

No Trade, Informed Trading, and Accuracy of Information*

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Abstract. We present a model in which there is uncertainty about realization of a risky asset value for an informed trader. Then, we show that the informed trader does not trade in equilibrium if the inside information the informed trader has is not sufficiently accurate. We use the framework presented by Glosten and Milgrom (1985) and extend the assumption that the informed trader knows the terminal value of the risky asset. Finally, we obtain the conditions under which the informed trader would not trade in equilibrium.

Key Words: Market microstructure; Glosten-Milgrom; Price formation; Asymmetric information; Bid–ask spreads.

JEL Classification Numbers: D82, G12.

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1. Introduction

Jones et al. (1994) suggested that future theoretical research needs to develop scenarios in which (i) both the frequency and size of trades are endogenously determined and yet, (ii) the size of trades has no information content beyond that contained in the number of transactions. Ozsoylev and Takayama (2010) presented a model in which the size of trades are endogenously determined. In this article, we present a model in which the frequency of trades is endogenously determined. The contribution of this article is to propose a model of “no-trade,” in which an informed trader does not trade as an equilibrium outcome. Milgrom and Stokey (1982) proposed a model in which the no-trade situation arises as the result of a violation of common knowledge. In this article, we propose a model such that even if common knowledge is assumed, an informed trader would not trade in equilibrium if the inside information that the informed trader has is not sufficiently accurate. We develop the model within the framework presented by Glosten and Milgrom (1985).

Here, we present a model that is very similar to the Glosten–Milgrom sequential trade model. There are two changes that we will make to the Glosten–Milgrom model. First, we allow the informed traders not to trade. Second, we assume that the informed trader has some uncertainty about the terminal value of the risky asset. This creates a situation where the market maker’s ask price is higher or the bid price is lower than the informed traders’ expected asset value. Moreover, we consider the static version of the sequential trade model. In this article, we focus on the situation where the informed trader does not trade in equilibrium. When we assume that all the random variables such as the value of the asset, the probability of informed trading, or the liquidity of the traders’ demand realizations are independent and identically distributed (i.i.d.) across all the periods, then each period is simply a replicate of other periods. Thus, we focus on one period as a representative of many identical periods and study the conditions under which there is a no-trade situation for the informed trader.

The model considers a market where a risky asset is traded between a market maker, strategic traders, and liquidity traders. First, the market maker, who is not informed of the risky asset payoff, quotes the bid and ask price. Then, either a strategic trader or a liquidity trader arrives in the market in a random manner. The liquidity trader’s trading motive is not related to the risky asset payoff at all, whereas the strategic trader has information on the risky asset payoff. In the model, there are two types of states. In one state called “a wide state”, the informed trader has more coarse information about the terminal value of the asset, whereas in the other state, “a narrow state”, the informed trader has better information. Then, we show that when the probability of a wide state is very low and the difference in the information accuracy is sufficiently large between the two states, the informed trader does not trade in

equilibrium. In other words, if the market is too volatile, even the informed trader does not trade.

The organization of our paper is as follows. The next section presents the model and the equilibrium concept. In Section 3, we study the conditions under which the informed trader does not trade and present our results.

2. The Model of Trading

There are three classes of risk-neutral market participants: a competitive market maker, an informed trader, and a liquidity trader. The game structure and the parameters of the joint distribution of the investor's state variables is common knowledge to all market participants. In each period, the market maker posts bid and ask prices equal to the expected value of the asset, conditional on the observed history of trades in equilibrium. The trader trades at those prices or possibly chooses not to trade. Trading occurs for T successive periods, after which all private information is revealed.

As noted above, there are two states of the world. These states, "wide" or "narrow", are denoted by W and N , respectively. A wide state occurs with probability ν and a narrow state occurs with probability $1 - \nu$. The terminal value of the asset is represented by $V \in \{0, 1\}$. The informed traders receive two signals, θ , about the asset's value, and τ , about the state of the world. The signal, θ , has two values, low (L) and high (H), where the probability that the signal is low is $1/2$. Let $p_\tau \in (0, 1/2)$ for each $\tau \in \{N, W\}$ such that $Pr(V = 1|H, \tau) = 1/2 + p_\tau$ in state τ and $Pr(V = 0|H, \tau) = 1/2 - p_\tau$ in state τ . Furthermore, $Pr(V = 1|L, \tau) = 1/2 - p_\tau$ in state τ and $Pr(V = 0|L, \tau) = 1/2 + p_\tau$ in state τ . We suppose that $p_W < p_N$. For the high signal, the conditional expected value of the asset is $\frac{1}{2} + p_\tau$, and for the low signal it is $\frac{1}{2} - p_\tau$. Thus, the conditional variance for the high signal is $(1/2 + p_\tau)(1 - 1/2 - p_\tau)^2 + (1/2 - p_\tau)(0 - 1/2 - p_\tau)^2 = \frac{1}{4} - p_\tau^2$ and the conditional variance for the low signal is $(1/2 - p_\tau)(1 - 1/2 + p_\tau)^2 + (1/2 + p_\tau)(0 - 1/2 + p_\tau)^2 = \frac{1}{4} - p_\tau^2$. Notice that in a wide state, the variance is larger than in a narrow state. In other words, in a narrow state, the probability of each signal is closer to certainty than is the case in a wide state. In this sense, we can think of p_τ as the accuracy of the informed trader's information. The market maker and the liquidity traders share common prior beliefs about the assets' values with a mean $V^* = 1/2$.

The probabilistic assumption about the states of the world can be interpreted as follows. The informed traders do not exactly know what is the terminal value of the asset. Instead, they obtain slightly better information about the value in the sense that they have "more accurate"

information prior to the market maker. If the state is wide, then, given a signal, the probability difference between the high and low values is wider than would be the case for the other state. Therefore, we can say that if a state is wider, the informed trader has less accurate information compared with a narrow state.

Let $E = \{S, N, B\}$ denote the set of possible trades available to the trader in each period, with e its generic element. That is, $e = B$ denotes a buy order, $e = S$ denotes a sell order, and $e = N$ denotes no trade. Let $\Delta(E)$ denote the set of probability distributions on E . The market maker posts an ask price and a bid price. The trader can choose to buy the asset at the ask price, or sell the asset at the bid price, or choose not to trade. Let α be the ask price posted and let β be the bid price. Let $p \equiv (\alpha, \beta) \in [0, 1]^2$.

We consider the following game: with probability μ , the informed trader will be chosen, and with probability $1 - \mu$, the liquidity trader will be chosen. If a trader is uninformed, his or her demand is determined by the random variable \tilde{Q} , which takes a value from E . We assume that $Pr(Q = e) = \gamma_e > 0$ for every $e \in E$. We suppose that all the random variables are mutually independent. The probability distribution is common knowledge to everybody in the model. In this paper, we simply consider a static model.

The timing structure of the trading game is as follows.

1. In period 0, nature chooses the realization $V \in \{0, 1\}$ of the risky-asset payoff \tilde{V} and the type of trader θ . The informed trader observes θ .
2. At the end of the game, the realization \tilde{V} is publicly disclosed, and consumption takes place.

For each type of trader, a trading strategy specifies a probability distribution over trades with respect to the ask and bid prices p . A strategy for the trader is defined as a function $\sigma : P \rightarrow \Delta(E)$. Let $\sigma(e|p)$ be the probability that σ assigns to action e conditional on p .

Now, we consider the market maker's belief. Let $\delta \in \Delta(\Theta)$ denote the market maker's prior belief; that is, $\delta(e)$ denotes the market maker's belief that the trader is the high type after observing trade e . Then, the (Bayesian) market maker's belief is updated through Bayes' rule; that is, for all $e \in E$:

$$\begin{aligned} \delta(e) &:= \Pr(\tilde{\theta} = H|e) \\ &= \Pr(\tilde{\theta} = H) \cdot \frac{\mu \sum_{\tau=N,W} \sigma_{He}^{\tau}(p) \cdot Pr(\tau) + (1 - \mu)\gamma_e}{\mu \sum_{\tau=N,W} \sum_{\theta \in \Theta} Pr(\tau) \Pr(\theta) \sigma_{\theta e}^{\tau}(p) + (1 - \mu)\gamma_e}. \end{aligned}$$

Definition 1 An informed trader's strategy profile σ^* is optimal for price rule p if it prescribes a probability distribution $\sigma_\theta^{\tau^*}$ over E for each $\theta \in \{L, H\}$ and $\tau \in \{N, W\}$ such that:

$$\sigma_\theta^{\tau^*}(p) \in \arg \max_{\sigma \in \Delta(E)} \left[\sum_{e \in E} \sum_{v \in \{0,1\}} Pr(V = v | \theta, \tau) \sigma_{\theta e}^\tau(p) (v - p) e \right].$$

Definition 2 An equilibrium consists of the market maker's prices, the informed traders' trading strategies, and posterior belief such that:

(P1) the bid and ask prices satisfy the zero-profit condition, given the posterior belief;

(P2) σ^* is the informed traders' optimal trading strategy for the price;

(B) the market maker's belief satisfies Bayes' rule.

3. No-trade in Equilibrium

We are interested in the situation where the informed trader in a wide state does not trade in equilibrium. If $\sigma_{HB}^N = 1$, $\sigma_{HN}^W = 1$, $\sigma_{LS}^N = 1$, and $\sigma_{LN}^W = 1$, then we obtain:

$$\begin{aligned} \delta(B) &= \Pr(\tilde{\theta} = H | B) = \frac{1}{2} \cdot \frac{(1-\mu)\gamma_B + \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)}; \\ \delta(S) &= \Pr(\tilde{\theta} = H | S) = \frac{1}{2} \cdot \frac{(1-\mu)\gamma_S}{(1-\mu)\gamma_S + 1/2 \cdot \mu(1-\nu)}. \end{aligned} \quad (1)$$

Observe that $\Pr(\tilde{\theta} = L | B) = 1 - \delta(B)$ and $\Pr(\tilde{\theta} = L | S) = 1 - \delta(S)$. Therefore, we obtain:¹

$$\begin{aligned} \alpha &= \sum_{\tau=N,W} Pr(\tau) (Pr(V = 1 | H, \tau) \cdot Pr(H | e = B) + Pr(V = 1 | L, \tau) \cdot Pr(L | e = B)) \\ &= \sum_{\tau=N,W} Pr(\tau) \left(\left(\frac{1}{2} + p_\tau \right) \cdot \frac{1}{2} \cdot \frac{(1-\mu)\gamma_B + \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)} + \left(\frac{1}{2} - p_\tau \right) \cdot \left[1 - \frac{1}{2} \cdot \frac{(1-\mu)\gamma_B + \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)} \right] \right) \\ &= \sum_{\tau=N,W} Pr(\tau) \left(\frac{1}{2} - p_\tau + p_\tau \cdot \frac{(1-\mu)\gamma_B + \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)} \right); \\ \beta &= \sum_{\tau=N,W} Pr(\tau) (Pr(V = 1 | H, \tau) \cdot Pr(H | e = S) + Pr(V = 1 | L, \tau) \cdot Pr(L | e = S)) \\ &= \sum_{\tau=N,W} Pr(\tau) \left(\left(\frac{1}{2} + p_\tau \right) \cdot \frac{1}{2} \cdot \frac{(1-\mu)\gamma_S}{(1-\mu)\gamma_S + 1/2 \cdot \mu(1-\nu)} + \left(\frac{1}{2} - p_\tau \right) \cdot \left[1 - \frac{1}{2} \cdot \frac{(1-\mu)\gamma_S}{(1-\mu)\gamma_S + 1/2 \cdot \mu(1-\nu)} \right] \right) \\ &= \sum_{\tau=N,W} Pr(\tau) \left(\frac{1}{2} - p_\tau + p_\tau \cdot \frac{(1-\mu)\gamma_S}{(1-\mu)\gamma_S + 1/2 \cdot \mu(1-\nu)} \right). \end{aligned} \quad (2)$$

¹To simplify notation, we write: $Pr(W) = \nu$ and $Pr(N) = 1 - \nu$.

Notice that if $\alpha > \frac{1}{2} + p_W$, then the high-type informed trader does not buy in state W and, similarly, if $\beta < \frac{1}{2} - p_W$, then the low-type informed trader does not sell in state W . By substituting α and β (2) into the two conditions and simplifying them, we obtain:

$$\begin{aligned} \sum_{\tau=N,W} Pr(\tau) \cdot \left(-p_\tau + p_\tau \cdot \frac{(1-\mu)\gamma_B + \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)} \right) &> p_W; \\ \sum_{\tau=N,W} Pr(\tau) \cdot \left(-p_\tau + p_\tau \cdot \frac{(1-\mu)\gamma_S}{(1-\mu)\gamma_S + 1/2 \cdot \mu(1-\nu)} \right) &< -p_W. \end{aligned} \quad (3)$$

In the end, we obtain:

$$p_W < (\nu p_W + (1-\nu)p_N) \cdot \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)} \quad (4)$$

$$-p_W > -(\nu p_W + (1-\nu)p_N) \cdot \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_S + 1/2 \cdot \mu(1-\nu)}. \quad (5)$$

Proposition 1 *In equilibrium, the following holds:*

(A) *the high-type informed trader buys in a narrow state but does not trade in a wide state if (4) holds;*

(B) *the low-type informed trader sells in a narrow state but does not trade in a wide state if (5) holds.*

Proof of Proposition 1: As the argument is symmetric, we will only prove the statement for (A). Similarly, with a wide state, we consider a narrow state. Then, the high-type informed trader buys if $\alpha < \frac{1}{2} + p_N$, and, similarly to the above, we obtain the following condition:

$$p_N > (\nu p_W + (1-\nu)p_N) \cdot \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)}. \quad (6)$$

Notice that $p_N > (\nu p_W + (1-\nu)p_N)$ as $p_N > p_W$ by assumption and also $\frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)} < 1$. Therefore, (6) always holds. Therefore, we can conclude that the high-type informed trader buys. This completes our proof. ■

Notice that $p_W < (\nu p_W + (1-\nu)p_N)$ and as $\frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)} < 1$, the condition (4) does not always hold. Whether it does hold depends on p_N , p_W , γ_B , or ν . If p_N is close to $\frac{1}{2}$, p_W is close to 0, and ν is low, then the informed trader's information is accurate with very high probability. Therefore, the prices are set very close to 0 or 1. Thus, the informed trader who has less accurate information would not trade. In other words, the informed trader does not trade in a wide state. This is true for both the buy and sell sides. The symmetry of the conditions (4) and (5) gives us the following corollary.

Corollary 1 *The informed trader does not trade in a wide state if $p_W < (\nu p_W + (1 - \nu)p_N) \cdot \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu) \max\{\gamma_B, \gamma_S\} + 1/2 \cdot \mu(1-\nu)}$.*

Proof of Corollary 1: By Proposition 1, the informed trader in a wide state does not trade if (4) and (5) both hold. The condition (5) can be rewritten as: $p_W < (\nu p_W + (1 - \nu)p_N) \cdot \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_S + 1/2 \cdot \mu(1-\nu)}$. Therefore, we can say that the informed trader in a wide state does not trade if $p_W < (\nu p_W + (1 - \nu)p_N) \cdot \min\left\{\frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_B + 1/2 \cdot \mu(1-\nu)}, \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_S + 1/2 \cdot \mu(1-\nu)}\right\}$. This gives us the desired result. ■

It is worth mentioning the relationship between the no-trade situation and the probability of informed trading (μ). If μ is sufficiently high, then $\frac{1/2 \cdot \mu(1-\nu)}{(1-\mu) \max\{\gamma_B, \gamma_S\} + 1/2 \cdot \mu(1-\nu)}$ is also close to 1. Therefore, the possibility of no-trade increases because $p_W < (\nu p_W + (1 - \nu)p_N) \cdot \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu) \max\{\gamma_B, \gamma_S\} + 1/2 \cdot \mu(1-\nu)}$ would hold for wider range of p_W . This is because if the probability of informed trading is higher, the prices reflect true values more. Therefore, the ask price is set higher and the bid price is set lower. Therefore, the informed trader who has more coarse information would not trade.

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