# Regime Shifts and Volatility Spillovers on International Stock Markets*

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**Preliminary**

**Abstract**

A standard capital asset pricing model is extended to allow for stochastic shifts in the volatility of the news process. This model is then estimated on bivariate stock market data to separate two exogenous news processes – a world and a domestic. The results indicate that the influence of the world news process on the Swedish stock market has increased significantly over the period 1970-1995. I also find that the foreign influence is much stronger when the volatility of the world news process is high. Furthermore, when the world state shifts to high risk, the Swedish stock market immediately reacts by a large fall, estimated to 7.0%. The bivariate model is also estimated on a set of other national stock markets.

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# 1. Introduction

It is now a well-established fact that financial volatility is a non-constant stochastic process with a non-negligible degree of persistence – if stock market volatility is high today it tends to be high also during the nearest future. This observation has received much attention from the finance profession due to its implications for asset pricing and portfolio management. The changing volatility and in particular its persistence, however, also has potential macroeconomic implications. In Hassler (1995) I show that there is evidence of a link between financial volatility and durables demand. When financial volatility increases durables purchases fall substantially while non-durables demand seems to be largely unaffected. In Hassler (1996a) this is given a structural interpretation: High stock market volatility implies a high current flow of information. Given that investments in durables involve some degree of irreversibility, the value of waiting to purchase a new durable increases in the current flow of information. Increased volatility should then lead to a higher tendency to postpone purchases.

In Hassler (1996b) I also show that there is evidence of a trend increase in the volatility on the Swedish stock market. The US and world aggregate stock markets do not show similar trends. An important issue to examine is whether the increase in Swedish financial volatility can be attributed to a increased volatility of purely domestic factors or to an increased sensitivity to world market conditions. If the latter is the case, we may also want to distinguish effects due to an increased internationalization of the stock market, for example due to capital market liberalization, from an increased sensitivity of the profitability of firms on the market to what happens on the world market.

Although little formal work has been done, it is often suggested that shifts in uncertainty can be of importance for the macroeconomic performance. Christina Romer (1990), for example, argues that increased uncertainty was one of the driving forces behind the great depression. Also the very sharp recessions in Sweden and Finland in the beginning of the 90's, in many respects comparable in size to the depression in the 30's, is often attributed to increased savings caused by a shift in uncertainty.

To understand the implications of time varying financial volatility, more knowledge of the processes that drive volatility is required. In particular, we would want to establish facts about the intra-market dependence of national stock markets. Here only a few studies have been conducted. Engle and Susmel (1993) and King et al (1994) estimate multivariate models with common factors. An often noted observation is that there appear to be regime shifts in the covariance matrix of different national stock markets (see Bollerslev, 1992, p. 30). During periods of high volatility there appears to be a tendency of higher international dependence. This observation calls for an attempt to apply Hamilton's regime switching model to a multivariate dataset of stock market returns. Using such a model, we can allow distinct shifts in both the first and higher order moments of the stochastic processes driving the stock markets.

I will estimate a multivariate Hamilton type model on data for the Swedish and world stock markets. This approach allows a fairly general specification of international interrelations of returns and volatility. I will assume that the world stock market is driven by a news process with two states – a low risk and a high risk state. The domestic stock market is affected by the world news process and by a domestic idiosyncratic process. Also the volatility of the latter may shift between two levels.

Observed covariances between different markets can be due to both a common news process and to comovements in endogenous state prices, due to, for example comovements in subjective discount rates. As we will see, the the model in this paper allows us to disentangle these two sources of comovements. We will see that data suggests that in particular the first of these two relations between Swedish and world stock market return have increased over time. I will show that there is a strong increase in the influence of the world news process on the Swedish stock market when the world goes into a high risk period. This foreign influence has increased substantially over time. There is some weaker evidence of an increased dependence of Swedish state prices on the world state. Expected returns on the Swedish stock market seems to have become more dependent on the world risk state.

Also the Swedish idiosyncratic news process show systematic variation in its volatility. We can distinguish periods of higher the usual volatility. A shift to the domestic high risk state is often followed by an immediate shift back. Given that this shift back do not occur – the domestic high risk state is fairly persistent with an expected length of over 7 months.

In section 2 I present a stylized asset pricing model that guide the specification of the econometric model. In Section 3 the model is subjected to some specification tests leading to some re-specifications. The results are then presented. In Section 4 I apply the basic model on a set of other stock markets. Section 5 concludes.

## 2. Model

### 2.1 The World Market

Suppose that there exists an exogenous stochastic state variable, denoted $s_t^w$ that determine the level of risk on a world stock market. Assume that the risk state only can take two values, 0 and 1, although extension to any finite number of states is straight forward in principle. We can think of the risk state as indicating "stable" or "unstable" weather. More precisely, assume that the world dividend process follows

$$\begin{aligned} d_{t+1}^w &= d_t^w e^{\overline{\mu}^w + \mu^w s_{t+1}^w + (\omega_1 + \omega_2 s_{t+1}^w)\varepsilon_{t+1}^w} \\ E_t \varepsilon_{t+1}^w &= 0,\ E_t\left((\varepsilon_{t+1}^w)^2\right) = 1 \end{aligned} \tag{2.1}$$

where $d_t^w$ is the aggregated dividend of the world market at time *t*. We do not need to interpret dividend literally but can think of it as representing news about the earning capacity of the firms noted on the stock market. The dividend process and the news process will thus be used interchangeably below. Furthermore, assume that the world risk state follows a first order stationary Markov chain with transition probabilities given by 1-$q^w(s_t^w)$.

The representative world agent maximizes a standard CRRA objective function $E_t \sum_{s=0} \beta^s c_{t+s}^{1-\alpha}$.

Assume that $c=d$ in each period and use the standard Lucas asset pricing equation. Then the price process of the world stockmarket index must satisfy

$$\tilde{p}\left(s_t^w, d_t^w\right) = E_t\left[\left(\tilde{p}\left(s_{t+1}^w, d_{t+1}^w\right) + d_{t+1}^w\right)\beta\left(\frac{d_{t+1}^w}{d_t^w}\right)^{-\alpha}\right]. \tag{2.2}$$

Since time $t$ expected future dividends are linear in $d_t^w$ and expected MRS are independent of $d_t^w$ it follows that stockmarket index is linear in $d_t^w$. Then the price dividend ratio only depends on the current state and satisfies

$$p\left(s_t^w\right) = E_t\left[\left(p\left(s_{t+1}^w\right)+1\right)\beta\left(\frac{d_{t+1}^w}{d_t^w}\right)^{1-\alpha}\right] \tag{2.3}$$

Furthermore, the return, defined as $r_{t+1} \equiv \ln\left(p\left(s_{t+1}^w\right)+1\right)d_{t+1}^w - \ln p\left(s_t^w\right)d_t^w$, satisfies

$$\begin{aligned} r_{t+1} &= \ln\left(p\left(s_{t+1}^w\right)+1\right) - \ln p\left(s_t^w\right) + \ln\frac{d_{t+1}^w}{d_t^w} \\ &= \ln\left(p\left(s_{t+1}^w\right)+1\right) - \ln p\left(s_t^w\right) + \overline{\mu}^w + \mu^w s_{t+1}^w + (\omega_1 + \omega_2 s_{t+1}^w)\varepsilon_{t+1}^w \end{aligned} \tag{2.4}$$

The real risk-free rate is also determined by the current state;

$$\bar{r}_{t+1}^w\left(s_t^w\right) = -\ln E_t\left[\beta\left(\frac{d_{t+1}^w}{d_t^w}\right)^{1-\alpha}\right] \tag{2.5}$$

Using (2.5) and (2.4) we can write the excess return in regression form

$$r_{t+1}^w - r_{t+1} = \mu_1 + \mu_2 s_t^w + \mu_3 \Delta s_{t+1}^w + (\omega_1 + \omega_2 s_{t+1}^w)\varepsilon_{t+1}^w. \tag{2.6}$$

Here it should be noted that the three $\mu_i$ in (2.6) are not sufficient to identify the two state prices and the drift parameters $\overline{\mu}^w$ and $\mu^w$. Nevertheless, (2.6) is a valid regression to run and $\omega_1$ and $\omega_2$ are identified.

## 2.2 The Domestic Market

Now turn to a domestic stock market. The return on the domestic market is assumed to be driven by both the world news to dividends and by an idiosyncratic domestic news process.

$$\begin{aligned} &d_{t+1}^d = d_t^d e^{\overline{\mu}^d + \mu^d s_{t+1}^w + (\omega_3 + \omega_4 s_{t+1}^w)\varepsilon_{t+1}^w + \mu^{d,w} s_{t+1}^d + (\omega_5 + \omega_6 s_{t+1}^d)\varepsilon_{t+1}^d}, \\ &E_t\varepsilon_{t+1}^d = E_t\varepsilon_{t+1}^d\varepsilon_{t+1}^w = 0,\ E_t\left((\varepsilon_{t+1}^d)^2\right) = 1 \end{aligned} \tag{2.7}$$

The influence of the foreign news process is allowed to shift with the international risk state. Also the intensity of the domestic news process is allowed to shift as a domestic risk state, denoted $s_{t+1}^d$ shifts. The domestic risk state follows a Markov chain with transition probabilities 1-$q^d(s_t^d)$, so it is assumed to be independent of, in particular, the world risk state.

The price of the domestic asset is going to be linear in $d^d$ so if the asset is priced by a world investor consuming $d^w$

$$p\left(s_t^w, s_t^d\right) = E_t\left[\left(p\left(s_t^w, s_t^d\right) + 1\right)\frac{d_{t+1}^d}{d_t^d}\beta\left(\frac{d_{t+1}^w}{d_t^w}\right)^{-\alpha}\right] \tag{2.8}$$

while if the price setter is domestic and restricted to consume $d^d$ it is

$$p\left(s_t^w, s_t^d\right) = E_t\left[\left(p\left(s_t^w, s_t^d\right) + 1\right)\beta\left(\frac{d_{t+1}^d}{d_t^d}\right)^{1-\alpha}\right]. \tag{2.9}$$

As noted in the introduction, there is evidence of a trend increase in the volatility on the Swedish stock market. To model such a trend in the simplest possible way, I allow a deterministic time trend in the volatility of the innovations to the domestic dividend process as well as to the drift terms and their sensitivity to the risk states. The final bivariate model is then given by

$$\begin{aligned}
r_{t+1}^w - r_{t+1} &= \mu_1 + \mu_2 s_t^w + \mu_3 \Delta s_{t+1}^w + (\omega_1 + \omega_2 s_{t+1}^w)\varepsilon_{t+1}^w, \\
r_{t+1}^d - \bar{r}_{t+1}^d &= \mu_4 + \mu_5 s_t^w + \mu_6 \Delta s_{t+1}^w + \mu_7 t/T + \mu_8 s_t^w t/T + \\
&\quad \mu_9 s_t^d + \mu_{10}\Delta s_{t+1}^d + \mu_{11} s_t^d t/T \\
&\quad +(\omega_3 + \omega_4 s_{t+1}^w + \omega_5 t/T)\varepsilon_{t+1}^w + (\omega_6 + \omega_7 s_{t+1}^d + \omega_8 t/T)\varepsilon_{t+1}^d
\end{aligned} \tag{2.10}$$

where $T$ is the total number of observations.

If the market structure changes so that, for example, capital controls are lifted we expect to find non-zero estimates of $\mu_7$, $\mu_8$ and/or $\mu_{11}$. The sensitivity to the domestic state, for example, should plausibly fall as domestic volatility is idiosyncratic from the point of view of an investor with access to the world market. This will change the covariance between domestic and foreign returns also if the news processes are invariant over time. If, on the other hand, the domestic influence of foreign news changes or if the strength of the domestic news flow changes, $\omega_6$ and/or $\omega_9$ should be non-negative.

## 2.3 Data and estimation

Stock market yields are calculated from the Morgan Stanley Capital International (MSCI) indices, which include re-invested dividends. The sample period is 1970:1-1995:8. The returns are calculated as the log-difference of month-end stock market index calculated in US dollar terms. Yields thus include an exchange rate term. Ideally we may want to model exchange rate fluctuations as a separate stochastic process. This is not done in this paper. The risk-free interest rate is the 30-days Eurodollar rate, provided by the Swedish Central Bank. The world market return is calculated from the MSCI value-weighted world index.

The model is estimated in the recursive way devised by Hamilton (1989). I maximize the likelihood function implied by (2.10), standard normality of the $\varepsilon$'s and the assumed Markov chain of the state variables. Standard errors are calculated from the Hessian of the loglikelihood function at the estimated parameters.

# 3. Diagnostic Tests and Estimation Results

## 3.1 Model Diagnostics

Before examining the results of the estimation of the model, we want to judge whether the model can be thought of as a reasonable description of the data. For this purpose I will, in some detail, first present the results from some diagnostic tests, based on work by Hamilton (1996). One could certainly test the statistical model in many dimensions and it is a priori clear that the probability of this model being exactly right is zero. The tests should thus be thought of as quantitative evaluations of how well the model, in some dimensions, describe data. Based on the purpose of this paper and the suggestions by Hamilton (1996), I have chosen to evaluate the model along the following dimensions:

1) We want to judge whether the volatilities of the information processes are reasonably well described by the two-state model. This will be tested against the hypothesis that there remains some autocorrelation in the volatility, i.e., that some ARCH-effects remain.
2) We also want to see if there is strong evidence against states follow independent first order Markov processes. Alternatively they may have higher order, be interdependent and/or depend on the level of the realized return.

3) Lastly I want to check whether the average return is constant in the two states, after potential linear time trends have been removed. Alternatively, there may remain some autocorrelation in the return.

The test are based on an examination of the derivatives of the loglikelihood function – the scores. Define as the vector of scores at time $t$ as

$$\mathbf{h}(t) \equiv \partial \ln f\left(\mathbf{r}_t \middle| \mathbf{r}_{t-1},\ldots,\mathbf{r}_1;\Phi\right) \Big/ \partial\Phi \qquad (3.1)$$

where $f$ is the log-likelihood function, $\mathbf{r}_t$ is the vector of excess returns at time $t$ and $\Phi$ is the vector of parameters to estimate. If the model is correctly specified, each element of $\mathbf{h}(t)$, is uncorrelated with all information in $t$-1. In particular, it should be uncorrelated with previous values of the itself and other scores. Intuitively, if this is violated we expect that our parameter estimates will change in some known direction when a data point is added to our sample. This could never be a feature of a reasonable estimator. By looking at linear relations between scores in $t$ and $t$-1 we may detect deviations from the assumptions in the model and may understand how they are violated. To this end I will study the following regressions

$$h_i(t) = \alpha_0 + \sum_{j \in J} \alpha_j h_j(t-1) + \varepsilon_t \qquad (3.2)$$

where $h_i(t)$ is the $i$th element of the score and $J$ is a subset of the parameters I estimate.

We should note that the residual in (3.2) in general is heteroschedastic, implying potentially serious small sample problems. These will be particularly severe for parameters that influence the likelihood function only at state realizations that occur with low probability. As we will soon see, the transition probabilities for both the domestic and the world state are low. This means that state switches are rare events. The scores for parameters that only affect the likelihood function at state switches, i.e., $\mu_3$, $\mu_6$, and $\mu_{10}$, will thus follow very heteroschedastic processes and tests based (3.2) will be quite unreliable[1]. We can understand this in the following way; despite the relatively large nominal number of degrees of freedom, we have relatively little information about of what happens at state shifts since these events are rare. I

[1] The scores for these parameters showed long periods of values close to zero interrupted by a small number of very large values.

will thus exclude the scores for theses parameters from the tests. I am in effect thus testing the model according how it behaves within the states, not what happens exactly at the state shifts.

The scores are calculated numerically at the estimated parameter vector. Using that $\partial \ln f(\mathbf{r}_t | \mathbf{r}_{t-1},\ldots,\mathbf{r}_1;\Phi) / \partial\Phi = \partial \ln f(\mathbf{r}_t,\ldots,\mathbf{r}_1;\Phi) / \partial\Phi - \partial \ln f(\mathbf{r}_{t-1},\ldots,\mathbf{r}_1;\Phi) / \partial\Phi$, we see that the scores can be calculated by first calculating $\partial \ln f(\mathbf{r}_t,\ldots,\mathbf{r}_1;\hat{\Phi}) / \partial\Phi$ for each $t$=1,...,T where $\hat{\Phi}$ is the ML estimate of the parameters given the full sample. The first differences of this series is the scores in (3.1).

To test the Markov assumption I have performed two tests. The first, Markow I, is to run one regression for each of the scores with respect to the probabilities $q^w(0)$, $q^w(1)$, $q^d(0)$ $q^d(1)$. In this test, one period lagged values of the four scores are used as regressors. This test is aimed at detecting deviations from the independent first order Markov assumption. If lagged values of scores predict scores for the same state variable, this is an indication of violation of the first order assumption. Similarly, if lagged scores can predict the score of the other state variable, this indicates non-independence between the two state variables.

In the second test, Markow II, the same dependent variables are used, but they are now regressed on the lagged scores for the drift parameters $\mu_1$, $\mu_2$, $\mu_4$, $\mu_5$, $\mu_7$, $\mu_8$, $\mu_9$ and $\mu_{11}$. This test can detect if the level of the stock return in the previous period contains information about the likelihood of staying in the state, which would violate the Markov assumption.

The third test, AR, is aimed at detecting deviations from the assumption of a constant expected return (except for the time trend) in each state. I run regressions for each of the scores for $\mu_1$, $\mu_2$, $\mu_4$, $\mu_5$, $\mu_7$, $\mu_8$, $\mu_9$ and $\mu_{11}$ against the lagged scores for the same parameters. If there is some autocorrelation in the return processes left unaccounted for by the model, the regressions contain some information.

The last test is an ARCH test. I run regressions for each of the scores with respect to the volatility parameters. $\omega_1$,..., $\omega_{9.}$ Significance here indicates remaining ARCH effects. If, for example the lagged score with respect to $\omega_1$ helps predict the current value of the score, there seems to be ARCH effects in state 1. Hamilton's (1996) propose the same four tests. The difference is only that he tests whether all regressions within a test simultaneously have zero

$R^2$. Studying the regressions separately can, however, give an indication of what causes a potential rejection and direct model re-specification.

**Table 1 Score test for Sweden**

| | Markov Test I | | | | Markov Test II | | | | AR Test | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $q^w(0)$ | $q^w(1)$ | $q^d(0)$ | $q^d(1)$ | $q^w(0)$ | $q^w(1)$ | $q^d(0)$ | $q^d(1)$ | $\mu_1$ | $\mu_2$ | $\mu_4$ | $\mu_5$ | $\mu_7$ | $\mu_8$ | $\mu_9$ | $\mu_{11}$ |
| $\bar{R}^2$ | 0.00 | 0.00 | -0.01 | 0.09 | -0.02 | 0.04 | 0.00 | -0.01 | 0.00 | 0.02 | -0.01 | 0.00 | -0.01 | 0.00 | 0.00 | 0.01 |
| p-value | 0.57 | 0.57 | 0.65 | 0.00 | 0.93 | 0.01 | 0.46 | 0.59 | 0.38 | 0.07 | 0.75 | 0.45 | 0.64 | 0.42 | 0.39 | 0.19 |

| | ARCH Test | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ | $\omega_6$ | $\omega_7$ | $\omega_8$ |
| $\bar{R}^s$ | -0.02 | -0.02 | 0.01 | -0.01 | 0.01 | -0.01 | 0.03 | 0.00 |
| Prob | 0.99 | 0.92 | 0.23 | 0.85 | 0.22 | 0.66 | 0.11 | 0.43 |

In *Table* 1 I present the results of the score test for the bivariate model for excess returns on the Swedish and the world market. I report the centered $R^2$ together with the asymptotic p-value for $R^2$=0. In the table we find evidence of violations of the Markov assumption. These are related to the scores with respect to the transition probabilities.

The assumption that the domestic state follow a first order, independent Markov chain is significantly rejected. In the regression of $q^d(1)$ in Markov test 1 we find a strong dependence on lagged scores with respect to the other transition probabilities. A closer inspection of the regression results indicates that the *t*-values in the regression are 0.03, -0.27, 5.79 and 2.77 for the scores for $q^w(0)$, $q^w(1)$, $q^d(0)$ $q^d(1)$.[2] This indicates that it is the first order assumption rather than the assumption of independence between the state variables, that is violated. I will thus re-estimate the model allowing the probability of staying in domestic state 1 to depend on the current as well as the lagged state. The probability of staying in domestic state 1 if the current and the previous state was 1 will be denoted by $q^d(1,1)$ and the probability of staying in state 1 if the previous state was 0 by $q^d(1,0)$.

[2] t-statistics for all regressions are presented in the Appendix.

There is also an indication that previous returns may influence the probability of staying in world state 1 – the second regression in Markov test II is significant. The significance is, however, due to the score with respect to $\mu_8$. This would mean that the realization of Swedish returns affects the probability of the world process to stay in its high risk state – which seems unreasonable. A closer inspection of the regression also shows that the significance is due to only one single observation – September 1990. If this is excluded the significance of the regressions is reduced to a marginal p-value of 0.20. The Swedish stock market had its lowest rate of return over the who sample this month – -25%. Apparently this occured one period before a realization on the world market that tended to push the estimate of $q^w(1)$ upwards. This seems to be a coincidence rather than a causal relationship. To handle this I include a dummy for the Swedish return in September 1990.

**Table 2 Score Tests for Final Model for Sweden**

| | Markov Test I | | | | Markov Test II | | | | AR Test | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $q^w(0)$ | $q^w(1)$ | $q^d(0)$ | $q^d(1)$ | $q^w(0)$ | $q^w(1)$ | $q^d(0)$ | $q^d(1,1)$ | $\mu_1$ | $\mu_2$ | $\mu_4$ | $\mu_5$ | $\mu_7$ | $\mu_8$ | $\mu_9$ | $\mu_{11}$ |
| $\overline{R}^2$ | 0.00 | 0.00 | 0.00 | 0.01 | -0.02 | 0.01 | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | 0.00 | 0.00 | 0.00 |
| P-value | 0.60 | 0.55 | 0.40 | 0.19 | 0.93 | 0.13 | 0.46 | 0.66 | 0.44 | 0.37 | 0.86 | 0.73 | 0.71 | 0.55 | 0.58 | 0.40 |

| | ARCH Test | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ | $\omega_6$ | $\omega_7$ | $\omega_8$ |
| $\overline{R}^2$ | -0.02 | 0.02 | 0.00 | 0.01 | 0.01 | -0.01 | 0.02 | 0.00 |
| P-value | 1.00 | 0.09 | 0.35 | 0.22 | 0.26 | 0.59 | 0.13 | 0.57 |

None of the regressions in the AR and the ARCH tests are significant at conventional levels. So there does not seem to be any significant AR or ARCH effect left in the data.

After re-estimating the model allowing $q^d(1,1) \neq q^d(1,0)$ and using a dummy for the Swedish return in September 1990 the model survives the tests in the sense that no regressions are significant at conventional significance levels. The p-values of the regressions are given in *Table* 2.

## 3.2 Results

The estimated parameters together with asymptotic standard errors, calculated from the Hessian of the loglikelihood function, are presented in *Table* 3.[3] We find that the low risk state is substantially more persistent than the high risk state, both for the world and the domestic state processes. This also means that economy spends most of the time in the low risk state. The unconditional probability of the low risk state is approximately 0.85 for both state processes. The probability of staying in the world high risk state is, although lower than the probability of staying in the low risk state, as high as 0.815. This means that a switch to the high risk state is something more than just one extreme realization of returns. This is also true for the domestic process. Here we find evidence that that it is more likely to stay in the high risk state given that it has continued at least two months. About half of the times a switch to domestic high risk state, is followed by an immediate shift back. If this does not occur, the high risk state is expected to continue for another 7 months.

The drift parameters for the world return are estimated with rather good precision. From the negative and large value of $\mu_3$ we see that there is a large fall in the stock market when the high risk state is entered. This adds a negative (positive) component to expected returns in the low (high) risk state. On the other hand, the drift term $\mu_2$ is negative adding a negative component to expected returns in the high risk state. This outweighs the effect due to $\mu_3$ so expected returns are lower in the high risk state.

The precision in the drift terms for Swedish returns is lower, only $\mu_6$, ($\mu_8$) and $\mu_{10}$ are significant at conventional significance levels. We find that the also the Swedish market falls when the world risk state shifts to high risk. A shift to the domestic high risk state has a quantitatively similar effect, the point estimated are -7% and -8.3%. The negative value for $\mu_8$ means that the effect of the world high risk state on expected Swedish returns has become more negative over time.

The parameters capturing the volatility of the news process are estimated with relatively good precision. The standard deviation of the news process at the world market increases by

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[3] The parameter estimates are very close to the estimates for the rejected model where the domestic state followed a first order Markov chain and no dummy for September 1990 was used.

1.8 % in the high risk state. Turning to the Swedish market and focusing on the influence of the world news process we find that $\omega_3$ is close to zero and insignificant. This implies a small foreign influence during the early period of the sample when the world market was in its low risk state. The importance of the foreign news, however, increases strongly when the world enters its high risk state, indicated by the positive and significant estimated value of $\omega_4$. As we can see, the sensitivity of the news process to the world state is more than twice as high for Sweden as for the world itself. Furthermore, the importance of the foreign news process has increased significantly over the sample period, $\omega_5$ is positive

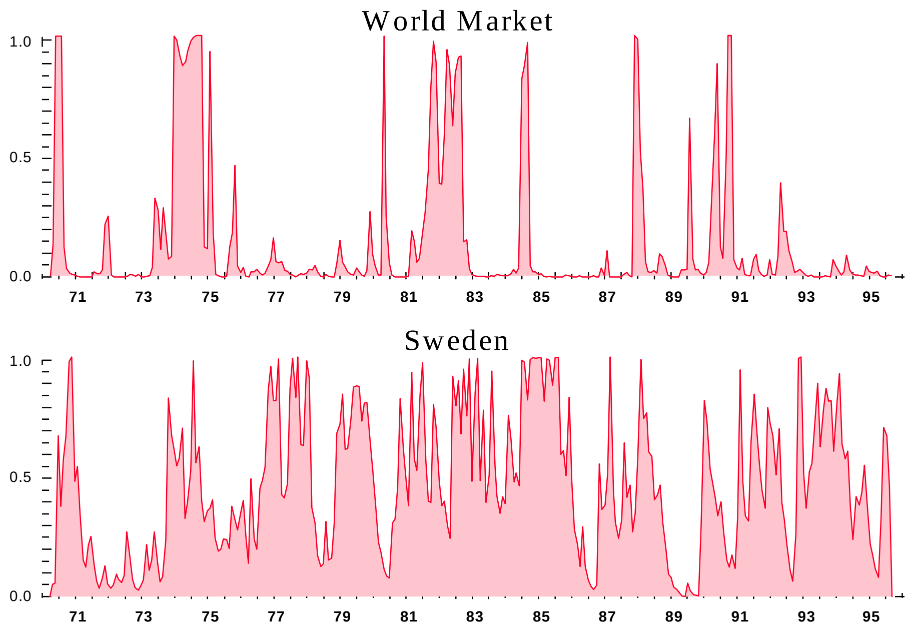
**Table 3 Estimated Parameters - Final Model for Sweden**

| Parameter | Estimate | Asymptotic St. dev | Estimate / St. dev | Parameter | Estimate | Asymptotic St. dev | Estimate / St. dev |
|---|---|---|---|---|---|---|---|
| $q^w(0)$ | 0.964 | 0.031 | 31.06 | $\mu_8$x100 | -5.170 | 2.993 | -1.73 |
| $q^w(1)$ | 0.815 | 0.061 | 13.28 | $\mu_9$x100 | -2.117 | 4.081 | -0.52 |
| $q^d(0)$ | 0.975 | 0.045 | 21.45 | $\mu_{10}$x100 | -8.307 | 2.631 | -3.16 |
| $q^d(1,0)$ | 0.471 | 0.254 | 1.85 | $\mu_{11}$x100 | 6.174 | 10.611 | 0.58 |
| $q^d(1,1)$ | 0.862 | 0.100 | 8.65 | $\omega_1$x100 | 3.162 | 0.182 | 17.34 |
| $\mu_1$x100 | 0.847 | 0.218 | 3.89 | $\omega_2$x100 | 1.801 | 0.804 | 2.24 |
| $\mu_2$x100 | -3.652 | 1.073 | -3.40 | $\omega_3$x100 | 0.982 | 0.638 | 1.54 |
| $\mu_3$x100 | -9.397 | 1.467 | -6.41 | $\omega_4$x100 | 3.095 | 1.060 | 2.92 |
| $\mu_4$x100 | 0.325 | 0.609 | 0.53 | $\omega_5$x100 | 3.169 | 1.010 | 3.14 |
| $\mu_5$x100 | 0.287 | 1.571 | 0.18 | $\omega_6$x100 | 3.860 | 0.483 | 7.99 |
| $\mu_6$x100 | -6.997 | 2.104 | -3.32 | $\omega_7$x100 | 3.358 | 1.202 | 2.79 |
| $\mu_7$x100 | 1.057 | 1.044 | 1.01 | $\omega_8$x100 | 0.767 | 0.874 | 0.88 |
| | | | | Dummy Sept. 90 | -10.048 | 5.065 | -1.98 |

Also the domestic news process is of strong importance for Swedish returns. Both $\omega_5$. and $\omega_5$ are large and significant. In the beginning of the sample, the correlation between Swedish and world returns during world low risk periods was thus very low. Also the domestic state process seems to have been of substantial importance throughout the sample. The standard deviation of domestic news is almost doubles in the domestic high risk state.

Contrary to the case of foreign news, there is no significant trend in the volatility of domestic news. This together with the positive trend in foreign influence implies that the correlation between Swedish and world returns, conditional on no state shifts, has increased over time.

**Figure 1 State Probabilities**




An output from the estimation of the regime switching model is probabilities of being in the high risk states, conditional on realized returns. The probabilities of being in the two high risk states for each period $t$, conditional on realization up to $t+1$ are plotted in *Figure* 1.[4,5]. We see that the inference regarding the world state is better than for the domestic state. This seems reasonable given that the return innovation on the world market more directly contain information about the world state than what is the case for the innovation on the domestic market.

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[4] The smoothed probabilities, based on the full sample, has for computational reasons not been calculated.

[5] The Swedish state probabilities are somewhat sensitive to the choice of model. The preliminary model, with first order state Markov chains. produced probabilities that generally where closer to zero.

## 4. A Multi Country Comparison

To be able to compare the results for Sweden in the previous section to other national stock markets I will in this section estimate the base model, described in Section 2, on a few other national stock market indices. The data come from MSCI and cover the same period as in the previous section. The national stock markets are Austria, Belgium, Canada, Denmark, France, Germany, Hongkong, Italy, Norway and Spain. I has been beyond the scope of this paper to repeat the rather careful examination of the diagnostic tests and re-specification of the model that was done for Sweden in the previous section. I will however, report the results from the 4 diagnostic tests for all studied countries. Nevertheless, some direction for future work could be gained by estimating the base model.

The diagnostic test results are presented in *Table* 4. We find that in no cases does the test fail to reject the assumptions of the model.. We thus have to careful when interpreting the findings here. A very common rejection is that the second regression in Markov test II, Markov II:2, is significant. As in the case of Sweden, it is the scores for domestic drift terms that are correlated with the scores for the world transition probability. It seems likely that this also here could be due to a one or few outliers, as for Sweden. Also the AR and ARCH tests are rejected in many cases, this calls for a closer inspection, possibly leading to a more general specification of expected returns and volatilities.

Bearing the fact the model is rejected by data in mind, we may look at the estimated parameters for each country. These are presented in *Table* 5[6]. First we note that some countries, Canada, Denmark and Norway have relatively low values of $q^d(1)$.This indicates a low degree of persistence in the domestic high risk state – rather than extended periods of high volatility we here seem to have some occurrences of extreme observations. This observation is further strengthened for Norway where we see that while $\mu_9$ and $\mu_{11}$ are large and with opposite signs $\omega_8$ is negative. This indicates that the estimation is dominated by some large positive

[6] The parameters for the world process are not given in the table. These estimates do not change more than marginally between the different data sets. The fact that also the domestic process contain information about the word innovation implies that we in small samples should get different estimates also of the parameters of the world process.

shocks early in the sample. The parameters for the other countries imply that the domestic high risk state have expected lengths of between 5 and 11 months.

We find that all domestic stock markets fall, in many cases quite dramatically, when the foreign state shifts to high risk ($\mu_6$ <0). In Sweden we found that $\omega_4$ was small relative to the other volatility parameters, indicating a low foreign influence in early low risk periods. The foreign influence the tended to increase over time. Similar patterns hold for most other countries in the sample except Canada and Hongkong. In the latter two the foreign influence rather tended to decrease. Canada seem to experience a strong foreign influence in both the world states wile Hongkong, on the other hand, appears extremely sensitive to the world state.

The domestic component of the return innovation seems to have quite different properties

**Table 4 Rejected Specification Tests of Base Model**

| Country | Rejected Tests† |
|---|---|
| Austria | Markov I:2*, Markov II:2*, AR 4*, ARCH 2,4* |
| Belgium | ARCH 7 |
| Canada | Markov I:1*, Markov II:2, ARCH 2*,3*,4*,5* |
| Denmark | Markov I:2, Markov II:2*, AR 2, 7* |
| France | AR 7, 8*, ARCH 7 |
| Germany | Markov I:2, Markov II:2*, 4, AR 7*,8, ARCH 2*, 4* |
| Hongkong | Markov I:4, Markov II:4, ARCH 2*, 4*, 5, 6, 8 |
| Italy | AR 4*, ARCH 4 |
| Norway | Markov II:2*, AR 2*, 4*, 6*, 8*, ARCH 5, 7* |
| Spain | Markov II:2*, 3, AR 4, 5, 6*, ARCH 5 |
| Sweden | Markov I:4*, Markov II:2 |

† Rejected at 5% nominal levels, * indicates rejection at 1%.

in different countries. In some countries there is a strong state dependence – idiosyncratic news volatility is much higher in the domestic high risk state. This is the case for Austria, and Hongkong. In Austria, the idiosyncratic component also seems more important relative to the world component than in most other countries. In Germany the volatility of the idiosyncratic news process is almost the same in the two states, so here it is inappropriate to denote state 1 as the high risk state. The latter is, as noted above, also the case for Norway. In the other countries the pattern is like in Sweden – idiosyncratic news are important and increase substantially in the high risk state $\omega_7 \approx \omega_8$. The idiosyncratic news process in Canada, however, seems to be of somewhat lower relative importance than in the other countries. Most countries, as Sweden, show no sign of a trend in the volatility of domestic news. However, Austria has a clear positive and France a clear negative trend.

## 5. Conclusion

I have in this paper applied the Hamilton regime switching model to bivariate stock market data. The model seems well suited to detect shifts in the assumed exogenous news processes that drive the stock market. As shown in Hassler (1995) such shifts may be of substantial importance for the timing of business investment and purchases of consumer durables. Sweden's stock market seems particularly sensitive to the world news process during high volatility periods. If the world market goes into a state of high volatility, the Swedish market reacts by a substantial fall followed by high volatility. The level shifts associated with international risk state shifts contribute largely to expected returns and their volatility, especially in the high risk state. This relatively high probability of large non-idiosyncratic shifts in the stock market may have substantial effects on the pricing of Swedish securities. I also find a significant trend in the foreign influence on the Swedish stock market.

The specification tests rejected the base model for Sweden as for all other countries. A careful application of the Hamilton approach thus requires testing and subsequent re-specification of the model before the results can be trusted with confidence. We immediately see that a basic two state Hamilton model may be inappropriate as a good description of data for many countries. The tests performed in this paper can, however. give good guidance in what way to re-specify the model after rejections of a tentative model.

## Table 5 Estimated Parameters of Base Model for Different Countries

| | Austria | | Belgium | | Canada | | Denmark | | France | | Germany | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | St. dev. | Estimate | St. dev. | Estimate | St. dev. | Estimate | St. dev. | Estimate | St. dev. | Estimate | St. dev. |
| $q^d(0)$ | 0.97 | 0.03 | 0.96 | 0.04 | 0.88 | 0.05 | 0.80 | 0.06 | 0.95 | 0.05 | 0.99 | 0.04 |
| $q^d(1)$ | 0.86 | 0.06 | 0.84 | 0.06 | 0.55 | 0.18 | 0.60 | 0.09 | 0.82 | 0.09 | 0.91 | 0.06 |
| $\mu_4$x100 | 0.25 | 0.41 | 0.72 | 0.51 | 0.60 | 0.52 | 0.05 | 0.71 | -0.28 | 0.97 | -0.02 | 0.63 |
| $\mu_5$x100 | 1.47 | 0.95 | -0.88 | 1.40 | -3.63 | 1.77 | -4.88 | 1.62 | -1.42 | 1.88 | 1.82 | 1.78 |
| $\mu_6$x100 | -7.32 | 1.50 | -4.58 | 1.37 | -3.58 | 2.25 | -3.87 | 1.95 | -5.21 | 1.70 | -3.68 | 2.19 |
| $\mu_7$x100 | -0.52 | 0.94 | -0.11 | 0.77 | -0.49 | 0.75 | 1.46 | 1.18 | 0.57 | 1.29 | 0.87 | 0.98 |
| $\mu_8$x100 | -11.83 | 2.82 | 0.84 | 1.92 | -2.39 | 3.61 | 6.79 | 2.73 | 2.40 | 2.72 | -8.87 | 3.54 |
| $\mu_9$x100 | 5.70 | 3.30 | -1.01 | 2.31 | 3.02 | 1.53 | 5.12 | 2.16 | 3.73 | 3.39 | 6.63 | 9.14 |
| $\mu_{10}$x100 | 10.17 | 2.79 | 5.68 | 1.71 | -3.08 | 1.23 | 3.12 | 1.37 | 1.15 | 3.36 | 14.12 | 2.87 |
| $\mu_{11}$x100 | -1.96 | 5.20 | 1.66 | 4.21 | -2.86 | 2.73 | -10.31 | 3.22 | -4.76 | 5.63 | -5.38 | 12.31 |
| $\omega_4$x100 | -0.04 | 0.37 | 1.19 | 0.56 | 3.63 | 0.47 | 1.23 | 0.57 | 2.36 | 0.81 | 1.01 | 0.78 |
| $\omega_5$x100 | 1.14 | 0.73 | 3.81 | 0.84 | 3.80 | 1.05 | 2.21 | 0.92 | 4.25 | 1.15 | 4.58 | 1.22 |
| $\omega_6$x100 | 1.86 | 0.79 | 1.35 | 0.78 | -1.75 | 0.70 | 1.39 | 0.88 | 0.73 | 1.03 | 2.02 | 1.10 |
| $\omega_7$x100 | 1.58 | 0.24 | 3.19 | 0.36 | 2.03 | 0.50 | 3.09 | 0.44 | 4.91 | 0.62 | 4.78 | 0.41 |
| $\omega_8$x100 | 4.51 | 1.02 | 3.14 | 0.88 | 1.60 | 0.65 | 3.15 | 0.59 | 4.22 | 1.09 | 1.46 | 1.24 |
| $\omega_9$x100 | 3.86 | 0.62 | -0.43 | 0.59 | 1.30 | 0.75 | -0.56 | 0.69 | -1.93 | 0.84 | -1.16 | 0.71 |

| | Hongkong | | Italy | | Norway | | Spain | | Sweden | |
|---|---|---|---|---|---|---|---|---|---|---|
| .Parameter | Estimate | St. dev. | Estimate | St. dev. | Estimate | St. dev. | Estimate | St. dev. | Estimate | St. dev. |
| $q^d(0)$ | 0.97 | 0.03 | 0.92 | 0.04 | 0.84 | 0.03 | 0.94 | 0.06 | 0.98 | 0.05 |
| $q^d(1)$ | 0.88 | 0.05 | 0.81 | 0.07 | 0.44 | 0.07 | 0.94 | 0.06 | 0.79 | 0.10 |
| $\mu_4$x100 | 3.19 | 1.03 | -0.78 | 0.85 | -2.08 | 0.82 | 1.43 | 0.64 | 0.39 | 0.63 |
| $\mu_5$x100 | -8.48 | 4.09 | -3.79 | 2.26 | -3.45 | 2.37 | -0.43 | 1.52 | 0.52 | 1.65 |
| $\mu_6$x100 | -17.53 | 3.33 | -6.24 | 2.70 | -14.13 | 2.42 | -2.79 | 2.03 | -7.38 | 2.19 |
| $\mu_7$x100 | -1.07 | 1.62 | 1.08 | 1.46 | 1.10 | 1.59 | -1.48 | 1.14 | 1.04 | 1.10 |
| $\mu_8$x100 | -0.65 | 5.44 | 1.75 | 3.97 | -7.09 | 4.81 | -0.84 | 2.58 | -7.05 | 3.22 |
| $\mu_9$x100 | 1.23 | 4.46 | 1.07 | 2.82 | 18.31 | 1.66 | -5.33 | 1.96 | -2.04 | 3.98 |
| $\mu_{10}$x100 | 1.03 | 5.92 | 4.93 | 2.92 | 3.99 | 0.99 | 1.39 | 3.37 | -8.54 | 2.66 |
| $\mu_{11}$x100 | -9.72 | 10.05 | 1.79 | 5.30 | -10.15 | 3.19 | 8.43 | 3.87 | 5.04 | 10.34 |
| $\omega_4$x100 | 2.59 | 0.93 | 1.55 | 0.84 | 1.51 | 0.89 | -0.48 | 0.66 | 0.98 | 0.61 |
| $\omega_5$x100 | 9.52 | 1.95 | 1.50 | 1.55 | 4.48 | 1.46 | 4.63 | 1.00 | 3.48 | 1.11 |
| $\omega_6$x100 | -2.04 | 1.49 | 2.06 | 1.28 | 2.01 | 1.39 | 4.03 | 0.99 | 3.25 | 1.00 |
| $\omega_7$x100 | 6.35 | 0.99 | 4.19 | 0.63 | 4.14 | 0.61 | 3.34 | 0.47 | 3.85 | 0.55 |
| $\omega_8$x100 | 11.88 | 2.05 | 5.22 | 1.04 | -0.94 | 0.73 | 3.27 | 0.70 | 3.42 | 1.20 |
| $\omega_9$x100 | -0.70 | 1.60 | 1.08 | 1.09 | 1.29 | 0.89 | 0.05 | 0.88 | 0.84 | 1.04 |

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# Appendix

**Table 6 Score test for Sweden**

| | Markov Test I | | | | Markov Test II | | | | AR Test | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $q^w(0)$ | $q^w(1)$ | $q^d(0)$ | $q^d(1)$ | $q^w(0)$ | $q^w(1)$ | $q^d(0)$ | $q^d(1)$ | $\mu_1$ | $\mu_2$ | $\mu_4$ | $\mu_5$ | $\mu_7$ | $\mu_8$ | $\mu_9$ | $\mu_{11}$ |
| $q^w(1)$ | -0.97 | -1.09 | 0.04 | 0.03 | | | | | | | | | | | | |
| $q^d(1)$ | -1.26 | 0.97 | 0.18 | -0.27 | | | | | | | | | | | | |
| $q^w(0)$ | -1.00 | 0.29 | -1.49 | 5.79 | | | | | | | | | | | | |
| $q^d(0)$ | -0.15 | -0.19 | -1.08 | 2.77 | | | | | | | | | | | | |
| $\mu_1$ | | | | | 1.21 | -0.77 | 0.43 | -0.54 | -0.64 | -1.50 | 0.76 | 1.27 | 1.45 | 1.24 | 1.33 | 1.78 |
| $\mu_2$ | | | | | 0.07 | 1.31 | -0..36 | 0.03 | 0.95 | 2.14 | 0.01 | -1.17 | -0.37 | -0.50 | -0.31 | -0.41 |
| $\mu_4$ | | | | | 0.33 | -0.46 | 1.45 | -1.94 | 1.69 | 0.54 | -0.83 | -0.29 | -0.20 | -0.69 | 0.40 | 0.02 |
| $\mu_5$ | | | | | 0.11 | -1.88 | -0.53 | 0.46 | 0.39 | 2.56 | -0.24 | -1.82 | -0.39 | -0.67 | -0.20 | 0.01 |
| $\mu_7$ | | | | | -0.29 | -0.16 | -0.12 | 1.09 | -0.84 | -0.53 | 0.37 | 0.51 | 0.01 | 1.00 | -0.09 | 0.35 |
| $\mu_8$ | | | | | -0.36 | 3.69 | -0.21 | -0.29 | -0.27 | -2.45 | 0.95 | 1.87 | 1.16 | 1.63 | 0.00 | -0.22 |
| $\mu_9$ | | | | | -0.05 | 0.07 | 0.75 | 1.65 | -1.17 | -0.07 | 0.59 | -0.03 | -0.32 | 0.10 | -0.87 | -0.50 |
| $\mu_{11}$ | | | | | 0.21 | -0.11 | -0.88 | -1.29 | 1.10 | 0.05 | 0.78 | -0.01 | 0.53 | -0.16 | 1.59 | 1.37 |
| $\bar{R}$ | 0.00 | 0.00 | -0.01 | 0.09 | -0.02 | 0.04 | 0.00 | -0.01 | 0.00 | 0.02 | -0.01 | 0.00 | -0.01 | 0.00 | 0.00 | 0.01 |
| Prob | 0.57 | 0.57 | 0.65 | 0.00 | 0.93 | 0.01 | 0.46 | 0.59 | 0.38 | 0.07 | 0.75 | 0.45 | 0.64 | 0.42 | 0.39 | 0.19 |

| | ARCH Test | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ | $\omega_6$ | $\omega_7$ | $\omega_8$ |
| $\omega_1$ | 0.23 | 0.18 | 2.07 | 0.20 | 1.66 | -1.08 | 0.10 | -1.85 |
| $\omega_2$ | -0.17 | 0.09 | 0.05 | 0.36 | 0.82 | 1.41 | 0.09 | 1.99 |
| $\omega_3$ | -0.36 | -0.94 | -1.62 | 0.27 | -1.01 | -0.16 | 0.29 | 0.51 |
| $\omega_4$ | -0.35 | 0.40 | -0.19 | -0.42 | 0.57 | 1.14 | 0.02 | 0.64 |
| $\omega_5$ | 0.19 | 0.26 | 2.23 | 0.46 | 1.88 | 0.05 | -0.50 | -0.57 |
| $\omega_6$ | -0.64 | -0.72 | -0.71 | 1.29 | -0.71 | -1.14 | 0.04 | -0.38 |
| $\omega_7$ | -0.64 | -0.14 | -0.60 | -0.02 | -0.32 | -0.78 | -3.56 | -0.67 |
| $\omega_8$ | 0.36 | 0.95 | 1.06 | -1.14 | 1.37 | 0.63 | 0.13 | 0.16 |
| $\bar{R}$ | -0.02 | -0.02 | 0.01 | -0.01 | 0.01 | -0.01 | 0.03 | 0.00 |
| Prob | 0.99 | 0.92 | 0.23 | 0.85 | 0.22 | 0.66 | 0.11 | 0.43 |