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Money Stock Control an Its Implications for Monetary Policy: Technical Appendices

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This working paper is a series of technical appendices that present a more detailed discussion of the procedures used in the article "Money Stock Control and Its Implications for Monetary Policy" which appeared in the October 1971 Review, and is available as reprint No. 72. The article presented a procedure the Federal Reserve could use to control the growth of the money stock, and a method for evaluating the effect of this control on the policymaker's ability to achieve GNP objectives. The net source base was used as the control variable for the money stock control process. To determine the amount of base to supply each month, the Federal Reserve forecast the money multiplier each month using a forecasting equation in which the previous three month's average multiplier and the reserve adjustment magnitude were the main independent variables. Using a simulation technique, evidence was presented for two sample periods (1962-65 and 1966-69) on the effect this money stock control procedure would have had on the policymaker's ability to achieve GNP objectives. To assess the effect of money stock control, moving outside the sample periods, the standard error of forecast statistic was developed to permit the construction of appropriate confidence intervals for GNP projections.

Appendix A presents evidence on the accuracy with which the Federal Reserve has forecast and measured the net source base. The first part of this appendix measures the accuracy with which the Federal Reserve has projected the net source base on a daily-average monthly basis. The second part assesses the accuracy with which the Federal Reserve has measured the previous day's net source base.

Appendix B presents the derivation of the standard error of forecast statistic assuming errors in money stock control. This appendix also discusses the computation of the error, and variance of the error in money stock control; and the sign of the covariance between the error in money stock control and the error in the GNP forecasting equation.

Appendix C presents a proof that the errors in predicting GNP are normally distributed. Appendix D presents 80 and 90 per cent confidence intervals for GNP projections and shows the effect of the errors in the money stock control procedure on these intervals.

Appendix E presents the monthly predicted and actual multipliers, and the parameter estimates of the 72 monthly forecasting regression equations. Also included are the R^2, Durbin Watson, and $\rho\mu_{t-1}$ values for each month. Appendix F gives monthly controlled values of the money stock for two sample periods, and charts comparing controlled and desired monthly levels.

TECHNICAL APPENDIX A

Forecast and Measurement of the Net Source Base

This Appendix presents some evidence on the accuracy with which the Federal Reserve has forecast the monthly daily-average net source base and measured the net source base on a daily basis. Two procedures, which are referred to as Part I and Part II, were used. Because the data were taken from confidential releases, we are able to report only a summary of our findings and a general outline of our procedure. At the end of this Appendix a table is presented which details the sources and uses of the net source base and shows its relation to the source base and monetary base.

Purpose: To measure the accuracy with which the Federal Reserve has projected the net source base on a daily-average basis monthly. As supplemental evidence, a sample of weekly daily-average errors is presented.

Source of data: Board of Governors of the Federal Reserve Confidential Daily Summary Release L.6.1, and Estimated Changes in Factors Affecting Free Reserves of All Member Banks, Release 76-L.

Sample period: April 1969 through February 1970.

Procedure: The projected and actual levels of the net source base, exclusive of Federal Reserve holdings of Government securities, were calculated for each business day of the month. Saturday, Sunday, and holiday dates were calculated by taking the values of the projected and actual figures for the preceding business day. For example, Friday data were carried for Saturday and Sunday. When data were missing for a day, the average monthly value was entered for the missing data. The daily figures were then averaged to get monthly daily-average projected and actual net source base figures.

For a given day, say 6/5/69, the following procedure was used:

(1) Projected levels for 6/5/69 were determined by taking the levels for 6/4/69 from the L.6.1 release for that data and adding the projected net change in each component for 6/5/69 from the 76-L release dated 6/5/69.

(2) Actual levels for 6/5/69 were obtained by moving forward one year and computing the final data for the levels from L.6.1 release.

(3) If 6/5/69 was a Saturday, then the actual and projected levels for 6/4/69 were used.

(4) If data for 6/5/69 were not available, then the monthly average computed from available data for June 1969 was used.

Results, Monthly basis:

The following table summarizes the results of this procedure. For each month the number of business days, and the number of days for which data were available, are given. April 1969 was excluded because we had only 9 days of data.

Table I

Monthly Daily Average Net Source Base Errors

		Number of days of available data	Business days	Difference between projected and actual daily-average net source base (millions $)
1969	May	16	21	+ 40
	June	21	21	- 95
	July	19	22	- 143
	August	21	21	+ 39
	September	14	21	- 113
	October	21	23	- 117
	November	17	17	- 33
	December	20	22	+ 58
1970	January	17	21	- 31
	February	18	19	- 2
		mean (with sign)	=	$ - 40
		mean (absolute value)	=	67

Results, Weekly basis:

The above procedure was repeated on the available weekly data. The sample included 28 weeks for which we had available consecutive business days. Each week is a 5-day week beginning Monday and ending Friday. For each sample week a weekly average of daily projected levels was compared to a weekly average of actual daily figures. Table II summarizes these results.

Table II

Weekly Daily Average Net Source
Base Projection Errors

Sample size: 28 weeks

Mean of absolute values of errors	$ 99 million
Median of absolute values of errors	70 million
Average of three largest values	315 million
Average of three smallest values	5 million

Part II

Purpose: To measure the accuracy with which the Federal Reserve has measured the previous day's net source base. In other words, how accurately does the Federal Reserve know today what was yesterday's net source base. We assume the Federal Reserve knows with complete accuracy its holdings of Government securities. The data are taken from the sources side of the base.

Source of data: Board of Governors of the Federal Reserve Confidential Daily Summary Release L.6.1.

Sample period: March 1969 through February 1970. Includes a total of 243 daily observations.

Procedure: For a given date, say June 5, 1969, the following procedure was used.

(1) List the levels for the source components of the net source base exclusive of Federal Reserve holdings of Government securities from the June 5, 1969 L.6.1 release.

(2) Move ahead one year and find the final figures for 6/5/69 from the L.6.1 release. This requires that net changes from the final column of the L.6.1 dated 6/5/70 are used to compute the final figure for 6/5/69.

(3) The final 6/5/69 figure and the original 6/5/69 figure are then compared to measure the error in determining the value of the net source base on 6/5/69.

Results:

The results of this procedure are summarized in Table III.

Table III

Daily Errors in Measuring the Previous Day's Net Source Base

Sample Size: 243 days

Mean of the absolute values of the errors	$ 66 million
Standard deviation of absolute values of the errors	48 million
Median of absolute values of the errors	57 million
Average of 25 largest errors	167 million
Average of 25 smallest errors	5 million

Virtually all of the errors were negative, indicating that the Federal Reserve consistently underestimated the net source base. Also, almost all of the errors were in the Federal Reserve float component of the sources of the base.

Table IV

Sources and Uses of the Net Source Base,
the Source Base,
and the Monetary Base, January 1971
(millions of dollars)*

Sources	
Federal Reserve holdings of Government securities	$ 62,141
Federal Reserve float	3,636
Gold stock plus special drawing rights	11,132
Treasury currency outstanding	7,157
Other Federal Reserve Assets	1,216
Less:	
Treasury cash holdings	445
Treasury deposits at Federal Reserve Banks	1,028
Foreign deposits at Federal Reserve Banks	155
Other deposits at F. R. plus F.R. liabilities and capital	2,894
Equals	
Net source base	$ 80,760
Plus:	
Federal Reserve discounts and advances	370
Equals:	
Source base	$ 81,130
Plus:	
Reserve adjustment	3,826
Equals:	
Monetary base	$84,956

Uses	
Member bank deposits at Federal Reserve Banks less discounts and advances	$ 24,568
Currency held by banks	7,092
Currency held by the public	49,100
Equals	
Net source base	80,760
Plus:	
Federal Reserve discounts and advances	370
Equals:	
Source base	$ 81,130
Plus:	
Reserve adjustment	3,826
Equals:	
Monetary base	$ 84,956

* Data not seasonally adjusted.

TECHNICAL APPENDIX B

B.1 Derivation of the Standard Error of Forecast Statistic

In order to derive the standard error of forecast, an expression for the error in the forecasting equation must be specified. This expression can be written in the following way:

$$\hat{f}_{t+n} = \hat{Y}_{t+n} - Y_{t+n}$$

$$= b_o + b_1 \cdot \Delta M^T_{t+n} + \sum_{i=2}^{k-1} \cdot b_i \cdot \Delta X^i_{t+n}$$

$$- \left[\beta_o + \beta_1 \cdot \Delta M_{t+n} + \sum_{i=2}^{k-1} \beta_i \cdot \Delta X^i_{t+n} + \varepsilon_{t+n} \right]$$

where ΔX^i_{t+n} is the i^{th} variable in the forecasting equation. The independent variables encompassed by this notation includes contemporaneous changes in expenditures, plus both lagged changes in money and expenditures.

ΔM^T_{t+n}: is the desired contemporaneous change in money

ΔM_{t+n}: is the actual contemporaneous change in money

β_i: population value of the i^{th} coefficient

b_i: estimated sample value of the i^{th} coefficient

ε_t: actual error in the GNP forecasting equation.

The standard error of forecast (S E F) is defined to equal the square root of the following expectation:1/

$$E\left(\hat{f}^2_{t+n} \mid X_1 \ldots X_t, \Delta M^T_{t+n}, \Delta X_{t+n}\right)$$

Before deriving the expression for the S E F, it is necessary to make the substitution $\Delta M_{t+n} = \Delta M^T_{t+n} + \Theta_{t+n}$. It is assumed that no errors exist in contemporaneous changes in government expenditures, ΔE_{t+n}, hence $\Delta E_{t+n} = \Delta E^T_{t+n}$. In addition, since the parameters must be considered relative to their expected values, it is appropriate to add and subtract the relevant expected values in the expression for the S E F. Carrying out these two procedures leads to the following expression:

$$E\left(\hat{f}^2_{t+n} / X_1 \ldots X_t, \Delta M^T_{t+n}, \Delta X_{t+n}\right) =$$

$$E\,\{[\{b_o - E(b_o/X_1 \ldots X_t)\} + \{E(b_o/X_1 \ldots X_t) - \beta_o\}$$

$$+ \{b_1 - E(b_1/X_1 \ldots X_t)\} \cdot \Delta M^T_{t+n} + \{E(b_1/X_1 \ldots X_t) - \beta_1\} \cdot \Delta M^T_{t+n}$$

$$- \beta_1 \Theta_{t+n} + \sum_{i=2}^{k-1} \{b_i - E(b_i/X_1 \ldots X_t)\} \cdot \Delta X^i_{t+n}$$

$$+ \sum_{i=2}^{k-1} \{E(b_i/X_1 \ldots X_t) - \beta_i\} \cdot \Delta X^i_{t+n} - \varepsilon_{t+n}]^2 / X_1 \ldots X_t, \Delta M^T_{t+n}, \Delta X_{t+n}\}$$

This can be rewritten as (eliminating terms equal to zero)

$$E(\hat{f}^2_{t+n}/X_1 \ldots X_t, \Delta M^T_{t+n}, \Delta X_{t+n}) =$$

$$E\{[\{b_o - E(b_o/X_1 \ldots X_t)\} + \{b_1 - E(b_1/X_1 \ldots X_t)\} \cdot \Delta M^T_{t+n} - \beta_1 \Theta_{t+n}$$

$$+ \sum_{i=2}^{k-1} \{b_i - E(b_i/X_1 \ldots X_t)\} \cdot \Delta X^i_{t+n} - \varepsilon_{t+n}]^2 / X_1 \ldots X_t, \Delta M^T_{t+n}, \Delta X_{t+n}\}$$

Expanding the above expression, taking expectations,[2/] and dropping terms equal to zero, gives the final result.[3/]

$$SEF^{*} = \{E(\hat{f}^{2}_{t+n}/X_1 \ldots X_t, \Delta M^T_{t+n}, \Delta X_{t+n})\}^{1/2}$$

$$= \{var(b_o) + 2\ cov\ (b_o, b_1) \cdot \Delta M^T_{t+n} + 2 \sum_{i=2}^{k-1} cov\ (b_o, b_i) \Delta X^i_{t+n}$$

$$+ var(b_1)\ (\Delta M^T_{t+n})^2 + 2 \sum_{i=2}^{k-1} cov(b_1, b_i)\ \Delta X^i_{t+n} \cdot \Delta M^T_{t+n}$$

$$+ \beta_1^{\ 2}\ var\ (\Theta_{t+n}) + 2\ \beta_1\ cov\ (\Theta_{t+n}, \varepsilon_{t+n})$$

$$+ \sum_{i=2}^{k-1} \sum_{j=2}^{k-1} cov\ (b_i, b_j) \cdot \Delta X^i_{t+n} \cdot \Delta X^j_{t+n} + var\ (\varepsilon_{t+n})\}^{1/2}$$

For the purposes of comparison, the "no errors-in-the-variables" standard error of forecast is written below.[4/]

$$SEF = \{E(\hat{f}^{2}_{t+n}/X_i \ldots X_t, \Delta_{t+n})\}^{1/2}$$

$$= \{var(b_o) + 2 \sum_{i=1}^{k-1} cov\ (b_o, b_i)\ \Delta X^i_{t+n}$$

$$+ \sum_{i=1}^{k-1} \sum_{j=1}^{k-1} cov(b_i, b_j) \cdot \Delta X^i_{t+n} \cdot \Delta X^j_{t+n} + var(\varepsilon_{t+n})\}^{1/2}$$

B.2 The Expected Values of Θ_t and $(\Theta_t)^2$

Starting with the achieved level of the money stock last period, M_{t-1}, a loss function is used in conjunction with the desired level in the money stock this period M_t^d to determine the targeted level of the money stock for the current period M_t^T. The targeted change in the money stock is then given as $\Delta M_t^T = M_t^T - M_{t-1}$. However, since the total differential of the base-multiplier relation applies $\Delta M^T = (\hat{m}_t - m_{t-1}) B_{t-1} + m_{t-1} (B_t - B_{t-1})$, the base in period (t), B_t, must be set to have the value

$$B_t = [\Delta M_t^T - (\hat{m}_t - m_{t-1}) B_{t-1} + m_{t-1} B_{t-1}] / m_{t-1}$$

in order to maintain consistency with the targeted change in the money stock, ΔM_t^T.

Yet, after the fact, the following relation must hold for the actual change in the money stock, ΔM_t.

$$\Delta M_t = (m_t - m_{t-1}) B_{t-1} + m_{t-1} (B_t - B_{t-1}).$$

Substituting the above equation for the base in period (t) into the expression for the change in money gives the following expression for the current change in money

$$\Delta M_t = \Delta M_t^T + (m_t - \hat{m}_t) B_{t-1}$$

Clearly, ΔM_t is stochastic, and should be viewed as a function of the fixed component ΔM_t^T and the stochastic component $(m_t - \hat{m}_t) B_{t-1}$.[5/] This result is valid for each of the following two types of estimation procedures which might be used to give $\hat{m}_t$.

(I) "OLS regression case"

(a) actual m_t is stochastic (relative to prediction period) where $m_t = m_t^e$ (fixed) + μ_t (stochastic) and

(b) predicted $\hat{m}$ is stochastic (relative to sample period)

(II) "measurement error case" (e.g., predictions using the definitional or behavioral approach within the Brunner-Meltzer multiplier framework).

(a) actual m_t is assumed fixed

(b) predicted $\hat{m}$ is stochastic (relative to prediction period

In setting up S E F statistic, the traditional approach[7/] would suggest that we deal with ΔM_t^T and ΔM_t. However, since the ΔM_t is constrained to equal

$$[\Delta M_t^T + (m_t^e + \mu_t - \hat{m}_t)\ B_{t-1}] \text{ then,}$$

in this case we are forced to deal with the independent quantities ΔM_t^T and $\hat{\Theta}_t = (m_t^e + \mu_t - \hat{m}_t)\ B_{t-1}$.

When the multiplier is predicted using a regression approach, several benefits result from carrying out the S E F derivation using ΔM_t^T and $\hat{\Theta}_t$.

First, since $\hat{\Theta}_t = (m_t^e + \mu_t - \hat{m}_t)\ B_{t-1}$

implies: $\hat{\Theta}_t = - z^* \ (z'z)^{-1}\ z' \quad \mu^s \cdot B_{t-1} + \mu_t\ B_{t-1}$

we find that $E(\Theta_t) = 0$[8/] (μ^s is a sample period vector of errors in the multiplier equation).

Second, it can be shown that the variance of $\hat{\Theta}_t$ is:

$$E[(\Theta_t)^2] = \qquad \sigma_m^2\ B_{t-1}^2 [z^* \ (z'z)^{-1}\ z^{*\prime} + 1]$$

where, since the multiplier is assumed to be normally distributed, Θ_t is also normally distributed (see Technical Appendix C).

For simplicity, it is assumed that there is no contemporaneous correlation between the sample period errors in the A-J equation and the multiplier equation. In practice, it is necessary to use the sample variance s_m^2, instead of σ_m^2, in order to estimate $E[(\Theta_t)^2]$.

B.3 The Sign of the Covariance of Θ_t and ε_t

The sign of the covariance between the error in money stock control (Θ_t) and the error in the GNP forecasting equation (ε_t) is not known in advance. However, estimates of both its sign and magnitude can be obtained from an appropriately designed simulation experiment.

The simulation deals with three distinct gross national product series: first, historic ΔGNP_t; second, estimated $Y(\Delta M_t)$[9/], using historic money; and third, $Y(\Delta M_t^{ac})$, based upon a simulation of the money stock control procedure where ΔM_t^{ac} is the achieved money stock change corresponding to the historic targeted change $\Delta M_t^T = \Delta M_t$. The two estimated GNP series will diverge from the historic series by different amounts.

In order to utilize the results of the above simulations, the following definitions of the error terms are given.

(a) $\Theta_t = [\Delta M_t^T - \Delta M_t^{ac}]$

(b) $\varepsilon_t = [Y(\Delta M_t) - \Delta GNP_t]$

For the sake of argument, assume that Θ_t and ε_t have the same signs "most" of the time, and in particular that the conditions (1) $\Theta_t > 0$ and (2) $\varepsilon_t > 0$ apply. First, from conditions (2) it follows that

(c) $Y(\Delta M_t) > \Delta GNP_t$.

Second, in the simulation period, the historic change in money will be the targeted change (i.e., $\Delta M_t = \Delta M_t^T$). And finally, from condition (1)

(d) $Y(\Delta M_t) > Y(\Delta M_t^{ac})$

holds because the contemporaneous coefficient of money (β_1) in Y is positive. As a consequence, estimated $Y(\Delta M_t^{ac})$ will tend to be closer to ΔGNP_t than will $Y(\Delta M_t)$, if the following condition holds:

$$\beta_1 \text{ avg.}|\Theta_t|<2\cdot\{\text{avg.}|\varepsilon_t|\}$$

This result suggests negative correlation between the errors. (If this were actually true, the terms in one of the error definitions would have to be interchanged.)

However, it is shown below that the above situation actually implies a positive correlation (and covariance) between the errors. <u>In the prediction period, the historic change in money will be the achieved change</u> (i.e., $\Delta M_t = \Delta M_t^{ac}$). As a result, from condition (1), the following inequality,

$$\text{(e)} \quad Y(\Delta M_t^T) > Y(\Delta M_t),$$

holds because the contemporaneous coefficient of money in Y is positive. Together with the previously stated implication of condition (2), it follows that

$$\text{(f)} \quad Y(\Delta M_t^T) > Y(\Delta M_t) > \Delta GNP_t.$$

Because estimates of GNP using targeted money stock changes will do worse than corresponding estimates using actual changes, it can be concluded that a positive correlation of errors exists for the prediction period. The case where the errors predominantly occur with opposite signs can be analyzed in a similar manner.

In practice, the covariance of Θ_t and ε_t is computed from the residuals, $\hat{\Theta}_i$ and $\hat{\varepsilon}_i$, generated by the simulations discussed above.

FOOTNOTES

1/ $X_1 \ldots X_t$ represent the vectors of the independent variables in the sample period. ΔX_{t+n} represents the vector of independent variables in the forecasting period, without the estimated contemporaneous change in the money stock, ΔM^T_{t+n}, which is indicated separately.

2/ Statistics derived from observations made in the first period (1 to t) are independent of those generated in the second period (t+n). Therefore, in those cases where statistics of the first period are multiplied by statistics from the second, the rule, the product of the expectations equals the expectation of the product, can be applied.

3/ In the standard error of forecast given below, the indicated moments are constructed around zero. Variances and co-variances are obtained from the matrix.

$$V^* = V - VR' \ (RVR')^{-1} \ RV \text{ where } V = \sigma_y^{\ 2} \ (X'X)^{-1}$$

The X matrix is (txk), and includes a vector of "ones" for the constant. R is the matrix of exact linear constraints which the Almon procedure imposed in the estimation of the A-J equation. However, since $\sigma_y^{\ 2}$ is unknown, the estimator $s_y^{\ 2} = \sum_i (e_i - \hat{e}_i)^2/(t-k)$ must be used. Similarly, the estimated b_1 must be used in place of the parameter β_1.

4/ See Christ, Carl F. Econometric Models and Methods, John Wiley & Sons, Inc., New York, 1966, pp. 549-564.

5/ The "active involvement" of the monetary authorities in the money stock control context is responsible for the reversal of Feldstein's 6/ more traditional assumptions about the estimated and actual independent predictors.

6/ Feldstein, Martin S., "The Error of Forecast in Econometric Models when the Forecast-Period Exogenous Variables are Stochastic." Econometrica, January 1971.

7/ See footnote 6.

8/ μ^s (nx1) and μ_t (1x1), respectively, are sample and prediction period errors. Z(nxk) and Z^*(1xk), respectively, are sample and prediction period independent variables in the multiplier equation.

9/ The notation Y( ) is a simplified representation of the A-J equation.

TECHNICAL APPENDIX C

Normal Distribution of Forecast Error

First,

$$\hat{f}_{t+n} = \hat{Y}_{t+n} - Y_{t+n} = \left[b_o + b_1 \Delta M^T_{t+n} + \sum_{i=2}^{L-1} b_i \Delta M_{t+n-i+1} + b_L \Delta E_{t+n} + \sum_{i=L+1}^{k-1} b_i \Delta E_{t+n-i+L} \right] - \left[\beta_o + \beta_1 \Delta M_{t+n} + \sum_{i=2}^{L-1} \beta_i \Delta M_{t+n-i+1} + \beta_L \Delta E_{t+n} + \sum_{i=L+1}^{k-1} \beta_i \Delta E_{t+n-i+1} + \varepsilon_{t+n} \right]$$

Second,

(1) the errors in the coefficient estimates $(b_i - \beta_i)$ are normally distributed since $(b - \beta) = (X'X)^{-1}X' \varepsilon_{ys}$

(2) the achieved (or historical) money stock in period t+n is normally distributed since in Technical B.2 Appendix, it is shown that

$$\Delta M_{t+n} = \Delta M^T_{t+n} - B_{t-1} Z^* (Z'Z)^{-1} Z' \mu^s + B_{t-1} \mu_{t+n}$$

Third,

$$\hat{f}_{t+n} = \left[(b_o - \beta_o) + (b_1 - \beta_1) \Delta M^T_{t+n} + \sum_{i=2}^{L-1} (b_i - \beta_i) \Delta M_{t+n-i+1} + (b_L - \beta_L) \cdot \Delta E_{t+n} + \sum_{i=L+1}^{k-1} (b_i - \beta_i) \Delta E_{t+n-i+1} + \beta_1 B_{t-1} Z^*_{t+n} (Z'Z)^{-1} Z' \mu^s - \beta_1 B_{t-1} \mu_{t+n} + \varepsilon_{t+n} \right],$$

where all normal variables are independent and appear alone or are multiplied by constants (no products of stochastic variables occur).

Fourth, $\hat{f}_{t+n}$ is normally distributed as can be shown using the variables' respective moment generating functions in the theorem dealing with sums of independent random variables.[1/]

1/ Fisz, Marek; Probability Theory and Mathematical Statistics, John Wiley and Sons, New York 1963, pp. 113, 149-150.

TECHNICAL APPENDIX D

80 and 90 Per Cent Confidence Intervals For GNP Projections

Because the errors in predicting changes in GNP can be shown to be normally distributed for large samples,[1] it is appropriate to set up confidence intervals, using a table of the standard normal distribution. A range of ± 1.96 standard deviations gives a 95 per cent confidence interval for the standard normal distribution. These results were presented in the body of our paper. Ranges of ± 1.65 and ± 1.28 standard deviations give 90 and 80 per cent confidence intervals. In this appendix the appropriate 80 and 90 per cent confidence intervals for the year 1970 are presented.

1/ The proof of this assertion is given in Technical Appencix C.

Table I

Quarter to Quarter GNP Projections
for the Year 1970, Using the A-J Equation [1/]
(Alternative Confidence Intervals)
(Billions of dollars)

	Confidence Interval	No Error in Money Stock Control	Error in Money Stock Control
I/1970	± 6.815	90 %	(82.9 - 89.0) %
	± 5.310	80 %	(71.1 - 78.5) %
II/1970	± 7.253	90 %	(87.6 - 89.0) %
	± 5.651	80 %	(77.0 - 78.9) %
III/1970	± 6.906	90 %	(87.4 - 89.0) %
	± 5.380	80 %	(76.6 - 78.5) %
IV/1970	± 7.006	90 %	(87.6 - 89.0) %
	± 5.459	80 %	(77.0 - 78.9) %

1/ Andersen-Jordan equation:

(a) Sample period I/1953 to IV/1969

(b) Almon specification: Both lag distributions employ fourth degree polynomials with lags t+1 and t-5 constrained to equal zero.

TECHNICAL APPENDIX E

Monthly Forecasts of the Money Multiplier

The money multiplier was forecasted with the following equation:

$$\hat{m}_t = b_0 + b_1 X_{1t} + b_2 X_{2t} + \sum_{i=1}^{11} b_{i+2} d_i + \rho\mu_{t-1}$$

where:

$\hat{m}_t$ = the predicted money multiplier,

X_1 = the average multiplier in the preceding three months,

X_2 = the reserve adjustment magnitude in month (t),

d_i = dummy variables to account for seasonal factors. Dummy variables equal to 1 for month (t). December is not accounted for by a dummy, since an intercept (b_0) was used in the equation.

$\rho\mu_{t-1}$ = The serial correlation adjustment term. ρ is equal to one minus the value of the Durbin-Watson value divided by two; and μ_{t-1} is the last residual in the regression equation.

b_i's = estimates made by multiple regression analysis using the last 36 months as observations.

Tables I and II present the monthly predicted and actual values of the money multiplier. Table II gives the estimates of the parameters used for making the 72 monthly forecasts of money multipliers made in the simulations of this paper. The numbers in parenthesis are t-values.

Table I

Money Multiplier: Predicted and Actual 1962-65

		Predicted	Actual			Predicted	Actual
1962	1	2.980	2.969	1964	1	2.948	2.943
	2	2.954	2.946		2	2.925	2.906
	3	2.930	2.921		3	2.887	2.871
	4	2.941	2.937		4	2.906	2.896
	5	2.903	2.878		5	2.852	2.836
	6	2.887	2.863		6	2.833	2.823
	7	2.869	2.843		7	2.817	2.832
	8	2.864	2.840		8	2.831	2.834
	9	2.874	2.850		9	2.847	2.850
	10	2.872	2.864		10	2.871	2.871
	11	2.891	2.912		11	2.893	2.873
	12	2.947	2.934		12	2.886	2.879
1963	1	2.975	2.968	1965	1	2.923	2.921
	2	2.959	2.936		2	2.889	2.869
	3	2.922	2.909		3	2.849	2.852
	4	2.933	2.936		4	2.880	2.882
	5	2.884	2.883		5	2.825	2.807
	6	2.868	2.873		6	2.804	2.813
	7	2.854	2.865		7	2.808	2.805
	8	2.859	2.872		8	2.805	2.803
	9	2.887	2.879		9	2.817	2.836
	10	2.893	2.904		10	2.846	2.848
	11	2.922	2.914		11	2.849	2.848
	12	2.944	2.893		12	2.853	2.861

Table II

Money Multiplier: Predicted and Actual 1966-69

Year	Month	Predicted	Actual	Year	Month	Predicted	Actual
1966	1	2.894	2.903	1968	1	2.816	2.811
	2	2.854	2.850		2	2.755	2.746
	3	2.830	2.850		3	2.742	2.755
	4	2.862	2.886		4	2.769	2.794
	5	2.813	2.805		5	2.710	2.749
	6	2.815	2.819		6	2.746	2.766
	7	2.805	2.763		7	2.733	2.752
	8	2.709	2.765		8	2.751	2.740
	9	2.723	2.779		9	2.763	2.764
	10	2.756	2.778		10	2.773	2.762
	11	2.767	2.769		11	2.761	2.781
	12	2.759	2.782		12	2.792	2.812
1967	1	2.802	2.785	1969	1	2.827	2.833
	2	2.733	2.734		2	2.783	2.784
	3	2.741	2.753		3	2.797	2.802
	4	2.785	2.774		4	2.834	2.834
	5	2.710	2.726		5	2.783	2.749
	6	2.733	2.753		6	2.779	2.783
	7	2.733	2.741		7	2.758	2.784
	8	2.754	2.746		8	2.761	2.753
	9	2.788	2.763		9	2.771	2.774
	10	2.796	2.771		10	2.771	2.776
	11	2.789	2.774		11	2.765	2.764
	12	2.798	2.793		12	2.777	2.776

Table III

PARAMETER ESTIMATES OF FORECASTING REGRESSION EQUATIONS AND SERIAL CORRELATION ADJUSTMENT

	Constant	Moving Average of the Multiplier	Reserve Adjustment Magnitude	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Durbin Watson	ρu_{t-1}	R^2
1962 Jan.	.92	.68	.01	.03											.61	-.01	.86
		(5.25)	(2.36)	(1.87)													
Feb.	.95	.67	.01		-.01										.59	-.01	.87
		(5.13)	(2.30)		(.47)												
March	.97	.67	.01			-.04									.57	-.01	.87
		(5.02)	(2.21)			(2.09)											
April	.92	.68	.01				-.03								.54	-.004	.88
		(5.18)	(2.28)				(1.46)										
May	.92	.69	.01					-.06							.53	-.003	.88
		(5.19)	(2.27)					(3.45)									
June	.89	.70	.01						-.05						.62	-.01	.88
		(5.34)	(2.35)						(3.29)								
July	.89	.70	.01							-.06					.71	-.01	.90
		(5.83)	(2.80)							(4.09)							
Aug.	.93	.68	.01								-.04				.74	-.01	.92
		(6.23)	(3.37)								(3.28)						
Sept.	.99	.66	.01									-.02			.63	-.01	.93
		(6.11)	(3.60)									(1.93)					
Oct.	.96	.67	.01										-.02		.62	-.01	.92
		(5.63)	(3.11)										(1.53)				
Nov.	.99	.66	.01											-.02	.60	-.01	.92
		(5.40)	(3.11)											(1.57)			
Dec.	1.19	.59	.01												.52	.01	.93
		(4.86)	(3.82)														

Table III

PARAMETER ESTIMATES OF FORECASTING REGRESSION EQUATIONS AND SERIAL CORRELATION ADJUSTMENT

		Constant	Moving Average of the Multiplier	Reserve Adjustment Magnitude	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Durbin Watson	ρu_{t-1}	R^2
1963	Jan.	1.09	.62 (4.98)	.01 (3.14)	.02 (1.51)											.60	-.004	.93
	Feb.	1.04	.64 (4.92)	.01 (2.70)		-.01 (.92)										.61	-.01	.93
	March	1.01	.65 (4.73)	.01 (2.30)			-.05 (2.76)									.61	-.01	.93
	April	.93	.68 (4.85)	.01 (1.83)				-.04 (1.92)								.58	-.01	.92
	May	.85	.71 (5.19)	.01 (1.36)					-.08 (4.72)							.65	-.01	.92
	June	.79	.74 (5.68)	.00 (.79)						-.09 (5.52)						.74	-.004	.92
	July	.81	.73 (5.95)	.00 (.40)							-.09 (6.47)					.83	-.002	.92
	Aug.	.90	.71 (5.93)	.00 (.10)								-.07 (5.43)				.85	-.002	.91
	Sept.	.94	.69 (5.60)	.00 (.25)									-.05 (3.47)			.78	.01	.90
	Oct.	.81	.73 (5.71)	.00 (.62)										-.04 (2.88)		.75	.00	.90
	Nov.	.76	.75 (5.39)	.00 (.85)											-.03 (1.94)	.67	.01	.89
	Dec.	1.04	.66 (4.53)	-.00 (.14)												.45	-.00	.89

Table III

PARAMETER ESTIMATES OF FORECASTING REGRESSION EQUATIONS AND SERIAL CORRELATION ADJUSTMENT

		Constant	Moving Average of the Multiplier	Reserve Adjustment Magnitude	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Durbin Watson	ρu_{t-1}	R^2
1964	Jan.	.86	.71	.00	.04											.70	-.02	.87
			(4.15)	(.20)	(3.01)													
	Feb.	.60	.80	.00		-.01										.73	-.01	.87
			(4.43)	(.03)		(.48)												
	March	.30	.90	.00			-.05									.77	-.01	.87
			(5.04)	(.38)			(3.02)											
	April	.17	.94	.00				-.02								.79	-.01	.88
			(5.53)	(.55)				(1.52)										
	May	.12	.96	.00					-.06							.82	-.00	.87
			(5.83)	(.65)					(4.65)									
	June	.14	.95	.01						-.06						.86	-.00	.90
			(6.36)	(.82)						(5.09)								
	July	.18	.94	.01							-.06					.86	-.00	.91
			(6.54)	(.88)							(5.49)							
	Aug.	.35	.88	.01								-.04				.87	.00	.92
			(6.10)	(.86)								(3.49)						
	Sept.	.39	.87	.01									-.02			.84	.00	.92
			(5.86)	(.86)									(1.88)					
	Oct.	.48	.84	.01										-.01		.82	.00	.93
			(5.49)	(.87)										(.64)				
	Nov.	.45	.84	.01											.01	.81	.00	.93
			(5.36)	(.87)											(.53)			
	Dec.	.27	.91	.00												.84	-.01	.92
			(5.63)	(.72)														

Table III

PARAMETER ESTIMATES OF FORECASTING REGRESSION EQUATIONS AND SERIAL CORRELATION ADJUSTMENT

		Constant	Moving Average of the Multiplier	Reserve Adjustment Magnitude	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Durbin Watson	ρu_{t-1}	R^2
1965	Jan.	.20	.93	.01	.03											.79	-.01	.92
			(5.78)	(.84)	(2.51)													
	Feb.	.15	.94	.01		-.02										.77	-.00	.91
			(6.05)	(.83)		(1.49)												
	March	.00	.99	.01			-.06									.81	-.01	.90
			(6.62)	(1.16)			(3.93)											
	April	.02	.98	.01				-.02								.80	.00	.91
			(7.08)	(1.48)				(1.87)										
	May	.06	.96	.01					-.07							.79	.00	.90
			(7.32)	(1.57)					(5.52)									
	June	-.02	.99	.01						-.06						.84	-.01	.91
			(7.76)	(1.57)						(5.11)								
	July	.00	.98	.02							-.05					.84	.00	.92
			(7.97)	(1.53)							(4.30)							
	Aug.	-.01	.99	.01								-.03				.89	-.00	.92
			(8.43)	(1.02)								(2.07)						
	Sept.	.00	1.00	.00									.01			.97	-.00	.93
			(8.99)	(.33)									(.87)					
	Oct.	.16	.96	-.01										.00		1.18	.00	.93
			(8.83)	(.29)										(.23)				
	Nov.	2.96	.52	-.31											.01	2.43	-.00	.98
			(5.99)	(6.79)											(2.01)			
	Dec.	2.99	.52	-.31												2.27	.00	.98
			(5.16)	(5.78)														

Table III

PARAMETER ESTIMATES OF FORECASTING REGRESSION EQUATIONS AND SERIAL CORRELATION ADJUSTMENT

		Constant	Moving Average of the Multiplier	Reserve Adjustment Magnitude	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Durbin Watson	ρu_{t-1}	R^2
1966	Jan.	1.93	.68 (6.57)	-.20 (3.34)	.04 (7.19)											2.50	-.00	.98
	Feb.	1.60	.73 (5.99)	-.16 (2.18)		-.01 (1.82)										2.40	-.00	.98
	March	1.31	.77 (6.07)	-.13 (1.61)			-.04 (6.49)									2.51	.00	.98
	April	.95	.82 (5.61)	-.08 (.88)				-.01 (1.25)								2.32	-.00	.97
	May	.42	.89 (5.36)	-.02 (.16)					-.06 (8.37)							1.76	.00	.96
	June	.05	.92 (6.38)	.03 (.36)						-.05 (6.39)						2.30	.00	.97
	July	-.28	.96 (8.09)	.08 (1.08)							-.03 (4.76)					2.35	-.00	.97
	Aug.	-1.81	1.20 (9.50)	.24 (3.43)								.00 (-.12)				1.68	-.00	.96
	Sept.	-.64	1.01 (10.43)	.12 (3.75)									.01 (1.37)			2.35	-.00	.96
	Oct.	-.19	.96 (9.33)	.06 (3.63)										.02 (2.39)		2.04	-.00	.96
	Nov.	.00	.91 (8.36)	.05 (3.68)											.01 (1.30)	1.86	.00	.96
	Dec.	.17	.85 (7.62)	.05 (4.19)												1.73	.00	.96

Table III

PARAMETER ESTIMATES OF FORECASTING REGRESSION EQUATIONS AND SERIAL CORRELATION ADJUSTMENT

		Constant	Moving Average of the Multiplier	Reserve Adjustment Magnitude	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Durbin Watson	ρu_{t-1}	R^2
1967	Jan.	.18	.87	.04	.03											1.71	.00	.96
			(7.23)	(3.53)	(2.85)													
	Feb.	.07	.90	.04		-.04										1.82	-.00	.96
			(6.97)	(3.44)		(3.31)												
	March	.25	.83	.05			-.05									1.78	.00	.96
			(6.45)	(3.79)			(4.63)											
	April	.34	.79	.05				-.01								1.75	.00	.96
			(7.69)	(4.44)				(1.16)										
	May	.18	.86	.05					-.08							1.77	-.00	.96
			(10.83)	(4.52)					(8.23)									
	June	.32	.80	.05						-.06						1.78	.00	.97
			(11.50)	(5.36)						(6.15)								
	July	.40	.77	.05							-.06					1.31	.00	.97
			(11.58)	(5.60)							(6.51)							
	Aug.	.51	.72	.06								-.04				1.53	.00	.97
			(12.75)	(6.86)								(4.19)						
	Sept.	.54	.71	.06									-.01			1.71	.00	.98
			(14.16)	(7.64)									(.70)					
	Oct.	.48	.74	.06										.01		1.70	-.00	.98
			(14.15)	(6.76)										(.80)				
	Nov.	.43	.76	.05											-.00	1.48	-.00	.97
			(14.02)	(5.92)											(.48)			
	Dec.	.41	.78	.05												1.34	-.00	.97
			(14.36)	(5.63)														

Table III

PARAMETER ESTIMATES OF FORECASTING REGRESSION EQUATIONS AND SERIAL CORRELATION ADJUSTMENT

		Constant	Moving Average of the Multiplier	Reserve Adjustment Magnitude	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Durbin Watson	$\rho\mu_{t-1}$	R^2
1968	Jan.	.39	.78	.05	.03											1.30	-.00	.97
			(14.71)	(5.58)	(2.73)													
	Feb.	.41	.78	.05		-.04										1.31	-.00	.96
			(14.89)	(5.54)		(3.68)												
	March	.42	.78	.05			-.04									1.25	-.00	.96
			(14.69)	(5.55)			(4.14)											
	April	.40	.79	.04				-.01								1.30	.00	.96
			(14.44)	(5.56)				(.82)										
	May	.41	.79	.04					-.07							1.23	.01	.95
			(13.00)	(4.84)					(6.78)									
	June	.42	.80	.03						-.04						1.04	.01	.94
			(11.01)	(3.61)						(3.56)								
	July	.40	.81	.03							-.06					.73	.01	.93
			(10.52)	(3.06)							(4.72)							
	Aug.	.43	.81	.02								-.03				.67	.02	.93
			(9.80)	(2.55)								(2.50)						
	Sept.	.44	.81	.02									-.01			.73	.00	.93
			(9.78)	(2.54)									(.49)					
	Oct.	.46	.80	.02										.00		.75	.00	.93
			(9.90)	(2.42)										(.14)				
	Nov.	.50	.79	.02											-.01	.81	.00	.93
			(9.95)	(2.37)											(.72)			
	Dec.	.54	.78	.02												.84	.01	.92
			(9.53)	(2.06)														

Table III

PARAMETER ESTIMATES OF FORECASTING REGRESSION EQUATIONS AND SERIAL CORRELATION ADJUSTMENT

		Constant	Moving Average of the Multiplier	Reserve Adjustment Magnitude	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Durbin Watson	ρu_{t-1}	R^2
1969	Jan.	.56	.78	.02	.01											.67	.01	.90
			(8.81)	(1.68)	(.78)													
	Feb.	.65	.76	.01		-.05										.69	.01	.88
			(8.62)	(1.16)		(3.92)												
	March	.77	.72	.00			-.04									.72	.01	.88
			(8.50)	(.51)			(2.93)											
	April	.87	.70	.00				.00								.73	.01	.86
			(7.76)	(.02)				(.27)										
	May	1.08	.63	-.01					-.05							.65	.01	.83
			(6.69)	(.69)					(4.04)									
	June	1.21	.57	-.00						-.03						.87	-.01	.82
			(4.97)	(.28)						(2.05)								
	July	1.43	.50	-.00							-.05					.71	.00	.81
			(3.39)	(.61)							(4.09)							
	Aug.	.66	.77	.00								-.04				.84	.01	.83
			(3.78)	(.18)								(3.60)						
	Sept.	.09	.96	.01									-.02			.95	-.00	.84
			(3.92)	(.92)									(1.56)					
	Oct.	-.19	1.06	.01										-.01		.97	-.00	.84
			(4.11)	(1.26)										(.90)				
	Nov.	-.17	1.05	.01											-.01	.95	.00	.84
			(4.19)	(1.31)											(1.18)			
	Dec.	-.40	.22	.01												1.06	-.00	.86
			(5.07)	(1.95)														

TECHNICAL APPENDIX F

Controlled Compared to Desired Monthly Averages of the Money Stock

Quarterly averages of the controlled and the desired (constant 4 per cent rate) money stock were presented in the body of this paper in the October 1971 Review. This appendix presents the monthly money stock levels achieved by the control process and charts these levels for the two sample periods 1962-65 and 1966-69. Quarterly averages are computed by averaging the monthly values.

Table I

Controlled Compared to Desired Monthly
Money Stock Levels: 1962-65
(billions of dollars)

Year	Month	Controlled	Desired	Controlled minus Desired
1962	1	$145.569	$146.085	$ -.516
	2	146.164	146.570	-.406
	3	146.589	147.056	-.467
	4	147.342	147.541	-.199
	5	146.755	148.026	-1.271
	6	147.263	148.512	-1.249
	7	147.689	148.997	-1.308
	8	148.226	149.483	-1.257
	9	148.754	149.968	-1.214
	10	150.052	150.453	-.401
	11	152.029	150.939	1.090
	12	150.750	151.424	-.674
1963	1	151.553	151.909	-.356
	2	151.210	152.395	-1.185
	3	152.190	152.880	-.690
	4	153.512	153.365	.147
	5	153.812	153.850	-.038
	6	154.646	154.336	.310
	7	155.454	154.821	.633
	8	156.050	155.307	.743
	9	155.371	155.792	-.421
	10	156.843	156.277	.566
	11	156.328	156.763	-.435
	12	154.519	157.248	-2.729
1964	1	157.455	157.733	-.278
	2	157.215	158.218	-1.003
	3	157.836	158.704	-.868
	4	158.651	159.189	-.538
	5	158.809	159.675	-.866
	6	159.598	160.160	-.562
	7	161.498	160.645	.853
	8	161.341	161.131	.210
	9	161.772	161.616	.156
	10	162.068	162.101	-.033
	11	161.432	162.587	-1.155
	12	162.688	163.072	-.384
1965	1	163.464	163.557	-.093
	2	162.860	164.042	-1.182
	3	164.700	164.528	.172
	4	165.170	165.013	.157
	5	164.429	165.498	-1.069
	6	166.497	165.984	.513
	7	166.284	166.469	-.185
	8	166.818	166.954	-.136
	9	168.606	167.440	1.166
	10	168.067	167.925	.142
	11	168.375	168.410	-.035
	12	169.375	168.896	.479

Controlled Compared to Desired Monthly
Money Stock Levels: 1966-69
(billions of dollars)

Year	Month	Controlled	Desired	Controlled minus Desired
1966	1	$168.232	$167.657	$ .575
	2	167.954	168.214	-.260
	3	169.957	168.771	1.186
	4	170.735	169.328	1.407
	5	169.401	169.885	-.484
	6	170.695	170.442	.253
	7	168.428	170.999	-2.571
	8	175.087	171.556	3.531
	9	175.664	172.113	3.551
	10	174.091	172.670	1.421
	11	173.352	173.227	.125
	12	175.206	173.784	1.422
1967	1	173.290	174.341	-1.051
	2	174.965	174.898	.067
	3	176.197	175.455	.742
	4	175.351	176.012	-.661
	5	177.637	176.569	1.068
	6	178.396	177.126	1.270
	7	178.245	177.683	.562
	8	177.759	178.240	-.481
	9	177.188	178.797	-1.609
	10	177.754	179.354	-1.600
	11	178.954	179.911	-.957
	12	180.159	180.468	-.309
1968	1	180.704	181.025	-.321
	2	180.977	181.582	-.605
	3	183.004	182.139	.865
	4	184.372	182.696	1.676
	5	185.891	183.253	2.638
	6	185.142	183.810	1.332
	7	185.682	184.367	1.315
	8	184.135	184.924	-.789
	9	185.565	185.481	.084
	10	185.333	186.038	-.705
	11	187.935	186.595	1.340
	12	188.504	187.152	1.352
1969	1	188.137	187.709	.428
	2	188.347	188.266	.081
	3	189.146	188.823	.323
	4	189.370	189.380	-.010
	5	187.605	189.937	-2.332
	6	190.733	190.494	.239
	7	192.831	191.051	1.780
	8	191.026	191.608	-.582
	9	192.380	192.165	.215
	10	193.083	192.722	.361
	11	193.221	193.279	-.058
	12	193.796	193.836	-.040

Table II

Deviations of Monthly Controlled Money
From Desired Money Stock
Levels: Summary of Results

		1962-65	1966-69
Mean value of deviations:	Absolute	.636	.972
	With Sign	-.331	.329
Variance of deviations:	Absolute	.254	.725
	With Sign	.550	1.561
Mean square deviations:		.659	1.669
Root mean square deviations:		.812	1.292

Controlled Compared to Desired Monthly Money Stock Levels 1962-1965 and 1966-1969