Forward Interest Rates as Indicators of Inflation Expectations

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May 28, 1995

Abstract

Forward interest rates have become popular indicators of inflation expectations. The usefulness of this indicator depends on the relative volatilty and the correlation of inflation expectations and expected real interest rates. This paper studies U.S. and U.K. data, using a range of different tools and data sets. The forward rate rule perfoms reasonably well, in spite of significant movements in the expected real interest rate. The reason is that the "noise" that movements in the expected real interest rate add to the inflation expectations is balanced by a tendency for expected real interest rates and inflation expectations to move in opposite directions.

Keywords: Inflation expectations, real interest rates, forward rates.

JEL Classification Numbers: E31, E43, E44, and G12.

1 Introduction

It has long been recognized that the yield curve contains information about inflation expectations. [1] Recently, Svensson [32] discussed the possibility of using the forward rate as an estimator of inflation expectations. The advantage of the forward rate is that it may give information about expectations for a future period, for instance, the year starting twelve months ahead, without blurring the picture with the expectations about what will happen

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up to that point in time.

According to the Fisher equation, interest rates have at least two components: inflation expectations and expected real interest rates. Svensson [32] uses forward rates to discuss inflation expectations under simple assumptions about the expected real interest rate. This method is now used by, among others, Bank of England [2] and Bank of Sweden [3], and a related approach is discussed in Ragan [29]. As a matter of presentation, I will focus on what may be a caricature of that approach, namely to attribute all movements in the forward rate to inflation expectations, which will henceforth be called "the forward rate rule.". The aim of this paper is to evaluate this approach.

The usefulness of the forward rate as an indicator of inflation expectations depends on the relative volatility and the correlation of inflation expectations and expected real interest rates. I try to calculate these magnitudes in several different ways. The first approach, which is related to Mishkin [25], is to generate inflation predictions from a VAR model of quarterly U.S. data since the mid 1950s and to relate them to forward rates on Treasury bonds. The results turn out to be sensitive to the sample period, in particular the early 1980s, so no firm conclusion can be drawn. The second approach is to use survey data on U.S. inflation expectations in order to get a more direct measure of expected inflation and expected real interest rates. The third approach, is to ask what a simple consumption asset pricing model would tell us. Hopefully, this will help us encircle the elusive covariance matrix of two unobservables. To make the exercise more realistic, the stochastic process driving bond prices (the time series representation of consumption, leisure, inflation, etc.) is taken from the same VAR model as the inflation expectations. The fourth and final approach is to study the relation between real interest rates of index-linked bonds in U.K., and the implied inflation expectations that falls out from comparing indexed-linked bonds with standard nominal bonds.

2 A Theoretical Framework for Inflation Expectations and Forward Rates

The Fisher equation states that the nominal interest rate is the sum of expected inflation and a real interest rate. More recent models for intertemporal optimization by risk averse consumers often generate the same kind of relation, but with a additional terms for risk premia. As an example, consider the standard asset pricing model with a time separable utility function with consumption (C_t) and leisure ($1 - H_t$) as its arguments

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left(C_t^{\psi} \left(1 - H_t\right)^{1-\psi} \right)^{1-\gamma}, \tag{1}$$

where γ is the relative risk aversion, and ψ the relative weight on consumption. The time period is assumed to be a quarter. The consumer can invest in, among other asset, nominal and real bonds with different maturities. The optimality conditions can be combined to give the following annualized gross yield to maturity on a k quarter nominal (discount) bond ($R_{k,t}$)

$$\left(R_{k,t}\right)^{k/4} = \frac{1}{\beta^k \mathrm{E}_t \frac{P_t}{P_{t+k}} \left(\frac{C_{t+k}}{C_t}\right)^{\psi(1-\gamma)-1} \left(\frac{1-H_{t+k}}{1-H_t}\right)^{(1-\psi)(1-\gamma)}}, \tag{2}$$

where P_t is the price level. A real bond, paying one unit of C_{t+k}, is priced in a similar way, but without the price ratio term.

The logarithm of (2) is easily evaluated if we approximate the unknown conditional distribution of consumption, leisure, and prices with a lognormal distribution. See Appendix for details. We express the result in terms of the log forward rate with maturity of four quarters and settlement k quarters ahead ($f_{k-4,4,t}$).[2] It can, as in Svensson [33], be interpreted as the sum of the expected real interest rate ($r^e_{k-4,4,t}$), the expected inflation ($\pi^e_{k-4,4,t}$), the nominal

[2]The subscripts for $f_{x,y,t}$ follows the convention that the third subscript (t) refers to the current date, the second subscript (y) to maturity (length of the investment period), and the first subscript (x) to the time until the start of the investment period.

(forward) term premium ($\varphi_{k-4,4,t}$), and the inflation risk premium ($\theta_{k-4,4,t}$)

$$f_{k-4,4,t} = r^e_{k-4,4,t} + \pi^e_{k-4,4,t} + \varphi_{k-4,4,t} + \theta_{k-4,4,t}. \tag{3}$$

The expected real interest rate is the expected interest rate on a real bond issued in $t+k-4$ and maturing in $t+k$

$$r^e_{k-4,4,t} := \mathrm{E}_t r_{4,t+k-4} = -\mathrm{E}_t \mu_{4,t+k-4} - \frac{1}{2}\mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4}\right), \tag{4}$$

where the log real intertemporal rate of substitution ($\mu_{k,t}$) is

$$\mu_{k,t} = k\ln\beta + \left[\psi\left(1-\gamma\right)-1\right]\Delta_k \ln C_{t+k} + \left(1-\psi\right)\left(1-\gamma\right)\Delta_k \ln\left(1-H_{t+k}\right). \tag{5}$$

In (5) Δ_k denotes a kth difference, for instance, $\Delta_k x_t = x_t - x_{t-k}$. (5) could also be interpreted as a first order approximation of the log real intertemporal rate of substitution for a more general time separable utility function than (1). Inflation expectations is defined by the rational expectation

$$\pi^e_{k-4,4,t} := \mathrm{E}_t \Delta_4 \ln P_{t+k}. \tag{6}$$

Rational expectations has mixed empirical support, see for instance Figlewski and Wachtel [15] or Pearce [28], so it is fortunate that the results for U.S. inflations expectations survey and U.K. index-linked bonds need no assumptions about how inflation expectations are formed.

The nominal (forward) term premium is the difference between the nominal forward rate and the expected future nominal interest rate ($\varphi_{k-4,4,t} = f_{k-4,4,t}$ - $\mathrm{E}_t \ln R_{4,t+k-4}$), and the inflation risk premium is the expected real excess return of a nominal bond over a real bond ($\theta_{k-4,4,t} = \mathrm{E}_t \ln R_{4,t+k-4}$ - $\pi^e_{k-4,4,t}$- $r^e_{k-4,4,t}$). Both are functions of variances and covariances, which are constant if the time series process for inflation, consumption and leisure is homoskedastic. See Appendix for details. I will use this assumption in most of the rest of the paper, which essentially is an assumption that the "expectations hypothesis" holds. The effects of relaxing this assumption, as well as of assuming other utility functions will be discussed. The model (3)-(6) is valid for both open and closed economies; nothing has been said about how the general equilibrium looks like.

3 Extracting Expected Inflation from Forward Rates

In deriving (3)-(6), we assumed a conditional normal distribution of inflation, consumption growth, and leisure growth. We now assume that all relevant variables in the information set are also normally distributed. To be precise, we assume that all relevant variables, including inflation, consumption growth, and leisure growth, follow a stationary linear time series process with homoskedastic normally distributed shocks.[3] As a consequence, the forward rate, the expected real interest rate, and the inflation expectations are random variables with a normal unconditional joint distribution. This is the distribution we need in order to evaluate the forward rate rule.

The forward rate rule is actually of the correct functional form, since the mathematical (rational) expectation of the (unobserved) inflation expectation is a linear prediction rule

$$\widehat{\pi^e_{k-4,4,t}} = a + b_k f_{k-4,4,t}. \tag{7}$$

The coefficient is the same as discussed in, among others, Mishkin [25] for the projection of actual inflation on nominal interest rates. It can be shown to be

$$\begin{aligned} b_k &= \tfrac{1+\rho_k\sigma_k}{1+\sigma_k^2+2\rho_k\sigma_k}, \text{ where} \\ \sigma_k^2 &= \mathrm{Var}\left(r^e_{k-4,4,t}\right)/\mathrm{Var}\left(\pi^e_{k-4,4,t}\right) \text{ and} \\ \rho_k &= \mathrm{Corr}\left(\pi^e_{k-4,4,t}, r^e_{k-4,4,t}\right). \end{aligned} \tag{8}$$

One way of thinking about this "signal extraction" problem is to assume that a central banker reads off the forward rate, but his limited talent does not allow him to calculate the rational inflation expectations; nor can he recall anything else about the state of the economy. He badly wants to get an estimate of the inflation expectations out there; he

[3]The normality assumption for inflation, growth in consumption and leisure cannot be rejected in formal (Jarque-Bera) tests on US data once the autocorrelation is taken into account. Data is described below.

decides to use the forward rate as an indicator. The rest of the paper will compare two central bankers of this basic type: one of them will use the forward rate rule, the other a slightly more sophisticated rule where he multiplies the forward rate with some coefficient b_k instead. It is clear that applying (7) uses only a very limited information set; no business cycle indicators are used and the time series properties are not exploited. The importance of increasing the information set is beyond the scope of this paper.[4]

The rational expectation is a natural benchmark since it gives the smallest mean squared error (MSE). There is also a strong case for the b_k coefficient in (8) even if we go beyond quadratic loss functions. Granger [19] showed that with a normal distribution, the rational expectation is the optimal forecasting rule for any well-behaved loss function which is symmetric around zero in the forecasting error. Asymmetric linear or linear plus exponential loss functions also give b_k, but require an adjustment of the constant (a) in (7).

As a stylized example of how (8) may work, suppose inflation (π_t) and the real interest rate (r_t) are AR(1) processes : $\pi_t = 0.9\pi_{t-1} + u_t$ and $r_t = 0.45 r_{t-1} + e_t$, where u_t and e_t are independently normally distributed with variances such that the unconditional variances of π_t and r_t are both one. The rational expectations are $\pi^e_{0,1,t} = 0.9\pi_t$ and $r^e_{0,1t+1} = 0.45 r_t$. The relative standard deviation of the expected real interest rate (σ_k) is 0.5, which reflects that r_{t+1} is more difficult to forecast than π_{t+1} (the same overall variance as π_t, but lower autocorrelation/higher variance of the innovation). The correlation (ρ_k) is zero, so $b_k = 1/\left(1+\sigma_k^2\right) = 0.8$. The point $(\sigma_k, \rho_k) = (0.5, 0)$ is given by the lower left corner of the letter "A" in *Figure 1.a.* The curves in this figure illustrates how b_k varies as the relative variance is increased, for three different values of the correlation.

The distance between b_k and one is one way of evaluating the forward rate rule. The relative efficiency, measured as the ratio of the mean squared error (MSE), is perhaps better. It can be shown that $\mathrm{MSE}(b_k)/\mathrm{MSE}(1) = \left(1-\rho_k^2\right)/\left(1+\sigma_k^2+2\rho_k\sigma_k\right)$, which is illustrated in *Figure 1.b.* Of course, both these measures can be pretty meaningless if both rules are almost useless, or for that matter, almost perfect. Extreme values of the R^2 for the

[4] For instance, Frankel and Lown [16] assumes a long run constant real interest rate and a parametric form of the adjustment mechanism. This improves the ability of the yield curve to forecast future inflation.

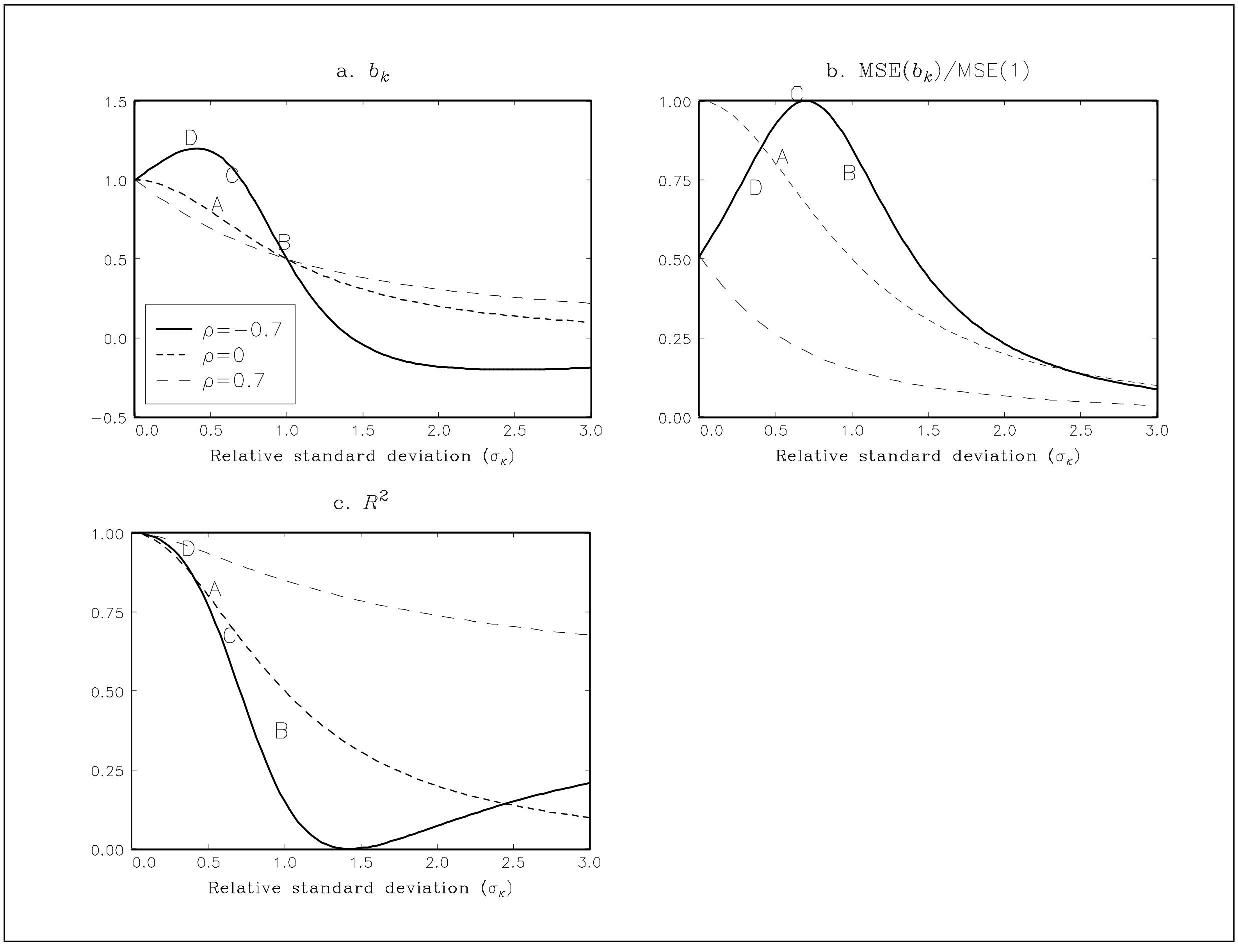

Figure 1: Projection coefficient (b_k), R^2, and relative efficiency of the forward rate rule.

rational expectation will signal if such problems are present. It can be shown that $R^2 = (1 + \rho_k\sigma_k)^2 / (1 + \sigma_k^2 + 2\rho_k\sigma_k)$, which is illustrated in *Figure 1.c.*

There are four basic cases to consider. *First*, the forward rate is a perfect signal when the expected real interest rate is constant ($\sigma_k = 0$); then $b_k = 1$ and $R^2 = 1$. This is the first possibility for the forward rate rule to be optimal. In terms of the model (3)-(6), this requires that both consumption and labor supply were random walks. Both rules have zero MSE in this case, but they converge a bit differently as $\sigma_k \to 0$, which explains why the relative MSE in Figure 1.b does not go to unity unless $\rho_k = 0$.

Second, the forward rate is a noisy and biased predictor whenever the expected real interest rate is not constant ($\sigma_k > 0$), even if inflation expectations and real interest rates vary independently ($\rho_k = 0$). This corresponds to a classical signal extraction problem, as in the simple example above. This would, for instance, be the case if inflation expectations are driven exclusively by monetary shocks and expectations about real variables by real shocks only. The rational expectations rule (8) would then set $0 \leq b_k < 1$ (some of the movements in the forward rate are rationally ascribed to the real interest rate). The relative MSE and R^2 happen to equal b_k, which is monotonically decreasing (to zero) in the relative variance. *Third*, a positive correlation between inflation expectations and expected real interest rates ($\rho_k > 0$) means that the forward rate tend to be a magnified version of the inflation expectations. It is then natural that $0 \leq b_k < 1$ as in the case with zero correlation, but that the forward rate becomes very informative (high R^2).

Fourth, for negative correlations, b_k is above one for small values of σ_k, but is even negative for large values of σ_k. R^2 is first decreasing in σ_k, but it is increasing for $\sigma_k > -1/\rho_k$. When σ_k is low and $\rho_k < 0$, then the forward rate tend to be a dampened version of inflation expectations; b_k is above one. The projection coefficient will eventually decrease as σ_k increases. At $\sigma_k = -\rho_k$, the noise (σ_k) and the information (ρ_k) in the movements of the expected real interest rates offset each other so $b_k = 1$; this is the second possibility for the forward rate rule to be optimal. This shows that an estimated value of b_k close to is not enough to claim that the real interest rate is almost constant.[5] At even higher values, movements in inflation expectations and real interest rates tend to cancel each to make the forward rate uninformative (think of this as a case where r^e tend to decrease with one whenever π^e increases with one). A negative correlation could, for instance, be explained by a combination of productivity shocks and a (more or less) constant velocity.[6]

[5] See for instance Fama [13] and the comment by Nelson and Schwert [27].

[6] One of the first models for a negative relation between expected inflation and real interest rates was set forward in Mundell [26].

4 U.S. Inflation Expectations: Rational Expectations, Survey Data, and an Asset Pricing Model

4.1 Data and VAR Estimation

The data used in this section are quarterly U.S. data for 1955:I to 1990:IV. Personal consumption of services and non-durables (C_t), consumer price index (P_t), and man-hours of employed labor force according to household data (H_t) are taken from Citibase. The money stock (M_t) for 1955:I-1959:I is the M1 series from Friedman and Schwartz [17], which is spliced with the Fed's M1 series (from Citibase) in 1959:I. All quantities are divided by the US population, and seasonally adjusted by taking out quarterly dummies for the quarterly growth rates. The level of H_t is adjusted to give a mean of 1/3, which should correspond to the average fraction of time spent on working. Interest rates for 1, 4, 8, and 12 quarters ($R_{1t}, R_{4t}, R_{8t}, R_{12t}$) are from McCulloch and Kwon [24]. These interest rates are estimated zero-coupon rates (per year, continuously compounded) for U.S. Treasury securities.

C_t, P_t, and M_t is expected to be non-stationary, which cannot be rejected in ADF tests, and also H_t has long swings. I therefore find it convenient to estimate a VAR system, with intercepts, of $\Delta \ln C_t$, $\Delta \ln P_t$, $\Delta \ln (1 - H_t)$, $\Delta \ln M_t$, and $\ln R_{1,t}$. The AIC favors 4 or 5 lags. The hypothesis that all coefficients for the sixth lag are zero cannot be rejected at the 10% level, but the same hypothesis for the fifth lag is easily rejected at the 5% level. [7] I therefore estimate a VAR(5). The hypothesis of no autocorrelation in the residuals cannot be rejected at the 5% level. The hypothesis of normality is rejected, because of excess kurtosis of the

[7] I have experimented a bit with the VAR system to see if the results are sensitive to the specification. A partial answer is no. Using total consumption expenditures instead of consumption of services and durables, or adding other variables like the oil price (Christiano, Eichenbaum and Evans [8] argue that commodity prices in order to understand the structure of monetary policy, but the current result shows that the oil price could well be merged with other exogenous shocks for the purpose of forecasting) and GNP, or fiddling with the lag order, or estimating an error correction model with real balances constrained to be stationary, or using monthly data give fairly sinilar results.

residuals for the interest rate (due to the early 1980s).

The implied forecast R^2 for *annual* consumption growth, annual inflation, annual leisure growth, and annual nominal interest rate are shown in *Table 1*.

Forecasting horizon (k) (quarters)	$\Delta_4 \ln C_t$	$\Delta_4 \ln P_t$	$\Delta_4 \ln (1 - H_t)$
4	0.46	0.85	0.49
8	0.23	0.60	0.24

Table 1: R^2 for forecasts based on estimated VAR.

Inflation is highly predictable, which is necessary for this exercise to make any sense. As a comparison, the R^2 obtained from forecasting inflation on forward rates are 0.29 ($k = 4$) and 0.07 ($k = 8$). Also growth in consumption and leisure are forecastable. *Figure 2* shows actual annual inflation, the inflation forecast for the same period made 4 quarters earlier, and the one-year interest rate for a bond maturing in the same quarter.

The inflation forecasts are fairly precise, apart from the initial surprises following the oil price shocks around 1974 and 1979. The high forecast R^2 for inflation means that the ex post real interest rate is well predicted by the VAR. The nominal interest rate is well synchronized with inflation expectations except for the early 1980s, where it increases at the same time as inflation is falling.

In the next two section I will pretend that the rational expectations for consumption, leisure, and prices equal the VAR predictions. Of course, the estimated VAR includes only a small subset of the available information, but we quickly run out of degrees of freedom as we add more variables. In order for the exercise to make sense, it is necessary that the VAR system is a reasonably good reduced form of the economy. There are at least two issues here: the effect of excluding available data series and parameter instability (probably reflecting things like "policy regimes" for which there are no data series, and also non-linearities). First, the point estimates presented in the rest of the paper are not particularly sensitive to including more variables, like GNP and the oil price. Second, there *are* signs of instability

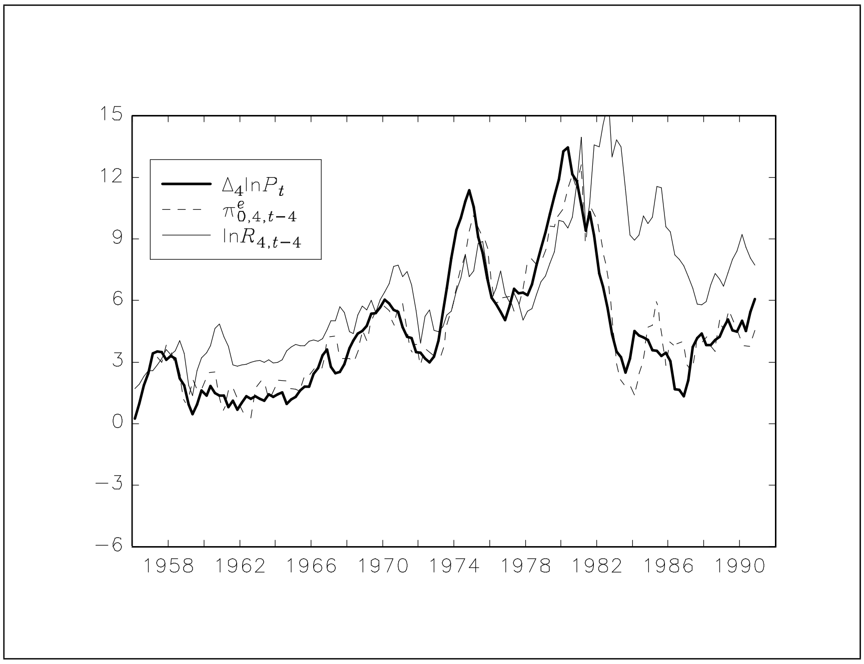

Figure 2: Inflation, interest rate, and expected inflationfrom VAR.

in the relation between the interest rates and the rest of the data, but not between the real data and inflation.[8] In particular, the events during the early 1980s seem atypical, but could heavily influence the results. This was probably a period with major shifts in the monetary policy; in 1979 the Fed signalled a tough anti-inflationary stance by switching to money stock as the only intermediate target; in 1982 interest rate targeting was resumed. It could perhaps be argued that the intervention policy has reverted to more traditional lines since then, and that this period should be excluded. In any case, I will report results for both the whole sample, but also discuss if they are sensitive to any specific periods.

The episode of high interest rates in 1984-1985 is also interesting, but for another reason: to demonstrate what the reduced form (VAR) should be able to capture. The VAR predicts

[8] This is reminiscent of the findings by Blanchard [4], who found that the regime shift in the early 1980s didn't affect the estimates of a Phillips curve, but did affect estimates of the yield curve. The VAR forecasts of inflation for the early 1980s are very similar to those of Sachs [30], and the out-of-sample forecasts by Gordon [18].

a rapid increase of the inflation rate for this period, as seen in Figure 2 (driven mainly by the sharp increases in real activity and money stock; we get the same peak if the interest rate used in the VAR forecast is set to a constant), which never materialized. One could perhaps argue that the inflation pressure was brought down just because the Fed hiked up the interest rates. A counter-argument is that the long rates increased considerably more than the short rates, which are more easily controlled by the Fed. In any case, provided that the VAR system is a reasonably good reduced form of the economy, the prediction error of the VAR should be due to *innovations* in price setting behavior, technology, Fed's behavior, etc. The point is that the VAR should capture any stable intervention rules of the Fed. It is possible to give clear labels to these innovations in only a few cases. For instance, the unexpected fall in the inflation rate in early 1986 was probably driven by the sudden and dramatic fall in oil prices in late 1985. However, a systematic classification of the innovations would require imposing a number of restriction in order to move from the reduced form (VAR) to a structural model, which is beyond the scope of this paper.

4.2 A Direct Attack on Forward Rates

This section studies the relation between the inflation predictions generated by the VAR ($\pi^e_{k-4,4,t}$), and data for forward rates ($f_{k-4,4,t}$) which were not part of the estimated VAR. The expected real interest rate is calculated as $f_{k-4,4,t} - \pi^e_{k-4,4,t}$. *Table 2* show the results for two different samples: the whole effective sample (1957:II-1990:IV for $k = 4$,1958:II-1990:IV for $k = 8$) and the sample with 1982:I-1985:IV excluded.[9] It shows implied projection coefficients (b_k), the relative standard deviation (σ_k) and the correlation (ρ_k) of the real interest rate and inflation expectations, the R^2 from using the b_k in the prediction rule, and the relative efficiency of the forward rate rule measured as MSE(b_k)/MSE(1).

The projection coefficients for the whole sample are 0.56 and 0.27 for the four- and eight-quarter horizons, respectively. Excluding 1982-1985 gives 0.98 and 0.60 instead. This is

[9] The sample for the levels start in 1955:I, so transforming into fourth differences and the generating inflation predictions implies that effective sample starts in 1957:II for the 4-quarters horizon and in 1958:II for the 8-quarters horizon.

Forecasting horizon (k) (quarters)	b_k	σ_k	ρ_k	R^2	$\frac{\text{MSE}(b_k)}{\text{MSE}(1)}$
4	0.56/0.98	0.93/0.59	-0.40/-0.57	0.35/0.65	0.75/1.00
8	0.27/0.60	1.32/0.90	-0.44/-0.47	0.11/0.34	0.51/0.81

Note: each cell shows results for whole sample/1982-1985 excluded.

Table 2: Results from forward rates and VAR predictions of inflation.

mainly driven by different estimates of the relative standard deviation (σ_k), while the estimates of the correlation (ρ_k) are less sensitive and consistently negative. The results are thus sensitive to the exclusion of the first years of the 1980s. Exclusion of any other period has only marginal effects.

Fama [14] also found a negative correlation between the real interest rate and inflation expectations when regressing nominal interest rates, inflation, and ex post real interest rates on the five-year yield spread. He argues that this is the reason why it is hard to predict nominal interest rates. Mishkin [25] study the relation between interest rates and future (actual) inflation rates, and conclude that interest rates contain information about inflation rates for periods between 6 and 12 months ahead (but not for shorter horizons). He estimates correlations to be below -0.5, and the projection coefficients to be between 0.7 and 1.5 (for the sample 1964-1979). Applying his method (regressing $\Delta_4 \ln P_t$ on $f_{k-4,4,t-k}$) on the present data set gives estimates of b_k which are very similar to those in Table 2. However, more information is needed in order to express the results in terms of the standard deviations and correlations of expected real interest rates and inflation expectations, which is precisely where the VAR system comes in.

The results in Table 2 are illustrated in *Figure 3*, which shows a scatter plot of the one-year interest rate ($\ln R_{4,t}$) and the inflation expectations for the same year ($\pi^e_{0,4,t}$), along with three regressions lines corresponding to the whole sample (middle), the sample with 1982-1985 excluded (upper curve, with the excluded data points marked by solid triangles), and the sample with 1979-1981 excluded (lower curve, with the excluded data points marked by solid crosses).

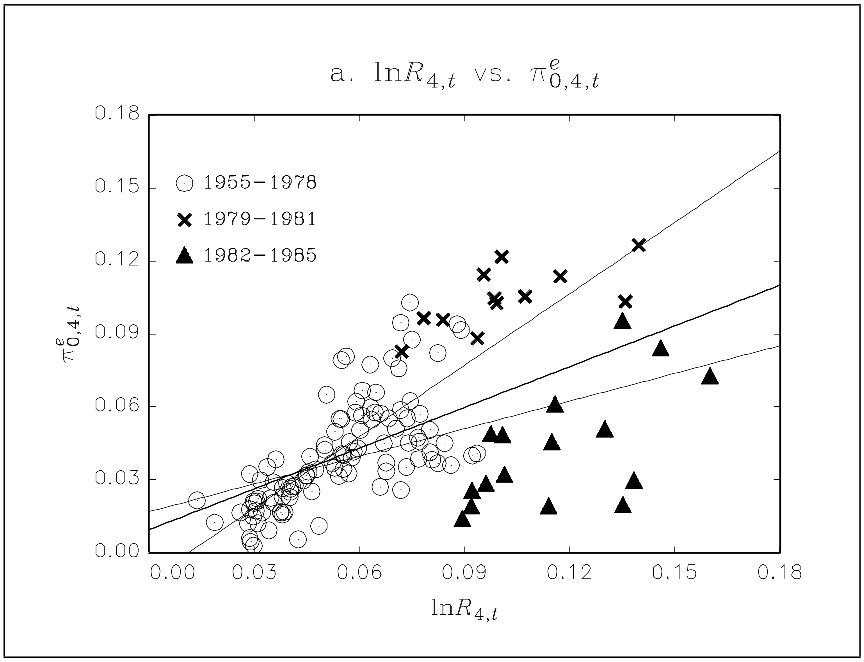

Figure 3: One-year interest rate vs. inflation predictions from VAR.

In terms of Figures 1.a-c, excluding the early 1980s entails a jump from the letter B to C: the real interest rate becomes less volatile and the correlation more negative. The lower relative volatility is also obvious from Figure 2, since excluding 1982-1985 shaves off the extreme real interest rates associated with the increasing nominal interest rates/falling inflation expectations in the aftermath of the "Volcker deflation." The result is a higher projection coefficient, a higher R^2 for the optimal rule, and a much higher relative efficiency of the forward rate rule.

The results in Table 2 suggest that the loss associated with using the forward rate are non-negligible for the whole sample, but negligible when the early 1980s are excluded. The sensitivity to excluding a few years means that no firm conclusion can be drawn, and that we may want to look at other pieces of evidence.

4.3 Results from Survey Data of U.S. Inflation Expectations

The Livingston survey of inflation expectations of some 50 business economists has been

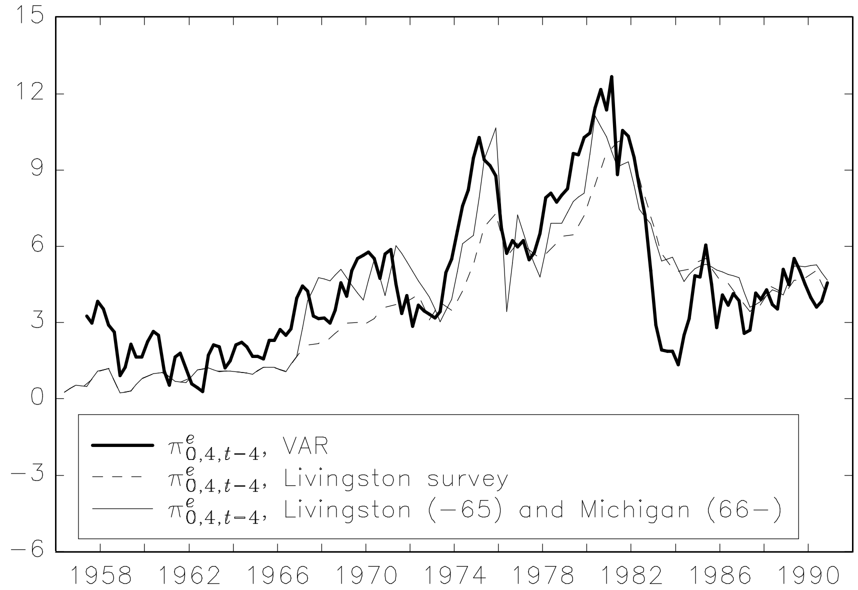

Figure 4: Comparison of inflation, and inflation expectations from surveys and VAR

conducted semi-annually (June and November) since 1946; the data used here is partly taken from Carlson [5] and partly from the Federal Reserve Bank of Richmond. The Michigan survey of approximately 500 randomly chosen persons has been conducted each quarter (and since 1978 each month) since 1946. The survey is based on qualitative questions (sign of price changes) up to the mid 1960s. To get a reasonably long quantitative series, the May and November figures of the Michigan survey for 1966-1990 are linked (without splicing) with the Livingston series for 1957-1965. These two series are shown in Figure 4 along with the inflation forecasts from the VAR.

The Michigan series and the VAR forecasts are rather similar, except for 1982-1984 when the Michigan survey is considerably higher than the VAR forecast. The Livingston series is also considerably higher than the VAR forecast for 1982-1984, but this is only one of many discrepancies: the Livingston series looks like a smooth moving average of the Michigan survey and the VAR forecasts.[10]

Survey	b_4	σ_4	ρ_4	R^2	$\frac{\text{MSE}(b_4)}{\text{MSE}(1)}$
Livingston (57-90):	0.73/0.84	0.62/0.51	-0.04/-0.17	0.71/0.77	0.74/0.89
Livingston (57-65) with					
Michigan (66-90):	0.71/0.86	0.72/0.60	-0.27/-0.39	0.57/0.66	0.82/0.95
VAR:	0.56/0.98	0.93/0.59	-0.40/-0.57	0.35/0.65	0.75/1.00

Note: whole sample/1982-1985 excluded.

Table 3: Results from the U.S. surveys of inflation expectations, 4 quarters.

The results for the survey series are shown in *Table 3*, and the results for the VAR predictions of inflation from Table 2 are reproduced on the last row. The real interest rate from the survey data are relatively less volatile than from the VAR, but at the same time also less negatively correlated with the inflation expectations. As a result, the optimal projection coefficients, which are essentially the same for the two surveys, are in the vicinity of the VAR results; b_4 is around 0.7 for the whole sample (0.56 for the VAR) and 0.85 when the early 1980s are excluded (0.98 for the VAR).

4.4 What Economists Should Expect of the Expected Real Interest Rate

This section shows what the asset pricing model in Section 2 would suggest about the relation between forward rates and inflation expectations. The idea is essentially to forget about the observed forward rates and instead look at the forward rates that are implied by VAR forecasts of consumption, leisure, and prices.

The estimated time series process of $\Delta_4 \ln C_t$, $\Delta_4 \ln (1 - H_t)$, and $\Delta_4 \ln P_t$ is combined with the asset pricing model (3)-(6), under the assumption of homoskedasticity (constant risk premia). *Figures 5.a-b* show 95% confidence intervals for the implied projection coefficients

[10] Darin and Hetzel [9] compare these survey series with some professional forecasts (DRI,Greenbook) for the period 1967-1993, and conlcude that these series are similar enough to be reliable indicators of actual inflation expectations. For a more general discussion about survey data, see for instance Kean and Runkle [22].

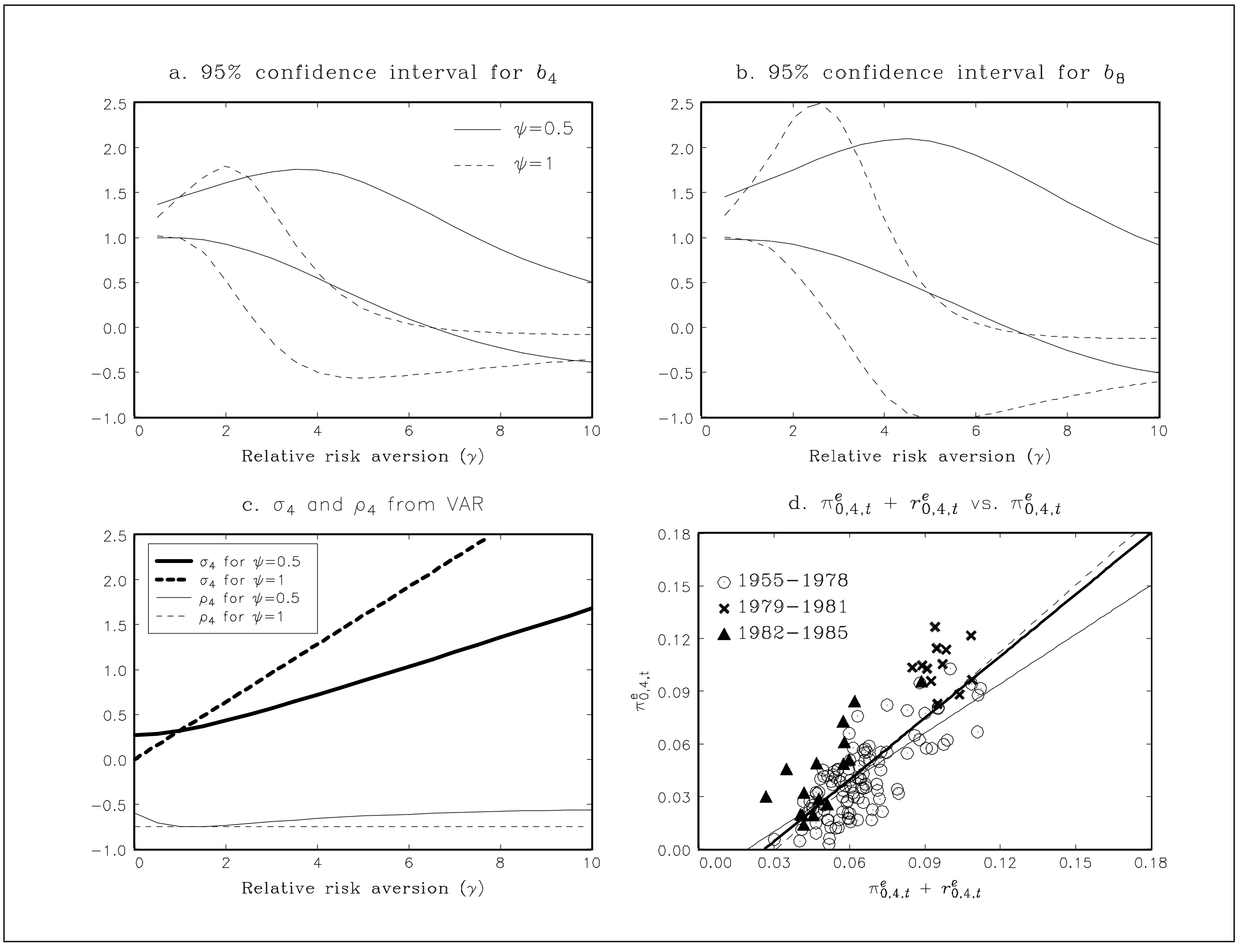

Figure 5: Results from asset pricing model.

(b_k). It shows results for values of the relative risk aversion (γ) between 0 and 10 (horizontal axis), by plotting curves corresponding to a relative weight (ψ) on consumption of a half (solid curves) and one (dashed curves).

The point estimates are mostly slightly above one for low and moderate values of the relative risk aversion. It takes $\gamma > 6$ to reject the hypothesis that $b_k = 1$ at the 5% level when $\psi = 0.5$, and $\gamma > 4$ when $\psi = 1$, but it should be admitted that the confidence intervals are fairly wide. The point estimates for the 12-quarters and 16-quarters horizons are fairly similar to b_8, but with much wider confidence intervals. It can be shown that the forward rate rule is often no more than 30%-50% worse (in terms of MSE) than the rational expectations, at least for low and moderate values of γ, and that the R^2 for the rational expectations rule is generally above 0.75 for values of $\gamma < 2$, but falls rapidly for higher values of γ, especially when the relative weight on consumption is large.

To develop an intuition for these results, the expected real interest rate from (4) and (5) is (disregarding constants)

$$r^e_{k-4,4,t} = [1 - \psi(1-\gamma)]\, \mathrm{E}_t\Delta_4 \ln C_{t+k} + (1-\psi)(\gamma-1)\, \mathrm{E}_t\Delta_4 \ln(1-H_{t+k}), \tag{9}$$

which is increasing in expected consumption growth, and decreasing (increasing) in expected growth in leisure when $\gamma < 1$ ($\gamma > 1$). The correlations and relative standard deviations of expected inflation and expected growth in consumption and leisure are shown in *Table 4*.

Forecasting horizon (k) (quarters)	Correlation with $\pi^e_{k-4,4,t}$		Standard deviation relative to standard deviation of $\pi^e_{k-4,4,t}$	
	$\mathrm{E}_t\Delta_4 \ln C_{t+k}$	$\mathrm{E}_t\Delta_4 \ln(1-H_{t+k})$	$\mathrm{E}_t\Delta_4 \ln C_{t+k}$	$\mathrm{E}_t\Delta_4 \ln(1-H_{t+k})$
4	-0.75	0.38	0.32	0.22
8	-0.86	0.49	0.27	0.19

Table 4: Time series properties from estimated VAR..

Table 4 implies that ρ_4 and ρ_8 must be negative for $\gamma < 1$, since $\mathrm{E}_t\Delta_4 \ln C_{t+k}$ is negatively correlated with $\pi^e_{k-4,4,t}$ and has always a positive coefficient, and $\mathrm{E}_t\Delta_4 \ln(1-H_{t+k})$ is positively correlated with $\pi^e_{k-4,4,t}$ but with a negative coefficient for $\gamma < 1$. ρ_4 and ρ_8 are likely to negative also for $\gamma > 1$, since $\mathrm{E}_t\Delta_4 \ln C_{t+k}$ varies much more than $\mathrm{E}_t\Delta_4 \ln(1-H_{t+k})$ and has a stronger correlation with $\pi^e_{k-4,4,t}$. Both series vary considerably less than expected inflation, so σ_4 and σ_8 are also likely to be less than or close to one, at least as long as $\gamma < 3$.

Values of σ_4 and ρ_4 are shown in *Figure 5.c*, which also uses the fact that the correlation of $\mathrm{E}_t\Delta_4 \ln C_{t+4}$ and $\mathrm{E}_t\Delta_4 \ln(1-H_{t+4})$ is -0.64. The correlation is between -0.75 and -0.5, and the relative standard deviation is almost linearly increasing in γ with a slope of 1/8 when $\psi = 0.5$ and 1/3 when $\psi = 1$. In terms of Figures 1.a-c, this leaves us close to the curves for $\rho_4 = -0.7$, and at $\gamma/8 < \sigma_4 < \gamma/3$. One such point, corresponding to $\gamma = 1$, is marked by the letter D in Figures 1.a-c. The basic insight from Figure 5.c is that all the results in Figures 5.a-b are driven by a scale factor: some linear combination of expected growth in consumption and leisure is scaled up a factor which drives all the results, while the exact weights in the linear combination is of less importance. The forward rate rule comes out reasonably well as long as this "scale factor" is not very large.

Which values of γ and ψ should we believe in, or how large is the scale factor? There is an extensive literature on this subject[11], but let us see which parameter values are consistent with the results from Section 4.2. For instance, suppose we want to match the $\sigma_k = 0.59$ and $\rho_4 = -0.57$ obtained when 1982-1985 is excluded; see Table 2. We see from Figure 5.c that we can match these moments closely by picking, for instance, $\gamma = 3.75$ and $\psi = 0.5$. These parameter values are used in *Figure 5.d*, which illustrates the importance of certain time periods for the results. Compared with Figure 3 the effect on the early 1980s is much smaller; the results from the asset pricing model are more stable than those using forward rates.

4.5 Exotic Utility Functions and Time-varying Risk Premia

The utility function (1) has been criticized for not being able to fit neither the equity premium nor the level of the risk free rate. Abel [1] suggested a utility function where average consumption in a previous period decreases utility of a representative consumer ("Catching up with the Jonses"). Adding this feature to (1) amounts to dividing C_t with $\bar{C}_{t-1}^{\delta}$, where $\bar{C}$ is average consumption of other consumers, and $\delta > 0$ measures the degree of "envy." It is straightforward to show that the log intertemporal rate of substitution in equilibrium (where $\bar{C}_t = C_t$) is as in (5) with the addition of the term $-\delta\psi\left(1-\gamma\right)\Delta_k \ln C_{t+k-1}$. The results are similar to those obtained in the standard model with $\delta = 0$, at least for low values of γ. For instance, for the 8 quarters horizon, $\delta = 0.5$, and $\psi = 1$, the values for $\gamma = \{0.5, 1, 3, 10\}$

[11] Campbell and Mankiw [7] and Mankiw, Rotemberg, and Summers [23] get very different results depending on instruments and the exact way the estimation is done and they reject the model. Hall [20] also gets quite different estimates, but argues in favor very low values of the intertemporal elasticity of subsitution. Hansen and Singleton [21] get values of γ slightly below one, and reject the fit of the model as the number of instruments are increased.

are $\{1.19, 1.26, 1.48, -0.38\}$ which should be compared with $\{1.13, 1.26, 1.15, -0.36\}$ for the standard model. The intuition for this result is that $\mathrm{E}_t \Delta_k \ln C_{t+k}$ and $\mathrm{E}_t \Delta_k \ln C_{t+k-1}$ are highly correlated, so the effect of the new term is essentially to adjust the coefficient of $\mathrm{E}_t \Delta_k \ln C_{t+k}$.

The utility function discussed by Epstein and Zin [12] is recursively defined as

$$U_t = \left\{ (1-\beta) C_t^{1-1/\nu} + \beta \left(\mathrm{E}_t U_{t+1}^{1-\gamma} \right)^{(1-1/\nu)/(1-\gamma)} \right\}^{1-1/\nu}, \tag{10}$$

where ν is the intertemporal elasticity of substitution. The arguments in Campbell [6] can be manipulated (see Appendix) to show that this utility function gives the same type of expression for the real interest rate as in (9) with $\psi = 1$ and $\gamma = 1/\nu$.

The assumption of homoskedastic shocks employed in the analysis thus far rules out movements in risk premia. According to (4) and (3), the expected real interest rate depends on the conditional variance of the log intertemporal rate of substitution, $-\frac{1}{2}\mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4}\right)$, and a nominal forward term premium $(\varphi_{k-4,4,t})$ rate and an inflation risk premium $(\theta_{k-4,4,t})$ enter the forward rate. The premia, given in Svensson [33] (see also Appendix), are[12]

$$\begin{aligned} \varphi_{k-4,4,t} &= -\tfrac{1}{2}\mathrm{Var}_t \left(\ln R_{4,t+k-4}\right) - \\ &\quad \mathrm{Cov}_t \left(\mu_{k-4,t} - \Delta_{k-4} \ln P_{t+k-4}, \mu_{4,t+k-4} - \Delta_4 \ln P_{t+k}\right), \text{ and} \\ \theta_{k-4,4,t} &= -\tfrac{1}{2}\mathrm{Var}_{t+k-4}\left(\Delta_4 \ln P_{t+k}\right) + \mathrm{Cov}_{t+k-4}\left(\mu_{4,t+k-4}, \Delta_4 \ln P_{t+k}\right). \end{aligned} \tag{11}$$

I use a particularly straightforward approach to calculate these risk premia. I estimate a time-varying covariance matrix by a rolling data window with ± 6 quarters over the residuals from the VAR estimation. These covariances are then treated as known by the agents, and used in (11) to calculate risk premia.

The implied nominal term premium is usually positive but small. For instance, even for a relative risk aversion (γ) of five it peaks at only 0.25%. Similarly, the inflation risk

[12]The intuition for the the inflation risk premium is that a positive covariance of the intertemporal rate of suvstition and inflation means that a nominal bond tend to have a low real return when consumption is already low, which requires a premium. For the intuition for the nominal (forward) term premia, note that the second term in the covariance operator is closely related to the future $\ln R_{4,t+k}$. Suppose this covariance is positive. Then, rolling over short bonds, instead of engaging in a forward contract, gives a low real return when consumption is already low; a forward rate is less risky and requires therefore a negative premium.

premium is mostly positive, but small: for $\gamma = 5$ it peaks at 0.3%. The intuition is that the prediction errors are generally small, and the squares tiny. Nor surprisingly, the effects on the projection coefficients (b_k) are almost trivial.[13]

5 Result from U.K. Indexed-linked Bonds

Index-linked bonds have been issued in U.K. since 1981, and been generally available on secondary markets since April 1982. These bonds have both coupons and principal linked to the Retail Price Index, so apart from some lags in the inflation compensation, they can essentially be regarded as a bond paying actual inflation plus a known real interest rate. Comparing prices of index-linked bond with prices of nominal bonds should make it relatively straightforward to infer the inflation expectations, as was done in, for instance, Woodward [34].

Bank of England, see Deacon and Derry [10], calculates implicit forward real interest rates and inflation expectations. I use their monthly series for the period 1982:04 to 1994:11. This is a short period, and U.K. both entered and (de facto) left the ERM, which may be regarded as "regime-shifts." The quality of the data can be expected to be poor for short horizons, since most of these bonds have had relatively long time to maturity; Bank of England recommends against using the results for shorter horizons than two years. Still, these data series are too interesting to leave without taking a look at them. The results for the for the investment periods [8,12] and [12,16] quarters ahead are shown in *Table 5*.

The estimated relative standard deviation is small, and the correlation is slightly negative. This makes the forward rate rule come out very well, and the loss in efficiency is less than 10%, as measured by the MSE.

[13]The asymptotic joint distribution of inflation expectations and forward rates cannot be normal in this case, so the forecasting rule (7)-(8) is not the mathematical expectation, but it still minimizes the MSE for the class of linear rules.

Forecasting horizon (k) (quarters)	b_k	σ_k	ρ_k	R^2	$\frac{\text{MSE}(b_k)}{\text{MSE}(1)}$
12	0.90	0.39	-0.11	0.86	0.91
16	0.93	0.34	-0.35	0.89	1.00

Table 5: Results from U.K. indexed-linked bonds.

6 Conclusions

The Fisher equation states that the nominal interest rate is the sum of expected inflation and a real interest rate. Recently, Svensson [32] suggested using forward rates as indicators of inflation expectations. The purpose of the present paper is to assess this suggestion.

The paper sets up a framework where inflation expectations and expected real interest rates have an asymptotic joint normal distribution, and asks the question: how does the optimal prediction rule look like and how much better is it than the forward rate? In theory, the answer depends on the relative volatility and the correlation of inflation expectations and expected real interest rates. These magnitudes are often not directly observable, and when they are (as with the U.K. index-linked bonds) available time series are short. I therefore try to encircle the answer by attacking from three different angles: comparing VAR forecasts of U.S. inflation and several different U.S. inflation expectations surveys,with both actual forward rates and implied forward rates from an asset pricing model over the period 1955 to 1990, and by studying implied real interest rates and inflation expectations from U.K. index-linked bonds over the period 1982 to 1994.

The main finding of is that the forward rate rule performs reasonably well compared with the optimal rule (rational expectation), in spite of the fact that expected real interest rates seem to vary quite a bit. The reason is that there are two forces counter balancing each other. First, as in a classical signal extraction problem, the volatility of the real interest rate (the "noise") would lead to a regression coefficient below one as some of the movements in the observed forward rate are attributed to movements in the real interest rate. Second, inflation expectations and expected real interest rates are negatively correlated, which tend to make the forward rate respond less than one-for-one to changes in inflation expectations.

7 Appendix: Derivations

7.1 The Nominal Interest Rate

The implicit gross annualized yield to maturity on a synthetic k quarter nominal bond $(R_{k,t})$

$$(R_{k,t})^{k/4} = \frac{1}{\mathrm{E}_t \frac{P_t}{P_{t+k}} \beta^k \frac{\partial U(C_{t+k}, H_{t+k})/\partial C_{t+k}}{\partial U(C_t, H_t)/\partial C_t}}. \tag{12}$$

Define the log real intertemporal rate of substitution (real discount factor or real pricing kernel) between t and $t+k$ as

$$\mu_{k,t} := \ln\left[\beta^k \frac{\partial U\left(C_{t+k}, H_{t+k}\right)/\partial C_{t+k}}{\partial U\left(C_t, H_t\right)/\partial C_t}\right], \tag{13}$$

which with the utility function (1) is

$$\mu_{k,t} = k \ln \beta + \left[\psi\left(1-\gamma\right) - 1\right] \Delta_k \ln C_{t+k} + \left(1-\psi\right)\left(1-\gamma\right) \Delta_k \ln\left(1 - H_{t+k}\right). \tag{14}$$

Using (13) in (12) gives

$$(R_{k,t})^{k/4} = \frac{1}{\mathrm{E}_t \exp\left(\mu_{k,t} - \Delta_k \ln P_{t+k}\right)}, \tag{15}$$

which is easily evaluated if we approximate the unknown conditional distribution of consumption, leisure, and prices with a lognormal distribution. The annualized nominal interest rate is

$$\ln R_{k,t} = -\frac{4}{k}\mathrm{E}_t \mu_{k,t} + \frac{4}{k}\mathrm{E}_t \Delta_k \ln P_{t+k} - \frac{4}{2k}\mathrm{Var}_t\left(\mu_{k,t} - \Delta_k \ln P_{t+k}\right), \tag{16}$$

and the expected annualized future nominal interest for an investment over the year $[t+k-4, t+k]$ is

$$\mathrm{E}_t \ln R_{4,t+k-4} = -\mathrm{E}_t \mu_{4,t+k-4} + \mathrm{E}_t \Delta_4 \ln P_{t+k} - \frac{1}{2}\mathrm{E}_t \mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4} - \Delta_4 \ln P_{t+k}\right). \tag{17}$$

7.2 The Real Interest Rate

A real bond gives a final payment of one unit of C_{t+k}. The annualized log real rate follows directly from (16) as

$$r_{k,t} = -\frac{4}{k}\mathrm{E}_t\mu_{k,t} - \frac{4}{2k}\mathrm{Var}_t\left(\mu_{k,t}\right), \tag{18}$$

and the expected annualized future real spot rate over the year $[t+k-4, t+k]$ is

$$\mathrm{E}_t r_{4,t+k-4} = -\mathrm{E}_t\mu_{4,t+k-4} - \frac{1}{2}\mathrm{E}_t\mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4}\right). \tag{19}$$

Note that the assumption of log normality at various dates implies that we assume that all variances are non-stochastic, so $\mathrm{E}_t\mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4}\right) = \mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4}\right)$. This simplified notation is used in the main text.

Example Let $\psi = 1$, then $\mu_{k,t} = k\ln\beta - \gamma\Delta_k \ln C_{t+k}$, so (16) and (18) become

$$\begin{aligned}
\ln R_{k,t} &= -4\ln\beta + \tfrac{4}{k}\gamma\mathrm{E}_t\Delta_k\ln C_{t+k} + \tfrac{4}{k}\mathrm{E}_t\Delta_k\ln P_{t+k} - \tfrac{4}{2k}\mathrm{Var}_t\left(-\gamma\Delta_k\ln C_{t+k} - \Delta_k\ln P_{t+k}\right), \\
r_{k,t} &= -4\ln\beta + \tfrac{4}{k}\gamma\mathrm{E}_t\Delta_k\ln C_{t+k} - \tfrac{4\gamma^2}{2k}\mathrm{Var}_t\left(\Delta_k\ln C_{t+k}\right).
\end{aligned}$$

7.3 The Nominal Forward Rate and the Nominal (forward) Term Premium

The annualized log forward rate is

$$f_{k-4,4,t} = \frac{k}{4}\ln R_{k,t} - \frac{k-4}{4}\ln R_{k-4,t}, \tag{20}$$

and from (16) and (20) we get the log forward rate with maturity of four quarters and settlement k quarters ahead

$$\begin{aligned}
f_{k-4,4,t} = {} & -\mathrm{E}_t\mu_{4,t+k-4} + \mathrm{E}_t\Delta_4\ln P_{t+k} - \tfrac{1}{2}\mathrm{Var}_t\left(\mu_{k,t} - \Delta_k\ln P_{t+k}\right) + \\
& \tfrac{1}{2}\mathrm{Var}_t\left(\mu_{k-4,t} - \Delta_{k-4}\ln P_{t+k-4}\right).
\end{aligned} \tag{21}$$

The nominal (forward) term premium for the period $[t+k-4, t+k]$, $\varphi_{k-4,4,t}$, is the difference between the nominal forward rate and the expected future nominal interest rate. It is obtained by combining (17) and (21)

$$\begin{aligned}
\varphi_{k-4,4,t} := f_{k-4,4,t} - \mathrm{E}_t\ln R_{4,t+k-4} = {} & -\tfrac{1}{2}\mathrm{Var}_t\left(\mu_{k,t} - \Delta_k\ln P_{t+k}\right) + \\
& \tfrac{1}{2}\mathrm{Var}_t\left(\mu_{k-4,t} - \Delta_{k-4}\ln P_{t+k-4}\right) + \\
& \tfrac{1}{2}\mathrm{E}_t\mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4} - \Delta_4\ln P_{t+k}\right).
\end{aligned} \tag{22}$$

This expression seems to differ from that in Svensson [33], but it is straightforward to show that they are actually identical. First, note that

$$\begin{aligned}\mathrm{Var}_t\left(\mu_{k,t}-\Delta_k \ln P_{t+k}\right) &= \mathrm{Var}_t\left(\mu_{k-4,t}-\Delta_{k-4}\ln P_{t+k-4}\right)+\mathrm{Var}_t\left(\mu_{4,t+k-4}-\Delta_4 \ln P_{t+k}\right)+ \\ & \quad 2\mathrm{Cov}_t\left(\mu_{k-4,t}-\Delta_{k-4}\ln P_{t+k-4},\mu_{4,t+k-4}-\Delta_4 \ln P_{t+k}\right).\end{aligned} \tag{23}$$

Second, note that (using x_{t+k} as short hand for $\mu_{k-4,t+k-4}-\Delta_{k-4}\ln P_{t+k}$)

$$\begin{aligned}\mathrm{E}_t\mathrm{Var}_{t+k-4}\left(x_{t+k}\right) &= \mathrm{E}_t\left(x_{t+k}\right)^2-\mathrm{E}_t\left[\mathrm{E}_{t+k-4}x_{t+k}\right]^2, \\ \mathrm{Var}_t\left(x_{t+k}\right) &= \mathrm{E}_t\left(x_{t+k}\right)^2-\left[\mathrm{E}_t x_{t+k}\right]^2.\end{aligned} \tag{24}$$

Third, and finally, note that

$$\mathrm{Var}_t\left(\ln R_{4,t+k-4}\right)=\mathrm{E}_t\left[\mathrm{E}_{t+k-4}\left(\mu_{4,t+k-4}-\Delta_4 \ln P_{t+k}\right)\right]^2-\left[\mathrm{E}_t\left(\mu_{4,t+k-4}-\Delta_4 \ln P_{t+k}\right)\right]^2. \tag{25}$$

Combining (22)-(25) gives

$$\begin{aligned}\varphi_{k-4,4,t} &= -\tfrac{1}{2}\mathrm{Var}_t\left(\ln R_{4,t+k-4}\right)- \\ & \quad \mathrm{Cov}_t\left(\mu_{k-4,t}-\Delta_{k-4}\ln P_{t+k-4},\mu_{4,t+k-4}-\Delta_4 \ln P_{t+k}\right),\end{aligned} \tag{26}$$

which is as in Svensson [33]. A constant nominal (forward) term premium is often called "the rational expectations hypothesis of the term structure of interest rates." Rejection of this hypothesis may, according to (26), be due to either irrationality or heteroskedasticity. The empirical evidence is mixed, see, for instance, Shiller [31].

7.4 Forward Inflation Risk Premium

The inflation risk premium is the expected real excess return of a nominal bond over a real bond. It is obtained by combining (17) and (19)

$$\begin{gathered}\theta_{k-4,4,t}:=\mathrm{E}_t \ln R_{4,t+k-4}-\mathrm{E}_t\Delta_4 \ln P_{t+k}-\mathrm{E}_t r_{4,t+k-4}= \\ -\tfrac{1}{2}\mathrm{E}_t\mathrm{Var}_{t+k-4}\left(\Delta_4 \ln P_{t+k}\right)+\mathrm{E}_t\mathrm{Cov}_{t+k-4}\left(\mu_{4,t+k-4},\Delta_4 \ln P_{t+k}\right)\end{gathered} \tag{27}$$

7.5 Accounting

Adding the expected real interest rate (19), the expected inflation, the nominal forward term premium (22), and the inflation risk premium (27) gives

$$\begin{aligned}
&\mathrm{E}_t r_{4,t+k-4} + \mathrm{E}_t \Delta_4 \ln P_{t+k} + \varphi_{k-4,4,t} + \theta_{k-4,4,t} = \\
&-\mathrm{E}_t \mu_{4,t+k-4} - \tfrac{1}{2}\mathrm{E}_t \mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4}\right) + \\
&\mathrm{E}_t \Delta_4 \ln P_{t+k} - \\
&\tfrac{1}{2}\mathrm{Var}_t\left(\mu_{k,t} - \Delta_k \ln P_{t+k}\right) + \tfrac{1}{2}\mathrm{Var}_t\left(\mu_{k-4,t} - \Delta_{k-4} \ln P_{t+k-4}\right) + \tfrac{1}{2}\mathrm{E}_t \mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4} - \Delta_4 \ln P_{t+k}\right) - \\
&\tfrac{1}{2}\mathrm{E}_t \mathrm{Var}_{t+k-4}\left(\Delta_4 \ln P_{t+k}\right) + \mathrm{E}_t \mathrm{Cov}_{t+k-4}\left(\mu_{4,t+k-4}, \Delta_4 \ln P_{t+k}\right).
\end{aligned} \tag{28}$$

This is equal to the forward rate in (21). To see this, note that

$$\begin{aligned}
&\tfrac{1}{2}\mathrm{E}_t \mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4}\right) - \tfrac{1}{2}\mathrm{E}_t \mathrm{Var}_{t+k-4}\left(\Delta_4 \ln P_{t+k}\right) + \mathrm{E}_t \mathrm{Cov}_{t+k-4}\left(\mu_{4,t+k-4}, \Delta_4 \ln P_{t+k}\right) = \\
&-\tfrac{1}{2}\mathrm{E}_t \mathrm{Var}_{t+k-4}\left(\mu_{4,t+k-4} - \Delta_4 \ln P_{t+k}\right).
\end{aligned} \tag{29}$$

7.6 The Nominal Interest Rate in Epstein-Zin

Equation (16) in Campbell [6]

$$\mathrm{E}_t \Delta \ln C_{t+1} = \text{constant} + \nu \mathrm{E}_t r^m_{t+1}, \tag{30}$$

can be used to substitute from the "market rate", r^m_{t+1}, in (17) to give

$$\mathrm{E}_t r^i_{t+1} = \text{constant} + \frac{1}{\nu}\mathrm{E}_t \Delta C_{t+1}, \tag{31}$$

for the return on any asset, r^i_{t+1}. Let r^i_{t+1} be the real holding return of a nominal bond, and recall that the average of the holding returns equal the yield to maturity. Disregarding constants, and applying the law of iterated expectations, we have

$$\mathrm{E}_t\left(\text{real yield to maturity}\right) = \frac{1}{k}\sum_{s=1}^{k} \mathrm{E}_t r^i_{t+s} = \frac{1}{\nu k}\sum_{s=1}^{k} \mathrm{E}_t \Delta C_{t+s} = \frac{1}{\nu k}\mathrm{E}_t \Delta_k C_{t+k}. \tag{32}$$

Multiply with 4 to convert from quarterly to annual rates, and note that

$$\mathrm{E}_t\left(\text{real yield to maturity}\right) * 4 = \ln R_{4,t} - \frac{4}{k}\mathrm{E}_t \Delta_k \ln P_{t+k}. \tag{33}$$

Combining (32) and (33) gives (16), which shows that, apart from constants, nominal bonds are priced in the same way in as with a time-separable utility function.

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