# Human Capital, Demographics and Growth across the US states 1920-1990 ${ }^{1}$ 

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#### Abstract

This paper finds robust evidence that age structure matters for subsequent growth in per capita income across the US states 1920-1990. The age groups 25-65 year are positively related to subsequent per capita income growth. Another conclusion is that the average years of schooling affects subsequent per capita income growth positively when age structure is controlled for. Moreover, the estimated speed of convergence (see e.g. Barro and Sala-i-Martin, 1992) increases substantially when schooling and age structure are held constant in the income growth regressions.


JEL classification: O18; O47
Keywords: Demographics; Human capital; Regions; Growth; Convergence

## 1. Introduction

The paper studies the effect of age structure on per capita income growth across the US States for the period 1920-1990. Despite the fact that the empirical literature on growth has blossomed during last ten years or so, the effect of age structure on growth has been subject to study only in a few papers. There are several reasons why age can be expected to matter for growth. Firstly, measures of human capital have always been a weak spot in growth empirics. As proxies for the human capital stock empirical growth studies typically use various educational variables such as literacy rates, school enrollment rates, average years of schooling, etc.. By using variables that only reflect formal education these studies neglect the human capital accumulation that is obtained by on-the-job-training. Age structure could be a proxy for this learning-by-doing process. Secondly, if life-cycle saving is present, then the age structure can be expected

[^0]to have growth effects, since aggregate savings typically is positively related to growth in cross-country studies (see e.g. Deaton, 1995). Thirdly, increased shares of dependent age groups may be expected to decrease growth because the work effort of adults would be directed more toward child rearing and care-taking of elderly.

Previous studies on age structure and growth include Sarel (1995) who studies the effect of age structure on per capita income growth for 121 countries using Summers and Heston data set. Sarel finds an inverted U-curve between the productivity of the age groups (on the $y$-axis), i.e., the estimated coefficients of the different age groups, and the age groups on the x -axis. Other studies on age and growth are Malmberg (1995) and Lindh and Malmberg (1995). Malmberg (1995) provides time-series evidence for Sweden 1950-1989 and Lindh and Malmberg (1995) is panel data study for the OECD-countries 1950-1990. Moreover, Lee and Ling (1994) partially analyze the effects of age structure on growth using the Summers-Heston data; they focus on the dependency ratios, i.e. the number of children and elderly as a share of the total population. No empirical study, that we know of, uses regional data.

One reason for studying the age structure effects on growth is the issue of convergence, which has been one of the most debated topics in the growth literature during the last ten years or so. Given that the age structure has growth effects and is correlated with initial income, excluding age structure will tend to bias the estimated convergence coefficient.

Our theoretical model is an extension of the model used by Mankiw, Romer and Weil (1992), hereinafter M-R-W, who augment the standard Solow model with human capital. Our extension of the M-R-W specification is that we argue that the stock of human capital consist of two parts, one part that can be accumulated and one part that depends on the age structure.

The paper is organized in the following way. Section 2 describes the model. Section 3 describes the data. Section 4 presents the empirical results and section 5 concludes.

## 2. A Solow-model augmented with age structure

In this section we analyze the effect of age on growth in a Solow-model. Our point of departure is Mankiw, Romer, and Weil (1992), who augment the standard Solow model with human capital. Our extension of the M-R-W specification is that we argue that the stock of human capital consists of two parts, one part that can be accumulated and one part that depends on the age structure. The part that depends on age structure could, e.g., reflect skills achieved through on-the-job-training and through learning by doing.

M-R-W use a Cobb-Douglas production function with labor-augmenting technological progress:
$Y(t)=K(t)^{\alpha} \tilde{H}(t)^{\lambda}(A(t) L(t))^{1-\alpha-\lambda}$
where $Y$ is output, $K$ physical capital, $\tilde{H}$ human capital, and $L$ is population, and $A$ is the level of technology. We extend the M-R-W formulation of the Solow model by assuming that the stock of human capital also depends on the age structure:
$\widetilde{H}(t)=H(t) \theta$
$\theta=\prod_{j} l_{j}^{\alpha_{j}}$
where $H$ is the part that can be accumulated and $\theta$ is the part that depends on the age structure. $l_{j}$ is the share of total population in age group $j$. The shares of the age groups sum to one. By using a Cobb-Douglas formulation we allow for interaction among age groups. For example, an increase in dependent age groups may affect the stock of human capital for goods production negatively because of e.g. increased childrearing.

To start with we make the simplifying assumption that $L$ is assumed to grow at the exogenously given rate $n$, i.e., the population growth rate is independent of the age structure. Moreover, $A$ is assumed to grow at the exogenously given rate $g$ so that

$$
\begin{equation*}
L(t)=L(0) e^{n t} \tag{4}
\end{equation*}
$$

$A(t)=A(0) e^{g t}$

Life-cycle saving is a potential mechanism through which age can affect growth. There is some support (although not universal) for explaining international differences in saving rates by international differences in population structure (see e.g. Deaton, 1995). However, here we stick to the conventional Solow framework and assume constant savings rates; that is, savings rates are assumed to be exogenous and independent of age. One motivation for this assumption is that we study regions across which physical capital tends to be more mobile than across countries. In fact, the close to perfect correlation between national investment and national saving at least when both are defined to exclude education that is found for countries (see Feldstein and Horioka, 1980) does not seem to hold for the US States (Barro and Sala-i-Martin, 1991). If this is the case, increased saving, due to e.g. shifts in the age structure, does not translate into increased accumulation of physical capital and increased growth. However, the age structure may also affect the savings for human capital accumulation. Despite this possibility we assume also that savings for human capital accumulation is independent of age: $s_{k}$ is the constant fraction of income invested in physical capital and $s_{h}$ is the constant fraction invested in human capital. Moreover, defining output, physical and human capital per unit of effective capita as $\hat{y}=Y / A L, \hat{k}=Y / A L$ and $\hat{h}=H / A L$, respectively, the dynamics of the economy is determined by

$$
\begin{align*}
& \dot{\hat{k}}(t)=s_{k} \hat{y}(t)-(n+g+\delta) \hat{k}(t) \\
& \dot{\hat{h}}(t)=s_{h} \hat{y}(t)-(n+g+\delta) \hat{h}(t) \tag{6}
\end{align*}
$$

where $\hat{y}(t)=\hat{k}(t)^{\alpha}(\hat{h}(t) \theta)^{\lambda}$ and $\delta$ is the constant rate of depreciation, which is assumed to be equal for the two types of capital.

Equations (6) imply that the economy converges to a steady state defined by:
$\hat{k}^{*}=\left(\frac{s_{k}^{1-\lambda} s_{h}^{\lambda} \theta^{\lambda}}{n+g+\delta}\right)^{1 /(1-\alpha-\lambda)}$
$\hat{h}^{*}=\left(\frac{s_{k}^{\alpha} s_{h}^{1-\alpha} \theta^{\beta}}{n+g+\delta}\right)^{1 /(1-\alpha-\lambda)}$

Estimates of speed of convergence toward steady state are typically low (see Barro and Sala-i-Martin (1991, 1992, 1995), which means that economies tend to be out of steady state. As a result, this paper focuses on the out of steady state dynamics. Approximating (see appendix) around the steady state, the rate of convergence is given by

$$
\begin{equation*}
\frac{d \ln \hat{y}}{d t}=-\beta \ln \left(\hat{y} / \hat{y}^{*}\right) \tag{8}
\end{equation*}
$$

where $\beta=(1-\alpha-\lambda)(n+g+\delta)$.

Equation (8) implies that
$\ln \hat{y}\left(t_{2}\right)=\left(1-e^{-\beta \tau}\right) \ln \hat{y}^{*}+e^{-\beta \tau} \ln \hat{y}\left(t_{1}\right)$
where $\hat{y}\left(t_{1}\right)$ is income per effective capita at some initial date and $\tau=\left(t_{2}-t_{1}\right)$.

Subtracting $\ln \hat{y}\left(t_{1}\right)$ from both sides yields:
$\ln \hat{y}\left(t_{2}\right)-\ln \hat{y}\left(t_{1}\right)=\left(1-e^{-\beta \tau}\right) \ln \hat{y}^{*}-\left(1-e^{-\beta \tau}\right) \ln \hat{y}\left(t_{1}\right)$

Finally, substituting for $\hat{y}^{*}$ :
$\ln \hat{y}\left(t_{2}\right)-\ln \hat{y}\left(t_{1}\right)=\left(1-e^{-\beta \tau}\right) \frac{\alpha}{1-\alpha-\lambda} \ln s_{k}+\left(1-e^{-\beta \tau}\right) \frac{\lambda}{1-\alpha-\lambda} \ln s_{h}$
$-\left(1-e^{-\beta \tau}\right) \frac{\alpha+\lambda}{1-\alpha-\lambda} \ln (n+g+\delta)+\left(1-e^{-\beta \tau}\right) \frac{\lambda}{1-\alpha-\lambda} \ln \theta-\left(1-e^{-\beta \tau}\right) \ln \hat{y}\left(t_{1}\right)$

We rewrite equation (11) in terms of income per capita. Note first that the $\log$ of income per effective capita is
$\ln \hat{y}(t)=\ln \left(\frac{Y(t)}{L(t)}\right)-\ln A(t)=\ln y(t)-\ln A(0)-g t$

Substituting (12) into (11) and rearrange somewhat, the growth rate of per capita income between $t_{1}$ and $t_{2}$ is
$\ln y\left(t_{2}\right)-\ln y\left(t_{1}\right)=\left(1-e^{-\beta \tau}\right) \frac{\alpha}{1-\alpha-\lambda} \ln s_{k}+\left(1-e^{-\beta \tau}\right) \frac{\lambda}{1-\alpha-\lambda} \ln s_{h}+\left(1-e^{-\beta \tau}\right) \frac{\lambda}{1-\alpha-\lambda} \ln \theta$
$-\left(1-e^{-\beta \tau}\right) \frac{\alpha+\lambda}{1-\alpha-\lambda} \ln (n+g+\delta)-\left(1-e^{-\beta \tau}\right) \ln y\left(t_{1}\right)+\left(1-e^{-\beta \tau}\right) \ln A(0)+g\left(t_{2}-t_{1} e^{-\beta \tau}\right)$

There is an alternative way to express the role of accumable human capital in determining income in this model. Combining (13) with the equation for the steadystate level of human capital given in equation (7) yields an equation for the growth rate of income as a function of the level of accumable human capital, instead of a function of the rate of investment in human capital that can be accumulated:
$\ln y\left(t_{2}\right)-\ln y\left(t_{1}\right)=\left(1-e^{-\beta \tau}\right) \frac{\alpha}{1-\alpha} \ln s_{k}+\left(1-e^{-\beta \tau}\right) \frac{\lambda}{1-\alpha} \ln h^{*}+\left(1-e^{-\beta \tau}\right) \frac{\lambda}{1-\alpha} \ln \theta$
$-\left(1-e^{-\beta \tau}\right) \frac{\alpha}{1-\alpha} \ln (n+g+\delta)-\left(1-e^{-\beta \tau}\right) \ln y\left(t_{1}\right)+\left(1-e^{-\beta \tau}\right) \ln A(0)+g\left(t_{2}-t_{1} e^{-\beta \tau}\right)$

Our statistical model is a discrete period version of equation (14), after both sides of the equation have been divided by $\tau$, that applies to an economy $i$ and is augmented by an error term:

$$
\begin{equation*}
\frac{1}{\tau} \ln \left(\frac{y_{i, t+\tau}}{y_{i, t}}\right)=a+\frac{\left(1-e^{-\beta \tau}\right)}{\tau} \frac{\lambda}{1-\alpha} \ln \theta_{i}-\frac{\left(1-e^{-\beta \tau}\right)}{\tau} \ln y_{i, t}+u_{i, t}, \tag{15}
\end{equation*}
$$

where $a=\frac{\left(1-e^{-\beta \tau}\right)}{\tau \cdot(1-\alpha)}\left(\alpha \ln s_{k}+\lambda \ln h^{*}-\alpha \ln (n+g+\delta)\right)+\left(1-e^{-\beta \tau}\right) \ln A(0)+$ $+g\left(t+\tau-t e^{-\beta \tau}\right)$ and $u_{i, t}$ is the error term.

Using equation (3) in (15) yields:

$$
\begin{equation*}
\frac{1}{\tau} \ln \left(\frac{y_{i, t+\tau}}{y_{i, t}}\right)=a+\frac{\left(1-e^{-\beta \tau}\right)}{\tau} \frac{\lambda}{1-\alpha} \cdot \sum_{j} \gamma_{j} \ln l_{j, i, t}-\frac{\left(1-e^{-\beta \tau}\right)}{\tau} \ln y_{i, t}+u_{i, t} \tag{16}
\end{equation*}
$$

where $a$ is the same as in equation (15).

When implementing this regression equation in a cross-section we assume as a starting point that the economies are equal with regard to the rate of investment in physical capital, the steady state level of accumable human capital, the population growth rate, the rate of depreciation, the technology and rate of technological progress or, at least, that these variables are uncorrelated with the regressors. This assumption implies that we can estimate (16) with ordinary least squares.

## 3. Data

The data on incomes for 1930-1990 for the 48 continental US States are from US Commerce Department. The income concept used is per capita personal income excluding government transfers. Data on incomes for 1920 are from Easterlin (1960). As a result, the data on incomes should be the same as Barro and Sala-i-Martin (1992) use. Moreover, following Barro and Sala-i-Martin (1992) we compute real income by dividing the nominal figures on personal income by the national values of the consumer price index $(1982-1984=100)$. (We use the figures from the Statistical Abstract of the US for all items since 1960. Before 1960 we use the overall index from US Commerce Department (1975), series E135.

The data on age structure are from US Department of Commerce (1975) and from the Statistical Abstract of the US. Data on average years of schooling are taken from Mulligan and Sala-i-Martin (1995). Data on labor earnings (including those from self-employment) broken down into nine sectors since 1929 is from US Department of Commerce. The share of the labor force engaged in agriculture is from Easterlin (1960).

## 4. Empirical Results

Figure 1 shows that there is a great deal of variation of demographic structure during the sample period. Moreover, we observe not only converging per capita incomes but also converging demographic profiles across the states.

In column 1 of table 1a we reproduce the results of Barro and Sala-i-Martin (1992) on absolute $\beta$-convergence for the US States for the period 1920-1990; that is, we estimate equation (16) without including age groups. A positive value of $\beta$ implies absolute convergence which means that poorer states tend subsequently to grow faster in per-capita terms than richer ones. A higher value of $\beta$ corresponds to a faster convergence rate. The sample period is split up into 10 -year long subperiods. The estimation method is nonlinear seemingly unrelated regression (SUR). Using SUR is a way to account for, e.g., long-lasting sectoral shocks, since it allows for correlation of disturbances over time, i.e. over equations. In row 8 of column $1 \beta$ is restricted to be the same over the periods but we allow for different constants. The joint estimate is 0.0166 (s.e.=0.0016). The hypothesis of equal $\beta \mathrm{s}$ over the subperiods is, however, strongly rejected; row 9 shows that the likelihood-ratio statistic is 41.8 with a p-value of 0.0000 . (Under the null hypothesis, the likelihood-ratio statistic is asymptotically distributed as chisquared with six degrees of freedom).

Column 2 of table 1a present the results from estimation of equation (16). We include the proportion of the population in the age groups 15-25, 25-45, 45-65 and over 65 years as regressors. The first age group has been dropped to avoid collinearity. The division of age groups is determined by the statistics (see the US Department of Commerce (1975)).

Rows $1-8$ of column 1 show the results of an estimation in which the coefficients of the age groups are restricted to be the same over the subperiods but the intercepts and $\beta \mathrm{s}$ are allowed to vary over the subperiods. The results can be summarized in the following way: The coefficients of the age groups, 15-65, are positive and significant. Hence, the age groups of the so-called working age appear to contribute positively to subsequent growth in per capita terms. However, at conventional levels of significance we reject the hypothesis that the coefficients of the age groups are the same over the periods;
the likelihood-ratio statistic in row 9 is 48.5 with a p-value of 0.0022 . (Under the null, the likelihood-ratio statistic is asymptotically distributed as chi-squared with twenty-four degrees of freedom).

The issue of convergence has generated a huge amount of research during the last ten years or so (see e.g. Barro and Sala-i-Martin, 1995). One issue has been what different estimates of $\beta$ imply for the shares of capital, which can be seen in equation (8). Comparing column 1 with column 2 we see that the estimates of $\beta$ increase when age structure is included in the regressions. This seems to indicate that not including age structure in growth regressions for the US states biases the estimate of $\beta$. However, only quantitative changes of the estimates of $\beta$ occur when age is included in the regressions. Furthermore, row 10 of column 2 in table shows that the joint estimate of $\beta$ increases to $0.0315(0.0022)$ when age structure is included in the regressions. The intercepts and the coefficients of the age groups are allowed to vary over the periods in these regressions. We continue, however, to reject the hypothesis that $\beta$ is the same over the periods on the basis of a likelihood-ratio test. The likelihood-ratio statistic is 26.7 and is presented in row 11. (Under the null, the likelihood-ratio statistic is asymptotically chi-squared distributed with six degrees of freedom).

## Sectoral Shocks

Following Barro and Sala-i-Martin $(1991,1992,1995)$ we include an auxiliary variable in our regression equations to hold constant aggregate shocks in order to obtain more accurate estimates of $\beta$ and of the coefficients of the age groups. This structural variable is
$s_{i, t}=\sum_{k=1}^{9} \omega_{k, i, t} \cdot \ln \left(y_{k, t+\tau} / y_{k, t}\right)$
where $\omega_{k, i, t}$ is the weight of sector $k$ in state $i$ 's labor earnings (including those from self employment) at time $t$, and $y_{k, t}$ is the national average of labor earnings per capita in sector $k$ at time $t$. The nine sector used are agriculture, mining, construction, manufacturing, trade, finance and real estate, transportation, services and government. For example, suppose that economy $i$ specializes in mining and that the aggregate mining sector does not grow over the period $t$ and $t+\tau$, then the value of $s_{i, t}$ is likely to be low which indicates that this region should not grow very fast because the mining industry has suffered from a shock.

Because of lack of data, we include the structural variable only for the periods after 1929. However, when estimating the income growth regression for the period 19201930 we obtain a rough measure of $s_{i, t}$ by using the share of the labor force engaged in agriculture in 1920.

The regressions behind table 1 b is identical to the regressions behind table 1a with the exception that we include the structural variable, $s_{i, t}$, in the regressions of table 1 b . The coefficient of $s_{i, t}$ is always allowed to vary over the subperiods. From our point of view; that is, focusing on the effect of age structure on subsequent growth, the main difference compared to table 1 a is that the age group 15-25 year turn insignificant.

## Schooling

Several cross-country studies find that various educational variables, such as literacy rates, school enrollment rates, and average years of schooling, are positively related to growth (see e.g. Mankiw, Romer and Weil (1992) and Barro and Sala-i-Martin (1995), ch. 12). Here we investigate the effect of education, measured by the average years of schooling, on growth for the US States for the period 1940-1990 by abandoning the assumption in equation (16) that the level of the part of human capital that can be accumulated is the same across the US States; that is, we estimate the equation
$\frac{1}{\tau} \ln \left(\frac{y_{i, t+\tau}}{y_{i, t}}\right)=a+\frac{\left(1-e^{-\beta \tau}\right)}{\tau} \frac{\lambda}{(1-\alpha)} \ln h_{i, t}^{*}+\frac{\left(1-e^{-\beta \tau}\right)}{\tau} \frac{\lambda}{1-\alpha} \cdot \sum_{j} \gamma_{j} \ln l_{j, i, t}-\frac{\left(1-e^{-\beta \tau}\right)}{\tau} \ln y_{i, t}+u_{i, t}$
where $a=\frac{\left(1-e^{-\beta \tau}\right)}{\tau \cdot(1-\alpha)}\left(\alpha \ln s_{k}-\alpha \ln (n+g+\delta)\right)+\left(1-e^{-\beta \tau}\right) \ln A(0)+g\left(t+\tau-t e^{-\beta \tau}\right)$
and $u_{i, t}$ is the error term.
Following e.g. Islam (1995) we let the average years of schooling be a proxy for $h_{i, t}^{*}$. Table 2a shows the results. In column 1 we include only schooling and initial per capita income as regressors; that is, the age groups are not included. The joint estimate of the coefficient of schooling, shown in row 6, is positive but insignificant when the intercepts and $\beta \mathrm{s}$ are allowed to vary over the periods. When we include the age groups; that is, when we estimate equation (18), the joint estimate of the coefficient of schooling is positive and significant. This is shown in row 6 in column 2. In the regressions of column 2 the intercept and $\beta$ are allowed to vary over the periods. Moreover, the coefficients of the age groups 25-45 and 45-65 are significant and positive. The age group 25-45 seems to contribute most to subsequent per capita growth when average years of schooling is held constant. This is a difference relative to table 1 .

As before we the reject the hypothesis that the coefficient of schooling and the coefficients of the age groups are the same over the periods: the likelihood-ratio statistic in row 7 is 53.6 with a p-value of 0.0001 (Under the null, the likelihood-ratio statistic is asymptotically distributed as chi-squared with 20 degrees of freedom).

Row 8 of column 2 shows the joint estimate of $\beta$ when the intercepts and the coefficient of the schooling variable and the coefficients of the age groups are allowed to vary over periods. The joint estimate of $\beta$ is 0.0642 ( 0.0074 ). However, we can reject the hypothesis that $\beta$ is the same over the periods.

In table 2 b we include the structural variable, $s_{i, t}$, as a regressor. The main results presented in table 2 b are similar to those presented in table 2 a .

## 5. Concluding Remarks

This paper finds robust evidence that age structure matters for subsequent growth in per capita income across the US states 1920-1990. The age groups 25-65 year are positively related to subsequent per capita income growth. Another conclusion is that the average years of schooling affects subsequent per capita income growth positively when age is held constant in the growth regressions. Moreover, the estimated speed of convergence increases substantially when schooling and age structure is held constant in the income growth regressions.

## Appendix: The Transitional Dynamics

To evaluate the dynamics we log-linearize the system around steady state. We start by substituting the production function into (6) and rewrite the dynamic system in terms of the logs of $\hat{k}$ and $\hat{h}$ :

$$
\begin{align*}
& \frac{d \ln \hat{k}}{d t}=s_{k} \theta^{\lambda} e^{-(1-\alpha) \ln \hat{k}} e^{\lambda \ln \hat{h}}-(n+g+\delta)  \tag{A.1}\\
& \frac{d \ln \hat{h}}{d t}=s_{h} \theta^{\lambda} e^{\alpha \ln \hat{k}} e^{-(1-\lambda) \ln \hat{h}}-(n+g+\delta)
\end{align*}
$$

We take a first-order Taylor expansion around the steady state values, $\ln \hat{k}^{*}$ and $\ln \hat{h}^{*}$, determined by equation (A.1):
$\frac{d \ln \hat{k}}{d t}=-(1-\alpha)(n+g+\delta) \ln \left(\hat{k} / \hat{k}^{*}\right)+\lambda(n+g+\delta) \ln \left(\hat{h} / \hat{h}^{*}\right)$

$$
\begin{equation*}
\frac{d \ln \hat{h}}{d t}=\alpha(n+g+\delta) \ln \left(\hat{k} / \hat{k}^{*}\right)-(1-\lambda)(n+g+\delta) \ln \left(\hat{h} / \hat{h}^{*}\right) \tag{A.2}
\end{equation*}
$$

To substitute away the two types of capital we take the $\log$ of the production function:

$$
\begin{equation*}
\ln \hat{y}=\alpha \ln \hat{k}+\lambda \ln \hat{h}+\lambda \ln \theta \tag{A.3}
\end{equation*}
$$

Differentiating (A.3) with respect to time yields:

$$
\begin{equation*}
\frac{d \ln \hat{y}}{d t}=\alpha \frac{d \ln \hat{k}}{d t}+\lambda \frac{d \ln \hat{h}}{d t} \tag{A.4}
\end{equation*}
$$

Inserting (A.2) into (A.4) and collecting terms:

$$
\begin{equation*}
\frac{d \ln \hat{y}}{d t}=-(1-\alpha-\lambda)(n+g+\delta)\left[\alpha \ln \left(\hat{k} / \hat{k}^{*}\right)+\lambda \ln \left(\hat{h} / \hat{h}^{*}\right)\right] \tag{A.5}
\end{equation*}
$$

Subtracting $\ln \hat{y}^{*}$ from (A.3) yields:

$$
\begin{equation*}
\ln \left(\hat{y} / \hat{y}^{*}\right)=\alpha \ln \left(\hat{k} / \hat{k}^{*}\right)+\lambda \ln \left(\hat{h} / \hat{h}^{*}\right) \tag{A.6}
\end{equation*}
$$

Combining (A.5) and (A.6) gives us:

$$
\begin{equation*}
\frac{d \ln \hat{y}}{d t}=-\beta \ln \left(\hat{y} / \hat{y}^{*}\right) \tag{A.7}
\end{equation*}
$$

where $\beta=(1-\alpha-\lambda)(n+g+\delta)$

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Table 1a. Growth Regressions with Age Structure for the US States 1920-1990

| Basic equation |  | Equation with demographic variables |  |  |  | 45-65 | 65- | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Period | (1) | $R^{2}$ | (2) | 15-25 | 25-45 |  |  |  |
|  | $\hat{\beta}$ | $\left[\hat{\sigma}_{]}\right]$ | $\hat{\beta}$ | years | years | years | years | $\left[\hat{\sigma}_{]}\right]$ |
| 1. 1920-1930 | -0.0151 | 0.14 | -0.0012 |  |  |  |  | 0.33 |
|  | (0.0044) | [0.0129] | (0.0053) |  |  |  |  | [0.0116] |
| 2. 1930-1940 | 0.0133 | 0.29 | 0.0272 |  |  |  |  | 0.25 |
|  | (0.0032) | [0.0077] | (0.0046) |  |  |  |  | [0.0080] |
| 3. 1940-1950 | 0.0424 | 0.67 | 0.0662 |  |  |  |  | 0.72 |
|  | (0.0054) | [0.0096] | (0.0075) |  |  |  |  | [0.0088] |
| 4. 1950-1960 | 0.0169 | 0.40 | 0.0346 |  |  |  |  | 0.50 |
|  | (0.0035) | [0.0055] | (0.0057) |  |  |  |  | [0.0050] |
| 5. 1960-1970 | 0.0262 | 0.48 | 0.0332 |  |  |  |  | 0.64 |
|  | (0.0038) | [0.0045] | (0.0054) |  |  |  |  | [0.0037] |
| 6. 1970-1980 | 0.0120 | 0.13 | 0.0187 |  |  |  |  | 0.09 |
|  | (0.0051) | [0.0059] | (0.0065) |  |  |  |  | [0.0060] |


| 7. 1980-1990 | 0.0048 | 0.01 | 0.0147 |  |  |  |  | $\begin{gathered} 0.22 \\ {[0.0084]} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (0.0074) | [0.0092] | (0.0080) |  |  |  |  |  |
| 8. Joint, seven | 0.0166 |  |  | 0.0243 | 0.0228 | 0.0395 | -0.0033 |  |
| subperiods | (0.0012) |  |  | (0.0097) | (0.0087) | (0.0068) | (0.0035) |  |
| 9. Likelihood-ratio | 58.6 |  | 48.5 |  |  |  |  |  |
| statistic (p-value) | (0.0000) |  | (0.0022) |  |  |  |  |  |
| 10. Seven periods, $\beta$ | -- | -- | 0.0315 | N.S. | N.S. | N.S. | N.S. | N.S |
| restricted |  |  | (0.0035) |  |  |  |  |  |
| 11. Likelihood-ratio | -- | - | 26.7 |  |  |  |  |  |
| statistic (p-value) |  |  | (0.0002) |  |  |  |  |  |

Note: N.S. = Not Shown. The estimation method is nonlinear SUR. The standard errors of the estimates are given in parenthesis and the standard errors of the regressions are given in brackets. Rows 1-7 in column 1 display estimated betas for the seven subperiods when age structure is not included in the regression equation, that is, we test for absolute convergence. Row 8 of column 1 shows the estimate of beta when it is restricted to be the same over the periods while allowing for individual intercepts. The likelihood-ratio statistic in row 9 of column 1 refers to a test of equality of beta over the subperiods. (Under the null, this statistic is asymptotically distributed as chi-squared with six degrees of freedom). Row 1-7 in column 2 present the estimated betas when age groups are included as regressors. We allow for individual intercepts (that are not shown in the table) but we restrict the coefficients of the age variables (presented in row 8) to be the same over the periods. The likelihood-ratio statistic in row 9 in column 2 refers to a test of equality of the coefficients of the age groups over the periods. (Under the null, this statistic is asymptotically distributed as chisquared with twenty-four degrees of freedom). Row 10 of column 2 presents the joint estimate of beta when we allow the intercepts and the coefficients of the age variables to vary over the subperiods. The likelihood-ratio statistic in row 11 of column 2 refers to a test of the convergence coefficient over the periods.

Table 1b. Growth Regressions with Age and Structural Variable for the US States 1920-1990
Equation with structural variable Equation with structural and demographic variables

| Period | (1) $\hat{\beta}$ | $\begin{gathered} R^{2} \\ {\left[\hat{\sigma}^{2}\right]} \end{gathered}$ | (2) <br> $\hat{\beta}$ | $15-25$ years | 25-45 <br> years | 45-65 <br> years | 65- <br> years | $\begin{aligned} & R^{2} \\ & {\left[\hat{\sigma}^{3}\right]} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. 1920-1930 | 0.0422 | 0.59 | 0.0581 |  |  |  |  | 0.65 |
|  | (0.0095) | [0.0089] | (0.0121) |  |  |  |  | [0.0083] |
| 2. 1930-1940 | 0.0160 | 0.39 | 0.0292 |  |  |  |  | 0.36 |
|  | (0.0031) | [0.0072] | (0.0045) |  |  |  |  | [0.0075] |
| 3. 1940-1950 | 0.0390 | 0.69 | 0.0584 |  |  |  |  | 0.73 |
|  | (0.0066) | [0.0092] | (0.0081) |  |  |  |  | [0.0086] |
| 4. 1950-1960 | 0.0268 | 0.68 | 0.0435 |  |  |  |  | 0.70 |
|  | (0.0031) | [0.0040] | (0.0051) |  |  |  |  | [0.0039] |
| 5. 1960-1970 | 0.0250 | 0.48 | 0.0342 |  |  |  |  | 0.63 |
|  | (0.0038) | [0.0045] | (0.0055) |  |  |  |  | [0.0038] |
| 6. 1970-1980 | 0.0124 | 0.13 | 0.0212 |  |  |  |  | 0.09 |
|  | (0.0053) | [0.0059] | (0.0066) |  |  |  |  | [0.0060] |


| 7. 1980-1990 | 0.0013 | 0.00 | 0.0107 |  |  |  |  | $\begin{gathered} 0.16 \\ {[0.0086]} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (0.0076) | [0.0092] | (0.0080) |  |  |  |  |  |
| 8. Joint, seven | 0.0244 |  |  | 0.0115 | 0.0204 | 0.0331 | -0.0043 |  |
| subperiods | (0.0017) |  |  | (0.0092) | (0.0090) | (0.0067) | (0.0035) |  |
| 9. Likelihood-ratio | 41.8 |  | 53.4 |  |  |  |  |  |
| statistic (p-value) | (0.0000) |  | (0.0005) |  |  |  |  |  |
| 10. Seven periods, $\beta$ | -- | -- | 0.0367 | N.S. | N.S. | N.S. | N.S. | N.S |
| restricted |  |  | (0.0040) |  |  |  |  |  |
| 11. Likelihood-ratio | -- | -- | 34.1 |  |  |  |  |  |
| statistic (p-value) |  |  | (0.0000) |  |  |  |  |  |

Note: The regressions behind table 1 b is identical to the regressions behind table 1 a with the exception that the structural variable in equation (17) is included in the regressions of table 1 b . The coefficient of the structural variable is allowed to vary in the all the specifications of table 1 b .

Table 2a. Growth Regressions with Age and Schooling 1940-1990

| 1. Equation with schooling |  |  |  | 2. Equation with schooling and age |  |  | 25-45 | 45-65 | $65-$ | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Period | (1) | School- | $R^{2}$ | (2) | School- | 15-25 |  |  |  |  |
|  | $\hat{\beta}$ | ing | $[\hat{\boldsymbol{\sigma}}]$ | $\hat{\beta}$ | ing | years | years | years | years | $\left[\hat{\sigma}_{]}\right]$ |
| 1. 1940-1950 | 0.0465 |  | 0.68 | 0.0787 |  |  |  |  |  | 0.75 |
|  | (0.0057) |  | [0.0094] | (0.0097) |  |  |  |  |  | [0.0084] |
| 2. 1950-1960 | 0.0171 |  | 0.40 | 0.0489 |  |  |  |  |  | 0.59 |
|  | (0.0042) |  | [0.0055] | (0.0082) |  |  |  |  |  | [0.0045] |
| 3. 1960-1970 | 0.0255 |  | 0.45 | 0.0488 |  |  |  |  |  | 0.61 |
|  | (0.0046) |  | [0.0046] | (0.0080) |  |  |  |  |  | [0.0039] |
| 4. 1970-1980 | 0.0157 |  | 0.13 | 0.0292 |  |  |  |  |  | 0.15 |
|  | (0.0057) |  | [0.0058] | (0.0079) |  |  |  |  |  | [0.0058] |
| 5. 1980-1990 | 0.0008 |  | 0.01 | 0.0233 |  |  |  |  |  | 0.12 |
|  | (0.0075) |  | [0.0092] | (0.0099) |  |  |  |  |  | [0.0087] |
| 6. Joint, seven |  | 0.0036 |  |  | 0.0174 | 0.0165 | 0.0473 | 0.0307 | -0.0000 |  |
| subperiods |  | (0.0058) |  |  | (0.0074) | (0.0110) | (0.0114) | (0.0083) | (0.0044) |  |


| 7.Likelihood |  | 24.2 | 53.6 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -ratio statistic |  | (0.0001) | (0.0001) |  |  |  |  |  |  |
| (p-value) |  |  |  |  |  |  |  |  |  |
| 8. Joint, seven | -- | -- | 0.0642 | N.S. | N.S. | N.S. | N.S. | N.S. | N.S |
| subperiods |  |  | (0.0074) |  |  |  |  |  |  |
| 9.Likelihood | -- | -- | 24.6 |  |  |  |  |  |  |
| -ratio statistic |  |  |  |  |  |  |  |  |  |
| (p-value) |  |  | (0.0001) |  |  |  |  |  |  |

Note: N.S. = Not Shown. The estimation method is nonlinear SUR. The standard errors of the estimates are given in parenthesis and the standard errors of the regressions are given in brackets. Rows 1-5 in column 1 display estimated betas for the five subperiods when the average years of schooling is included in the regression equation. The coefficient of the schooling variable is restricted to be the same over the periods. This joint estimate is presented in row 6 of column 1 . The intercepts are allowed to vary over the periods. The likelihood-ratio statistic in row 7 of column 1 refers to a test of equality of the coefficients of the schooling variable over the subperiods. (Under the null, this statistic is asymptotically distributed as chi-squared with four degrees of freedom). Row 1-5 in column 2 present the estimated betas when schooling and age groups are included as regressors. We allow for individual intercepts (that are not shown in the table) but we restrict the coefficient of the schooling variable and the coefficients of the age variables (presented in row 6) to be the same over the periods. The likelihood-ratio statistic in row 7 in column 2 refers to a test of equality of the coefficient of the schooling variable and of the coefficients of the age groups over the periods. (Under the null, this statistic is asymptotically distributed as chi-squared with twenty degrees of freedom). Row 8 of column 2 presents the joint estimate of beta when we allow the intercepts, the coefficient of the schooling variable and the coefficients of the age variables to vary over the subperiods. The likelihood-ratio statistic in row 9 of column 2 refers to a test of equality of the convergence coefficient over the periods.

Table 2b. Regressions with Age, Structural Variable and Schooling 1940-1990

1. Equation with structural variable and schooling 2. Equation with structural variable, schooling and age

| Period | (1) $\hat{\beta}$ | School- <br> ing | $\begin{gathered} R^{2} \\ {\left[\hat{\sigma}^{2}\right]} \end{gathered}$ | $\begin{aligned} & (2) \\ & \hat{\boldsymbol{\beta}} \end{aligned}$ | School- <br> ing | $15-25$ <br> years | $25-45$ <br> years | $45-65$ <br> years | 65- <br> years | $\begin{aligned} & R^{2} \\ & {\left[\hat{\sigma}^{\prime}\right]} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. 1940-1950 | 0.0395 |  | 0.72 | 0.0684 |  |  |  |  |  | 0.76 |
|  | (0.0068) |  | [0.0088] | (0.0105) |  |  |  |  |  | [0.0081] |
| 2. 1950-1960 | 0.0318 |  | 0.70 | 0.0615 |  |  |  |  |  | 0.73 |
|  | (0.0042) |  | [0.0039] | (0.0087) |  |  |  |  |  | [0.0038] |
| 3. 1960-1970 | 0.0287 |  | 0.42 | 0.0492 |  |  |  |  |  | 0.59 |
|  | (0.0050) |  | [0.0048] | (0.0085) |  |  |  |  |  | [0.0040] |
| 4. 1970-1980 | 0.0165 |  | 0.14 | 0.0301 |  |  |  |  |  | 0.12 |
|  | (0.0059) |  | [0.0058] | (0.0080) |  |  |  |  |  | [0.0059] |
| 5. 1980-1990 | 0.0048 |  | 0.00 | 0.0208 |  |  |  |  |  | 0.16 |
|  | (0.0081) |  | [0.0092] | (0.0096) |  |  |  |  |  | [0.0086] |
| 6. Joint, seven |  | 0.0105 |  |  | 0.0195 | 0.0101 | 0.0393 | 0.0323 | -0.0016 |  |
| subperiods |  | (0.0065) |  |  | (0.0074) | (0.0110) | (0.0124) | (0.0085) | (0.0046) |  |


| 7.Likelihood |  | 29.2 | 43.9 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -ratio statistic |  | (0.0000) | (0.0015) |  |  |  |  |  |  |
| (p-value) |  |  |  |  |  |  |  |  |  |
| 8. Joint, seven | -- | -- | 0.0583 | N.S. | N.S. | N.S. | N.S. | N.S. | N.S |
| subperiods |  |  | (0.0076) |  |  |  |  |  |  |
| 9.Likelihood | -- | -- | 23.3 |  |  |  |  |  |  |
| -ratio statistic |  |  |  |  |  |  |  |  |  |
| (p-value) |  |  | (0.0001) |  |  |  |  |  |  |

Note: The regressions behind table 2 b is identical to the regressions behind table 2 a with the exception that the structural variable in equation (17) is included in the regressions of table 1 b . The coefficient of the structural variable is allowed to vary in the all the specifications of table 1 b .


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