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Misleading Advertising in Duopoly

Keisuke Hattori
Faculty of Economics, Osaka University of Economics

Keisaku Higashida
School of Economics, Kwansei Gakuin University

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SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

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Keisuke Hattori† and Keisaku Higashida‡

Abstract

In this paper, we build a model of strategic misleading advertising in duopolistic markets with horizontal product differentiation and advertising externality between firms. We investigate the effects of regulating misinformation on market competition, behavior of firms, and social welfare. We show that the degree of advertising externality and the magnitude of advertising costs are crucial for determining the welfare effects of several regulations, including prohibiting misleading advertising, educating consumers, taxing production, and taxing misleading advertising. We then extend the model by introducing two types of heterogeneities; heterogeneous consumers and heterogeneous production costs between firms.

Keywords Misleading Advertising; Regulation; Duopoly; Product Differentiation; Advertising Externality;

JEL Code L13, L15, M37

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†Faculty of Economics, Osaka University of Economics, 2-2-8, Osumi, Higashiyodogawa-ku, Osaka 533-8533, Japan. Email: *hattori@osaka-ue.ac.jp*

‡School of Economics, Kwansei Gakuin University, 1-155, Ichiban-cho, Uegahara, Nishinomiya, Hyogo 662-8501, Japan. Email: *keisaku@kwansei.ac.jp*

1 Introduction

In many developed countries, governments regulate misleading or false advertising and encourage the provision of sufficient information to allow consumers to make informed choices. For example, the U.S. Food and Drug Administration (FDA) is responsible for advancing public health by helping the public receive accurate, scientifically based information they need to consume medicines and food to maintain and improve their health.[1] The Canadian Competition Bureau aims at ensuring veracity in advertising and prohibits any deceptive representations for the purpose of promoting a product or a business interest.[2] In Japan, the Consumer Affairs Agency, officially established in 2009, promotes awareness of product quality to protect consumer benefits.[3] Directive 2006/114/EC of European legislation directly regulates and controls misleading and comparative advertising in the interests of consumers, competitors, and the general public.[4]

Nevertheless, misleading advertising is ubiquitous. It can be observed on TV, newspaper, and other media because there exist loopholes in regulations.[5] Firms engage in misleading advertising to make consumers believe that the quality of their products is higher than they truly are. For example, some menu photographs of fast food products look better than the actual products. Another example includes vague description of the use of food additives, chemicals in children's toys, and dubious promises of travel services and accommodations.[6]

When misleading advertising makes consumers buy products that they would not have bought without its influence, it seems reasonable that the government should strictly prohibit such advertising. However, when a product market is imperfectly competitive, there is a problem of under-consumption of products, which implies that a positive amount of misinformation may increase consumption and improve welfare (Glaeser and Ujhelyi (2010)). Therefore, the welfare effects of some regulatory policies (e.g., prohibiting misleading advertising, educating consumers, taxing advertising or production, and prohibiting cooperative advertising between firms) may depend on the degree of product market competition as well as on the magnitude of advertising costs. How should the government regulate misleading advertising? Under what conditions would these regulatory policies increase welfare? We attempt to address these questions in our study.

We build a model of strategic misleading advertising in duopolistic markets with hor-

[1] See http://www.fda.gov/AboutFDA/WhatWeDo/default.htm.

[2] See http://www.competitionbureau.gc.ca/eic/site/cb-bc.nsf/eng/02776.html.

[3] See http://www.caa.go.jp/en/index.html.

[4] Directive 2006/114/EC defines misleading advertising as "advertising which, potentially or actually, misleads or affects the judgment of the consumer or which, for these reasons, is detrimental to a competitor." For further detail, visit the website of this directive (http://europa.eu/legislation summaries/consumers/consumer information/l32010 en.htm).

[5] Byrd-Bredbenner and Grasso (2001) investigated the effect of food advertising policy on televised nutrient content claims and health claims of the FTC, which was implemented in 1994. Hansen and Law (2008) analyzed the effect of truth-in-advertising regulation in the early twentieth century.

[6] Glaeser and Ujhelyi (2010) provides several other examples of misleading advertising.

izontal product differentiation and positive advertising externality between firms. Our model modifies and extends Glaeser and Ujhelyi (2010), which builds a model of Cournot oligopoly with homogenous goods. Specifically, we consider the following two-stage game: In the first stage of the game, each firm simultaneously chooses the amount of misleading advertising that will make its own products appear to be of higher quality than they really are.[7] The misleading advertising carried out by one firm is assumed to have positive external effects on the demand for its rival's products to a certain degree. In the second stage, firms compete in price (Bertrand) for *horizontally differentiated* products.[8] Consumers are assumed to be *naive* in the sense that they are not aware of the true benefits of consuming the products,[9] and are easily deceived by misleading advertising. For example, they may be misinformed about the effectiveness of some medicine or the health effects of eating fast food.

Within the above framework, we investigate the effects of various types of regulating misinformation on market competition, firm behavior, and social welfare to address the aforementioned issues. We show that the degree of product differentiation between firms and the magnitude of advertising costs (or level of consumer gullibility) play a crucial role in determining regulation policies. In particular, the policies for prohibiting misleading advertising, educating consumers, and taxing production are more likely to reduce welfare when the degree of product homogeneity is high and/or the magnitude of advertising cost is large. On the other hand, taxing misleading advertising and prohibiting cooperative advertising between firms necessarily improve welfare.

We then extend the basic model by including two types of heterogeneities. First, we consider a case of heterogeneous consumers, where naive and smart consumers coexist in society, and investigate the impacts caused by changes in naive consumers. Interestingly, we find that an increase in the proportion of naive consumers stimulates firms' misleading advertising and therefore reduces not only naive consumers' utility but also smart consumers' utility, although smart consumers are not at all deceived by misleading advertising. This is because the increase in misleading advertising raises product prices, and thus even smart consumers suffer a loss from price increases. In other words, government policy for educating naive consumers is also beneficial to smart consumers because it will reduce misleading advertising, which in turn lower the equilibrium price of a product. However, an increase in smart consumers does not necessarily benefit the society because it exacerbates the problem of under-provision of products associated with an oligopoly market.

Second, we incorporate the heterogeneous costs of production into the basic model. We find that a low-cost firm is likely to engage in misleading advertising more aggressively

[7]Quality investments are not considered in this study as they are considered to be relatively long-term decisions. Instead, we focus on misleading advertising strategies, which are short-term decisions, taken by wily entrepreneurs to deceive consumers.

[8]The results obtained in this study are also robust when we consider a quantity competition (Cournot) instead of Bertrand competition.

[9]For a comprehensive survey of the economic analyses of advertising, see Bagwell (2007).

than a high-cost firm. The difference in the amount of misleading advertising may lead to a seemingly paradoxical result of product misallocation: the price of low-cost firm's product is higher than that of high-cost firm. In addition, we find that social welfare may decrease with the dispersion of marginal production costs due to the misallocation of products.

The welfare effects of advertising and optimal regulatory policies have been extensively investigated by Nelson (1974), Dixit and Norman (1978), Kotowitz and Mathewson (1979), Becker and Murphy (1993), Glaeser and Ujhelyi (2010), among many others. Our study is complementary to the recent contribution by Glaeser and Ujhelyi (2010), which investigates the welfare effects of different regulatory responses to misinformation in a model of Cournot oligopoly with homogenous goods and misleading advertising. In their model, advertising is a pure public good among firms because they produce homogenous goods. Our current study deals with horizontal product differentiation, and thus the degree of positive externality in advertising among the firms is related to the degree of product differentiation. Our study also considers cost differentials and other types of policy responses, and finds significant results regarding the relationship between welfare effects of regulatory policies and the degree of horizontal product differentiation.

Our model structure is similar to that of Garella and Petrakis (2008) who analyzes the effects of minimum quality standards in oligopoly in a model of pure vertical differentiation, in which product qualities are enhanced by a firm's ex-ante investment. In our model, however, product qualities are made out to be enhanced (not really enhanced) by a firm's ex-ante advertising. This difference is crucial to evaluate the welfare consequences of firm behaviors.

The remainder of the paper is organized as follows. Section 2 sets up the basic model and characterizes the subgame-perfect Nash equilibrium. Section 3 examines several regulations: complete prohibition of misleading advertising, educating consumers, taxing misleading advertising, taxing production, and prohibiting cooperative advertising campaign. Section 4 extends the basic model by introducing two factors: heterogeneous consumers and heterogeneous production costs. Section 5 presets the conclusion.

2 The Model and its Equilibrium

Consider two firms who each produces a horizontally differentiated good and engages in advertising activities. Profits of firm i $(i = 1, 2)$ are $\pi_i = (p_i - c)x_i - ks_i^2$, where p_i represents the price of good i, c the (constant) marginal cost of the production, s_i the advertising investment made by firm i, and k the cost parameter of advertising. The advertising we consider is classified as *persuasive advertising* in that it may affect consumers' preferences by enhancing the product's value in the consumers' eyes. However, it differs from traditional advertising described in industrial organization literature in that it is *misleading advertising*, which deludes consumers into thinking that the quality of the advertised product is higher that it actually is. Therefore, we refer to s_i as the firm's

misleading advertising. In addition, the cost parameter k is also considered as the level of consumer gullibility, where small (large) k implies that consumers are easily (not easily) deceived by the firms' misleading advertising.

Following Singh and Vives (1984) and Garella and Petrakis (2008), consumers have identical tastes, summarized by the following utility function:

$$U(x_1, x_2) = (\alpha + e_1)x_1 + (\alpha + e_2)x_2 - (x_1^2 + x_2^2 + 2\gamma x_1 x_2)/2 + m,$$

where x_i represents the consumption of good i, α the true quality of the good i, e_i perceived incremental quality (i.e., misinformation) as a result of two firms' misleading advertisement, m the consumption of other composite goods (or money), and $\gamma \in [0, 1]$ the parameter indicating the degree of homogeneity (or substitutability) between the two goods. The misinformation regarding good i, e_i, is yielded from misleading advertising investment of firm i and firm j. We assume

$$e_i = e_i(s_i, s_j, \gamma),$$

where $\partial e_i/\partial s_i > 0$, $\partial e_i/\partial s_j \geq 0$, $\partial e_i/\partial s_j = 0$ when $\gamma = 0$, and $\partial e_i/\partial s_j = \partial e_i/\partial s_i$ for $\gamma = 1$. The last two assumptions implies that one firm's misleading advertisement is more likely to make its rival's goods look better as the degree of homogeneity between two goods becomes higher. When the two goods are perfect substitutes ($\gamma = 1$), the misleading advertisement is public good for both firms. Specifically, we assume a tractable form of the function:

$$e_i(s_i, s_j, \gamma) = s_i + \gamma s_j.$$

The budget constraint of consumers is $\sum_{i=1}^{2} p_i x_i + m = y$, where y is the exogenously given income of consumers. Then the utility maximization yields the following familiar demand functions:

$$x_i = \frac{\alpha}{1+\gamma} + \frac{e_i - p_i - \gamma(e_j - p_j)}{1-\gamma^2}, \quad \text{for } i = 1, 2,\ i \neq j. \tag{1}$$

Note that misinformation about good j partly decreases the demand for good i.

The timing of events is as follows: in the first stage, each firm decides on the amount of misleading advertising needed to deceive consumers and persuade them to buy more; in the second stage, firms compete a la Bertrand.

2.1 Equilibrium

We solve the model by backward induction. In the second stage, taking the other firm's price as given, each firm simultaneously sets its price to maximize its profits. Then, the second stage Nash equilibrium is characterized as follows:

$$p_i = \frac{(2+\gamma)[(1-\gamma)\alpha + c] + (2-\gamma^2)e_i - \gamma e_j}{4-\gamma^2},$$
$$x_i = \frac{(2+\gamma)(1-\gamma)(\alpha - c) + (2-\gamma^2)e_i - \gamma e_j}{(1-\gamma^2)(4-\gamma^2)}.$$

Thus, p_i and x_i are increasing in e_i but partly decreasing in e_j.

In the first stage, each firm simultaneously chooses its level of misleading advertising s_i, given the rival's choice. From the first-order condition of profit maximization, we have the following reaction function of firm i in choosing s_i against s_j:

$$s_i = R_i(s_j) \equiv s_0 + \frac{2\gamma(1-\gamma^2)}{k(4-\gamma^2)^2 - 4(1-\gamma^2)}\, s_j, \tag{2}$$

where $s_0 = \dfrac{2(1-\gamma)(2+\gamma)(\alpha-c)}{k(4-\gamma^2)^2 - 4(1-\gamma^2)}$. In order to ensure a unique and stable Nash equilibrium,[10] we assume the following:

Assumption 1 *For all* $\gamma \in [0,1]$, $k > \dfrac{2(1-\gamma^2)}{(2-\gamma)^2(2+\gamma)}$.

Using this assumption, we find that the intercept of the reaction function (2), s_0, is always positive, and that the misleading advertising investments are strategic complements.[11]

In the subgame-perfect Nash equilibrium, misleading advertising, misinformation, output, and price are as follows:

$$s_i^* = \frac{2(\alpha-c)(1-\gamma)}{\Delta}, \tag{3}$$

$$e_i^* = \frac{2(\alpha-c)(1-\gamma^2)}{\Delta}, \tag{4}$$

$$x_i^* = \frac{k(\alpha-c)(4-\gamma^2)}{(1+\gamma)\Delta}, \tag{5}$$

$$p_i^* = \frac{k(4-\gamma^2)[\alpha(1-\gamma)+c] - 2c(1-\gamma^2)}{\Delta}, \tag{6}$$

where $\Delta \equiv k(2-\gamma)^2(2+\gamma) - 2(1-\gamma^2) > 0$ from Assumption 1.

Result 1
The equilibrium misleading advertising s^* is monotonically decreasing in γ. The equilibrium misinformation e^* is inverted U-shaped maximized at $\gamma \approx 0.34$. Both s^* and e^* become zero when $\gamma = 1$.

Proof: Differentiating (3) with γ, we have

$$\begin{aligned}
\frac{ds^*}{d\gamma} &= -\frac{2(\alpha-c)\Big[k(2-\gamma)(2-\gamma+2\gamma^2) - 2(1-\gamma)^2\Big]}{\Delta^2} < 0, \\
\frac{de^*}{d\gamma} &= -\frac{2k(\alpha-c)(12\gamma - 4 - \gamma^2 - \gamma^4)}{\Delta^2} \gtreqless 0 \;\Leftrightarrow\; \gamma \lesseqgtr 0.34.
\end{aligned}$$

[10] The second-order condition is $-\frac{2(k(4-\gamma^2)^2-4(1-\gamma^2))}{(4-\gamma^2)^2}$, therefore $k > \frac{4(1-\gamma^2)}{(4-\gamma^2)^2}$ to ensure interior solutions of s_i for $i = 1, 2$. Furthermore, the slope of reaction function should be less than unity to assure the stability of Nash equilibrium, thus, $k > \frac{2(1-\gamma^2)}{(2-\gamma)^2(2+\gamma)}$, which is more restrictive to the former conditions.

[11] In detail, $k > 0.274$ is sufficient for the equilibrium to be unique and stable.

In addition, substituting $\gamma = 1$ into (3), we have $s^* = e^* = 0$. $\square$

An increase in the degree of homogeneity between two products (γ) affects the firms' incentives to engage in misleading advertising in two ways: it lowers the product prices through increased price competition, which reduces the return on advertising; it increases the incentives for free riding on the rival firm's misleading advertising through an increase in its spillover effects. Thus, the misleading advertising is maximized when $\gamma = 0$, which indicates that a monopolist engages in the greatest amount of misleading advertising.

Now, we derive the equilibrium profits, consumer surplus, and welfare. The equilibrium profits π^* can be obtained as:

$$\pi^* = \frac{k(\alpha - c)(1-\gamma)[k(4-\gamma^2)^2 - 4(1-\gamma^2)]}{(1+\gamma)\Delta^2}. \tag{7}$$

We distinguish between *ex-ante* and *ex-post* consumer surpluses. The ex-ante consumer surplus $\widehat{CS}$ is defined as:

$$\widehat{CS} = (\alpha + e_1^*)x_1^* + (\alpha + e_2^*)x_2^* - (x_1^{*2} + x_2^{*2} + 2\gamma x_1^* x_2^*)/2 + y - p_1^* x_1^* - p_2^* x_2^*. \tag{8}$$

Consumers can realize the true quality of the goods only after consuming them. Thus, ex-post consumer surplus CS is defined as:

$$CS^* = \alpha\, x_1^* + \alpha\, x_2^* - (x_1^{*2} + x_2^{*2} + 2\gamma x_1^* x_2^*)/2 + y - p_1^* x_1^* - p_2^* x_2^*. \tag{9}$$

Note that $CS^* = \widehat{CS} - (e_1^* x_1^* + e_2^* x_2^*)$, where the second term represents the consumers' disappointment regarding the quality of goods. Substituting (5) and (6) into (9), we obtain the ex-post consumer surplus in equilibrium:

$$CS^* = \frac{k(\alpha - c)^2(4-\gamma^2)[k(4-\gamma^2) - 4(1-\gamma^2)]}{(1+\gamma)\Delta^2} + y. \tag{10}$$

The consumer surplus should be measured based on the true quality of the product because the advertising is misleading. Therefore, we define social welfare as the sum of equilibrium profits and ex-post consumer surplus: $SW^* = CS^* + \sum_i^2 \pi_i^*$.[12] We obtain

$$SW^* = \frac{k(\alpha - c)^2\, \Phi}{(1+\gamma)\Delta^2} + y, \tag{11}$$

where

$$\Phi = k(4-\gamma^2)^2(3-2\gamma) - 4(1-\gamma^2)(6-2\gamma+\gamma^2).$$

2.2 The second-best misleading advertising

Here we investigate the socially optimal (the second-best) level of misleading advertising given the duopoly market structure. This can be obtained by maximizing the sum of

[12] We assume that the exogenously given income (y) is acquired from sources other than firms 1 and 2. Thus, $\sum_i^2 \pi_i^*$ is not included in y.

ex-post consumers surplus and firms' profits in the second-stage equilibrium with respect to s_1 and s_2. Then, we obtain the second-best amount of misleading advertising s^{SB} as

$$s^{SB} = \frac{(1-\gamma)(\alpha - c)}{2k(2-\gamma)^2 + (1+\gamma)} \geq 0. \tag{12}$$

Thus, we obtain the following results:

Result 2
The second-best misleading advertising s^{SB} is monotonically decreasing in γ. The second-best misinformation e^{SB} is inverted U-shaped maximized at $\gamma = \frac{10k+1-6\sqrt{k^2+k}}{8k-1}$. Both s^* and e^* become zero when $\gamma = 1$.

Proof: $ds^{SB}/d\gamma < 0$ is shown by differentiating (12) in γ:

$$\frac{ds^{SB}}{d\gamma} = -\frac{2(\alpha - c)[k\gamma(2-\gamma)+1]}{[2k(2-\gamma)^2 + (1+\gamma)]^2} < 0.$$

The effect of changes in γ on the second-best misinformation $e^{SB} = (1+\gamma)s^{SB}$ is:

$$\frac{de^{SB}}{d\gamma} = \frac{(\alpha - c)[4k(2-5\gamma+2\gamma^2) - (1+\gamma)^2]}{[2k(2-\gamma)^2 + (1+\gamma)]^2} \gtreqless 0 \;\Leftrightarrow\; \gamma \lesseqgtr \frac{10k+1-6\sqrt{k^2+k}}{8k-1}. \;\square$$

Result 3
$s^* > s^{SB} > 0$ holds for all $\gamma \in [0,1)$. $s^* = s^{SB} = 0$ for $\gamma = 1$.

Proof: Comparing (3) with (12) yields:

$$s^* - s^{SB} = \begin{cases} \dfrac{(\alpha - c)(1-\gamma)(2-\gamma)[k(2-\gamma)^2 + 2(1+\gamma)]}{[2k(2-\gamma)^2 + (1+\gamma)]\,\Delta} > 0 & \text{for } \gamma \in [0,1) \\ 0 & \text{for } \gamma = 1 \end{cases}$$

Thus, $s^* > s^{SB}$ necessarily holds for $\gamma \in [0,1)$. $\square$

Firms engage in inefficiently excessive misleading advertising from the social welfare point of view. Nevertheless, the second-best misleading advertising is positive. Although misinformation causes consumers disappointment and thus reduces ex-post consumer surplus, it mitigates the problem of under-provision of goods that results from oligopolistic competition. Misinformation increases consumers' willingness to pay, raises the prices of goods, and thus necessarily reduces ex-post consumer surplus. However, as long as the misinformation is smaller than the second-best level, it will increase the profit of firms and thereby override the negative effect of reducing consumer surplus.[13] Figure 1 illustrates the comparison of misleading advertising and misinformation between laissez-faire and the second-best equilibrium.

[13]Needless to say, if all profits accrue to consumers, a small amount of misinformation improves welfare without decreasing consumer surplus.

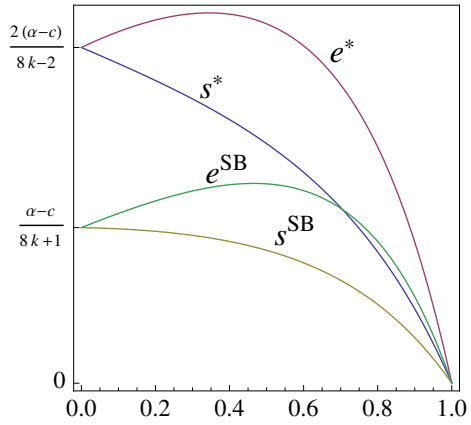

Figure 1: Misleading advertising and misinformation: Equilibrium and the second-best

3 Regulating Misleading Advertising

In this section, we consider different policies for regulating misleading advertising: (i) prohibiting misleading advertising; (ii) educating consumers; (iii) taxing misleading advertising; (iv) taxing production; and (v) prohibiting cooperative misleading advertising between firms.

3.1 Complete prohibition of misleading advertising

Here we investigate the welfare consequences of complete prohibition of misleading advertising. In reality, it is difficult for policymakers to directly assign firms the upper limit of misleading advertising to the second-best level due to its informational requirements. Therefore, prohibiting or banning any misleading advertising is a viable policy option for the government.

Under the policy, firms cannot engage in any misleading advertising, i.e., they set $s_i = 0$ for $i = 1, 2$. Then, equilibrium profits, consumer surplus, and welfare can be obtained by

$$\pi^{CP} = \frac{(1-\gamma)(\alpha-c)^2}{(1+\gamma)(2-\gamma)^2} \tag{13}$$

$$CS^{CP} = \frac{(\alpha-c)^2}{(1+\gamma)(2-\gamma)^2} + y \tag{14}$$

$$SW^{CP} = CS^{CP} + 2\pi^{CP} = \frac{(3-2\gamma)(\alpha-c)^2}{(1+\gamma)(2-\gamma)^2} + y, \tag{15}$$

where superscript CP refers to the case of complete prohibition.

Result 4

Compared to the laissez-faire case, the complete prohibition of misleading advertising

(a) reduces firms' profits,

(b) increases ex-post consumer surplus,

(c) enhances (reduces) social welfare when k and/or γ are smaller (larger). Formally,

$$SW^{CP} \gtreqless SW^* \quad \Leftrightarrow \quad k \lesseqgtr \zeta^{CP} \equiv \frac{(1+\gamma)(3-2\gamma)}{\gamma(2-\gamma)^2} \quad \text{for } \gamma \in [0,1).$$

Proof: From (7), (10), (11), (13), (14), and (15), we have

$$\begin{aligned}
\pi^{CP} - \pi^* &= -\frac{4(\alpha-c)^2(1+\gamma)(1-\gamma)^2[k(2-\gamma)^2-(1-\gamma)]}{(2-\gamma)^2\Delta^2} < 0, \\
CS^{CP} - CS^* &= \frac{4(\alpha-c)^2(1-\gamma)^2[k(2-\gamma)^2(2+\gamma)+(1+\gamma)]}{(2-\gamma)^2\Delta^2} > 0, \\
SW^{CP} - SW^* &= -\frac{4(\alpha-c)^2(1-\gamma)^2[k\gamma(2-\gamma)^2-(1+\gamma)(3-2\gamma)]}{(2-\gamma)^2\Delta^2} \gtreqless 0.
\end{aligned}$$

When $\gamma = 1$, the above three inequalities become zero. □

When the government prohibits any misleading advertising engaged by duopolists, consumers are better off and firms are worse off unambiguously. However, the welfare consequences of the policy depend on the value of k and γ. When consumers are easily deceived by misinformation (i.e., k is small) and/or the products are highly differentiated (i.e., γ is small), the difference between s^* and s^{SB} is large, and thus the complete prohibition policy works well. This is because the positive effect of the policy on ex-post consumer surplus dominates the negative effect on firms' profits. On the other hand, the degree of excessive advertising is mild when consumers are hardly deceived and/or the products are highly substitutable. In this case, the policy's social costs of exacerbating the under-provision of goods that results from oligopolistic competition outweigh its social benefits from the disappearance of consumers disappointment. Figure 2 depicts ζ^{CP} curve in $\gamma - k$ plane. The region above (below) ζ^{CP} curve indicates $SW^{CP} < SW^*$ ($SW^{CP} > SW^*$).

3.2 Educating consumers

We then investigate the effects of a policy for educating consumers. As discussed in the introduction, in many developed countries, governments and specialized agencies implement policies to spread accurate information about product qualities to educate consumers. The policy for educating consumers here is described as a marginal increase in k. In other words, the policy costs more for firms to deceive consumers.[14]

From (11), we have the following result:

Result 5

Educating consumers

[14]We assume that no implementation costs are attached to the policy here, but the implications of the results do not change qualitatively when considering such costs.

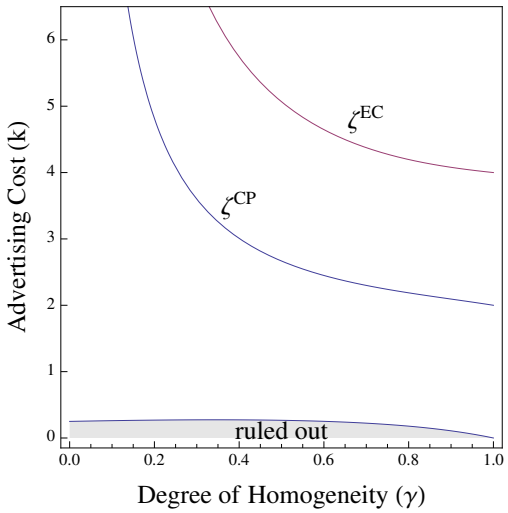

Figure 2: Welfare effect of complete prohibition and educating consumers

(a) reduces firms' profits,

(b) increases ex-post consumer surplus,

(c) enhances (reduces) social welfare when k and/or γ are smaller (larger). Formally,

$$\frac{dSW^*}{dk} \gtreqless 0 \;\Leftrightarrow k \lesseqgtr \zeta^{EC} \equiv \frac{2(1+\gamma)(6-2\gamma-\gamma^2)}{\gamma(2+\gamma)(2-\gamma)^2}.$$

Proof: Differentiating (7), (10), and (11) in k, we obtain

$$\begin{aligned}
\frac{\partial \pi^*}{\partial k} &= -\frac{4(\alpha-c)^2(1-\gamma)^2(1+\gamma)[k(2-\gamma)^2(2+\gamma)-2(1-\gamma)]}{\Delta^3} < 0, \\
\frac{\partial CS^*}{\partial k} &= \frac{4(\alpha-c)^2(1-\gamma)(4-\gamma^2)[k(4-\gamma^2)+2(1+\gamma)]}{\Delta^3} > 0, \\
\frac{\partial SW^*}{\partial k} &= -\frac{4(\alpha-c)^2(1-\gamma)^2\Big[k\gamma(2+\gamma)(2-\gamma)^2-2(1+\gamma)(6-2\gamma-\gamma^2)\Big]}{\Delta^3} \gtreqless 0.
\end{aligned}$$

The last inequality proves assertion (c). □

Two points should be emphasized here. First, the result of $\partial SW^*/\partial k > 0$ cannot be obtained if s_i would really enhance product qualities (e.g., quality investment like Garrella and Petrakis (2008)) or if s_i would be informative advertising (e.g., Kotowitz and Mathewson (1979)). On the other hand, in our model setting, the amount of misinformation is large in the case of lower k and/or γ, which indicates the positive effects of the policy on consumer surplus are substantial. This positive effects dominate the negative effects of increasing advertising costs and of reducing outputs. The situation is depicted by the region below ζ^{EC} curve in Figure 2. Second, the result of $\partial SW^*/\partial k < 0$ is significant

because $s^* > s^{SB}$ necessarily holds in equilibrium. A decrease in k spurs firms to misinform more, but the benefit of cost reduction dominates the loss of misinformed consumers when k and/or γ are large (initially small misinformation). The situation is depicted by the region above ζ^{EC} curve in Figure 2.

3.3 Taxing misleading advertising

We consider a direct taxation of misleading advertising. Following Glaeser and Ujehlyi (2010), it is modeled by the taxation of profits gross of advertising costs.[15] When advertising costs are not be deducted from the tax base, firms' profits after tax are $\bar{\pi}_i = (1-\tau_\pi)[(p_i - c)x_i] - k\,s_i^2$, where τ_π is the rate of profit tax. Tax revenues are rebated to consumers in a lump-sum form. Then, the welfare maximizing profit tax τ_π^{SB} becomes

$$\tau_\pi^{SB} = \frac{k(2-\gamma)^2 + 2(1+\gamma)}{4k(2-\gamma) + 2(1+\gamma)}. \tag{16}$$

Not surprisingly, each firm's misleading advertising under the tax scheme equals to the second-best level s^{SB}. Thus, the government can attain the second-best allocation by using the profit taxation that does not deduct the advertising costs from the tax base.

Result 6
The second-best tax τ_π^{SB} is

(a) decreasing in k,

(b) increasing (decreasing) in γ for small (large) k.

Proof: Differentiating (16) in k and γ yields

$$\frac{d\tau_\pi^{SB}}{dk} = -\frac{(1+\gamma)(4-\gamma^2)}{2[2k(2-\gamma)+(1+\gamma)]^2} < 0,$$
$$\frac{d\tau_\pi^{SB}}{d\gamma} = \frac{k[4+2\gamma+\gamma^2-2k(2-\gamma)^2]}{2[2k(2-\gamma)+(1+\gamma)]^2} \gtreqless 0 \;\Leftrightarrow\; k \lesseqgtr \frac{4+2\gamma+\gamma^2}{2(2-\gamma)^2},$$

where the first and second equations prove assertion (a) an (b), respectively. □

The assertion (a) is intuitive. The more the cost for firms to persuade consumers, the smaller their misleading advertising and the lower the tax to attain the second-best allocation. The assertion (b) is slightly difficult to understand. The amount of misleading advertising becomes small as γ becomes large (Result 1). Therefore, the optimal advertising tax seems to be decreasing in γ. However, the total output increases with the degree of substitutability between goods γ. Thus, the disutility from misinformation also increases with γ for the given s. When the advertising costs are rather small, the amounts of misinformation are large, so the disutility from the increase in outputs dominates the utility gain from the decrease in misleading advertising.

[15] As an example of this type of tax schemes, Glease-Ujehlyi (2010) quotes the example of Prescription Price Regulation Scheme (PPRS) in the U.K.

3.4 Taxing production

We investigate whether or not output taxation enhances welfare. The profit function of firm i is modified by $\pi_i = (p_i - c - t)x_i - k\, s_i^2$ for $i = 1, 2$. Then, the equilibrium amount of misleading advertising can be obtained by

$$s_i^{**} = \frac{2(\alpha - c - t)(1 - \gamma)}{\Delta},$$

which indicates that output taxation reduces firms' misleading advertising. If the tax revenues are rebated to consumers in a lump-sum form, then the welfare maximizing output tax is given by

$$t_x^* = \frac{(\alpha - c)(1 - \gamma)[2(1 + \gamma)(8 - \gamma(4 + \gamma)) - k(4 - \gamma^2)^2]}{8(1 - \gamma)^2(1 + \gamma) + k(4 - \gamma^2)^2} \gtreqless 0 \Leftrightarrow k \lesseqgtr \frac{2(1 + \gamma)[8 - \gamma(4 + \gamma)]}{(4 - \gamma^2)^2}.$$

The above inequalities indicate that the welfare maximizing rate of output tax is positive (negative) when the advertising costs are relatively small (large).[16] Thus, we have the following result.

Result 7
Taxing production improves (reduces) welfare if the advertising costs are relatively small (large).

Note that in the absence of misleading advertising activities, taxing production in oligopoly necessarily reduces welfare because it exacerbates the problem of under-provision of products. However, taking firms' misleading advertising into account, taxing production also reduces misleading advertising, which mitigates the problem of over-advertising. When the advertising costs are small, the problem of over-advertising is more severe than that of under-provision, and thus taxing production improves welfare.

3.5 Prohibiting cooperative misleading advertising

Here we consider whether or not cooperative misleading advertising between two firms is beneficial to society. We consider a *horizontal* cooperative advertising where the advertising is jointly projected by a group of firms that are horizontal competitors. For example, different hotels in the same vacation city cooperatively advertise to improve the public *image* of the city. We consider a case where firms cooperate in misleading advertising in the first stage, but they do not cooperate in price setting in the second stage. Sometimes a certain industry or a production district as a whole engages in cooperative advertising among many firms because advertising has positive externalities that enhance the image of goods in the same category but provided by other firms.

[16] The welfare maximizing output tax (or subsidy) certainly converges to zero as γ is close to 1, i.e., perfect competition case. For example, the critical value of k is 1 for $\gamma = 0$, 1.23 for $\gamma = 0.5$, 1.34 for $\gamma = 0.9$.

As a result of maximizing joint profits in the first stage, we obtain the following cooperative misleading advertising:

$$s^C = \frac{(1-\gamma)(\alpha - c)}{k(2-\gamma)^2 - (1-\gamma^2)}.$$

From (3), we have

$$s^C - s^* = \frac{k(\alpha - c)(2-\gamma)^2}{[k(2-\gamma)^2 - (1-\gamma^2)]\Delta}\Big[(1-\gamma)\gamma\Big],$$

which yields $s^C > s^*$ for $\gamma \in (0,1)$, and $s^C = s^*$ for $\gamma = 0$ or $\gamma = 1$. Thus, we have the following result for welfare evaluation of cooperative misleading advertising:

Result 8
Because $s^C > s^* > s^{SB}$ holds for $\gamma \in (0,1)$, the cooperative misleading advertising between firms necessarily increases their profits, reduces consumers surplus, and reduces social welfare.

The result implies that the government should regulate cooperative advertising campaigns of firms if the advertising is considered misleading.

4 Extensions

In this section, we extend the model by considering two factors: heterogeneous consumers and heterogeneous costs of production.

4.1 Heterogeneous consumers: naive and smart consumers coexist

We consider a case where a proportion $\delta \in [0,1]$ of consumers are naive as in the previous section, but $(1-\delta)$ of consumers are smart in the sense that they can identify the true quality of goods without being influenced by misleading advertising. We investigate the effects of changes in the proportion of naive consumers on firms' misleading advertising activities, prices of goods, and social welfare.

The aggregate demand for good i (the sum of the demand of naive and smart consumers) is

$$\bar{x}_i = \frac{\alpha}{1+\gamma} + \frac{\delta(e_i - \gamma e_j) - p_i + \gamma p_j}{1-\gamma^2}.$$

The case of $\delta = 1$ corresponds to (1). The subgame-perfect Nash equilibrium of this game is characterized by

$$\bar{s}^* = \frac{2\delta(\alpha - c)(1-\gamma)}{\Theta}, \tag{17}$$

$$
\begin{aligned}
\bar{x}_i^* &= \frac{k(\alpha-c)(4-\gamma^2)}{(1+\gamma)\Theta}, \\
\bar{x}_i^{sm} &= \frac{(\alpha-c)[k(4-\gamma^2)-2\delta^2(1-\gamma^2)]}{(1+\gamma)\Theta}, \\
\bar{x}_i^{na} &= \frac{(\alpha-c)[k(4-\gamma^2)-2\delta(1-\delta)(1-\gamma^2)]}{(1+\gamma)\Theta}, \\
\bar{p}_i^* &= \frac{k(\alpha(1-\gamma)+c)(4-\gamma^2)-2c\delta^2(1-\gamma^2)}{\Theta}, \qquad (18) \\
\overline{CS}^* &= \frac{(\alpha-c)^2[k^2(4-\gamma^2)^2-4k\delta^2(4-\gamma^2)(1-\gamma^2)-4\delta^3(1-\delta)(1-\gamma^2)^2]}{(1+\gamma)\Theta^2}+y, \\
\overline{CS}^{sm} &= \frac{(\alpha-c)^2[(k(4-\gamma^2)-2\delta^2(1-\gamma^2)]^2}{(1+\gamma)\Theta^2}+y, \\
\overline{CS}^{na} &= \frac{(\alpha-c)^2[k(4-\gamma^2)+2\delta(1-\delta)(1-\gamma^2)][k(4-\gamma^2)-2\delta(1+\delta)(1-\gamma^2)]}{(1+\gamma)\Theta^2}+y, \\
\bar{\pi}_i^* &= \frac{k(\alpha-c)^2(1-\gamma)[k(4-\gamma^2)^2-4\delta^2(1-\gamma^2)]}{(1+\gamma)\Theta^2}, \qquad (19) \\
\overline{SW}^* &= \frac{(\alpha-c)^2\Psi}{(1+\gamma)\Theta^2}+y, \qquad (20)
\end{aligned}
$$

where

$$
\begin{aligned}
&\Theta = k(2-\gamma)^2(2+\gamma)-2\delta^2(1-\gamma^2)>0, \\
&\Psi = k^2(3-2\gamma)(4-\gamma^2)^2-4\delta^2(1-\gamma^2)\Big[k(6-2\gamma-\gamma^2)+\delta(1-\delta)(1-\gamma^2)\Big].
\end{aligned}
$$

The superscripts sm and na respectively represent the equilibrium variables of smart and naive consumers. Note that both consumers surpluses are evaluated from an ex-post point of view. Certainly, $\delta\bar{x}_i^{na}+(1-\delta)\bar{x}_i^{sm}=\bar{x}_i^*$ and $\delta\overline{CS}^{na}+(1-\delta)\overline{CS}^{sm}=\overline{CS}^*$. In addition, the case of $\delta=1$ is equivalent to the model analyzed in the previous section.

We find from (17), that $\bar{s}^*=0$ for $\delta=0$, and $d\bar{s}^*/d\delta>0$, which indicates that firms' misleading advertising is increasing in the number of naive consumers. We also find the following result:

Result 9
An increase in the number of naive consumers necessarily raise equilibrium prices ($d\bar{p}_i^*/d\delta>0$). Thus, it harms smart consumers as well as naive consumers, but is beneficial to firms.

Proof: Differentiating (18) and (19) in δ respectively yields

$$
\begin{aligned}
\frac{d\bar{p}_i^*}{d\delta} &= \frac{4k\delta(\alpha-c)(4-\gamma^2)(1-\gamma)^2(1+\gamma)}{\Theta^2}>0, \\
\frac{d\bar{\pi}_i^*}{d\delta} &= \frac{8k\delta(\alpha-c)^2(1-\gamma)^2(1+\gamma)[k(2-\gamma)^2(2+\gamma)-2(1-\gamma)\delta^2]}{\Theta^3}>0.
\end{aligned}
$$

The price increases necessarily reduce surplus of naive and smart consumers. □

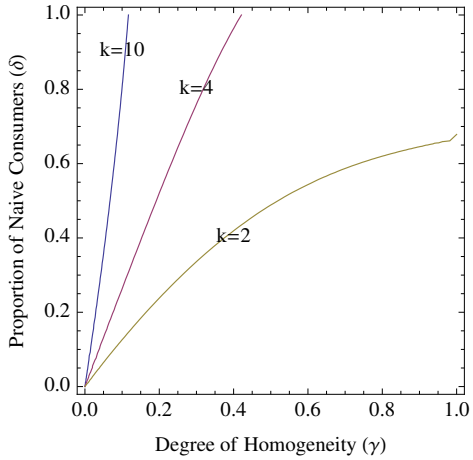

Figure 3: Welfare consequences of an increase in the number of naive consumers

An increase in the number of naive consumers makes firms engage in more misleading advertising, while causing price increases. An interesting point here is that an increase in the number of naive consumers affects the utility of smart consumers indirectly through price changes although smart consumers never fall for misleading advertising.

Then we examine the effects of a change in the number of naive consumers on social welfare. Differentiating (20) in δ yields

$$\frac{d\overline{SW}^*}{d\delta} = \frac{4\delta(\alpha - c)^2(1-\gamma)^2\Gamma}{\Theta^3} \gtreqless 0,$$

where

$$\begin{aligned} \Gamma \equiv\ & 2\gamma k^2(2+\gamma)(2-\gamma)^2 - 2\delta^3(1-\gamma)(1+\gamma)^2 \\ & -k\delta[3(1+\gamma)(2+\gamma)(2-\gamma)^2 - 4\delta(1-\gamma^2)(2-\gamma^2)] \gtreqless 0. \end{aligned} \tag{21}$$

The welfare consequence of a change in δ depends on the sign of Γ. The sign of Γ cannot be analytically solved easily. Therefore, we present the numerical results graphically in Figure 3.

In Figure 3, $d\overline{SW}^*/d\delta = 0$ curves for various value of k are depicted in $\gamma - \delta$ plane. The region above (below) each curve indicates $d\overline{SW}^*/d\delta < 0$ ($d\overline{SW}^*/d\delta > 0$). Thus, we have the following result.

Result 10

An increase in the number of naive consumers reduces (increases) social welfare when the proportion of naive consumers is large (small), the degree of product homogeneity is small (large), and the costs for misleading advertising are small (large).

The intuition is simple but revealing. When all consumers are naive ($\delta = 1$), the amount of misinformation is socially excessive (Result 3). On the other hand, when all consumers are smart ($\delta = 0$), the amount of misinformation becomes to zero and

is socially insufficient (Result 3). Thus, there exists a socially desirable proportion of naive consumers in this environment. Likewise, the amount of misinformation is more likely to be excessive when the products are more differentiated or when the cost of misleading advertising are smaller. The point to be emphasized here is that an increase in the number of smart consumers does not necessarily benefit the society because it reduces firms' misleading advertising and thus exacerbates the under-provision of goods associated with oligopoly outputs.

4.2 Cost heterogeneity

Here we extend the basic model by introducing the heterogeneity of marginal production costs c, and examine its effect on the amount of misleading advertising, prices of goods, and social welfare.

Suppose there are two firms l and h, where firm l (h) is the low-cost (high-cost) firm who has efficient (inefficient) production technologies. The profit functions of firms l and h are respectively defined as $\pi_l = [(p_l - (c - \epsilon)]x_l - k\, s_l^2$ and $\pi_h = [p_h - (c + \epsilon)]x_h - k\, s_h^2$, where $\epsilon \in [0, c)$ represents firm h's cost disadvantages against firm l. The model is similar to that in the previous section, except for the cost heterogeneity of firms.[17] Consumers are also assumed to be homogeneous and naive as in the basic model.

Then we obtain the difference between equilibrium misleading advertising of firms l and h:

$$s_l^* - s_h^* = \frac{2(2-\gamma^2)\epsilon}{k(2-\gamma)^2(2+\gamma) - (1-\gamma)(2-\gamma^2)} > 0.$$

Thus, we obtain the following result:

Result 11
Ceteris paribus, low-cost firm engages in misleading advertising more than high-cost firm.

The result implies that the smaller the marginal cost of production, the larger the marginal revenue of misleading advertising. Then, we compare the resulting prices of the two goods:

$$p_l^* - p_h^* = -\frac{2\epsilon(1-\gamma)[k(4-\gamma^2) - (2-\gamma^2)]}{k(2-\gamma)^2(2+\gamma) - (1-\gamma)(2-\gamma^2)} \gtreqless 0 \;\Leftrightarrow\; k \lesseqgtr \frac{2-\gamma^2}{4-\gamma^2}.$$

Thus, we have

Result 12
When the advertising costs are small (large), ceteris paribus, the price of a low-cost firm's good is higher (lower) than that of a high-cost firm's good.

[17] Note that even a small cost difference makes high-cost firm exit the market when the products are highly substitutable. We consider the cost heterogeneity only in the case of interior solution (i.e., the case where both $x_h^* > 0$ and $x_l^* > 0$ hold in equilibrium.

In the absence of misleading advertising, ceteris paribus, a low-cost firm sets the price lower than its high-cost rival. In that case, the relatively low price would help consumers understand that the firm has efficient production technologies. However, as shown in result 12, the price of low-cost firm's good is higher than that of high-cost firm's good when the costs of misleading advertising are relatively small. Thus, in this case, naive consumers cannot know whether the apparently high prices are attributed to the firm's inefficient production or to its relatively large misleading advertising.

Next, we investigate the effects of the dispersion of marginal costs on total outputs. The sum of equilibrium output of goods l and h is

$$x_l^* + x_h^* = \frac{2k(\alpha - c)(4 - \gamma^2)}{k(2-\gamma)(2+\gamma)^2 - (1+\gamma)(2-\gamma^2)},$$

which is independent of ϵ. Individual outputs (x_l and x_h) are affected by the dispersion of marginal costs, but total outputs are not. In fact, total outputs are a function of average efficiency, $c = \frac{1}{2}[(c-\epsilon) + (c+\epsilon)]$.[18] In addition, the sum of misinformation of goods l and h is

$$e_l^* + e_h^* = \frac{2(\alpha - c)(1+\gamma)(2-\gamma^2)}{k(2-\gamma)(2+\gamma)^2 - (1+\gamma)(2-\gamma^2)},$$

which is also independent of ϵ. Thus, the sum of outputs and misinformation of two goods are both independent of the cost variations between firm l and h.

Finally, we investigate the effects of cost dispersion on welfare.

$$\begin{aligned} \frac{dSW}{d\epsilon} &= \frac{2\epsilon\, k[k(3-\gamma)(4-\gamma^2)^2 - 2(2-\gamma^2)(6-4\gamma-2\gamma^2+\gamma^3)]}{[k(2-\gamma)^2(2+\gamma) - (1-\gamma)(2-\gamma^2)]^2} \gtreqless 0 \\ &\Leftrightarrow\ k \gtreqless \frac{2(2-\gamma^2)(6-4\gamma-2\gamma^2+\gamma^3)}{(3-\gamma)(4-\gamma^2)^2} \end{aligned}$$

Result 13

Social welfare increases (decreases) with the dispersion of marginal production costs when the advertising costs are relatively large (small).

In the absence of firms' misleading advertising activities (i.e., $s_l = s_h = 0$ in equilibrium), consumers surplus, producers surplus, and the social welfare increase with the dispersion of marginal costs.[19] This is because the cost variation allows the efficient allocation of goods by increasing the demand for low-cost firm's goods and by reducing the demand for high-cost firm's goods. However, in the presence of firms' misleading advertising, the demand for low-cost firm's goods becomes much greater because the misleading advertising of low-cost firm's products is larger than that of high-cost firm's products (Result 11). The effect becomes even stronger so that $p_l > p_h$ if k is small (Result 12). In that case, the more misleadingly advertised goods are allocated to consumers, leading to

[18] The result is generally known in I.O. literature. See Bergstrom and Varian (1985).

[19] This result is also shown by Salant and Shaffer (1999), Février and Linnemer (2004), and Jurgan (2009).

excessive consumer disappointment. Although the total outputs and misinformation are independent of cost dispersion, the misallocation effects reduce ex-post consumer surplus, and thus decrease welfare.

5 Conclusion

In this study, we examine the effects of regulatory policies on misleading advertising in duopolistic markets with horizontal product differentiation. One important feature of the model is positive advertising externality between firms, and the degree of externality correlating with the degree of product homogeneity. We show that the degree of substitutability between goods and the magnitude of advertising costs (or level of consumer gullibility) are crucial factors in determining the welfare consequences of regulation policies. The welfare effects of prohibiting misleading advertising, educating consumers, and taxing production with misleading advertising are not necessarily positive and depend on these factors. On the other hand, taxing misleading advertising and prohibiting cooperative advertising between firms necessarily improve welfare. The model is extended by including two types of heterogeneities: heterogeneous consumers and heterogeneous production costs between firms. We show that an increase in the number of naive consumers makes smart consumers worse off due to price changes, but it may improve social welfare. In addition, the welfare effects of the dispersion of production costs depend again on the magnitude of advertising costs (or level of consumer gullibility).

One conceivable extension of our analysis would be to investigate the effect of misleading advertising in a model of vertical product differentiation. The incentives for engaging in misleading advertising may be different between firms producing different quality levels. In this case, regulations on misleading advertising may produce results different from those found in this study. This awaits future investigation.

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