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Growth and Welfare under Endogenous Lifetime*

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Abstract: We develop a perpetual youth model to investigate how longevity affects economic growth and welfare. Life expectancy is determined by individuals' investments in healthcare. We find that improvements in the healthcare technology always increase the steady state growth rate. Although the effect is small, even for large increases in longevity, welfare gains may be substantial depending on the type of the technological improvement. We identify two externalities associated with healthcare investments and provide a condition when healthcare expenditures are inefficiently low in the market equilibrium. Finally, we discuss our results with respect to alternative spillover specifications in the production sector.

Keywords: economic growth, endogenous longevity, healthcare expenditures, healthcare technology, quality-quantity trade-off

JEL-Classification: O40, I10, J10

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1 Introduction

Over the last decades human longevity has increased substantially. Higher expected lifetime has been accompanied, at least in the developed world, by a significant rise of healthcare expenditures. For example, between 1960 and 2000 life expectancy in the U.S. rose from 69.8 to 77.1 years, while health expenditures, as a share of GDP, increased from 5.2% to 13.4% (according to OECD data). Even though empirical evidence on the relationship between health expenditures and life expectancy is ambiguous, there is no doubt that expected lifetime is not given per se but can be influenced by investments in healthcare, such as improving sanitation, buying medication and inoculation, consulting a physician, etc. (Lichtenberg 2004, Cutler et al. 2006, Hall and Jones 2007, Caliskan 2009).

In this paper, we analyze the relationship between endogenous investments in longevity, economic growth and welfare. Therefore, we develop a model that combines the household side of perpetual youth models in the tradition of Yaari (1965) and Blanchard (1985) with the production side of an endogenous growth model in the style of Romer (1986). The key novelty in our model is a healthcare sector that allows individuals to reduce their risk of dying by purchasing health services. Thus, instead of varying longevity exogenously, we focus on how economic growth and welfare reacts to endogenous changes in life expectancy induced by improvements in the healthcare technology.[1] In fact, we consider two different types of healthcare improvements. The first type decreases the baseline mortality, which is independent of individual investments in healthcare. One could think of improvements in the sanitary infrastructure or behavioral changes such as reduced smoking. The second type increases the marginal productivity of healthcare expenditures. Examples include better medication or therapeutic breakthroughs, such as new diagnostic tools or surgeries.

In our standard model specification, we find that improvements in the healthcare technology always lead to higher steady state growth rates. Intuitively, higher life expectancy increases the share of old ("rich") relative to young ("poor") households and leads ceteris paribus to higher capital per capita accumulation. However, our numerical calculations suggest that the effect on the growth rate is rather small. Yet, higher life expectancy may yield considerable welfare gains. The type of healthcare improvements matters, however. For a given increase in longevity, welfare improvements are substantially higher if higher longevity is induced by improvements of the first type (baseline mortality decreases) compared to improvements of the second type (higher efficiency of healthcare treatment). Using the data set of Becker et al. (2005) amended by data on healthcare expenditures, we obtain higher

[1] Our model emphasizes that increases in healthcare expenditures and longevity are mainly driven by the availability of better healthcare technologies. Newhouse (1992), Cutler et al. (2006) and Fonseca et al. (2009), among others, support this view. Alternative explanations focus on income effects (Hall and Jones 2007) or greater health-insurance coverage.

welfare improvements than Becker et al. (2005) in low developed regions, where healthcare improvements were predominantly of the first type, and lower welfare improvements in highly developed regions, where longevity increases were mainly triggered by healthcare improvements of the second type.

We identify two externalities associated with healthcare investments. The first, which is well known (see, for example, Philipson and Becker 1998), stems from the price taking behavior of households with respect to annuities and leads to over-investment in healthcare. The second externality is a consequence of our growth model. Households under-invest in healthcare, as they do not take into account that increased longevity on the aggregate level induces a positive effect on the economy's growth rate. We show that healthcare investments in the market equilibrium cannot be inefficiently high and give a condition under which they are inefficiently low.

Finally, we emphasize the importance of the spillover specification in the production sector for the growth effects of healthcare investments. Generalizing the specification of the spillover effect in our basic model set-up, we show that when technological improvements trigger higher healthcare expenditures, longevity increases may even lead to negative growth and welfare effects. By contrast, increases in expected lifetime enjoyed without healthcare expenditures always induce positive growth and welfare effects independent of the spillover specification. This further highlights the importance of the source of longevity increases.

Our paper combines three strands of the literature. The first is the literature on the willingness to pay for lower mortality risk including Grossman (1972), Arthur (1981), Shepard and Zeckhauser (1984), Rosen (1988), Ehrlich and Chuma (1990), Murphy and Topel (2003). These papers study the trade-off of an individual household between consumption and longevity, but do not consider the effects on economic growth and also neglect potential externalities of healthcare expenditures.[2] A standard result in this literature is that healthcare investments decrease in the household's intertemporal substitution elasticity. The reason is that there exists a trade-off between the quality (consumption level) and the quantity of life (length of life). The household is the more willing to sacrifice current consumption in order to increase its lifetime the stronger are the diminishing returns from consumption (i.e., the more concave is the instantaneous utility function). We find that this trade-off between the quality and the quantity of life is only present if the households' consumption elasticities are low but vanishes if consumption elasticities are higher. Households with high consumption elasticities invest more in their longevity with the aim to maximize their quality of life,

[2] The latter is examined by Kuhn et al. (2010) who explicitly account for negative externalities such as congestion of medical facilities or positive spillovers of preventive activities. By contrast, the externalities in our paper are induced by households not taking into account the aggregate effects of health expenditures on annuity returns and the economy's growth rate.

i.e. their level of lifetime consumption. The reason is that when the households also decide on capital savings, the consumption profile for a given lifetime becomes steeper for higher intertemporal substitution elasticities. This gives an incentive to live longer to enjoy more periods with high consumption levels. Thus, the relationship between healthcare expenditures and the intertemporal elasticity of substitution is U-shaped rather than monotonically declining.

The second strand is the literature on the growth effects of increased life expectancy. By now, there exists a considerably body of literature that examines *exogenous* variations in expected lifetime on economic development (see, for example, de la Croix and Licandro 1999, Kalemli-Ozcan et al. 2000, Zhang et al. 2001 Kalemli-Ozcan 2002, Azomahou et al. 2009, Prettner 2010). However, we are aware of only three other papers which consider *endogenous* expected lifetime in the sense that healthcare investments are directly chosen either by the government or the individual household.[3] Chakraborty and Das (2005) examine the effects of endogenous longevity choices on inequality, Bhattacharya and Qiao (2006) argue that the presence of public input in private longevity may lead to chaotic dynamics in a neoclassical growth model, and Finlay (2006) investigates individuals' incentives to invest in human capital and higher expected lifetime dependent on the agents' degree of risk aversion. In contrast to these papers, we investigate the growth effects of changes in the healthcare technology rather than assuming a static one. In addition, these contributions employ a two period overlapping generations model, where households have a certain probability to live in the second period and growth is driven by human capital, while we consider a continuous time perpetual youth model. This allows us to connect to the first strand of the literature mentioned above in a natural way.

Finally, our numerical exercises are closely related to the papers on the welfare consequences of increased longevity by Becker et al. (2005) and Jones and Klenow (2010). They neglect effects of longevity on (consumption) growth in their welfare estimations. Our basic model set-up supports such an approach, as growth effects of longevity are rather small. However, we emphasize that reasonable changes in the specification of spillover effects may yield growth effects of increased longevity that substantially (and negatively) affect welfare.

The paper is organized as follows. The next section introduces the model and provides a detailed discussion of the household's maximization problem with respect to healthcare. Section 3 determines the market equilibrium and derives the dynamics of the aggregate

[3] Other papers including Blackburn and Cipriani (2002), Aisa and Pueyo (2004) and Chakraborty (2004) endogenize the individual's life expectancy which, however, is not chosen fully independently. More remotely related to our paper is the literature on the effect of health expenditures on growth in the presence of epidemics such as AIDS (Bell et al. 2006, Bell and Gersbach 2009). In these papers health expenditures may have a substantial effect on longevity and growth by alleviating or eliminating the negative consequences of the epidemic.

economy. In Sections 4 and 5 we discuss the quality-quantity trade-off and the relationship between healthcare expenditures and the economy's age structure. We investigate the effects of improvements in the healthcare technology on the economy's steady state dynamics and provide numerical examples that illustrate the growth and welfare effects in Section 6. In Section 7, we identify the externalities of the market equilibrium. Section 8 extends the analysis to different spillover specifications. Finally, Section 9 concludes.

2 The Model

The model comprises a continuum of households. Like in Blanchard (1985), households born at time $s \in (-\infty, \infty)$ face a hazard rate $p(s)$ of dying that is constant throughout the lifetime of each household. In our model, however, the hazard rate may vary among households of different cohorts, as it is determined by the level of medical treatment the household gets throughout its lifetime. At time of birth, households choose a level of medical treatment $h(s)$, which is fixed over the entire lifetime and determines the hazard rate via a healthcare technology $H(h(s))$.

As the hazard rate is constant over the entire lifetime, all households born at time s and still alive face an expected remaining lifetime $T(s)$ at any time $t \geq s$ given by

$$T(s) = \int_t^\infty (t' - t)p(s) \exp[-p(s)(t' - t)]dt' = \frac{1}{p(s)} \ . \tag{1}$$

Although the lifetime of each household is stochastic, we assume that the size of each cohort is large enough for cohort sizes to decline deterministically over time. At all times a new cohort is born. We abstract from fertility choices of households and assume that cohort size grows at the constant and exogenously given rate ν.[4] Normalizing the cohort size at time $t = 0$ to unity, we obtain for the size of the population at time t

$$N(t) = \int_{-\infty}^t \exp[\nu s] \exp[-p(s)(t - s)]ds \ . \tag{2}$$

Households exhibit identical ex ante preferences and face equal hazard rates for the same levels of medical treatment. Households born at time s maximize expected discounted lifetime utility derived from consumption U

$$U(s) \equiv \int_s^\infty V(c(t, s)) \exp\left[-(\rho + p(s))(t - s)\right] dt \ , \tag{3}$$

[4] The parameter ν can be mapped into the economy's fertility rate, which specifies how many children are born on average by each woman (or by our abstract genderless individual). The fertility rate is independent of the size of the actual population.

where $V(c(t,s))$ denotes the instantaneous utility derived from consumption $c(t,s)$ at time t of the household born at time s, and ρ is the constant rate of time preference. We impose standard curvature properties on the instantaneous utility function ($V' > 0$ and $V'' < 0$). Rosen (1988) showed that optimal investments in healthcare crucially depend on two characteristics of the instantaneous utility function: (i) the intertemporal elasticity of substitution and (ii) the difference in instantaneous utility between being alive and dead. As our definition of lifetime utility (3) normalizes instantaneous utility of being dead to zero, a utility representation with $V(c) > 0$ for all $c > 0$ avoids that households may wish to be rather dead than alive. As a consequence, we employ an instantaneous utility function with intertemporal substitution elasticity of σ larger than one[5]

$$V(c(t,s)) \equiv \frac{c(t,s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} , \quad \sigma > 1 . \tag{4}$$

At any time alive households are endowed with one unit of labor each that is supplied inelastically to the labor market at wage $w(t)$. In addition, households can save and borrow assets $b(t,s)$ at the interest rate $r(t)$. Households are born without assets and we assume that they can contract against the risk of leaving unanticipated requests on a perfectly competitive life insurance market. Each unit of assets buys a life annuity paying the return a as long as the household is alive. We further assume that the insurance company can learn the probability of dying $p(s)$ of each cohort at no costs. Due to the large cohort sizes insurance companies can offer risk free annuities. Perfect competition among insurance companies leads to fair annuity payments $a(t,s) = r(t) + p(s)$. In line with Philipson and Becker (1998) and Eeckhoudt and Pestieau (2008), among others, we assume that households take $a(t,s)$ as given. Given that negative bequests are prohibited, households hold their entire wealth in fair annuities. Denoting the costs of healthcare by $M(h(s))$, the households' budget constraint reads

$$\dot{b}(t,s) = a(t,s)b(t,s) + w(t) - c(t,s) - M(h(s)) , \qquad t \geq s , \tag{5}$$

with $b(s,s) = 0$.

The economy comprises two production sectors: the consumption-good and the healthcare sector. We assume that both sectors operate at perfectly competitive conditions. As all firms have access to the same constant returns to scale production technologies, we restrict the

[5] Rosen (1988), Hall and Jones (2007) and Becker et al. (2005) use $V(c(t,s)) = c(t,s)^{1-\frac{1}{\sigma}}/(1-1/\sigma) + \alpha$ with some positive constant α. This allows either to employ intertemporal substitution elasticities of $\sigma < 1$ (Hall and Jones 2007) or to calibrate the model to different values of a statistical life without changing the intertemporal elasticity of substitution (Becker et al. 2005, Hall and Jones 2007). We use the functional form (4) representing homothetic preferences which allow for a balanced growth path.

analysis to one representative firm in each sector.

2.1 Consumption-good production

The representative firm in the consumption-good sector produces a homogeneous consumption good, that can also be used as physical capital, via a Cobb-Douglas production technology $Y(t) \equiv K(t)^{\alpha}\big(A(t)L^F(t)\big)^{1-\alpha}$, where $\alpha \in (0,1)$ and $K(t)$ and $L^F(t)$ denote the aggregate amount of capital and labor employed in consumption-good production, respectively. $A(t)$ characterizes the technological level of the economy that is exogenous to the representative firm. We assume a "learning-by-investing" externality similar to Romer (1986) but corrected for scale effects: $A(t) \equiv K(t)/L^F(t)$.[6] Capital depreciates at the constant rate δ. Profit maximization of the representative firm yields factor prices equal to the marginal productivities

$$r(t) = \alpha - \delta \ , \tag{6a}$$

$$w(t) = (1-\alpha)\frac{K(t)}{L^F(t)} \ . \tag{6b}$$

As the interest rate is constant due to the "learning-by-investing" externality, we introduce the notation $r \equiv r(t) = \alpha - \delta$ and $a(s) \equiv a(s,t) = r + p(s)$.

2.2 Healthcare sector

The representative firm in the healthcare sector provides medical treatment by employing labor. Without loss of generality, we assume that one unit of labor produces one unit of medical treatment. Given the levels $h(s)$ of medical treatment of each cohort alive, the total amount of labor employed in the healthcare sector equals

$$L^H(t) = \int_{-\infty}^{t} h(s)\exp[\nu s]\exp[-p(s)(t-s)]ds \ . \tag{7}$$

In the labor market equilibrium, labor employed in the healthcare sector has to earn the same wage as labor employed in the consumption-good sector. As a consequence, the healthcare sector offers medical treatment at marginal costs $w(t)$ and we obtain $M(h(s)) = h(s)w(t)$ for the costs of medical treatment of a household born at time s.

[6] In Romer (1986) it is assumed that $A(t) \equiv K(t)$. By specifying the learning externality to be proportional to capital per worker, we correct for scale effects.

The level of medical treatment $h(s)$ determines the hazard rate $p(s)$ via the healthcare technology $H(h(s))$

$$p(s) = H(h(s)) \equiv p_{max} - \psi h(s) \ . \tag{8}$$

Without medical treatment ($h = 0$) households face the hazard rate $p(s) = p_{max}$ of dying. The hazard rate $p(s)$ decreases with constant returns ψ in the level of medical treatment $h(s)$. The parameter $\psi < p_{max}$ reflects the productivity of healthcare investments and may be interpreted as the quality level of the health system or the state of the art in medical treatment. It denotes the maximum amount by which one may reduce the hazard rate against p_{max} by spending all wage income on healthcare. While p_{max} reflects, for example, the sanitary infrastructure of the economy, ψ increases with the human capital of physicians, the efficiency of hospitals and the like.

Improvements in the healthcare technology may come in two qualitatively different ways. First, the maximal hazard rate p_{max} may decrease implying that all households, independently of their levels of healthcare spending, experience a lower hazard rate of dying. In fact, a decrease in p_{max} offers higher life expectancy for free (at least for the individual household). Historic examples in this respect could be new knowledge about germ theory leading to better hygienic standards and a change in personal behavior. We also interpret the introduction of most vaccines and drugs as a decrease in p_{max}, because these drugs are usually not very expensive. As an example, think of penicillin which led to substantial declines in mortality in the last century. Second, the state of the art in medical treatment ψ may increase implying that the same amount of healthcare spending reduces the hazard rate more than before. However, only households with positive healthcare spending benefit from the improved healthcare technology. One may think of improvements such as magnetic resonance imaging, coronary heart bypass grafting, transplantation, and the like.[7] In Sections 6 and 8 we shall see that these two channels of healthcare technology improvements differently affect the economy's growth rate and the households' lifetime utility.

2.3 The individual household's problem

Inserting $M(h(s)) = h(s)w(t)$ into the households' budget constraint (5) yields

$$\dot{b}(t,s) = a(s)b(t,s) + (1 - h(s))w(t) - c(t,s) \ , \qquad t \geq s \ . \tag{9}$$

[7] Although it makes perfect sense to distinguish conceptionally the two different channels of improvements in the healthcare technology, we want to emphasize that most real world improvements impact simultaneously on p_{max} and ψ. For example, knowledge about germ theory led to better hygienic standards not only in every day life, thereby decreasing p_{max}, but also in medical treatment, which increased ψ.

Thus, we can interpret the level of medical treatment $h(s)$ as the fraction of labor income a household spends throughout its entire life for healthcare services. This implies $h(s) \in [0, 1]$, as households are born without assets and must not be indebted when dying.

Households maximize expected intertemporal utility (3) subject to conditions (9) and $b(s, s) = 0$ by choosing an optimal level of medical treatment $h(s)$ and an optimal consumption path $c(t, s)$. To solve this maximization problem we interpret it as a sequence of two maximization problems. The first is to find the optimal consumption path for some given level of healthcare expenditures $c^{\star}(t, s, h)$. We obtain the second maximization problem by inserting $c^{\star}(t, s, h)$ back into the expected intertemporal utility function (3) and solve for the optimal healthcare expenditures $h^{\star}(s)$.

Assuming a given level of medical treatment $h(s)$, which implies a given hazard rate $p(s)$ via the healthcare technology (8), the first maximization problem yields the Euler equation

$$\frac{\dot{c}(t, s)}{c(t, s)} = \sigma[a(s) - \rho - p(s)] \ , \qquad t \geq s \ . \tag{10}$$

For given $h(s)$ the behavior of a household born at time s is characterized by the system of differential equations (9) and (10), the initial condition $b(s, s) = 0$ and the transversality condition for the stock of assets $\lim_{t \to \infty} b(t, s) \exp\left[-a(s)(t - s)\right] = 0$. Under the assumptions that $(1 - \sigma)a(s) + \sigma(\rho + p(s)) > 0$ and the long-run growth rate of wages $w(t)$ is smaller than $a(s)$, we obtain for the optimal paths of consumption $c^{\star}(t, s, h)$ and assets $b^{\star}(t, s, h)$[8]

$$c^{\star}(t, s, h) = c^{\star}(s, s, h) \exp\left[\sigma\big(a(s) - \rho - p(s)\big)(t - s)\right] \ , \tag{11a}$$

$$b^{\star}(t, s, h) = \frac{c^{\star}(t, s, h)}{(1 - \sigma)a(s) + \sigma\big(\rho + p(s)\big)} - \ \big(1 - h(s)\big)W(t, s) \ , \tag{11b}$$

$$c^{\star}(s, s, h) = \left[(1 - \sigma)a(s) + \sigma\big(\rho + p(s)\big)\right] \big(1 - h(s)\big)W(s, s) \ . \tag{11c}$$

where $W(t, s) \equiv \int_t^{\infty} w(t') \exp\left[-a(s)(t' - t)\right] dt'$ denotes the expected net present value of the household's future labor income at time t.

We now turn to the second maximization problem. Given an optimal consumption path $c^{\star}(t, s, h)$ and recalling that $p(s) = H(h(s))$, the necessary condition for the optimal level

[8] Without these assumptions, the household's problem is not well defined. We shall see in Section 3 that the condition that the long-run growth rate of wages $w(t)$ is smaller than $a(s)$ always holds in the market equilibrium.

of healthcare expenditures $h^\star(s)$ reads

$$\begin{aligned} &\int_s^\infty \frac{\partial V(c^\star(t,s,h))}{\partial h(s)} \exp\left[-(\rho+p(s))(t-s)\right] dt \\ &\leq \int_s^\infty (t-s)H'(h(s))V(c^\star(t,s,h)) \exp\left[-(\rho+p(s))(t-s)\right] dt \ . \end{aligned} \tag{12}$$

For an interior solution the equality sign holds, while the inequality sign applies in case of the corner solution $h(s) = 0$. The intuition for this condition is straightforward. In the optimum the decrease in expected lifetime utility from a marginal increase of healthcare expenditures due to decreasing lifetime consumption has to equal the increase in expected lifetime utility due to an increasing expected lifetime. If the marginal benefits of an investment in healthcare do not outweigh the corresponding costs for any level of healthcare expenditures, the optimal level of healthcare is given by the corner solution $h(s) = 0$.[9]

Inserting equations (4), (11a), and (11c) into equation (12), we obtain for the first-order condition

$$F(h(s)) \equiv \frac{\sigma\psi}{(\sigma-1)\left[(1-\sigma)a(s)+\sigma(\rho+p(s))\right]} - \frac{1}{1-h(s)} \leq 0 \ . \tag{13}$$

This determines a unique optimal level of healthcare expenditures $h^\star(s)$.

Proposition 1 (Existence and uniqueness of household optimum)
There exists a unique optimal level of healthcare expenditures $h^\star(s)$ which is given by

$$h^\star(s) = \begin{cases} \max\left[0, \dfrac{\sigma\psi-(\sigma-1)\left[(1-\sigma)a(s)+\sigma(\rho+p_{max})\right]}{\sigma\psi(2-\sigma)}\right] & , \quad \text{if} \quad \sigma < 2 \ , \\ 0 & , \quad \text{if} \quad \sigma \geq 2 \ . \end{cases}$$

In the proof of Proposition 1, provided in the appendix, we show that there is no interior solution for $\sigma \geq 2$. The corner solution $h(s) = 1$ cannot be optimal, as this implies no consumption and zero lifetime utility, while both are positive for any other value $h(s) \in [0, 1)$. As a consequence, the corner solution $h(s) = 0$, i.e. no healthcare expenditures, is the optimal solution. For $\sigma < 2$, there exists a local maximum, which is the optimal solution if it occurs in the feasible range $h(s) \in [0, 1)$. Otherwise again the corner solution $h(s) = 0$ is optimal.

[9] In the first-order condition (12), the trade-off between investments in healthcare and average lifetime involves not only marginal but also absolute welfare comparisons. This illustrates the importance of the difference in instantaneous utility between being alive and dead, as mentioned earlier.

3 Market Equilibrium and Aggregate Dynamics

To investigate the aggregate economy, we introduce aggregate household variables per capita derived by integrating over all living individuals and dividing by the population size of the economy

$$z(t) \equiv \frac{\int_{-\infty}^{t} z(t,s) \exp[\nu s] \exp[-p(t-s)]\, ds}{N(t)} , \tag{14}$$

where $z(t)$ and $z(t,s)$ denote aggregate per capita respectively individual household variables.[10] The economy consists of five markets: the labor market, the capital market, the consumption good market, the market for annuities and the market for healthcare. We assume the economy to be in market equilibrium at all times t. In particular, this implies that labor demand equals the population size, $L^F(t) + L^H(t) = N(t)$ and capital per capita equals aggregate assets per capita, $k(t) = b(t)$.

As the interest rate r is constant, the equilibrium on the market for annuities, $a(s) = r + p(s)$, implies that the first-order condition (13) is identical for all households irrespective of their date of birth. Consequently, all households spend the same fraction $h(s) = h$ of income for medical treatment implying that the hazard rate $p(s) = p$ and the population growth rate $\dot{N}(t)/N(t) = \nu$ are also constant. By setting $a \equiv r + p$, we obtain for the optimal healthcare expenditures in the market equilibrium:

$$h^{\star} = \begin{cases} \max\left[0, \sigma - \dfrac{(\sigma-1)\left[(1-\sigma)r + \sigma\rho + p_{max}\right]}{\psi}\right] & , \quad \text{if} \quad \sigma < 2 , \\ 0 & , \quad \text{if} \quad \sigma \geq 2 . \end{cases} \tag{15}$$

Via the healthcare technology (8), the optimal level of healthcare expenditures $h^{\star}$ in the market equilibrium, which is completely determined by the set of exogenous parameters, maps into the optimal hazard rate $p^{\star}$. Introducing the abbreviation[11]

$$x(p) \equiv (1-\sigma)a(s) + \sigma(\rho + p) = r + p - \sigma(r - \rho) > 0 , \tag{16}$$

which reflects the difference between the return on annuities $r + p$ and the growth rate of the households' consumption $\sigma(r - \rho)$, we can characterize the aggregate dynamics dependent on the hazard rate p.

[10] We are aware of the slight abuse of notation, which we consider to be justified to keep notation at a minimum.

[11] Note that $x(p) > 0$ is necessary for the household's maximization problem to be well defined.

Proposition 2 (Aggregate system dynamics)

(i) The dynamics of the aggregate economy is characterized by:

$$\dot{c}(t) = \sigma(r-\rho)c(t) - (p+\nu)x(p)k(t) \ , \tag{17a}$$

$$\dot{k}(t) = (1-\delta-\nu)k(t) - c(t) \ . \tag{17b}$$

(ii) The dynamics of the aggregate economy is governed by a balanced growth path, i.e. aggregate consumption $c(t)$ and aggregate capital $k(t)$ are growing at the same constant rate g. The sign of the growth rate g is determined by

$$g \gtreqqless 0 \quad \Leftrightarrow \quad x(p)(p+\nu) \lesseqqgtr \sigma(r-\rho)(1-\delta-\nu) \ .$$

The proof is given in the appendix. Note that we neglect trivial steady states where $c(t) = k(t) = 0$, for all t. The proposition establishes that there is a unique balanced growth path for any given hazard rate p. As shown in the proof, the growth rate g equals

$$g(p) = \frac{1}{2}\left[1+\sigma(r-\rho)-\delta-\nu-\sqrt{\left[1-\delta-\nu-\sigma(r-\rho)\right]^2+4x(p)(p+\nu)}\right] \ . \tag{18}$$

Inserting the optimal hazard rate in the market equilibrium yields the growth rate in the market equilibrium $g^\star = g(p^\star)$. Thus, the aggregate system dynamics is fully characterized by the set $(h^\star, p^\star, g^\star)$.

4 The quality-quantity trade-off

The literature on the value of life and the willingness to pay for lower mortality risk emphasizes that the value of the intertemporal substitution parameter is of key importance for understanding trade-offs between the quantity and quality of life (see, for example, Arthur 1981, Shepard and Zeckhauser 1984, Rosen 1988, 1994, Ehrlich and Chuma 1990). A standard result of this literature is that the value of increases in longevity declines monotonically with the intertemporal substitution elasticity σ. In our growth model, however, the equilibrium healthcare expenditures follow a U-shaped curve in σ, as shown in the next proposition and explained in the subsequent paragraphs.

Proposition 3 (Effect of σ on optimal healthcare)
For an interior market equilibrium, the optimal level of healthcare $h^\star(s)$ follows a U-shaped curve in σ. Formally

$$\frac{dh^\star(s)}{d\sigma} \lesseqqgtr 0 \quad \Leftrightarrow \quad \sigma \lesseqqgtr 1 + \frac{\rho + p_{max} - \psi}{2(r-\rho)} \ .$$

Proof: The claim follows directly by differentiating the interior solution for $h^{\star}$ in (15) with respect to σ. □

The intuition for Proposition 3 is that there exist two motives for healthcare investments that depend differently on the intertemporal elasticity of substitution σ. First, consider the household could not save but only spend labor income on either consumption or healthcare investments. Then, the household invests more in healthcare, the lower is the intertemporal elasticity of substitution σ. To see this, recall that instantaneous utility (4) is more concave, the smaller is σ. In fact, for σ close to one instantaneous utility is insensitive to changes in the consumption level. As a consequence, the household has an incentive to invest in the quantity of life in order to spread out consumption over more periods, thereby counteracting the strong diminishing returns of instantaneous utility with respect to consumption. If σ becomes larger, instantaneous utility becomes more sensitive to changes in the consumption level, i.e. the quality of life becomes more important. As a consequence, the household invests less in healthcare, thereby enjoying higher levels of consumption over a shorter lifetime.

Second, if households can also save income in order to reap capital rents, the relationship between the intertemporal elasticity of substitution σ and the investments in healthcare is no longer monotonically decreasing but U-shaped. The reason is that now σ also influences the growth rate of consumption. The household smoothes consumption less the higher is σ leading to a higher growth rate of consumption $\sigma(r-\rho)$. As the level of consumption becomes more relevant the higher is σ, living longer can increase the expected lifetime utility if the consumption loss in early periods is outweighed by the consumption gain in later periods.

In summary, there are two motives for households to invest in healthcare. The first is to *spread out consumption* over time to counteract the diminishing returns of instaneous utility. It is decreasing in σ and particularly strong for low values of σ. The second motive is to *maximize total lifetime consumption*, i.e. the total quality of life. This motive exploits the fact that the consumption levels increase over time and an additional period of life comes with high levels of consumption. This motive is increasing in σ and particulary strong for high values of σ. For small values of σ the first motive outweighs the second, leading to a declining relationship between the intertemporal elasticity of substitution and healthcare expenditures. Eventually, the second motive dominates the first and investments into healthcare increase with σ. While the first motive is well understood (see, for example, the references mentioned at the beginning of this section), the second motive can only be captured if households simultaneously decide on consumption, savings, and healthcare investments and if $r > \rho$.[12]

Using (18), the following proposition investigates how the economy's growth rate is affected by different values of the intertemporal elasticity of substitution σ.

Proposition 4 (Effect of σ on growth rate)
For interior levels of healthcare in the market equilibrium, if $\sigma < 1+(\rho+p_{max}-\psi)/[2(r-\rho)]$, implying $dh^\star/d\sigma < 0$, the sign of $dg^\star/d\sigma$ is ambiguous. Otherwise, $dg^\star/d\sigma > 0$.

The proof is given in the appendix. The intertemporal elasticity of substitution σ exerts two opposing effects on the growth rate. First, the direct effect of an increase in σ for given hazard rate p is positive: everything else equal, households save more the higher is σ resulting in an increasing growth rate. The indirect effect of σ via the households' optimal choice of healthcare expenditures $h^\star$ is ambiguous. If the indirect effect is negative, which is the case if $h^\star$ is decreasing in σ, then the overall effect is ambiguous. Otherwise the economy's growth rate is increasing with σ.

5 Age Structure of the Economy

We now turn to the effects of the growth rate of the cohort size ν and the hazard rate p on the economy's growth rate g. To sharpen intuition, consider an identical model but with a standard infinitely lived Ramsey consumer instead of finitely lived individuals on the household side. Then, the system dynamics were characterized by $\dot{c}(t) = \sigma(r-\rho)c(t)$ and (17b). Consequently, the economy's growth rate would neither depend on ν and p. The overlapping generations structure of our model is captured by the term $(p+\nu)x(p)k(t)$ in equation (17a), which reflects the population-weighted difference between per capita consumption and the consumption of the newly born individuals at time t. To see this,

[12] Restricting attention to interior solutions, we can re-write the first-order condition (12) to yield

$$\frac{c^\star(s,s,h)^{1-\frac{1}{\sigma}}}{x(p)}\left[\frac{1}{1-h}\right] = \frac{c^\star(s,s,h)^{1-\frac{1}{\sigma}}}{x(p)}\left[\psi\frac{\sigma}{(\sigma-1)}\frac{1}{x(p)}\right],$$

where the left-hand side reflects the marginal cost of longevity and the right-hand side the benefits. Neglecting the identical terms in front of the brackets, we can observe that the relative marginal costs (the expression in brackets on the left-hand side) does not depend on σ. The relative marginal benefits of longevity (the expression in brackets on the right-hand side) decrease with σ for low values of σ and then increase at higher σ-values, thereby reflecting the two investment motives for longevity. If $r = \rho$, $x(p)$ is independent of σ and consequently the second motive vanishes. Then, the willingness to pay for a longer life only depends on the monotonically declining term $\sigma/(\sigma-1)$, as in Shepard and Zeckhauser (1984). Relaxing the assumption $r = \rho$ in Shepard and Zeckhauser (1984) should uncover the second motive for investments to prolong life. We also expect the second motive to be present in Ehrlich (2000). However, the comparative statics with respect to the intertemporal elasticity of substitution assume an exogenously given wealth level, thereby excluding this motive.

differentiate the aggregation rule (14) and insert equations (10) and (17a) yielding

$$(p+\nu)\,[c(t)-c(t,t)] = (p+\nu)x(p)k(t)\ . \tag{19}$$

Thus, $(p+\nu)x(p)k(t)$ reflects the economy's age structure. When p decreases, the share of older households rises. Ceteris paribus, this leads to higher capital per capita and, as can be inferred from (18), to a higher growth rate of the economy. Further, (18) indicates that the growth effect of the age structure is independent of the interest rate r. An increase in the growth rate of the cohort size ν leads to a larger share of ("poor") young households relative to old ("rich") ones implying, ceteris paribus, lower values of capital per capita. Thus, the economy's growth rate depends negatively on the long-run population growth rate ν.[13]

Proposition 5 (Effect of p and ν on growth rate)
Everything else equal, the economy's growth rate decreases with the hazard rate p and the growth rate of the cohort size ν.

The proof is given in the appendix.

6 Improvements in the Healthcare Technology

We are particularly interested in how the aggregate economy is affected by changes in the healthcare technology. As discussed earlier, the healthcare technology (8) exhibits two parameters influencing the hazard rate p of the households. A decline in the parameter p_{max} reduces the hazard rate that households face without investments in healthcare. A rise in the parameter ψ increases the reduction of the hazard rate that is purchased for any given healthcare investment h. As stated in the following proposition, an improvement of the healthcare technology either via a decrease in p_{max} or an increase in ψ leads to a higher rate of growth.

Proposition 6 (Improvements in the healthcare technology)
For interior levels of healthcare in the market equilibrium, the following conditions hold:

(i) $$\frac{dh^\star}{dp_{max}} < 0\ , \qquad \frac{dp^\star}{dp_{max}} > 0\ , \qquad \frac{dg^\star}{dp_{max}} < 0\ ,$$
$$\frac{dh^\star}{d\psi} > 0\ , \qquad \frac{dp^\star}{d\psi} < 0\ , \qquad \frac{dg^\star}{d\psi} > 0\ ,$$

[13] Note that we hold ν fixed when taking the derivative with respect to p. This implies that the birth rate varies. Defining the measure of newborns at time t by $\beta N(t)$, where β reflects the birth rate, we obtain in equilibrium that $\nu = \beta - p^\star$. Inserting this expression into $g(p)$ as given in (18), it can be easily verified that the growth rate increases with p holding β fixed and decreases with the birth rate β.

$$(ii) \qquad -\frac{dp^\star}{dp_{max}} = \frac{dp^\star}{d\psi} = -\sigma \ , \qquad\qquad -\frac{dh^\star}{dp_{max}} < \frac{dh^\star}{d\psi} \ .$$

The proof of Proposition 6, given in the appendix, shows that better healthcare technology affects the equilibrium growth rate $g^\star$ in two ways. First, there is a direct effect. Ceteris paribus, a decrease in p_{max} or an increase in ψ lowers the hazard rate $p^\star$, which implies an increase in the equilibrium growth rate $g^\star$, as detailed in Section 5. Second, there is an indirect effect. When the healthcare technology is more productive, households invest a higher share of income in healthcare implying an increase in $h^\star$. This additional reduction in the hazard rate $p^\star$ further increases the equilibrium growth rate $g^\star$.

An important insight is conveyed by the conditions in Proposition 6 (ii). In an interior equilibrium ($h^\star > 0$), the magnitude by which a marginal improvement in the healthcare technology increases expected lifetimes is determined by the households' intertemporal elasticity of substitution. The higher is the intertemporal elasticity of substitution, the larger is the effect of a marginal improvement in the healthcare technology on life expectancy. The magnitude of the effect is independent of whether the improvement results from a decrease in p_{max} or an increase in ψ. However, the channel by which the healthcare technology improves is crucial for the effect on equilibrium healthcare expenditures $h^\star$: An increase in longevity via a marginal increase in ψ incurs higher costs in equilibrium relative to a marginal decrease in p_{max}.

6.1 Magnitude of effects

To get an idea of the magnitude of the comparative static effects from an improvement in the healthcare technology we provide a numerical example. Table 1 illustrates the case where $p_{max} = 1/60$ (implying 60 years of expected lifetime without healthcare investments) and improvements in the productivity of healthcare expenditures ψ are such that – in equilibrium – life expectancy is increased in steps of 5 years. For each state of technology characterized by p_{max} and ψ, the table gives the corresponding equilibrium levels of healthcare spending $h^\star$, the expected lifetime $T = 1/p^\star$ and the corresponding growth rate of the economy $g^\star$. For all parameters we choose plausible real world values.[14]

We observe that the growth rate $g^\star$ increases very little in response to a higher productivity of healthcare investments ψ accompanied by a higher expected lifetime T. The relative

[14] For the intertemporal elasticity of substitution σ we follow Murphy and Topel (2003) who suggest a value of $\varepsilon = (u'(c)c)/u(c) = 0.346$ which is also used by Becker et al. (2005). For our instantaneous utility function (4) this translates into $\sigma = 1.529$, which we round to $\sigma = 1.5$. The remaining parameters are set to $\alpha = 0.33$, $r = 3.5\%$, $\rho = 2\%$, $\nu = 0$. In addition, we normalize the wage level at time $t = 0$ to $w(0) = 1$. A sensitivity analysis (available upon request) shows that our results are qualitatively very robust to reasonable changes in the parameter values.

p_{max}	1/60						
ψ	0.97%	1.06%	1.13%	1.19%	1.25%	1.30%	1.34%
$h^\star$	0	12.12%	21.05%	27.91%	33.33%	37.74%	41.38%
$T = 1/p^\star$	60	65	70	75	80	85	90
$g^\star$	2.179%	2.187%	2.194%	2.200%	2.204%	2.208%	2.212%
$\Delta g^\star/g^\star$	0%	0.383%	0.310%	0.255%	0.212%	0.179	0.153%
$U(0)$	102.03	102.29	102.80	103.46	104.20	104.98	105.77
$\Delta U_T(0)/U(0)$	0%	0.249%	0.502%	0.640%	0.713%	0.746%	0.755%
$\Delta U_p(0)/U(0)$	0%	0.152%	0.419%	0.569%	0.651%	0.692%	0.707%
$\Delta U_g(0)/U(0)$	0%	0.098%	0.083%	0.071%	0.062%	0.054%	0.047%

Table 1: Equilibrium values for healthcare expenditures $h^\star$, life expectancy T, growth rate $g^\star$ and lifetime utility $U(0)$ for different parameters of ψ.

increase of the growth rate induced by an additional five years of expected lifetime is shown in the row labeled $\Delta g^\star/g^\star$ and ranges between 0.383% for an increase of expected lifetime from 60 to 65 and 0.153% for an increase in expected lifetime from 85 to 90 years.

The row of Table 1 labeled $U(0)$ shows the expected lifetime utility of an individual household born at time $t = 0$ (with the wage at birth normalized to unity). We observe that households living in an economy with a better healthcare technology (and everything else equal) live longer and enjoy higher expected lifetime utility. The relative increase is shown in the row labelled $\Delta U_T(0)/U(0)$ and ranges between 0.249% for an increase of expected lifetime from 60 to 65 and 0.755% for an increase in expected lifetime from 85 to 90 years. Our previous analysis showed that these utility gains of longevity originate from two different sources: (i) the direct utility of a longer lifetime and (ii) the utility gain associated with a higher growth rate. Decomposing the relative utility gain of five year increases in longevity into these two sources, the row labeled $\Delta U_p(0)/U(0)$ shows the relative utility increase stemming from the first source. More precisely, it captures the relative utility gain of a household that experiences an additional five years of expected lifetime but the same growth rate of the economy.[15] The residuum to the total relative utility gain from increased life expectancy is presented in the last row labeled $\Delta U_g(0)/U(0)$. It reflects the utility gain due to the increase in the growth rate compared to the economy in which households live five years less in expectation.[16] We observe that most of the utility gain is due to the direct

[15] One may consider two hypothetical households. Household 1's expected lifetime, $1/p_1$, is 5 years higher than the one of household 2, $1/p_2$. Then $\Delta U_p(0)/U(0)$ represents the relative utility difference between these two households given they both experience growth rate $g(p_2)$.

[16] The results change minimally when $\Delta U_p(0)/U(0)$ is derived as the residuum and $\Delta U_g(0)/U(0)$ is calculated as follows. Both households exhibit the same life expectancy $1/p_2$, but household 1 experiences the growth rate $g(p_1)$ and household 2 the growth rate $g(p_2)$, where $1/p_1 - 1/p_2 = 5$ years.

p_{max}	1/60	1/65	1/70	1/75	1/80	1/85	1/90
ψ	0.5%						
$h^\star$	0	0	0	0	0	0	0
$T = 1/p^\star$	60	65	70	75	80	85	90
$g^\star$	2.179%	2.187%	2.194%	2.200%	2.204%	2.208%	2.212%
$\Delta g^\star/g^\star$	0%	0.383%	0.310%	0.255%	0.212%	0.179	0.153%
$U(0)$	102.03	106.79	111.23	115.38	119.28	122.93	126.38
$\Delta U_T(0)/U(0)$	0%	4.662%	4.157%	3.734%	3.375%	3.067%	2.800%
$\Delta U_p(0)/U(0)$	0%	4.560%	4.071%	3.661%	3.311%	3.012%	2.752%
$\Delta U_g(0)/U(0)$	0%	0.102%	0.086%	0.073%	0.063%	0.055%	0.048%

Table 2: Equilibrium values for healthcare expenditures $h^\star$, life expectancy T, growth rate $g^\star$ and lifetime utility $U(0)$ for different parameters of p_{max}.

effect of a longer expected lifetime and only a small fraction is attributable to the increase of the growth rate.

Overall, the utility gains from longevity due to a higher productivity of healthcare investments seem rather limited. This may change when we consider five-year increases of expected lifetime resulting from decreases in p_{max}, as – according to Proposition 6 – a marginal decrease in p_{max} induces a smaller rise in the healthcare expenditures compared to a marginal increase in ψ. In Table 2 we demonstrate the case where ψ is so low that no investments in healthcare are always optimal and, thus, increases of expected lifetime solely stem from the reduction of p_{max}. As a consequence, improvements in longevity come without direct costs to households. We concentrate on this polar case for two reasons. First, it highlights the difference between increases in longevity via the two different channels, as utility gains are highest if longevity increases come without healthcare costs. Second, this case is methodologically identical to models with exogenous changes in longevity and, hence, allows us to compare our results with this literature. Apart from ψ and p_{max} all parameter values are identical to the example shown in Table 1 (see footnote 14).

We observe the same growth rates as in the previous example, as in our model the growth rate only depends on the equilibrium life expectancy $p^\star$ and the other exogenous parameters. However, the relative utility gain from a five-year increase in expected lifetime is now substantially higher (more than threefold). As a consequence, the share of the utility gain attributable to an increasing growth rate is now even smaller compared to the scenario where the longevity increase stems from improvements in the productivity of healthcare investments.

Thus, our model indicates that improvements in healthcare technology may have a large impact on overall welfare. However, this impact is rather driven by increasing lifetime utility due to an increasing life expectancy than by the effects of longevity on economic growth. Therefore, our model supports the assumption (implicitly) made by several papers on the welfare aspects of longevity (see, for example, Becker et al. 2005 and Jones and Klenow 2010) that the welfare gains from an increase in longevity are not well reflected in the GDP-growth rate. However, our numerical example also shows that the magnitude of the welfare gains due to a higher expected lifetime strongly depends on the channel by which this increase in longevity is reached, and, in particular, by the accompanied rise in health expenditures. Welfare gains are considerably higher if increases in expected lifetime come as windfall gains from a decrease in the maximal hazard rate p_{max} together with no healthcare expenditures compared to improvements in the productivity of healthcare treatment ψ.

6.2 Welfare gains between 1960 and 2000

The previous discussion indicates that the welfare consequences of increased longevity depend substantially on the healthcare costs associated with it. To elicit the welfare gains that have been realized by the longevity increases over the last decades, we apply our model to the development of healthcare expenditures and average lifetime between the years 1960 and 2000 for seven world regions.[17] The results for all seven regions and details on the data for the numerical exercise are given in the appendix. In the following discussion, we concentrate on a developed (North America) and a developing (South Asia) region.

The levels of h given in Table 3 are the observed health expenditures per GDP multiplied by 3/4. This factor has been chosen for the following reason: On the one hand, h in our model is the share of labor income spent on healthcare rather than the share of total GDP. Assuming a labor share of 2/3, we divide data on health expenditures per GDP by this number. On the other hand, not all health expenditures are effective in prolonging life. Assuming that half of the expenditures affect the individuals' life expectancy leads to the factor of 3/4 given above. We have no data on health expenditures for South Asia in 1960, which we estimate to be (close to) zero.[18] In line with our theoretical model, we assume that increases in average lifetime stem from improvements of the healthcare technology. This implies that the growth rate increases accordingly. We assume that between 1960 and 1980 the respective world region experienced growth of income per capita consistent with the healthcare expenditure and average lifetime data of 1960, and between 1980 and 2000 income per capita grew consistently with 2000 data. Using the parameters $\alpha = 0.33$, $\sigma = 1.5$

[17] To be able to compare our results with previous studies, we use the original data set of Becker et al. (2005), which we amend by data on healthcare expenditures.

[18] As further discussed in the appendix, our qualitative results are very robust with respect to variations of the level of health expenditures.

Region	N. America		N. Am. (exo. p)		S. Asia	
Year	1960	2000	1960	2000	1960	2000
$T = 1/p^\star$	69.9	77.3	69.9	77.3	44.0	62.7
$h^\star$	3.9%	9.8%	0%	0%	0%	3.1%
$g^\star$	2.44%	2.45%	2.44%	2.45%	2.38%	2.45%
r	3.69%		3.69%		3.74%	
ν	1.14%		1.14%		2.22%	
$g_\varnothing$	2.44%		2.44%		2.42%	
$\hat{g}$	2.57%		2.65%		3.30%	
$\Delta U/U(1960)$	43.25%		46.29%		71.04%	
$\Delta U_T/\Delta U$	11.24%		17.06%		47.43%	
$\Delta U_g/\Delta U$	0.67%		0.64%		3.19%	
$\Delta U_p/\Delta U$	10.57%		16.43%		44.24%	

Table 3: Utility gains for North America and South Asia from 1960 to 2000.

and $\rho = 0.02$ of the previous numerical example and normalizing the wage level in 1960 to 1, the depreciation rate has been adjusted such that the simulated average growth rate in each region is identical to the observed average growth rate between 1960 and 2000.

Table 3 shows the results. In North America average life expectancy increased from 69.9 in 1960 to 77.3 years in 2000. At the same time the healthcare expenditures (in percentage of labor income) increased from 3.9% to 9.8%. In South Asia, life expectancy rose from 44.0 in 1960 to 62.7 years in 2000. Healthcare expenditures equaled 3.1% in 2000 and are estimated (close to) zero for 1960. Over this 40 year period the average annual growth rate of income per capita equaled 2.44% for North America and 2.42% for South Asia. Population grew by an average annual rate of 1.14% in North America and 2.22% in South Asia.

We are now interested in the utility gain of a person born under the conditions of the year 2000 relative to a person born in 1960 in the same region. The result is given in the row labeled $\Delta U/U(1960)$: In North America the expected lifetime utility increased by 43.25%. Of course, due to economic growth a person born in 2000 was better off than a person born in 1960 even without increases in life expectancy. To elicit which share of the utility gain is attributable to the increase in average lifetime, we deduct the utility gain that a person would enjoy if born in 2000 while expected lifetime remained unchanged relative to 1960. We find that a share of 11.24% of the total welfare gains in North America between 1960 and 2000 is attributable to the increase in average lifetime (see row labeled $\Delta U_T/\Delta U$). The last two rows recall our previous finding that almost all the utility gains originate directly from a longer expected lifetime ($\Delta U_p/\Delta U$) rather than indirectly via an increased growth rate of GDP per capita ($\Delta U_g/\Delta U$). The row labelled $\hat{g}$ reports the average annual "full-income" growth rate of GDP per capita that would have been necessary between 1960 and 2000 to give a person born in 2000 the same utility without the increase in longevity. The table indicates that the growth rate had to be 2.57% instead of 2.44% to compensate for the utility gain of increased expected lifetime.

To get an idea of the role played by healthcare expenditures, we contrast our results with a thought experiment in which longevity increases come without costs. The results are reported in the second column headed "North America (exogenous p)". Without healthcare expenditures the total relative welfare gains between 1960 and 2000 would be higher and 17.06% of these larger welfare gains would be due to increased longevity, an increase of more than 50% relative to the real world scenario. This is also reflected in the higher full-income growth rate of 2.65% compared to 2.57% when healthcare costs are considered.

These results might indicate that relative welfare gains due to increased longevity are considerably higher in developing countries where relatively cheap measures (such as better sanitation, better access to standard vaccines, etc.) involve relatively high increases in p_{max} compared to developed countries where further increases in average lifetime are mainly due to improvements of expensive cutting-edge medical treatment. As an example for a developing region, we report in the third column of Table 3 the relative welfare gain of 71.04% in South Asia between 1960 and 2000 of which almost half (47.43%) are attributable to increases in life expectancy. We obtain a full-income growth rate of 3.3% while on average GDP per capita only grew by 2.42% per annum in this region.

For the period between 1960 and 2000, Becker et al. (2005) report full-income growth rates for North America of 2.7% and for South Asia of 3.1%. Further they calculate shares of welfare improvements due to mortality reductions compared to overall welfare improvements between 1960 and 2000 for North America of about 12% and for South Asia of about 30.4%. They conclude that when additionally considering longevity improvements the world's welfare inequality has become lower than solely GDP-based measures suggest. The re-examination of their data set in light of our theoretical model supports this conclusion. However, the numbers given in Table 3 suggest an even stronger convergence of welfare for two reasons. First, in contrast to Becker et al. (2005), we consider a growth model and take lifetime utility and not instantaneous utility as yardstick for welfare. The strong diminishing returns of consumption due to the concavity of the instantaneous welfare function (3) lead to a lower lifetime utility increase in high-income countries compared to low-income countries for the same average growth rate of income per capita. Second, in our model we explicitly consider costs of improvements in life expectancy.[19] As a consequence,

[19] While Becker et al. (2005) did not include healthcare expenditures in their analysis, they conjectured that this would make a difference in the relative welfare gains from longevity increases enjoyed by developed and developing countries.

welfare gains for a given increase in longevity are higher if this increase is accompanied by no or small changes in healthcare expenditures.

7 Inefficient Market Equilibria

An important policy question is whether the market equilibrium, as analyzed in the previous sections, is efficient. In fact, the model comprises three externalities that are not accounted for in the market equilibrium.

First, there is a "learning-by-investing" externality (Romer 1986). At any time, firms take the technological level $A(t)$ of the economy as given, neglecting the positive spillovers the employment of capital exerts on the economy's production output $Y(t)$ via an increase in the technological level. As is well known, this leads to an inefficiently low level of asset holdings that could be corrected, for example, by subsidizing household savings.

Second, there are two additional externalities associated with healthcare expenditures. To identify these, we take the total derivative of an individual's lifetime utility with respect to healthcare expenditures and use (11a), (13), and $w(t) = \exp[gt]$ to obtain

$$\frac{dU(s)}{dh} = \frac{c^{\star}(s,s,h)^{1-\frac{1}{\sigma}}}{x(p)} \left[F(h) + \frac{da}{dh}\left(\frac{1}{x(p)} - \frac{1}{y(p)}\right) + \frac{dg(p)}{dh}\left(\frac{1}{y(p)} + s\right)\right] . \tag{21}$$

The term $y(p) \equiv r+p-g$ represents the difference between the return of annuities a and the economy's growth rate g. The function $F(h)$ constitutes the individual's first-order condition with respect to healthcare expenditures (12). In the decentralized market equilibrium with an interior solution the first-order condition equals zero. The remaining two summands in brackets denote the impact of healthcare investments on the equilibrium return for annuities, a, and the economy's growth rate, g, which the households do not take into account. The existence of these two externalities is independent of the positive spillovers from capital accumulation. The reason is that internalizing the "learning-by-investing" externality increases the effective interest rate. The additional two externalities, however, do not disappear for any the level of the interest rate. They occur because households do not take into account that increased longevity reduces the equilibrium return of annuities (for a given interest rate) and increases the economy's growth rate via the age structure of the economy, as described in Section 3. The externality of healthcare spending on the rate of return for annuities a, reflected by the expression $da/dh\,(1/x(p) - 1/y(p))$, is negative because $da/dh = -\psi$ and $y(p) - x(p) = \sigma(r-\rho) - g > 0$.[20] Hence, households tend to over-

[20] The term $y(p) - x(p)$ represents the difference between the growth rate of individual household consumption and the growth rate of per capita consumption, which is positive due to equation (17a).

invest in healthcare (Philipson and Becker 1998). However, the last expression in brackets in (21) representing the externality on the economy's equilibrium growth rate g is positive implying under-investment in healthcare. Thus, a decrease in healthcare expenditures has a positive effect on the rate of return from annuities a, but a negative effect on g and vice versa. Whether the equilibrium level of healthcare expenditures is inefficient in the sense that there exists a steady state in which all households are better off by investing either more or less in healthcare depends on the magnitude of the welfare losses associated with the two opposed externalities.

All terms in brackets in equation (21) are independent of time except for the last term that reflects the increase of the wage-level of a generation born at time s due to a marginal change in the growth rate. This term increases with s,[21] implying that the welfare loss due to a lower than optimal steady-state growth rate is larger the later a household is born. Even small changes in the economy's growth rate g have huge welfare effects for generations living in the far distant future. As the term is linear in s, there exists some $\bar{s}$ for any decrease in healthcare expenditures such that all generations born at $s > \bar{s}$ are worse off, although early generations may benefit. Thus, it cannot occur that healthcare expenditures in the market equilibrium are inefficiently high in the sense that there exists a lower level of healthcare expenditures for which all households born at $s \geq 0$ would be better off. Whether healthcare expenditures are inefficiently low in the market equilibrium depends on the relative strengths of the two externalities for the generation born at $s = 0$. Proposition 7 gives a condition for which under-investment in healthcare occurs.

Proposition 7 (Inefficient levels of healthcare expenditures)
For interior levels of healthcare expenditures, households invest inefficiently low amounts in healthcare in the market equilibrium if $dg(p)/dp < 1 - y(p)/x(p)$.

The proof is given in the appendix.

8 Spillover Effects

In our model we have specified the "learning-by-investing" spillovers as a function of the capital-stock per worker. Although this is a reasonable assumption, one may ask how our results would change with a different specification. To answer this question, we consider the definition

$$A(t) \equiv \frac{K(t)}{L^F(t) + (1-\eta)L^H(t)}, \qquad \eta \in [0,1] \ , \tag{22}$$

[21] Note that by setting $w(t) = \exp[gt]$, we normalize the wage rate at $t = 0$ to unity. This implies that we compare the lifetime utility of all generations born at $s \geq 0$.

which captures all spillover magnitudes between the two polar cases $\eta = 1$ representing our previous model and $\eta = 0$ reflecting spillovers depending on capital per capita $A(t) = k(t)$. While condition (13) for the individual household's optimal choice of healthcare expenditures remains unchanged, as households take prices as given, the new specification of $A(t)$ affects the interest and the wage rate

$$r(t) = \alpha \left[\frac{L^F(t)}{L^F(t) + (1-\eta)L^H(t)} \right]^{1-\alpha} - \delta \ , \tag{23a}$$

$$w(t) = (1-\alpha)\frac{K(t)}{L^F(t)} \left[\frac{L^F(t)}{L^F(t) + (1-\eta)L^H(t)} \right]^{1-\alpha} \ . \tag{23b}$$

Given the equilibrium levels of $h(s)$, $L^H(t)$ can still be expressed by (7). For $\eta < 1$ the interest rate declines with the level of healthcare expenditures, as $L^H(t)$ increases and $L^F(t) = N(t) - L^H(t)$ declines. On the one hand, a decline in labor employed in consumption-good production reduces the marginal productivity of capital. On the other hand, for $\eta > 0$ the technological level $A(t)$ increases, as the capital intensity in consumption-good production increases which amplifies the learning externality.[22] The first effect dominates the second implying a negative influence of health-care expenditures on the interest rate. Note that in the basic version of our model, which corresponds to $\eta = 1$, the two effects cancel out leading to the constant interest rate $r(t) = \alpha - \delta$. This interest rate also results in the general set-up if no healthcare expenditures are optimal in the market equilibrium for all generations, i.e. $h(s) = 0$ for all s, implying $L^F(t) = N(t)$ and $L^H(t) = 0$ at all times t.

In the following, we restrict attention to the steady state market equilibrium in which the interest rate $r(t) = r$ is constant. For this to hold, optimal healthcare expenditures h have to be constant implying that also the hazard rate p and the population growth rate $\dot{N}(t)/N(t) = \nu$ are constant. We obtain for the interest rate and the wage rate in steady state[23]

$$\bar{r}(h) = \alpha \left[\frac{1-h}{1-\eta h} \right]^{1-\alpha} - \delta \ , \tag{24a}$$

[22] An alternative interpretation of the effect of healthcare expenditures on $A(t)$ would be that better health increases the workers' productivity. A more precise implementation of this idea would specify $A(t)$ as a function of $L^H(t)$, $L^F(t)$, p_{max}, ψ, and $K(t)$. A simple way to augment our basic model in this direction is to multiply $A(t)$, as given in Section 2.1, by a constant $\zeta(p_{max}, \psi)$ that increases with ψ and declines with p_{max}. This would not change our analysis substantially but leads to larger effects of improvements in the healthcare technology on the growth rate. A more general way is to include the average health status reflected by average lifetime into the model. Such an extension yields similar effects of health on production as described in the main text. On the one hand, better health increases the productivity of capital in consumption-good production. On the other hand, if this higher level of health incurs healthcare expenditures, the labor force decreases leading to a lower marginal productivity of capital.

[23] We indicate steady state values in the general spillover setting by a bar and, where applicable, use h as an additional argument to highlight the difference to the basic version of the model.

$$\bar{w}(h,t) = \frac{1-\alpha}{1-h} k(t) \left[\frac{1-h}{1-\eta h} \right]^{1-\alpha} \quad . \tag{24b}$$

Inserting $\bar{a}(h,p) = \bar{r}(h) + p$ into (13) determines the healthcare expenditures in the steady state market equilibrium.

Proposition 8 (Existence of steady state healthcare level)
Given equation (22) *for the technological level of the economy, there exists a steady state equilibrium level of healthcare expenditures,* $\bar{h}^\star$.

The proof is given in the appendix. In contrast to the basic model, the aggregate dynamics in the steady state depends not only on the hazard rate p but also on the level of healthcare expenditures h.

Proposition 9 (Steady state aggregate dynamics)
Given equation (22) *for the technological level of the economy, the steady state dynamics of the aggregate economy*

(i) is characterized by:

$$\begin{aligned} \dot{c}(t) &= \sigma \left[\bar{r}(h) - \rho \right] c(t) - \bar{x}(h,p)(p+\nu) k(t) \ , \\ \dot{k}(t) &= \left[\frac{\bar{r}(h)}{\alpha} + \frac{1-\alpha}{\alpha}\delta - \nu \right] k(t) - c(t) \ , \end{aligned}$$

(ii) is governed by a balanced growth path given by

$$\begin{aligned} \bar{g}(h,p) =& \frac{1}{2} \left\{ \frac{\bar{r}(h)}{\alpha} + \frac{1-\alpha}{\alpha}\delta - \nu + \sigma \left[\bar{r}(h) - \rho \right] \right\} \\ & - \frac{1}{2} \sqrt{ \left\{ \frac{\bar{r}(h)}{\alpha} + \frac{1-\alpha}{\alpha}\delta - \nu - \sigma \left[\bar{r}(h) - \rho \right] \right\}^2 + 4\bar{x}(h,p)(p+\nu) } \ , \end{aligned}$$

where $\bar{x}(h,p) \equiv \bar{r}(h) + p - \sigma(\bar{r}(h) - \rho)$.

The proof is given in the appendix. Equation (9) shows that not only the hazard rate p matters for the steady state growth rate, as it is the case in the basic model, but also the level of healthcare expenditures by which it is achieved. If there are no healthcare investments in steady state, $\bar{h}^\star = 0$, we are back in the basic model implying $\bar{g}(0,p) = \bar{g}(0,p_{max}) = g(p_{max})$. By virtue of Proposition 5, for $\bar{h}^\star = 0$ a decreasing $p = p_{max}$ leads to an increasing equilibrium growth rate $\partial \bar{g}(0,p)/\partial p < 0$. This does not necessarily hold if the reduction in p is accompanied by an increase in h. We obtain for the total derivative of

$\bar{g}(h,p)$ with respect to h

$$\frac{d\bar{g}(h,p)}{dh} = \frac{\partial \bar{g}(h,p)}{\partial p}\frac{dp}{dh} + \frac{\partial \bar{g}(h,p)}{\partial \bar{r}(h)}\frac{d\bar{r}(h)}{dh} \ . \tag{26}$$

The first term is positive, as $\partial\bar{g}(h,p)/\partial p < 0$ (Proposition 5) and $dp/dh < 0$. The sign of the second term is ambiguous, as the sign of $\partial\bar{g}(h,p)/\partial\bar{r}(h)$ is ambiguous and $d\bar{r}(h)/dh < 0$. In contrast to our basic model, it is now possible that the steady state growth rate declines in response to an increase in longevity. This happens, if $\partial\bar{g}(h,p)/\partial r > 0$ and sufficiently large so that the second term outweighs the first. Thus, the difference between improvements in longevity originating from increases in p_{max} accompanied by zero healthcare expenditures and those from an increase of ψ is even further pronounced compared to the basic model. In the basic model, both types of longevity increases had (small) positive growth effects but substantial welfare differences. With the alternative spillover mechanism and $\eta < 1$ we may even experience negative growth effects in response to longevity increases.

This also affects the inefficiency result given in Proposition 7. It is now possible that the steady state investments in healthcare are inefficiently high in the sense that all households born at $s \geq 0$ were better off in a steady state where all households invest less in healthcare.

Proposition 10 (Inefficient levels of healthcare expenditures)
Given equation (22) *for the technological level of the economy, in an interior steady state, healthcare expenditures* $\bar{h}^\star$ *are inefficiently high if* $d\bar{g}(h,p)/dh|_{h=\bar{h}^\star} < 0$.

The proof of Proposition 10, provided in the appendix, shows that under the given condition both externalities connected with healthcare investments – on the return of annuities and on the steady state growth rate – have a negative impact on expected lifetime utility.

Finally, we illustrate how the results of our numerical example given in Section 6.2 change under the alternative spillover specification. More precisely, we show results for η ranging from 1 (which corresponds to our previous setup) to 0.9. In addition, we also show the results for a hypothetical North America in which no healthcare expenditures are undertaken and all improvements in longevity solely stem from an increase in p_{max}. In this case the new specification of the spillover effect collapses to the basic model for any value of η. Thus, the results are identical to the second row in Table 3.

For both values of $\eta < 1$ given in Table 4, we observe that the steady state growth rate declines in response to the increase of the healthcare expenditures from 3.9% of labor income in 1960 to 9.8% in 2000. The effect is more pronounced the smaller is η. As in Section 6.2, we adjust the depreciation rate such that the average annual growth rate of GDP per capita matches the observed value of 2.44%. This implies that the growth rate drops from 2.49% in 1960 to 2.39% in 2000 for $\eta = 0.95$ and from 2.55% in 1960 to 2.34% in 2000 for $\eta = 0.9$. In

Region	North America			
T	$T(1960) = 69.9$ $T(2000) = 77.3$			
h	0	$h(1960) = 3.9\%$ $h(2000) = 9.8\%$		
η	$\in [0, 1]$	1	0.95	0.9
$r(1960)$	3.69%	3.69%	3.73%	3.76%
$r(2000)$	3.69%	3.69%	3.65%	3.62%
$g^\star(1960)$	2.44%	2.44%	2.49%	2.55%
$g^\star(2000)$	2.45%	2.45%	2.39%	2.34%
$\hat{g}$	2.65%	2.57%	2.54%	2.51%
$\Delta U/U(1960)$	46.29%	43.25%	41.16%	39.11%
$\Delta U_T/\Delta U$	17.06%	11.24%	4.18%	−3.48%
$\Delta U_g/\Delta U$	0.64%	0.67%	−7.16%	−15.64%
$\Delta U_p/\Delta U$	16.43%	10.57%	11.33%	12.16%

Table 4: Utility gains for North America from 1960 to 2000 for different values of η.

contrast to the results in Section 6.2, the welfare effects of these changes in the growth rate may be substantial, as can be seen in the row labeled $\Delta U_g/\Delta U$. In fact, while for $\eta = 0.95$ the total welfare gain attributable to increases in longevity is still positive, for $\eta = 0.9$ the utility loss from the decreasing growth rate outweighs the direct utility increase due to increased average lifetime so that the welfare gain between 1960 and 2000 had been higher by 3.48% without the increase in life expectancy.

These results emphasize that the growth effects of increases in longevity may drastically depend on the associated healthcare costs. In any case, welfare and growth effects are the highest if the increases in longevity stem from a decrease of the maximal hazard rate p_{max} together with zero healthcare expenditures. If increases in longevity are accompanied by increases in healthcare expenditures, as is always the case for increases in the efficiency of healthcare ψ, the growth effects of increased longevity are either small (in case of our basic model) or may even be negative (in case of our alternative spillover specification). In fact, the negative growth effects may even outweigh the direct utility gains from increased longevity. At least for the richest regions like North America or Western Europe, this might challenge the conclusions of Becker et al. (2005) and Jones and Klenow (2010) who argue that recent increases in longevity induced non-negligible positive welfare gains.

It is important to recognize that the negative welfare effects in the case of $\eta = 0.9$ result from the negative externality of healthcare expenditures on the economy's growth rate. In general, these externalities call for governmental action. In most countries, the healthcare system is heavily regulated. Usually health insurance systems result in inefficiently high demand for healthcare implying higher healthcare spending relative to the pure market equilibrium considered in our model (see, for example, Manning et al. 1987, Cutler and Zeckhauser 2000, Cutler 2002, Gruber 2008). As a consequence, when government subsidies for healthcare are taken into account welfare losses may be even higher than those shown for the case $\eta = 0.9$ in Table 4.

Which model specification applies is an important empirical question. To answer this question, one could test whether the return on capital is affected by healthcare expenditures. One of the challenges of such an exercise would be to isolate healthcare expenditures that prolong life from those that do not. For the existing empirical literature, our model is consistent with recent results suggesting that the long-run effects of longevity on GDP per capita are either moderately positive (Ashraf et al. 2008), insignificant or even negative (Acemoglu and Johnson 2007). By contrast, many (earlier) contributions to this literature usually found substantial positive effects of longevity on economic growth.[24] In an attempt to reconcile these different findings, Aghion et al. (2010) argue that both the level of life expectancy and the increase in life expectancy have to be considered and find that both have significantly positive effects on per-capita GDP growth. However, they also indicate that when restricting attention to OECD-countries in the post-1960 period, the effects weaken. Our model allows for two different interpretations of this result. First, in our basic model the growth rate is increasing and concave with respect to longevity. Hence, the model predicts that the effect of an increase in life expectancy on the growth rate becomes smaller the higher is the level of longevity. Second, in the alternative spillover specification the growth rate of the economy may decline in response to higher healthcare costs. The longevity increases in the developed countries in the recent past have mainly originated from "big medicine" involving expensive and intensive personal interventions rather than the eradication of infectious diseases with relatively cheap hygienic measures (see, for example, Becker et al. 2005, Cutler et al. 2006). As a consequence, our theory would predict smaller growth and welfare gains derived from increased life expectancy.[25]

9 Conclusion

We developed an overlapping generations endogenous growth model in continuous time to investigate the link between life expectancy, which is the result of endogenous investments

[24] See, for example, Gallup and Sachs (2001), Bhargava et al. (2001), Barro (1996), Azomahou et al. (2009), Lorentzen et al. (2008). Only very few studies find small negative or no effects of longevity of growth such as Caselli et al. (1996). An overview can be found in Bloom et al. (2004).

[25] Aghion et al. (2010) hypothesize that gains in life expectancy at young age mattered more than gains in life expectancy at old age supposedly for reasons of labor market participation and education. We do not refute these reasons but emphasize the importance of healthcare expenditures associated with the increase in expected lifetime.

in healthcare, economic growth and welfare. We find that improvements in the healthcare technology increase both life expectancy and steady state growth rates of the economy. However, simulation results suggest that the magnitude of the latter effect is small, even for substantial improvements in the healthcare technology. Hence, for the most part welfare gains stem from an increase in average lifetime directly. The magnitude of the welfare gains, however, strongly depends on the channel by which the healthcare technology improves. A reduction of the baseline mortality yields higher welfare gains than an increase in the productivity of healthcare expenditures. Our finding tends to be reinforced when considering more general specifications of the spillover effects in the production sector.

This has implications for the future development of healthcare expenditures and welfare. Several authors have argued that the recent increases in longevity in the developed countries are mainly the result of "big medicine" rendering healthcare expenditures more productive in treating life-threatening diseases rather than a decrease in the baseline mortality level via cheap measures such as improved sanitation. Extrapolating this development, our model suggests that the prospects for future welfare gains from increased longevity are rather modest.

We have shown that the decentralized market solution exhibits several externalities that call for government action. It would be an interesting venue for future research to augment the model with realistic features of national health systems to examine their effects on growth and welfare and to be able to evaluate potential policy interventions. Other interesting extensions would be to incorporate age-dependent mortality, retirement decisions or endogenous fertility. Finally, we only considered exogenous improvements in the healthcare technology. Endogenizing these improvements is a further challenge for future research.

Appendix

A.1 Proof of Proposition 1

First, the corner solution $h(s) = 1$ cannot be an optimal solution, as consumption and lifetime utility would drop to zero, while both are positive for any value $h(s) \in [0, 1)$.

Second, there exists at most one $h^\star(s)$ with $F\big(h^\star(s)\big) = 0$. To see this, set $F\big(h(s)\big) = 0$ and re-arrange terms to yield[26]

$$1 - h(s) = \frac{\sigma(\sigma-1)(\rho+p_{max}) - (\sigma-1)^2 a(s)}{\sigma\psi} - (\sigma-1)h(s)\ . \tag{A.1}$$

Both, the left-hand and the right-hand side are linear equations in $h(s)$, which intersect at most once and are identical in the special case that $\sigma = 2$ and $\sigma(\sigma-1)(\rho+p_{max}) - (\sigma-1)^2 a(s) = \sigma\psi$ hold simultaneously. This special case is precluded, however, as the latter condition contradicts the necessary condition $(1-\sigma)a(s) + \sigma(\rho + p(s)) > 0$ for all $p(s) \in [p_{max}, p_{max} - \psi]$ for the household's problem to be well defined.

Third, the local extremum given by $F\big(h(s)\big) = 0$ is a local maximum only if $\sigma < 2$. Differentiating $F\big(h(s)\big)$ with respect to $h(s)$ and evaluating at the local extremum yields:

$$\left.\frac{\partial F\big(h(s)\big)}{\partial h(s)}\right|_{F(h(s))=0} = \frac{\sigma^2\psi^2}{(\sigma-1)^2\tilde{x}^2\big(h(s)\big)}(\sigma-2) \gtreqless 0 \quad \Leftrightarrow \quad \sigma \gtreqless 2\ , \tag{A.2}$$

where $\tilde{x}\big(h(s)\big) = \big[(1-\sigma)a(s) + \sigma\big(\rho + p_{max} - \psi h(s)\big)\big]$. As a consequence, an interior optimal solution can only exist for $\sigma \in (1, 2)$ and thus $h^\star(s) = 0$ if $\sigma \geq 2$. Even for $\sigma \in (1, 2)$, the optimal solution may be the corner solution $h(s) = 0$. This holds if $F\big(h(s)\big) < 0$ for all $h(s) \in [0, 1)$. □

A.2 Proof of Proposition 2

(i) Aggregate dynamics: To derive the aggregate system dynamics, we evaluate equation (11b) in the market equilibrium, aggregate according to equation (14) and differentiate with respect to t:

$$\dot{c}(t) = x(p)\left[\dot{k}(t) + (1-h)\dot{W}(t)\right]\ , \tag{A.3}$$

[26] Obviously, this re-arrangement is only identical to $F\big(h(s)\big) = 0$ if $h(s) \neq 1$. However, we have already seen that $h(s) = 1$ cannot be an optimal solution.

where $W(t) \equiv \int_t^\infty w(t') \exp[-(r+p)(t'-t)]\, dt'$ denotes the net present value of the household's lifetime labor income. Evaluating the budget constraint in the market equilibrium and aggregating according to equation (14), we obtain

$$\dot{b}(t) = (r-\nu)b(t) + (1-h)w(t) - c(t) \ . \tag{A.4}$$

Inserting $\dot{W}(t)$ and equation (A.4) into equation (A.3) yields equation (17a). We derive (17b) by observing that in the market equilibrium $w(t) = k(t)(1-\alpha)/(1-h)$ and inserting it into equation (A.4).

(ii) Balanced growth path: We prove the existence of a unique balanced growth path (BGP) given a fixed hazard rate p by contradiction. Suppose there is no BGP, but capital would grow at a constant rate g_k and consumption at the constant rate g_c, with $g_k \neq g_c$. Observe in equations (17a) and (17b) that $g_c > g_k$ implies $g_k = -\infty$ and $g_c < g_k$ implies $g_c = -\infty$. As both cases yield economically infeasible solutions the only remaining possibility is a BGP with $g_c = g_k = g$.

To verify uniqueness and the second part of the proposition, we calculate the growth rate g and show that under the given conditions it possesses the respective sign. First, we solve the equations of motion for $c(t)/k(t)$ given that $g_c = g_k = g$. As $x(p)(p+\nu) > 0$ for all $p > 0$, there is only one economically feasible solution (with $c(t)/k(t) > 0$)

$$\frac{c(t)}{k(t)} = \frac{1}{2}\left[(1-\delta-\nu) - \sigma(r-\rho) + \sqrt{[(1-\delta-\nu)-\sigma(r-\rho)]^2 + 4x(p)(p+\nu))}\right] \ . \tag{A.5}$$

This establishes the uniqueness of the BGP for a given value of p.

Inserting (A.5) into $g = (1-\delta-\nu) - c(t)/k(t)$, we obtain for the growth rate g

$$g = \frac{1}{2}\left[(1-\delta-\nu) + \sigma(r-\rho) - \sqrt{[(1-\delta-\nu)-\sigma(r-\rho)]^2 + 4x(p)(p+\nu)}\right] \ . \tag{A.6}$$

After some minor manipulations, we observe that the growth rate on the BGP is positive if and only if $x(p)(p+\nu) < \sigma(r-\rho)(1-\delta-\nu)$. Consequently, $g < 0$ if $x(p)(p+\nu) > \sigma(r-\rho)(1-\delta-\nu)$ and $g = 0$ if $x(p)(p+\nu) = \sigma(r-\rho)(1-\delta-\nu)$. □

A.3 Proof of Proposition 4

The total derivative of $g(p)$ with respect to σ equals:

$$\frac{dg^\star}{d\sigma} = \left.\frac{\partial g(p)}{\partial \sigma}\right|_{p=p^\star} + \left.\frac{\partial g(p)}{\partial p}\right|_{p=p^\star} \frac{dp^\star}{d\sigma} \ . \tag{A.7}$$

The first term denoting the direct effect of σ on the growth rate g is always positive

$$\frac{\partial g(p)}{\partial \sigma} = \frac{1}{2}(r-\rho)\left[1 + \frac{x(p) + p + \nu + 1 - \alpha}{\sqrt{[1-\delta-\nu-\sigma(r-\rho)]^2 + 4(p+\nu)x(p)}}\right] > 0 \ . \tag{A.8}$$

For the second term constituting the indirect effect via changes in the hazard rate p, we differentiate equation (18) with respect to the hazard rate p to obtain

$$\frac{\partial g(p)}{\partial p} = -\frac{x(p) + p + \nu}{\sqrt{[1-\delta-\nu-\sigma(r-\rho)]^2 + 4(p+\nu)x(p)}} < 0 \ . \tag{A.9}$$

We can decompose $dp^\star/d\sigma$: $dp^\star/d\sigma = dp^\star/dh^\star \cdot dh^\star/d\sigma$. By virtue of the healthcare technology, we find that $dp^\star/dh^\star = -\psi < 0$. We also know from Proposition 3 that the sign of $dh^\star/d\sigma$ is ambiguous, depending on the value of σ. Combining these findings yields the claim of the proposition. □

A.4 Proof of Proposition 5

The total derivative of the equilibrium growth rate $g(p)$ with respect to ν reads

$$\frac{dg^\star}{d\nu} = \left.\frac{\partial g(p)}{\partial \nu}\right|_{p=p^\star} + \left.\frac{\partial g(p)}{\partial p}\right|_{p=p^\star} \frac{dp^\star}{dh^\star}\frac{dh^\star}{d\nu} \ . \tag{A.10}$$

According to equation (15), $\partial h^\star/\partial \nu = 0$ implying that the second summand equals zero. We obtain for the partial derivative of $g(p)$ with respect to ν

$$\frac{\partial g(p)}{\partial \nu} = -\frac{1}{2}\left[1 + \frac{x(p) + p + v - (1-\alpha)}{\sqrt{[1-\delta-\nu-\sigma(r-\rho)]^2 + 4(p+\nu)x(p)}}\right] \ , \tag{A.11}$$

which is smaller than zero if

$$\begin{aligned} &[1-\delta-\nu-\sigma(r-\rho)]^2 + 4(p+\nu)x(p) > [x(p) + p + v - (1-\alpha)]^2 \\ &\qquad \Leftrightarrow \qquad 4x(p)(1-\alpha) > 0 \ . \end{aligned} \tag{A.12}$$

The last condition is always satisfied as $\alpha \in (0,1)$ and $x(p) > 0$. In addition, we know from the proof of Proposition 4 that $\partial g(p)/\partial p < 0$. □

A.5 Proof of Proposition 6

(i) Differentiating the equilibrium level of healthcare expenditures, as given in (15), with respect to ψ and p_{max} yields:

$$\frac{dh^\star}{d\psi} = \frac{(\sigma-1)\left[(1-\sigma)r+\sigma\rho+p_{max}\right]}{\psi^2} > 0 \ , \qquad \frac{dh^\star}{dp_{max}} = -\frac{\sigma-1}{\psi} < 0 \ . \tag{A.13}$$

From the healthcare technology (8), we obtain

$$\frac{dp^\star}{d\psi} = \frac{\partial p^\star}{\partial \psi} + \frac{\partial p^\star}{\partial h^\star}\frac{dh^\star}{d\psi} = -\left(h^\star + \psi\frac{dh^\star}{d\psi}\right) < 0 \ , \tag{A.14a}$$

$$\frac{dp^\star}{dp_{max}} = \frac{\partial p^\star}{\partial p_{max}} + \frac{\partial p^\star}{\partial h^\star}\frac{dh^\star}{dp_{max}} = 1 - \psi\frac{dh^\star}{dp_{max}} > 0 \ . \tag{A.14b}$$

For the growth rate $g^\star$ we obtain

$$\frac{dg^\star}{d\psi} = \left.\frac{\partial g(p)}{\partial p}\right|_{p=p^\star}\frac{dp^\star}{d\psi} > 0 \ , \qquad \frac{dg^\star}{dp_{max}} = \left.\frac{\partial g(p)}{\partial p}\right|_{p=p^\star}\frac{dp^\star}{dp_{max}} < 0 \ . \tag{A.15}$$

The inequalities follow from $\partial g(p)/\partial p < 0$, as shown in Proposition 5.

(ii) Inserting the derivatives in (A.13) and the interior solution for $h^\star$ from (15) into equations (A.14) gives $dp^\star/d\psi = -\sigma$ and $dp^\star/dp_{max} = \sigma$. Using (A.13), the condition $-dh^\star/dp_{max} < dh^\star/d\psi$ translates to

$$\frac{\sigma-1}{\psi} < \frac{(\sigma-1)\left[(1-\sigma)r+\sigma\rho+p_{max}\right]}{\psi^2} \quad \Leftrightarrow \quad (1-\sigma)r+\sigma\rho+p_{max}-\psi > 0 \ . \tag{A.16}$$

The latter condition is satisfied by assumption, as otherwise the households' maximization problem is not well defined. □

A.6 Proof of Proposition 7

Healthcare investment in the market equilibrium is inefficiently low if $dU(s)/dh|_{h=h^\star} > 0$ for all $s \geq 0$, as this would imply that a marginal increase in healthcare expenditures would increase the lifetime utility of all generations born at s ≥ 0. For an interior solution $F(h^\star) = 0$. As a consequence, the condition is satisfied for all $s \geq 0$ if

$$\frac{da}{dh}\left(\frac{1}{x(p)} - \frac{1}{y(p)}\right) + \frac{dg(p)}{dh}\frac{1}{y(p)} > 0 \ . \tag{A.17}$$

As $da/dh = dp/dh = -\psi$ and $dg(p)/dh = dg(p)/dp \cdot dp/dh$, (A.17) holds if the inequality given in the proposition is satisfied. □

A.7 Proof of Proposition 8

The individual household's choice of optimal healthcare expenditures, as given in Proposition 1, remains unchanged by the new definition of the technological level of the economy. As a consequence, the optimal level of healthcare in the steady state market equilibrium equals $\bar{h}^\star = 0$ for $\sigma \geq 2$. For $\sigma < 2$, we insert $\bar{a}(h,p) = \bar{r}(h) + p$ into the first-order condition (13) and obtain

$$\bar{F}(h) \equiv \frac{\sigma\psi}{(\sigma-1)\left[(1-\sigma)\bar{r}(h) + \sigma\rho + p\right]} - \frac{1}{1-h} \ . \tag{A.18}$$

Note that $\lim_{h\to 1} \bar{F}(h) = -\infty$, as the first term remains finite[27] and the second term diverges to $-\infty$ for $h \to 1$. Thus, there exists an $\bar{h}^\star$ with $\bar{F}(\bar{h}^\star) = 0$ and $d\bar{F}(h)/dh|_{h=\bar{h}^\star} < 0$ if $\bar{F}(h) > 0$ for some $h \in [0,1)$. Otherwise the optimal level of healthcare equals $\bar{h}^\star = 0$.

□

A.8 Proof of Proposition 9

(i) Aggregate dynamics: Using equations (24) instead of (6), we derive the aggregate steady state dynamics analogously to part (i) of the proof of Proposition 2. To derive the equation of motion for the capital stock $k(t)$ insert $b(t) = k(t)$ and $\bar{w}(h,t) = (1-\alpha)/[\alpha(1-h)] \cdot \bar{r}(h)k(t)$.

(ii) Balanced growth path: The existence and uniqueness of a balanced growth path can be shown as in part (ii) of the proof of Proposition 2. Replacing $(1-\delta-\nu)$ in equation (A.6) by $\bar{r}(h)/\alpha + \delta(1-\alpha)/\alpha - \nu$ yields $\bar{g}(h,p)$ as given in the proposition. □

A.9 Proof of Proposition 10

Analogously to the proof of Proposition 7 the steady state level of healthcare expenditures $\bar{h}^\star$ is inefficiently high if $\partial U(s)/\partial h|_{h=\bar{h}^\star} < 0$ for all $s \geq 0$. The equation corresponding to equation (21) in the case of the general spillover specification reads

$$\frac{dU(s)}{dh} = \frac{c^\star(s,s,h)^{1-\frac{1}{\sigma}}}{\bar{x}(h,p)} \left[\bar{F}(h) + \frac{d\bar{a}(h,p)}{dh}\left(\frac{1}{\bar{x}(h,p)} - \frac{1}{\bar{y}(h,p)}\right) + \frac{d\bar{g}(h,p)}{dh}\left(\frac{1}{\bar{y}(h,p)} + s\right)\right] ,$$

[27] Note that $(1-\sigma)\bar{r}(h)+\sigma\rho+p > 0$ for all $p \in [p_{max}, p_{max} - \psi]$ is necessary for the household's maximization problem to be well defined.

(A.19)

where $\bar{y}(h,p) = \bar{r}(h) + p - \bar{g}(h,p)$. For an interior level of healthcare expenditures $\bar{F}(h) = 0$. Moreover, $d\bar{a}(h,p)/dh = d\bar{r}(h)/dh + dp/dh < 0$ and $\bar{y}(h,p) - \bar{x}(h,p) = \sigma[\bar{r}(h) - \rho] - \bar{g}(h,p) > 0$. Thus, $\partial U(s)/\partial h|_{h=\bar{h}^\star} < 0$ if $d\bar{g}(h,p)/dh|_{h=\bar{h}^\star} < 0$. □

A.10 Details on the numerical exercises

We use the original data set of Becker et al. (2005)[28] and amend it by data on health expenditures from 1960 and 2000. For the year 2000 the WHO[29] provides data of healthcare expenditures per GDP for all countries. Data for healthcare expenditures in 1960 is limited. The OECD[30] provides data on healthcare expenditures per capita in 1960 for some of their members. In fact, complete data on healthcare expenditures is only available for North America. Assuming the missing values to be zero, the data listed in Table 5 for healthcare expenditures in 1960 is a lower bound for real healthcare expenditures. According to Becker et al. (2005) income per capita is GDP per capita in 1996 international prices adjusted for terms of trade (Penn World Tables 6.1). Data on life expectancy at birth is taken from the World Bank Development Indicators. The average growth rate $g_\varnothing$ has been calculated from income levels in 1960 and 2000. Accordingly, the average population growth rate ν has been calculated from population data in 1960 and 2000. All regional aggregates are population weighted sums of country data.

Table 5 shows the calculation results for seven world regions (E&CA: Europe and Central Asia, EA&P: East Asia and Pacific, LA&C: Latin America and the Caribbean, ME&NA: Middle East and North Africa, NAM: North America, SASIA: South Asia and SSA: Sub-Saharan Africa). NAM (exo) denotes a hypothetical North America in which no healthcare expenditures are undertaken and increases in longevity solely stem from a decrease in p_{max}.

To estimate the sensitivity of our results with respect to the missing data on healthcare expenditures in 1960, Table 6 shows the same calculations for the seven world regions but with rather high estimates on the healthcare expenditures in 1960. We observe that our results are robust to reasonable changes in the healthcare expenditures.

[28] Available at http://www.aeaweb.org/aer/data/mar05_data_becker.zip.
[29] Available at http://www.who.int/gho/en/.
[30] Available at http://puck.sourceoecd.org.

Region	E & CA	EA & P	LA & C	ME & NA	NAM	NAM (exo)	SASIA	SSA
inc.(1960)	6810.37	1316.87	3459.36	1935.13	12379.8	12379.8	892.08	1470.48
inc.(2000)	18280.6	5866.24	7161.46	5524.89	32880.2	32880.2	2345.84	1573.02
$T(1960)$	67.99	42.05	56.26	47.89	69.89	69.89	44.04	40.55
$T(2000)$	76.22	70.71	70.46	68.94	77.25	77.25	62.73	46.02
$h(1960)$	1.64%	1.03%	0%	0%	3.91%	0%	0%	0%
$h(2000)$	6.02%	4.05%	4.97%	4.03%	9.77%	0%	3.12%	4.84%
r	3.70%	4.58%	3.33%	3.87%	3.69%	3.69%	3.74%	2.32%
ν	0.71%	1.64%	2.24%	2.54%	1.14%	1.14%	2.22%	2.73%
$g_{\varnothing}$	2.47%	3.73%	1.82%	2.62%	2.44%	2.44%	2.42%	0.17%
$g^{\star}(1960)$	2.46%	3.69%	1.80%	2.59%	2.44%	2.44%	2.38%	0.15%
$g^{\star}(2000)$	2.47%	3.78%	1.84%	2.66%	2.45%	2.45%	2.45%	0.19%
$\hat{g}$	2.65%	5.10%	2.25%	3.47%	2.57%	2.65%	3.30%	0.4%
$\Delta U/U(1960)$	45.82%	137.92%	41.83%	75.95%	43.25%	46.29%	71.04%	8.09%
$\Delta U_T/\Delta U$	15.16%	53.91%	35.12%	45.74%	11.24%	17.06%	47.43%	75.32%
$\Delta U_g/\Delta U$	0.69%	3.26%	2.29%	3.02%	0.67%	0.64%	3.19%	7.74%
$\Delta U_p/\Delta U$	14.47%	50.65%	32.89%	42.72%	10.57%	16.43%	44.24%	67.58%

Table 5: Numerical results for all seven world regions.

Region	E & CA	EA & P	LA & C	ME & NA	NAM	SASIA	SSA
$h(1960)$	3.0%	1.5%	2.0%	1.5%	3.91%	1.0%	2.0%
$h(2000)$	6.02%	4.05%	4.97%	4.03%	9.77%	3.12%	4.84%
r	3.70%	4.58%	3.33%	3.87%	3.69%	3.74%	2.32%
ν	0.71%	1.64%	2.24%	2.54%	1.14%	2.22%	2.73%
$g_{\varnothing}$	2.47%	3.73%	1.82%	2.62%	2.44%	2.42%	0.17%
$g^{\star}(1960)$	2.46%	3.69%	1.80%	2.59%	2.44%	2.38%	0.15%
$g^{\star}(2000)$	2.47%	3.78%	1.84%	2.66%	2.45%	2.45%	0.19%
$\hat{g}$	2.67%	5.10%	2.28%	3.49%	2.57%	3.31%	0.44%
$\Delta U/U(1960)$	46.50%	138.30%	42.79%	76.84%	43.25%	71.62%	8.45%
$\Delta U_T/\Delta U$	16.39%	54.04%	36.57%	46.37%	11.24%	47.85%	76.38%
$\Delta U_g/\Delta U$	0.68%	3.25%	2.19%	3.00%	0.67%	3.17%	7.43%
$\Delta U_p/\Delta U$	15.71%	50.78%	34.38%	43.36%	10.57%	44.68%	68.95%

Table 6: Sensitivity analysis with respect to healthcare expenditures in 1960.

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