MARKET BASED COMPENSATION, PRICE INFORMATIVENESS AND SHORT-TERM TRADING

by Riccardo Calcagno
and Florian Heider

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In 2007 all ECB publications feature a motif taken from the €20 banknote.

CONTENTS

Abstract

This paper shows that there is a natural trade-off when designing market based executive compensation. The benefit of market based pay is that the stock price aggregates speculators' dispersed information and therefore takes a picture of managerial performance before the long-term value of a firm materializes. The cost is that informed speculators' willingness to trade depends on trading that is unrelated to any information about the firm. Ideally, the CEO should be shielded from shocks that are not informative about his actions. But since information trading is impossible without non-information trading (due to the "no-trade" theorem), shocks to prices caused by the latter are an unavoidable cost of market based pay. This trade-off generates a number of insights about the impact of market conditions, e.g. liquidity and trading horizons, on optimal market based pay. A more liquid market leads to more market based pay while short-term trading makes it more costly to provide such incentives leading to lower CEO effort and worse firm performance on average. The model is consistent with recent evidence showing that market based CEO incentives vary with market conditions, e.g. bid-ask spreads, the probability of informed trading (PIN) or the dispersion of analysts' forecasts.

Non-technical summary

Why does CEO pay include measures of stock price performance? What are the limits to such market based pay? To what extent do trading conditions, e.g. trading horizons, matter for the cost and benefit of market based CEO pay and, ultimately, real firm performance? Despite a large literature on executive compensation and debates about market based CEO pay, there is hardly any research on how trading conditions in the stock market matter for market based compensation. This paper attempts to fill the gap and shows that there is a natural trade-off when designing market based pay.

The benefit of market based pay is that the market as a whole, i.e. the sum of all traders' information, takes a picture of managerial performance at an early stage before the final long-term value of a firm materializes. In a competitive market in which the ownership of listed companies is dispersed, it is impossible to access all traders' information directly. Self-interested trading overcomes this communication problem, albeit imperfectly. It is well known that in order to make room for information trading, some people must trade for reasons other than information about the value of the firm, e.g. stochastic life cycle motives or the need to fulfil margin calls. The cost of market based pay therefore is that it will necessarily be contingent on such noise trading although noise trading per se is not informative about managerial effort. The pay of a risk averse CEO should ideally be shielded from shocks that are not related to information about his performance, but this is impossible with market based pay.

Market based CEO pay will be proportional to the liquidity of the market for the company's shares since a more liquid market allows more information based trading. More non-information or noise trading increases the liquidity of the market but it also adds pure noise to the stock price. The paper shows that the overall impact is negative since noise trading reduces the information content of the stock price.

The paper then examines the impact of shortening the investment horizon of traders on the balance between the benefit and the cost of market based pay. Speculators with short horizons act less on their private information since the stock price reflects this information only imperfect at the time they need to sell the asset. This reduces the amount of information trading relative to noise trading. The stock price becomes less

informative about managerial performance, market based incentives weaken and managerial effort subsides leading to worse firm performance on average.

Recent empirical research establishes a significant cross-sectional link between the extent of stock-based CEO pay and trading conditions that is consistent with our analysis. The paper's prediction that shorter investment horizons of traders should lead to less market based CEO pay and lower CEO effort has not yet been tested directly. There is however a sharp increase in the positive link between measures of stock price informativeness, e.g. PIN (probability of informed trading), volume or bid-ask spreads, and the sensitivity of market based CEO pay to shareholder value after the stock market bubble burst in 2000. If traders acted more myopically in the run up to the stock market bubble then our model provides a possible rationale for the increase. Finally, the analysis speaks to issue of how stock markets affect real economic performance since there is cross-country cross-industry evidence that liquid stock markets promote economic efficiency via market based governance.

1 Introduction

Why does CEO pay include measures of stock price performance? What are the limits to such market based pay? To what extent do trading conditions, e.g. trading horizons, matter for the cost and benefit of market based CEO pay and, ultimately, real firm performance? Despite a large literature on executive compensation and debates about market based CEO pay (see for example Bertrand and Mullainathan (2001) and Bebchuk and Fried (2004)), there is hardly any research on how trading conditions in the stock market matter for market based compensation (see for example the surveys by Murphy (1999) and Core et al. (2003a)).

This paper attempts to fill the gap and shows that there is a natural trade-off when designing market based pay. Whenever the stock price contains useful information for incentive contracting, it must also contain useless information. The trade-off between useful and useless information in designing optimal market based CEO pay and empirical implications about the role of trading conditions arise naturally from combining two well known but hitherto unconnected insights from the incentive literature and the literature on information aggregation in asset markets. On the one hand Holmström (1979)'s informativeness principle says that any signal that is not informative about managerial effort should not be used to condition a manager's compensation scheme. On the other hand, the no-trade theorems of Grossman and Stiglitz (1980) and Milgrom and Stokey (1982) stipulate that information trading by speculators is impossible without noise trading, i.e. trading that is unrelated to any information about the value of the asset being traded, e.g. the shares of a firm.

The benefit of market based pay is that the market as a whole, i.e. the sum of all speculators' information, takes a picture of managerial performance at an early stage before the final long-term value of a firm materializes. The stock market therefore provides value neutral information that has no direct impact on future managerial decisions but provides an assessment of past decisions.[1] In a competitive market in which the ownership of listed companies is dispersed, it

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[1]Stock markets also provides value enhancing information, i.e. information about future corporate strategy.

is impossible to access all speculators' information directly. Self-interested trading overcomes this communication problem, albeit imperfectly. In order to make room for information trading by speculators, some people must trade for reasons other than information about the value of the firm, e.g. stochastic life cycle motives or the need to fulfill margin calls. The cost of market based pay therefore is that it will necessarily be contingent on such noise trading although noise trading per se is not informative about managerial effort. Our contribution is to point out that the cost of market based pay is inextricably linked to its benefit.

Combining a multi-period trading model with efficient pricing (we follow the formulation of Vives (1995)) and a standard incentive contracting framework, we show how speculators' trading horizons affect the balance between the benefit and the cost of market based pay. Speculators with short horizons act less on their private information since the stock price reflects this information only imperfect at the time they need to sell the asset.[2] This reduces the amount of information trading relative to noise trading. The stock price becomes less informative about managerial performance, market based incentives weaken and managerial effort subsides leading to worse firm performance on average.

Recent empirical research by Kang and Liu (2005) establishes a significant positive link between the extent of market based CEO pay in the US and the informativeness of the stock price (see also Garvey and Swan (2002) for a related result). Moreover, they find that the sensitivity of CEO pay to stock price movements increases after the stock market bubble burst in 2000. If there was more short-term trading in the run up of the bubble than after it had burst, then our analysis is consistent with their findings.

To the best of our knowledge, our trade-off between useful and useless information in designing marked based CEO pay due to trading and examining the impact of short trading horizons

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Dow and Gorton (1997) analyze the interaction between these two types of information when managers make investment decisions.

[2]Shleifer and Vishny (1990) and Allen and Gorton (1993) give reasons why traders may have short investment horizons. Short-horizons also limit information trading in Froot et al. (1992b) and Dow and Gorton (1994).

is new. Holmström and Tirole (1993) examine the role of noise trading in a static model. They show that more noise trading motivates a single large insider to collect additional information about management. More noise trading allows him to better hide his informed trades and therefore to make larger profits that offset the cost of collecting more precise information. Some of his information nevertheless flows into the stock price and improves its information content. More noise trading always leads to more market based pay via the indirect monitoring by the insider. In contrast, we show that more noise trading leads to *less* market based pay. This more pessimistic view of the role of uninformed trading on CEO incentives is a direct consequence of the fundamental difficulty that the stock price cannot aggregate useful but dispersed information without including also useless information.[3]

Kim and Suh (1993) point to a measurement problem when examining market based CEO pay. They argue that using the "raw" price to construct market measures is problematic since the stock price impounds public information from earnings reports in addition to private information. As a result empirical studies may exaggerate the importance of market based pay.[4] Paul (1992) shows that stock prices do not provide efficient multi-task incentives. To do that, stock prices would have to measure the value-added of the manager for each activity. But stock prices only convey information about the total value of the firm. The disadvantage of market based pay is that it may skew manager's incentives towards particular activities.[5] If one of management's activities can be the exaggeration of performance, then Goldman and Slezak (2006) show how stock based performance contracts induce CEOs to waste resources by manipulating the information transmitted to investors. Bolton et al. (2006) also take up the multi-tasking issue in a static model and ask: what if the market is inefficient so that the

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[3]We discuss Holmström and Tirole (1993) and its empirical relevance further in section 4.

[4]In an earlier paper without trading, Diamond and Verrecchia (1982) analyze a related filtering issue for the use of stock prices in CEO pay. In their model, all investors receive the *same* signal and the stock price perfectly reveals the common signal. Since the stock price conveys information about noise that is unrelated to management effort, the optimal CEO pay in Diamond and Verrecchia depends positively of final output and *negatively* on the stock price.

[5]Bushman and Injejikian (1993) combine the arguments of Kim and Suh (1993) and Paul (1992) within a single framework.

stock price no longer reflects the expected long-run fundamental value of a firm? In that case, a CEO has an incentive to wastefully increase the risk of his firm to play up the speculative component of the stock price.

The organization of the paper is as follows. Section 2 presents a static benchmark model to introduce the trade-off between useful and useless information when designing market based CEO pay. The static benchmark prepares the ground for the dynamic extension to which we turn in section 3 and show how speculators' trading horizons affect managerial pay and effort. Section 4 discusses our results in the light of existing empirical evidence. Section 5 concludes. All proofs are contained in the appendix.

2 The static benchmark: market based compensation and price informativeness

The model assumes a standard moral-hazard problem between the owners and the management of a publicly traded firm. We introduce active trading of the firm's shares in a large competitive market where speculators have heterogenous, dispersed and imperfect information about the future value of the firm. Speculators' self-interested trading leads to an aggregation of information in the stock price that may be useful for incentivizing management.

A publicly traded firm is run by a risk-averse manager (the agent) whose unobservable effort drives the expected value of the firm. A collective of risk-neutral inside owners (the principal) owns the firm. They are value oriented investors in the sense that they hold the firm's shares until the firm is liquidated.

The company stock is traded by a continuum of informed risk-averse speculators, indexed by $i \in [0, 1]$. Each speculator possesses different imperfect information about the value of the firm. Moreover, there are noise traders who trade for reasons that are not related to any information about the firm. Finally, there is a risk-neutral market making sector that ensures that the stock price will be efficient and reflects all publicly available information.

We first present a static benchmark that illustrates the trade-off between useful and useless information when designing market based CEO pay. The static case sets the stage for exploring the impact of short trading horizons in the dynamic extension of the model. The sequence of events is as follows. First, the principal hires a manager to run the firm and signs an incentive contract with him. Second, the manager exerts an unobservable effort e that determines the expected future value of the firm, $v = e + \theta$, where $\theta \sim N(0, \sigma_\theta^2)$. Third, each speculator privately receives imperfect information about the value of the firm $s_i = v + \varepsilon_i$, where ε_i are i.i.d. random variables, $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. Trading then results in a share price p. Fourth, the manager quits the firm and is paid according to his incentive contract. His income contains a fixed wage, a market based element contingent on the stock price p and a non-market based element contingent on a signal y that is available at the moment the manager quits the firm and that contains unbiased but noisy information about the future value of the firm: $y = v + \eta$, where $\eta \sim N(0, \sigma_\eta^2)$. The manager's total income I therefore is:[6]

$$I = a_0 + a_p p + a_y y \tag{1}$$

Finally, the value of the firm v realizes and the firm is liquidated at a net value π. The difference between the value of the firm v and its liquidation value π results from the cost of compensating the manager.

The manager's preferences are represented by a CARA utility function defined over income minus the (monetary) cost of effort: $U_m(e) = -\exp[-r_m(I - \frac{1}{2}e^2)]$, where r_m is the coefficient of constant absolute risk-aversion for the manager.

Owners choose an incentive contract (a_0, a_p, a_y) that maximizes their expected wealth,

$$\max_{a_0, a_p, a_y} E[v - I] \tag{2}$$

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[6]We conform to the standard practice that the contract is linear in the signals.

subject to the manager acting in his own interest,

$$e = \arg\max_{e'} E[U_m(e')] \tag{3}$$

and subject to the manager's participation constraint:

$$E[U_m(e)] \geq 0 \tag{4}$$

where we have normalized the manager's outside opportunity to zero.

The firm's shares are traded in a standard competitive noisy rational expectations market (we follow the model of Vives (1995)). A speculator i maximizes the expected CARA utility of his return from buying x_i shares of the company stock at a price p:

$$U_i(x_i) = -\exp[-rx_i(\pi - p)] \tag{5}$$

where r is a speculator's coefficient of constant absolute risk aversion.

Speculators have rational expectations, i.e. they use all information available to them. This means that they condition their trading not only on their private signal s_i but also on the publicly observable price p. A speculator's strategy therefore maps his private information s_i into a demand function $x_i(s_i, p)$.

As is standard in the literature on informed speculative trading, there are noise or uninformed traders who trade the company stock for exogenous reasons. Their demand u is assumed to be random according to $u \sim N(0, \sigma_u^2)$ and independent of all other random variables in the model. The idea is that there are factors other than information about the company that cause its stock price to vary. Examples are stochastic life cycle motives, margin calls or requirements for investors to hold certain assets in fixed proportions.

The stock price is determined by a competitive risk neutral market making sector. It

observes the aggregate limit order book, i.e. the joint demand caused by information and non-information trading,

$$L(p) = \int_0^1 x_i(s_i, p)di + u \tag{6}$$

and sets the price efficiently:

$$p = E[\pi|L(p)] \tag{7}$$

The sequence of events is summarized in figure 1.

| Owners give an incentive contract (a_0, a_p, a_y) to manager. | Manager exerts unobservable effort e. | Speculators receive information s_i. Trading in a competitive market results in stock price p . | The manager quits the firm. He is paid income I that is based on the stock price p and the non-price signal y. | Firm is liquidated for a gross value v. |

Figure 1: The timing of events

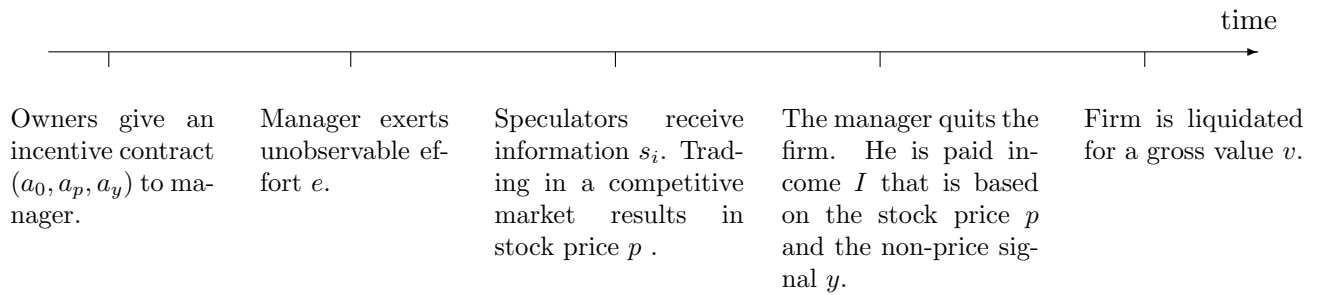## 2.1   Incentives and information

The manager's problem in equation (3) is equivalent to:

$$e = \arg\max_{e'} E[I] - \frac{r_m}{2} Var[I] - \frac{1}{2} e'^2 \tag{8}$$

The first-order condition characterizing optimal managerial effort is:

$$e = a_p \frac{\partial E[p]}{\partial e} + a_y \tag{9}$$

Since the market price will reflect the speculators' inference process about the value of the firm

v given their signals s_i, the information in the equilibrium price p and the amount of non-information based trading u, we use the general notation $E[p]$ for the moment. The condition shows that any appropriate linear combination of market based compensation a_p and non-market based compensation a_y induces the same effort level.

The cheapest way to induce effort is to minimize the income risk borne by the risk-averse manager. An optimal contract must therefore choose a_p and a_y to minimize the variance of managerial income, $Var[I]$, subject to effort being optimal for the manager (equation (9). The first-order condition for this optimization program is:

$$a_y[Var[y]\frac{\partial E[p]}{\partial e} - Cov[p,y]] = a_p[Var[p] - Cov[p,y]\frac{\partial E[p]}{\partial e}] \qquad (10)$$

We can use this condition to illustrate the information structure of the model. Since the speculators' individual errors ε_i cancel out, the market as a whole has early information about the future value of the firm and thus managerial performance, $\int s_i di = v$. The value of the firm v itself is not available for contracting since it is realized only in the future and after the manager has left the firm. If the incentive contract could include total market information, $\int s_i di$ then the contract should not use the non-market information y as this would only add extra noise to the manager's pay. Replacing p with $\int s_i di$ in equation (10) implies that $a_y = 0$. In our set-up, market information is a sufficient statistic for effort (see Holmström (1979)).[7] But an incentive contract cannot include the information of all the speculators operating in a competitive market. Instead, a contract can only include the stock price that is the outcome of decentralized self-interested trading. The competitive market as a whole therefore has useful but not directly accessible information about management performance.

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[7]This is not the case in Holmström and Tirole (1993). There the market does not take an early picture of future firm value. Instead, the market (in their case a single large insider) provides *additional* information about the value of the firm.

2.2 Trading

A speculator's demand x_i that maximizes the expected CARA utility in (5) is given by the following standard condition:

$$x_i(s_i, p) = \frac{E[\pi - p|s_i, p]}{rVar[\pi - p|s_i, p]} \tag{11}$$

We follow Holmström and Tirole (1993) and normalize the price and the incentive contract in order to separate the trading and the incentive problem. The manager is paid $a_0 + a_p p$ in cash and the amount $a_y y$ is paid in shares transferred from long-term inside owners to the manager. This accounting convention leaves payoffs unchanged and the net liquidation value of the firm is $\pi = v - a_0 - a_p p$. The fraction of shares α that must be transferred is given by $a_y y = \alpha E[v - a_0 - a_p p|y, p]$ since this is the fair price given public information.

Letting $\hat{p}$ be the normalized share price[8]

$$\hat{p} = a_0 + (1 + a_p)p \tag{12}$$

we can write the manager's income as follows:

Proposition 1 *Managerial income is linear in the normalized price $\hat{p}$ and the non-price signal y.*

$$I = \hat{a}_0 + \hat{a}_{\hat{p}}\hat{p} + \hat{a}_y y \tag{13}$$

where $\hat{a}_0 = \frac{(1-\alpha)a_p a_0}{1+a_p}, \hat{a}_y = \alpha\frac{\tau_\eta}{\tau_\eta + \tau}$ *and* $\hat{a}_{\hat{p}} = 1 - \hat{a}_y$.

The efficient pricing of shares (7) becomes $\hat{p} = E[v|L(\hat{p})]$ and a speculator's demand (11) now is:

$$x_i(s_i, \hat{p}) = \frac{E[v|s_i, \hat{p}] - \hat{p}}{rVar[v|s_i, \hat{p}]} \tag{14}$$

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[8]The prices p and $\hat{p}$ are informationally equivalent.

The standard linear-normal framework admits linear equilibria so we write speculators' demand as:

$$x_i(s_i, \hat{p}) = \beta s_i + f(\hat{p}) \tag{15}$$

where β is the trading intensity of an informed trader on his private information and $f(\hat{p})$ is a linear function of the price.

The aggregate limit order book then is:

$$
\begin{aligned}
L(\hat{p}) = \int_0^1 x_i(s_i, \hat{p}) di + u &= \beta(e + \theta) + u + f(\hat{p}) \\
&= z + f(\hat{p})
\end{aligned}
$$

where $z = \beta(e + \theta) + u$ is the part of the aggregate limit order book that is informative about the value of the firm v. The price setting condition $\hat{p} = E[v|L(\hat{p})]$ can therefore be written as $\hat{p} = E[v|z]$. The following proposition shows the equilibrium price $\hat{p}$ and the trading intensity β:

Proposition 2 *The equilibrium price $\hat{p} = E[v|z]$ is given by:*

$$\hat{p} = (1 - \lambda\beta)e^* + \lambda\beta(e + \theta) + \lambda u \tag{16}$$

where e^* is the hypothesized equilibrium effort, e is the actual effort and

$$
\begin{aligned}
\lambda &= \frac{\beta\tau_u}{\tau} \tag{17} \\
\beta &= \frac{\tau_\varepsilon}{r} \tag{18}
\end{aligned}
$$

where $\tau_j = 1/\sigma_j^2$ denotes the precision of random variable j and $\tau = Var[v|\hat{p}]^{-1} = \beta^2\tau_u + \tau_\theta$ is the informativeness of the price.

In a rational expectations equilibrium the actual effort e and the hypothesized equilibrium

effort e^* must coincide. In equilibrium the price therefore is

$$\hat{p} = e^* + \lambda\beta\theta + \lambda u \tag{19}$$

The share price is affected by two random shocks, one that is useful for incentive contracting while the other is not. The key issue is that they are inextricably linked. One shock is due to information trading by speculators. It provides information about θ, the shock that garbles the impact of managerial effort on firm value. This information is useful for incentive contracting as it allows to give better incentives to risk-averse management. The other shock is due to non-information trading u. It adds extra noise to the stock price that is unrelated to the moral hazard problem and that should ideally not affect managerial incentives. But without non-information trading, there will be no information trading: if $\sigma_u^2 = 0$ then $\lambda\beta = 1$ and the price is $\hat{p} = v = e + \theta$. The price then provides more accurate information about the value of the firm than a speculator's signal s_i. Speculators would disregard their own signals and only use the information conveyed by the price. But this begs the question of how information can flow into the price in the first place (this is a version of the "no-trade" or "no speculation" results of Grossman and Stiglitz (1980) and Milgrom and Stokey (1982)).

2.3   Market based compensation and price informativeness

Lambert and Larcker (1987) show that in order to carry out cross-sectional analyses of the attributes of compensation contracts, it is preferable to focus on the relative weights placed on performance measures in order to reduce the confounding factors of CEO risk aversion, their outside opportunities or disutility of effort. We therefore present the following result:

Proposition 3 *The ratio of market based to non-market based compensation is given by:*

$$\frac{\hat{a}_{\hat{p}}}{\hat{a}_y} = \frac{\beta}{\lambda}\frac{\tau_u}{\tau_\eta} = \frac{\tau}{\tau_\eta} \tag{20}$$

CEO effort is given by:

$$e = [1 + r(\tau_\theta^{-1} + (\tau_\eta + \tau - \tau_\theta)^{-1})]^{-1} \tag{21}$$

Proposition 3 shows that the ratio of market based compensation relative to non-market based compensation is given by the ratio of price informativeness $\tau = Var[v|\hat{p}]^{-1}$ to the precision of non-market information τ_η. Equation (20) also shows that the ratio is proportional to the liquidity of the market λ^{-1} ceteris paribus.[9] A competitive stock price aggregates dispersed and heterogenous information about the firm via self-interested speculative trading, and a more liquid market allows more information based speculative trading.[10] Reacting to his incentive pay, the CEO exerts higher effort and thus increases expected firm value when the stock price is more informative (higher τ).

Using the results from proposition 2, the following corollary collects the comparative statics of the ratio of market to non-market based pay and CEO effort with respect to the parameters of the model.

Corollary 1 *The relative weight on market based pay increases if i) speculators have better information (lower σ_ε^2), ii) speculators are less risk averse (lower r), iii) the non-market information is less precise (higher σ_η^2), iv) future firm value is less volatile (lower σ_θ^2) and v) there is less noise trading (lower σ_u^2). CEO effort increases when vi) there is less noise trading, vii) a more precise non-market signal, viii) a lower volatility of final firm value, ix) speculators have better information and x) they are less risk averse.*

More non-information trading u increases the liquidity of the market λ^{-1} but it also adds noise to the stock price that is unrelated to managerial effort. The overall impact of more noise trading on market based pay and effort is negative since it reduces the informativeness of the price $\tau = Var[v|\hat{p}]^{-1}$.

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[9]Kyle (1985) introduced the inverse of the resilience of the price to order shocks, λ^{-1}, as an intuitive measure of market liquidity.

[10]This corresponds to Bagehot (1971)'s classic intuition that a market is more liquid if informed speculators trade more with each other.

The negative effect of noise trading on market based pay in our model stands in contrast to the positive effect in Holmström and Tirole (1993). They do not consider the aggregation of useful but dispersed information via the stock price. Instead they examine how noise trading motivates a single large insider to collect additional costly information about the performance of management. More noise trading allows the trader to better hide his informed trades, to make more profits and to better recoup the cost of collecting information. It is an indirect effect of noise trading that operates via the precision of speculators' information τ_ε, and, according to result i) of the corollary, more precise information for speculators leads to more market based pay.[11]

A lower variance of θ increases the relative weight the contract places on the stock price and CEO effort since it increases the liquidity of the market. A more liquid market is one that allows a better aggregation of information leading to a more informative stock price and hence more market based incentives.

3 The dynamic case: market-based compensation and speculators' horizon

Having shown that there is natural trade-off between useful and useless information when making managerial pay contingent on the firm's stock price, we now examine how shortening speculators' investment horizons affects this trade-off. A short investment horizon reduces the aggressiveness with which speculators trade on their information and makes the aggregation of dispersed information via competitive trading less efficient. The stock price will therefore take

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[11]If we allow the precision of dispersed private information to increase with the amount of noise trading to incorporate the indirect effect of Holmström and Tirole, the overall impact of noise trading on market-based pay is still negative as long as there are no increasing returns to scale in information collection that are stronger than inversely proportional. Suppose that $\tau_\varepsilon = \tau_\varepsilon(\tau_u)$ with $\tau_\varepsilon' < 0$. Then $\frac{\partial(\hat{a}_p)/\hat{a}_y}{\partial \tau_u} > 0$ iff $\tau_\varepsilon'/\tau_\varepsilon > -\tau_u^{-1}$. This holds as long as $\tau_\varepsilon(\tau_u)$ is less steeply curved than $\tau_\varepsilon(\tau_u) = \tau_u^{-1}$. Our direct negative effect of noise trading on the informativeness of the price is therefore likely to outweigh the indirect positive effect of Holmström and Tirole that operates via the precision of speculators' information .

a less precise image of future firm value making it more costly for the owners of the firm to give market based incentives to management.

3.1 Adding a round of trading

We extend the static benchmark by adding an extra round of trading before the manager quits the firm (see figure 2).

time

| Owners give an incentive contract $(a_0, a_{p_1}, a_{p_2}, a_y)$ to the manager. | Manager exerts unobservable effort e. | Speculators receive information s_i. Trading in a competitive market results in stock price p_1. | The manager quits the firm. He is paid income I that is based on the stock prices p_1 and p_2, and the non-price signal y. | Firm is liquidated for a gross value v. |

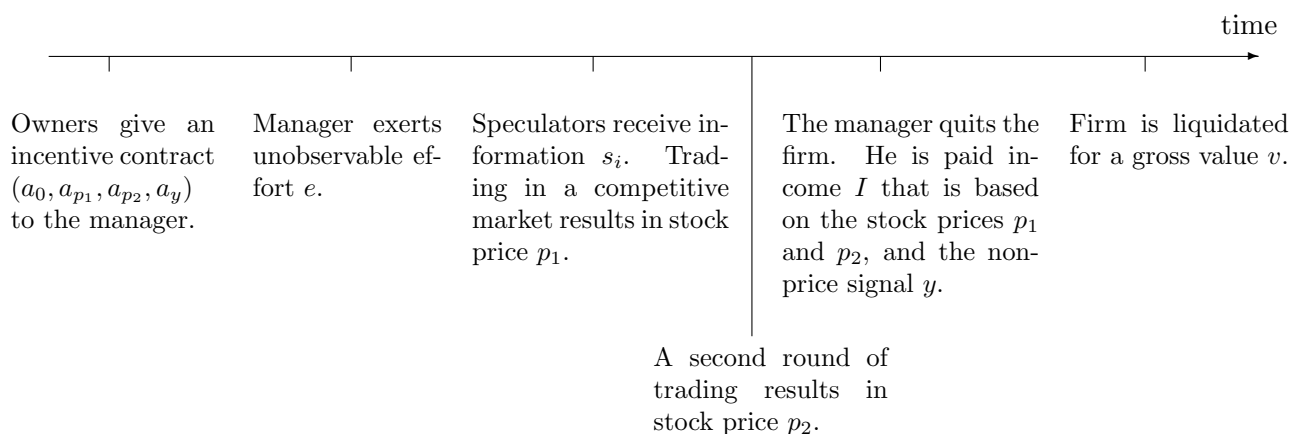A second round of trading results in stock price p_2.

Figure 2: The timing of events

Managerial pay income is now given by

$$I = a_0 + a_1 p_1 + a_2 p_2 + a_y y$$

and noise trading will be i.i.d. across the two periods, $t = 1, 2$.

$$u_t \sim N(0, \sigma_u^2)$$

Let

$$L_1 = \int_0^1 x_{i1} di + u_1$$

be the order flow the market makers observe in the first trading round when an informed trader

i takes the position x_{i1} in the first trading period. Analogously,

$$L_2 = \int_0^1 x_{i2} di - \int_0^1 x_{i1} di + u_2$$

is the net order flow in the second trading round.

As in the static case, a competitive risk-neutral market making sector observing the aggregate limit order book ensures efficient pricing:

$$p_1 = E[\pi|L_1] \tag{22}$$

$$p_2 = E[\pi|L_1, L_2] \tag{23}$$

As before, we focus on linear symmetric equilibria in which a speculator's demand x_{it} is linear in prices p_t and his signal s_i, and we write the informative part of the order book as $z_1 = \beta_1 v + u_1$ and $z_2 = (\beta_2 - \beta_1)v + u_2$. We again normalize prices

$$\hat{p}_1 = a_0 + (1 + a_1 + a_2)p_1 \tag{24}$$

$$\hat{p}_2 = a_0 + a_1 p_1 + (1 + a_2)p_2 \tag{25}$$

to rewrite (22) and (23) as:[12]

$$\hat{p}_1 = E[v|z_1] \tag{26}$$

$$\hat{p}_2 = E[v|z_1, z_2] \tag{27}$$

The next proposition characterizes the pricing functions in the dynamic case.

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[12]As in the static case, we follow Holmström and Tirole (1993). The manager is paid his fixed and market based pay in cash and the remainder is paid by transferring shares from inside owners. This accounting convention yields a net liquidation value of the firm $\pi = v - a_0 - a_{p_1} p_1 - a_{p_2} p_2$.

Proposition 4 *The first and second period stock price are given by:*

$$\hat{p}_1 = (1 - \lambda_1\beta_1)e^* + \lambda_1\beta_1(e + \theta) + \lambda_1 u_1$$

$$\hat{p}_2 = (1 - \frac{\tau_1}{\tau_2}\lambda_1\beta_1 - \lambda_2(\beta_2 - \beta_1))e^* + (\frac{\tau_1}{\tau_2}\lambda_1\beta_1 + \lambda_2(\beta_2 - \beta_1))(e + \theta) + \frac{\tau_1}{\tau_2}\lambda_1 u_1 + \lambda_2 u_2$$

where e^* is the hypothesized equilibrium effort, e is the actual effort, $\tau_1 = Var[v|z_1]^{-1} = \tau_\theta + \beta_1^2\tau_u$, $\tau_2 = Var[v|z_1, z_2]^{-1} = \tau_1 + (\beta_2 - \beta_1)^2\tau_u$ and

$$\lambda_1 = \beta_1\frac{\tau_u}{\tau_1}$$

$$\lambda_2 = (\beta_2 - \beta_1)\frac{\tau_u}{\tau_2}$$

The speculators' trading aggressiveness β_t that determines the liquidity of the market λ_t^{-1} in each period depends on the trading horizon. We consider two cases. First, we present the benchmark case of speculators with long investment horizons who can trade in both periods and show that this case will be identical to the static model above. We therefore confirm that adding another trading round by itself, i.e. without shortening trading horizons, is innocuous since a competitive market with long investing horizons incorporates information into the stock price immediately. We then consider the case of two generations of short-term or myopic speculators who trade only for one period.

A speculator with a long investing horizon maximizes the expected utility of wealth from gains in both trading periods:

$$U_i(x_{i1}, x_{i2}) = -\exp[-r(x_{i1}(p_2 - p_1) + x_{i2}(\pi - p_2))]$$

The following proposition describes such a speculator's trading aggressiveness:

Proposition 5 *With long investing horizons, speculators' trading aggressiveness is constant and identical to the static case: $\beta_1 = \beta_2 = \frac{\tau_\varepsilon}{r}$.*

There is no information trading in the second period so that speculators with long investment horizons pursue a buy-and-hold strategy. Consequently, any non-information trading in the second period u_2 is absorbed by the competitive risk-neutral market making sector: the market is infinitely liquid in the second period, $\lambda_2 = (\beta_2 - \beta_1)\tau_u/\tau_2 = 0$, and the first and second period price are the same, $\hat{p}_1 = \hat{p}_2$. The manager's contract and effort are the same as in propositions 3 where $\tau = \tau_1 = \tau_2 = \beta^2\tau + \tau_\theta$ and $\beta = \beta_1 = \beta_2 = \tau_\varepsilon/r$.

3.2 The effect of speculators' short-termism on market based compensation

Speculators with short trading horizons maximize

$$E[-\exp(-r(x_{i1}(p_2 - p_1)))|s_i, p_1] \tag{28}$$

in the first period and

$$E[-\exp(-r(x_{i2}(\pi - p_2)))|s_i, p_1, p_2] \tag{29}$$

in the second period.

We assume that speculators in the second period have access to all the information of the first period. The set-up therefore represents either a situation where speculators live for two periods but undertake successive myopic one-period investments, or a situation where a new generation of short-lived speculators enters the market in the second period inheriting the knowledge of the previous generation.

Proposition 6 *With short investing horizons, speculators' trading aggressiveness increases over time:* $\beta_1 = \frac{\tau_\varepsilon \tau_2}{r(\tau_\varepsilon + \tau_2)} < \beta_2 = \frac{\tau_\varepsilon}{r}$.

Speculators with a short investment horizon hold back in the first period because they have information about the final value of the firm v but cannot hold the asset until this value realizes.

Instead they need to close their position early at a price $\hat{p}_2$, which is only an imperfect estimate of future firm value v. Speculators have therefore fewer incentives to trade aggressively on their information in the first period.

A first consequence of less aggressive information trading in the first period is that an optimal incentive contract is not contingent on the first period stock price.

Proposition 7 *Optimal CEO pay will not be based on the stock price in the first period.*

Since the optimal incentive contract for the CEO does not include the first period price $\hat{p}_1$ as a performance measure, the analysis of managerial incentives parallels the one carried out in the static case. The result in propositions 3 carries over with the informativeness of the stock price now being $\tau_2 = \tau_\theta + (\beta_1^2 + (\beta_2 - \beta_1)^2)\tau_u$, and speculators' trading aggressiveness being $\beta_1 = \tau_\varepsilon \tau_2 / (r(\tau_\varepsilon + \tau_2))$ and $\beta_2 = \tau_\varepsilon / r$ (propositions 4 and 6).

The next proposition summarizes the impact of short-termism in the stock market via market-based pay on CEO effort.

Proposition 8 *When speculators have short trading horizons then CEO pay is less contingent on the stock price and the CEO exerts less effort than when speculators have long trading horizons.*

Speculators with shorter trading horizons trade less aggressively on their information. This reduces the information content of the stock price, worsens the trade-off between useful and useless information and makes it more costly to provide market based incentives to management, which in turn leads to less managerial effort and ultimately to lower expected firm value.

4 Discussion and empirical implications

Recent empirical research by Garvey and Swan (2002) and Kang and Liu (2005) establishes a significant cross-sectional link between the extent of stock-based CEO pay and trading conditions in the market market for a sample of publicly traded US corporations that is consistent

with our analysis. Kang and Liu (2005) find that CEO pay is more sensitive to changes in shareholder value when more information is impounded into stock prices. They measure the informativeness of the stock price using the PIN of Easley et al. (1997) (see for example Chen et al. (2006)) for an application of the PIN as a measure of stock price informativeness in a different context) and also using the dispersion and error of analysts' forecasts. Similarly, Garvey and Swan (2002) find a negative link between both the bid-ask spread and the ratio of turnover to market capitalization and the extent of market-based CEO pay. They argue that the impact of these two measures of market liquidity on CEO pay is at least as large as the effect of traditional cross-sectional determinants such as size, risk or industry.[13]

Our paper provides a suitable theoretical background for these results. Proposition 3 shows that a more informative stock price and a more liquid market lead to more market based pay ceteris paribus. It is more difficult to reconcile these empirical findings with the analysis of Holmström and Tirole (1993). First, they focus on the role of a single large insider in monitoring management indirectly via strategic trading against uninformed traders. Hartzell and Starks (2003) find evidence against such indirect monitoring since more institutional investor concentration leads to subsequent changes in CEO pay but not vice versa as one would expect if investor concentration and incentive compensation arose simultaneously and endogenously.[14] Large insiders such as institutional investors appear to influence CEO pay more directly, e.g. through shareholder activism. Second, Holmström and Tirole (1993) focus only on role of noise traders and their positive impact on market-based CEO pay. Both Garvey and Swan (2002) and Kang and Liu (2005) however examine the general, informal

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[13]Schipper and Smith (1986) provide indirect evidence for the positive link between liquidity and market based CEO pay by examining carve-outs. After selling a subsidiary to the public equity market, management typically receives compensation contracts that include the new company's stock.

[14]An information monopolist trading strategically in the US stock market may also run the danger of violating section 10(b) of the Securities Exchange Act. Courts have interpreted this section in conjunction with Rule 10b-5 to prohibit insider trading by a corporate "outsider" (see http://www.sec.gov/answers/insider.htm for more information.) Moreover, Laffont and Maskin (1990) show that a single large trader with private information typically finds it more profitable to conceal his private information and to trade in such a way that the price does not reflect his private information at all.

but more testable intuition that a more informative stock price should lead to more market based pay. Our analysis shows that the link between noise trading, price informativeness and market based pay is not straight-forward. While a more informative price always leads to more market based pay ceteris paribus (proposition 3), this is not the case for the amount of noise trading (corollary 1). Third, an issue of analyzing market-based CEO pay in a model with a single informed trader is its robustness to the threat of collusion between management and the outside source of information about management performance. This issue does not arise in our analysis since information is highly dispersed across a competitive market.

Our model may also provide a new perspective on the debate on the relationship between risk and incentives (see Prendergast (2002)). Core et al. (2003b) for example find that counter to the standard predictions of agency models, the variation in the relative weight on price and non-price measures in total CEO compensation is an increasing function of their relative variances. One possible explanation is based on the observation that a more informative stock price is also more volatile.[15] Adding trading to a standard agency problem, as in our paper, can generate a positive relationship between relative incentives and the volatility of price based performance measures via the informativeness of the stock price.

According to proposition 8, shorter investment horizons of traders should lead to less market based CEO pay and lower CEO effort. To our knowledge, this prediction has not yet been tested directly. Kang and Liu (2005) however show in a robustness test a sharp increase in the positive link between measures of stock price informativeness and the sensitivity of market based CEO pay to shareholder value after the stock market bubble burst in 2000. If traders acted more myopically in the run up to the stock market bubble then our model provides a possible rationale for the increase. Short-termism in the market made stock prices less good at aggregating dispersed information in the years prior to 2000 so that the market took a blurrier picture of future firm performance weakening market based incentives in equilibrium.[16]

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[15]This can be seen from $Var[\hat{p}] = Var[E[v|\hat{p}]] = Var[v] - Var[v|\hat{p}]$.

[16]Note that our analysis and the cited evidence examine the composition of CEO pay but not its level.

The role of the stock market in our model is to provide information about the performance of firm management. Decentralized speculative trading aggregates dispersed information about the firm into its stock price. More speculative information trading therefore promotes economic efficiency via more efficient incentive contracting. Using industry level data across 38 countries, Tadesse (2004) finds that liquid stock markets promote economic performance via market based governance. Gupta (2005) identifies the positive role of financial markets as information producers on firm performance. She studies partial privatization programs in which government sells only non-controlling shares to the public. This approach allows to eliminate the confounding effect of direct shareholder control on the relationship between stock market trading and firm performance.

5   Conclusion

This paper presents a model where the benefit of market based CEO pay is that the stock market aggregates useful but dispersed private information about past managerial performance via self-interested trading. But speculators only trade on their private information if there is non-information trading, e.g. trade due to margin calls or life-cycle motives. Non-information or noise trading is unrelated to management's action and should therefore not affect their incentive schemes according to Holmström's "informativeness principle". But since such noise trading makes room for information trading, it is a necessary cost of market based pay.

The balance between useful and useless information when designing market based pay is reflected in the liquidity of the market. A more liquid market allows more information trading and is therefore better at aggregating dispersed information. Short trading horizons reduce liquidity. They lower the aggressiveness with which speculators trade on their private information since the stock price reflects their information only imperfectly at the time they need to close their positions. Short-termism makes the stock price less informative about management performance and weakens market based incentives.

A natural extension of our set-up is to have a richer moral-hazard problem, perhaps one in which management performs several different tasks. Such a model could formalize the intuition of Froot et al. (1992a) that tying CEO pay to stock prices when traders have short horizons induces CEOs to focus on short-term earnings at the expense of long-term corporate strategy.

References

Allen, F. and Gorton, G. (1993). Churning bubbles. *Review of Economic Studies*, Vol. 60, No. 4, pp. 813-836.

Bagehot, W. (1971). The only game in town. *Financial Analysts Journal*, Vol. 22, pp. 12-14.

Bebchuk, L. and Fried, J. (2004). *Pay without performance.*, Harvard University Press.

Bertrand, M. and Mullainathan, S. (2001). Are CEOs rewarded for luck? The ones without principals are. *Quarterly Journal of Economics*, Vol. 116, No. 3, pp. 901-932.

Bolton, P., Scheinkman, J. and W. Xiong, (2006). Executive compensation and short-termist behaviour in speculative markets. *Review of Economic Studies, forthcoming.*

Bushman, R. and Indjejikan, R. (1993). Accounting income, stock price and managerial compensation. *Journal of Accounting and Economics*, Vol. 16, No. 1-3, pp. 3-23.

Chen, Q., Goldstein, I. and W. Jiang, (2006). Price informativeness and investment sensitivity to stock price. *Review of Financial Studies, forthcoming*

Core, J., Guay, W. and D. Larcker, (2003a). Executive equity compensation and incentives: a survey.*Federal Reserve Bank of New York Economic Policy Review*, Vol. 9, pp. 27-50.

Core, J., Guay, W. and R. Verrecchia (2003b). Price vs. non-price performance measures in optimal CEO compensation contracts.*The Accounting Review*, Vol. 78, pp. 957-981.

Danthine, J.-P. and Moresi, S. (1993) Volatility, information and noise trading. *European Economic Review*, Vol. 37, No. 5, pp. 961-982.

Diamond, D. and Verrecchia, R. (1982). Optimal managerial contracts and equilibrium security prices. *Journal of Finance*, Vol. 37, No. 2, pp. 275-287.

Dow, J. and G. Gorton (1994). Arbitrage chains. *Journal of Finance*, Vol. 49, No. 3, pp. 819-849.

Dow, J. and G. Gorton (1997). Stock market efficient and economic efficiency: Is there a connection? *Journal of Finance*, Vol. 52, No. 3, pp. 1087-1129.

Easley, D., Kiefer, N. and M. O'Hara (1997). One day in the life of a very common stock. *Review of Financial Studies*, Vol. 10, No. 3, pp. 805-835.

Froot, K., Perold, A. and J. Stein (1992a). Shareholder trading practices and corporate investment horizons. *Journal of Applied Corporate Finance*, Vol. 5, No. 3, pp. 42-58.

Froot, K., Scharfstein, D. and J. Stein (1992b). Herd on the street: informational inefficiencies in a market with short-term speculation. *Journal of Finance*, Vol. 47, Vol. 4, pp. 1461-1484.

Garvey, G. and P. Swan, (2002). What can market microstructure contribute to explaining executive incentive pay? Liquidity and the use of stock-based compensation. *University of New South Wales Working Paper.*

Goldman, E. and S. Slezak, (2006). An equilibrium model of incentive contracts in the presence of information manipulation. *Journal of Financial Economics, forthcoming*

Grossman, S. and Stiglitz, J. (1980). On the impossibility of informationally efficient markets. *American Economic Review*, Vol. 70, pp. 393-408.

Gupta, N. (2005). Partial privatization and firm performance. *Journal of Finance*, Vol. 60, No. 2, pp. 987-1015.

Hartzell, J. and L. Starks, (2003). Institutional investors and executive compensation. *Journal of Finance*, Vol. 58, No. 6, pp. 2351-2374.

Holmström, B. (1979). Moral hazard and observability. *Bell Journal of Economics*, Vol. 10, pp. 74-91.

Holmström, B. and J. Tirole (1993). Market liquidity and performance monitoring. *Journal of Political Economy*, Vol. 101, No. 4, pp. 678-709.

Kang, Q. and Liu, Q. (2005). Stock market information production and CEO incentives. *University of Hong Kong Working Paper*

Kim, O. and Suh, Y. (1993). Incentive efficiency of compensation based on accounting and market performance. *Journal of Accounting and Economics*, Vol. 16, pp. 25-53.

Kyle, A. (1985). Continuous auctions and insider trading. *Econometrica*, Vol. 53, No. 6, pp. 1315-1336.

Laffont, J.-J. and Maskin, E. (1990). The efficent market hupothesis and isider trading on the stock market. *Journal of Political Economy*, Vol. 98, No. 1, pp. 70-93.

Lambert, R. and Larcker, D. (1987). An analysis of the use of accounting and market measures of performance in executive compensation contracts. *Journal of Accounting Research*, Vol. 25, pp. 85-125.

Milgrom, R. and Stokey, N. (1982) Information, trade and common knowledge. *Journal of Economic Theory*, Vol. 26, No. 1, pp.17-27.

Murphy, K. (1999). Executive compensation, in: Ashenfelter, O. and D. Card (eds). *Handbook of Labour Economics*, North-Holland.

Paul, J. (1992). On the efficiency of stock-based compensation. *Review of Financial Studies*, Vol. 5, No. 3, pp. 471-502.

Prendergast, C. (2002). The tenuous trade-off between risk and incentives. *Journal of Political Economy*, Vol. 110, No. 5, pp. 1071-1102.

Schipper, K. and Smith, A. (1986). A comparison of equity carve-outs and seasonal equity offerings: Share price effects and corporate restructuring. *Journal of Financial Economics*, Vol. 15, pp. 153-186.

Shleifer, A. and Vishny, R. (1990). Equilibrium short horizons of investors and firms. *American Economic Review*, Vol. 80, No. 2, pp. 148-153.

Tadesse, S. (2004). The allocation and monitoring role of capital markets: theory and international evidence. *Journal of Financial and Quantitative Analysis*, Vol. 39, No. 4, pp. 701-730.

Vives, X. (1995). Short-term investment and the informational efficiency of the market. *Review of Financial Studies*, Vol. 8, No. 1, pp. 125-160.

A   Appendix

In order to calculate the conditional distribution, we use the following standard result for normally distributed variables:

Result 1 *Let Y_i be a $(n_i \times 1)$ vector with mean μ_i, i=1,2, and variance-covariance matrices Σ_{ij}, then*

$$Y_2|Y_1 = y_1 \sim N([\mu_2 + \Sigma_{21}\Sigma_{11}^{-1}(y_1 - \mu_1)], [\Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}])$$

We also make use of the following technical result from Danthine and Moresi (1993):

Result 2 *Let $z \sim N(0, \sigma^2)$ and $W = az^2 + bz + c$ then*

$$E[-\exp(-rW)] = -\frac{1}{\sigma\sqrt{\frac{1}{\sigma^2} + 2ra}} \exp\left(-r\left(c - \frac{rb^2}{2(\frac{1}{\sigma^2} + 2ra)}\right)\right)$$

When $a = 0$ the result describes the familiar certainty equivalent of normally distributed wealth W with mean c and variance $b^2\sigma^2$.

Proof of proposition 1

It is easier to calculate the conditional expectation and variance using the following information equivalent of price $\hat{p}$,

$$\hat{\hat{p}} = \frac{\hat{p} - (1 - \lambda\beta)e^*}{\lambda\beta} = e + \theta + \frac{1}{\beta}u \tag{30}$$

Using result (1) we have

$$
\begin{aligned}
E[v|y,\hat{\hat{p}}] &= e^* + (\sigma_\theta^2, \sigma_\theta^2) \begin{pmatrix} \sigma_\theta^2 + \sigma_\eta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \frac{1}{\beta^2\sigma_u^2} \end{pmatrix}^{-1} \left(\begin{pmatrix} y \\ \hat{\hat{p}} \end{pmatrix} - \begin{pmatrix} e^* \\ e^* \end{pmatrix} \right) \\
&= \frac{\beta^2\sigma_\eta^2\sigma_\theta^2\hat{\hat{p}} + \sigma_u^2\sigma_\theta^2 y + \sigma_u^2\sigma_\eta^2 e^*}{\beta^2\sigma_\eta^2\sigma_\theta^2 + \sigma_u^2(\sigma_\eta^2 + \sigma_\theta^2)}
\end{aligned}
$$

Substituting back and using $\tau = \beta^2 \tau_u + \tau_\theta$ we obtain:

$$E[v|y, \hat{p}] = \frac{\tau_\eta y + \tau \hat{p}}{\tau_\eta + \tau}$$

We can therefore rewrite managerial income as

$$
\begin{aligned}
I &= a_0 + a_p p + a_y y \\
&= a_0 + a_p p + \alpha E[v - a_0 - a_p p | y, p] \\
&= (1 - \alpha)a_0 + (1 - \alpha)\frac{\hat{p} - a_0}{1 + a_p} + \alpha E[v|y, \hat{p}] \\
&= \frac{(1 - \alpha)a_p a_0}{1 + a_p} + \left(\frac{(1 - \alpha)a_p}{1 + a_p} + \frac{\alpha \tau}{\tau_\eta + \tau} \right) \hat{p} + \frac{\alpha \tau_\eta}{\tau_\eta + \tau} y
\end{aligned}
$$

Proof of proposition 2

Let $Y_1 = z$ with mean $\mu_1 = \beta e^*$ and $\Sigma_{11} = Var(z) = \beta^2 \sigma_\theta^2 + \sigma_u^2$, $Y_2 = v$ with mean $\mu_2 = e^*$ and $\Sigma_{22} = \sigma_\theta^2 + \sigma_\eta^2$, and $\Sigma_{21} = \Sigma_{12} = Cov[v, z] = \beta \sigma_\theta^2$. Hence

$$E[v|z] = e^* + \frac{\beta \sigma_\theta^2}{\beta^2 \sigma_\theta^2 + \sigma_u^2}(z - \beta e^*)$$

Letting $\lambda = \frac{\beta \sigma_\theta^2}{\beta^2 \sigma_\theta^2 + \sigma_u^2}$ and substituting for $z = \beta(e + \theta) + u$ gives the result for $\hat{p} = E[v|z]$.

To solve for β we need to characterize the distribution of $v|s_i, \hat{p}$. Using again result 1, let $Y_1 = (s_i, \hat{\hat{p}})$ with mean $\mu_1 = (e^*, e^*)$ and $Y_2 = v$ with mean $\mu_2 = e^*$, where we use again the information equivalent $\hat{\hat{p}}$ instead of $\hat{p}$. The covariance-variance matrices are:

$$
\Sigma_{11} = \begin{pmatrix} \sigma_\theta^2 + \sigma_\varepsilon^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \frac{1}{\beta^2 \sigma_u^2} \end{pmatrix}
$$

and $\Sigma_{22} = \sigma_\theta^2 + \sigma_\eta^2$, and $\Sigma_{21} = \Sigma_{12} = (Cov[s_i, v], Cov[\hat{\hat{p}}, v]) = (\sigma_\theta^2, \sigma_\theta^2)$.

Hence

$$
\begin{aligned}
E[v|s_i,\hat{\hat{p}}] &= e^* + (\sigma_\theta^2, \sigma_\theta^2) \begin{pmatrix} \sigma_\theta^2 + \sigma_\varepsilon^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \frac{1}{\beta^2 \sigma_u^2} \end{pmatrix}^{-1} \left(\begin{pmatrix} s_i \\ \hat{\hat{p}} \end{pmatrix} - \begin{pmatrix} e^* \\ e^* \end{pmatrix} \right) \\
&= \frac{\beta^2 \sigma_\varepsilon^2 \sigma_\theta^2 \hat{\hat{p}} + \sigma_u^2 \sigma_\theta^2 s_i + \sigma_u^2 \sigma_\varepsilon^2 e^*}{\beta^2 \sigma_\varepsilon^2 \sigma_\theta^2 + \sigma_u^2 (\sigma_\varepsilon^2 + \sigma_\theta^2)}
\end{aligned}
$$

Substituting $\hat{p}$ for $\hat{\hat{p}}$ and writing the expression in terms of precision $\tau_j = 1/\sigma_j^2$, we obtain:

$$
E[v|s_i,\hat{p}] = \frac{\tau_\varepsilon s_i + (\beta^2 \tau_u + \tau_\theta)\hat{p}}{\tau_\varepsilon + (\beta^2 \tau_u + \tau_\theta)}
$$

Next we need to calculate

$$
\begin{aligned}
Var[v|s_i,\hat{\hat{p}}] &= \sigma_\theta^2 - (\sigma_\theta^2, \sigma_\theta^2) \begin{pmatrix} \sigma_\theta^2 + \sigma_\varepsilon^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \frac{1}{\beta^2 \sigma_u^2} \end{pmatrix}^{-1} \begin{pmatrix} \sigma_\theta^2 \\ \sigma_\theta^2 \end{pmatrix} \\
&= \frac{1}{\tau_\varepsilon + (\beta^2 \tau_u + \tau_\theta)}
\end{aligned}
$$

Last, substituting $E[v|s_i,\hat{p}]$ and $Var[v|s_i,\hat{\hat{p}}] = Var[v|s_i,\hat{p}]$ into (11) yields

$$
x_i(s_i,\hat{p}) = \frac{\tau_\varepsilon}{r}(s_i - \hat{p})
$$

This means that $\beta = \tau_\varepsilon/r$.

Proof of proposition 3

We use proposition 1 and replace the contract (a_0, a_p, a_y) in (1) with $(\hat{a}_0, \hat{a}_p, \hat{a}_y)$ and the stock price p with $\hat{p}$ (equation (12)). The optimal contract still has to satisfy (10), but now with the normalized weights $\hat{a}_p$ and $\hat{a}_y$ and the normalized price $\hat{p}$:

$$
\hat{a}_y [Var[y]\frac{\partial E[\hat{p}]}{\partial e} - Cov[\hat{p}, y]] = \hat{a}_{\hat{p}} [Var[\hat{p}] - Cov[\hat{p}, y]\frac{\partial E[\hat{p}]}{\partial e}]
$$

Using proposition 2 to substitute for $\hat{p}$ and rearranging yields equation (20). To derive the expression for optimal CEO effort we need to calculate the absolute weights $(\hat{a}_{\hat{p}}, \hat{a}_y)$ the contract places on the stock price and the non-price signal.

Since the manager's participation constraint (4) will be binding at the optimum, the optimal managerial contract $(\hat{a}_0, \hat{a}_{\hat{p}}, \hat{a}_y)$ maximizes

$$E[v] - \frac{r_m}{2} Var[I] - \frac{1}{2}e^2$$

subject to managerial effort being optimal:

$$e = \hat{a}_{\hat{p}} \lambda \beta + \hat{a}_y \tag{31}$$

Substituting for e, v, I, taking first-order conditions with respect to $\hat{a}_{\hat{p}}$ and $\hat{a}_y$, and rearranging gives the absolute weights on the stock price $\hat{p}$ and the signal y.

$$\hat{a}_y = \frac{\sigma_u^2}{\sigma_u^2 + \beta^2 \sigma_\eta^2 + r((\sigma_u^2 + \beta^2 \sigma_\eta^2)\sigma_\theta^2 + \sigma_\eta^2 \sigma_u^2)}$$

$$\hat{a}_{\hat{p}} = \frac{\sigma_\eta^2(\sigma_u^2 + \beta^2 \sigma_\theta^2)}{\sigma_\theta^2 \left[\sigma_u^2 + \beta^2 \sigma_\eta^2 + r((\sigma_u^2 + \beta^2 \sigma_\eta^2)\sigma_\theta^2 + \sigma_\eta^2 \sigma_u^2) \right]}$$

Now we substitute for the absolute weights in (31) and use the definition of λ in proposition 2 to write optimal effort as

$$e = \left[1 + r(\sigma_\theta^2 + \frac{\sigma_\eta^2 \sigma_u^2}{\beta^2 \sigma_\eta^2 + \sigma_u^2}) \right]^{-1}$$

which is the expression in the proposition after writing variances as precisions and using the definition of τ in proposition 2.

Proof of proposition 4

Applying result 1 and writing expression in terms of precisions $\tau_j = 1/\sigma_j^2$:

$$E[v|z_1] = e^*(1 - \frac{\beta_1^2 \tau_u}{\beta_1^2 \tau_u + \tau_\theta}) + \frac{\beta_1 \tau_u}{\beta_1^2 \tau_u + \tau_\theta} z_1$$

Letting $\lambda_1 = \frac{\beta_1 \tau_u}{\beta_1^2 \tau_u + \tau_\theta}$, substituting $z_1 = \beta_1(e + \theta) + u$ and denoting $\tau_1 = \beta_1^2 \tau_u + \tau_\theta$ gives the result for the first period price.

Applying result 1 again and using notation from above:

$$E[v|z_1, z_2] = e^*(1 - \frac{\beta_1^2 \tau_u + (\beta_2 - \beta_1)^2 \tau_u}{\beta_1^2 \tau_u + (\beta_2 - \beta_1)^2 \tau_u + \tau_\theta}) + \frac{\beta_1^2 \tau_u}{\hat{\beta}_1^2 \tau_u + (\beta_2 - \beta_1)^2 \tau_u + \tau_\theta} z_1 + \frac{(\beta_2 - \beta_1)^2 \tau_u}{\beta_1^2 \tau_u + (\beta_2 - \beta_1)^2 \tau_u + \tau_\theta} z_2$$

Letting $\lambda_2 = \frac{(\beta_2 - \beta_1)^2 \tau_u}{\hat{\beta}_1^2 \tau_u + (\beta_2 - \beta_1)^2 \tau_u + \tau_\theta}$ and $\tau_2 = \tau_1 + (\beta_2 - \beta_1)^2 \tau_u$ gives the result for the second period price.

Proof of proposition 5

We follow the proof of proposition 4.1 in Vives (1995) and adapt it to our setting. Rewriting wealth using the normalized prices defined in the proof of proposition 4:

$$U_i(x_{i1}, x_{i2}) = -exp[-r(\frac{1}{1 + a_2} x_{i1}(\hat{p}_2 - \hat{p}_1) + x_{i2}(v - \hat{p}_2))]$$

At time t=2 the speculator chooses his second period holding so that

$$x_{i2} = \underset{x'_{i2}}{\operatorname{argmax}} E[-\exp(-r x'_{i2}(v - \hat{p}_2))|s_i, \hat{p}_2]$$

since the first period wealth $\frac{1}{1 + a_2} x_{i1}(\hat{p}_2 - \hat{p}_1)$ is known at t=2 and $\hat{p}_1$ does not add information given that $\hat{p}_2$ is observed. Hence, the second period problem reduces to the static one and following the proof of proposition 1 with $v = e + \theta$ and $s_i = e + \theta + \varepsilon_i$ yields $x_{i2} = \beta_2(s_i - \hat{p}_2)$ and

$$\beta_2 = \frac{\tau_\varepsilon}{r} \tag{32}$$

Using i) the optimal second period trading strategy in (32), ii) the fact that $E[v|s_i, \hat{p}_2] = \frac{\tau_\varepsilon s_i + \tau_2 \hat{p}_2}{\tau_\varepsilon + \tau_2}$ (from result 1) and iii) result 2 with $z = v - E[v|s_i, \hat{p}_2]$ so that $z|s_i, \hat{p}_2 \sim N(0, Var[v|s_i, \hat{p}_2])$, and:

$$a = 0; \quad b = \beta_2(s_i - \hat{p}_2); \quad c = \beta_2(s_i - \hat{p}_2)^2 \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_2} \tag{33}$$

we can rewrite the utility of expected second period trading profits as:

$$E[\exp(-r\beta_2(s_i - \hat{p}_2)(v - \hat{p}_2))|s_i, \hat{p}_2] = \exp\left(-\frac{r^2}{2}\frac{\beta_2^2(s_i - \hat{p}_2)^2}{\tau_\varepsilon + \tau_2}\right) \tag{34}$$

At t=1, the speculator chooses his first period holding x_{1i} to maximise

$$E\left[E\left[-\exp\left(-r(\frac{1}{1+a_2}x_{i1}(\hat{p}_2 - \hat{p}_1) + x_{i2}(v - \hat{p}_2))\right)|s_i, \hat{p}_2\right]|s_i, \hat{p}_1\right]$$

or

$$E\left[-\exp\left(-r\frac{1}{1+a_2}x_{i1}(\hat{p}_2 - \hat{p}_1)E[x_{i2}(v - \hat{p}_2)|s_i, \hat{p}_2]\right)|s_i, \hat{p}_1\right]$$

or, using (34)

$$E\left[-\exp\left(-r(\frac{1}{1+a_2}x_{i1}(\hat{p}_2 - \hat{p}_1) + \frac{r}{2}\frac{\beta_2^2(s_i - \hat{p}_2)^2}{\tau_\varepsilon + \tau_2})\right)|s_i, \hat{p}_1\right]$$

or, using (32)

$$E\left[-\exp\left(-r(\frac{1}{1+a_2}x_{i1}(\hat{p}_2 - \hat{p}_1) + \frac{\tau_\varepsilon^2}{2r(\tau_\varepsilon + \tau_2)}(s_i - \hat{p}_2)^2)\right)|s_i, \hat{p}_1\right] \tag{35}$$

We now again invoke result 2 with $z = \hat{p}_2 - E[\hat{p}_2|s_i, \hat{p}_1]$ and:

$$a = \frac{\tau_\varepsilon^2}{2r(\tau_\varepsilon + \tau_2)}$$

$$b = x_1 - \frac{\tau_\varepsilon^2}{r(\tau_\varepsilon + \tau_2)}(s_i - E[\hat{p}_2|s_i, \hat{p}_1])$$

$$c = x_1(E[\hat{p}_2|s_i, \hat{p}_1] - \hat{p}_1) + \frac{\tau_\varepsilon^2}{2r(\tau_\varepsilon + \tau_2)}(s_i - E[\hat{p}_2|s_i, \hat{p}_1])^2$$

so that the expression in the exponent in (35) is of the form $W = c + bz + az^2$ with $z|s_i, \hat{p}_1 \sim N(0, Var[p_2|s_i, \hat{p}_1])$.

In order to derive the optimal first period holding x_{1i}, we only need to consider the part in the

exponent of result 2. Substituting for a, b and c, the first-order condition with respect to x_{i1} is:

$$E[\hat{p}_2|s_i, \hat{p}_1] - \hat{p}_1 - \frac{r x_{i1} - \frac{\tau_\varepsilon^2}{\tau_\varepsilon + \tau_2}(s_i - E[\hat{p}_2|s_i, \hat{p}_1])}{\frac{1}{Var[p_2|s_i, \hat{p}_1]} + \frac{\tau_\varepsilon^2}{\tau_\varepsilon + \tau_2}} = 0$$

or

$$r x_{i1} = (E[\hat{p}_2|s_i, \hat{p}_1] - \hat{p}_1)\left(\frac{1}{Var[p_2|s_i, \hat{p}_1]} + \frac{\tau_\varepsilon^2}{\tau_\varepsilon + \tau_2}\right) + \frac{\tau_\varepsilon^2}{\tau_\varepsilon + \tau_2}(s_i - E[\hat{p}_2|s_i, \hat{p}_1])$$

or, after some rearranging,

$$x_{i1} = \frac{\tau_\varepsilon}{\tau_2 + \tau_\varepsilon}\frac{\tau_\varepsilon}{r}(s_i - \hat{p}_1) + \frac{E[p_2|s_i, \hat{p}_1] - \hat{p}_1}{r Var[p_2|s_i, \hat{p}_1]} \tag{36}$$

Using result 1 with the pricing equations from proposition 4, one finds that:

$$E[\hat{p}_2|s_i, \hat{p}_1] = \hat{p}_1 + \frac{(\tau_2 - \tau_1)\tau_\varepsilon}{\tau_2(\tau_1 + \tau_\varepsilon)}(s_i - \hat{p}_1)$$

$$Var[\hat{p}_2|s_i, \hat{p}_1] = \frac{(\tau_2 - \tau_1)(\tau_\varepsilon + \tau_2)}{\tau_2^2(\tau_1 + \tau_\varepsilon)}$$

Substitution these expressions into (36) yields

$$x_{i1} = \frac{\tau_\varepsilon}{r}(s_1 - \hat{p}_1)$$

which means that $\beta_1 = \beta_2 = \frac{\tau_\varepsilon}{r}$.

Proof of proposition 6

The situation in the second period with a short investing horizon is identical to the one with a long horizon. Thus we know from proposition 5 that $\beta_2 = \tau_\varepsilon/r$.

Using the normalizes prices $\hat{p}_1$ and $\hat{p}_2$ we can rewrite (29) as

$$E\left[-\exp\left(-r x_{i1}\frac{1}{1 + a_2}(\hat{p}_2 - \hat{p}_1)\right)|s_i, \hat{p}_1\right]$$

Maximizing with respect to x_{i1} yields

$$\frac{1}{1+a_2}x_{i1} = \frac{E[\hat{p}_2 - \hat{p}_1 | s_i, \hat{p}_1]}{Var[\hat{p}_2 - \hat{p}_1 | s_i, \hat{p}_1]} \tag{37}$$

Using the pricing functions of proposition 4 we can write

$$E[\hat{p}_2 - \hat{p}_1 | s_i, \hat{p}_1] = \lambda_2(\beta_2 - \beta_1)E[v - \hat{p}_1 | s_i, \hat{p}_1]$$
$$Var[\hat{p}_2 - \hat{p}_1 | s_i, \hat{p}_1] = \lambda_2^2((\beta_2 - \beta_1)^2 Var[v - \hat{p}_1 | s_i, \hat{p}_1] + \sigma_u^2)$$

Result 1 allows to calculate

$$E[v|s_i, \hat{p}_1] = \frac{\tau_\varepsilon s_i + \tau_1 \hat{p}_1}{\tau_\varepsilon + \tau_1}$$
$$Var[v|s_i, \hat{p}_1] = \frac{1}{\tau_\varepsilon + \tau_1}$$

Substituting back into (38) and (38), and then into (37) using also the result for β_2 yields

$$\frac{1}{1+a_2}x_{i1} = \frac{\tau_\varepsilon \tau_2}{r(\tau_\varepsilon + \tau_2)}(s_i + \hat{p}_1)$$

so that $\beta_1 = \frac{\tau_\varepsilon \tau_2}{r(\tau_\varepsilon + \tau_2)}$.

Proof of proposition 7

As in the static case, maximizing expected net firm value (2) subject to the incentive constraint (3) and the participation constraint (4) means that the dilution free contract $(\hat{a}_1, \hat{a}_2, \hat{a}_y)$ solves

$$\min_{\hat{a}_1, \hat{a}_2, \hat{a}_y} Var[I]$$

subject to

$$e = \hat{a}_1 \frac{\partial E[\hat{p}_1]}{\partial e} + \hat{a}_2 \frac{\partial E[\hat{p}_2]}{\partial e} + \hat{a}_y$$

From the first-order conditions, $\hat{a}_1, \hat{a}_2$ and $\hat{a}_y$ must satisfy

$$\hat{a}_1 \left[\frac{\partial E[\hat{p}_2]}{\partial e} Var[\hat{p}_1] - \frac{\partial E[\hat{p}_1]}{\partial e} Cov[\hat{p}_1, \hat{p}_2] \right] + \hat{a}_2 \left[\frac{\partial E[\hat{p}_2]}{\partial e} Cov[\hat{p}_1, \hat{p}_2] - \frac{\partial E[\hat{p}_1]}{\partial e} Var[\hat{p}_2] \right]$$

$$+ \hat{a}_y \left[\frac{\partial E[\hat{p}_2]}{\partial e} Cov[\hat{p}_1, y] - \frac{\partial E[\hat{p}_1]}{\partial e} Cov[\hat{p}_2, y] \right] = 0$$

The first period stock price is therefore not included if

$$\frac{\partial E[\hat{p}_2]}{\partial e} Cov[\hat{p}_1, \hat{p}_2] = \frac{\partial E[\hat{p}_1]}{\partial e} Var[\hat{p}_2]$$

and

$$\frac{\partial E[\hat{p}_2]}{\partial e} Cov[\hat{p}_1, y] = \frac{\partial E[\hat{p}_1]}{\partial e} Cov[\hat{p}_2, y]$$

Some algebra (available on request from the authors) shows that after substituting β_1 and β_2 from proposition 6 into the pricing functions of proposition 4 the conditions hold.

Proof of proposition 8

We need to compare the informativeness of the stock price $Var[v|\hat{p}_1, \hat{p}_2]^{-1}$ with long and short investment horizons. With long investment horizons, the information content is

$$\tau_2 = \tau_\theta + \left(\frac{\tau_\varepsilon}{r} \right)^2 \tau_u$$

and with short investment horizons it is

$$\tau_2 = \tau_\theta + \left(\frac{\tau_\varepsilon}{r} \right)^2 \left[\left(\frac{\tau_2}{\tau_2 + \tau_\varepsilon} \right)^2 + \left(\frac{\tau_\varepsilon}{\tau_2 + \tau_\varepsilon} \right)^2 \right] \tau_u$$

Thus, the information content of the stock price is lower when speculators have a short investment horizon. The conclusions on the relative weight of market based pay and on CEO effort then follow directly from proposition 3.

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