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A REVIEW OF NONFUNDAMENTALNESS AND IDENTIFICATION IN STRUCTURAL VAR MODELS

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Abstract

We review, under a historical perspective, the development of the problem of nonfundamentalness of Moving Average (MA) representations of economic models. Nonfundamentalness typically arises when agents' information space is larger than the econometrician's one. Therefore it is impossible for the latter to use standard econometric techniques, as Vector AutoRegression (VAR), to estimate economic models. We restate the conditions under which it is possible to invert an MA representation in order to get an ordinary VAR and identify the shocks, which in a VAR are fundamental by construction. By reviewing the work by Lippi and Reichlin [1993] we show that nonfundamental shocks may be very different from fundamental shocks. Therefore, nonfundamental representations should not be ruled out by assumption and indeed methods to detect nonfundamentalness have been recently proposed in the literature. Moreover, Structural VAR (SVAR) can be legitimately used for assessing the validity of Dynamic Stochastic General Equilibrium models only if the representation associated with the economic model is fundamental. Factor models can be an alternative to SVAR for validation purposes as they do not have to deal with the problem of nonfundamentalness.

Keywords: Nonfundamentalness, Structural VAR, Dynamic Stochastic General Equilibrium Models, Factor Models.

JEL-classification: C32, C51, C52.

Non-technical summary

We review, under a historical perspective, the development of the problem of nonfundamentalness of Moving Average (MA) representations of economic models, starting from the work by Hansen and Sargent [1980]. Nonfundamentalness has to do with identification in Structural Vector AutoRegressions (SVARs), which are a popular tool for the empirical validation of structural models, in particular Dynamic Stochastic General Equilibrium models. In a SVAR, linear combinations of structural shocks are estimated as residuals of an unrestricted VAR and the structural shocks are then identified by rotating the VAR innovations in a suitable way, i.e. by imposing restrictions. However, if the structural model has an MA component, the VAR representation is admissible only under some conditions which may not be verified in the structural model. In particular, the MA representation is invertible in the past, i.e. the VAR representation is admissible only if no root of the determinant of the matrix of the MA is inside the unit circle. If at least one root is smaller than one in modulus, we have a problem of nonfundamentalness of the structural shocks: VAR estimation will not allow to recover them because we would need to invert the MA in the future. This is a consequence of the fact that the agents' information set is bigger that the econometrician's one. The aim of this paper is twofold. Firstly, we would like to convince the reader that ruling out nonfundamental representations by assumption is not harmless: indeed, there are many meaningful economic models which generate nonfundamental representations. We describe examples of rational expectations models, models with heterogeneous information, but also very simple models, for example a permanent income model and a trend-cycle decomposition, where nonfundamentalness arises or may arise in a very natural manner. Moreover, models with a nonfundamental structural representation might be able to explain puzzles, for example in the analysis of financial markets, which standard models are not able to account for. Secondly, once explained why nonfundamental representations cannot be ignored, we review the literature proposing how to deal with the issue of nonfundamentalness. One option is enlarging the econometrician's information set: we cannot include future observations but we can still extend the cross-section dimension. To handle the estimation problems deriving from the inclusion of many variables in the analysis, we might for example assume a factor structure in the data: indeed, it is possible to show that dynamic factor models are able to retrieve the structural shocks even when a SVAR, because of nonfundamentalness, can not. A second alternative is to estimate the nonfundamental representations associated with the VAR.

1 Introduction

Structural Vector AutoRegressions (SVARs) are a popular tool for the empirical validation of structural models, in particular Dynamic Stochastic General Equilibrium (DSGE) models. In a SVAR, linear combinations of structural shocks are estimated as residuals of an unrestricted VAR and the structural shocks are then identified by rotating the VAR innovations in a suitable way, i.e. by imposing restrictions. If different theoretical models imply the same restrictions, their predictions can be compared by evaluating how close they are to the empirical impulse responses obtained in the SVAR. However, if the structural model has a Moving Average (MA) component, the VAR representation is admissible only under some conditions which may not be verified in the structural model. If this is the case, we have a problem of nonfundamentalness of the structural shocks and VAR estimation will not allow to recover them. Therefore, the SVAR impulse responses will not be consistent with the theoretical impulse responses.

In this paper we summarize and organize existing results on nonfundamentalness in macroeconomics with a twofold objective. Firstly, we would like to convince the reader that there are many meaningful economic models which generate nonfundamental representations. We briefly describe examples of rational expectations models, models with heterogeneous information, models with control rules, but also very simple models, for example a permanent income model and a trend-cycle decomposition, where nonfundamentalness arises or may arise in a very natural manner. Once explained why nonfundamental representations cannot be ignored, we review methods to detect it and survey the literature proposing how to deal with the issue of nonfundamentalness. Basically, either we enlarge the econometrician's information set – for example, we might assume a factor structure in the data and estimate a Dynamic Factor model on a large cross-section – or we generate and estimate the nonfundamental representations associated with the VAR of interest. We review some empirical applications where nonfundamental shocks are found to be markedly different from fundamental shocks: indeed, virtually every time we estimate a SVAR we should check whether the results coming from the associated nonfundamental representations significantly differ from those obtained by means of standard techniques. This double check yields a more robust validation procedure: if fundamental and nonfundamental shocks are similar the results coming from the SVAR are endorsed, while if they are different the results coming from the SVAR are not reliable.

The point on nonfundamentalness was first made by Hansen and Sargent [1980] and Hansen and Sargent [1991] in a purely theoretical setting, while Lippi and Reichlin [1993] and Lippi and Reichlin [1994] pioneered the empirical analysis of nonfundamental representations. The debate on the usefulness of SVARs for discriminating among competing structural models has been recently brought back in the macroeconomic debate by Chari et al. [2005], Christiano et al. [2006] and Fernández-Villaverde et al. [2007]: the first paper concludes that SVARs are not suitable for model validation, the second paper argues that they are, while the third explains which is the condition a structural model has to satisfy in order for a SVAR to be consistent. The condition is precisely fundamentalness of the structural representation of the model.

The paper is structured as follows. In the next section we give the main definitions of nonfundamentalness. In section 3 we illustrate the debate between Blanchard and Quah [1989] and Lippi and Reichlin [1993] as a textbook example of how an economically meaningful model can generate nonfundamental representations. In section 4 we look at another case of nonfundamentalness generated by rational expectations, we discuss the role of nonfundamentalness in models with feedback control rules and we briefly review some examples where nonfundamental representations arise as a consequence of heterogeneous information. In Section 5 we recall Blanchard and Quah [1993] argument for nonfundamentalness in cointegrated models. In section 6 we consider briefly DSGE models and a recent method proposed by Fernández-Villaverde et al. [2007] to check for nonfundamentalness in these models. Then we deal with a different method for detecting nonfundamentalness put forward by Giannone and Reichlin [2006] and based on Granger causality. In section 7 the Dynamic Factor model is proposed as an alternative tool for identification. First we introduce the model as a consequence of DSGE models with measurement errors and then we show how to deal with nonfundamentalness in this case. Section 8 concludes and suggests developments for future research on nonfundamental representations.

2 Nonfundamentalness

Consider an N-dimensional covariance stationary zero-mean vector stochastic process x_t of observable variables, driven by a q-dimensional unobservable vector process u_t of structural (i.e. with economic meaning) shocks. We can always write

$$x_t = C(L)u_t \ , \tag{1}$$

where $C(L) = \sum_{k=0}^{\infty} C_k L^k$ is a one-sided polynomial in the lag operator L, in principle of infinite order. The shocks are orthogonal white noises: $u_t \sim \text{w.n.}(0, \Gamma_0^u)$, with Γ_0^u diagonal. In all what follows we assume that x_t has rational spectral density and therefore the entries of $C(L)$ are rational functions of L. We define the k-th lag impulse response of the variable x_{it} to the shock u_{jt} as the (i, j)-th element of the matrix C_k. Whenever $u_t \in \overline{\text{span}}\,\{x_{t-k}, k \geq 0\}$, we say that u_t is fundamental with respect to x_t. If $N < q$ then it is almost impossible to obtain u_t from the present and past values of observed data, since we observe fewer series than the shocks that we want to recover. Thus a necessary condition for fundamentalness is that $N \geq q$.

We start by considering square systems (i.e. $N = q$) and we provide the sufficient condition for fundamentalness.

Definition 1 (Fundamentalness in square systems) *Given a covariance stationary vector process x_t, the representation $x_t = C(L)u_t$ is fundamental if*

1. *u_t is a white noise vector;*

2. *$C(L)$ has no poles of modulus less or equal than unity, i.e. it has no poles inside the unit disc;*

3. *$\det C(z)$ has no roots of modulus less than unity, i.e. all its roots are outside the unit disc*

$$\det C(z) \neq 0 \quad \forall\, z \in \mathbb{C} \quad \text{s.t.} \ \ |z| < 1 \ .$$

If the roots of $\det C(z)$ are outside the unit disc, we have invertibility in the past (i.e. the inverse representation of (1) depends only on nonnegative powers of L) and we have fundamentalness. Usually the literature considers only this kind of invertibility. However, if at least one of the roots of $\det C(z)$ is inside the unit disc, we still have invertibility, and we also have nonfundamentalness. Since in this case the inverse representation of (1) depends also on negative powers of L, we can speak of invertibility in the future. Finally if there is one root on the unit circle, the representation is still fundamental but it is not invertible.

The inverse representation of (1) is

$$D(L)x_t = u_t \ . \tag{2}$$

where in principle $D(L)$ is an infinite order, two-sided polynomial. In the case in which $D(L)$ is a one-sided polynomial, then the shocks u_t are fundamental by construction, and the finite order approximation of (2) is called VAR. However there may be models in which the shocks are nonfundamental, thus $D(L)$ is not one-sided. There is no way to identify nonfundamental shocks by means of VAR techniques. Economic theory and models, in general, do not provide support for fundamentalness so that all representations that fulfill the same economic statements but are nonfundamental are ruled out by VAR estimation without justification. Nonfundamentalness can typically be restated as a case where the agents' information space is larger than the econometricians' one. For example, when agents have expectations of future variables, they can use additional information to form such expectations, while the econometrician estimating a VAR makes use only of a limited amount of information.
In VAR literature, identification of the structural shocks is accomplished by estimating a one-sided, finite order approximation of (2) and by imposing restrictions derived from economic theory. The old literature used to impose such restrictions directly on the lag coefficients, however Sims [1980] dubbed them as "incredible" and proposed to put weaker identifying restrictions generally on the covariance matrix of the residuals of a VAR, or on the impact multiplier $C(0)$ and on the long run multiplier $C(1)$.[1] A VAR with structural restrictions is usually called Structural VAR. In any case, whatever the identification scheme used, the identified shocks are still fundamental for the VAR representation given that they are simple rotations of the ones estimated in (2).

Summarizing, if $\det C(z)$ has roots outside the unit disc and we estimate a VAR for x_t, the residuals, once identified, are the real economic shocks we are looking for. On the opposite, if at least one root is inside the unit circle, there is a problem of nonfundamentalness and we cannot use standard techniques as VAR to identify the model. The problem of nonfundamentalness is a problem only for the estimation of Structural VAR models. When instead we use VAR models for forecasting, we are not concerned about nonfundamentalness since in this case we are not interested in recovering the structural shocks, but we just care about exploiting all the information available. Notice that fundamental representations arise naturally with linear prediction, being the prediction error $u_t = x_t - \text{Proj}\ (x_t | x_{t-1}, x_{t-2}, \ldots)$, by construction, fundamental for x_t. Therefore when estimating a VARMA with forecasting purposes, the MA matrix polynomial is always chosen to be fundamental.

Note that fundamental and nonfundamental representations may imply the same covariance structure. We illustrate this point with a simple example. Consider the two univariate representations for x_t

$$\text{A)} \quad x_t = (1 - bL)u_t \quad u_t \sim \text{i.i.d}(0, \sigma_u^2) \ ,$$

$$\text{B)} \quad x_t = (1 - \tfrac{1}{b}L)\tilde{u}_t \quad \tilde{u}_t \sim \text{i.i.d}(0, \sigma_{\tilde{u}}^2) \ ,$$

[1]For a survey on Structural VAR see Watson [1994].

with $|b| > 1$ and $\sigma^2_{\tilde{u}} = b^2\sigma^2_u$, so that in both cases the variance of x_t is $\sigma^2_u(1 + b^2)$. Representation A is nonfundamental but the first two moments of x_t are not enough to discriminate between this model and model B which instead is fundamental. Suppose model A is the true one, a researcher using the VAR representation is forced to estimate B, recovering $\tilde{u}_t$ in place of the true u_t as the structural shocks.

Nonfundamentalness appears in the literature in two ways: endogenously or exogenously. In the first case the model is by definition nonfundamental – this is the case of permanent income models (see Blanchard and Quah [1993] and Fernández-Villaverde et al. [2007]) and rational expectations (see Hansen and Sargent [1980]) – while in the exogenous case it is the way in which the dynamics of exogenous variables is specified which makes the model fundamental or not. We start with an example of this latter case by Lippi and Reichlin [1993].

3 Why do nonfundamental representations matter?

Our intention is to review the development of the problem of nonfundamentalness under a historical perspective. Therefore, we start the excursus from the work which represents the origin of the debate on nonfundamentalness, i.e. Lippi and Reichlin [1993] (LR henceforth). In a comment to the well known VAR model by Blanchard and Quah [1989] (BQ henceforth), LR clearly highlight the possible existence of nonfundamental representations that, although not recoverable with a VAR, may still give rise to economic meaningful representations. Both these works take, as a starting point, the following model based on Fischer [1977]:

$$\begin{aligned} y_t &= m_t - p_t + a\theta_t\,, \\ y_t &= n_t + \theta_t\,, \\ p_t &= w_t - \theta_t\,, \\ w_t &= w|\left[E_{t-1}(n_t = \overline{n})\right]\,, \end{aligned}$$

where y, n, and θ denote the logs of output, employment, and productivity; $\overline{n}$ is full employment; w, p and m are the logs of nominal wage, price level, and money supply; $a\theta$ is investment demand with $a > 0$. In the last equation nominal wages at t are set so that the expectation at $t-1$ of employment at t equals full employment. The evolution of money supply and productivity is given by:

$$\begin{aligned} m_t &= m_{t-1} + u^d_t\,, \\ \theta_t &= \theta_{t-1} + d(L)u^s_t\,. \end{aligned}$$

There are two types of uncorrelated shocks, one that has a permanent effect on output through productivity, while the other has not. The former can be interpreted as supply disturbances (u^s_t) while the latter as demand disturbances (u^d_t). This model for output growth rate ($(1-L)y_t$) and unemployment (U_t) has the structural form

$$\begin{bmatrix} \Delta y_t \\ U_t \end{bmatrix} = \begin{bmatrix} (1-L) & d(L) + (1-L)a \\ -1 & -a \end{bmatrix} \begin{bmatrix} u^d_t \\ u^s_t \end{bmatrix} = C(L) \begin{bmatrix} u^d_t \\ u^s_t \end{bmatrix}. \tag{3}$$

The only difference between the models by BQ and the model by LR is on the impact of the supply shock on output growth rate. The model by BQ assumes no dynamics in productivity except for the instantaneous response to the supply shock, therefore they implicitly assume

$d(L) = 1$. The model by LR assumes learning-by-doing dynamics such that the coefficients d_k of the $d(L)$ polynomial sum to 1, therefore in their model the rate of increase of productivity at time $t + k$ is $d_k u_t^s$.
We now review in detail the implications of these two choices.

Fundamental representations

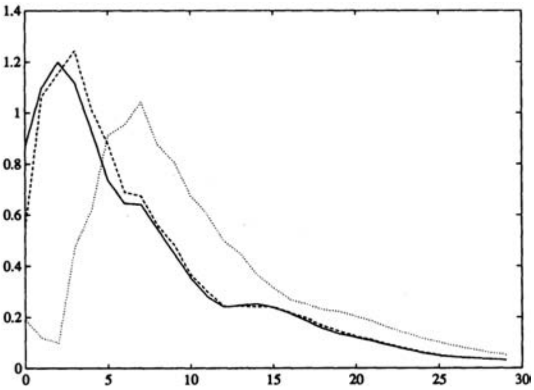
BQ estimate the following SVAR

$$D(L) \begin{bmatrix} \Delta y_t \\ U_t \end{bmatrix} = \begin{bmatrix} u_t^d \\ u_t^s \end{bmatrix}. \quad (4)$$

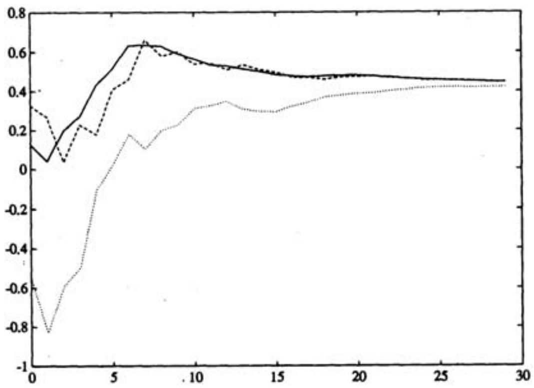
The inverse representation of (4) is given in (3) with $d(L) = 1$. The structural shocks u_t are thus estimated from the innovations of a reduced form VAR by imposing long-run neutrality of the demand shock on y_t, i.e. $C_{11}(1) = 0$. By estimating the model with real data the following impulse responses $C(L)$ are obtained: the effect of the demand shock is hump-shaped for both variables, while the effect of the supply shock on output increases steadily over time before reaching a plateau (solid lines in figure 1).
Note that the issue of nonfundamentalness is always present when dealing with VAR models, even when it is not explicitly mentioned as in the work by BQ. Indeed all their procedure is correct provided that $C(L)$ is invertible in the past. From (3), with the condition $d(L) = 1$, we have that $\det C(z) = 1$, and definition 1 is trivially satisfied. Therefore the VAR of equation (4) is a correct representation of the model. Note that if this were not the case, then the estimated innovations e_t would not be a simple linear combination of u_t since the latter ones would be nonfundamental for x_t. Therefore, the econometrician would estimate nonfundamental shocks as if they were fundamental, thus committing a possibly fatal error.

Nonfundamental representations

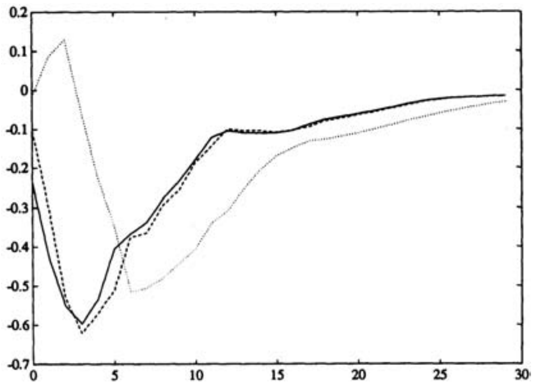
As mentioned above, LR assume nontrivial dynamics for productivity and this simple and very realistic assumption generates a variety of other possible impulse responses. Indeed in this case $\det C(z) = d(z)$, therefore invertibility of (3) (i.e. fundamentalness of u_t) is no more automatically guaranteed unless we impose additional restrictions on the process of learning-by-doing. However, economic theory does not provide sufficient restrictions for θ_t in order to satisfy definition 1. For instance, the typical case of learning-by-doing characterizing the diffusion of technological innovations can be modeled by assuming a bell-shaped pattern for the coefficients d_k, which generates an S-shaped long-run impulse response of the output growth rate to a supply shock. LR show that such a choice may imply that some roots of $\det C(z)$ are inside the unit disc. The bottom line of the work by LR consists in the possibility of producing economically sensible models in which the standard assumption of fundamentalness is violated.
In fact we can still estimate a VAR for such a model but we will face two problems: the usual problem of determining the matrix $D(0)$ through identification restrictions, plus the problem of establishing the position of the zeroes of the representation (3). The key point of the whole procedure lies in the fact that by inverting the estimated VAR we will obtain a fundamental representation, but it is possible to obtain many other nonfundamental representations that we cannot rule out since some of them may have meaningful economic interpretation as the learning-by-doing example.
To show how this can happen, LR use the same data as in BQ and first estimate a VAR, then



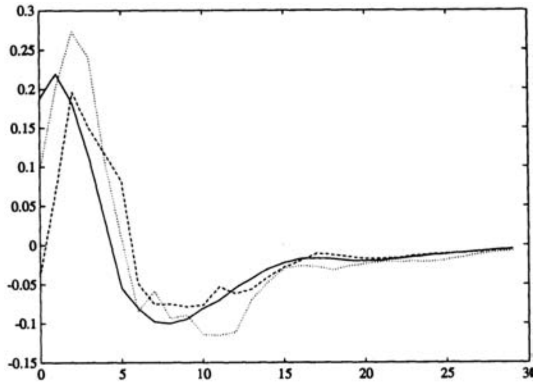
(a) Output response to demand



(b) Output response to supply



(c) Unemployment response to demand



(d) Unemployment response to supply

Figure 1: Solid line: impulse response to fundamental shocks; dashed and dotted lines: impulse responses to nonfundamental shocks. *Source*: Lippi and Reichlin [1993].

they invert it to get its MA representation, and starting from its roots, that are by definition outside the unit disc, they generate many different nonfundamental representations and their impulse responses (the procedure they use is reviewed in detail in the appendix). The task of switching from fundamental to nonfundamental representations is accomplished by means of Blaschke matrices: these are complex-valued filters which take the zeroes of a representation from outside to inside the unit disc, thus generating a nonfundamental representation from a fundamental one. The main property of Blaschke transformations is that they take orthonormal white noises into orthonormal white noises: this ensures that the requirement of uncorrelated structural shocks is fulfilled also in the case of nonfundamental representations. Some of the impulse responses obtained by LR are immediately rejected as implausible, while others can be given an economic interpretation. Figure 1 compares the impulse responses obtained by BQ (solid line) and the impulse responses which LR obtain from two different nonfundamental representations. While one of the experiments generates responses to nonfundamental shocks which do not substantially differ from responses to fundamental shocks, in the other nonfundamental case the shape of the responses is considerably different from the fundamental case. Indeed the responses to the supply shock can be interpreted as responses to a technology shock which does not have an instantaneous one-to-one impact on the variables of interest, while the response of output to the demand shock exhibits a shift in the lag structure. Moreover, the variance decomposition also changes ascribing less importance to demand than in the fundamental case.

In general the literature does not provide support for fundamentalness, so that all representations that fulfill the same economic statements but are nonfundamental are ruled out with no justification. Although skeptical about the economic usefulness of nonfundamental representations, Blanchard and Quah [1993] recognize that we cannot neglect this problem just by assuming that it is not present. As another example of nonfundamentalness, they consider the model of permanent income by Friedman-Muth where income y_t is decomposed in a permanent part y_{1t} and a transitory part y_{0t} which are independently affected by uncorrelated shocks

$$\begin{aligned} \Delta y_{1t} &= u_{1t}\,, \\ y_{0t} &= u_{0t}\,. \end{aligned}$$

If consumption follows the permanent income hypothesis, as in Hall [1978], we have: $\Delta c_t = u_{1t} + (1-\beta)u_{0t}$ where β is the agent discount factor. Therefore we have

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \end{bmatrix} = \begin{bmatrix} 1 & 1-L \\ 1 & 1-\beta \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{0t} \end{bmatrix} = C(L)u_t\,.$$

In this case $\det C(z) = (z-\beta)$ and hence it has the only root in β, which by definition is inside the unit disc. The representation is nonfundamental. Permanent and transitory components of income are not recoverable just by considering only income and consumption as in a VAR. This is a typical case of endogenous nonfundamentalness, in that this property does not depend on any exogenous variable, it is instead a property of the model that cannot be eliminated. The model by LR is instead a case in which nonfundamentalness is exogenously generated by the way in which the technological shock hits the economy. However, exogeneity is not a good reason for considering nonfundamentalness an innocuous problem. Indeed, as we just showed, we can generate nonfundamental but meaningful economic models, that SVARs cannot identify. Unless we knew the real economic model, we must take into account all the possible representations including the nonfundamental ones.

Both examples in this section show how nonfundamental representations can arise even in very simple models where no expectations are present, as it is instead the case for the models that we will consider in the next sections. Since evidence of economic meaningful nonfundamental representations is accumulating, it is useful to find a way for considering such representations every time that we have to deal with identification issues.

4 Nonfundamentalness in rational expectations models

Hansen and Sargent [1980] introduced the issue of nonfundamentalness while trying to set up a method for formulating and estimating dynamic linear econometric models with rational expectations. In these models, the problem lies with the fact that estimation is usually run by estimating agents' decision rules jointly with the model of the stochastic process they face, subject to the restrictions implied by the rational expectations rules. These in turn imply that agents observe and respond to more data than those the econometrician possesses, i.e. agents' information space is larger than the econometrician's one. Hansen and Sargent [1980] express the problem as follows:

"[...] the dynamic economic theory implies that agents' decision rules are *exact* (non-stochastic) functions of the information they possess about the relevant state variables governing the dynamic process they wish to control. The econometrician must resort to *some* device to convert the exact equations delivered by economic theory into inexact (stochastic) equations susceptible to econometric analysis."

To fix ideas, let us take the simple example by Hansen and Sargent [1991]. Suppose that one set of economic variables w_t, representing the true process, is generated by a fundamental moving average process, while another set x_t, representing the estimated process, is made of expectational variables. Namely,

$$
\begin{aligned}
w_t &= u_t - \theta u_{t-1} = \tilde{C}(L) u_t \, , \\
x_t &= E_0 \left[\sum_{t=0}^{\infty} \beta^t w_t \right] = (1 - \beta\theta) u_t - \theta u_{t-1} = C(L) u_t \, .
\end{aligned}
$$

The only root of $C(z)$ is $(1 - \beta\theta)/\theta$ which can be inside the unit disc even if $\tilde{C}(z)$ has its root outside the unit disc. If only x_t are available to the econometrician then she may not be able to recover the structural shocks u_t that generate w_t.

A recent strand of literature studies the characteristics of the equilibria in dynamic rational expectations models when the assumption of homogeneous information across agents is relaxed in favor of symmetric information. Representative of this literature are the works by Kasa [2000], Kasa et al. [2006], and Rondina [2007]. In the heterogeneous information setting, nonfundamental representations correspond to nonrevealing equilibria. The mechanism at work in these models is the following: agents do not directly observe the structural shocks and the equilibrium price is not fully revealing of the true state of the economy. In this case, the formation of agents' rational expectations involves a component related to the "average" market expectation, which in turn implies "forecasting the forecast of others". In other words, heterogeneous information breaks the law of iterated expectations and gives rise to higher order beliefs. When agents observe shocks with noise, the solution of the fixed point problem posed by the assumption of consistency between beliefs and outcomes, implied in turn by the infinite regress in expectations, produces or may produce nonfundamental MA representations.

To put it differently, nonfundamentalness is linked to nonrevealing equilibria because in order to prevent the aggregate price to be a sufficient statistic of the state of the economy in equilibrium, the model must be such that agents cannot retrieve structural shocks from observations: in other words, the model must have a nonfundamental MA representation.

By taking into account the difference between fundamental and nonfundamental shocks, these models are able to explain puzzles which standard theory does not account for. Figure 2 reports the impulse response function of asset prices to a shock in market fundamentals (e.g. dividends) in both the full-information standard asset pricing model and in the dynamic asset pricing model with persistent heterogeneous beliefs developed by Kasa et al. [2006]. The response of asset prices in the heterogeneous information case, where the MA representation is nonfundamental, is more than twice as large as the standard response at impact, and the effects are persistent. This explains the empirically observed persistence in aggregate price dynamics and asset prices systematic violation of the linear present value model standard variance bounds. Figure 3 shows how in the model by Rondina [2007], where nonfundamentalness arises endogenously via informational heterogeneity, the equilibrium aggregate price might

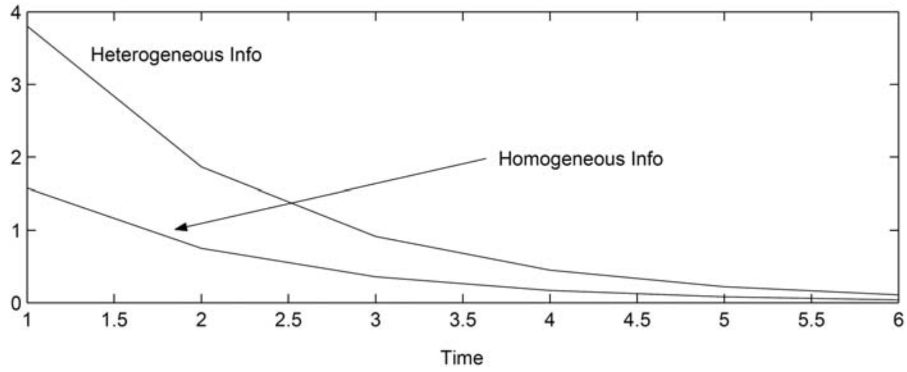


Figure 2: Asset price response to a shock in market fundamentals when the MA representation is fundamental (homogeneous information case) and when it is nonfundamental (heterogeneous information case). *Source*: Kasa et al. [2006].

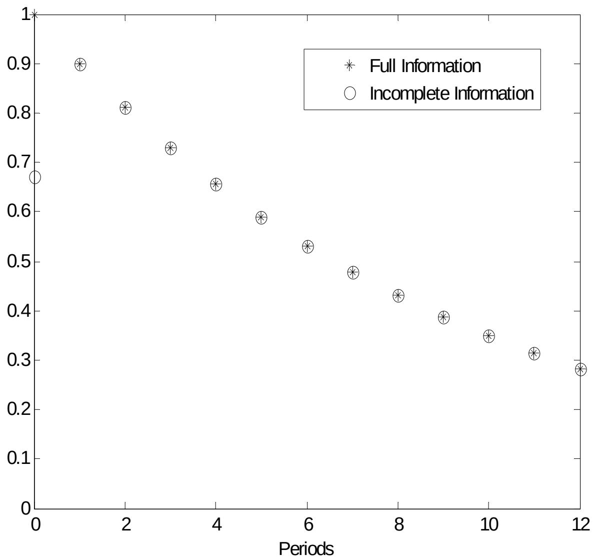


Figure 3: Aggregate price response to a shock in aggregate productivity when the MA representation is fundamental (full information case) and when it is nonfundamental (incomplete information case). *Source*: Rondina [2007].

underreact to structural aggregate technology shocks and might not allow to recover them. The incomplete information response plotted in figure 3 corresponds to the case in which the incentive for firms to coordinate price adjustments is strong enough to turn the MA component from fundamental to nonfundamental: in this case the effect of a productivity shock on the aggregate price is dampened at impact, which explains the propagation of transitory shocks throughout the economy.

Finally, let us outline an example of how forward-looking systems with rational expectations may give origin to nonfundamental representations. What follows is taken from a recent work by Brock et al. [2008], where the authors analyze the role of rational expectations in the framework of frequency domain analysis of linear systems with feedback control rules. They show that by means of an appropriate choice of the control, e.g. monetary policy, it is possible to take the roots of the characteristic polynomial outside the unit circle, thereby turning a nonrevealing equilibrium into a revealing one (and vice versa).

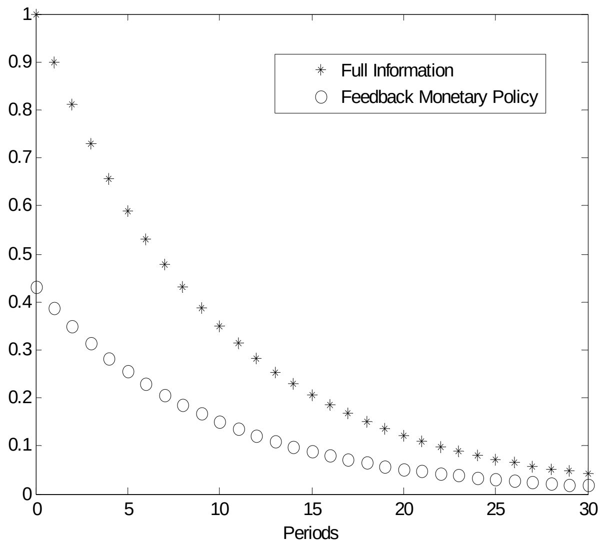


Figure 4: Impulse response of the equilibrium aggregate price in the homogeneous-full information case and when a control is implemented to turn the representation from nonfundamental to fundamental. *Source*: Rondina [2007].

Formally, a forward-looking system with controls is written as

$$D_0 x_t = \beta E_t(x_{t+1}) + D(L)x_{t-1} + P(L)c_t + \varepsilon_t \, ,$$

where x_t are the state variables, c_t are the control variables and $\varepsilon_t = W(L)u_t$ with u_t being the structural shocks. A generic linear feedback rule is written as

$$c_t = K(L)x_{t-1} \, .$$

Finally, we denote with $x_t = C(L)u_t$ the equilibrium moving average representation of the system. The key point is that $C(L)$ depends on the choice of the control rule, i.e on the polynomial matrix $K(L)$. Indeed, the choice of different control rules has an impact on the spectral density matrix of the state variable x_t, which is

$$f_x(\theta) = \frac{1}{2\pi} C(e^{-i\theta})\Sigma_u(\theta)C(e^{-i\theta})' \, ,$$

$\Sigma_u(\theta)$ being the spectral density of the structural shocks. The control enters the expression for $C(e^{-i\theta})$ as follows

$$C(e^{-i\theta}) = D_0 - (D(e^{-i\theta}) + P(e^{-i\theta})K(e^{-i\theta})e^{-i\theta})^{-1}W(e^{-i\theta}) \, .$$

It is possible to show that the application of a given control can have an impact on the value of $C(L)$ and on the location of the zeroes of its determinant. This is crucial in the case of forward-looking systems when the structural shocks cannot be recovered by current and past values of the state variables. These latter constitute the policymaker information set, while the agents also observe the structural disturbances and know their process $W(L)u_t$. However, with an appropriate choice of the feedback control, the policymaker is able to turn a nonrevealing equilibrium into a revealing one, and vice versa.

In appendix to their paper, Brock et al. [2008] provide an example in the univariate case with $D(L) = D$, $P(L) = P$, $K(L) = K$ and $W(L) = 1 + wL$. In this case, the solution of the system is

$$C(L) = \frac{\frac{1}{\lambda_1}\left(1 + \frac{w}{\lambda_1}\right)\left(1 + \frac{\lambda_1 w}{\lambda_1 + w} L\right)}{\beta(1 - \lambda_2 L)}$$

where $1/\lambda_1$ and $1/\lambda_2$ are the roots of the characteristic polynomial $(\beta - L + (D + PK)L^2)$. The representation is fundamental if

$$\left|\frac{\lambda_1 w}{\lambda_1 + w}\right| < 1.$$

For given values of D, P and w, the above condition might be not satisfied in absence of a control, while it might be satisfied by choosing an appropriate value for the control K. Figure 4 shows the impulse response function of the equilibrium aggregate price in the full-information model and in a system with a nonfundamental MA representation, in which a feedback control rule (monetary policy) is designed to eliminate a problematic MA component. Interestingly, it shows that the use of a control to turn a nonrevealing (nonfundamental) equilibrium into a revealing (fundamental) equilibrium will however introduce permanent distortions.

5 Nonfundamentalness and cointegration

In this section we briefly illustrate another reason for which nonfundamental representations can arise, which is explained in Blanchard and Quah [1993] and has to do with cointegrated models. Assume to have a bi-dimensional vector $x_t = (x_{1t}\, x_{2t})'$ of integrated time series which has a fundamental MA representation in first difference: $\Delta x_t = C(L)u_t$, where u_t are structural shocks. By applying the Beveridge-Nelson decomposition into trend and cycle, i.e. into long- and short-run dynamics, we obtain

$$\Delta x_t = K(1)\sum_{j=1}^{t} u_j + (I - L)K^*(L)u_t \tag{5}$$

where $(I - L)K^*(L) = K(L) - K(1)$. If $\operatorname{rank} K(1) = 1$ then the two components of x_t are cointegrated, therefore they have a common trend and it is enough to include a sufficient number of lags of Δx_t in the empirical analysis in order to identify the short-run dynamics, provided that $K^*(L)$ is invertible. Actually, if we consider decomposition (5) for I(1) variables we are sure that $\det K^*(z)$ has no roots for $|z| = 1$ since we have taken differences. However, it remains the possibility to have roots for $|z| < 1$ as illustrated in a numerical example by Blanchard and Quah [1993]. If indeed some roots of $\det K^*(z)$ happen to be inside the unit disc, then there is no way to recover the short-run structural shocks u_t from decomposition (5).

6 Detecting nonfundamentalness

In this section we review two methods recently proposed by Fernández-Villaverde et al. [2007] and by Giannone and Reichlin [2006] to detect nonfundamentalness. Before introducing these two methods, however, it is worth discussing why one should check for nonfundamentalness before estimating a SVAR and what are the consequences in terms of validation of economic models, Dynamic Stochastic General Equilibrium (DSGE) models in particular, if nonfundamentalness is not recognized. Indeed, the bottom line is that the whole debate on the effectiveness of traditional VAR techniques for DSGE model evaluation is rooted into nonfundamentalness.

The procedure, used for example in a couple of recent papers by Chari et al. [2005] and by Christiano et al. [2006], for assessing the reliability of VAR as a tool to discriminate among competing models, is the following:

1. consider a DSGE model (e.g. a real business cycle model or a nominal rigidities model);
2. reformulate it in a state space form usually obtained by log-linearizing about the non-stochastic steady state;
3. estimate the parameters of the state space form (e.g. by Maximum Likelihood or with Bayesian methods);
4. compute the impulse response functions of the DSGE variables to the economic shocks as given in the state space form;
5. generate new data from the state space model, using parameters estimated at step 3 (this and the following steps are repeated thousands of times in a Monte Carlo experiment);
6. using the data generated in the previous step, estimate a VAR jointly with economically meaningful identification restrictions, and compute the same impulse responses, together with their confidence intervals;
7. compare these simulated VAR impulse responses with the ones obtained in step 4.

The last step is crucial since, if there is no bias in the estimated impulse responses and in their confidence intervals, we can say that VARs are indeed a useful tool for discriminating among different models, i.e. we can estimate the VAR with real data and, from its impulse responses, we can say which is the more correct economic model.

Let us now show how the problem of nonfundamentalness arises in dealing with DSGEs. When using real data to estimate the impulse responses, observations for many state variables (usually stocks as e.g. capital) are typically not available. Therefore, it is not possible to estimate the same impulse responses as the simulated ones since some of the variables of the DSGE are omitted when using real data. Whenever we omit a variable we do not have anymore a VAR representation but we typically end up writing a VARMA representation of the linearized DSGE solution. When estimating a VARMA we must always consider the possibility of having a nonfundamental MA part before transforming it in a VAR.

The following fiscal policy example (see Pagan [2007]) relates to noninvertibility rather than to nonfundamentalness but still illustrates the argument. x_t is the primary deficit and the level of debt is defined as a gap relative to its desired equilibrium value. Debt accumulates as $\Delta d_t = x_t$ where we set the interest rate on past debt to zero. In order to stabilize debt we need a fiscal rule that relates to the past debt level and responds to an output gap y_t, i.e.

$$\Delta d_t = x_t = a d_{t-1} + c y_t + u_t \quad \text{with} \quad a < 0\,.$$

Typically we drop debt from the VAR thus we need to solve the previous equation for d_t and substitute it in the fiscal policy equation, obtaining

$$x_t = (1 + a)x_{t-1} + c\Delta y_t + \Delta u_t \,.$$

This is no more a VAR but a VARMA where the MA part $\Delta u_t = (1 - L)u_t$ has its root in $z = 1$, thus it is not invertible. Indeed, Favero and Giavazzi [2007] show that omitting the level of debt from the VAR can result in biased estimates of the effects of fiscal policy shocks: in particular, if debt dynamics are unstable the impulse response functions will eventually diverge.

Nonfundamentalness generated by omitted variables is often considered innocuous provided that we estimate a VAR with a sufficient number of lags. However, the feasibility of writing a VAR representation of a particular DSGE model is never seriously considered. Indeed, given the presence of expectations in such models, it is not unlikely to face a problem of nonfundamentalness already when solving and linearizing the DSGE. When this happens, the entire procedure of validation of a DSGE model through a VAR is invalid, given that it will recover fundamental representations of a nonfundamental structural model.
A general DSGE model is formulated as follows:

$$\begin{array}{ll} \max\limits_{Y_t} & \mathrm{E}_0\left[\sum\limits_{t=0}^{\infty} \beta^t U(Y_t)\right] \\ \text{s.t.} & g(Y_t, Y_{t-1}, \ldots, Z_t, Z_{t-1}) \leq 0 \,. \end{array}$$

The model includes p endogenous variables Y_t and q exogenous variables Z_t, which are usually modeled as functions of q serially uncorrelated orthonormal structural shocks u_t.[2] Therefore, the system contains $N = p + q$ variables $X_t = (Y_t' \; Z_t')'$. Let us indicate with small letters the difference between the log of the variables and their non-stochastic steady state. We have the linearization of the model

$$\begin{aligned} y_t &= \Theta(L) z_t \,, \\ \Psi(L) z_t &= u_t \,. \end{aligned}$$

The system can be transformed into a state space form by defining the state variables as $f_t = (z_t', \ldots, z_{t-s}')$ where s is the maximum degree between $\Psi(L)$ and $\Theta(L)$ (see Giannone et al. [2006] for details). Therefore we have

$$\begin{aligned} x_t &= \Lambda f_t \,, \\ A(L) f_t &= B u_t \,. \end{aligned} \tag{6}$$

We have a system with an N-dimensional vector of observable variables x_t and a q-dimensional vector of economic shocks u_t such that $u_t \sim \text{w.n.}\,(0, I_q)$. Note that the dimension of f_t is $r = q(s + 1)$ and that $r \leq N$.

Analogously to Hannan and Deistler [1988] and Fernández-Villaverde et al. [2007], we can state the conditions under which we can write a VAR as a linearized solution of a DSGE

[2]For simplicity we omit the distinction between non-predetermined and predetermined enodgenous variables as the conclusion does not change.

model as (6). Let us assume that $A(L) = (I - AL)$ and that Λ has maximum rank N so that $r = N$. The matrices A, B, and Λ are functions of the parameters that define preferences, technology, and, in general, economic shocks. They contain the typical cross-equation restrictions embedded in macroeconomic models. It is then possible to find conditions on these matrices that allow for the existence of a VAR representation for x_t. Indeed, since we are in the maximum rank case, we can write

$$f_t = \Lambda^{-1} x_t \, .$$

Plugging this into the second equation of (6) we get

$$(I - \Lambda A \Lambda^{-1} L) x_t = \Lambda B u_t \, .$$

If we now compare this theoretical expression with the VAR that an econometrician will estimate, say for example $D(L)x_t = e_t$, we realize that the VAR representation that an econometrician will estimate is consistent with the theory only if the eigenvalues of $\Lambda A \Lambda^{-1}$ lie all inside the unit circle. This condition, analogous to the one stated by Fernández-Villaverde et al. [2007], gives us a practical way to check for fundamentalness of the economic shocks u_t when we have a system of N observable variables with maximum rank $r = N$. Such a criterion might be useful for small systems in the case we have a state space form of our model but we do not have a structural representation for it as in (1), so that we cannot check directly definition 1.[3] However, in many cases DSGE models consider a large number of variables and therefore are likely to have reduced rank $r < N$. In the next section we deal with this case.

Giannone and Reichlin [2006] propose a criterion to detect nonfundamentalness in VAR representations that is based on the concept of Granger causality. They consider the well known VAR firstly estimated by Galí [1999], which can be derived from very different DSGE models such as real business cycle models or New-Keynesian models

$$\begin{bmatrix} \Delta a_t \\ \Delta l_t \end{bmatrix} = C(L) \begin{bmatrix} u_t^s \\ u_t^d \end{bmatrix} , \tag{7}$$

where a_t is the log of aggregate labor productivity and l_t is the log of aggregate labor supply. There are two structural shocks: a technological shock u_t^s and a shock u_t^d which is neutral for productivity in the long-run, being thus interpretable as a labor income (or demand) shock or a monetary shock. Let us call $x_t = (\Delta a_t \, \Delta l_t)'$ the vector of observable variables which we augment with other variables x_t^*, so that (7) for the larger system becomes

$$\begin{bmatrix} x_t \\ x_t^* \end{bmatrix} = \begin{bmatrix} C(L) & 0 \\ C^*(L) & \Psi(L) \end{bmatrix} \begin{bmatrix} u_t \\ v_t \end{bmatrix} ,$$

with v_t as additional structural shocks orthogonal to u_t. If u_t is fundamental for x_t then there exists a one-sided filter $D(L)$ such that $u_t = D(L)x_t$, therefore

$$x_{it}^* = C_{i\cdot}^*(L) D(L) x_t + \Psi_{i\cdot}(L) v_t \qquad \text{for } i = 1, \ldots, N \, ,$$

[3] The state-space form considerd by Fernández-Villaverde et al. [2007] reads as

$$\begin{aligned} x_t &= \Lambda f_{t-1} + D u_t \, , \\ f_t &= A f_{t-1} + B u_t \, , \end{aligned}$$

and the condition for fundamentalness requires the eigenvalues of $(A - BD^{-1}\Lambda)$ to lie all inside the unit circle.

where $C^*_{i\cdot}$ indicates the i-th row of C^*. Hence, each x^*_{it} depends only on the past of x_t and does not incorporate any further information useful for forecasting x_t, i.e. none of the x^*_{it} Granger causes x_t. This result was firstly introduced by Forni and Reichlin [1996]. It follows that nonfundamentalness can be detected empirically by checking whether the variables of interest x_t are weakly exogenous with respect to potentially relevant additional blocks of variables that are likely to be driven by shocks which are common to the variables belonging to the block of interest. In the model above, Giannone and Reichlin [2006] consider as additional variables labor productivity and labor input at sectoral level and they indeed reject the hypothesis of weak exogeneity, thus giving a clue for nonfundamentalness.

By exploiting the additional information contained in the large system, we are able not only to check for nonfundamentalness but also to identify the nonfundamental shocks u_t. Indeed, this is what Giannone and Reichlin [2006] do by assuming a factor structure in the data (see the next section for details on factor models and nonfundamentalness)

$$\begin{bmatrix} x_t \\ x^*_t \end{bmatrix} = \begin{bmatrix} \Lambda \\ \Lambda^* \end{bmatrix} f_t + \Psi(L) v_t\,,$$

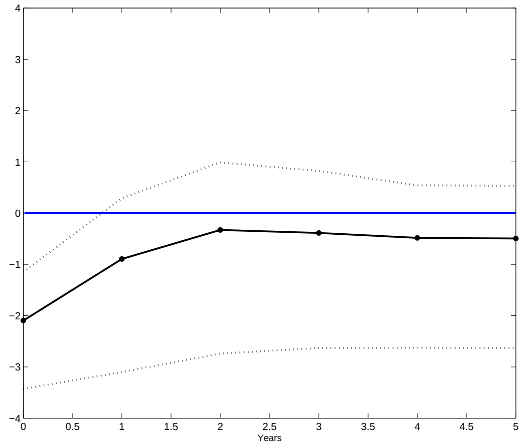
where $\Lambda = (C_0 \dots C_s)$, $\Lambda^* = (C^*_0 \dots C^*_s)$ and $f_t = (u'_t \dots u'_{t-s})'$ are the (static) common factors. Figure 5 (a) reports the estimated response of first-differenced hours, together with its 5% confidence bands, to a technology shock in the bivariate VAR: the point estimate exhibits a significant and persistent decline in hours, the bulk of the variation taking place at impact. Giannone and Reichlin [2006] estimate the same impulse response by means of the Dynamic Factor model, including different numbers of common factors (up to 8) and imposing the same identification restrictions as in the SVAR. Figure 5 (b) reports the value at impact of this response together with 5% confidence bands, for different numbers of common factors (on the x-axis): the more factors are included in the model, i.e. the more sectoral information gets captured, the more the response is shifted upward and the contemporaneous response of hours becomes not significantly different from zero. This result confirms that in this case there is a problem of nonfundamentalness: nonfundamental shocks are different from the fundamental shocks estimated in the SVAR, or in other words the shocks estimated in the SVAR are actually non-structural shocks. Therefore, nothing can be said about the dispute between real business cycle models and models with nominal rigidities by looking only at labor productivity and labor input as it is usually done in the literature.

7 Large cross-sections for structural identification

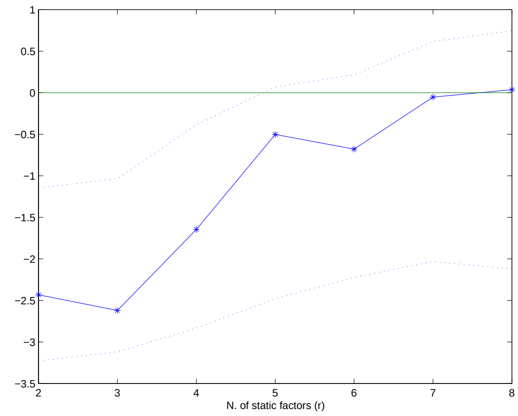
Although the literature often considers nonfundamentalness as a minor problem at least in all practical cases, we tried to convince the reader that ruling out nonfundamental representations might hide the econometrician a large number of alternative possible meaningful representations of a given model. We would like to find econometric models that do not have to bother with the problem, but still are able to achieve identification of structural shocks.

Nonfundamentalness is ultimately a problem of missing information. As we have seen, if we have nonfundamentalness the inverse of the MA representation involves future observations. Indeed, this is in principle a first approach we might take: by estimating a VAR we will never be able to retrieve u_t, since we would need x_{t+s}, but we could still estimate u_{t-s} for some $s > 0$.

What if we want to retrieve the contemporaneous structural shocks? We need to enlarge



(a) Response from a bivariate VAR



(b) Response at impact from a large scale system

Figure 5: Estimated impulse response of hours to a technology shock. *Source*: Giannone and Reichlin [2006].

the econometrician's information set in some other manner which is not including future observations. The alternative to the time dimension is the cross-section dimension. However, including many variables in the econometric model poses a problem of estimation, therefore we need to use tools, such as Bayesian VARs, Global VARs and Dynamic Factor models, which are able to handle large cross-sections of time series. In this section we focus on Dynamic Factor models and outline how they are built and how they deal with nonfundamentalness. Notice, however, that estimating a Dynamic Factor model on a large cross-section is a good alternative to estimating a VAR on a few variables of interest only if we are willing to assume a factor structure in the data, where the number of primitive shocks (dynamic common factors) is equal to the number of shocks in the VAR despite the large number of variables included.

Dynamic Factor models as representations of DSGE models

Giannone et al. [2006] and Boivin and Giannoni [2006] provide the motivation for considering a factor structure in validating DSGE models. Typical theoretical macroeconomic models have few shocks driving the business cycle, e.g. only one technology shock in first generation real business cycle models, two or three in second generation ones. We say that these models have reduced stochastic rank. Usually in DSGE models also measurements errors are considered and in this case it can be shown that the model can have a factor structure, since factor models separate out measurement errors by their own nature. Indeed, in these models the spectral density matrix of the observed variables is decomposed into two orthogonal parts: the spectral density of the common component, of reduced rank, that contains all the relevant information of covariances (at all leads and lags), and the spectral density of the idiosyncratic component, of full rank, that represents non correlated or mildly correlated measurement errors. This approach wipes away measurement errors, which heavily affect VAR impulse responses. Therefore factor models seem to be a good alternative tool to validate DSGE models, as formally discussed in this section.

Let us recall from the previous section the state-space form (6) of a linearized DSGE model:

$$\begin{aligned} x_t &= \Lambda f_t\,, \\ A(L) f_t &= B u_t\,. \end{aligned}$$

Remember also that the dimension of f_t is $r = q(s+1)$. Thus, the static rank of the system (i.e. the rank of the covariance of x_t) is at most r and it is given by the restrictions imposed on the VAR (the q shocks) and on the number of lags included in the model s, therefore it depends on the structure of the economy. In most DSGE models we have reduced static rank i.e. $r < N$, which is also empirically found in the form of common cycles. In this case, it is therefore impossible to use the technique by Fernández-Villaverde et al. [2007] to detect nonfundamentalness. From (6) we obtain the MA representation

$$x_t = \Lambda A(L)^{-1} B u_t = C(L) u_t\,. \tag{8}$$

From this equation is clear that the dynamic rank of x_t (i.e. the rank of its spectral density matrix) is q, and therefore it depends on the number of exogenous forces. In general for macroeconomic datasets $q < N$, which means that there is collinearity among the N variables. The reduced static and dynamic ranks are restrictions that come from the theory and that could be tested. In principle we could now estimate the VAR $D(L)x_t = e_t$ and then identify the economic shocks u_t as simple rotations of e_t. However, to estimate this VAR we need the covariance of x_t to have maximum rank $r = N$, which as we said it is almost never the case. Thus VAR estimation is not possible due to the reduced static rank of macroeconomic datasets. There are two alternatives: either we estimate a VAR only on blocks of r variables, or we add measurement errors. In the latter case we eliminate the collinearity among variables and we can estimate the full system, thus either we estimate a VARMA on the whole system, or we estimate a Dynamic Factor model. This last case is the one that we are interested in (see Giannone et al. [2006] for details on all the cases).[4]

[4]About the ranks notice that

$$\Sigma^x(\theta) = C(e^{-i\theta})\Gamma_0^u C(e^{i\theta})'\,,$$

and since $\operatorname{rank} C(L) = q$ the dynamic rank is q, while

$$\Gamma_0^x = \Lambda f_t f_t' \Lambda'\,.$$

Therefore the maximum static rank is r.

Introducing measurement errors

When adding orthogonal measurement errors ξ_t, we lose collinearity of the variables and we can write (8) for a covariance stationary process x_t as a Dynamic Factor model

$$x_t = C(L)u_t + \xi_t = \chi_t + \xi_t \,, \tag{9}$$

where u_t is the q-dimensional vector of common shocks s.t. $u_t \sim$ w.n.$(0, I_q)$, and ξ_t is an idiosyncratic N-dimensional process of measurement errors s.t. ξ_{it-k} is orthogonal to u_{jt} for any i, j, and k. Two assumptions are made for the factor model: for all frequencies $\theta \in [-\pi, \pi]$, the q largest eigenvalues of the spectral density matrix of x_t diverge as $N \to \infty$, while the $(q+1)$-th is bounded almost everywhere. These assumptions are reasonable since measurement errors are supposed to vanish when considering linear combinations of many collinear variables. As a consequence, the common component χ_t has reduced dynamic rank $q < N$, while ξ_t has full dynamic rank: this is how we break collinearity. Notice that the need of large cross sections to apply the factor model is perfectly consistent with the standard practice of central banks, which use all the available information when making decisions.
We can also add measurement errors to the state space form (6)

$$\begin{aligned} x_t &= \Lambda f_t + \xi_t \,, \\ A(L) f_t &= B u_t \,. \end{aligned} \tag{10}$$

Once again, given the previous assumptions, we have a common part with reduced static rank and an idiosyncratic part with asymptotically vanishing covariance that has full static rank. Therefore, when dealing with large cross sections we still have reduced dynamic and static ranks of the whole dataset x_t. We can estimate a factor structure on every model with reduced static and dynamic ranks, which are typical properties of macroeconomic datasets. Hereafter we will call f_t the static factors while u_t will be the dynamic factors that correspond to the structural shocks of the economy. We want to identify u_t and the impulse responses they generate.
The most general factor model is the Generalized Dynamic Factor Model by Forni et al. [2000], where some cross-correlation between the elements of ξ_t is allowed. This model in its state-space form can be estimated by using the one-sided estimator proposed by Forni et al. [2005] and by applying the procedure suggested in Giannone et al. [2004].

To sum up, the two main advantages from imposing a factor structure on the linearized solution of a DSGE model are the following:

1. given the properties of the estimator by Forni et al. [2000] we need a large cross section ($N \to \infty$) and to have a good estimation of the spectral density we require also a large time dimension. This seems a perfectly realistic requirement in agreement with the practice followed by central banks, where usually DSGE models are applied;

2. x_t contains the observed variables of the DSGE model and some proxies of the state variables which are often unobserved and can be estimated as the latent static factors f_t. Indeed, the typical macroeconomic variables included in the panel are indicators of economic activity built by aggregation, which can be seen as linear combinations of unobserved state variables (and their lags) plus some measurement errors. It is possible to impose structural relations between the observed x_t and the unobserved f_t, i.e. to impose restrictions on Λ. The two-step procedure for estimating the restricted model is

the following: (i) carry out a non-parametric estimation of f_t as in Forni et al. [2000]; (ii) apply a Quasi-ML Kalman filter estimator as the one proposed by Doz et al. [2006].

Fundamentalness in Dynamic Factor models

Why in the previous section, when considering factor models as a tool for validating DSGE models, have we not raised the issue of fundamentalness, that is pervasive when dealing with VAR? Because we can show that actually nonfundamentalness is not a generic problem in factor models, and, under reasonable assumptions, we can always guarantee that the dynamic factors u_t are fundamental for x_t (see Forni et al. [2007]). In factor models we always have $N > q$, therefore we first need a definition of nonfundamentalness that generalizes definition 1 to the case of singular systems. It is indeed the singularity of dynamic factor models that makes the property of nonfundamentalness non generic.

Definition 2 (Fundamentalness in singular models) *Given a covariance stationary vector process x_t, the representation $x_t = C(L)u_t$ is fundamental if:*

1. *u_t is a white noise vector;*
2. *$C(L)$ has no poles of modulus less or equal than unity, i.e. it has no poles inside the unit disc;*
3. *$C(L)$ has full rank inside the unit disc*

$$\text{rank } C(z) = q \quad \forall\, z \in \mathbb{C} \quad \text{s.t. } |z| < 1\,.$$

Alternatively, we can restate this last condition in terms of the roots of $\det C(z)$. We ask that the determinants of all the $q \times q$ submatrices of $C(z)$ have no common roots inside the unit disc. More precisely, if we call $C_j(L)$ the submatrices contained in $C(L)$ and we define the set of indexes $\mathbb{I} = \left\{ j \in \mathbb{N} \text{ s.t. } j = 1, \ldots, \binom{N}{q} \right\}$, the definition of nonfundamentalness requires that

$$\nexists\, z \in \mathbb{C} \;\text{ s.t.} \begin{cases} |z| < 1 \\ \\ \det C_j(z) = 0 \quad \forall\, j \in \mathbb{I}\,. \end{cases}$$

As an example, consider the case $q = 1$. If $N = 1$ we are back to definition 1 and for fundamentalness we require that no root of $C(z)$ is smaller than one in modulus. If instead $N > 1$ we have N polynomials $C_j(z)$ and from definition 2 the representation is nonfundamental if they have a common root smaller than one in modulus. Thus, if $N = q$, nonfundamentalness is generic since if it holds in a point then, for continuity of the roots of $C(z)$, it holds also in its neighborhood; while if $N > q$ nonfundamentalness is non-generic because to have a common root we must satisfy $\binom{N}{q} - 1$ equality constraints. In singular models we usually have highly heterogeneous impulse responses of the variables to the few structural shocks, therefore it is highly improbable to have a common root for all of them, although it is not unlikely to have common roots for some submatrices of $C(L)$. Roughly speaking, although in principle the econometrician has a smaller information set than the agents' one (i.e. there is nonfundamentalness), she can include additional series in the system, and if dynamic heterogeneity is guaranteed then these series contain useful information. In macroeconomic datasets this is very likely to happen, thus it is reasonable to assume fundamentalness in factor models.

Fernández-Villaverde et al. [2007] provide an economic example, used also by Forni et al. [2007], that clarifies this point. Let us consider the permanent income consumption model

$$\begin{aligned} c_t &= c_{t-1} + \sigma_u(1-\rho^{-1})u_t\,, \\ s_t &= y_t - c_t = -c_{t-1} + \sigma_u\rho^{-1}u_t\,, \end{aligned}$$

where c_t is consumption, y_t is labor income, s_t are savings, u_t is a white noise process and ρ is the gross interest rate. Fernández-Villaverde et al. [2007] assume that s_t is observable while c_t is not. From equations above, we have

$$s_t - s_{t-1} = \sigma_u\rho^{-1}(1-\rho L)u_t = d(L)u_t\,.$$

Since $d(z) = 0$ for $z = \rho^{-1} < 1$, u_t is nonfundamental for s_t. Thus a VAR(1) estimated by the econometrician will produce innovations which are not the structural shocks. However, if the econometrician observes also some additional variables such that $z_{it} = b_i(L)u_t$, then u_t is fundamental for the whole system $(s_t\, z_t)'$ unless $d(z)$ and $b_i(z)$ have the same root, i.e. unless $b_i(\rho^{-1}) = 0$ for every variable z_i added, which is extremely unlikely.

In what follows we formalize the ideas shown in this example. Together with the usual assumptions of the Dynamic Factor model, Forni et al. [2007] assume also that the dynamic factors u_t are fundamental for the static factors f_t. This assumption can be formally stated as follows: there exists a squared-summable one-sided $r \times q$ filter $N(L)$ such that

$$C(L) = \Lambda N(L) \qquad \text{and} \qquad f_t = N(L)u_t\,. \tag{11}$$

As shown in the previous section, it easy to meet this requirement. Indeed, it is enough to choose $N(L) = (I_q\,(I_qL)\ldots(I_qL^s))'$ so that the following identities hold

$$\begin{aligned} f_t &= (u_t'\,u_{t-1}'\ldots u_{t-s}')'\,, \\ \Lambda &= (C_0\ \ldots C_s)\,, \\ r &= q(s+1)\,. \end{aligned} \tag{12}$$

Fundamentalness of u_t for χ_t is equivalent to left-invertibility of $N(L)$, i.e. to the existence of a $q \times r$ one-sided filter $G(L)$ such that $G(L)N(L) = I_q$. Indeed, if we define $S(L) = G(L)(\Lambda'\Lambda)^{-1}\Lambda'$, we have

$$S(L)x_t = G(L)(\Lambda'\Lambda)^{-1}\Lambda'\Lambda F_t + S(L)\xi_t \xrightarrow{m.s.} G(L)N(L)u_t = u_t \qquad \text{for } N \to \infty\,.$$

where convergence is given in mean-square. Therefore, u_t lies in the space spanned by the present and past values of χ_t. Given (12), the dynamic and static representations (9) and (10) are equivalent for a given lag length s, choosing $G(L) = (I_q\,0_q\ldots 0_q)$.

Why can we safely make the assumptions of fundamentalness in Dynamic Factor models?

Consider the state-space representation (6) together with (12). As said in the previous section, in empirical applications with large cross sections we often have reduced static and dynamic ranks, i.e. $r < N$ and $q < N$. Dynamic Factor models are a useful way to model systems with reduced rank. The main assumption of these models is that only the largest r eigenvalues of the covariance of x_t diverge as $N \to \infty$, the others being bounded. This in turn implies that $\text{rank}\,(\Lambda'\Lambda)/N = r$ for large N, i.e. factors are pervasive. Such a condition is equivalent to ask for no restrictions on the entries of $C(L)$ which are the elements of Λ. Therefore this is equivalent to ask for heterogeneity of the impulse responses. Dynamic heterogeneity is indeed a reasonable property of a factor model with large cross sectional dimension N as economic variables react differently to structural shocks. Thanks to this property, whenever we face missing information that creates nonfundamentalness, we can provide the system with new information coming from additional series. More precisely, in Dynamic Factor models also if u_t is fundamental for the whole χ_t, it may not be fundamental for some subsamples of series. However, this is not a major problem in this context. Indeed, thanks to dynamic heterogeneity, the missing information due to local nonfundamentalness is completed with additional cross sectional information from other series and in this way we are able to recover u_t. Therefore, dynamic heterogeneity, which is natural in Dynamic Factor models, is precisely what we need for considering nonfundamentalness a non-generic problem.

Formally, let us consider the projection

$$f_t = \text{Proj}\,(f_t | f_{t-1}, f_{t-2}, \ldots, f_{t-m}) + w_t \quad \text{with} \quad m > 0\,, \tag{13}$$

where the prediction error w_t is fundamental by construction. From assumption (11), u_t is fundamental for f_t, therefore the representation $f_t = N(L)u_t$ has an equivalent VAR representation $A(L)f_t = Bu_t$. By comparing this last representation with (13) we get $w_t = Bu_t$. In many cases when there is dynamic heterogeneity, the information contained in the lagged values of f_t can be substituted by using cross sectional information, therefore one lag for $A(L)$ seems to be enough and we have the VAR(1) specification

$$f_t = Af_{t-1} + Bu_t\,.$$

Finally, we must notice that, when estimating a Dynamic Factor model, only the spaces spanned by the factors f_t and u_t are identified. However, given the property of fundamentalness of u_t for the whole χ_t, the true dynamic factors (interpreted as structural shocks) can be easily identified by imposing economic restrictions as in SVARs. Identification is then reduced to the choice of an orthogonal matrix R with only $q(q-1)/2$ parameters. Notice that, in contrast with the SVAR case, in order to achieve identification we simply need a small fixed number of restrictions without having to impose any limitation on the size of the panel.

The following simple example is taken from Forni et al. [2007]. Consider the case with only one dynamic factor loaded with one lag, therefore $q = 1$, $s = 1$, and $r = 2$. The common part of the i-th series is

$$\chi_{it} = (1 - c_i L)u_t = \Lambda_{i\cdot} f_t\,.$$

If we had homogeneous responses to the static factors f_t we would have $c_i = c$ for any i. In this case, we can easily see that

$$\text{rank}\,(\Lambda'\Lambda) = \text{rank} \begin{bmatrix} N & -Nc \\ -Nc & Nc^2 \end{bmatrix} = 1\,,$$

hence $\Lambda'\Lambda$ has not full rank. Since $N(L) = (1 - cL)$, fundamentalness is guaranteed only if we impose $|c| < 1$. In this case the problem of nonfundamentalness is pervasive.

Horizon (quarters)	Dynamic Factor Model			SVAR		
	Output	Consumption	Investment	Output	Consumption	Investment
1	0.37	0.30	0.07	0.45	0.88	0.12
4	0.37	0.30	0.07	0.45	0.88	0.12
8	0.78	0.87	0.72	0.68	0.83	0.40
12	0.86	0.90	0.80	0.73	0.83	0.43
16	0.89	0.91	0.83	0.77	0.85	0.44
20	0.91	0.92	0.86	0.79	0.87	0.46

Table 1: Fraction of the forecast-error variance attributed to the permanent shock. *Source*: Forni et al. [2007].

In order to have fundamentalness without any additional restrictions we need heterogeneity in the dynamics of the responses, i.e. $c_i \neq c_j$ for $i \neq j$. In this case $(\Lambda'\Lambda)$ has full rank since

$$\text{rank}\,(\Lambda'\Lambda) = \text{rank}\begin{bmatrix} N & -\sum_i c_i \\ -\sum_i c_i & \sum_i (c_i)^2 \end{bmatrix} = 2\,.$$

Moreover, now

$$f_t = \begin{bmatrix} 1 \\ L \end{bmatrix} u_t = N(L)u_t\,,$$

hence fundamentalness is always satisfied with $G(L) = (1\ 0)$. In this case indeed we can recover u_t from any couple of series as

$$u_t = \frac{c_j \chi_{it} - c_i \chi_{jt}}{c_j - c_i}\,.$$

Therefore u_t is fundamental for (χ_{it}, χ_{jt}) even if $c_i > 1$ for any i, i.e. even if u_t is not fundamental for χ_{it}.

Finally, let us review the empirical application in Forni et al. [2007]. The idea is the same as the one behind the empirical application by Giannone and Reichlin [2006] discussed in the previous section. In this case the benchmark SVAR is the one by King et al. [1991], which comprises output, consumption and investment. Forni et al. [2007] include these three variables into a much larger system composed by 89 variables in addition to the three of interest, and estimate a Dynamic Factor model with 3 dynamic common factors in analogy to the 3 shocks in the SVAR. Again, the model is estimated for different numbers of static factors and identified by imposing the same long-run restrictions as in the SVAR. Table 1 reports variance decomposition results for the Dynamic Factor model with 15 static common factors and for the SVAR by King et al. [1991]: the impulse response functions from the large system imply a larger effect of the permanent shock on output and investment than in the SVAR. This means that the typical VAR puzzle concerning the small amount of investment variance explained by supply shocks in the medium-long run might be due to the fact that the structural shocks associated with output, consumption and investment are nonfundamental.

8 Concluding remarks and further research

The standard practice in structural VAR analysis consists in assuming that the innovations of the estimated VAR are linear combinations of structural shocks, i.e. the zeroes of the matrix of the moving average representation of the model are not smaller than one in modulus.

However, as pointed out by Hansen and Sargent [1980, 1991], Lippi and Reichlin [1993, 1994], and Blanchard and Quah [1993], there may exist economically sensible theoretical models whose associated MA representation does not fulfill the above hypothesis. We have described examples of meaningful economic models which generate, endogenously or exogenously, nonfundamental representations: indeed, nonfundamental representations can arise in rational expectations models, in heterogeneous information models, in cointegrated models, but also in extremely simple models. In these cases, since VAR representations are fundamental by construction, the nonfundamental structural shocks cannot be identified by estimating and inverting a VAR. In other words, SVARs do not allow to recover the structural shocks in all those cases in which the structural shocks are functions not only of present and past values of observed variables, but also of future values, i.e. when the agent's information space is larger than the econometrician's one.

When there is an issue of nonfundamentalness, SVARs are not useful for discriminating among competing economic models. We have recalled Fernández-Villaverde et al. [2007] alternative definition of nonfundamentalness, which can be used as a test to check whether a given DSGE model produces a fundamental representation and therefore the impulse responses of the associated VAR are consistent with the theoretical impulse responses. If this is not the case, one can resort to Dynamic Factor models: indeed, as shown in Forni et al. [2007], if the data follow a factor structure the nonfundamentalness issue can be tested and made non generic by exploiting cross-sectional information.

An alternative strategy is to generate infinite nonfundamental representations from the only fundamental representation, estimated with a VAR, by means of Blaschke matrices, which are filters capable to flip the roots of a fundamental representation inside the unit circle. Lippi and Reichlin [1993] have applied this procedure, but much can still be done as far as the search for nonfundamental representations is concerned. For instance, we would like to identify a correspondence between the roots of a given MA representation of an economic model and the associated impulse responses by exploring whole regions of the parameter space. The same method would allow us to find theoretical impulse responses which may derive also from nonfundamental representations and are consistent both with the data and with the structural model. This is the subject of our current research.

Moreover, it would be interesting to discuss the identification of structural shocks within the framework of consensus VARs of the monetary transmission mechanism (MTM) as well as standard SVARs used for the estimation of the effects of fiscal policy shocks. These are cases in which it might not be possible to solve the identification problem by means of standard techniques. Clearly, monetary policy and fiscal policy are fields in which, in general, fundamentalness has no economic justification since structural models can produce nonfundamental representations insofar as agents are characterized by rational expectations. Indeed, being agents forward-looking they possess a wider information set than the econometrician and anticipate the effects of any foreseen future intervention by the Central Bank or the Government, let alone the large amount of information available to the agents but not included in a low-dimensional VAR. As a consequence, an unexpected intervention by the Central Bank might not necessarily coincide with the fundamental monetary shock identified in a VAR, and a VAR might yield misleading results on the effects of tax policies. As far as we know, very few studies are available where nonfundamental representations of the MTM are investigated: adopting such an approach, Klaeffing [2003] explains some puzzles concerning the effects of a monetary shock on output while Giannone et al. [2008], identifying nonfundamental shocks in the pre-Volcker period, show that the cause of the Great Moderation is not a decline in the volatility of the shocks. Beyer and Farmer [2007a] and Beyer and Farmer [2007b] show via simulations that determinate and indeterminate models may be observationally equivalent: indeed, nonfundamentalness is precisely the counterpart of indeterminacy of equilibria in the economic model since structural shocks can be nonfundamental (i.e. sunspot) if equilibria are indeterminate. On fiscal policy, a recent work by Leeper et al. [2008] provides evidence of fiscal foresight and shows that this intrinsic feature of the tax policy process implies nonfundamentalness in the econometric model.

References

A. Beyer and R. E. Farmer. What we don't know about the monetary transmission mechanism and why we don't know it. *Macroeconomic Dynamics*, 12(S1):60–74, October 2007a.

A. Beyer and R. E. A. Farmer. Testing for indeterminacy: An application to U.S. monetary policy: Comment. *American Economic Review*, 97(1):524–529, 2007b.

O. J. Blanchard and D. Quah. The dynamic effects of aggregate demand and supply disturbances. *American Economic Review*, 79(4):655–73, September 1989.

O. J. Blanchard and D. Quah. The dynamic effects of aggregate demand and supply disturbances: Reply. *American Economic Review*, 83(3):653–58, June 1993.

J. Boivin and M. Giannoni. DSGE models in a data-rich environment. NBER Working Papers 12772, National Bureau of Economic Research, Inc, December 2006.

W. A. Brock, S. N. Durlauf, and G. Rondina. Design limits and dynamic policy analysis. Unpublished, University of Wisconsin-Madison, 2008.

V. V. Chari, P. J. Kehoe, and E. R. McGrattan. A critique of structural VARs using business cycle theory. Staff Report 364, Federal Reserve Bank of Minneapolis, July 2005.

L. J. Christiano, M. Eichenbaum, and R. Vigfusson. Assessing structural VARs. NBER Working Papers 12353, National Bureau of Economic Research, Inc, July 2006.

C. Doz, D. Giannone, and L. Reichlin. A quasi maximum likelihood approach for large approximate dynamic factor models. Working paper series no 674, European Central Bank, 2006.

C. Favero and F. Giavazzi. Debt and the effects of fiscal policy. Working Papers 07-4, Federal Reserve Bank of Boston, 2007.

J. Fernández-Villaverde, J. Rubio-Ramírez, T. J. Sargent, and M. W. Watson. A, B, C's (and D)'s for understanding VARs. *American Economic Review*, 97(3):1021–1026, 2007.

S. Fischer. Long-term contracts, rational expectations, and the optimal money supply rule. *Journal of Political Economy*, 85(1):191–205, February 1977.

M. Forni and L. Reichlin. Dynamic commmon factors in large cross-sections. *Empirical Economics*, 21(1):27–42, 1996.

M. Forni, M. Hallin, M. Lippi, and L. Reichlin. The generalized dynamic factor model: identification and estimation. *The Review of Economics and Statistics*, 82(4):540–554, November 2000.

M. Forni, M. Hallin, M. Lippi, and L. Reichlin. The generalized dynamic factor model: one-sided estimation and forecasting. *Journal of the American Statistical Association*, 100(471):830–840, September 2005.

M. Forni, D. Giannone, M. Lippi, and L. Reichlin. Opening the black box: structural factor models with large cross-sections. Working Paper Series 712, European Central Bank, January 2007.

J. Galí. Technology employment and the business cycle: do technology shocks explain aggregate fluctuations? *The American Economic Review*, 89(1):249–271, March 1999.

D. Giannone and L. Reichlin. Does information help recovering structural shocks from past observations? *Journal of European Economic Association*, 4(2/3):455–465, April/May 2006.

D. Giannone, L. Reichlin, and L. Sala. Monetary policy in real time. In M. Gertler and K.Rogoff, editors, *NBER Macroeconomic Annual*. MIT Press, 2004.

D. Giannone, L. Reichlin, and L. Sala. VARs, common factors, and the empirical validation of equilibrium business cycle models. *Journal of Econometrics*, 132(1):257–279, May 2006.

D. Giannone, M. Lenza, and L. Reichlin. Explaining the great moderation - it is not the shocks. Working Paper Series 865, European Central Bank, Feb. 2008.

R. E. Hall. Stochastic implications of the life cycle-permanent income hypothesis. *Journal of Political Economy*, 86(6):971–987, December 1978.

E. Hannan and M. Deistler. *The statistical theory of linear systems*. Wiley, New York, 1988.

L. P. Hansen and T. J. Sargent. Formulating and estimating dynamic linear rational expectations models. *Journal of Economic Dynamics and Control*, 2(2):7–46, May 1980.

L. P. Hansen and T. J. Sargent. Two difficulties in interpreting vector autoregressions. In L. P. Hansen and T. J. Sargent, editors, *Rational Expectations Econometrics*, pages 77–120. Westview Press, Boulder, 1991.

K. Kasa. Forecasting the forecasts of others in the frequency domain. *Review of Economic Dynamics*, 3(4):726–756, October 2000.

K. Kasa, T. B. Walker, and C. H. Whiteman. Asset prices in a time series model with perpetually disparately informed, competitive traders. Caepr working papers, Center for Applied Economics and Policy Research, Economics Department, Indiana University Bloomington, September 2006.

R. G. King, C. I. Plosser, J. H. Stock, and M. W. Watson. Stochastic trends and economic fluctuations. *American Economic Review*, 81(4):819–40, September 1991.

M. Klaeffing. Monetary policy shocks - a nonfundamental look at the data. Working Paper Series 228, European Central Bank, May 2003.

E. M. Leeper, T. B. Walker, and S.-C. S. Yang. Fiscal foresight: Analytics and econometrics. NBER Working Papers 14028, National Bureau of Economic Research, Inc, May 2008.

M. Lippi and L. Reichlin. The dynamic effects of aggregate demand and supply disturbances: Comment. *American Economic Review*, 83(3):644–52, June 1993.

M. Lippi and L. Reichlin. VAR analysis, nonfundamental representations, Blaschke matrices. *Journal of Econometrics*, 63(1):307–325, July 1994.

A. Pagan. Techniques for building small macroeconomic models. Lecture notes, CIDE Summer School in Econometrics, June 2007.

G. Rondina. Incomplete information and informative pricing: Theory and application. Mimeo, University of Wisconsin, Madison, January 2007.

C. A. Sims. Macroeconomics and reality. *Econometrica*, 48(1):1–48, January 1980.

M. Watson. Vector autoregressions and cointegration. In D. L. McFadden and R. F. Engle, editors, *Handbook of Econometrics, Vol. IV*, pages 2843–2915. Elsevier Science, 1994.

Appendix: the search for nonfundamental representations

MA representations and Blaschke matrices

Nonfundamental representations can be generated by means of Blaschke matrices, which are defined as follows (see Lippi and Reichlin [1994] for additional details)

Definition A-1 (Blaschke Matrix) *A complex-valued matrix $B(z)$ is a Blaschke matrix if:*

1. *it has no poles inside the unit disc;*
2. $B(z)^{-1} = \overline{B'}(z^{-1})$, *where the bar indicates the matrix obtained by taking conjugate coefficients.*

Whenever we apply a Blaschke matrix to an MA process we get the new nonfundamental representation defined as

$$x_t = D(L)v_t = C(L)B(L)B(L)^{-1}u_t\,. \tag{A-1}$$

The main property of Blaschke transformations is that if u_t is an orthonormal white noise then $v_t = B(L)u_t$ is an orthonormal white noise if and only if $B(L)$ is a Blaschke matrix. This ensures that also for nonfundamental representations structural shocks will be uncorrelated, which is a necessary condition in all structural models. Thus (A-1) together with usual identification restrictions is still a valid structural model with new impulse responses that are not recoverable with an ordinary VAR.
As examples of Blaschke matrices we have the orthogonal matrices and the matrices with a Blaschke factor as one of the entries. A generic Blaschke matrix can be always written as the product of these two.

Theorem A-1 *Let $B(z)$ be an $N \times N$ Blaschke matrix then $\exists\ m \in \mathbb{N}$ and $\exists\ \alpha_i \in \mathbb{C}$ s.t. $|\alpha_i| < 1$ for $i = 1, \ldots, m$ and*

$$B(z) = \prod_{i=1}^{m} K(\alpha_i, L)\, R_i = \prod_{i=1}^{m} \begin{pmatrix} \frac{z-\alpha_i}{1-\overline{\alpha}_i z} & 0 \\ 0 & I_{N-1} \end{pmatrix} R_i\,, \tag{A-2}$$

where $R_i \overline{R}_i' = I_N$.

Note that $B(z)$ has poles in $(\overline{\alpha}_i)^{-1}$, i.e. outside the unit disc as required from definition 1.
With reference to (A-1), given a fundamental representation $x_t = C(L)u_t$, let us consider the zeroes of $\det C(z)$, which by definition are all outside the unit disc, and call them γ_i. We can build a nonfundamental representation just by applying a Blaschke matrix $B(L)$ to $C(L)$ with $\alpha_i = (\overline{\gamma_i})^{-1}$ for $i = 1, \ldots, m$ and $1 \le m \le N$. Theorem 1 tells us that $B(L)$ is taking zeroes of $C(L)$, that are outside the unit disc ($|\gamma_i| > 1$), into zeroes of $D(L)$ which are inside the unit disc ($|\alpha_i| = |(\overline{\gamma_i})^{-1}| < 1$).
Finally, note that $x_t = C(L)B(L)v_t$, therefore $B(L)^{-1}C(L)^{-1}x_t = v_t$, but, although $C(L)$ is invertible in the past (i.e. is fundamental) by construction, the inverse of a Blaschke matrix requires the use of L^{-1} (the forward operator), therefore it is impossible to recover v_t only from the past of x_t: this is nonfundamentalness.

ARMA representations

We now move to ARMA representations $M(L)x_t = C(L)u_t$, where $\det M(z)$ has no zeroes inside the unit disc in order to guarantee stationarity and causality for the AR part. The ARMA representation is fundamental if its MA part, $C(L)u_t$, is fundamental. Lippi and Reichlin [1994] look for different ARMA specifications where, while the AR part is completely identified, the MA part is identified up to a Blaschke matrix transformation. They point out how many examples of intertemporal maximization under rational expectations produce indeed such a situation, as discussed in section 4. If $C(L)$ is fundamental then its determinant has all $h \leq N$ roots α_i outside the unit disc, hence we can build nonfundamental representations $D(L)$ just by moving one or more roots of $\det C(z)$ from outside to inside the unit circle by means of a Blaschke matrix.
In order to do so, first define the subset $\Omega \in \mathbb{R}^h$ such that $\Omega = \{\omega = (\omega_1 \dots \omega_h) \,\text{s.t.}\, \omega_i = \pm 1\}$. We have the following theorem:

Theorem A-2 *For any possible $\omega \in \Omega$ there exist representations $M(L)x_t = P(L)v_t$ such that $\det P(z)$ has h roots β_i defined as*

$$\begin{array}{rcll} \beta_i & = & \alpha_i & \text{if } \omega_i = 1\,, \\ \beta_i & = & (\overline{\alpha_i})^{-1} & \text{if } \omega_i = -1\,. \end{array}$$

Moreover, if $P(L)$ and $Q(L)$ correspond to the same ω, then $P(L) = KQ(L)$ with K orthogonal, i.e. the two representations are unique up to a rotation.

Note that if at least one of the elements of ω is -1 then $P(L)$ will be a nonfundamental representation. All the nonfundamental representations obtained in this way are called basic. They come from an ARMA just by transforming the MA part while leaving untouched the AR part. Moreover, if we start from an ARMA(p,q) then all its basic representations are ARMA(p,q). Non-basic representations are obtained by multiplying the MA part $C(L)$ by an arbitrary Blaschke matrix. By doing so we increase the order of the MA and AR matrices and if γ is a nonfundamental root of the MA, then $(\overline{\gamma})^{-1}$ is a root of the AR part. Both common sense and literature suggest that this latter case is not likely to occur, thus it makes sense to search only for basic nonfundamental representations.

VAR representations

In general we always start from an estimated VAR, and, once inverted, we get an MA representation that by definition will be fundamental. However, from the latter we can always get nonfundamental representations that generate the impulse responses of our alternative theoretical model. This is the procedure followed by Lippi and Reichlin [1993] to generate impulse responses that represent technological diffusion under learning-by-doing dynamics. Such method is clearly explained by Lippi and Reichlin [1994]. If the true fundamental MA representation $x_t = C(L)u_t$ were known then all its nonfundamental counterparts would easily be recovered just by applying a Blaschke matrix as in (A-1). However, from an estimated VAR, $A(L)x_t = u_t$, we can only get the approximate ARMA representation as

$$(\det A(L))\, x_t = A_{ad}(L)u_t\,.$$

Its associated approximate MA representation is $x_t = T(L)u_t$ with $T(L) = (\det A(L))^{-1}\, A_{ad}(L)$. We have approximations because these are all finite order representations, although in theory they should have an infinite MA part or, viceversa, if the true MA were of finite order, then we should estimate an infinite VAR.

As an example, Lippi and Reichlin [1994] consider the following two-dimensional MA representation:

$$x_t = C(L)v_t = (I - CL)u_t\,.$$

They assume that $\det(I - Cz)$ has two roots α_1 and α_2, which by fundamentalness are both outside the unit disc ($|\alpha_i| > 1$). The VAR representation that we estimate is only the order p approximation

$$A(L) = I + \sum_{k=1}^{p} C^k L^k \simeq (I - CL)^{-1}\,.$$

It is possible to show that the $2p$ complex roots of $\det A(z)$ are

$$\alpha_i \exp\left(k\frac{2\pi i}{p+1}\right) \quad \text{for } i = 1, 2 \text{ and } k = 1, \ldots, p\,.$$

Therefore, the roots of the VAR are all on circles of radius $|\alpha_i| > 1$. If the roots of the MA are complex we have only one circle of roots, if instead they are real we have two circles. Here we consider the case of two complex conjugate roots $\alpha_1 = \overline{\alpha}_2$.
Actually, we are able only to get an estimate of $A(L)$, thus we cannot estimate directly the roots of $C(L)$. But we can determine the radius ρ of the circle where the roots of $A(L)$ lie. For every complex β such that $|\beta| = \rho$, we proceed as though β were a root of $T(L)$, which is only an approximation of $C(L)$. We therefore apply theorems 1 and 2 by multiplying $T(L)$, which is by construction a fundamental representation, by a Blaschke matrix in order to obtain a nonfundamental representation. First, we look for a rotation R such that $T(z)R$ has in its first column the factor $(z - \beta)$. R has to satisfy

$$[T(z)R]e_1 = (z - \beta)e_1 \tag{A-3}$$

where $e_1 = (1 \;\; 0)'$. Note that (A-3) is a condition only on the first column of R, while the second column is obtained just by using the orthogonality condition: $R\overline{R}' = I$. If the system were N-dimensional we would determine unambiguously only the first column of R while no rule exists for fixing all other columns besides the orthogonality condition. After rotating $T(z)$, we can move the root, that now is in the first column, from β to $(\overline{\beta})^{-1}$ with

$$K\left((\overline{\beta})^{-1}, L\right) = \begin{bmatrix} \frac{z-(\overline{\beta})^{-1}}{1-\beta^{-1}z} & 0 \\ 0 & 1 \end{bmatrix}. \tag{A-4}$$

We thus obtain a nonfundamental representation

$$x_t = T(L)B(L)B(L)^{-1}u_t \quad \text{where} \quad B(L) = RK\left((\overline{\beta})^{-1}, L\right)\,. \tag{A-5}$$

Actually, since we know only ρ, we need to repeat this procedure n times in order to explore all the circle of roots of $A(L)$. We choose $\beta = \rho\exp(ik\theta)$ with $\theta = \pi/n$, and $k = 1, \ldots, n-1$, n being the number of roots. Note however that since we consider all β on the circle we are taking in account not only the roots of $C(L)$ but also other values, therefore we are looking also for non-basic representations. This in turn implies that no uniqueness result as in theorem 2 holds in this case. Finally, we can study the impulse responses of the nonfundamental representations, see if some of them are economically sensible and possibly assess differences with the fundamental impulse responses $T(L)$. Although this is only an approximate procedure, it has delivered promising results in Lippi and Reichlin [1994].

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