An Econometric Analysis of the Hungarian Sovereign Yield Curve

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Abstract:

This paper analyses the Hungarian sovereign yield curve via econometric methods. First I apply Principal Component Analysis (PCA) on my panel data consisting of zero coupon interest rates derived from government bond trading. This decomposition of the yield curve highlights important relationsips between identified factors and metrics of the term structure shape. As a second step, I implement a semi non-parametric model, as suggested by Gallant and Tauchen (1996). This way, one can understand governing processes of the sovereign yield curve without making arbitrary parametric assumptions. My empirical findings support statistical similarities between the Hungarian yield curve and, in the literature most often analysed, US term structure.

Keywords: term structure of interest rates, principal component analysis, semi non-parametric modelling

1   Introduction: who is interested in the evolution of the sovereign yield curve?

Government bonds represent claims for future cash flows, thus aggregate information regarding the time value of money. The government bond yield curve (also called the sovereign term structure, as one can construct a term structure from market instruments other than bonds) summarizes all available market expectations of a given time regarding the aforementioned time value of money, as perceived by investors. The idea is based on substitution: if a representative investor lends money to the treasury of the country, they charge a certain amount of interest as required compensation. The charged interest should compensate for several risks, the investor is running when being lent, including: inflation risk in the local currency (how quickly does the purchasing power of, let us say

100 Forints, erode over time), foreign exchange risk of the local currency (if a foreign investor wants to repatriate their investment, how much – sticking to our recent example – 100 Forints are going to be worth in their foreign currency), and sovereign credit risk (what is the likelihood that the chosen country will default on, or restructure their debt?).
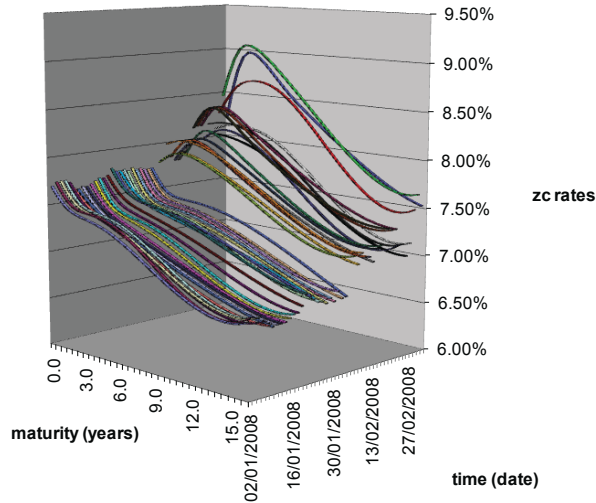

Figure 1

GDMA zero coupon term structure between 2 Jan and 3 Mar 2008, source: Government Debt Management Agency (GDMA)

These risks, or to be precise, their perception are often strongly fluctuating in the market, which adds volatility to the trading of these instruments (a volatile 2-month period of the Hungarian sovereign yield curve is depicted on Figure 1). A recent example might well be Greece, where investors suddenly changed their attitude towards the country, after long years of utmost patience. As the underlying macroeconomic situation hardly changed at all during investors' awakening, Greece's case is a prime example of how markets are driven by psychology[1]. Apart from crisis scenarios where no one seems to be exempt from money market repercussions, the extensive knowledge of the term structure is key in the following areas:

1.   forecasting future interest rates, decision making support for economic actors (investment decisions of corporates, saving decisions of individuals),

2.   monetary policy and its mechanisms,

3.   debt management of treasuries (e.g. maturity profile),

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1   One important point to note here is that, although there are often sudden changes in market perceptions, low and high volatile periods are natural in the history of markets. In fact efficient markets assume that price changes are driven by surprises (as everyting else had already been priced in before), and therefore occur in a sudden fashion per se. Having said that, these market swings are by all means compatible with statistical models relying on the assumption that there are no sudden changes in the data generating process.

4. pricing and hedging of interest rate derivatives (e.g. the value of both the most complicated interest rate derivatives and plain vanilla bonds (see: Arrow-Debreu prices) depend on interest rates).

The goal of this paper is to investigate Hungarian government bond term structure dynamics via econometric methods. After analysing the relationship between principal components of the sovereign yield curve and measures of its shape, I apply a semi non-parametric model, as suggested by Gallant and Tauchen (1996). This way, one can understand governing processes of the sovereign yield curve without making arbitrary parametric assumptions. My empirical findings support similarities in structural dynamics of the Hungarian yield curve and, in the literature most often analysed US term structure.

2   The data set and its Principal Component Analysis

For my empirical research I used a zero coupon sample of government bond data, collected on a daily basis between 1998 and 2008 by the Government Debt Management Agency (GDMA). It is important to note that zero coupon rates are not directly observable in the market, but are calculated from noisy data of secondary market trading of coupon bonds on the Budapest Stock Exchange. The GDMA uses a cubic spline fitting technique to have a smooth curve complying with no-arbitrage restrictions. My goal to keep the method to obtain zero coupon rates consistent over the whole sample was very well achieved here as the GDMA exhibited a high level of proficiency to generate their zero coupon curves. Recorded maturities are 2 week, 1 month, 3 month, 6 month, 9 month, 1 year, 2 year, 3 year, 4 year, 5 year, 6 year, 7 year, 8 year, 9 year and 10 year. Selected maturities are displayed on Figure 2.
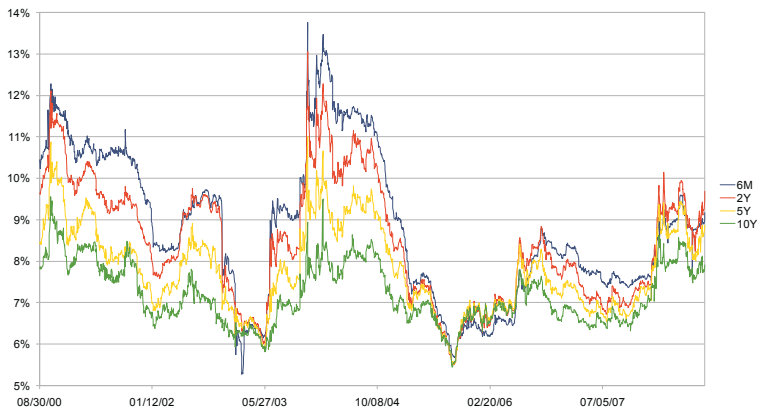

Figure 2

Time series of sample data (N=2007), source: Government Debt Management Agency (GDMA)

As the chart on Figure 2 shows, Hungarian interest rates have been volatile over the observation period. The bond market experienced several paradigm shifts, leading to a complete repricing of risks. The histogram of the 10 year tenor[2] (shown on Figure 3)

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2   I have chosen the 10 year tenor for its stability. Since longer maturities are the averages of

perfectly highlights this dual mode nature of the Hungarian market. The question whether the bipolar nature of Hungarian economic processes warrant bimodal yield levels remains open to be answered by a macroeconomic study. One thing is for sure, though: both fiscal and monetary policies should aim at reducing excess volatility from bond markets, as thus they reduce risk premia, leading to cheaper financing of the country's debt.

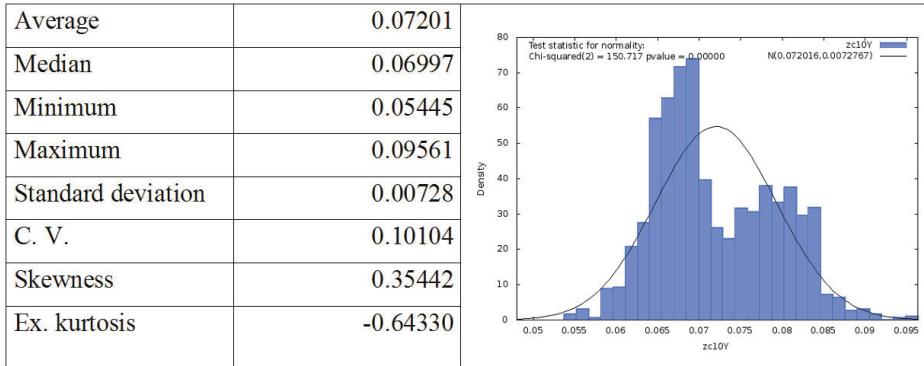|	
Average	0.07201
Median	0.06997
Minimum	0.05445
Maximum	0.09561
Standard deviation	0.00728
C. V.	0.10104
Skewness	0.35442
Ex. kurtosis	-0.64330

Figure 3

Descriptive statistics of the 10 year tenor, source: own calculations

For a better understanding of the Hungarian sovereign yield curve, I decomposed its structural dynamics. After the often cited paper of Litterman and Scheinkman (1991), it is common to assume that three factors, namely level, steepness and curvature drive the whole spectrum of the yield curve. Applying Principal Component Analysis (PCA) on the Hungarian data set revealed that the first three factors cumulatively explain 99.81% of yield levels' covariance; results for daily yield changes show 95.71% cumulative explained covariance for the first three principal components.

Principal components[1]	Cumulated explained covariance (yield levels)	Cumulated explained covariance (daily yield changes)
Factor 1	0.9470	0.6692
Factor 2	0.9933	0.8994
Factor 3	0.9981	0.9571
Factor 4	0.9994	0.9848
Factor 5	0.9998	0.9970

Table 1

PCA of yield levels and their first differences, source: own calculations

expected short maturities (this is due to forward interest rates and no-arbitrage rules), they are less influenced by "market noise".

Now we can conclude that 3 factors are sufficient to describe structural dynamics of the Hungarian sovereign yield curve. But how does these factors relate to yield curve metrics identified by Litterman and Scheinkman (1991)? A good way to investigate this is to compare the second and third principal components obtained in the PCA study with yield curve metrics suggested by Litterman and Scheinkman, namely steepness and curvature. The market standard (i.e. market participants use these proxies the most) defines yield curve steepness by the difference between the 10 year and 2 year tenors, and the curvature as the 2/5/10 year butterfly structure (a duration-neutral[3] structure which consists of a borrowed position at the belly, and lent position at the wings). I applied the market practice with regards to the yield curve steepness, but slightly amended the curvature's definition (I took the 6 year tenor instead of the 5 year one) to get more symmetrical weights for the duration-neutral structure. This choice is underlined by the fact, that in zero coupon space the 6 year point and the equally weighted portfolio consisting of the 2 year and 10 year tenors have identical duration.

Figures 4 and 5 plot the yield curve steepness with the second, and yield curve curvature with the third principal component. Co-movements are particularly visible in case of the curvature.
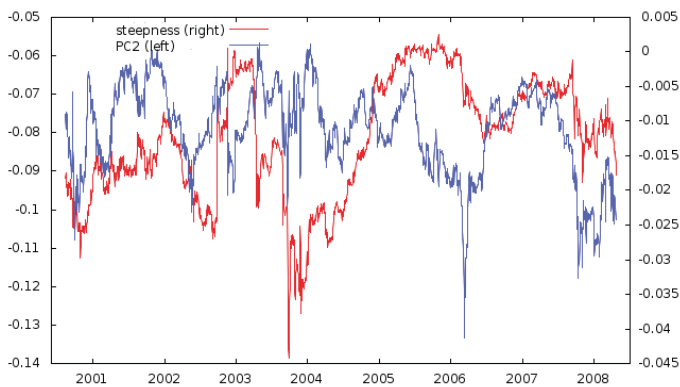

Figure 4

Yield curve steepness and the 2nd principal component, source: own calculations

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3   Duration is the measure of interest rate risk, i.e. shows how much an investor will be exposed to the risk of a 1 basispoint (1/100 of a %p) change in the yield-to-maturity of the selected instrument.
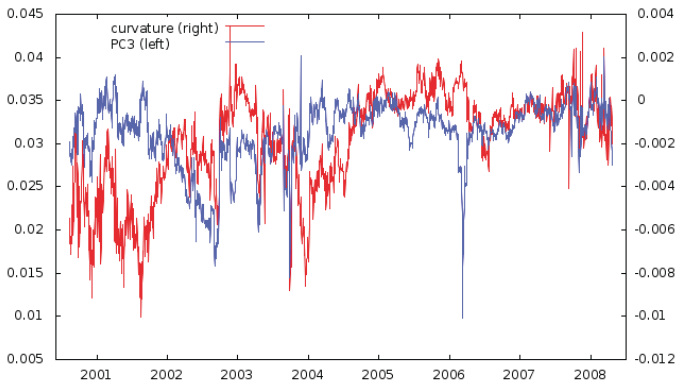
Figure 5

Yield curve curvature and the 3rd principal component, source: own calculations

3   Semi non-parametric model calibration

Following the advice of Gallant and Tauchen (1996) I calibrated a semi non-parametric (SNP) model to gain better understanding of structural dynamics of the Hungarian sovereign yield curve without making parametric assumptions. The lack of a-priori model selection is an appealing feature of SNP modelling, as a) there is no universal interest rate model[4], and b) the researcher does not have to make an arbitrary choice. Instead, the SNP employs an expansion in Hermite functions to approximate the conditional density of a multivariate process. As a nonlinear non-parametric model, it directly nests a wide range of models from the Gaussian VAR model to several different semiparametric ARCH and GARCH models. The unrestricted SNP expansion is more general than any of these underlying models. The SNP model is fitted with conventional maximum likelihood together with a model selection strategy that determines the appropriate order of expansion.

After Gallant and Tauchen's recommended order of expansion, I started with plain VAR models (with lags 1, 2, 3 and 4), first these have been calibrated to the sample. Going further I continuously extended parameterisation of the SNP model (e.g. with ARCH and GARCH processes) and judged their significance by using two information criteria (AIC and BIC). Optimisations have been carried out with a C++ program by Gallant and Tauchen (1996), using control runs to ensure robust results (i.e. not falling in the trap of a local minimum). The applied order of SNP expansion and estimation results are shown in Table 2.

Results showed that Hungarian government bond term structure dynamics are driven by a semi non-parametric GARCH process. The conditional variance of the SNP model is a VAR(1), GARCH(1,1) process, innovations are given by a 6th order polynomial with a

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4   Bookshelves groan under the number of parametric models proposed to fit term structure dynamics, the discussion of which lies beyond this paper. See Aït-Sahalia (1996), Brigo and Mercurio (2006) or Duffie and Singleton (1997) for further details.

time lag of 1. I investigated if using polynomials as coefficients of the innovation polynomial or introducing asymmetric volatility (also called leverage effect in literature) into the model improve SNP model fit; but I did not get a confirmation in any of the two cases. SNP model calibration has been carried out for the tenors 6 month, 2 year, 5 year and 10 year in a pure time series approach, and for all these maturities combined in a panel approach. The four individual pure time series approaches and the panel data approach led to the same results and conclusions.

VAR(lag)	GARCH(autoregressive part)	GARCH(moving average part)	Lag of Hermite expansion	Degree of Hermite polynom	Degree of coefficients of Hermite polynom	BIC (* denotes best so far)
1	0	0	1	0	0	1.41687
2	0	0	1	0	0	1.41622*
3	0	0	1	0	0	1.41764
4	0	0	1	0	0	1.41930
2	0	1	1	0	0	1.25810
2	0	2	1	0	0	1.20764
2	0	3	1	0	0	1.17220
2	0	4	1	0	0	1.16947*
2	0	4	1	4	0	1.03674
2	0	4	1	5	0	1.03470
2	0	4	1	6	0	1.00986*
2	0	4	1	6	1	1.01168
2	0	4	1	6	2	1.01340
1	1	1	1	0	0	1.15599
1	1	1	1	6	0	0.99975*
1	1	1	1	6	2	1.00047

Table 2

SNP expansion order and results, source: own calculations

Figures 6 and 7 display SNP model fit in the Hungarian zero coupon sample, with data on the 6-month tenor being plotted.
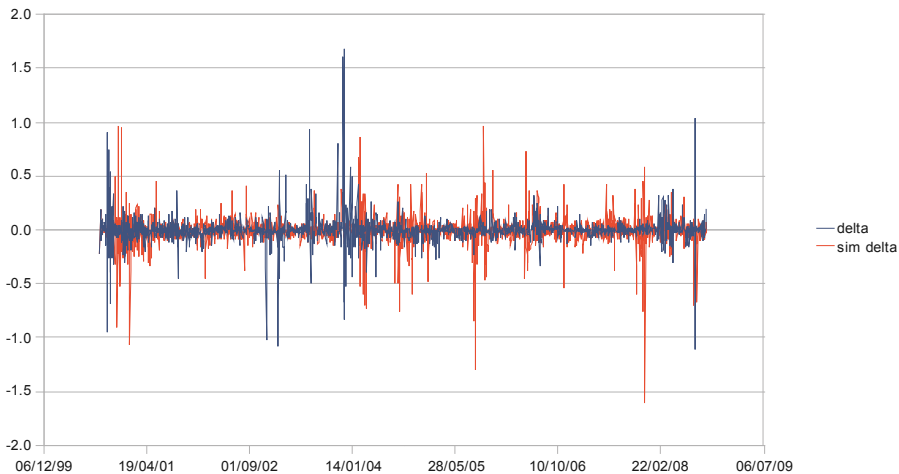

Figure 6

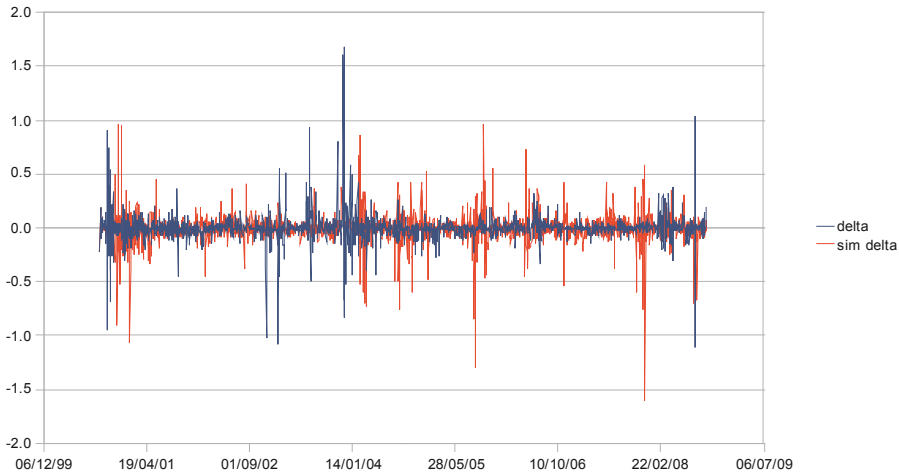SNP model fit as applied for daily yield changes, source: own calculations

Figure 7

SNP model fit as applied for daily yield levels, source: own calculations

My results for the Hungarian market are easily comparable with a study by Dai and Singleton (2000) referring to US markets. Authors there found the best score for a VAR(1), GARCH(1,2) SNP model, with innovations represented by a 4th order polynomial with a time lag of 1. This leads to the conclusion that, from a purely statistical perspective, the American and Hungarian yield curves are to a high degree similar, despite the fact that a) the Hungarian term structure had been an inverted one throughout the entire observation period, and b) Hungary is an emerging market, not like the US which is the most important core market of the world.

4   Concluding thoughts

Genuine understanding and modelling the sovereign yield curve is of utmost importance for a wide range of economic actors, including corporates, banks and sovereign treasuries. In crisis scenarios, like nowadays in peripheral Europe, bond market swings affect lives of the whole population.

What sort of model should one construct to fit yield curve dynamics well? One can find dozens of suggested parametrisation forms in literature, but the choice amongst them leads the researcher to an immense dilemma. Semi non-parametric models, applied in this paper, require no a-priori model selection, but the selected parametrisation is result of the estimation procedure. My empirical calculations showed that the Hungarian yield curve is driven by a semi non-parametric GARCH process with no leverage effect. These results are confirmed in several pure time series runs and the panel data approach, which ulitises the whole data set.

Comparing my results with available empirical evidence in literature, I conclude that the Hungarian government bond yield curve and the US sovereign bond market share similar statistical patterns.

Acknowledgements

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