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ECONOMETRIC ESTIMATION OF DISTANCE FUNCTIONS AND ASSOCIATED MEASURES OF PRODUCTIVITY AND EFFICIENCY CHANGE[1]

by

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Abstract: The economically-relevant characteristics of multi-input multi-output production technologies can be represented using distance functions. The econometric approach to estimating these functions typically involves factoring out one of the outputs or inputs and estimating the resulting equation using maximum likelihood methods. A problem with this approach is that the outputs or inputs that are not factored out may be correlated with the composite error term. Fernandez, Koop and Steel (2000, p. 58) have developed a Bayesian solution to this so-called 'endogeneity' problem. O'Donnell (2007) has adapted the approach to the estimation of directional distance functions. This paper shows how the approach can be used to estimate Shephard (1953) distance functions and an associated index of total factor productivity (TFP) change. The TFP index is a new multiplicatively-complete index that satisfies most, if not all, economically-relevant tests and axioms from index number theory. The fact that it is multiplicatively-complete means it can be exhaustively decomposed into a measure of technical change and various measures of efficiency change. The decomposition can be implemented without the use of price data and without making any assumptions concerning either the optimising behaviour of firms or the degree of competition in product markets. The methodology is illustrated using state-level quantity data on U.S. agricultural inputs and outputs over the period 1960 to 2004. Results are summarised in terms of the characteristics (e.g., means) of estimated probability density functions for measures of TFP change, technical change and output-oriented measures of efficiency change.

KEYWORDS: Markov Chain Monte Carlo, Gibbs Sampler, total factor productivity, Bayes.

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1. INTRODUCTION

Improvements in productivity are a fundamental precondition for sustainable improvements in standards of living. Empirical analysis in this area often involves estimating the frontier of the production possibilities set. O'Donnell (2008; 2010a) shows how estimated production frontiers can be used to identify the main drivers of productivity change: a technical change component that measures movements in the production frontier, a technical efficiency change component that measures movements towards or away from the frontier, and scale and mix efficiency change components that measure productivity gains associated with economies of scale and scope. O'Donnell (2008; 2010a) shows how these components can be estimated without any restrictive assumptions concerning the structure of the technology, the degree of competition in input or output markets, or the optimizing behavior of firms – all that is required is an estimate of the production frontier.

Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) are the two main techniques available for estimating production frontiers. The idea behind DEA is to identify a surface that envelops the data points as closely as possible without violating any assumed properties of the production technology (e.g., convexity). The main advantages of DEA are that it does not require any explicit[2] assumptions concerning the functional form of the unknown production frontier, it does not require any explicit[3] assumptions concerning error terms, there are no statistical issues (esp. endogeneity) associated with estimating multiple-input multiple-output technologies, and fast computer packages are available for computing different measures of efficiency. The main weaknesses of DEA are that it does not allow for statistical noise and so cannot distinguish inefficiency from noise, it is difficult to compute elasticities of output response and associated economic quantities that involve partial derivatives (e.g., shadow prices), it is computationally difficult to obtain measures of reliability for efficiency scores, results may be sensitive to outliers, and technical efficiency estimates are upwardly biased in small samples. SFA is an alternative econometric methodology that involves the use of an arbitrary function to approximate the unknown production frontier. The main advantages of SFA are that it accommodates errors of approximation and other sources of statistical noise (e.g., measurement errors, omitted exogenous variables) and it is reasonably straightforward to conduct statistical inference (e.g., construct confidence intervals and test hypotheses). The main weaknesses of SFA are that results may sensitive to the choice of approximating functional form and associated assumptions concerning error distributions, and results may be unreliable if sample sizes are small. SFA estimation of primal representations of multiple-input multiple-output production technologies may also be complicated by the fact that the explanatory variables in the econometric model may be correlated with the error term. This problem is known as the 'endogeneity' problem.

Primal representations of multiple-input multiple-output production technologies include input and output distance functions. In the econometric approach to estimating these functions it is common to assume that either the inputs or the outputs are endogenous. Estimation then involves factoring out one of the endogenous variables and expressing the distance function in the form of a conventional stochastic frontier

[2] DEA implicitly assumes the production frontier is locally linear (e.g., O'Donnell 2010a).

[3] DEA implicitly assumes all error terms are zero.

model (e.g. Lovell et al. 1994). If the endogenous variables that are not factored out remain correlated with the error term then estimates of the parameters of the production frontier and associated measures of productivity and efficiency change will generally be biased and inconsistent.

A common solution to the endogeneity problem is to estimate the parameters of the model using the generalized method of moments (GMM) (e.g. Kopp and Mullahy 1990; Atkinson, Cornwell and Honerkamp 2003). GMM involves the arbitrary selection of instrumental variables that are uncorrelated with the error term. A problem with this approach is that GMM estimates are often sensitive to the choice of instruments, and the finite sample properties of the estimator are unknown. An alternative solution that does not involve the use of instruments has been suggested by Fernandez et al. (2000). This approach involves the specification of a system of equations in which the all but one of the dependent variables is unobserved. Bayesian methods are used to estimate the latent dependent variables and draw exact finite sample inferences concerning the parameters of the model and associated measures of efficiency. O'Donnell (2007) has adapted the approach to the estimation of directional output distance functions. This paper adapts the approach to the estimation of Shephard (1953) output distance functions and associated measures of productivity change.

The outline of the paper is as follows. Section 2 describes a multiple-input multiple-output production technology that satisfies a set of regularity conditions that are quite common in the productivity literature (e.g., monotonicity). Section 3 uses distance function representations of this technology to define a spatially- and temporally-transitive total factor productivity (TFP) index that satisfies important axioms and tests from index number theory (e.g., identity, transitivity). Section 4 shows how this index can be decomposed into various measures of technical change and efficiency change. Section 5 specifies an empirical output distance function and describes how the unknown parameters of the function can be estimated using the Bayesian methodology of Fernandez et al. (2000). Section 6 illustrates the methodology using a well-known panel of state-level data on outputs and inputs in U.S. agriculture. The paper is concluded in Section 7.

2. THE PRODUCTION TECHNOLOGY

I follow Fernandez et al. (2000) and assume the production technology available to firms in period t can be represented by the separable transformation function

(1) $$T^t(x,q) = g(q) - f^t(x) \leq 0$$

where $x = (x_1,...,x_K)' \in \mathbb{R}_+^K$ and $q = (q_1,...,q_J)' \in \mathbb{R}_+^J$ denote vectors of input and output quantities. Two alternative representations of this production technology are the Shephard (1953) output and input distance functions:

(2) $$D_O^t(x,q) = \min_{\delta}\{\delta > 0 : T^t(x, q/\delta) \leq 0\} \quad \text{and}$$

(3) $$D_I^t(x,q) = \max_{\rho}\{\rho > 0 : T^t(x/\rho, q) \leq 0\}.$$

The output distance function gives the inverse of the largest factor by which a firm can scale up its output vector while holding its input vector fixed. The input distance function gives the maximum factor by which a firm can scale down its input vector and still produce the same output vector. If the technology exhibits constant returns to scale then $D_O^t(x,q) = D_I^t(x,q)^{-1}$. Technically-feasible and efficient input-output combinations are defined by $T^t(x,q) = 0$ and $D_O^t(x,q) = D_I^t(x,q) = 1$. A local measure of returns to scale is the elasticity of scale (e.g., Krivonozhko and Forsund 2010, p. 160):

$$\eta(x,q,t) \equiv -\left(\sum_{k=1}^{K} \frac{\partial T^t(x,q)}{\partial x_k} x_k\right)\left(\sum_{j=1}^{J} \frac{\partial T^t(x,q)}{\partial q_j} q_j\right)^{-1}. \tag{4}$$

The technology exhibits (local) decreasing, constant or increasing returns to scale as the elasticity of scale is less than, equal to, or greater than one.

I assume the transformation function satisfies the following standard regularity conditions (e.g., Chambers 1988, p. 260-261):

T.1 non-decreasing in outputs: $T^t(x,q^1) \geq T^t(x,q^0)$ for $q^1 \geq q^0$.

T.2 non-increasing in inputs: $T^t(x^1,q) \leq T^t(x^0,q)$ for $x^1 \geq x^0$.

Associated properties of the output distance function are

O.1 nonincreasing in inputs: $D_O^t(x^1,q) \leq D_O^t(x^0,q)$ for $x^1 \geq x^0$.

O.2 nondecreasing in outputs: $D_O^t(x,q^1) \geq D_O^t(x,q^0)$ for $q^1 \geq q^0$.

O.3 linearly homogenous in outputs: $D_O^t(x,\lambda q) = \lambda D_O^t(x,q)$ for $\lambda > 0$.

It is convenient to let

$$\ln g(q) = \theta^{-1} \ln\left(\sum_{j=1}^{J} \alpha_j^{\theta} q_j^{\theta}\right) + v \quad \text{and} \tag{5}$$

$$\ln f^t(x) = \gamma_0 + \gamma_1 t + \sum_{k=1}^{K} \beta_k \ln x_k + \varepsilon \tag{6}$$

where ε and v are errors of approximation. Then the regularity properties O.1 to O.3 will be satisfied if

$$\theta > 1, \tag{7}$$

$$\alpha_j \in (0,1) \text{ for } j = 1,...,J, \tag{8}$$

$$\beta_k \geq 0 \text{ for } k = 1,...,K \text{ and} \tag{9}$$

$$\alpha' \iota_J = 1 \tag{10}$$

where $\alpha = (\alpha_1, ..., \alpha_J)'$ and ι_J is a $J \times 1$ unit vector. Equation (5) is a constant elasticity of substitution (CES) function with elasticity of transformation between any two outputs equal to $1/(1-\theta) < 0$. Equation (6) is a Cobb-Douglas (CD) function that allows for Hicks-neutral technical change. Equations (1), (5) and (6) can be used to write the logarithms of the output and input distance functions as[4]:

$$\text{(11)} \qquad \ln D_O^t(x,q) = \theta^{-1} \ln\left(\sum_{j=1}^{J} \alpha_j^\theta q_j^\theta\right) - \gamma_0 - \gamma_1 t - \sum_{k=1}^{K} \beta_k \ln x_k + v$$

$$\text{(12)} \qquad \ln D_I^t(x,q) = \eta^{-1}\left[\gamma_0 + \gamma_1 t + \sum_{k=1}^{K} \beta_k \ln x_k - \theta^{-1} \ln\left(\sum_{j=1}^{J} \alpha_j^\theta q_j^\theta\right) + \varepsilon\right]$$

where $\eta = \sum_k \beta_k$ is the elasticity of scale. This paper shows how estimates of the parameters of these distance functions can be used to estimate a spatially- and temporally-transitive index of productivity change.

3. A TRANSITIVE PRODUCTIVITY INDEX

It is convenient at this point to introduce a firm subscript i and a time subscript t into the notation and let $x_{it} = (x_{1it}, ..., x_{Kit})'$ and $q_{it} = (q_{1it}, ..., q_{Jit})'$ denote the input and output quantity vectors of firm i in period t $(i = 1, ..., N;\ t = 1, ..., T)$. In the aggregate price-quantity framework of O'Donnell (2008), the TFP of firm i in period t is $TFP_{it} = Q_{it} / X_{it}$ where $Q_{it} = Q(q_{it})$ is an aggregate output, $X_{it} = X(x_{it})$ is an aggregate input, and $Q(.)$ and $X(.)$ are non-negative, non-decreasing and linearly homogeneous aggregator functions. It follows that the index that compares the TFP of firm i in period t with the TFP of firm h in period s is

$$\text{(13)} \qquad TFP_{hs,it} \equiv \frac{TFP_{it}}{TFP_{hs}} = \frac{Q_{it} / X_{it}}{Q_{hs} / X_{hs}} = \frac{Q_{hs,it}}{X_{hs,it}}$$

where $Q_{hs,it} \equiv Q_{it} / Q_{hs}$ is an output quantity index and $X_{hs,it} \equiv X_{it} / X_{hs}$ is an input quantity index. Thus, within this framework, TFP change is a measure of output growth divided by a measure of input growth. TFP index numbers that can be written in the form of (13) are said to be multiplicatively-complete (O'Donnell, 2008).

Different multiplicatively-complete TFP indexes are obtained by choosing different (non-negative, non-decreasing and linearly homogeneous) aggregator functions. For example, Laspeyres TFP indexes are obtained by choosing price-weighted linear aggregator functions with reference firm/period prices as weights. Other

4 If $\delta = D_O(x,q)$ then $T(x, q/\delta) = \left(\sum_{j=1}^{J} \alpha_j^\theta \left(q_j/\delta\right)^\theta\right)^{1/\theta} e^v - f(x) = 0$ which can be solved for $\ln\delta = \theta^{-1}\ln\left(\sum_{j=1}^{J}\alpha_j^\theta q_j^\theta\right) - \ln f(x) + v.$ If $\rho = D_I(x,q)$ then $T(x/\rho, q) = g(q) - \exp\left(\gamma_0 + \gamma_1 t + \sum_{k=1}^{K}\beta_k\left[\ln x_k - \ln\rho\right]\right)e^\varepsilon = 0$ which can be solved for $\ln\rho = \eta^{-1}\left[\gamma_0 + \gamma_1 t + \sum_{k=1}^{K}\beta_k \ln x_k - \ln g(q) + \varepsilon\right].$

members of the class of multiplicatively-complete TFP indexes include Paasche, Laspeyres, Fisher, Tornquist, Lowe, Walsh and Hicks-Moorsteen indexes.

A fundamentally important property of multiplicatively-complete TFP indexes is that if the aggregator functions are fixed for all possible binary comparisons then the resulting TFP index satisfies a set of commonsense axioms and tests. Included among these are a transitivity test, which says that a direct comparison of the TFP of two firms should yield the same estimate of TFP change as an indirect comparison through a third firm (i.e., $TFP_{hs,it} = TFP_{hs,kr} \times TFP_{kr,it}$). Seemingly through force of habit, many applied economists (implicitly) use different aggregator functions from one binary comparison to the next, leading to sets of TFP index numbers that fail the transitivity test. In contrast, O'Donnell (2010b) computes temporally- and spatially-transitive Lowe TFP indexes using linear aggregator functions defined over fixed vectors of representative output and input prices. This paper supposes that such price vectors may be unavailable and instead computes transitive TFP indexes using period-*T* distance functions defined over fixed vectors of representative output and input quantities:

(14) $$Q(q_{it}) = D_O^T(x_0, q_{it}) \quad \text{and}$$

(15) $$X(x_{it}) = D_I^T(x_{it}, q_0)$$

where q_0 and x_0 are finite non-zero vectors. The associated output, input and TFP indexes are:

(16) $$Q_{hs,it} = \frac{D_O^T(x_0, q_{it})}{D_O^T(x_0, q_{hs})}$$

(17) $$X_{hs,it} = \frac{D_I^T(x_{it}, q_0)}{D_I^T(x_{hs}, q_0)} \qquad \text{and}$$

(18) $$TFP_{hs,it} = \frac{Q_{hs,it}}{X_{hs,it}} = \frac{D_O^T(x_0, q_{it})}{D_O^T(x_0, q_{hs})} \frac{D_I^T(x_{hs}, q_0)}{D_I^T(x_{it}, q_0)}.$$

The indexes (16) and (17) are closely related[5] to the Malmquist output and input quantity indexes of Caves et al. (1982), and the TFP index given by (18) is closely related to the Hicks-Moorsteen TFP index discussed by Bjurek (1996). All three indexes satisfy the monotonicity, linear homogeneity, identity, homogeneity of degree zero, commensurability and proportionality axioms of Eichhorn (1978). They also satisfy the transitivity and time and space reversal tests of Fisher (1922). Other tests that are occasionally discussed in the index number literature make for convenient computations but do not appear to have any economic relevance. For further insights into the properties of fixed-weight multiplicatively-complete TFP indexes, see O'Donnell (2010b).

[55] For example, if $\bar{x} = x_{hs}$ and the period-*s* and period-*T* technologies are identical then (16) would correspond to a "firm-*hs*" Malmquist output index as defined by Caves, Christensen and Diewert (1982, p. 1400).

If the (antilogarithms of the) CES-CD approximating functions defined by (11) and (12) are used to approximate the aggregator functions (14) and (15) then, if the errors of approximation are fixed[6], the output and input indexes given by (16) and (17) become

$$(19) \qquad Q_{hs,it} = \left(\frac{\sum_{j=1}^{J} \alpha_j^{\theta} q_{jit}^{\theta}}{\sum_{j=1}^{J} \alpha_j^{\theta} q_{jhs}^{\theta}} \right)^{1/\theta} \quad \text{and}$$

$$(20) \qquad X_{hs,it} = \prod_{k=1}^{K} \left(\frac{x_{kit}}{x_{khs}} \right)^{\eta^{-1}\beta_k} .$$

Observe that these output and input indexes, and therefore the associated TFP index, do not depend on the arbitrarily-chosen input and output vectors x_0 and q_0, nor on the unknown parameters γ_0 and γ_1. One implication is that the output and input indexes will still be given by (19) and (20) even if the period-*s* distance functions are used in (14) and (15) instead of the period-*T* distance functions. A further implication is that γ_0 and γ_1 can be permitted to vary across (groups of) observations (e.g., to reflect changes in the production environment) and the TFP index will still be given by the ratio of the indexes defined in (19) and (20)[7]. Also observe that the output and input indexes given by (19) and (20) could have been obtained using the following non-negative, non-decreasing and linearly homogeneous aggregator functions:

$$(21) \qquad Q(q_{it}) = \left(\sum_{j=1}^{J} \alpha_j^{\theta} q_{jit}^{\theta} \right)^{1/\theta} \quad \text{and}$$

$$(22) \qquad X(x_{it}) = \prod_{k=1}^{K} x_{kit}^{\eta^{-1}\beta_k} .$$

This illustrates that different aggregator functions can be used to motivate the same multiplicatively-complete TFP index. More important for empirical work is the fact that if outputs are aggregated using (21) then the output distance function given by (11) can be rewritten in the form of a conventional stochastic frontier model:

$$(23) \qquad \ln Q_{it} = \gamma_0 + \gamma_1 t + \sum_{k=1}^{K} \beta_k \ln x_{kit} + v_{it} - u_{it}$$

where $u_{it} = -\ln D_O^t(x_{it}, q_{it}) \geq 0$. If inputs are aggregated using (22) then the input distance function (12) can also be written in the form of a conventional stochastic frontier model. Indeed, if inputs are aggregated using (22) then it is also straightforward to show that

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[6] If the errors of approximation are not fixed then (19) and (20) can be derived as the antilogarithms of the expected values of the logarithms of the output and input indexes.

[7] It is also possible to allow other parameters of the technology to be observation varying. However, when it comes to computing TFP indexes they must be held fixed (at possibly arbitrarily-chosen values) if the TFP index is to satisfy the transitivity test.

(24) $$\ln TFP_{hs,it} = \ln TFP_{it} - \ln TFP_{hs} = \gamma_1 (t-s) - \left[u_{it} - u_{hs}\right] + (\eta - 1)\left[\ln X_{it} - \ln X_{hs}\right] + \left[v_{it} - v_{hs}\right].$$

This equation illustrates how the (logarithm of) a multiplicatively-complete TFP index can be broken into (logarithms of) measures of technical change, technical efficiency change, scale-mix efficiency change[8], and noise.

4. THE COMPONENTS OF TFP CHANGE

O'Donnell (2008) shows how any multiplicatively-complete TFP index can be decomposed into a measure of technical change and various measures of efficiency change. The decomposition methodology does not rely on any restrictive assumptions concerning the production technology, nor does it involve any assumptions concerning firm behavior or the level of competition in input or output markets. The methodology can be used to motivate an infinite number of economically-meaningful decompositions of TFP change.

For illustrative purposes, O'Donnell (2008) considers a technology that exhibits variable returns to scale. For such technologies it is generally possible to find finite non-zero input and output vectors that maximize TFP. Then it is meaningful to compare the TFP of the firm with the maximum TFP that is possible. Let TFP_t^* denote the maximum TFP that is possible in period *t*. O'Donnell (2008) defines the TFP efficiency (TFPE) of firm *i* in period *t* as

(25) $$TFPE_{it} = \frac{TFP_{it}}{TFP_t^*} = \frac{Q_{it} / X_{it}}{Q_t^* / X_t^*}$$

where Q_t^* and X_t^* denote aggregates of the output and input vectors that maximize TFP. Figure 1 illustrates this measure of overall productive performance in two-dimensional aggregate quantity space. In this figure, the curve passing through point E is a production frontier that envelops all aggregate-output aggregate-input combinations that are technically feasible in period *t*. In aggregate quantity space, the TFP at any point is the slope of the ray from the origin to that point. For example, the TFP at point A is $TFP_{it} = Q_{it} / X_{it} = \text{slope 0A}$, while the maximum productivity possible using the technology is the TFP at point E: $TFP_t^* = Q_t^* / X_t^* = \text{slope 0E}$. It follows that the measure of TFP efficiency given by (25) can be expressed in terms of slopes of rays in aggregate quantity space: $TFPE_{it} = TFP_{it} / TFP_t^* = \text{slope 0A/slope 0E}$.

Many other measures of efficiency can be expressed in terms of aggregate quantities and therefore as slopes of rays in aggregate quantity space. For example, the measure of overall efficiency given by (25) can be decomposed into measures of output-oriented technical efficiency (OTE) and output-oriented scale-mix efficiency (OSME). Mathematically, $TFPE_{it} = OTE_{it} \times OSME_{it}$ where

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[8] If the technology is represented by (12) then it exhibits Hicks–neutral technical change. In this special case the aggregator functions (21) and (22) map to the frontier surface and all input and output combinations are mix efficient. Thus, the scale-mix efficiency change component in (24) is, in fact, a pure measure of scale efficiency change. If the technology exhibits constant returns to scale then this component disappears.

(26) $$OTE_{it} = \frac{Q_{it}}{\bar{Q}_{it}} = D_O^t(x_{it}, q_{it})$$

(27) $$OSME_{it} = \frac{\bar{Q}_{it} / X_{it}}{TFP_t^*}$$

and $\bar{Q}_{it} \equiv Q_{it} / D_O^t(x_{it}, q_{it})$ denotes the maximum aggregate output that can be produced by firm *i* in period *t* if it holds its input vector and output mix fixed. The OTE measure given by (26) is attributed to Farrell (1957) and is a measure of the productivity shortfall associated with operating below the production frontier. The OSME measure given by (27) is defined in O'Donnell (2010b) and is a measure of the productivity shortfall associated with diseconomies of scale and scope. Figure 1 depicts the relationship between these various measures of efficiency in aggregate quantity space: $OTE_{it} = Q_{it} / \bar{Q}_{it} =$ slope 0A/slope 0C, $OSME_{it} = (\bar{Q}_{it} / X_{it})/(Q_t^* / X_t^*)$ $=$ slope 0C/slope 0E and $TFPE_{it} = OTE_{it} \times OSME_{it} =$ slope 0A/slope 0E. See O'Donnell (2008; 2010a) for more details concerning these and related measures of efficiency.

It is useful to rearrange equation (25) and express the TFP of the firm as a proportion of maximum possible TFP: $TFP_{it} = TFP_t^* \times TFPE_{it}$. A similar equation holds for firm *h* in period *s*: $TFP_{hs} = TFP_s^* \times TFPE_{hs}$. It follows that the index number that compares the TFP of firm *i* in period *t* with the TFP of firm *h* in period *s* can be decomposed as

(28) $$TFP_{hs,it} \equiv \frac{TFP_{it}}{TFP_{hs}} = \left(\frac{TFP_t^*}{TFP_s^*}\right)\left(\frac{TFPE_{it}}{TFPE_{hs}}\right) = \left(\frac{TFP_t^*}{TFP_s^*}\right)\left(\frac{OTE_{it}}{OTE_{hs}}\right)\left(\frac{OSME_{it}}{OSME_{hs}}\right).$$

The first term on the far-right-hand side of (28) is a natural measure of technical change. The remaining terms are measures technical efficiency change and scale-mix efficiency change. Unlike the decomposition given by equation (24), there is no noise component in equation (28) because in this particular section of the paper the production technology has been treated as known.

If the production technology everywhere exhibits strictly increasing (decreasing) returns to scale then the maximum TFP that is possible using the technology will be infinitely large (zero) and the decomposition given by (28) will not be mathematically well-defined. In such cases, any number of local measures of technical change can be used to effect a decomposition of the TFP index. For example, let $TFP_t^{\#} = \max_i \{TFP_{it} : i \in \mathbb{Z}_N^{++}\}$ denote the maximum observed TFP of any firm in the sample in period *t*. The associated local measure of TFPE is

(29) $$TFPE_{it}^{\#} = \frac{TFP_{it}}{TFP_t^{\#}} = \frac{Q_{it} / X_{it}}{Q_t^{\#} / X_t^{\#}}$$

where $Q_t^{\#}$ and $X_t^{\#}$ are aggregate outputs and inputs associated with $TFP_t^{\#}$. Figure 2 uses a scatter of sample observations to illustrate this local measure of efficiency in aggregate quantity space. In this figure, the frontier passing through point G exhibits strictly decreasing returns to scale. Observe that the most productive firm in the sample is the firm operating at point H. The TFP at this point (and, incidentally, the point at which the ray intersects the frontier) is $TFP_t^{\#} = Q_t^{\#}/X_t^{\#} =$ slope 0H. The associated local measure of TFP efficiency is $TFPE_{it}^{\#} = TFP_{it} / TFP_t^{\#}$ = slope 0A/slope 0H. It is clear from Figure 2 that $TFPE_{it}^{\#}$ can be decomposed into the product of the measure of OTE given by (26) and the following local measure of output-oriented scale-mix efficiency:

$$\text{(30)} \qquad OSME_{it}^{\#} = \frac{\bar{Q}_{it} / X_{it}}{TFP_t^{\#}}.$$

It is also clear that the index number that compares the TFP of firm *i* in period *t* with the TFP of firm *h* in period *s* can still be decomposed into measures of technical change and different types of efficiency change:

$$\text{(31)} \qquad TFP_{hs,it} \equiv \frac{TFP_{it}}{TFP_{hs}} = \left(\frac{TFP_t^{\#}}{TFP_s^{\#}}\right)\left(\frac{TFPE_{it}^{\#}}{TFPE_{hs}^{\#}}\right) = \left(\frac{TFP_t^{\#}}{TFP_s^{\#}}\right)\left(\frac{OTE_{it}}{OTE_{hs}}\right)\left(\frac{OSME_{it}^{\#}}{OSME_{hs}^{\#}}\right).$$

This particular decomposition is available whenever the technology everywhere exhibits strictly increasing or strictly decreasing returns to scale.

Observe that the measures of TFP change and OTE change in equations (28) and (31) are identical. This suggests that any plausible measure of technical change can be used to effect a decomposition of a given TFP index. For example, if the technology is represented by the CES-CD approximating functions defined by (11) and (12) then the logarithm of the index that compares the TFP of firm *i* in period *t* with the TFP of firm *h* in period *s* is given by equation (24). In that equation the term $\gamma_1(t-s)$ is the (firm-invariant) logarithm of a measure of Hicks-neutral technical change. In this case the following alternative decomposition of the TFP index is available:

$$\text{(32)} \qquad TFP_{hs,it} \equiv \frac{TFP_{it}}{TFP_{hs}} = \left(\frac{TFP_t^{\dagger}}{TFP_t^{\dagger}}\right)\left(\frac{OTE_{it}}{OTE_{hs}}\right)\left(\frac{OSME_t^{\dagger}}{OSME_t^{\dagger}}\right)$$

where[9]

$$\text{(33)} \qquad TFP_t^{\dagger} \propto \exp(\gamma_1 t) \quad \text{and}$$

$$\text{(34)} \qquad OSME_t^{\dagger} = \frac{\bar{Q}_{it} / X_{it}}{TFP_t^{\dagger}}.$$

Note that if the technology is represented by the CES-CD approximating functions defined by (11) and (12) then, if the errors of approximation are fixed, it will everywhere exhibit decreasing, constant or increasing returns to scale depending on whether the observation-invariant $\eta = \sum_k \beta_k$ is less than, equal to, or greater than one. Thus, the decomposition given by (28) is unavailable. This paper estimates the CES-CD model and

[9] The notation $\propto$ means "is proportional to".

implements the decomposition given by (32) instead of the decomposition given by (31) because the estimated measure of technical change in the former equation is less likely to be affected by outliers.

5. ECONOMETRIC MODEL

I assume that firms choose input-output combinations to maximize a benefit function that is increasing in net returns. I also assume the time horizon is sufficiently short that input levels can be treated as pre-determined (exogenous). The *J* outputs are treated as endogenous and I focus on estimating the parameters of the output distance function given by (11). The empirical version of the model is given by equations (21) and (23):

(21) $$Q_{it} = \left(\sum_{j=1}^{J} \alpha_j^{\theta} q_{jit}^{\theta} \right)^{1/\theta}$$ and

(23) $$\ln Q_{it} = \gamma_0 + \gamma_1 t + \sum_{k=1}^{K} \beta_k \ln x_{kit} + v_{it} - u_{it}$$

where $u_{it} = -\ln D_O^t(x_{it}, q_{it})$ represents technical inefficiency and v_{it} represents approximation errors and other sources of statistical noise. The unknown parameters could be estimated by substituting (21) into (23) and estimating the resulting model using GMM. However, the choice of moment conditions is not obvious and the finite sample properties of the GMM estimator are unknown. Moreover, GMM methods for imposing the inequality constraints given by (7) to (9) are unsatisfactory, not least because binding inequality constraints lead to parameter estimates with standard errors of zero (implying we know their values with certainty). This paper solves the problem using the Bayesian methodology of Fernandez et al. (2000). The methodology has previously been used to estimate multiple-input multiple-output directional distance functions by O'Donnell (2007). Bayesian estimation involves sampling from the joint posterior probability density function (pdf) of the unknown parameters and unobserved inefficiency effects. This section presents the likelihood function, prior pdf, and conditional posterior pdfs needed for a Markov Chain Monte Carlo (MCMC) sampling algorithm.

5.1 The Likelihood Function

The set of all *NT* observations represented by (23) can be compactly written

(35) $$y = X\beta + v - u$$

where $y = (y_{11}, y_{12}, ..., y_{NT})'$, $y_{it} \equiv \ln Q_{it}$, $\beta = (\gamma_0, \gamma_1, \beta_1, ..., \beta_K)'$ and the remaining definitions are obvious, although it is worth noting that X is $NT \times (K+2)$. I assume the elements of v are independently and identically distributed normal random variables[10]:

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10 The notation $f_N(a \mid b, C)$ is used for a normal pdf with mean vector *b* and covariance matrix *C*.

(36) $$p(v \mid h) = f_N\left(v \mid 0_{NT}, h^{-1} I_{NT}\right)$$

where 0_{NT} denotes a zero vector of dimension *NT* and I_{NT} denotes an identity matrix of order *NT*. The conditional joint density for the unobserved dependent variable vector is

(37) $$p(y \mid \beta, u, h) = f_N\left(y \mid X\beta - u, h^{-1} I_{NT}\right)$$

where, for notational convenience, the conditioning on *X* has been suppressed. Unfortunately, this *NT*-variate density is not enough to define a sampling density for the $J \times NT$ matrix of observed outputs $Q = (q_{11}, q_{12}, ..., q_{NT})$. Such a density can only be defined by introducing $J-1$ new random variables into the model to generate stochastics in another $J-1$ dimensions. In this paper I introduce elasticities of distance with respect to outputs:

(38) $$\varepsilon_{kit} \equiv \frac{\partial \ln D_O^t(x_{it}, q_{it})}{\partial \ln q_{kit}} = \frac{\alpha_k^\theta q_{kit}^\theta}{\sum_{j=1}^{J} \alpha_j^\theta q_{jit}^\theta} \quad \text{for } k = 1, ... J.$$

Observe that these elasticities sum to one. Accordingly, I follow Fernandez et al. (2000) and assume that $\varepsilon_{it} = (\varepsilon_{1it}, ..., \varepsilon_{Jit})'$ is independently distributed with a Dirichlet pdf[11]:

(39) $$p(\varepsilon_{it} \mid s) = f_D\left(\varepsilon_{it} \mid s\right) \quad \text{for } i = 1, ..., N \text{ and } t = 1, ..., T$$

where $s = (s_1, ..., s_J)' \in \mathbb{R}_+^J$. Given θ and α there is a one-to-one mapping between the observed output vector $q_{it} \in \mathbb{R}_+^J$ and the unobserved vector $(\varepsilon_{1it}, ..., \varepsilon_{J-1,it}, y_{it})' \in \mathbb{R}^J$. Thus, the conditional likelihood function for the matrix of observed outputs $Q = (q_{11}, q_{12}, ..., q_{NT})$ is (Fernandez et al. 2000, p. 55, eq. 2.7):

(40) $$p(Q \mid \theta, \alpha, \beta, h, s, u) = f_N\left(y \mid X\beta - u, h^{-1} I_{NT}\right) \prod_{i=1}^{N} \prod_{t=1}^{T} f_D\left(\varepsilon_{it} \mid s\right) \prod_{i=1}^{N} \prod_{t=1}^{T} |J_{it}|$$

where

(41) $$|J_{it}| = \theta^{J-1} \prod_{j=1}^{J} \frac{\varepsilon_{jit}}{q_{jit}}$$

is the absolute value of the Jacobian of the transformation from $(\varepsilon_{1it}, ..., \varepsilon_{J-1,it}, y_{it})'$ to q_{it}.

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[11] The notation $f_D(a \mid b)$ the notation for a Dirichlet pdf used by Poirier (1995, p. 132). If $a = (a_1, ..., a_J)'$ and $b = (b_1, ..., b_J)'$ then $E(a_j) = b_j / b_0$ and $Var(a_j) = b_j(b_0 - b_j)/(b_0^3 + b_0^2)$ where $b_0 = b'\iota_J$. Other distributional assumptions are possible, including the additive logistic model.

5.2 The Joint Prior

Fernandez, Osiewalski and Steel (1997) show that proper priors on the parameters of frontier models are generally needed to ensure the existence of the posterior density. I follow Fernandez et al. (2000) and specify a prior of the form

$$(42) \qquad p(\theta,\alpha,\beta,h,s,u) = p(\theta)p(\alpha)p(\beta)p(h)p(s)p(u)$$

where each of the component priors is proper[12]. To be specific:

$$(43) \qquad p(\theta) = f_G(\theta \mid 1, k_1) I(\theta > 1)$$

$$(44) \qquad p(\alpha) = f_D(\alpha \mid \iota_J)$$

$$(45) \qquad p(\beta) = f_N(\beta \mid 0_{K+2}, k_2 I_{K+2}) I(\beta \in R)$$

$$(46) \qquad p(h) = f_G(h \mid 1, k_1)$$

$$(47) \qquad p(s) = \prod_{j=1}^{J} f_G(s \mid 1, k_3)$$

$$(48) \qquad p(u \mid \lambda) = \prod_{i=1}^{N}\prod_{t=1}^{T} f_G(u_{it} \mid 1, \lambda) \quad \text{and}$$

$$(49) \qquad p(\lambda) = f_G(\lambda \mid 1, -\ln(\tau))$$

where R is the region of the parameters space where the constraints (9) are satisfied. In the empirical example I set $k_1 = k_3 = 10^{-4}$ and $k_2 = 10^4$ to ensure the priors for θ, β, h and s are relatively non-informative. The prior given by equations (42) to (49) is a special case of the noninformative prior used by Fernandez et al. (2000). The pdf (49) is centred on $-\ln(\tau)$ where τ is a prior estimate of the mean level of efficiency. In the empirical example I set $\tau = 0.9$.

5.3 Posterior Inference

The likelihood function combines with the joint prior to yield a joint posterior for the unknown parameters and the unobserved inefficiency effects. Analytical integration of this posterior appears impossible, so posterior inference is conducted using MCMC simulation methods. The Gibbs sampling algorithm partitions the vector of unknown parameters and inefficiency effects into blocks, then simulates sequentially from the conditional posterior distribution for each block. In the present case, the conditional posteriors are (Fernandez et al. 2000, p. 58-61)[13]:

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[12] The notation $f_G(a \mid b,c)$ is used for a gamma pdf with mean b/c and variance b/c^2. If $b = 1$ then $f_G(a \mid b,c)$ is an exponential pdf.

[13] If we were to let $\sigma^2 \equiv h^{-1}, D \equiv I_{NT}, z \equiv u, V \equiv X, \delta \equiv y$ and $\lambda(\phi) \equiv \lambda$ then equation (4.2) in Fernandez et al. (2000) is identical to (53), except that Fernandez et al. (2000) write λ in the mean function instead of λ^{-1}. This appears to be a typographical error on their part.

(50) $$p\left(\beta \mid \theta,\alpha,h,s,\lambda,u,Q\right) \propto f_N\left(\beta \mid hVX'(y+u),V\right) I\left(\beta \in R\right)$$

(51) $$p\left(h \mid \theta,\alpha,\beta,s,\lambda,u,Q\right) \propto f_G\left(h \mid 1+.5NT, k_1+.5e'e\right)$$

(52) $$p\left(\lambda \mid \theta,\alpha,\beta,h,s,u,,Q\right) \propto f_G\left(\lambda \mid NT+1, u'\iota_{NT} - \ln(\tau)\right)$$

(53) $$p\left(u \mid \theta,\alpha,\beta,h,s,\lambda,Q\right) \propto f_N\left(u \mid X\beta - y - h^{-1}\lambda^{-1}\iota_{NT}, h^{-1}I_{NT}\right) I\left(u \geq 0_{NT}\right)$$

(54) $$p\left(\theta \mid \alpha,\beta,h,s,\lambda,u,Q\right) \propto \theta^{NT(J-1)} \exp\left(-k_1\theta - 0.5h^{-1}e'e - \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{j=1}^{J} s_j \ln \varepsilon_{jit}^{-1}\right) I(\theta > 1)$$

(55) $$p\left(s_j \mid \theta,\alpha,\beta,h,s_{-j},\lambda,u,Q\right) \propto \Gamma\left(\sum_{m=1}^{J} s_m\right)^{NT} \Gamma\left(s_j\right)^{-NT} \exp\left[-s_j\left\{k_1 + \sum_{i=1}^{N}\sum_{t=1}^{T} \ln \varepsilon_{jit}^{-1}\right\}\right] I(s_j \geq 0)$$

(56) $$p\left(\alpha \mid \theta,\beta,h,s,\lambda,u,Q\right) \propto \prod_{j=1}^{J} \alpha_j^{s_j\theta NT} \prod_{i=1}^{N}\prod_{t=1}^{T}\left(\sum_{j=1}^{J} \alpha_j^{\theta} q_{jit}^{\theta}\right)^{-\sum_{j=1}^{J} s_j} \exp\left(-0.5h^{-1}e'e\right) I(\alpha \in R)$$

where $e \equiv y - X\beta + u$; $V \equiv \left(hX'X + k_2^{-1}I_{K+2}\right)^{-1}$; and s_{-j} is the vector comprising all elements of s except s_j. Simulating from the densities (51) to (53) is straightforward using non-iterative simulation methods. Indeed, simulating from (53) can be accomplished by sampling independently from *NT* univariate truncated normal distributions. Although the remaining densities are nonstandard, they can be simulated using a Metropolis-Hastings (M-H) algorithm. A simple accept-reject algorithm that can be used for sampling from (56) involves drawing $J-1$ elements of α, computing the J^{th} element from the adding up constraint (10), then rejecting the entire vector if any elements lie outside the unit interval.

6. EMPIRICAL ILLUSTRATION

This section illustrates the methodology using a state-level panel dataset obtained from the Economic Research Service (ERS) of the U.S. Department of Agriculture (USDA). The panel covers the N = 48 contiguous states over the T = 45 years from 1960 to 2004. The data file records the quantities of J = 3 agricultural outputs (livestock, crops, other outputs) and K = 4 inputs (capital, land, labour, materials) in a particular state in a particular year relative to Alabama in 1960. Details concerning the construction of the data can be accessed from Ball, Hallahan and Nehring (2004).

All results presented in this section were generated using MATLAB. Starting values for the MCMC algorithm described in Section 5 included $\theta = 1.1$ and $\alpha_j = 1/J$ for $j = 1,...,J$. These values were used in (22) to compute an aggregate output series and then starting values for the remaining parameters in the model were obtained by applying least squares to equation (23). The MCMC algorithm was used to obtain 12,000 draws on the unknown parameters and technical inefficiency errors. The first 2,000 draws were used to tune the M-H components of the simulator and were then discarded as a burn-in. The M-H algorithms were tuned so that the acceptance rates were between 0.2 and 0.6. The chains of retained observations are presented in Figure 3 and show no signs of nonstationarity.

The estimated posterior means, standard deviations and 95% highest posterior density (HPD) interval limits for $(\theta, \alpha, \beta, h, s, \lambda, \eta)$ are presented in Table 1. These values are estimates obtained from the 10,000 post burn-in posterior draws. The joint prior incorporates the economic regularity constraints given by equations (7) to (10) so the estimates reported in Table 1 are guaranteed to be "correctly" signed. Interpretation of the estimates is straightforward: for example, the posterior mean for η is 1.035 > 1 indicating that the technology everywhere exhibits increasing returns to scale; the HPD interval limits for γ_1 reveal that the annual rate of technical change in U.S. agriculture lies between 0.8% and 2.2% with probability 0.95. One of the advantages of the Bayesian approach is that it is also straightforward to draw valid finite-sample inferences about the unknown parameters in ways that are often more informative than simple point and interval estimates: for example, the estimated pdf depicted in panel (a) of Figure 4 gives a very clear picture of likely and unlikely values of the elasticity of scale; 4.7% of the area under this pdf is below one indicating there is a 4.7% chance the technology exhibits decreasing returns to scale..

The estimated pdfs in the remaining panels in Figure 4 are representative of our post-sample beliefs about other economic quantities of interest. They depict levels of productivity and efficiency in California in 2004 relative to levels in Alabama in 1960: panel b) presents the estimated pdf of the TFP index defined by (18) and indicates that TFP in California in 2004 was two to four times higher than TFP in Alabama in 1960; panel (c) presents the estimated pdfs of the Farrell (1957) measure of output-oriented technical efficiency (26) and indicates that OTE in California in 2004 (solid line) was higher than OTE in Alabama in 1960 (dashed line); panel (d) presents the estimated pdf for the associated index of OTE change and confirms that California was more than twice as likely to have had higher levels of OTE in 2004 than Alabama had in 1960 (the posterior odds ratio is 2.2); panel (e) presents the estimated pdf for the change in the output-oriented scale-mix efficiency measure (34) and indicates that OSME was higher in California in 2004 than in Alabama in 1960 (with probability 0.70); and panel (f) presents the estimated pdf for the measure of technical change defined by (33) and reveals that the maximum productivity possible using the technology available in 2004 was 1.5 to 2.5 times higher than the maximum productivity possible in 1960.

It is useful to assess the plausibility of the estimated pdfs presented in Figure 4 in terms of measures of central tendency and dispersion and by comparison with results from other years. Figure 5 presents the geometric mean and 95% HPD interval limits for indexes comparing levels of productivity and efficiency in California with levels in Alabama in 1960. Panel (a) in Figure 5 presents results for TFP change (ΔTFP) while panels (b) to (d) present results for technical change (ΔTech), technical efficiency change (ΔOTE) and scale-mix efficiency change (ΔOSME). These panels suggest that (smooth) technical change appears to be driving long run increases in the TFP index (and the HPD limits). They also reveal there is considerable uncertainty concerning the estimates of OTE and OSME change.

A clearer picture of the drivers of productivity change in California is given in Figure 6. Panel (a) in this figure simply reproduces the mean series' from Figure 5 on a single diagram with a common vertical scale. This figure reveals that in the first two decades of the sample period productivity increases due to technical progress and technical efficiency improvement were roughly offset by productivity declines due to changes in scale and mix. These results are consistent with the U.S. results obtained by O'Donnell (2010a, p. 553) using DEA methodology and a different agricultural dataset. O'Donnell (2010a, p. 552) explains that firms who have benefit functions that are increasing in net returns will rationally change the scale and mix of their operations in response to (anticipated) changes in relative output and input prices. One way of assessing this argument is to estimate the shadow (or support) prices faced by agricultural producers in California over this period. Grosskopf, Margaritis and Valdmanis (1995) use duality theory to show that revenue-deflated shadow prices are equal to the derivatives of the output distance function with respect to output quantities: $p_{kit}^* / p_{it}' q_{it} = \partial D_O^t(x_{it}, q_{it}) / \partial q_{kit}$. If the output distance function is given by (11) then the shadow price ratios are:

(57) $$\frac{p_{kit}^*}{p_{lit}^*} = \left(\frac{q_{lit}}{q_{kit}}\right)\left(\frac{\alpha_k q_{kit}}{\alpha_l q_{lit}}\right)^{\theta} \quad \text{for } k, l = 1, \ldots K.$$

Panel (b) in Figure 6 reveals that the years from 1960 to 1980 wer characterised by a significant fall in the estimated shadow price of crops relative to the estimated shadow price of other crops, and this was associated with a significant fall in the observed output of crops relative to the observed output of other crops. It was also plausibly associated with a fall in OSME.

7. CONCLUSION

Measures of productivity and efficiency are generally well-defined and understood, especially in the case of single-output single-input firms. In those cases it is common to draw simple diagrams to illustrate relationships between the concepts of productivity, technical efficiency, scale efficiency and technical change. Matters become slightly more complicated in the case of multiple-output multiple-input firms where it is usually possible to capture productivity dividends through economies of scope. In those cases it is common to draw diagrams to illustrate the concepts of technical, cost and allocative efficiency, but only recently has O'Donnell (2008) shown how simple diagrams can also be used to illustrate important relationships between measures of efficiency and common measures of productivity change. This provides for some simple decompositions of common productivity index numbers. Implementing the O'Donnell (2008) decomposition methodology involves estimating production frontiers using conventional DEA and/or SFA techniques.

O'Donnell (2010b) has shown how DEA techniques can be used to decompose Paasche, Laspeyres, Fisher, Lowe and Hicks-Moorsteen TFP indexes. This paper shows SFA methodology can be used to decompose a new TFP index that satisfies most, if not all, economically-relevant axioms and tests from index number theory. Estimating and decomposing this new index involves estimating the parameters of output and input distance functions.

SFA estimation of distance functions is complicated by the fact that the explanatory variables in the standard SFA formulation of the model may be correlated with the error term. This paper overcomes the problem using a Bayesian systems approach developed by Fernandez et al. (2000). One of the advantages of the Bayesian approach is that it is possible to draw valid finite-sample inferences concerning nonlinear functions of the model parameters. To illustrate, this paper draws inferences concerning returns to scale and measures of TFP and efficiency change in U.S. agriculture. The results indicate that the primary drivers of agricultural productivity change in California have been technical progress and improvements in scale-mix efficiency. These results are consistent with the US results obtained by O'Donnell (2010a) using DEA methodology and an OECD agricultural dataset.

This paper shows how to compute and decompose TFP indexes in an econometric framework when only quantity data are available (i.e., when there are no prices). The methodology does not rely on assumptions concerning the optimising behaviour of firms (e.g., cost minimisation) or the degree of competition in product markets (e.g., perfect competition), except insofar as they may be necessary to determine which variables in the model are determined endogenously and which are not. Nor does the methodology rely on any particular assumptions concerning the functional form of the output or input distance functions (e.g., translog, CES, CD) or the distribution of random inefficiency effects (e.g., time-varying, half-normal). Thus, the method appears to be applicable many empirical contexts where mainstream efficiency estimation methods are now used.

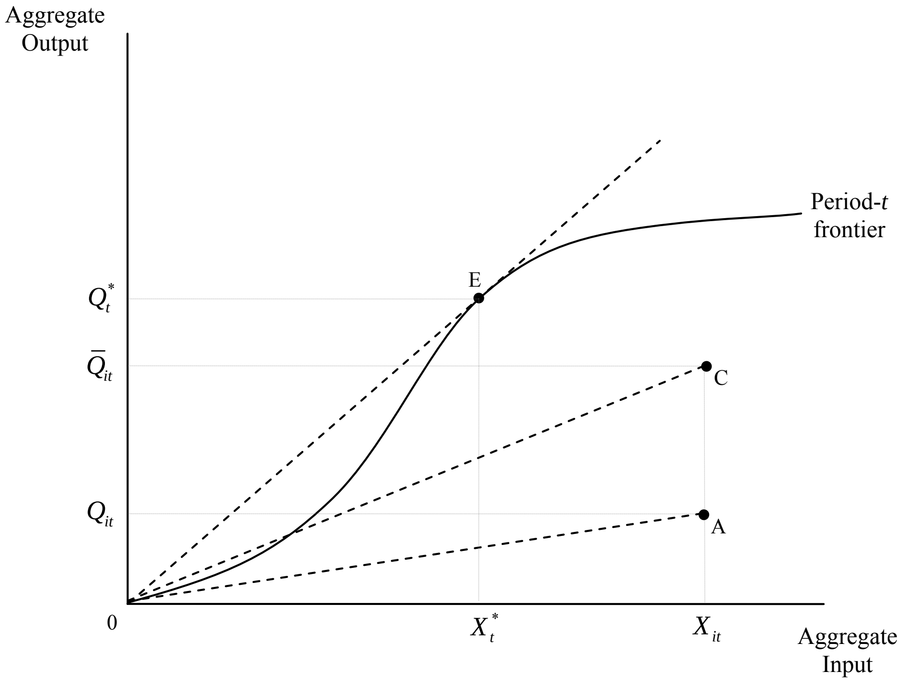


Figure 1. Measures of Efficiency

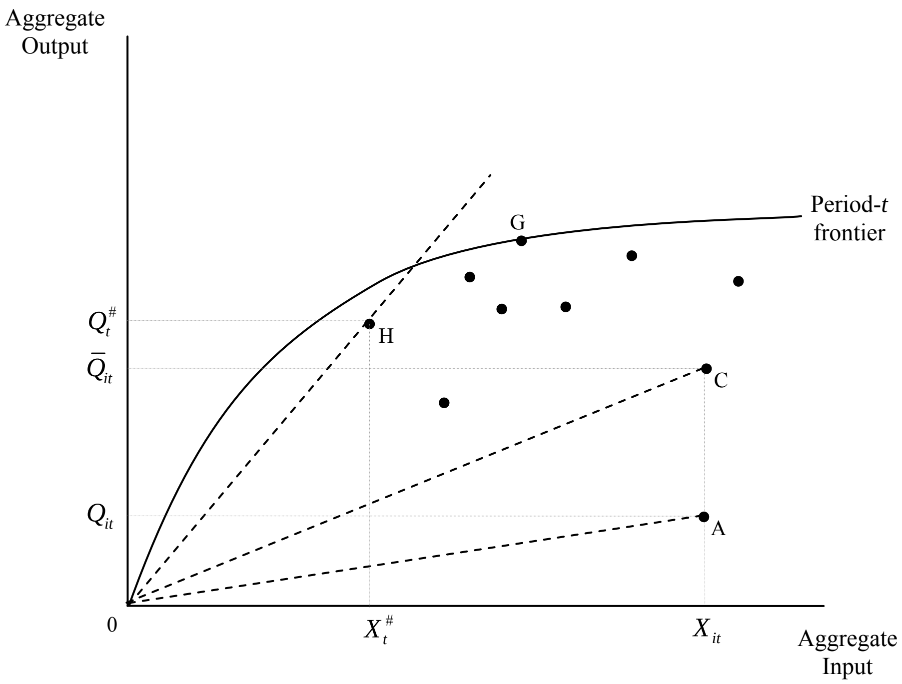


Figure 2. Measures of Efficiency

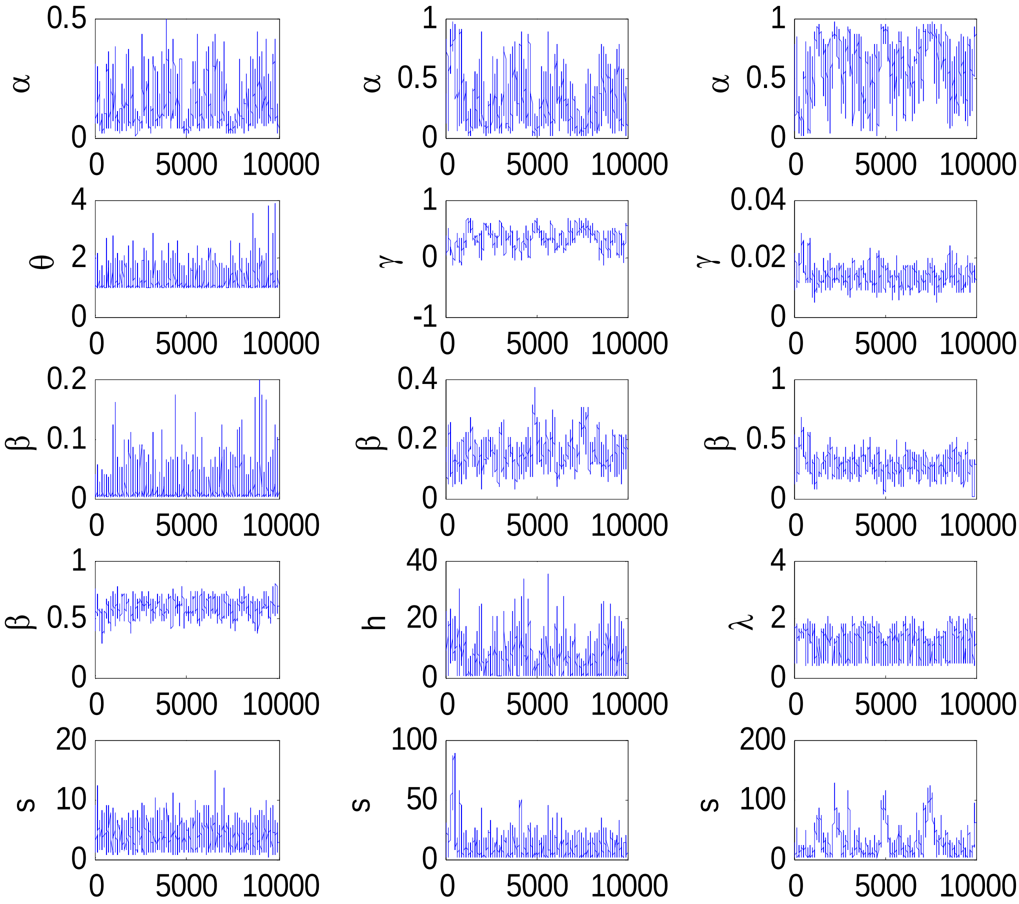


Figure 3. MCMC chains

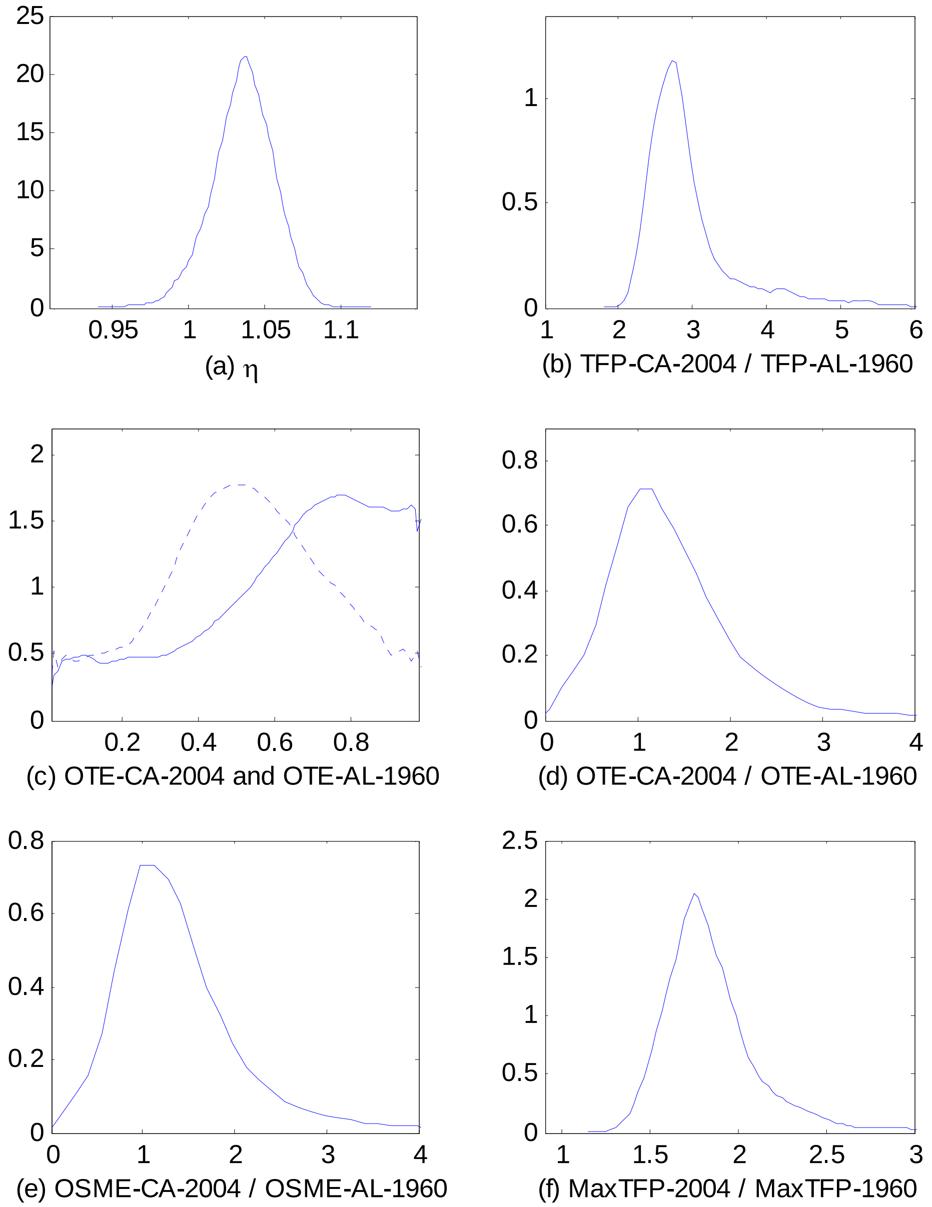
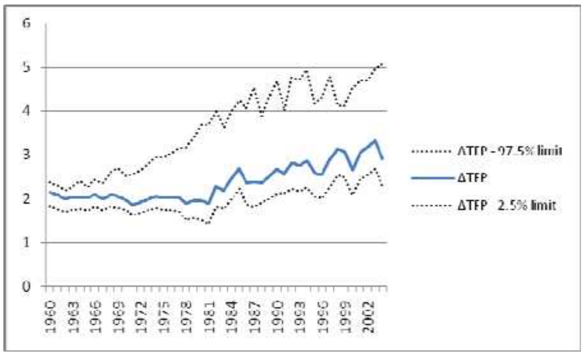
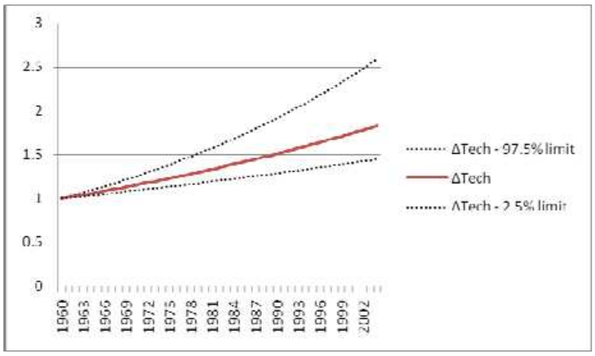


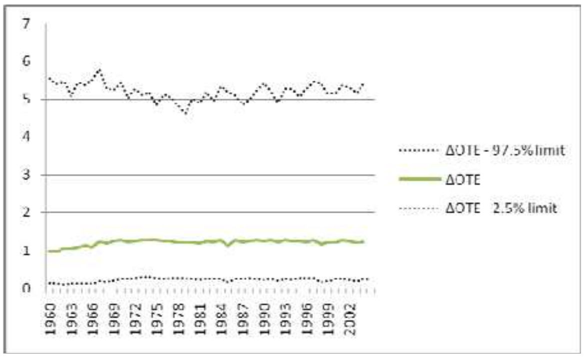
Figure 4. Posterior Pdfs



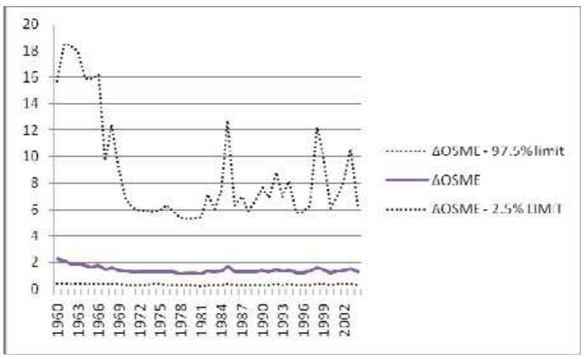
(a) TFP Change



(b) Technical Change

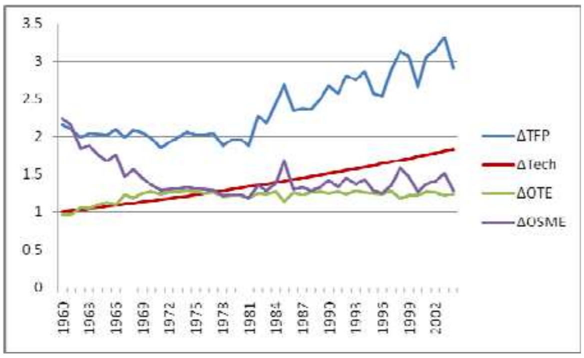


(c) OTE Change

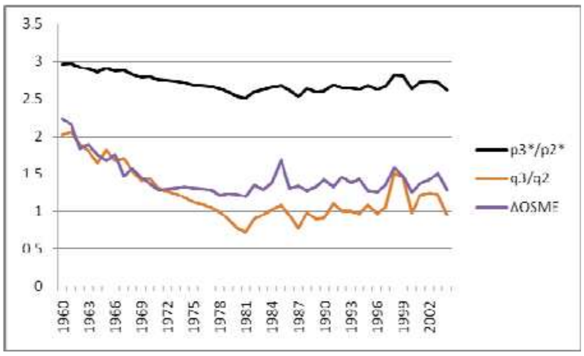


(d) OSME Change

Figure 5. Components of TFP Change: California cf. Alabama in 1960



(a) TFP Change, Technical Change and Efficiency Change



(b) Shadow Price Ratio, Observed Output Mix and OSME Change

Figure 6. Components of TFP Change: California cf. Alabama in 1960

Table 1. Parameter Estimates

	MEAN	STDEV	2.50% HPD limit	97.50% HPD limit
θ	1.161	0.255	1.000	1.854
α_1	0.123	0.075	0.027	0.304
α_2	0.297	0.204	0.042	0.812
α_3	0.580	0.231	0.108	0.915
γ_0	0.324	0.159	-0.003	0.600
γ_1	0.014	0.003	0.008	0.022
β_1	0.011	0.019	0.000	0.065
β_2	0.150	0.049	0.067	0.264
β_3	0.283	0.084	0.139	0.491
β_4	0.591	0.068	0.447	0.714
h	6.575	4.558	0.254	16.456
s_1	3.770	1.487	1.242	7.005
s_2	10.368	10.845	0.813	42.632
s_3	27.703	26.061	1.554	98.235
λ	1.259	0.388	0.472	1.810
η	1.035	0.020	0.992	1.073

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