When Less is More: Rationing and Rent Dissipation in Stochastic Contests*

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September 8, 2010

Abstract

This paper shows how to maximize revenue when a contest is noisy. We consider a case where two or more contestants bid for a prize in a stochastic contest with proportional probabilities, where all bidders value the prize equally. We show that by fixing the number of tickets, thus setting a limit to total expenditures, it is possible to maximize the auctioneer's revenue and obtain (almost) full rent dissipation. We test this hypothesis with a laboratory experiment. The results indicate that, as predicted, revenue is significantly higher in a lottery with rationing than in a standard lottery. On the other hand, an alternative rationing mechanism that does not limit total expenditures fails to increase revenue relative to a standard lottery.

Keywords: Stochastic Contests; Rent Seeking; Laboratory Experiments.
JEL codes: C91; D44.

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*We would like to thank Gabriele Camera, Jeffrey Carpenter, Tim Cason, John Duffy, William Masters, Peter Matthews, Vai-Lam Mui and Lise Vesterlund for helpful comments and suggestions. We are also grateful to seminar participants at the University of Pittsburgh, Purdue University and Middlebury College.

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1 Introduction

Gordon Tullock (1980) famously conceived rent seeking as a lottery where lobbysts compete for a prize held by a politician. The prize could be, for instance, a monopoly priviledge, favourable legislation or a government contract, while lobbysts' bids consist of non-refundable investments that could take the form of campaign contributions, gifts or explicit briberies. Each lobbyist's probability of winning is equal to her lobbying expenditure divided by total lobbying expenditure. Similarly, lotteries are commonly employed to model, for example, rivalry and conflict (see, for example, Abbink et al., 2010), R&D tournaments, or market competition (e.g. Morgan et al., 2010). Moreover, a number of recent papers have explored the use of proportional contests as incentive mechanisms (Cason et al., 2010; Masters, 2005; Masters and Delbecq, 2008, among others). These tournaments have interesting applications as schemes to reward workers in firms or elicit effort among suppliers. It is worth noticing that, if contestants are risk neutral, proportional tournaments are isomorphic to a lottery.

The other typical way in which contests have been modeled is as all-pay auctions, in which the highest bid is awarded the prize with certainty (see, for instance, Lazear and Rosen, 1981). Both a lottery and an all-pay auction can be interpreted as extreme cases of a class of contests described by a Tullock's success function where the exponent ρ varies between 1 and ∞, respectively. Which type of contest is a more appropriate description of, say, rent seeking activities or political competition is an interesting empirical question which has not been fully addressed yet. It is well known, though, that contrary to an all-pay auction, rents are not fully dissipated in lotteries. Under-dissipation has been commonly interpreted as a result of the stochastic element of the lottery.[1]

Despite their inefficiency, lotteries are ubiquitous as fundraising mechanisms and their origins are so old that they can hardly be traced back in time. There exist virtually innumerable examples of lotteries used to raise funds for civic or charitable purposes.[2] Just like national lotteries today help fund charitable causes, the Great Wall of China, the Republic of Milan's

[1] See, for example, in the context of fundraising mechanisms, Goeree et al. (2005, p. 903): "A major difference [between lotteries and all-pay auctions] is that lotteries are not, in general, efficient; i.e., they do not necessarily assign the object for sale to the bidder who values it the most. Indeed, even in symmetric complete information environments in which efficiency plays no role, lotteries tend to generate lower revenues because the highest bidder is not necessarily the winner."

[2] A recent literature has analyzed the use of lotteries as fundraising mechanisms for the provision of public goods, both theoretically and through experiments (see Morgan, 2000; Morgan and Sefton, 2000; Goeree et al., 2005; Landry et al., 2006; Lange et al., 2007; Orzen, 2008; Schram and Onderstal 2009; Corazzini et al., 2010, among others).

ongoing war against Venice in the fifteenth century, and the bridges, canals and fortifications of Burgundian and Dutch cities were all financed through public lotteries (see Welch, 2008). However, it would be wrong to exaggerate the importance of their public good component. As reported by the historian Evelyn Welch, "after the failure of the Milanese lottery to appeal to a sense of public duty, no Italian lottery (even those run by religious organizations) referred to either civic pride or spiritual devotion when encouraging ticket purchases. Buyers were simply enticed by the chance of winning wealth" (Welch, 2008, p. 97).

Indeed, although it is perhaps less known, it was equally frequent in the past for individuals or private institutions to hold lotteries and raffles to sell objects or raise revenue for private causes. In 1446, the widow of the Flemish painter Jan Van Eyck held one of the first recorded European lotteries to sell her late husband's expensive paintings, for which buyers could not be readily found. In Venice, during the sixteenth century, private lotteries were held daily for speculative reasons to sell objects as well as silver and gold. In seventeenth-century London, lotteries were commonly used to sell "books, maps and other goods" (Welch, 2008). The "running lotteries" of the Virginia Company are perhaps one of the most notorious examples of private lotteries. Between 1612 and 1621 the Virginia Company of London, a joint stock company, funded its enterprise through a series of local lotteries throughout England (Johnson, 1960).[3]

The question we address in this paper is how an auctioneer, a fundraiser[4] or a politician[5] can maximize revenue when the contest is noisy. We show that the intuition that under-dissipation derives from the inefficiency of a stochastic contest is incorrect. The reason why rents are not fully dissipated in a Tullock lottery is that the total number of tickets is determined endogenously. Hence, each additional ticket purchased has a positive but decreasing marginal effect on the probability of winning the prize. As a consequence, in equilibrium, each contestant bids less than the expected value of the prize, and the overall rate of dissipation is less then one. For example, denoting the number of contestants with N and the prize with Π, in equilibrium each

[3]The lotteries were extremely successful, so much so that the annual financial report for 1620 states that they brought to the company a profit of £7,000. This was a very substantial sum, considering that the cost of furnishing one ship amounted at the time to about £791 (see Johnson, 1960).

[4]Although the literature on fundraising mechansisms has focused on situations where the contestants (significantly) value the public good, as we mentioned earlier, it is plausible to think of scenarios in which society values the project financed through their bids, but the bidders' valuation of it is actually zero.

[5]Typically the literature on rent seeking has considered that contestants' effort to earn rents is wasteful and it is therefore socially optimal to minimize it. Nevertheless, it is obvious that the politician's goal is to maximize the revenue acquired from the lobbyists.

participant bids $\frac{N-1}{N}\frac{\Pi}{N}$, total expenditures are $\frac{N-1}{N}\Pi$ and the overall rate of dissipation is $\frac{N-1}{N} < 1$.[6]

On the other hand, by exogenously setting the number of existing tickets, each additional ticket has a constant marginal effect on the probability of winning the prize. Provided the total cost of the tickets is less than the value of the prize, contestants will have an incentive to buy all the tickets. We show that by appropriately setting the total number of tickets, the auctioneer can increase rent seeking expenditures. To the limit, by setting the total value of the tickets arbitrarily close to the value of the prize, it is possible to obtain full dissipation of rents. This is why *less is more*: in equilibrium, setting a limit to total expenditures by rationing tickets leads to an *increase* in revenue relative to a standard lottery.[7]

In order to define a rationing scheme, we need a rule specifying how tickets are allocated when demand exceeds supply. We consider a simultaneous move game, and assume that, when the capacity constraint is binding, each player is allocated a share of the tickets equal to her share of total demand. Given this allocation rule, the most intuitive scheme is one where bidders only pay for the tickets they actually receive. We call this mechanism *fixed price rationing*. It should be noted that, although this rationing scheme may lead to an increase in revenue relative to the standard lottery, it limits the maximum revenue a seller can potentially raise, albeit off equilibrium. In order to address this issue, we also consider an alternative rationing scheme where bidders pay for the tickets they demand, while only receiving a share of the tickets equal to their share of total demand. We call this scheme *variable price rationing*. This mechanism shares the features of both fixed price rationing and a standard lottery. When demand is less than supply it is identical to fixed price rationing, but when demand exceeds supply it works just like a standard lottery. Consequently, it does not limit potential revenue. We show that both rationing schemes lead to the same dissipation rate in equilibrium. However, while in fixed price rationing there is a unique equilibrium in dominant strategies, in variable price rationing there exists a continuum of multiple equilibria.

We test these theoretical predictions with a laboratory experiment, fo-

[6] Note that the marginal effect of buying an extra ticket on the probability of winning the prize decreases faster as the number of contestants is small. Therefore, ceteris paribus, the higher the number of contestants, the higher the expected rate of dissipation. As $N \to \infty$, the expected rate of dissipation tends to 1.

[7] It should be observed that, by fixing supply, the auctioneer sets both a ceiling *and* a floor for the number of tickets. Although it is the presence of the floor that drives the result of (almost) full rent dissipation, we use the term rationing to emphasize the fact that the maximum number of tickets is fixed and there is an upper limit to potential revenue. This is in contrast to a standard lottery, where the maximum number of tickets is determined by demand and revenue is potentially unlimited.

cusing on a case where two contestants bid for a prize with a common value in a stochastic contest with proportional probabilities. Using a between-subjects experimental design, we test the hypothesis that expenditures are higher with either rationing mechanism than with the standard lottery. The results indicate that, as predicted by the theory, expenditures with fixed price rationing are significantly higher than in the standard lottery. However, contrary to the theoretical predictions, the variable price rationing mechanism does not increase expenditures relative to the standard lottery. The results also lead to reject the hypothesis of revenue equivalence for the two rationing schemes: expenditures are significantly higher with fixed price rationing than with variable price rationing. We interpret this result as reflecting strategic uncertainty, given the existence of multiple equilibria under variable price rationing.

Our work is related to a growing body of experimental research on rent seeking (Millner and Pratt, 1989, 1991; Shogren and Baik, 1991; Davis and Reilly, 1998; Potters et al., 1998; Anderson and Stafford, 2003; Shupp, 2004; Schmitt et al., 2004; Schmidt et al., 2006; Bullock and Rutström, 2007; Herrmann and Orzen, 2008; Kong, 2008; Matros and Lim, 2009, among others).[8] With relatively few exceptions (see Shogren and Baik, 1991; Shupp, 2004; Schmidt et al., 2004), these experiments typically report higher rent seeking expenditures than predicted by the theory. In the standard lottery treatment, we too find higher rent dissipation than predicted. However, rent dissipation is significantly higher when fixed price rationing is applied.

There also exists a parallel theoretical literature that considers a very similar issue in the case of all-pay auctions (Che an Gale,1998; Kaplan and Wettstein, 2006, among others). In a seminal paper, Che and Gale (1998) analyze a lobbying game between two lobbyists and a politician, and show that if the lobbyists have different valuations, then rents are not fully dissipated. They demonstrate that setting caps on *individual* bids can produce the perverse effect of increasing rent dissipation. Interestingly, in a subsequent paper, Fang (2002) shows that limiting individual bids does not decrease rent dissipation in a lottery. The difference between Che and Gale's idea and ours is that under-dissipation is driven by heterogeneous valuations in their setting, and by the decreasing marginal benefit of bidding in ours.

The remainder of the paper is organized as follows. Section 2 outlines the theoretical framework. Section 3 describes the experimental design and procedures. Section 4 presents the results. Section 5 concludes.

[8]For an excellent survey of this literature see Morgan et al. (2010).

2 Theory

Consider a Tullock contest where n risk-neutral players compete for a prize Π they all equally value. Players bid for the prize. We call b_i individual i's bid, while we will refer to $B = \sum_{i=1}^{n} b_i$ and $B_{-i} = \sum_{j \neq i}^{n} b_j$ as the total bids and the sum of all bids but i's, respectively. Player i's probability of winning the prize is $\frac{b_i}{B}$ and her expected payoff from bidding b_i is given by $\frac{b_i}{B}\Pi - b_i$. It is well known that in equilibrium $B^* = \frac{n-1}{n}\Pi$, meaning that rents are not fully dissipated. Differentiating i's expected payoff with respect to b_i we obtain

$$\frac{B_{-i}}{(b_i + B_{-i})^2}\Pi - 1. \tag{1}$$

Notice that in equilibrium it must be the case that $b_i + B_{-i} > 0$, otherwise an agent would have an incentive to bid ε arbitrarily close to zero and win the prize for sure. Setting (1) equal to zero and rearranging it we have $(b_i + B_{-i})^2 = B_{-i}\Pi$. Replacing B_{-i} with $(n-1)b_i$ we obtain $b^* = \frac{n-1}{n^2}\Pi$ and $B^* = \frac{n-1}{n}\Pi$. As proved by Fang, this equilibrium is unique (see Theorem 1 in Fang, 2002).

The first term in expression (1) represents i's marginal benefit of bidding an extra unit. Note that it is decreasing in b_i and is equal to the marginal cost when $b_i = \sqrt[2]{B_{-i}\Pi} - B_{-i}$. This means that, for a given $B_{-i} < \Pi$, the optimal bid for player i is smaller than $\Pi - B_{-i}$, resulting in less than full dissipation. The intuition for this result is that bidding an extra unit not only increases an individual's bid, but also the sum of all bids, thus making the marginal increase in the chance of winning decreasing in b_i.

Such a contest has been typically interpreted as a lottery in which every player is entitled to a number of lottery tickets equal to her bid. Once players have submitted their bids, a ticket is randomly drawn and the holder wins the prize. Note that no limit is placed to players' bids and thus to the number of tickets that can be purchased.

We modify the above game by fixing the total number of existing tickets, so that if some tickets remain unsold there is a positive probability that no one wins the prize. Call κ the fixed number of tickets, with $\kappa < \Pi$. If agent i buys $b_i \leq \kappa$ tickets, her expected payoff is now defined by $\frac{b_i}{\kappa}\Pi - b_i$, independently of other players' bids. Note that the marginal benefit of buying an extra ticket is always greater than the marginal cost. Hence, although the number of tickets a player can purchase will be subject to their availability, each agent would want to buy all the tickets. As a result, in equilibrium no ticket will remain unsold. Since κ can be set arbitrarily close to Π, rents can be (almost) fully dissipated.[9] Notice that this mechanism maintains the inefficiency of a

[9]Note that the increase in rent dissipation is not driven by rationing per se, but by the

Tullock contest, in that the highest bid is not necessarily awarded the prize. This illustrates that under-dissipation of rents in a lottery is not a result of inefficiency, but rather a consequence of the decreasing marginal benefit of bids.

We consider a simultaneous move game. We focus on the case in which $\kappa \in (\frac{n-1}{n}\Pi, \Pi)$,[10] and we assume that each individual has the same endowment $\omega \geq \frac{\kappa}{n}$. Notice that it is necessary to define a rule specifying how to allocate the tickets in case total demand exceeds the fixed supply ($B > \kappa$). We apply the following rule. If total bids exceed κ, then each agent receives a share of the κ tickets equal to her share of the total demand ($\frac{b_i}{B}$). This implies that individual i's expected prize from bidding b_i, when the sum of all other players' bids is B_{-i}, is given by:

$$E[\Pi, \kappa, b_i, B_{-i}] = \begin{cases} \frac{b_i}{\kappa}\Pi & \text{if } B \leq \kappa \\ \\ \frac{b_i}{B}\Pi & \text{if } B > \kappa \end{cases}.$$

Finally, we need to define an expenditure rule specifying how much players pay for the tickets they purchase as a function of the tickets they demand, conditional on the other players' demands. Call $x(b_i \mid B_{-i})$ such function. Hence a player's expected payoff is defined by

$$\omega + E[\Pi, \kappa, b_i, B_{-i}] - x(b_i \mid B_{-i})$$

We consider two rationing mechanisms, defined by the different expenditure function they apply.

Definition 1 *Fixed price rationing: $x(b_i \mid B_{-i})$ is equal to b_i if $B \leq \kappa$, while it is equal to $\frac{b_i}{B}\kappa$ if $B > \kappa$.*

Definition 2 *Variable price rationing: $x(b_i \mid B_{-i})$ is always equal to b_i.*

With fixed price rationing agents pay only for the tickets they receive and the price of each ticket is equal to 1. With variable price rationing each player pays her own bid, independently of the number of tickets actually allocated to her. If total bids are less than or equal to κ, the price of each ticket is 1.

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winning ticket being drawn from the exogenously fixed set of tickets. Hence, unless all of the tickets are bought, there is a positive probability that no one wins the prize.

[10] Clearly, if $\kappa \leq \frac{n-1}{n}\Pi$ the auctioneer would raise a revenue less than or equal to the amount raised with a standard lottery, while if $\kappa > \Pi$ no one would bid anything. If $\kappa = \Pi$, it is easy to prove that in equilibrium total expenditures could assume any value less than or equal to Π.

But, if total demand exceeds supply, each agent receives $\frac{b_i}{B}\kappa$ tickets, which is equivalent to letting the price of each ticket vary to accommodate demand. Indeed, note that if $B > \kappa$ then a ticket's price is equal $\frac{B}{\kappa}$.

As stated in the following proposition, with fixed price rationing there exists a unique equilibrium in dominant strategies.

Proposition 1 *With fixed price rationing there exists a unique Nash equilibrium in dominant strategies in which each player bids her whole endowment and total expenditure is equal to κ.*

Proof. It is easy to show that bidding ω is a dominant strategy for this game. Let us first consider the case $\omega + B_{-i} \leq \kappa$. An individual's expected payoff from bidding ω is equal to $\frac{\omega}{\kappa}(\Pi - \kappa)$, while, if she bids $z < \omega$, her expected utility is $\frac{z}{\kappa}(\Pi - \kappa)$. Since $\kappa < \Pi$, the latter is strictly less than the former.

Suppose now that $\omega + B_{-i} > \kappa$. Clearly, if $B_{-i} = 0$ then bidding $z < \omega$ would either guarantee the same utility $\Pi - \kappa$, if $z \geq \kappa$, or a lower expected payoff otherwise. Hence suppose $B_{-i} > 0$. The expected payoff from bidding ω is equal to $W = \frac{\omega}{\omega + B_{-i}}(\Pi - \kappa)$. Vice versa a player's expected utility from bidding $z < \omega$ is represented either by $X = \frac{z}{z + B_{-i}}(\Pi - \kappa)$, if $z + B_{-i} > \kappa$, or by $Y = \frac{z}{\kappa}(\Pi - \kappa)$ if $z + B_{-i} \leq \kappa$.

Let us compare W and X. Their difference is given by

$$W - X = \frac{B_{-i}(\omega - z)(\Pi - \kappa)}{(\omega + B_{-i})(z + B_{-i})} > 0$$

which means that bidding ω strictly dominates any lower bid z when $z + B_{-i} > \kappa$.

When $z + B_{-i} \leq \kappa$, we have to study the sign of $W - Y$, given by

$$\begin{aligned} W - Y &= \frac{\omega}{\omega + B_{-i}}(\Pi - \kappa) - \frac{z}{\kappa}(\Pi - \kappa) \\ &= (\Pi - \kappa)\frac{\omega(\kappa - z) - zB_{-i}}{(\omega + B_{-i})\kappa} \end{aligned}$$

The above expression is greater than zero if $\omega > \frac{zB_{-i}}{(\kappa - z)}$. Since $B_{-i} \leq \kappa - z$, this inequality is always true. This proves that bidding ω is a dominant strategy. Finally, as we are assuming that $\omega \geq \frac{\kappa}{n}$, total expenditure is equal to κ. ■

The next proposition shows that also with variable price rationing total expenditure in equilibrium is equal to κ. In this case though, there exist multiple equilibria. We provide a full characterization of the pure strategy equilibria of the game.

Proposition 2 *The complete set of pure strategy equilibria of the variable price rationing mechanism is represented by the strategy profiles* $\{b_1^*, b_2^*, ..., b_n^*\}$ *such that* $\sum_{i=1}^{n} b_i^* = \kappa$ *and* $\frac{\kappa(\Pi-\kappa)}{\Pi} \leq b_i^* \leq \omega \; \forall i \in \{1, ..., n\}$*. There always exists a symmetric equilibrium in which every player bids* $\frac{\kappa}{n}$*. Such equilibrium is unique if* $\omega = \frac{\kappa}{n}$*.*

Proof. We first show that in equilibrium the sum of all bids cannot be strictly less or greater than κ. If $b_i + B_{-i} < \kappa$ then player i would have an incentive to increase her bid from b_i to $\kappa - B_{-i}$, since the marginal benefit is greater than the marginal cost. If $b_i + B_{-i} > \kappa$ player i's utility is equal to $\Pi \frac{b_i}{b_i + B_{-i}} - b_i$, which is the payoff of a standard Tullock contest. Since we know that in a Tullock game in equilibrium $B = \frac{n-1}{n}\Pi$, and we assumed that $\frac{n-1}{n}\Pi < \kappa < \Pi$, it follows that B cannot be greater than κ.

It remains to explore the case $B = \kappa$. In order for a strategy profile $\{b_1, b_2, ..., b_n\}$, with $\sum_{i=1}^{n} b_i = \kappa$, to be an equilibrium we have to verify under what conditions no player has an incentive to deviate. Clearly, an agent who is submitting a positive bid has no incentive to decrease it. However, it is possible that a player may be better off by increasing her bid. Recall that, keeping the strategies of the other agents constant, if i raises her bid to $\hat{b}_i > b_i$ her payoff is $\Pi \frac{\hat{b}_i}{\hat{b}_i + B_{-i}} - \hat{b}_i$. Differentiating i's payoff with respect to $\hat{b}_i$ we obtain

$$\Pi \frac{B_{-i}}{(\hat{b}_i + B_{-i})^2} - 1$$

Setting the above expression equal to zero and solving for $\hat{b}_i$ we obtain i's best response function

$$\hat{b}_i = \sqrt[2]{\Pi B_{-i}} - B_{-i}$$

This means that i will not have an incentive to raise her bid above b_i provided that $b_i \geq \sqrt[2]{\Pi B_{-i}} - B_{-i}$. Since $\sum_{i=1}^{n} b_i = \kappa$, we can substitute B_{-i} with $\kappa - b_i$ and, rearranging, we obtain

$$b_i \geq \frac{\kappa(\Pi - \kappa)}{\Pi}$$

It follows that the complete set of pure strategy equilibria for this game is represented by the strategy profiles $\{b_1^*, b_2^*, ..., b_n^*\}$ such that $\sum_{i=1}^{n} b_i^* = \kappa$ and $\frac{\kappa(\Pi-\kappa)}{\Pi} \leq b_i^* \leq \omega \forall i \in \{1, ..., n\}$. Notice that the symmetric equilibrium in which every player bids $\frac{\kappa}{n}$ is always an element of this set, and it is the unique equilibrium if $\omega = \frac{\kappa}{n}$. ■

Note that variable price rationing resembles a coordination game in which players want to coordinate on a total demand equal to κ, although each one of them would prefer to demand as many tickets as possible, provided that B does not exceed κ. Due to this strategic uncertainty, the equilibrium prediction is clearly less robust than in fixed price rationing, where it is a dominant strategy to bid all the endowment. Yet, it is worth noting that while fixed price rationing limits to κ the total revenue that can be extracted from the bidders, variable price rationing does not place any limit to the total revenue, albeit off equilibrium. This makes the latter mechanism directly comparable with a standard lottery, where the auctioneer can potentially earn a higher revenue than predicted.

3 The Experiment

The experiment is designed to test the hypothesis that by exogenously setting the number of tickets in a lottery it is possible to increase contestants' expenditures. The experimental task is a standard rent seeking game as in Tullock (1980). We implement three treatments: a standard lottery, used as a benchmark, a lottery with fixed price rationing and a lottery with flexible price rationing. In this section, we describe the design of the experiment, the hypotheses to be tested and the experimental procedures.

3.1 Baseline Game

Two agents compete by expending resources (buying lottery tickets) to influence the probability of acquiring a given rent. Each session consists of 20 rounds. In each round, subjects have an endowment of 800 points and have to decide simultaneously how many lottery tickets they want to buy. Each ticket costs 1 point. At the end of each round the computer selects randomly the winning ticket among all the existing tickets. The owner of the winning ticket wins a prize of 1600 points. In case no tickets are purchased, no one wins the prize. Actual earnings are determined on the basis of one round randomly selected at the end of the session.

3.2 Treatments

The experimental design is based on three treatments, implemented between subjects, aimed at comparing the effects of alternative allocation mechanisms that differ in the way participants can determine their probability of winning the prize (see Section 2 for details):

1. **Standard lottery** (LOT). Subjects can buy any number of tickets. At the end of each round the computer selects randomly the winning ticket *among all the tickets purchased*.

2. **Lottery with fixed price rationing** (RF). The total number of tickets is κ. If total bids (B) are not greater than κ, each subject receives a number of tickets equal to her bid (b_i) and wins with probability $\frac{b_i}{\kappa}$. If $B > \kappa$, each subject receives *a share of the* κ *tickets* equal to $\frac{b_i}{B}$, *pays only for the number of tickets she receives*, and wins with probability $\frac{b_i}{B}$.

3. **Lottery with variable price rationing** (RV). The total number of tickets is κ. If $B \leq \kappa$, each subject receives b_i tickets and wins with probability $\frac{b_i}{\kappa}$. If $B > \kappa$, each subject receives *a share of the* κ *tickets* equal to $\frac{b_i}{B}$, *pays for the number of tickets she demanded*, and wins with probability $\frac{b_i}{B}$.

3.3 Hypotheses

Table 1 presents the theoretical predictions for the three treatments in our experimental design, where we set the prize $\Pi = 1600$, the number of tickets under rationing $\kappa = 1200$, and the endowment of each subject $\omega = 800$. Note that κ is set as the average between the theoretical prediction for total expenditures in the standard lottery (800) and the value of the prize (1600). As a consequence, while in LOT total expenditures are 50% of the prize, in both RF and RV, total expenditures are 75% of the prize. Note that the sum of individual endowments within a group is equal to the value of the prize, so that individual expenditures as a percentage of the endowment are equivalent to group-level expenditures as a percentage of the prize.

Table 1: Theoretical predictions for the experiment

	LOT	RF	RV
Individual expenditure	400	600	600
Total expenditures	800	1200	1200
Dissipation rate	0.50	0.75	0.75

Note: $\Pi = 1600$, $\omega = 800$, $\kappa = 1200$. LOT=Standard lottery. RF=Fixed-price rationing. RV=Variable-price rationing. Figures are expressed in points (experimental units). The prediction for RV at individual level (row 1) is expected expenditure.

Defining μ_i as average expenditures in treatment i, the hypotheses to be tested can be stated as follows:

Hypothesis 1 - Less is more: Expenditures are higher with either rationing mechanism than with the standard lottery:

$$H_0 : \mu_{LOT} = \mu_{RF} \text{ vs } H_1 : \mu_{LOT} < \mu_{RF}$$

$$H_0 : \mu_{LOT} = \mu_{RV} \text{ vs } H_1 : \mu_{LOT} < \mu_{RV}$$

Hypothesis 2 - Revenue Equivalence: Expenditures are the same under fixed and variable price rationing:

$$H_0 : \mu_{RF} = \mu_{RV} \text{ vs } H_1 : \mu_{RF} \neq \mu_{RV}$$

3.4 Procedures

We implemented one session for each treatment, with 32 subjects participating in each session, for a total of 96 subjects. In each session, subjects were randomly assigned to a computer terminal at their arrival. To ensure public knowledge, instructions were distributed and read aloud (see the Appendix for the instructions). Moreover, to ensure understanding of the experimental design, sample questions were distributed and the answers privately checked and, if necessary, individually explained to the subjects.

A strangers matching mechanism was adopted to avoid strategic incentives. In each round, subjects were randomly and anonymously rematched in pairs. At the end of each round, subjects were informed of their own payoff, given by the initial endowment, minus the expenditure for buying the tickets, plus the prize if won. At the end of the last round, subjects were informed of their total payoff for each of the twenty rounds, and of the actual earnings in points and Euros determined on the basis of a randomly selected round. They were then asked to answer a short questionnaire on the understanding of the experiment and socio-demographic information, and were paid in private using an exchange rate of 100 points per Euro.

Subjects earned 14.5 Euro on average for sessions lasting about 50 minutes, including the time for instructions. Participants were undergraduate students of Economics recruited by e-mail using a list of voluntary potential candidates. The experiment took place in March 2010 in the Experimental Economics Lab of the University of Milan Bicocca. The experiment was computerized using the z-Tree software (Fischbacher, 2007).

4 Results

This section presents the experimental results. We start with a descriptive analysis of the main features of the data in the three treatments. We then present formal tests of the theoretical predictions, focusing on average expenditures over all subjects. Finally, we compare behavior at individual level across treatments.

4.1 Overview

Figure 1 compares overall mean and median relative expenditures (as a percentage of the endowment) for each of the three treatments.[11] Mean relative expenditures are 55% in LOT, 65% in RF and 49% in RV. Focusing on median relative expenditures, the results for LOT and RF are strikingly consistent with the theoretical predictions (50% and 75%, respectively). Contrary to the theoretical predictions, median relative expenditures in RV are the same as in the standard lottery (50%).

Figure 1: Overall expenditures, by treatment

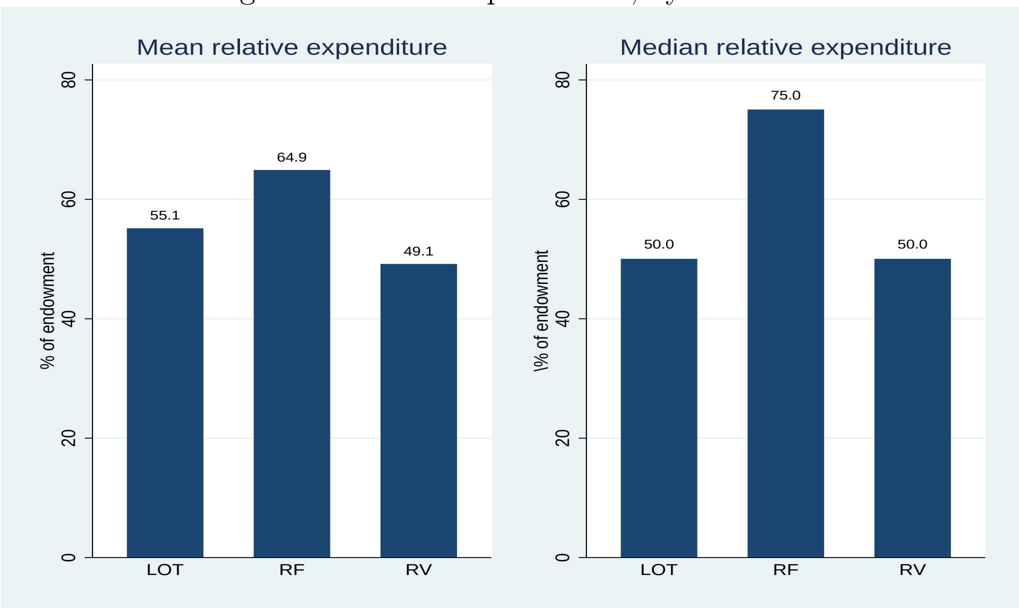
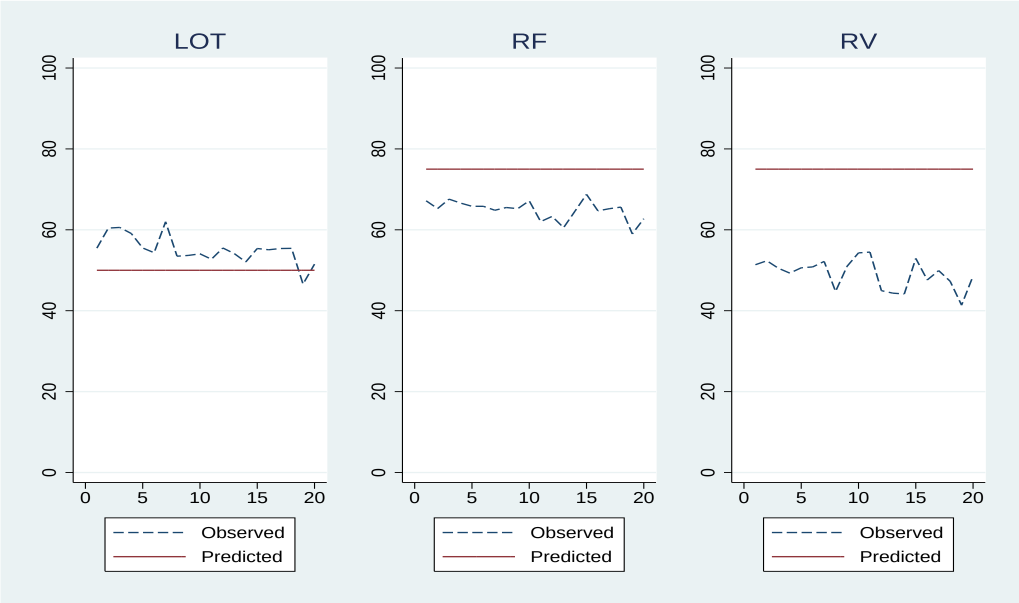


Figure 2 compares observed and predicted average expenditures, as a percentage of the endowment, over rounds for each treatment. Average expenditures are relatively stable in all treatments, displaying only a slight decline over successive rounds. In LOT, average relative expenditures are initially above the theoretical predictions, but gradually converge to 50% over successive rounds. In RF, average expenditures are relatively stable and slightly lower than the theoretical prediction throughout the session. In RV,

[11]Note that in presenting the results we refer to individual-level expenditures as a percentage of the endowment. Given the parameter calibration ($N = 2$, $\omega = 800$, $\Pi = 1600$), this is equivalent to referring to group-level expenditures as a percentage of the prize.

average expenditures are substantially lower than the 75% prediction and declining over rounds.

Figure 2: Average expenditures over rounds, by treatment



Overall, these descriptive results indicate that, as predicted by the theory, while the standard lottery produces substantial under-dissipation, fixed price rationing provides an effective mechanism to increase revenue. On the other hand, contrary to the theoretical predictions, the variable price rationing mechanism does not increase expenditures relative to the standard lottery.

4.2 Average Expenditures

In order to assess the statistical significance of the differences observed in expenditures under alternative allocation mechanisms, Table 2 presents results of Wilcoxon rank-sum tests of the null hypothesis that median expenditures are the same across treatments. The first two rows test the effect of rationing (Hypothesis 1), by comparing expenditures in each of the two rationing mechanisms (fixed and variable price) with the benchmark standard lottery. Given that the theory predicts the direction of departure from the null hypothesis, we use the relevant one-sided tests. The third row tests revenue equivalence (Hypothesis 2) by comparing expenditures in RF and RV, using a two-sided alternative.

It is important to note that the random rematching mechanism implies that subject-level observations might be dependent in rounds beyond the first, due to repeated interaction with a strangers matching mechanism and (limited) feedback. However, given that there are 32 subjects in each session, subjects know that there is a very small probability of interacting with the same subject as in the previous round(s). In addition, at the end of each round subjects only learn whether they won the prize, without being informed of the actual bid of the other subject. The dependence across individual observations can therefore be considered negligible, so that the test results we present are based on the assumption of independent individual observations (see Morgan and Sefton, 2000, for a similar approach). As a robustness check, we also present test results based on the first round only, so that individual observations are independent by construction.

Table 2: Expenditures across treatments: tests of hypotheses

	1-20	1	1-10	11-20
RF - LOT	1.46	1.40	1.38	1.32
(p-value)	(0.07)	(0.08)	(0.08)	(0.09)
RV - LOT	-0.77	-0.53	-1.01	-0.54
(p-value)	(0.78)	(0.70)	(0.84)	(0.70)
RF - RV	2.87	2.29	2.67	2.74
(p-value)	(0.00)	(0.02)	(0.01)	(0.01)

Note: The table reports rank-sum test statistics and corresponding p-values for the relevant one-sided (rows 1-4) or two-sided (rows 5-6) hypotheses, based on 32 independent observations for each treatment.

In the comparison between RF and LOT, the rank-sum test statistics for the whole session (rounds 1 to 20, column 1) are positive and significant at the 10 per cent level (p-value=0.07). Similar results are obtained when considering only the first round (p-value=.008). The results are also virtually unchanged when focusing on either the first or the last 10 rounds, indicating that repetition has a negligible effect on the comparison between expenditures in the two treatments.

> **Result 1**: Expenditures in the lottery with fixed price rationing are significantly higher than in the standard lottery.

The results for the difference between RV and LOT indicate that, contrary to the theoretical predictions, the variable price rationing mechanism actually *exacerbates* under-dissipation relative to the standard lottery. This result holds irrespective of the sub-sample considered.

> **Result 2**: The variable price rationing mechanism does not increase expenditures relative to the standard lottery.

The test results for the comparison between the two rationing schemes indicate that the positive difference between RF and RV is not only large but also strongly statistically significant, irrespective of the sub-set of rounds considered.

> **Result 3**: Expenditures with fixed price rationing are significantly higher than with variable price rationing.

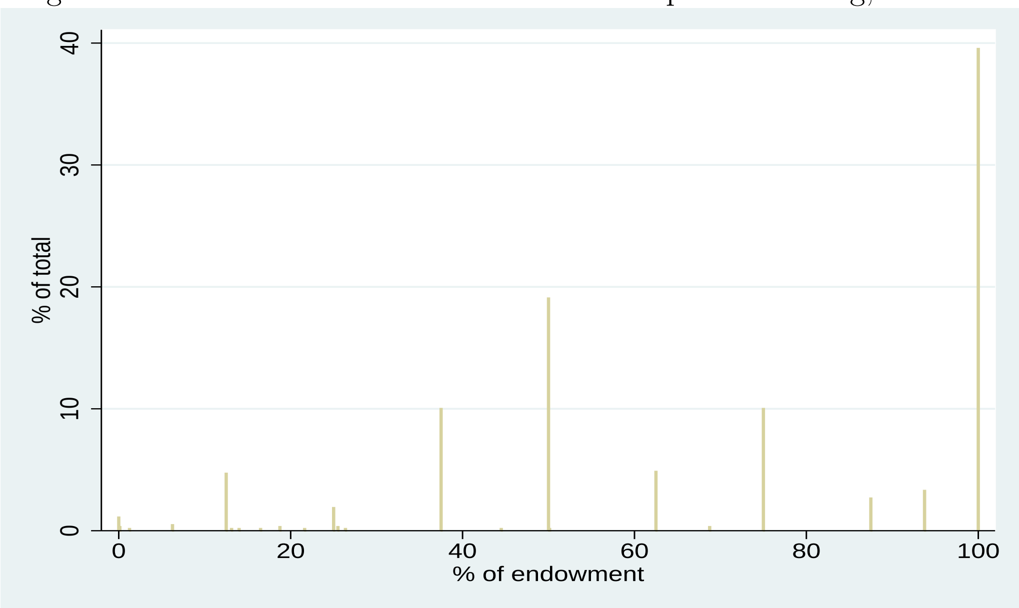
4.3 Individual Choices

Figure 3 compares the distribution of individual expenditures for the three treatments. In LOT, 15.9 per cent of the subjects spend half of their endowment, as predicted by theory. Although a relatively large fraction of subjects spend their whole endowment (27.2 per cent), about 41 per cent of the subjects spend less than 50 per cent of their endowment. In RF, the expenditure predicted by the theory (0.75) is indeed the modal value (24.1 per cent). The high overall expenditures in this treatment are also explained by a large fraction of subjects spending their whole endowment (18.1 per cent) or half their endowment (19.1 per cent). Interestingly, only a fifth of the subjects spend less than 50 per cent of their endowment. The comparison with the distribution for LOT indicates that the fixed price rationing mechanism is indeed effective in discouraging low expenditures. In RV too the 0.75 prediction represents the modal value (18.1 per cent). However, relatively few subjects spend their whole endowment (10.9 per cent) or half of it (13.6 per cent), whereas 46.4 per cent of the subjects spend less then 50 per cent of their endowment. In particular, RV is characterized by a much higher fraction of zero expenditures (8.3 per cent) than either LOT or RF (2.8 and 1.1 per cent, respectively). This result is consistent with the effect of strategic uncertainty, given the existence of a continuum of multiple equilibria in RV.

It should be observed that, in RF, actual expenditures are different from original bids in all the cases where the constraint on the supply of tickets is binding. This implies that the 0.75 modal value for individual expenditures for RF in Figure 3 may comprise both actual bids of 0.75 and higher bids rationed ex post. This also explains the clustering of density around 0.75 for expenditures in RF. In order to assess strategic behavior before the effect of rationing, Figure 4 displays the distribution of individual bids (tickets demanded) over all rounds. Interestingly, about 40 per cent of the subjects bid their whole endowment, as opposed to 27.2 and 10.9 per cent in LOT or RV, respectively. An additional 10 per cent of the subjects bid 0.75 of their endowment. Overall, the distribution of individual bids provides further support to the theoretical predictions for fixed price rationing.

Figure 3: Distribution of individual expenditures, all rounds

Figure 4: Distribution of individual bids: fixed price rationing, all rounds



5 Conclusions

This paper formulated the hypothesis that, in a stochastic contest, setting a limit to total expenditures by fixing the number of tickets may lead to an increase in total revenue. We characterized the theoretical properties of two rationing schemes, where the price of each ticket is either fixed or variable. We tested the theoretical predictions with a laboratory experiment where two contestants bid for a prize they equally value in a stochastic contest with proportional probabilities. The results indicate that, as predicted by the theory, revenue is significantly higher in a lottery with rationing than in a standard lottery. On the other hand, contrary to the theoretical predictions, an alternative rationing mechanism that fixes the number of tickets without limiting potential revenue does not increase revenue relative to a standard lottery.

At the theoretical level, our results indicate that under-dissipation is not related to the stochastic nature of the lottery. By appropriately setting the number of tickets, the auctioneer can increase total expenditures relative to a standard lottery. Provided the total cost of the tickets is less than the value of the prize, contestants will have an incentive to buy all the tickets. To the limit, by setting the total value of the tickets arbitrarily close to the value of the prize, it is possible to obtain full dissipation of rents in a stochastic contest.

At the empirical level, the effectiveness of rationing in raising total expenditures is strongly supported by our experimental results for the case of fixed price rationing, where contestants only pay for the tickets they receive. However, it is not supported for variable price rationing, where contestants pay for the tickets they demand. This result can be interpreted as a consequence of strategic uncertainty. An apparently advantageous feature of variable price rationing is that, contrary to fixed price, potential revenue is unlimited. On the other hand, while fixed price rationing has a unique equilibrium in dominant strategies, there exists a continuum of multiple equilibria with variable price rationing, thus introducing a coordination problem for the contestants.

Our results also have relevance beyond theory and the lab. House raffles are an interesting example of the mechanism we propose.[12] In a house raffle, the home owner sells her property by selling a fixed number of lottery tickets to the public, the total value of the tickets being equal to the appraisal value of the house. Once all of the tickets are sold, one ticket is randomly drawn and the holder wins the property prize. Such lotteries are becoming more and more popular in the US, as well as in the UK, where they are attracting the attention of the media.[13] Interestingly, in these real life examples the price

[12] See, for example, http://winahouseraffles.com.

[13] A famous antecedent of these modern raffles is the case of the US President Thomas

of each ticket is fixed, although this sets a limit to the revenue the seller can raise. Our experimental results shed light on this apparently puzzling decision, indicating a possible explanation why sellers may choose to opt for a fixed price scheme.

Jefferson, who attempted to sell his property, including the residence of Monticello, through a lottery with a fixed number of tickets. Only a series of unfortunate events prevented Jefferson from running the lottery (see Welch, 2008).

Appendix: Instructions

[In italics parts that are common across treatments]

Welcome. Thanks for participating in this experiment. If you follow the instructions carefully and make good decisions you can earn an amount of money that will be paid to you in cash at the end of the experiment. During the experiment you are not allowed to talk or communicate in any way with other participants. If you have any questions raise your hand and one of the assistants will come to assist you.

General rules

- *There are 32 subjects participating in this experiment.*
- *The experiment will consist of 20 rounds.*
- *In each round you will interact in a group of 2 with another participant selected randomly and anonymously by the computer.*
- *In each round each participant will be assigned an endowment of 800 points. You will make decisions that will determine the number of points you can earn.*
- *At the end of each round the computer will display your payoff in points.*
- *When the 20 rounds are completed, one round will be randomly chosen by the computer. The number of points that you have obtained in the selected round will be converted in euros at a rate of 100 points per euro and the resulting amount will be privately paid to you in cash.*

How earnings are determined

Treatment 1 (Standard Lottery)

- In each round, you and the other participant will play a two-person lottery. The lottery prize is 1600 points. You and the other participant will be given the chance to purchase lottery tickets at 1 point per ticket. As you purchase tickets, your point endowment will be reduced by the value of the tickets purchased.
- At the end of each round the computer will select randomly the winning ticket among all the tickets purchased. The owner of the winning ticket wins the prize.

- Thus, your probability of winning is given by the number of points you purchase divided by the total number of tickets purchased by you and the other participant in your group.
- For example, if you have purchased x tickets and the other participants has bought y tickets, the probability that you will win the prize is $\frac{x}{x+y}$.
- In case no tickets are purchased, no one wins the price.

  Treatment 2 (Fixed Price Rationing)

- In each round, you and the other participant will play a two-person lottery. The lottery prize is 1600 points. You and the other participant will be given the chance to purchase lottery tickets at 1 point per ticket. As you purchase tickets, your point endowment will be reduced by the value of the tickets purchased.
- There is a total number of 1200 tickets available.
- At the end of each round the computer selects randomly the winning ticket among the 1200 tickets. The owner of the winning ticket wins the prize.
- In case no tickets are purchased, or the winning ticket is not purchased, no one wins the price.
- Note that here are two possible cases:

  1. If the total number of tickets demanded by your group is not greater than 1200, you will receive the number of tickets you demanded. In this case, it is possible that no one bought the winning tickets, in which case no one wins the price.

     Example: if you have purchased 800 tickets and the other participant has purchased 200 tickets, the probability that you will win the prize is $\frac{800}{1200} = \frac{2}{3}$
  2. If the total number of tickets demanded by your group exceeds 1200, each participant receives a share of the 1200 tickets equal to the number of tickets she purchased divided by the total number of tickets purchased. In this case the number of points you will spend will be equal to the number of tickets you actually received.

     Example: if you have demanded 800 tickets and the other participant has demanded 800 tickets, you will receive $\frac{800}{1600} * 1200 = 600$ tickets, at a cost of 600 points, and the probability that you will win the prize is $\frac{600}{1200} = \frac{1}{2}$

Treatment 3 (Variable Price Rationing)

- In each round, you and the other participant will play a two-person lottery. The lottery prize is 1600 points. You and the other participant will be given the chance to purchase lottery tickets at 1 point per ticket. As you purchase tickets, your point endowment will be reduced by the value of the tickets purchased.

- There is a total number of 1200 tickets available.

- At the end of each round the computer selects randomly the winning ticket among the 1200 tickets. The owner of the winning ticket wins the prize.

- In case no tickets are purchased, or the winning ticket is not purchased, no one wins the price.

- Note that here are two possible cases:

  1. If the total number of tickets demanded by your group is not greater than 1200, you will receive the number of tickets you demanded. In this case, it is possible that no one bought the winning tickets, in which case no one wins the price.

     Example: if you have purchased 800 tickets and the other participant has purchased 200 tickets, the probability that you will win the prize is $\frac{800}{1200} = \frac{2}{3}$

  2. If the total number of tickets demanded by your group exceeds 1200, each participant receives a share of the 1200 tickets equal to the number of tickets she purchased divided by the total number of tickets purchased. In this case the number of points you will spend will be equal to the number of tickets you demanded.

     Example: if you have demanded 800 tickets and the other participant has demanded 800 tickets, you will receive $\frac{800}{1600} * 1200 = 600$ tickets, at a cost of 800 points, and the probability that you will win the prize is $\frac{600}{1200} = \frac{1}{2}$

References

Abbink, K., Brandts, J., Herrmann, B., Orzen, H., 2010. Inter-group conflict and intra-group punishment in an experimental contest game, *American Economic Review*, forthcoming.

Anderson, L.R., Stafford, S.L., 2003. An experimental analysis of rent seeking under varying competitive conditions, *Public Choice*, 115, 199-216.

Bullock, D.S., Rutström, E.E., 2007. Policy making and rent-dissipation: An experimental test, *Experimental Economics*, 10, 21-36.

Cason, T.N., Masters, W.A., Sheremeta R.M., 2010. Entry into winner-take-all and proportional-prize contests: An experimental study, *Journal of Public Economics*, forthcoming.

Che, Y.K., Gale, I., 1998. Caps on political lobbying, *American Economic Review,* 88, 643–651.

Corazzini, L., Faravelli, M., Stanca, L., 2010. A prize to give for: An experiment on public good funding mechanisms, *Economic Journal*, 120, 944–967.

Davis, D., Reilly, R., 1998. Do too many cooks always spoil the stew? An experimental analysis of rent-seeking and the role of a strategic buyer, *Public Choice*, 95(1), 89-115.

Fang, H., 2002. Lottery versus all-pay auction models of lobbying, *Public Choice*, 112, 351-71.

Fischbacher, U., 2007. z-Tree: Zurich toolbox for ready-made economic experiments, *Experimental Economics*, 10(2), 171-178.

Fonseca, M. A., 2006. It's not how you play the game, it's winning that matters: An experimental investigation of asymmetric contests. Mimeo, Royal Holloway University of London.

Goeree, J., Maasland, A., Onderstal, S., Turner, J., 2005. How (not) to raise money, *Journal of Political Economy*, 113, 897-918.

Herrmann, B., Orzen, H., 2008. The appearance of homo rivalis: social preferences and the nature of rent seeking. CeDEx Discussion Paper No. 2008-10.

Johnson, R.C., 1960. The "running lotteries" of the Virginia Company, *The Virginia Magazine of History and Biography*, 68-2, 156-165.

Kaplan, T., Wettstein, D., 2006. Caps on political lobbying: Comment, *American Economic Review*, 96, 1351-1354.

Kong, X., 2008. Loss aversion and rent-seeking: An experimental study. Working Paper, University of Nottingham.

Landry, C., Lange, A., List, J.A., Price, M.K., Rupp, N., 2006. Toward an understanding of the economics of charity: Evidence from a field experiment, *Quarterly Journal of Economics*, 121(2), 747-782.

Lange, A., List, J. A., Price, M. K., 2007. Using lotteries to finance public goods: Theory and experimental evidence, *International Economic Review*, 48, 901-927.

Lazear, E.P., Rosen, S., 1981. Rank-order tournaments as optimum labor contracts, *Journal of Political Economy*, 89, 841-864.

Masters, W.A., 2005. Paying for prosperity: How and why to invest in agricultural research and development in Africa, *Journal of International Affairs*, 58(2), 35-64.

Masters, W.A., Delbecq, B., 2008. Accelerating innovation with prize rewards, IFPRI Discussion Paper 00835.

Matros, A., Lim, W., 2009. Contests with a stochastic number of players, *Games and Economic Behavior*, 67-2, 584-597.

Millner, E., Pratt, M., 1989. An experimental investigation of efficient rent-seeking, *Public Choice*, 62(2), 139-151.

Millner, E., Pratt, M., 1991. Risk aversion and rent-seeking: An extension and some experimental evidence, *Public Choice*, 69(1), 81-92.

Morgan, J., 2000. Financing public goods by means of lotteries, *Review of Economic Studies*, 67, 761-784.

Morgan, J., Orzen., H., Sefton, M., 2010. Endogenous entry in contests, *Economic Theory*, forthcoming.

Morgan, J. and Sefton, M., 2000. Funding public goods with lotteries: Experimental evidence, *Review of Economic Studies*, 67, 785-810.

Orzen, H., 2008. Fundraising through competition: Evidence from the lab. Discussion Papers 2008-11, The Centre for Decision Research and Experimental Economics, School of Economics, University of Nottingham.

Potters, J., de Vries, C., van Winden, F., 1998. An experimental examination of rational rent-seeking, *European Journal of Political Economy*, 14, 783-800.

Schmidt, D., Shupp, R., Walker, J., 2006. Resource allocation contests: Experimental evidence. CAEPR Working Paper, No. 2006-004.

Schmitt, P., Shupp, R., Swope, K., Cadigan, J., 2004. Multi-period rent-seeking contests with carryover: Theory and experimental evidence, *Economics of Governance*, 5, 187-211.

Schram, A., Onderstal, S., 2009. Bidding to give: An experimental comparison of auctions for charity, *International Economic Review*, 50(2), 431-457.

Shogren, J.F., Baik, K.H, 1991. Reexamining efficient rent-seeking in laboratory markets, *Public Choice*, 69(1), 69-79.

Shupp, R., 2004. Single versus multiple winner rent-seeking contests: An experimental investigation. Working Paper, Ball State University.

Tullock, G. 1980. Efficient rent-seeking, in J.M. Buchanan, R.D. Tollison and G. Tullock (Eds.), *Towards a theory of the rent-seeking society*, 97-112, College Station: Texas A&M University Press.

Welch, E., 2008. Lotteries in early modern Italy, *Past and Present*, 199, 71-111.