibit properties expected from the examined series. For instance, it could reject cyclical behavior of a series even though such a behavior does really exist. Unfortunately, finding a "correct" model specification inevitably also remains a problem in the case of structural time models.

In this paper, we adopt the model as in eq. (1) and assume that the irregular component ε_t is normally, independent and identically distributed with variance σ_ε^2:

$$\varepsilon_t \sim \mathcal{NID}(0, \sigma_\varepsilon^2) \tag{4}$$

The stochastic trend component y_t^g can be formulated as follows:[8]

$$\begin{aligned} y_{t+1}^g &= y_t^g + \beta_t + \eta_t, & \eta_t &\sim \mathcal{NID}(0, \sigma_\eta^2) \\ \Delta^m \beta_{t+1} &= (1-L)^m \beta_{t+1} + \zeta_t, & \zeta_t &\sim \mathcal{NID}(0, \sigma_\zeta^2) \end{aligned} \tag{5}$$

The variable β_t is the slope of the trend and the scalar $m\,(m = 1, 2, 3, ...)$ is the order of the slope. If the slope follows an I(m) process then the trend is I($m+1$). In case of $m = 1$ the trend is called local linear trend. Imposing restrictions on the variances σ_η^2 and σ_ζ^2 leads to various trend forms. With $m = 1$ and $\sigma_\eta^2 = \sigma_\zeta^2 = 0$ one obtains a deterministic linear trend. The assumption $\sigma_\zeta^2 = 0$ together with $m = 1$ implies that the trend is a random walk, whereas $\sigma_\eta^2 = 0$ along with $m = 1$ results in a relatively smooth I(2) trend component. If one needs to model an even smoother trend component the trend is supposed to be of higher order ($m > 1$). The cycle y_t^c

[8]See Koopman et al. (2009, S. 55-56).

is defined as:[9]

$$\begin{bmatrix} y^c_{t+1} \\ y^{c*}_{t+1} \end{bmatrix} = \rho \begin{bmatrix} \cos(\omega) & \sin(\omega) \\ -\sin(\omega) & \cos(\omega) \end{bmatrix} \begin{bmatrix} y^c_t \\ y^{c*}_t \end{bmatrix} + \begin{bmatrix} \chi_t \\ \chi^*_t \end{bmatrix}, \qquad (6)$$
$$\begin{bmatrix} \chi_t \\ \chi^*_t \end{bmatrix} \sim \mathcal{NID}(\mathbf{0}, \sigma^2_\chi \boldsymbol{I}_2),$$

where y^{c*}_t is an auxiliary variable, ω denotes the frequency ($0 \leq \omega \leq \pi$) and ρ is the damping factor ($0 \leq \rho \leq 1$). The period p of the cycle is therefore $p = 2\pi/\omega$. If $\omega = 0$ or $\omega = \pi$, the VAR(1) process in (6) collapses into an AR(1) process. The variance σ^2_χ is given as $\sigma^2_\chi = \sigma^2_c(1 - \rho^2)$, where σ^2_c is the variance of the cycle so that with $\rho = 1$ the cycle is reduced to a deterministic and covariance stationary process. For all three series we start with the general formulation of the model with no variance restriction (model 1), but we constrain the trend specification to the local linear trend ($m = 1$). The whole model is estimated by maximum likelihood with the Kalman filter. The Kalman smoothing provides the estimates of the trend and cycle component. The estimated model parameters, called hyperparameters, that refer to real GDP are reported in Table 2. A high value of σ^2_η relative to σ^2_χ indicates an erratic trend component and a damped cycle component. Since this result seems rather implausible, in the next step we apply the restriction $\sigma^2_\eta = 0$ ensuring bigger deviations of the cycle than in the general model (see Table 2, model 2). The cycle extracted from the restricted model does coincide better with the German history of booms and recessions. The results of a likelihood ratio (LR) test also confirm the validity of the variance restriction. For the consumer real wage, the initial model leads to a deterministic cyclical component so in this case we reject the general model, too (see Table 3). We enforce the cyclical component to be stochastic

[9] See Koopman et al. (2009, p. 63), Koopman et al. (2008, p. 23) und Harvey and Streibel (1998). Clark (1989) suggests to describe the cycle as an AR(2) process. Harvey and Trimbur (2003) generalize the trigonometric version in (6) to cycles of higher order.

Table 2: Estimation of the general and restricted trend-cycle model for real GDP

model	hyperparameter[a)]						R_D^2 [b)]
	σ_ε^2	σ_η^2	σ_ζ^2	σ_χ^2	ρ	ω	
1) no restrictions	0,865	74,98	0,215	11,734	0,976	0,234	0,0545
2) $\sigma_\eta^2 = 0$	5,032	–	0,449	67,683	0,929	0,194	0,0469

[a)] The estimated variances have been multiplied by 10^6.

[b)] The coefficient of determination R_D^2 is based on the first differences of the observed series.

Table 3: Estimation of three trend-cycle models for the consumer real wage

model	hyperparameter[a)]						R_D^2 [b)]
	σ_ε^2	σ_η^2	σ_ζ^2	σ_χ^2	ρ	ω	
1) no restrictions, $m = 1$	24,187	22,883	1,862	0	1	0,499	0,0964
2) $\sigma_\eta^2 = 0$, $m = 1$	33,166	–	3,298	1,102	0,966	0,468	0,0697
3) $\sigma_\eta^2 = 0$, $m = 2$	22,482	–	0,004	26,537	0,957	0,153	0,0498

[a)] The estimated variances have been multiplied by 10^6

[b)] The coefficient of determination R_D^2 is based on the first differences of the observed series

by assuming $\sigma_\eta^2 = 0$. However, the irregular term becomes the most important component in explaining the consumer real wage variation and the cycle has a high frequency (see Table 3, model 2). These problems are eliminated if we allow for a smoother trend by setting $m = 2$ (see Table 3, model 3). We proceed similarly with the model identification for the producer real wage, hence we choose the trend-cycle model with $\sigma_\eta^2 = 0$ and $m = 2$. The estimation results are summarized in Table 4. Following e.g. Harvey and Koopman (1992) and Commandeur and

Table 4: Estimation of three trend-cycle models for the producer real wage

model	**hyperparameter**[a)]						$\boldsymbol{R_D^2}$ [b)]
	σ_ε^2	σ_η^2	σ_ζ^2	σ_χ^2	ρ	ω	
1) no restrictions, $m = 1$	14,894	61,212	9,687	0	1	0,542	0,119
2) $\sigma_\eta^2 = 0$, $m = 1$	30,67	–	12,293	10,776	0,928	0,505	0,0901
3) $\sigma_\eta^2 = 0$, $m = 2$	10,39	–	0,009	73,623	0,947	0,203	0,0914

a) The estimated variances have been multiplied by 10^6

b) The coefficient of determination R_D^2 is based on the first differences of the observed series

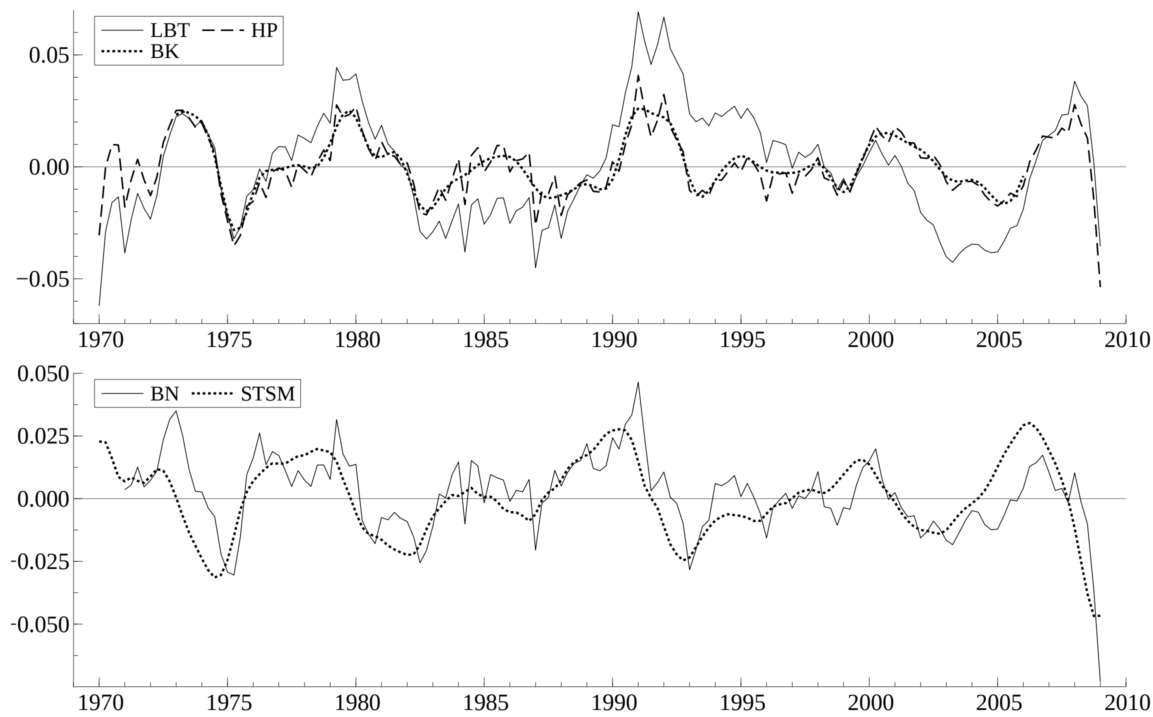
Koopman (2007), we then check all selected models with the following diagnostic tests: the Ljung-Box autocorrelation test, the Goldfeld-Quandt heteroscedasticity test and the Bowman-Shenton normality test. In all cases, we cannot reject the hypothesis of no autocorrelation at the 5% significance level. The heteroscedasticity test finds evidence against the homoscedasticity assumption only for the consumer real wage, whereas the Bowman-Shenton test indicates violation of the normality assumption for all series. Nevertheless, since the main concern of time series analysis is the autocorrelation problem we can conclude that these models provide a satisfying specification of the data generating process.

In Figure 1, we depict the cyclical component for real GDP for the different detrending methods outlined above. One can easily recognize the periods of booms and recessions that Germany has experienced since 1970. The first recessions occur as a result of the first and second oil crisis 1974–75 and 1980–82.[10] After a relatively weak recovery in the second half of the 1980s one can observe a clear boom phase

[10] We do not interpret the large negative values at the beginning of the sample in the case of the LBT and HP cycles since they could be caused by problematic behavior of these methods at the bounds.

in the early 1990s that is due to German reunification. The next turning point is reached in 1993 with the beginning of a recession phase initiated by the restrictive monetary policy of the Deutsche Bundesbank. In the late 1990s one observes a recovery that may have been caused by the IT boom followed by a recessionary phase 2001–2005. Afterwards, the economy expands again. This positive course ends in 2008 because of the world economic and financial crisis leading to a severe downturn. The various cyclical components of the consumer real wage and the

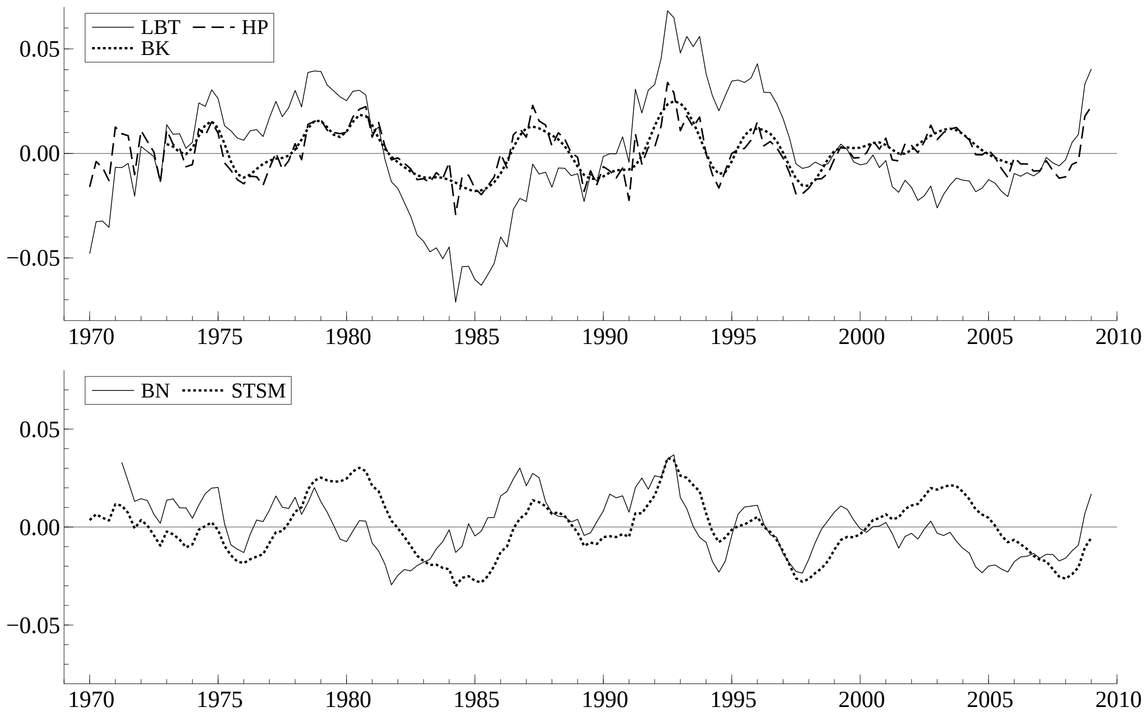
Figure 1: Cycles of real GDP



producer real wage are compared in Figure 2 and Figure 3, respectively.

For all series, the LBT cycles exhibit the most striking peaks and troughs. It is

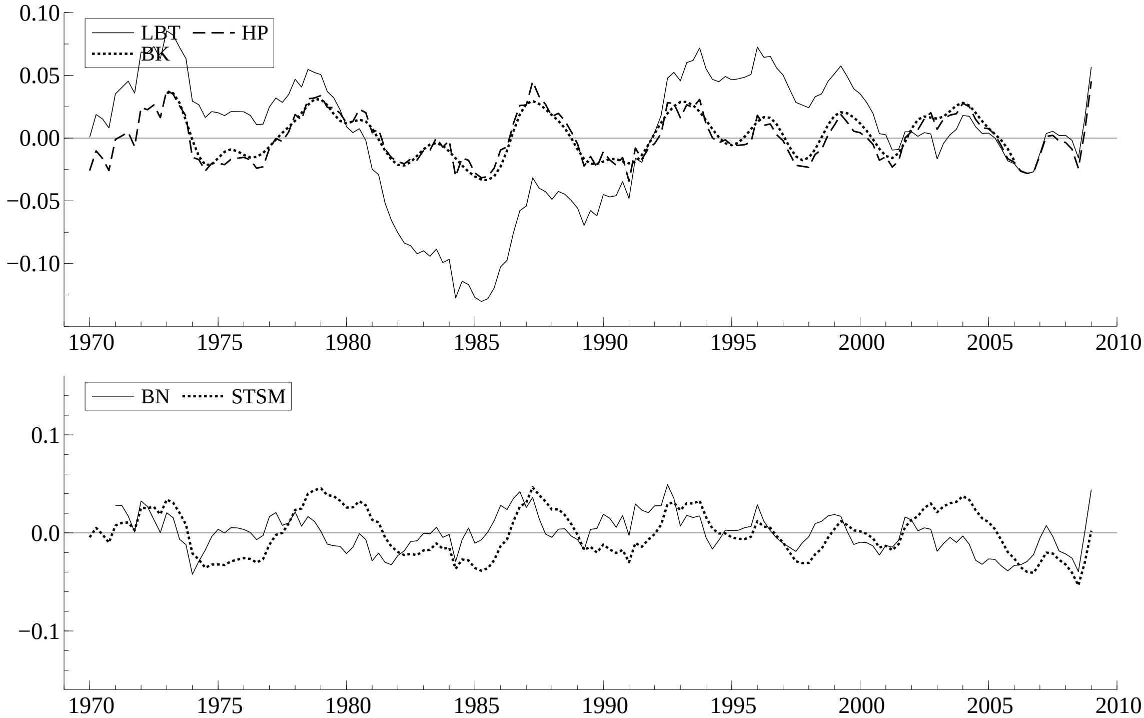
Figure 2: Cycles of the consumer real wage



apparent that the BN cycles of both real wages are shifted relative to the STSM, HP and BK cycles. Moreover, the STSM cycles of both real wages are almost in line with the HP cycles. This can be explained by the fact that the structural time series model with a trend of higher order can be associated with a Butterworth filter.[11]

[11]Gomez (2001) shows that the HP filter is a Butterworth filter.

Figure 3: Cycles of the producer real wage



4 Comovements of real GDP and real wages

4.1 Time Domain

The analysis of comovements in the time domain between real wage cycles and real GDP cycles as a reference for the business cycle is a natural approach to detect the cyclical behavior of real wages. The measure of comovements we adopt here are the sample cross–correlations between the cycle of each of the real wage series and the real GDP cycle. We consider not only the contemporaneous relationship but also analyze whether real wages react with delay or run ahead of cyclical movements in real GDP. We find it a bit misleading that in the literature wages are often classified as pro– or countercyclical by solely focusing on the contemporaneous correlation.[12] Using quarterly data, one can not seriously expect that the main adjustment of real wages to cyclical changes in GDP will take place in the same period. Therefore, we classify the considered real wage as procyclical (countercyclical) if the estimated correlation coefficients are positive (negative) taking into account the lead–lag structure of the examined series. If the estimated correlation coefficients are close to zero the particular real wage is defined to be acyclical. If the largest sample cross–correlation occurs at any lead (lag) relative to the GDP cycle we say that the particular real wage is lagging (leading) the cycle.

Table 5: Contemporaneous and largest sample cross–correlations between the real GDP cycle and the particular real wage cycle by various decomposition methods

correlation of GDP with	**methods**			
	BN	HP	BK	STSM
consumer real wage	0,1169	0,0124	0,1438	$-0,1677^*$
	$0,4879^*(+6)$	$0,4572^*(+6)$	$0,6346^*(+5)$	$0,4099^*(+11)$
producer real wage	0,0279	$-0,0423$	0,0314	$-0,0362$
	$0,2718^*(+6)$	$0,2381^*(+7)$	$0,315^*(+7)$	$0,2163^*(+10)$

Notes: “ * ” indicates statistical significance at the 5% level

The findings are summarized in Table 5. Each cell contains in the first row the contemporaneous sample cross–correlation between the cycle of the real wage series and the corresponding real GDP cycle. The value below is the maximum sample

[12]This focus of the literature on the contemporaneous correlation has also been criticized by Messina et al. (2009).

cross–correlation at the kth lead or lag of the real wage cycle relative to the real GDP cycle, where $k \in \{-12, -11, ..., 0, ..., 11, 12\}$. The number in brackets along with the "+" or "−" sign specifies at which lead or lag of the real wage cycle this maximum cross–correlation occurs.[13] We first consider the results for the consumer real wage. Except for the STSM cycle, the estimates of the contemporaneous cross–correlation are positive but statistically insignificant at the 5% level. The low practical significance is most apparent in the case of the HP cycle. Considering the leads of the real wage cycles, we find that for all cycles except for the STSM cycles, the relationship with the corresponding real GDP cycles is still positive but now becomes significant. The sample cross–correlations reach their maximum values at the 6th lead (BN and HP cycles) or 5th lead (BK cycles). In the case of the STSM cycles, there is first a significant negative sample cross–correlation until the 3rd lead. From the 6th lead, it takes high positive values that are statistically significant. We find the greatest cross–correlation at the 11th lead. Examination of the lags of the real wage cycles reveals that almost all sample cross–correlations are statistically insignificant in the case of the HP, BK and BN cycles. The significant ones are small compared to the significant sample cross–correlations at the leads.[14] To sum up, the consumer real wage displays a procyclical pattern and lags the business cycle. The strongest reaction to the actual economic situation can be observed between the 5th and the 11th quarter.

The behavior of the producer real wage differs somewhat from that of the consumer real wage. All estimated contemporaneous cross–correlations are statistically insignificant at the 5% level. Furthermore, although there is a similar cyclical pattern

[13]For clarity reasons, we do not present detailled figures of the lead–lag structure and instead describe some results verbally.

[14]In the case of the STSM cycles, the significant negative sample cross–correlations emerge at the first 3 lags. In contrast, the BN cycles are characterized by positive cross–correlations which, though, are insignificant.

as in the case of the consumer real wage, the sample cross–correlations at the leads of the real wage are not as high. In Table 5 this is evident from the differences in the maximum cross–correlations between both wages. The sample cross–correlations at the lags of the producer real wage, with the exception of some lags in the case of the BN cycles, are statistically insignificant. The analysis leads to the conclusion that the producer real wage behaves procyclically and lags the business cycle. The main reaction to the actual economic situation occurs after 6 (BN cycle) to 10 (STSM cycle) quarters.

4.2 Frequency domain

The above analysis of the comovements between real wages and the cycle in the time domain might give the impression that the cyclicality of real wages has been sufficiently characterized. However, the observed behavior of real wages in the time domain results from the countervailing or/and reinforcing influences of cycles of different length. As a consequence, if we want to learn something about the behavior of real wages *over the business cycle*, we could be misled by looking at the time domain results alone. In this section, we resort to some spectral analysis concepts that enable us to assess the relative importance of cycles of different length and strength and therefore provide a comprehensive picture on the cyclical behavior of real wages.

We will first give a short introduction to these concepts. The central one is the spectral representation of a covariance stationary process, also called the Cramér representation, as a frequency domain counterpart to the Wold representation of such a process. According to the spectral representation a time series Y_t, which is a single realization of a zero–mean covariance stationary process y_t, is regarded as comprising various superimposed cosine and sine waves each having different frequency and amplitude. If such a stochastic process y_t is discrete and real–valued, it can be described by:[15]

$$y_t = \int_0^\pi \alpha(\omega)\cos(\omega t)d\omega + \int_0^\pi \delta(\omega)\sin(\omega t)d\omega, \tag{7}$$

where $\alpha(\omega)$ and $\delta(\omega)$ are orthogonal complex–valued stochastic processes with zero mean and equal variances. The variable ω denotes the (angular) frequency. The coefficients $\alpha(\omega)$ and $\delta(\omega)$ give rise to the stochastic nature of the process in (7), in that $|\alpha(\omega)|$ and $|\delta(\omega)|$ are random amplitudes, whereas $\arg\{\alpha(\omega)\}$ and $\arg\{\delta(\omega)\}$ describe random phases of the particular cosine and sine wave. Each wave contributes to the explanation of the overall variance (power) of this process. This information is given by the real–valued function $s(\omega)$, the so–called spectral density function or, in short, spectrum:

$$\begin{aligned} s(\omega) &= \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\gamma_j\; e^{-i\omega j} \\ &= \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\gamma_j\cos(\omega j) \\ &= \frac{1}{2\pi}\big[\gamma_0 + 2\sum_{j=1}^{\infty}\gamma_j\cos(\omega j)\big], \end{aligned} \tag{8}$$

where γ_j is the jth autocovariance of the process and i is the imaginary number. The area under the graph of $s(\omega)$ for $\omega \in [-\pi, \pi]$ describes the total variance of the process.

Since we are primarily interested in the interactions between time series, we now turn to the multivariate case and consider two series Y_{kt} and Y_{lt} $(k, l = 1, 2, .., n,\ k \neq l)$. The frequency by frequency relationship between the underlying processes y_{kt} and y_{lt} can be measured by the cross–spectrum $s_{kl}(\omega)$:

$$\begin{aligned} s_{kl}(\omega) &= \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\gamma_{kl}^j\; e^{-i\omega j} \\ &= \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\gamma_{kl}^j[\cos(\omega j) - i\sin(\omega j)], \end{aligned} \tag{9}$$

[15] See, for example, DeJong and Dave (2007, pp. 41–42) and Priestley (1981, pp. 251–252).

where γ_{kl}^{j} is the jth cross–covariance of the two processes defined as

$$\gamma_{kl}^{j} = E[(y_{kt} - \mu_k)(y_{l,t-j} - \mu_l)] \tag{10}$$

The cross–spectrum, which is a complex–valued function of ω, can be decomposed into the real part $c_{kl}(\omega)$ called cospectrum and the imaginary part $q_{kl}(\omega)$ called quadrature spectrum. In analogy to the spectrum of an individual process, the area under the cross–spectrum in the range $[-\pi, \pi]$ gives the overall covariance of the two processes. Additionally, as the quadrature spectrum integrates to zero in this interval, the area under the cross–spectrum is equal to the area under the cospectrum. According to this, the cospectrum at frequency ω can be interpreted as the marginal contribution of the components with frequency $\omega + d\omega$ to the overall covariance between the processes. The quadrature spectrum at this frequency can serve as an indicator for the out–of–phase covariance since it measures the portion of the covariance between two processes shifted relative to one another by $\pi/2$ which is attributable to the waves with this frequency. The information contained in both the cospectrum and the quadrature spectrum is useful in establishing the lead–lag relationship between two processes. For this purpose, the inferences from both parts of the cross–spectrum at any frequency can be combined into one quantity, the so–called phase angle, denoted by $\theta(\omega)$:

$$\theta(\omega) = \arctan\left[\frac{q_{kl}(\omega)}{c_{kl}(\omega)}\right] \tag{11}$$

Because of the properties of arctangent, the phase angle $\theta(\omega)$ is a multivalued function, but it is common to limit its values to the interval $(-\pi, \pi)$. The unique value in $(-\pi, \pi)$ and therewith the sign of the phase angle can, however, only be determined by the signs of the cospectrum and the quadrature spectrum. If $\theta(\omega)$ takes on positive values, we say that the component of y_{kt} with frequency ω leads the corresponding component of y_{lt}. The opposite case is implied by $\theta(\omega) < 0$. Both components are in phase if $\theta(\omega)$ equals zero. Based on the values of the phase angle we can also make statements about the correlation between y_{kt} and y_{lt}. If the values of the phase angle range between $[-\pi/2, \pi/2]$, we say that y_{kt} and y_{lt} are positively correlated (procyclical behavior), whereas the values of $\theta(\omega)$ in the interval $[\pi/2, \pi]$ or $[-\pi/2, -\pi]$ indicate a negative relationship (countercyclical behavior) between them.

In this paper, we focus on the nonparametric approach to the estimation of spectra and cross–spectra.[16] For that purpose, we have to choose a suitable spectral window and the truncation point of the window. Since, as pointed out by Jenkins and Watts (1968, p. 280), the spectral estimates are barely affected by the functional form of the window, we use the Bartlett window and concentrate on finding an appropriate truncation point. We allow the window lag size to be 20 resting upon the technique of window closing which enables a researcher to make her choice in the process of learning about the shape of the spectrum instead of relying on any rules of thumb.[17] The estimated spectra of the HP, BK, BN and STSM cycles of real GDP, consumer real wages and producer real wages, and the cross–spectra between all GDP cycles and the real wage cycles are shown in Appendix B.

In the following, we focus on the interpretation of the estimated phase angle. In Figures 4 and 5, the point estimates of the phase lead of the real GDP cycle over the corresponding consumer and producer real wage cycle, respectively, for all decomposition methods along with the respective confidence bounds are drawn.[18] The frequency range presented here covers all business cycle periodicities, i.e. periods between 1.5 (frequency of about 1.0) and 8 years (frequency of about 0.2).[19] The relationship between frequency ω and period p is given by the formula: $p = 2\pi/\omega$. It

[16]We refer those readers who are not familiar with univariate and multivariate spectral estimation to, e.g. , Koopmans (1974, Ch. 8) and Priestley (1981, Ch. 6 and 9.5).

[17]The method of window closing is described in Jenkins and Watts (1968, pp. 280–282).

[18]We construct the 90% confidence intervals as described in Koopmans (1974, pp. 285–287).

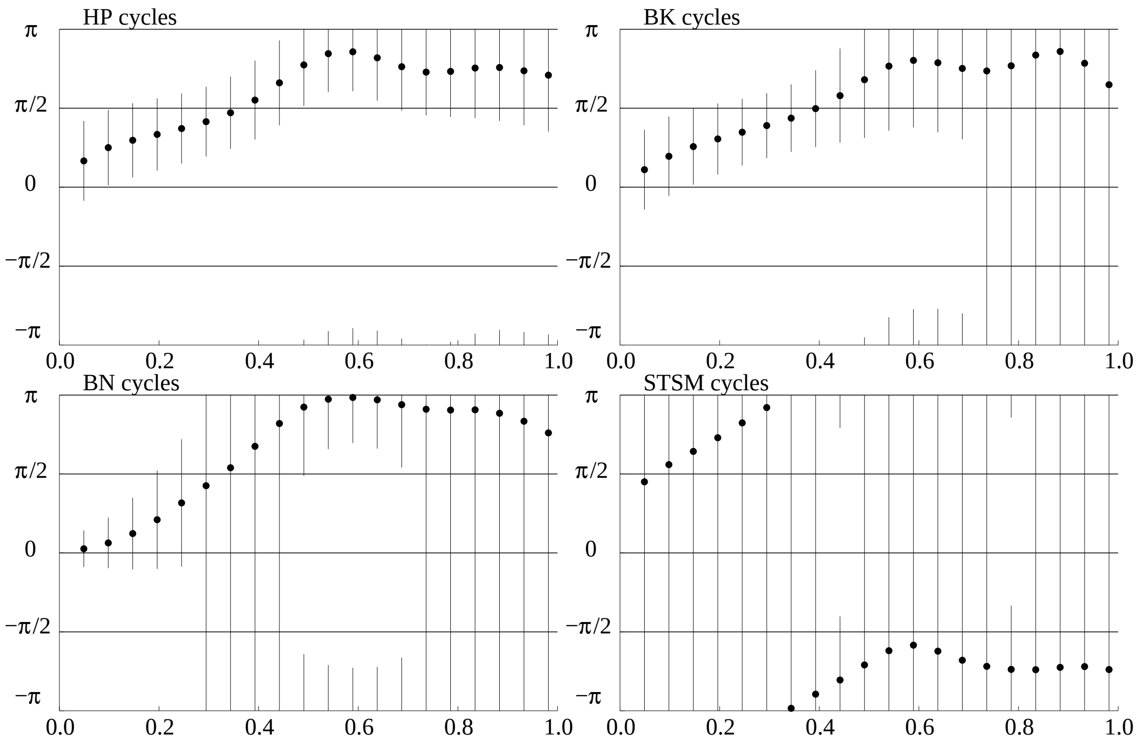
[19]Following the seminal paper of Burns and Mitchell (1946) this is the commonly used range for the business cycle length.

should be noticed that the vertical axis representing the values of the phase angle is divided into four regions.[20] If the confidence interval covers one of two upper regions we say that the real GDP cycle significantly leads the real wage cycle. The opposite holds true if the confidence interval lies in one of the two lower regions. A significant procyclical behavior of the real wage cycle is indicated by the confidence interval in the two regions around 0. If, on the other hand, the confidence interval covers the top or the bottom region we conclude that the real wage behaves countercyclically. If the confidence interval covers at least three regions, we interpret it as being a "no information confidence interval".

In Figure 4, it is apparent that for the consumer real wage the point estimates of the phase angle display a similar pattern in the case of the HP, BK and BN cycles. At all frequencies, the estimated phase angle takes on positive values which suggests a lagging behavior of cycles of the real wage characterized by business cycle frequencies with respect to the corresponding cycles of real GDP. However, statistical significance of such a behavior pertains rather to lower business cycle frequencies. We also observe that for these three decomposition approaches the lower frequencies (up to about 0.35) are associated with estimates of the phase angle in the interval $[0, \pi/2]$. The significant ones are confined to the frequencies up to 0.25 thereby indicating significant procyclical pattern of the consumer real wage at these frequencies. In contrast, shorter cycles of the real wage are negatively correlated with the particular real GDP cycle as shown by the estimated phase angle values lying above $\pi/2$. For the STSM cycles we obtain positive point estimates at lower frequencies as well. However, we cannot make any statement about the statistical significance of any estimated phase angle in the whole frequency range. Taking all findings into account we can conclude that, in general, longer consumer real wage

[20] The results for each frequency are illustrated on a linear scale which can be obtained through "straightening" a circular scale connected by the points representing angles π and $-\pi$.

Figure 4: Phase angle: real GDP and consumer real wage cycles

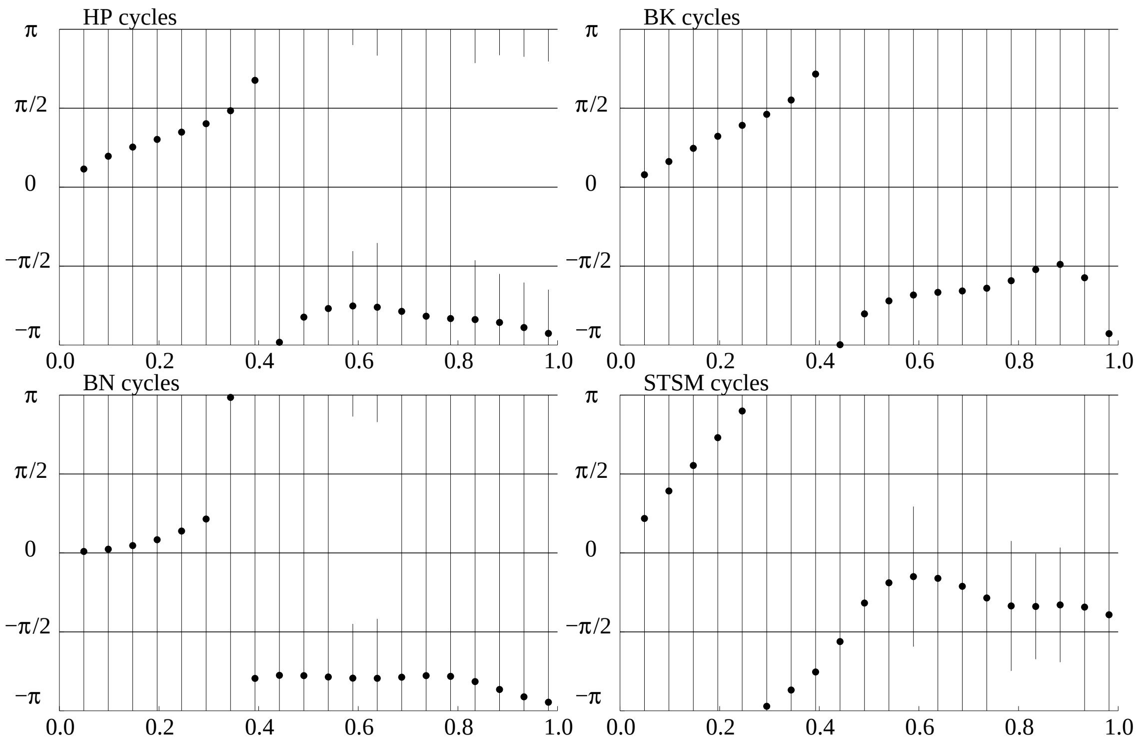


Notes: The horizontal axis represents (angular) frequency ω.

cycles seem to exhibit a procyclical and lagging behavior, whereas the shorter ones evolve countercyclically and also react with delay to the actual economic situation.

As for the producer real wage, the estimation results presented in Figure 5 look almost identical to the ones for the consumer real wage if we consider the estimated values of the phase angle at lower frequencies. Also, the values in the interval $[-\pi, -\pi/2]$ (HP, BK and BN cycles) at the frequencies above 0.4 (below 4 years) could serve as an indicator for a countercyclical behavior of the producer real wage. Despite this similarity to the correlation scheme of the shorter cycles of the consumer real wage, it can be noted that for shorter cycles the producer real wage seems to lead

Figure 5: Phase angle: real GDP and producer real wage cycles



Notes: The horizontal axis represents (angular) frequency ω.

the corresponding real GDP cycle. The main problem, however, is the insignificance of almost all estimates. Hence, the results for producer real wages in the frequency domain remain inconclusive.

5 Summary and Conclusions

This paper provides stylized facts about the cyclical behavior of consumer and producer real wages in Germany. In order to see whether a robust empirical picture on real wage behavior emerges, several detrending methods have been applied to both real wage series and real GDP, including the deterministic trend model, the Beveridge–Nelson decomposition, the Hodrick–Prescott filter, the Baxter–King filter and the structural time series model. The stochastic properties of the original time series and the derived cyclical components were analyzed using a set of unit root tests and other diagnostic tests. Since the cycles generated by the deterministic trend model violated the stationarity condition, they were excluded from further analysis.

We then analyzed the comovements of the detrended real wage series with real GDP in the time domain and in the frequency domain. For both approaches not only the contemporaneous correlation between real wages and GDP, but also the lag–lead structure has been taken into account. In the time domain the sample cross–correlations between the cycle of each of the real wage series and the GDP cycle have been evaluated. According to our results in the time domain, the contemporaneous correlation between the real wage and GDP is statistically insignificant, with the exception of the cycles from the structural time series model. In the latter case we found a negative contemporaneous correlation. Regarding the lead–lag structure, the consumer real wage displays a procyclical pattern and lags behind the business cycle. The strongest reaction to the actual economic situation can be observed between the 5th and the 11th quarter. For the producer real wage all estimated contemporaneous cross–correlations are statistically insignificant. Furthermore, although there is a similar cyclical pattern as in the case of the consumer real wage, the sample cross–correlations at the leads of the real wage are not as high.

In the next step, we analyzed the comovements in the frequency domain. The great advantage of an analysis in the frequency domain is that it allows to assess the relative importance of particular frequencies for the behavior of real wages. We followed the non-parametric approach to the estimation of spectra and cross–spectra. The analysis of the phase angle for the consumer real wage shows that the observed cyclicality depends on the frequency range under consideration. All decomposition methods for which we got statistically significant results reveal a similar pattern. The consumer real wage is lagging the real GDP cycle. For shorter time periods up to about three years, the consumer real wage shows an anticyclical behavior, whereas for longer time spans a procyclical behavior can be observed. However, for the producer real wage the results in the frequency domain remain inconclusive.

Our results for consumer real wages are in line with an economy that is characterized by wage stickiness in the short run. For example, an economic upswing could first lead to a decline in real wages because of rising prices and rigid nominal wages. In the longer run, nominal wages are adjusted upwards eventually leading to a rise in real wages as well.

A Data Selection

We use quarterly data for Germany from 1970.Q1 to 2009.Q1 (157 observations). All series that served to generate the project data, except for working hours, have already been available as seasonally adjusted data based on the Census-X12-ARIMA procedure.

Real GDP

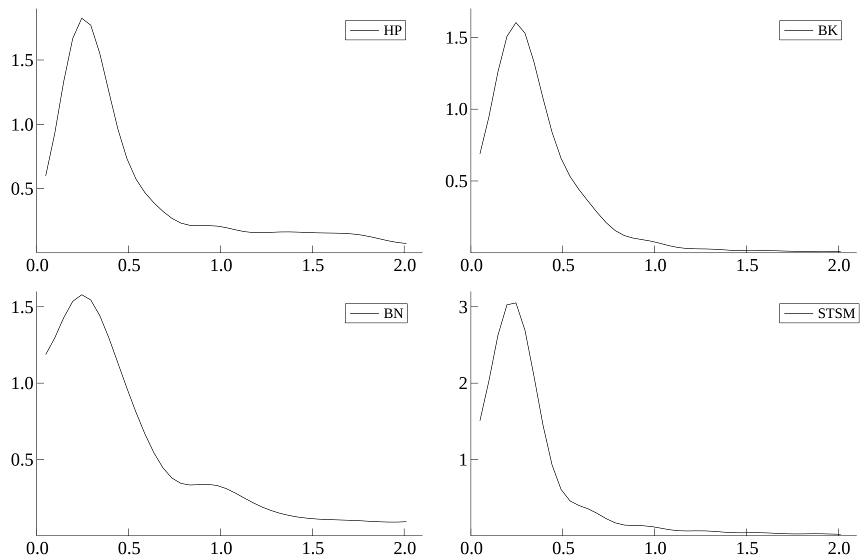
In order to obtain the real GDP series we used the price adjusted chain index with the base year 2000 (source: Deutsche Bundesbank, series JB5000). The raw data before 1991.Q1 referred to West Germany and after 1991.Q1 to unified Germany. The index series has already been linked over the annual average for 1991. We multiplied the index with the nominal GDP in 2000 and divided it by 100 (source of nominal GDP: Statistisches Bundesamt, GENESIS online database).

Real wages

We obtained the real wage series on the basis of gross wages and salaries (source: prior to 1991.Q1 Statistisches Bundesamt, Beiheft zur Fachserie 18, Reihe 3; from 1991.Q1 on Statistisches Bundesamt, GENESIS online database). Since we were interested in hourly real wages, we divided this series by total working hours of the domestic labor force. The data for working hours from 1970.Q1 to 1991.Q4 referred to West Germany (source: Statistisches Bundesamt, Ergänzung zur Fachserie 13, Reihe S.12) and from 1991.Q1 on to unified Germany (source: Statistisches Bundesamt, GENESIS online database). After seasonal adjustment with the Census-X12-ARIMA procedure we linked both series over the annual average for 1991. The nominal hourly wage has been deflated with the consumer price index (CPI) or the producer price index (PPI) in order to generate the respective real wage series. The source of both price indices is Deutsche Bundesbank (CPI: series USFB99, PPI: series USZH99).

B Figures: nonparametric spectral estimates

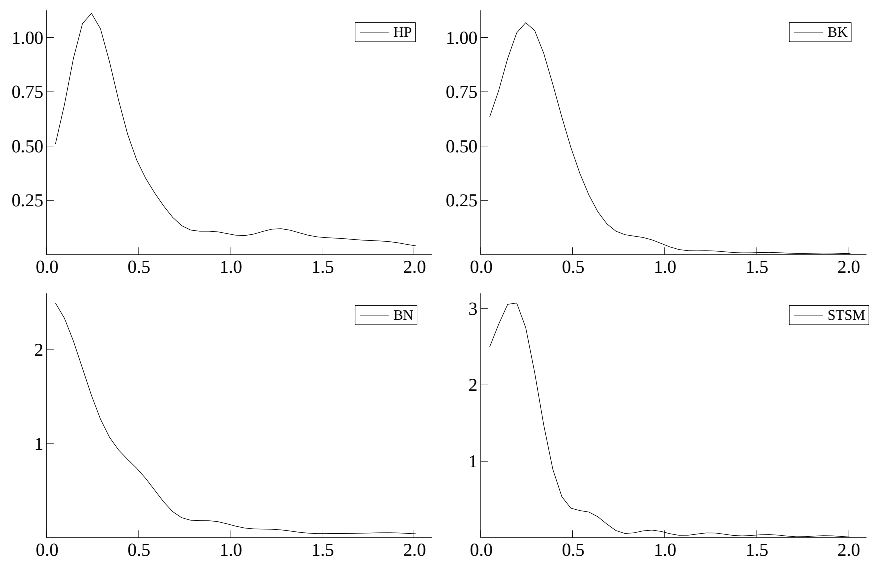
Figure B.1: Spectra of the real GDP cycles



Notes: The horizontal axis represents the (angular) frequency ω.

The values on the vertical axis have been multiplied by 10^4.

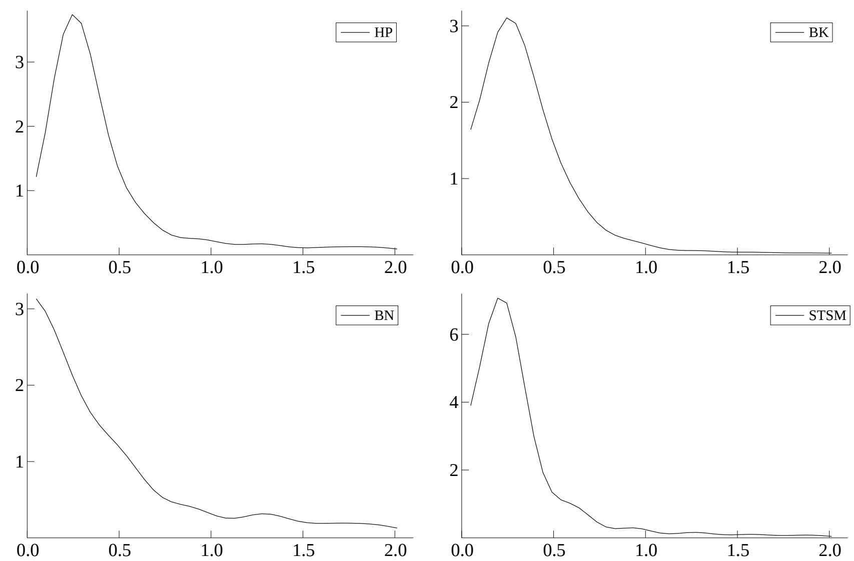
Figure B.2: Spectra of the consumer real wage cycles



Notes: The horizontal axis represents the (angular) frequency ω.

The values on the vertical axis have been multiplied by 10^4.

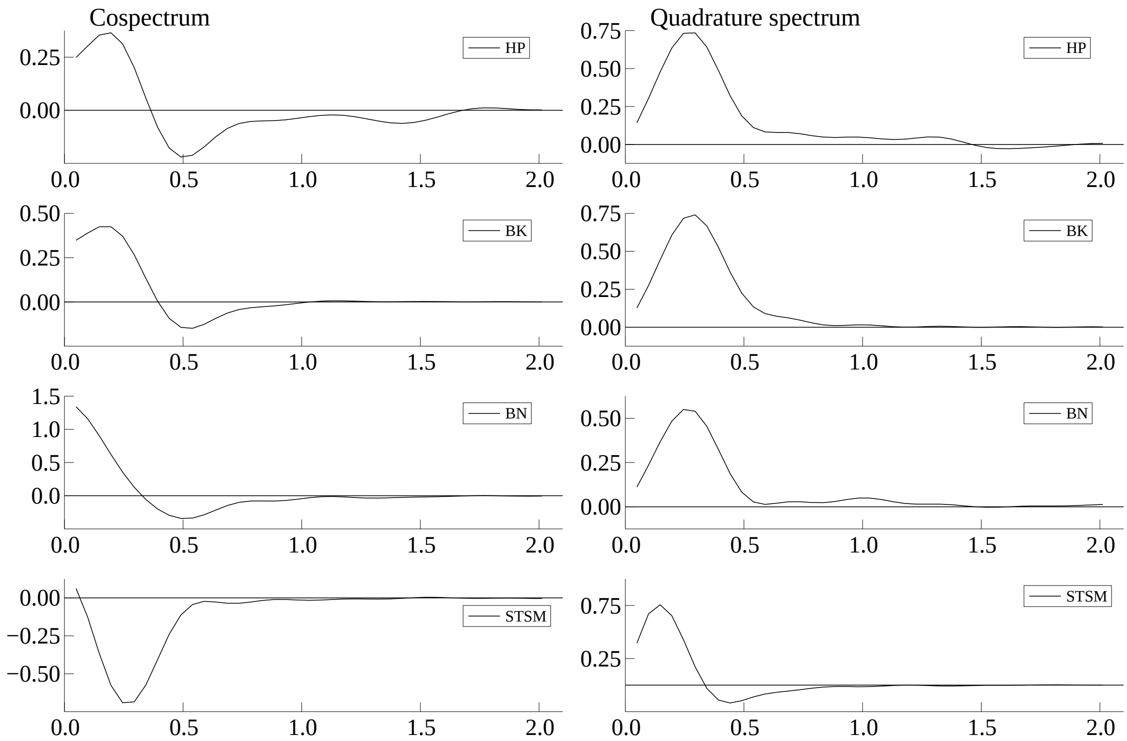
Figure B.3: Spectra of the producer real wage cycles



Notes: The horizontal axis represents the (angular) frequency ω.

The values on the vertical axis have been multiplied by 10^4.

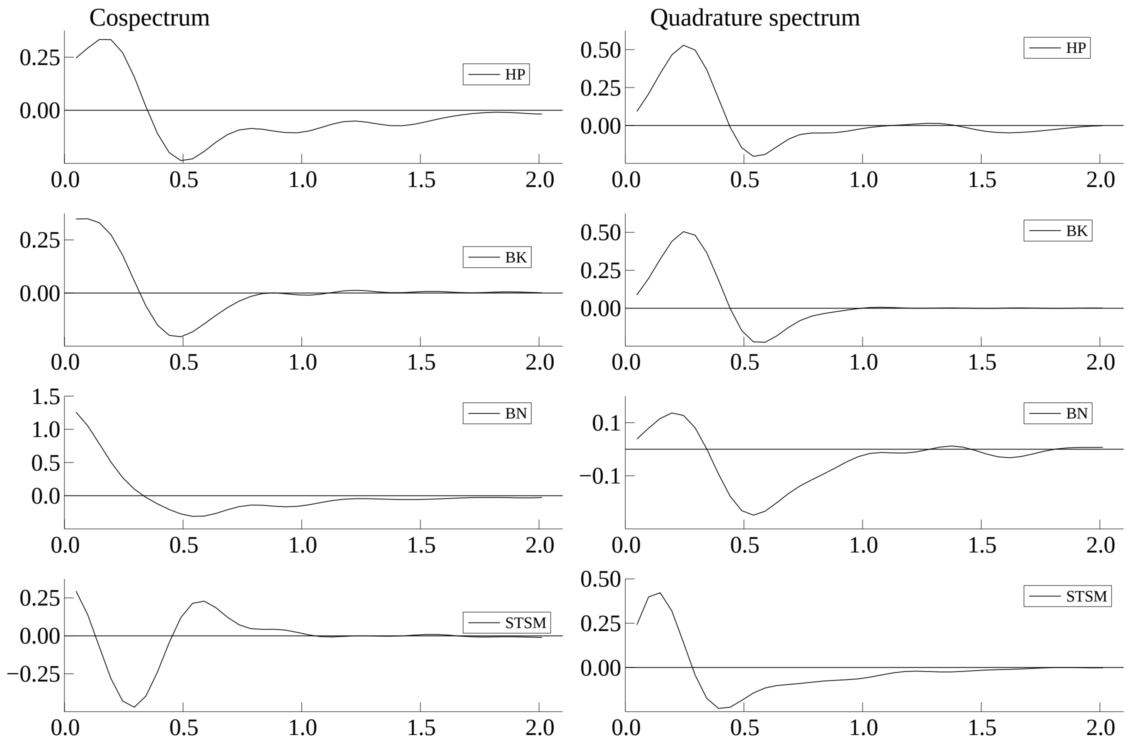
Figure B.4: Cospectra and quadrature spectra between the real GDP cycles and the consumer real wage cycles



Notes: The horizontal axis represents the (angular) frequency ω.

The values on the vertical axis have been multiplied by 10^4.

Figure B.5: Cospectra and quadrature spectra between the real GDP cycles and the producer real wage cycles



Notes: The horizontal axis represents the (angular) frequency ω.

The values on the vertical axis have been multiplied by 10^4.

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