

# Country Size, Trade, and Productivity: An analysis of heterogenous firms and differential beachhead costs\*

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## Abstract

This paper modifies the heterogenous firms and trade model by Melitz (2003) by explicitly modelling the beachhead cost of a firm in a new market as a function of market size. This leads to several new predictions compared to the standard model. In particular, the productivity of non exporters and exporters depends on market size. Moreover, manufacturing export shares vary inversely with market size. However, export shares converge (upwards) as markets are integrated. The empirical part of the paper offers support for our model specification.

*JEL Classification:* H32, P16

*Keywords:* heterogenous firms, market size, beachhead costs

## 1 Introduction

It is empirically well established that there are systematic productivity differences among firms; see Tybout (2003) for a survey.<sup>1</sup> In particular, exporting firms tend to be more productive, larger, and live longer than domestic firms. There is also evidence that multinational firms tend to be more productive than exporters (Helpman et al. (2004)).

These empirical results have spurred the development of a new theoretical literature on trade with heterogenous firms. The explanation for the empirical finding that exporters are

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<sup>1</sup>Other studies include Aw et al. (2000), Bernard och Jensen (1995, 1999a, 1999b, 2001), Clerides et al. (1998) as well as Eaton et al. (2004).

more productive than non-exporters is either iceberg trade costs associated with exports, as in Bernard et al. (2003), or higher fixed costs associated with market entry into a foreign market, as in Melitz (2003) and Yeaple (2004). Only the most productive firms will find it profitable to pay the additional cost necessary for exports, and export firms will therefore, on average, be more productive.

We here investigate whether patterns of heterogeneity across firms and differences between non-exporters and exporters vary systematically with country size. That country or market size is of importance is indicated by Syverson (2004, 2006) who present empirical evidence of firms being more productive in larger (denser) markets. There are also stylized facts indicating that country size affects the relative performance of exporters to non-exporters. Schank et al. (2006) offer a literature overview where they measure the wage premium of exporter firms compared to non-exporter firms. Typically, a regression is run on firm level data with some measure of wages as the dependent variable, and with a dummy variable indicating whether the firm is an exporter or not. The estimated coefficient for this dummy variable is the exporter wage premium as compared to non-exporters. We interpret this wage premium to indicate productivity differences between exporters and non-exporters.<sup>2</sup> Figure 1 plots the exporter wage premium versus population size of countries in the studies surveyed in the appendix of Schank et al. (2006). We have also added an observation for Sweden using data provided by Statistics Sweden. Naturally, it must be acknowledged that all regressions are not done with exactly the same methodology or fully comparable data. Nevertheless, Figure 1 shows a negative correlation between export premium and population size. Running a regression on this data gives a slope of  $-0.605$  with a  $t$  value of  $-3.68$ .

This paper suggests one channel through which country size can affect exporter productivity premium in a way consistent with Figure 1; namely, that country size affects the size of the beachhead cost that firms must pay when entering a new market. (We will use the term beachhead cost for the domestic as well as the foreign market.) In particular, we assume that the beachhead cost in a market has a fixed and a market size dependent component. The fixed part may e.g. be related to standardization of the product for the market or to creating a marketing message for this particular market. The market size dependent component of the beachhead cost is interpreted as the marketing cost of introducing a new variety in a market. It is natural that this cost depends on the size of the market and, for instance, the marketing cost of establishing a new product in a large market such as the U.S. is much higher than in a small market. That the fixed entry cost depends on market size is normally taken for granted in the marketing literature, where the marketing cost over sales ratio is a key variable<sup>3</sup>.

We introduce the market size dependent beachhead cost into the Helpman et al. (2004) (HMY) version of the Melitz (2003) model. HMY analyse a model version with a freely traded

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<sup>2</sup>Naturally, this interpretation implies non-competitive wage setting. E.g. efficiency wages a la Shapiro and Stiglitz (1984) combined with frictions preventing free worker mobility between firms.

<sup>3</sup>See e.g. Buzzell et al. (1975).

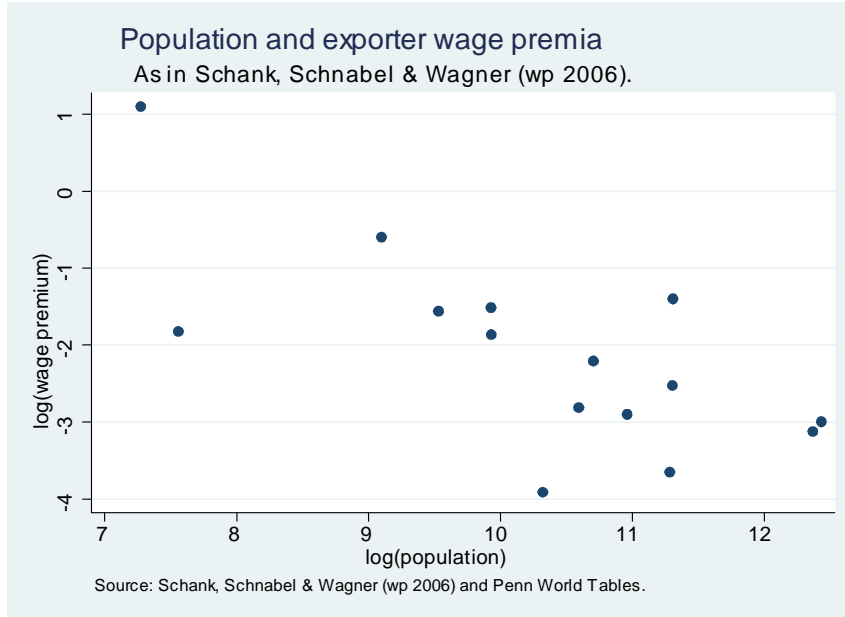


Figure 1: Export premiums decrease in country size.

homogenous good which fixes the factor price (wage). This allows for an analytical treatment of countries of asymmetric size. Since our focus is precisely on country size, we employ the HMY framework. Several new results emerge from our analysis. First, exporters as well as non-exporters in a large market are, on average, more productive than in a smaller market. Second, as in Melitz (2003), exporters are more productive than non-exporters. However, in line with the stylised evidence above, the productivity premium between exporters and non-exporters decreases with the home country size. Finally, we derive a set of new results related to trade volume. Contrary to what would be the case in the HMY framework, the manufacturing export share decreases in the size of the exporting country. Moreover, it is shown that as the result of globalization, for instance, export shares converge as the fixed entry cost of exporters into each market declines.

The theoretical results are supported by the empirical section of the paper. Manufacturing export shares are affected by market size in accordance with our theoretical predictions, and we also find strong evidence of manufacturing export shares converging over time. Finally, we show how productivity is positively associated with market size in line with our theoretical model.

Our analysis is related to Melitz and Ottaviano (2005) who introduce firm heterogeneity *a la* Melitz (2003) in the model by Ottaviano et al. (2002) with a linear demand system and where the endogenous mark-ups of monopolistically competitive firms depend on market size. Melitz and Ottaviano (2005) find that firms selling to large markets are larger and more productive, since higher competition forces down the mark-ups in a large market. The same holds in our model, but the mechanism leading to higher productivity in a large market is instead that firms need to be more productive to afford the higher beachhead cost associated with a larger market.

A difference as compared to Melitz and Ottaviano (2005) is that the productivity of firms in a market also depends on the size of other markets in our model. E.g. a larger foreign market implies more competition from imports, which forces up the productivity of domestic firms. One consequence of this dependence of the foreign market size is that export shares will vary with market size. The result that trade shares converge as the entry cost into foreign markets falls is naturally not present in Melitz and Ottaviano (2005), since they do not employ any beachhead costs.

Arkolakis (2006) presents a model of heterogenous firms, related to ours, where the marketing cost of each firm is convex in the share of consumers to be reached by the marketing message in a given market. The set-up implies scale economies in marketing so that the marginal firm to survive in a larger market is less productive than the corresponding firm in a smaller market. Average firm productivity is therefore lower in a larger market. Our model, on the contrary, implies that firms are more productive in large markets, since the variable component of the beachhead cost is higher in such a market. This feature is supported by the empirical part of our paper. Our results regarding the effect of falling fixed export entry costs on export shares have no correspondence in the model by Arkolakis (2006).

The paper is organized as follows: Section 2 contains the model and section 3 presents the theoretical results. Section 4 contains empirical tests of our theoretical predictions. Finally, section 5 concludes.

## 2 The Model

This paper employs a modified Helpman et al. (2004) version of Melitz' (2003) monopolistic competition trade model with heterogeneous firms.

### 2.1 Basics

There are two countries, home and foreign (denoted by  $**$ ), and a single primary factor of production labour,  $L$ , used in the A-sector and the M-sector. The A-sector is a Walrasian, homogenous-goods sector with costless trade. The M-sector (manufactures) is characterized by increasing returns, Dixit-Stiglitz monopolistic competition and iceberg trade costs. M-sector firms face constant marginal production costs and three types of fixed costs. The first fixed cost,  $F_E$ , is the standard Dixit-Stiglitz cost of developing a new variety. The second and third fixed costs are 'beachhead' costs reflecting the one-time expense of introducing a new variety into a market. These costs are here assumed to depend on the size of the market.

There is heterogeneity with respect to firms' marginal costs. Each Dixit-Stiglitz firm/variety is associated with a particular labour input coefficient – denoted as  $a_j$  for firm  $j$ . After sinking  $F_E$  units of labour in the product innovation process, the firm is randomly assigned an ' $a_j$ ' from a probability distribution  $G(a)$ .

Our analysis exclusively focuses on steady-state equilibria and intertemporal discounting is

ignored; the present value of firms is kept finite by assuming firms to face a constant Poisson hazard rate  $\delta$  of ‘death’.

Consumers in each nation have two-tier utility functions with the upper tier (Cobb-Douglas) determining the consumer’s division of expenditure among the sectors and the second tier (CES) dictating the consumer’s preferences over the various differentiated varieties within the M-sector.

All individuals in country  $k$  have the utility function

$$U_k = C_M^\mu C_A^{1-\mu}, \quad (1)$$

where  $k = H, F$ ,  $\mu \in (0, 1)$ , and  $C_A$  is consumption of the homogenous good. Manufactures enter the utility function through the index  $C_M$ , defined by

$$C_M = \left[ \int_0^n c_i^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}, \quad (2)$$

$n$  being the mass of varieties consumed,  $c_i$  the amount of variety  $i$  consumed and  $\sigma > 1$  the elasticity of substitution.

Each consumer spends a share  $\mu$  of his income on manufactures, and demand for a domestically produced variety  $i$  is therefore

$$x_i = \frac{p_i^{-\sigma}}{P^{1-\sigma}} \mu Y, \quad (3)$$

where  $p_i$  is the consumer price of variety  $i$ ,  $Y$  is income, and  $P \equiv \left( \int_0^n p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$  the price index of manufacturing goods.

The unit factor requirement of the homogeneous good is one unit of labour. This good is freely traded, and since it is chosen as the numeraire

$$p_A = w = 1, \quad (4)$$

$w$  being the nominal wage of workers in all countries.

Shipping the manufactured good involves a frictional trade cost of the ‘iceberg’ form: for one unit of good from country  $j$  to arrive in country  $k$ ,  $\tau > 1$  units must be shipped. Trade costs are assumed to be equal in both directions. Profit maximization by manufacturing  $i$  firms leads to price

$$p_i = \frac{\sigma}{\sigma - 1} a_i, \quad p_i = \frac{\sigma}{\sigma - 1} \tau a_i \quad (5)$$

in the domestic and foreign market, respectively.

Manufacturing firms draw their marginal cost,  $a$ , from the probability distribution  $G(a)$  after having sunk  $F_E$  units of labour to develop a new variety.

Having learned their productivity, firms decide on entry in the domestic and foreign market. Firms will enter a market as long as the operating profit in this market is sufficiently large to cover the fixed beachhead cost associated with this market. Because of the constant mark-up

pricing, it is easily shown that operating profits equal sales divided by  $\sigma$ . Using this and (3), the critical 'cut-off' levels of the marginal costs for the two countries are given by:

$$a_D^{1-\sigma} B = F_D(L), \quad (6)$$

$$a_X^{1-\sigma} \phi B^* = F_X(L^*), \quad (7)$$

$$a_D^{*1-\sigma} B^* = F_D(L^*), \quad (8)$$

$$a_X^{*1-\sigma} \phi B = F_X(L), \quad (9)$$

where  $F_D \equiv \delta\sigma\tilde{F}_D$ ,  $F_X \equiv \delta\sigma\tilde{F}_X$ ,  $B = \frac{\mu L}{P^{1-\sigma}}$ ,  $B^* = \frac{\mu L^*}{P^{*(1-\sigma)}}$ , and  $\phi \equiv \tau^{1-\sigma} \in [0, 1]$  represents trade freeness. It is assumed that the fixed market entry cost (beachhead cost) increases in the size of the market  $\frac{dF_D}{dL} > 0$ ,  $\frac{dF_X}{dL} > 0$ . We will parametrize how the beachhead cost depends on market size below. Note, however, that it is natural that  $F$  depends on  $L$ , since the marketing costs of establishing a new brand in a large market, such as e.g. the US, are much higher than in a small country.

Finally, free entry ensures that the ex-ante expected profit of developing a new variety equals the investment cost in both countries:

$$\int_0^{a_D} (a_D^{1-\sigma} B - F_D(L)) dG(a) + \int_0^{a_X} (\phi a_X^{1-\sigma} B^* - F_X(L^*)) dG(a) = F_E, \quad (10)$$

$$\int_0^{a_D^*} (a_D^{*(1-\sigma)} B^* - F_D(L^*)) dG(a) + \int_0^{a_X^*} (\phi a_X^{*(1-\sigma)} B - F_X(L)) dG(a) = F_E. \quad (11)$$

## 2.2 Solving for the Long-run Equilibrium

We follow HMY in assuming that the probability density function is Pareto<sup>4</sup>:

$$G(a) = a^k. \quad (12)$$

Substituting the cut-off conditions (6), (7), (8), and (9) into the free-entry conditions (10) and (11) gives  $B$ , and  $B^*$ ,

$$B = \left( \frac{F_E F_D^{\beta-1}(L) \cdot (\beta-1)(1-\Omega(L^*))}{1-\Omega(L)\Omega(L^*)} \right)^{\frac{1}{\beta}} \quad (13)$$

$$B^* = \left( \frac{F_E F_D^{\beta-1}(L^*) \cdot (\beta-1)(1-\Omega(L))}{1-\Omega(L)\Omega(L^*)} \right)^{\frac{1}{\beta}}, \quad (14)$$

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<sup>4</sup>This assumption is consistent with the empirical findings by Axtell (2001).

where  $\beta \equiv \frac{k}{\sigma-1} > 1$ , and  $\Omega(L^j) \equiv \phi^\beta \left( \frac{F_X(L^j)}{F_D(L^j)} \right)^{1-\beta} \in [0, 1]$  is an index of trade costs.

Using (13), (14) and the cut-off conditions, gives the cut-off marginal costs:

$$a_D^k = \frac{(\beta-1)F_E}{F_D(L)} \left( \frac{(1-\Omega(L^*))}{1-\Omega(L)\Omega(L^*)} \right), \quad a_D^{*k} = \frac{(\beta-1)F_E}{F_D(L^*)} \left( \frac{(1-\Omega(L))}{1-\Omega(L)\Omega(L^*)} \right), \quad (15)$$

$$a_X^k = \frac{(\beta-1)\Omega(L^*)F_E}{F_X(L^*)} \left( \frac{(1-\Omega(L))}{1-\Omega(L)\Omega(L^*)} \right), \quad a_X^{*k} = \frac{(\beta-1)\Omega(L)F_E}{F_X(L)} \left( \frac{(1-\Omega(L^*))}{1-\Omega(L)\Omega(L^*)} \right). \quad (16)$$

From these it is seen that, contrary to the standard model by Melitz (2003), the market size will typically affect the cut-off marginal costs. We will assume that  $\frac{F_X^j}{\Omega^j} > F_D^k$  for all  $j, k$ . As shown **below**, this assumption implies that  $a_X^j < a_D^j \forall j$ .

The price indices may be written as

$$P^{1-\sigma} = \frac{\beta}{\beta-1} \left( na_D^{1-\sigma} + n^* \phi a_D^{*(1-\sigma)} \left( \frac{a_X^*}{a_D^*} \right)^{k+1-\sigma} \right), \quad (17)$$

$$P^{*(1-\sigma)} = \frac{\beta}{\beta-1} \left( n \phi a_D^{(1-\sigma)} \left( \frac{a_X}{a_D} \right)^{k+1-\sigma} + n^* a_D^{*(1-\sigma)} \right), \quad (18)$$

and the mass of firms in each country can be calculated using (13), (14), (15), and (16) together with the fact that  $B = \frac{\mu L}{P^{1-\sigma}}$ , and  $B^* = \frac{\mu L^*}{P^{*(1-\sigma)}}$ :

$$n = \frac{\mu(\beta-1)L(1-\Omega(L)) - L^*\Omega(L)(1-\Omega(L^*))}{F_D(L)\beta(1-\Omega(L)\Omega(L^*))} \quad (19)$$

$$n^* = \frac{\mu(\beta-1)L^*(1-\Omega(L^*)) - L\Omega(L^*)(1-\Omega(L))}{F_D(L^*)\beta(1-\Omega(L)\Omega(L^*))} \quad (20)$$

Welfare may be measured by indirect utility, which is proportional to the real wage  $\frac{w}{p_A^{1-\mu} P^u}$ . Since  $p_A = w = 1$ , it suffices to examine  $P$ . Using (17), (15), (16), (19), and (20) we have

$$P = \left( \mu^{-\beta} L^{-\beta} F_D^{\beta-1}(L) F_E (\beta-1) \cdot \frac{1-\Omega(L^*)}{1-\Omega(L)\Omega(L^*)} \right)^{\frac{1}{\beta(\sigma-1)}}. \quad (21)$$

This expression shows that, as in the Melitz (2003) model, welfare always increases ( $P$  decreases) with trade liberalisation; that is with higher  $\phi$  or lower  $\frac{F_X}{F_D}$ .

### 2.2.1 Parametrisation of the beachhead cost

In the following text, we parametrise the beachhead costs as:

$$\tilde{F}_D(L^j) = f_D + (L^j)^\gamma, \quad \tilde{F}_X(L^j) = f_X + (L^j)^\gamma, \quad \gamma > 0. \quad (22)$$

The variable component of the beachhead cost increases in market size, while the constant term picks up costs that are independent of market size. It is quite natural that the beachhead cost would have one fixed and one variable component. The constant  $f$  could be the fixed cost

of standardizing a product for a particular market or the cost of producing an advertisement tailored to a particular market with its culture and language. The variable cost term  $L^\gamma$  represents the fact that the cost of spreading an advertising message increases with the number of consumers targeted. For instance, the number of free product samples or advertising posters increases with the size of the population. Likewise, the cost of television advertising increases with the number of viewers. We do not put any restriction on the shape of the variable cost term except  $\gamma > 0$ .

### 3 Results

A large number of comparative static results may be derived. Here, we focus on the more novel aspects of our model, which are related to the effects of market size. From now on, the simplified notation  $F_D^j \equiv F_D(L^j)$ ,  $F_X^j \equiv F_X(L^j)$ , and  $\Omega^j \equiv \Omega(L^j)$  is adopted.

#### 3.1 Productivity

The first set of results concerns the productivity of exporters and non-exporters in the two countries. From (6), and (7)

$$a_D^{\sigma-1} = \frac{B}{F_D}, \quad a_X^{\sigma-1} = \frac{\phi B^*}{F_X^*}. \quad (23)$$

A higher  $L^j$  affects the cutoffs via two channels: First, it changes the demand facing each firm (via  $B$  respective  $B^*$ ) and, second, it increases the market size dependent beachhead costs.

The effect of the foreign market size on non-exporters

$$\frac{\partial a_D}{\partial L^*} < 0, \quad (24)$$

from (23), since  $\frac{\partial B}{\partial L^*} < 0$  by inspection of (13). The intuition is that a larger foreign market implies a larger mass of foreign firms competing in the home market, which decreases the market shares of domestic non-exporters.

The effect of a larger home market on non-exporters is

$$\frac{\partial a_D}{\partial L} < 0 \quad \text{for } \phi < 1, \quad (25)$$

as shown in appendix 6.2. The negative signs imply that the higher beachhead cost due to a larger market dominates the effect of higher demand.

Next, from (23)

$$\frac{\partial a_X}{\partial L} < 0, \quad (26)$$

since  $\frac{\partial B^*}{\partial L} < 0$ . A larger mass of domestic exporters implies stronger competition in the foreign market, and the marginal exporter must consequently be more productive.



The effect of foreign market size on the productivity of domestic exporters is, as shown in appendix 6.3, ambiguous:

$$\begin{aligned} \frac{\partial a_X}{\partial L^*} &\leq 0 \quad \text{for} \quad \psi^{\beta-1} \psi^{*(\beta-1)} (\beta - (\beta - 1) \psi^*) \leq \phi^{2\beta} \\ \frac{\partial a_X}{\partial L^*} &> 0 \quad \text{for} \quad \psi^{\beta-1} \psi^{*(\beta-1)} (\beta - (\beta - 1) \psi^*) > \phi^{2\beta}, \end{aligned} \quad (27)$$

where  $\psi^j \equiv \frac{F_X^j}{F_D^j}$  measures relative market access (relative beachhead cost) of foreign versus domestic firms. As is easily shown, the left-hand side of the inequality, determining the sign of the derivative, decreases in  $\psi^*$ . This means that  $a_X$  will always decrease in the foreign market size when the relative beachhead cost in the foreign market is sufficiently high. Referring back to (23),  $a_X$  will fall when the effect from a higher beachhead cost dominates. For  $\psi^*$  close to one, on the contrary, the effect of larger sales dominates, which implies that the marginal exporter becomes less productive as the export market increases in size.

The effects of market size on the productivity of exporters and non-exporters are summarized in Result 1.

*Result 1: The average productivity of exporters as well as non-exporters increases in the size of the domestic market as long as  $\phi < 1$ . The average productivity of non-exporters also increases in the size of the foreign market. The average productivity of exporters increases in the foreign market size if the beachhead cost of exporters is sufficiently higher than the beachhead cost of domestic firms in this market.*

The next question is how the relative productivity of firms in the two countries is affected by market size. Note that the productivity of non-exporters in both countries increases as one of the markets grows. As shown in appendix 6.4

$$\left(\frac{a_D}{a_D^*}\right)^k = \frac{F_D^*}{F_D} \left(\frac{1 - \Omega^*}{1 - \Omega}\right) > 1 \quad \text{for} \quad L^* > L, \quad \text{and} \quad \Omega^*, \Omega < 1, \quad (28)$$

meaning that domestic producers are more productive in a larger economy. It is also the case that the productivity difference between domestic producers in the two economies increases with the difference in market size:

$$\frac{\partial \left(\frac{a_D}{a_D^*}\right)}{\partial L^*} > 0, \quad \text{for} \quad L^* > L, \quad \text{and} \quad \Omega^*, \Omega < 1, \quad (29)$$

as shown in appendix 6.1.

*Result 2: Non-exporters in a large market are, on average, more productive than non-exporters in a smaller market, and this difference increases with the difference in country size.*

Next using (15) and (16), the relative cut-off productivity for non-exporters and exporters in the home country is

$$\left(\frac{a_D}{a_X}\right)^k = \frac{F_X^*}{F_D \Omega^*} \left(\frac{1 - \Omega^*}{1 - \Omega}\right) > 1, \quad \text{for} \quad \frac{F_X^j}{\Omega^j} > F_D^k \forall j, k, \quad \text{and} \quad \Omega^*, \Omega < 1. \quad (30)$$

There is strong empirical support for exporters being more productive than domestic firms, and we follow Melitz (2003) by making parameter assumptions for this to hold:  $\frac{F_X^j}{\Omega^j} > F_D^k$ .<sup>5</sup>

The market size is of importance for the relative productivity of exporters to non-exporters:

$$\frac{\partial \left( \frac{a_D}{a_X} \right)}{\partial L} < 0 \text{ for } \Omega < 1, \quad (31)$$

as shown in appendix 6.5. The larger is the home country, the less productive are exporters as compared to non-exporters. Essentially, the higher fixed cost associated with the larger home market will push up the relative productivity of domestic firms, which makes exporters look less productive in comparison.

*Result 3: Exporters are more productive than producers for the domestic market. However, this effect decreases in the size of the home country.*

### 3.2 Trade volume

The next set of results concerns the relationship between country size and manufacturing export share. A home exporting firm with marginal cost  $a$ , sells  $a^{1-\sigma} \phi B^*$  in the foreign market. Using (7), the total export volume from home is

$$V_X = \int_0^{a_X} a^{1-\sigma} dG(a | a_D) \cdot \frac{F_X^*}{a_X^{1-\sigma}} = \left( \frac{a_X}{a_D} \right)^k \frac{\beta}{\beta - 1} F_X^* n. \quad (32)$$

Similarly, the total production volume for the home market is

$$V_D = \int_0^{a_D} a^{1-\sigma} dG(a | a_D) \cdot \frac{F_D}{a_D^{1-\sigma}} = \frac{\beta}{\beta - 1} F_D n. \quad (33)$$

The export share may now be written as

$$S_X = \frac{V_X}{V_X + V_D} = \frac{\Omega^*(1 - \Omega)}{1 - \Omega^* \Omega}. \quad (34)$$

Differentiating with respect to country size gives

$$\frac{\partial S_X}{\partial L} = \frac{\Omega^* (\Omega^* - 1) \partial \Omega}{(1 - \Omega^* \Omega)^2 \partial L} < 0, \quad (35)$$

$$\frac{\partial S_X}{\partial L^*} = \frac{1 - \Omega}{(1 - \Omega^* \Omega)^2} \frac{\partial \Omega^*}{\partial L^*} > 0, \quad (36)$$

which implies that a smaller country has a higher manufacturing export share than a larger one.

*Result 4: The manufacturing export share of a country decreases in its own size, and increases in the trade partner's size.*

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<sup>5</sup>The corresponding condition in Melitz (2003) is that  $\frac{F_X}{\phi} > F_D$ .

Next, note that for  $f_X = f_D$ ,  $\Omega^* = \Omega = 1$ . This means, from (34), that  $S_X = S_X^*$ ; i.e., manufacturing export shares converge as  $f_X$  approaches  $f_D$ . Moreover, since a falling  $f_X$  makes export easier, export shares converge upwards.

*Result 5: Falling relative beachhead costs ( $f_X$  converging to  $f_D$ ) imply (upwards) converging manufacturing export shares.*

The intuition for *Result 4* and *Result 5* is helped by expressing the export share as

$$S_X = \frac{1}{1 + \frac{B}{B^*} \frac{1}{\phi} \left(\frac{a_D}{a_X}\right)^k \frac{\beta-1}{\beta}}. \quad (37)$$

The export share decreases in the relative size of the home market ( $\frac{B}{B^*}$ ), in the ratio of non-exporters to exporters  $\left(\frac{a_D}{a_X}\right)^k$ , and increases in trade freeness. A larger home market has two opposing effects; it increases ( $\frac{B}{B^*}$ ) but decreases  $\left(\frac{a_D}{a_X}\right)$  (from Result 3). As it turns out, the first effect dominates so that export shares always decrease in the size of the exporting country. The convergence result stems from the fact that the fixed component of the beachhead cost  $f_X$  is relatively more important in a small market, where the variable component is low. A falling  $f_X$  therefore increases market access relatively more in a small market. This means that  $\frac{B}{B^*}$  increases when home is the smaller country and decreases when home is the larger country, which implies converging trade shares. Second, a fall in  $f_X$  makes export easier while increasing import competition in both countries. This results in a decreasing  $a_D^j$ , and an increasing  $a_X^j$  and, as a result,  $\frac{a_D^j}{a_X^j}$  falls in both countries.

It may be useful to compare our results to the standard set-up. We here use the Melitz (2003) model with a homogenous good and a freely traded A-sector a la Helpman et al. (2004), which fixes factor prices. This allows us to analytically handle the asymmetric country case. Without our assumption of a market size dependent beachhead cost, the terms  $\left(\frac{B^*}{B}\right)$  and  $\left(\frac{a_X}{a_D}\right)^k$  are independent of country size in this model, which implies that also manufacturing export shares are independent of country size. Another comparison may be made against the standard Dixit-Stiglitz trade model without a homogenous good A-sector (see e.g. Helpman (1987)). Like our model, trade shares are negatively related to market size in this model. However, in contrast to our model, manufacturing trade shares diverge as trade costs fall: trade shares increase from zero in autarky to the share of the foreign market in total demand at free trade.<sup>6</sup>

Below, the prediction of converging manufacturing export shares will be empirically tested.

## 4 Empirical Analysis

In this section, we empirically test several predictions of our model related to the effects of market size. These predictions should ideally be tested in a cross-country firm level data set, but this type of data is not yet available. To focus on the effects of market size, we use cross-

<sup>6</sup>Naturally, in this model there is no beachhead cost that can be affected by trade liberalization.

country data rather than e.g. firm level data for an individual country. We work with the OECD’s STAN industrial database which includes sectoral production and trade data for 29 manufacturing sectors in OECD countries from 1980 to 2003.

## 4.1 Country size and manufacturing export shares

We start by focusing on implications of the model related to country size and manufacturing export shares. First, we check that manufacturing export shares are negatively correlated with country size in our dataset, as predicted by Result 4.

Second, Result 5 states that the export share of the manufacturing sector across countries converges as the fixed component of the exporting beachhead cost,  $f_X$ , approaches the value for the fixed component of the domestic beachhead cost,  $f_D$ . Given that this has been happening over time, we should observe converging manufacturing trade shares over time. The assumption that the relative access cost to foreign markets, as compared to that of the domestic market, has been falling over time is very much in line with the often cited effect of globalization making the world more alike. A concrete example supporting this assumption is the process of product standardization and removal of non-tariff barriers to trade within the European Union during the last 20-30 years. GATT and WTO negotiations have also aimed at not only reducing tariffs but also nontariff barriers to trade during this period. Finally, the rapid improvement of telecommunications, including the internet, simplifies business contacts and information gathering about foreign markets, which may be interpreted as a fall in  $f_X$ .

We look at the evolution of manufacturing export shares over time, on a sectoral level within the OECD using the STAN database with yearly observations from 1980 to 2003. Accepting the assumption that the process of falling access costs to foreign markets has occurred gradually over time during the period investigated, we should observe converging manufacturing export shares. We apply four different methods of analysis as outlined in the following sections.

### 4.1.1 Country size and manufacturing export shares

Result 4 implies that large countries should have relatively lower manufacturing export shares than smaller countries. We investigate this by running the simple regression

$$s_{ist} = \beta_0 + \beta_1 l_{it} + \varepsilon_{ist}, \tag{38}$$

where  $s_{ist} \equiv \log\left(\frac{X}{Y}_{ist}\right)$ ,  $l_{it} \equiv \log L_{it}$ . The regression is run at the sectoral level. Table 1 shows the regression of export shares over GDP on a sectoral level in 2001. The regression includes fixed effects for sectors. The coefficient for population, which can be interpreted as a standard elasticity, is highly significant and of the expected sign.

Year 2001	
Dependent variable	$\frac{X}{Y}$
	(1)
$l_{it}$	-0.16*** (0.025)
Sector dummies	Yes
Observations	605
R squared	0.38

Note: Standard errors in parentheses. Errors are clustered on country and year pairs.

\* significant at 10%

\*\* significant at 5%

\*\*\* significant at 1%.

Table 1: EXPORT SHARES AND COUNTRY SIZE

#### 4.1.2 Convergence of manufacturing export shares

Next, we proceed to test Result 5 predicting an upward convergence in manufacturing export shares when  $f_X$  approaches  $f_D$ , and, as argued above, we assume time to be a good proxy for this process.

The first approach is to simply regress the annual change (first difference) in the manufacturing export shares in a specific sector on a dummy,  $D_{ist}$ , which takes the value of 1 if that sector has a lower export share than the average, interacted with a time variable:

$$\Delta s_{ist} = \beta_0 + \beta_1 \Delta l_{it} + \beta_2 D_{ist} + \beta_3 * t + \beta_4 * D_{ist} * t + \gamma \Gamma_s + \varepsilon_{ist}. \quad (39)$$

The implication of Result 5 it that we would find a positive value for  $\beta_4$  (with fixed effects for all sectors,  $\Gamma_s$ ), that is, that those countries with a sector below average tend to increase, on average, while the countries above do the opposite. The result is reported in Table 2. Errors are clustered around country and year pairs. The coefficient on the interacted variable is significantly positive as predicted, which indicates convergence on average over time. Moreover, the coefficient on  $t$  is significantly positive, consistent with upward convergence.

One source of convergence in export shares may simply be that countries are converging in size. This is controlled for by the term  $\Delta l_{it}$ . The negative and significant estimate for  $\beta_1$

indicates that there is indeed some convergence due to converging population sizes.

Years 1980 to 2003	
Dependent variable	$\Delta s_{ist}$
	(1)
$\Delta l_{it}$	-1.52** (0.72)
$D_{ist}$	-1.87***
$t$	0.031***
$D_{ist} * t$	0.025*** (0.005)
Sector dummies	Yes
Observations	11203
R squared	0.67

Note: Standard errors in parentheses. Errors are clustered on country and year pairs.

\* significant at 10%

\*\* significant at 5%

\*\*\* significant at 1%.

Table 2: CONVERGENCE (DUMMY APPROACH)

Our second approach is to check for mean reversion in the manufacturing export share series by regressing the first difference in export shares on its own lagged value in levels:

$$\Delta s_{ist} = \beta_0 + \beta_1 s_{ist-1} + \beta_2 \Delta l_{it} + \beta_3 D_s + \varepsilon_{ist}, \quad (40)$$

with fixed effects for sectors,  $D_s$ . Also in this case do we cluster on country-year pairs. The model would predict a negative value of  $\beta_1$  for convergence. To deal with the possibility of serially correlated errors, lags up to the degree of  $p = 3$  are included<sup>7</sup> 41:

<sup>7</sup>To include the possibility of the errors following an AR(1) process, we run a regression of the residuals from (41) in the following way

$$\hat{\varepsilon}_{ist} = \rho \hat{\varepsilon}_{ist-1} + u_{ist}$$

increasing  $p$  in (41) by one each time. We find that there is evidence for  $\rho$  being positive and significant for

$$\Delta s_{ist} = \beta_0 + \sum_{i=1}^p \beta_{1i} s_{ist-i} + \beta_2 \Delta l_{it} + \beta_3 D_s + \varepsilon_{ist}. \quad (41)$$

Our model predicts that the sign of  $\beta_1$  in (40) is negative. This means that the higher was the export share in the previous period, the less of an increase there is in the current period. The results are shown in Table 3. The sign on the first lag of the export share is negative and significant, suggesting convergence. The result is upheld also in the regressions with three lags, suggesting that serial correlation only produces a positive bias, if any.

Years 1980 to 2003			
Dependent variable	$\Delta s_{ist}$	$\Delta s_{ist}$	$\Delta s_{ist}$
	(1)	(2)	(3)
$s_{ist-1}$	-0.038*** (0.006)	-0.038*** (0.006)	-0.134*** (0.039)
$\Delta l_{ist}$		0.062 (1.364)	0.673 (1.284)
$s_{ist-2}$			0.066
$s_{ist-3}$			0.036
Sector dummies	Yes	Yes	Yes
Observations	10932	10932	9618
R squared	0.03	0.03	0.04

Note: Standard errors in parentheses. Errors are clustered on country and year pairs.

\* significant at 10%

\*\* significant at 5%

\*\*\* significant at 1%.

Table 3: CONVERGENCE (LAGGED VALUES)

Our third approach follows the standard empirical growth literature. We use the initial value of the manufacturing export share for which we have data and regress the first differences

$p = 1$  and  $2$  but not for  $p = 3$ . That is why we include three lags in Table 3.

in export shares on the initial level of trade shares:

$$\Delta s_{ist} = \beta_0 + \beta_1 s_{is0} + \beta_2 \Delta l_{it} + \beta_3 D_s + \varepsilon_{ist}.$$

Country-year pairs are clustered as previously and sectors dummies are included. Once more, the model predicts that  $\beta_1$  should be negative since the higher was the initial level, the lower would be the average change over time if convergence holds.

In Table 4, it is seen that the growth rate of export shares depends negatively on the initial level in 1980, suggesting convergence within the OECD at the sectoral level.

Years 1980 to 2002		
Dependent variable	$\Delta s_{ist}$	$\Delta s_{ist}$
	(1)	(2)
$s_{i,1980}$	-0.016*** (0.004)	-0.016*** (0.004)
$\Delta l_{it}$		-0.625 (0.936)
Sector dummies	Yes	Yes
Observations	8542	8542
R squared	0.01	0.01

Note: Standard errors in parentheses. Errors are clustered on country and year pairs.

\* significant at 10%

\*\* significant at 5%

\*\*\* significant at 1%.

Table 4: CONVERGENCE (INITIAL VALUES)

For robustness, we have performed the same analysis as above also with five-year averages. However, this does not alter the results in any of the regressions above.

Finally, the effect is visible by graphically examining the shift in the distribution of manufacturing export shares globally, as shown in Figure 2. We restrict the STAN sample to only include countries for which there is data from 1970 until 2002 and construct five histograms displaying the distribution of manufacturing export shares globally in the years 1970, 1978, 1986,



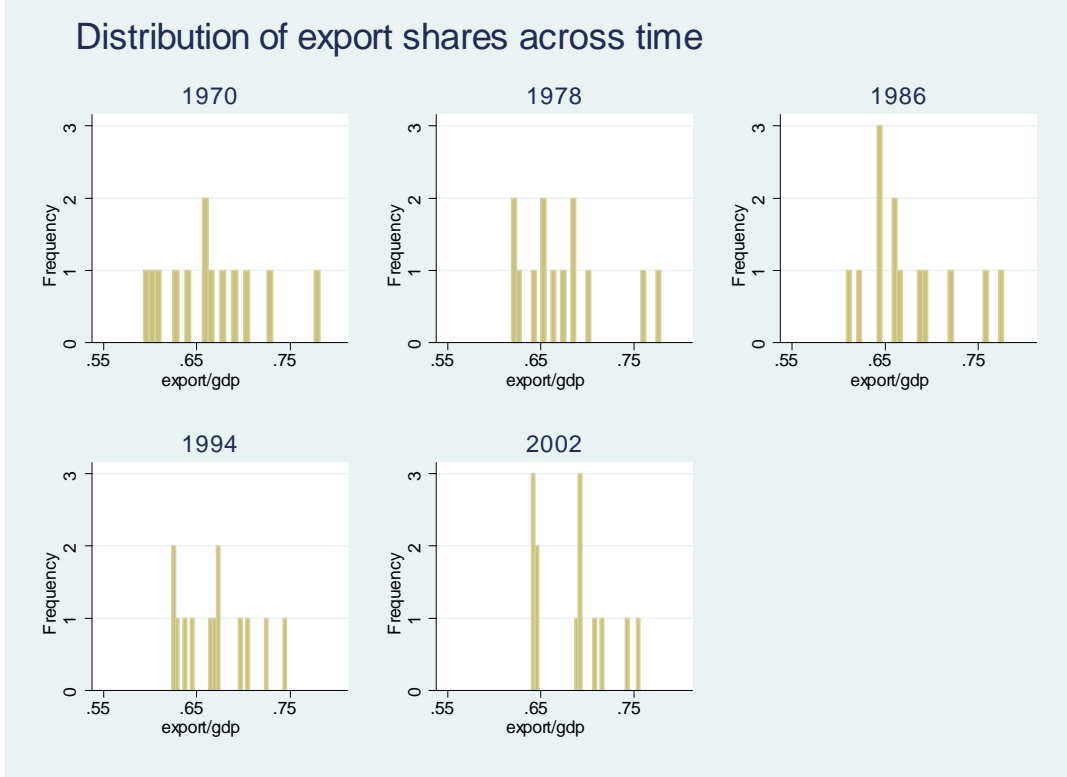


Figure 2: The distribution of export shares becomes more narrow as time progresses. Source: OECD STAN.

1994 and 2002. It can be seen that the distribution becomes more narrow as time progresses. A list of countries included in this graph is found in the appendix.

An alternative way of examining this graphically is to calculate how the coefficient of variation changes over time. This variable is neutral to units and therefore, we rather use this than the variance. In Figure 3, the result can be seen for the same sample period over time. There appears to be a notable decline in the variable from 1970 until 2002.

## 4.2 Productivity and market size

Result 1 implies that the average productivity of non-exporters as well as exporters increases in the home market size. To see its implications on average overall (aggregate) productivity in the model, aggregate productivity is expressed as<sup>8</sup>:

$$\bar{\varphi} = \left( s_D \int_0^{a_D} a^{1-\sigma} dG(a|a_D) + s_X \int_0^{a_X} a^{1-\sigma} dG(a|a_D) \right)^{\frac{1}{\sigma-1}}, \quad (42)$$

<sup>8</sup>See Melitz (2003).

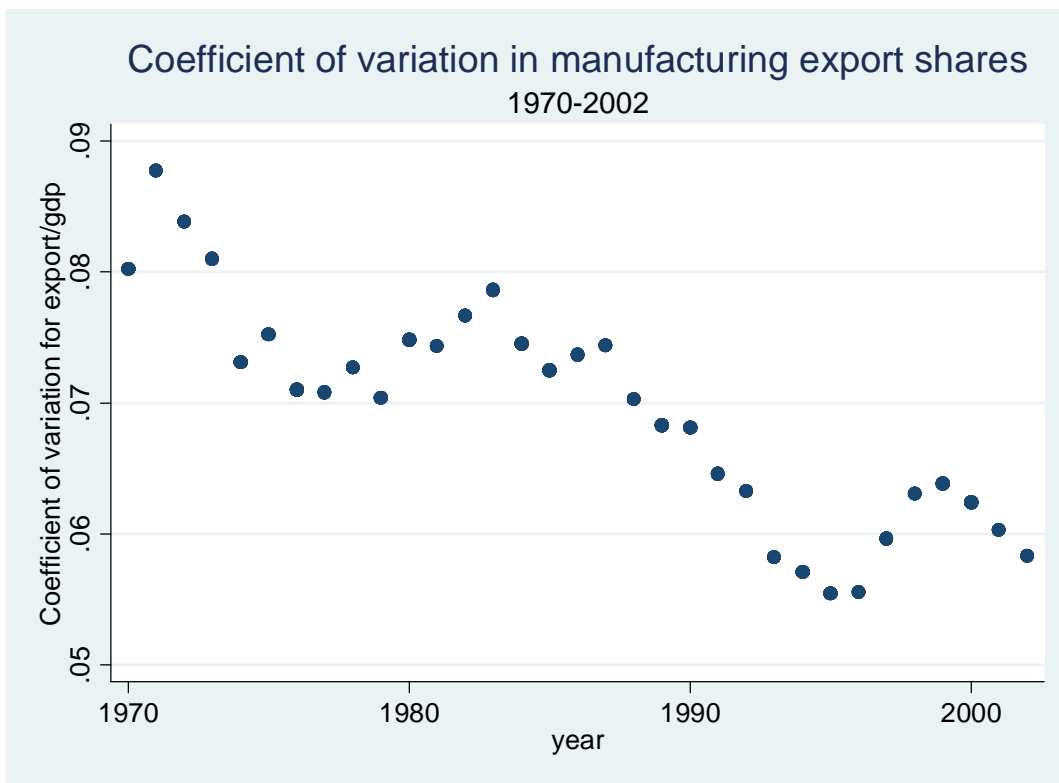


Figure 3: The coefficient of variation for export shares decreases over time. Source: OECD STAN.

where  $s_D$  is the share of home producers that sells domestically only and  $s_X$  is the share that exports. Since the ratio of exporters to non-exporters is  $\left(\frac{a_X}{a_D}\right)^k$ ,  $s_D = \frac{1}{1 + \left(\frac{a_X}{a_D}\right)^k}$ , and

$s_X = \frac{\left(\frac{a_X}{a_D}\right)^k}{1 + \left(\frac{a_X}{a_D}\right)^k}$ , we can rewrite (42) as:

$$\bar{\varphi} = \frac{1}{a_D} \left( \frac{k}{k - \sigma + 1} \right)^{\frac{1}{\sigma-1}} \left( \frac{1 + \left(\frac{a_X}{a_D}\right)^{2k+1-\sigma}}{1 + \left(\frac{a_X}{a_D}\right)^k} \right)^{\frac{1}{\sigma-1}}. \quad (43)$$

From (43), it is seen that average productivity increases in  $L$  since from (25)  $\frac{\partial a_D}{\partial L} < 0$ ,  $\frac{\partial \left(\frac{a_X}{a_D}\right)}{\partial L} > 0$  from (31), and  $k - \sigma + 1 > 0$ .

Therefore, we arrive at the prediction that aggregate productivity in manufacturing increases in country size, mainly due to the fact that both domestic and foreign producers face a higher beachhead cost in the larger market, which restricts sales to this market to the most productive firms. To test this prediction, we run the following regression<sup>9</sup>:

$$\log \tilde{\varphi}_{ist} = \beta_0 + \beta_L \log L_{it} + \beta_K \log K_{ist} + \beta_D \log D + \varepsilon_{ist}. \quad (44)$$

Here,  $\tilde{\varphi}_{ist}$  denotes aggregate labour productivity in country  $i$  in sector  $s$  and year  $t$ .  $L_{it}$  is the national population size of country  $i$  in year  $t$ .  $K_{ist}$  is the amount of capital used and  $D$  is a set of dummies that will be explained.

We control for sectors by using the set  $D_s$  in all regressions, since  $f_D$ ,  $f_X$ , and  $\gamma$  are expected to vary among sectors. Table 5 reports the estimated coefficients for a regression done only for the year of 2002, since this is the most recent year for which there is much data. This analysis captures cross-sectional effects of population on productivity. We use two measures of labour productivity: (1) output divided by employment and (2) value added divided by employment.<sup>10</sup> Population is used as a measure of country size when estimating the effect of country size on productivity. This is because population can be considered an exogenous variable for our purposes and, second, it is consistent with the treatment of country size in our model. Were we instead to use GDP, for example, this would depend both on population size and aggregate productivity. Errors are clustered on country and year pairs. The results are according to the model. Table 5 shows that, on average across sectors adjusted for sectoral dummies, labour productivity is higher in larger countries.

To also look at other years, we plot in Figure 4 and Figure 5 the specific values of  $\beta_L$  over time with a 95% confidence interval around it, starting in 1980 for the regression in columns (3)

<sup>9</sup>Pavcnik (2002) uses the semiparametric method from Olley and Pakes (1996) to estimate productivity. However, we do not have any firm level data which would be required for this method.

<sup>10</sup>A problem is that employment is reported by different countries in different (but similar) ways. We will use the standard measure that covers most countries, which is called total employment in the database.

Year 2002				
Dependent variable	$\frac{\text{Output}}{\text{Worker}}$	$\frac{\text{Output}}{\text{Worker}}$	$\frac{\text{V.A.}}{\text{Worker}}$	$\frac{\text{V.A.}}{\text{Worker}}$
Units	Values	Values	Values	Values
	(1)	(2)	(3)	(4)
Population	0.241*** (0.097)	0.491*** (0.196)	0.250*** (0.097)	0.479*** (0.215)
Capital		0.845*** (0.214)		0.851*** (0.237)
Sector dummies	Yes	Yes	Yes	Yes
Observations	412	86	412	86
R squared	0.06	0.55	0.06	0.55

Note: Standard errors in parentheses. Errors are clustered on country and year pairs.

\* significant at 10%

\*\* significant at 5%

\*\*\* significant at 1%.

Table 5: PRODUCTIVITY AND COUNTRY SIZE (VALUES)

and (4). Figure 4 shows that the coefficient is positive and significant over time when capital is not included. In Figure 5, where capital is included, population is insignificant except in 2002. The regressions including capital should be interpreted with caution, however. First, when including capital, the sample shrinks to only seven countries. Moreover, there is an obvious endogeneity problem associated with capital, since it would tend to move towards more productive locations.

Finally, Table 6 displays regressions with country dummies to use within country variation in population size and test whether such variation affects aggregate productivity differently than cross-sectional differences in population. Here, the population turns out to be significant in all specifications.

We interpret our results as being consistent with firms being more productive in large markets. This is also consistent with e.g. Syverson (2004, 2006) who finds a positive association between productivity and market density using firm level data.

Naturally, an alternative explanation for the observed higher productivity in larger countries is that we are picking up productivity spillovers or agglomeration rents in line with e.g. the new

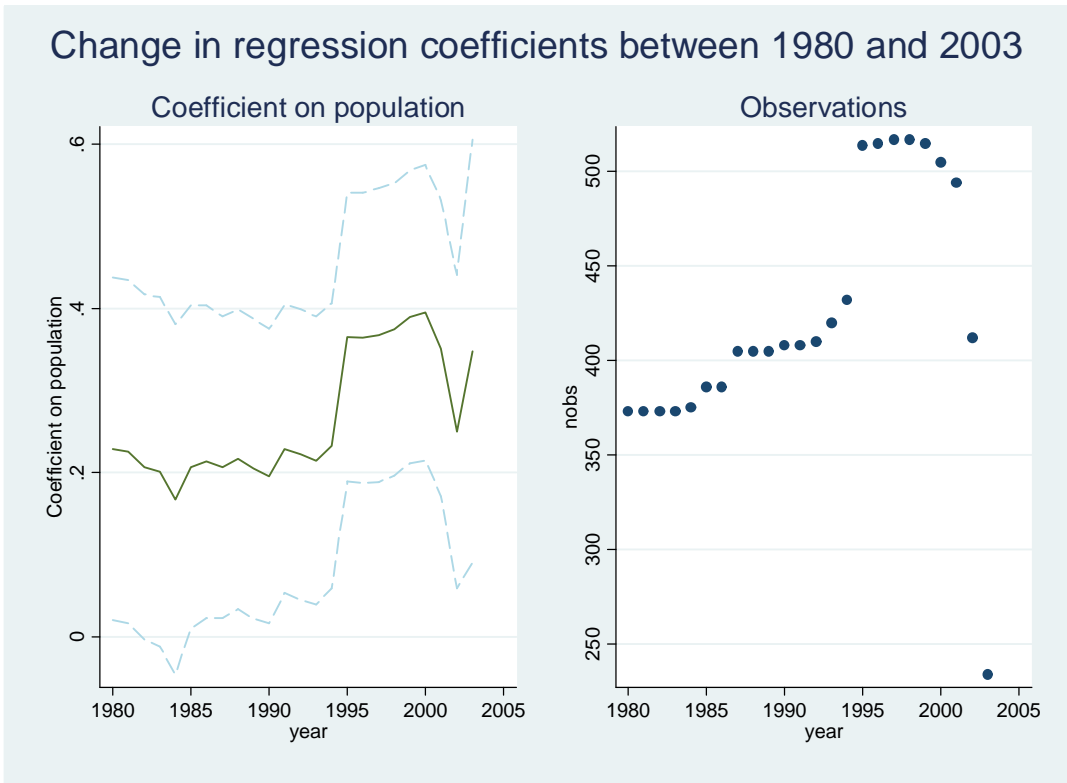


Figure 4: Regression of productivity on population at the sectoral level, with 95% confidence interval. Sector dummies are used. Source: OECD.

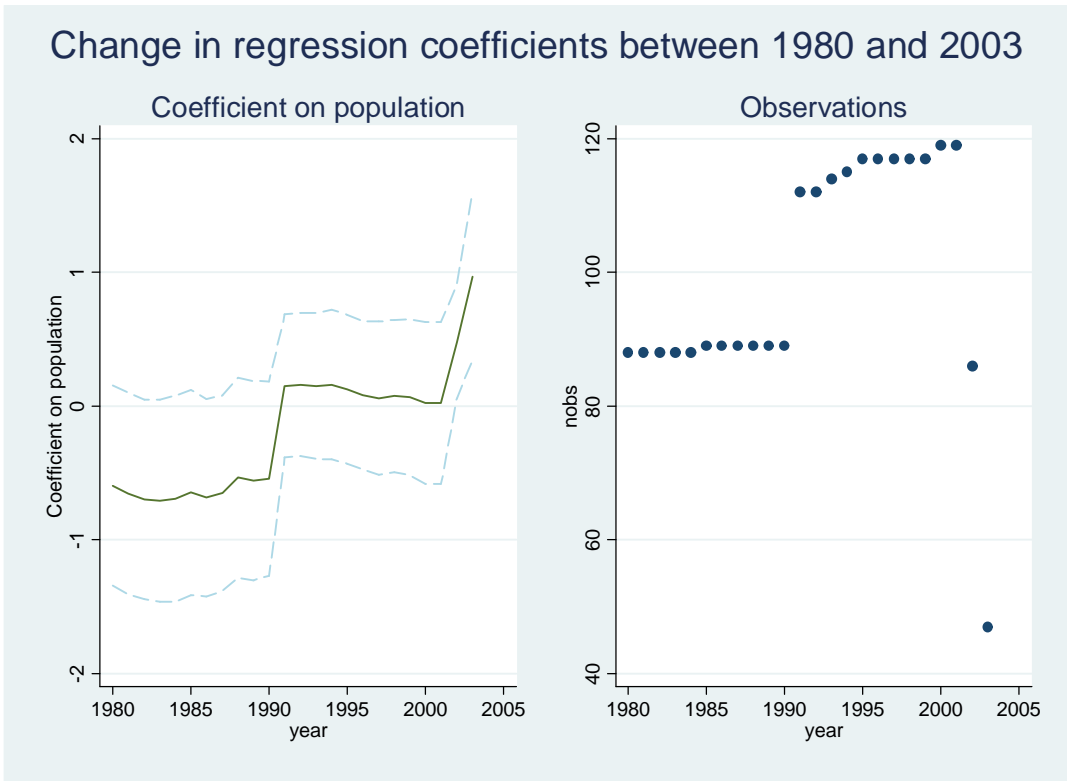


Figure 5: Regression of productivity on population at the sectoral level controlling for capital, with 95% confidence intervals. Sector dummies are used. Source: OECD.

Years 1980 to 2003

Dependent variable	$\frac{\text{Output}}{\text{Worker}}$	$\frac{\text{Output}}{\text{Worker}}$	$\frac{\text{V.A.}}{\text{Worker}}$	$\frac{\text{V.A.}}{\text{Worker}}$
Units	Volumes	Volumes	Volumes	Volumes
	(1)	(2)	(3)	(4)
Population	7.083*** (0.359)	8.48** (0.862)	2.393*** (0.219)	2.666** (0.237)
Capital		0.234*** (0.075)		0.474*** (0.039)
Dummies				
Country and sector	Yes	Yes	Yes	Yes
Observations	4172	1164	6544	2320
R squared	0.98	0.98	0.97	0.97

Note: Standard errors in parentheses. Errors are clustered on country and year pairs.

\* significant at 10%

\*\* significant at 5%

\*\*\* significant at 1%.

Table 6: PRODUCTIVITY AND COUNTRY SIZE (VOLUMES)

economic geography models (See e.g. Krugman (1991), and Krugman and Venables (1995)). However, empirical studies do not show any clear pattern of agglomeration in OECD data during the period of interest (See e.g. Knarvik and Overman (2002)). More importantly, agglomeration of the manufacturing sector in large countries would imply that manufacturing export shares increase in small countries and decrease in large ones. That is, such a scenario would imply diverging manufacturing export shares, which is not consistent with our theoretical model, and is rejected by our empirical results.

## 5 Conclusion

This paper has explicitly modelled a market size dependent market access or beachhead cost in the heterogenous firms and trade model by Melitz (2003). We model this cost as having one variable component that increases in market size, and one fixed component. The fixed component could e.g. be interpreted as the cost of standardizing a product for a particular

market, while the variable cost term e.g. represents that the advertising cost of introducing a new product increases with the size of the market (number of consumers).

The introduction of market size dependent beachhead costs leads to a number of new results. The productivity of non-exporter as well as exporter firms will depend on market size, and so will manufacturing export shares. In particular, we show that non-exporter firms in a large market are more productive than non-exporters in a smaller market. Second, as in the standard model, exporters are more productive than non-exporters, but this productivity premium decreases in the size of the home country. Finally, we show that the manufacturing export share of a country decreases in its own size, and increases in the trade partner's size. This last effect decreases as markets are integrated (in the sense that the fixed beachhead cost of foreign markets declines). Accepting that market access costs into foreign markets have been falling over time as a result of globalization, the model predicts converging manufacturing export shares over time.

In the empirical section, we focus on testing results related to country size, which are new compared to the standard model. This implies that we need to use cross-country data. First, it is shown how manufacturing export shares are negatively correlated with market size, in accordance with the model. Second, a number of tests generate support for the model generated hypothesis that manufacturing export shares should converge over time. Finally, it is shown how average productivity is generally positively correlated with country size, as predicted by the model.



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## 6 Appendix

**6.1**  $\frac{\partial}{\partial L^*} \left( \frac{a_D}{a_D^*} \right)^k > 0$  for  $\Omega^*, \Omega < 1$

Proof:

From (28)

$$\left( \frac{a_D}{a_D^*} \right)^k = \frac{F_D^*}{F_D} \left( \frac{1 - \Omega^*}{1 - \Omega} \right).$$

Differentiating w.r.t.  $L^*$  gives:

$$\frac{\partial}{\partial L^*} \left( \frac{F_D^*}{F_D} \left( \frac{1 - \Omega^*}{1 - \Omega} \right) \right) = \frac{\gamma L^{*\gamma-1}}{F_D(1 - \Omega)} \left( 1 - \Omega^* - (\beta - 1) \Omega^* \left( 1 - \Omega^{*\frac{1}{\beta-1}} \phi^{\frac{\beta}{1-\beta}} \right) \right). \quad (45)$$

The sign of the derivative depends on the sign of the term:

$$\left( 1 - \Omega^* - (\beta - 1) \Omega^* \left( 1 - \Omega^{*\frac{1}{\beta-1}} \phi^{\frac{\beta}{1-\beta}} \right) \right). \quad (46)$$

The first- and second-order conditions for a minimum of this term w.r.t.  $\Omega(L^*)$  are:

$$\begin{aligned} \frac{\partial}{\partial \Omega^*} \left( \Omega^* \left( 1 + (\beta - 1) \left( 1 - \Omega^{*\frac{1}{\beta-1}} \phi^{\frac{\beta}{1-\beta}} \right) \right) \right) &= \beta \left( \Omega^{*\frac{1}{\beta-1}} \phi^{\frac{\beta}{1-\beta}} - 1 \right) = 0 \\ \frac{\partial^2}{\partial \Omega^{*2}} \left( \Omega^* \left( 1 + (\beta - 1) \left( 1 - \Omega^{*\frac{1}{\beta-1}} \phi^{\frac{\beta}{1-\beta}} \right) \right) \right) &= \frac{\beta}{\beta - 1} \Omega^{*\frac{1}{\beta-1}-1} \phi^{\frac{\beta}{1-\beta}} > 0. \end{aligned} \quad (47)$$

The minimum is, thus, given by  $\Omega^* = 1$  (since  $\Omega^* = 1 \iff \phi = 1$ ). Substituting  $\Omega^* = 1$  into (45) gives  $\frac{\partial}{\partial L^*} \left( \frac{a_D}{a_D^*} \right)^k = 0$ . Consequently, it must be the case that  $\frac{\partial}{\partial L^*} \left( \frac{a_D}{a_D^*} \right)^k > 0$  for  $\Omega^*, \Omega < 1$ .

## 6.2 $\frac{\partial a_D^j}{\partial L^j} < 0$

From (6.1), we have that

$$\frac{\partial \left( \frac{a_D}{a_D^*} \right)}{\partial L^*} = \frac{\partial a_D}{\partial L^*} \frac{1}{a_D^*} - \frac{a_D}{(a_D^*)^2} \frac{\partial a_D^*}{\partial L^*} > 0. \quad (48)$$

Since from (15)  $\frac{\partial a_D}{\partial L^*} < 0$ , (48) holds iff  $\frac{\partial a_D^*}{\partial L^*} < 0$

## 6.3 $\frac{\partial a_X}{\partial L^*}$

From (16)

$$a_X^k = \frac{(\beta - 1) \Omega^* F_E}{F_X^*} \left( \frac{(1 - \Omega)}{1 - \Omega \Omega^*} \right) = (\beta - 1) F_E (1 - \Omega) \frac{1}{F_X^* \left( \frac{1}{\Omega^*} - \Omega \right)}. \quad (49)$$

The sign of  $\frac{\partial a_X}{\partial L^*}$  is therefore determined by the sign of

$$\frac{\partial}{\partial L^*} [F_X^* \left( \frac{1}{\Omega^*} - \Omega \right)] \quad (50)$$

$$= \frac{\partial}{\partial L^*} [F_X^{*\beta} F_D^{*1-\beta} \phi^{-\beta} - F_X^* \Omega]. \quad (51)$$

Now

$$\begin{aligned} \frac{\partial}{\partial L^*} [F_X^{*\beta} F_D^{*1-\beta} \phi^{-\beta} - F_X^* \Omega] &\leq 0 \\ &\Leftrightarrow \\ \left( \frac{F_X^*}{F_D^*} \right)^\beta \left( \beta \frac{F_D^*}{F_X^*} - (\beta - 1) \right) &\leq \Omega \phi^\beta \\ &\Leftrightarrow \\ \beta - \frac{F_X^*}{F_D^*} (\beta - 1) &\leq \Omega \Omega^*. \end{aligned}$$

**6.4**  $\left(\frac{a_D}{a_D^*}\right)^k > 1$  iff  $L^* > L$  for  $\Omega^*, \Omega < 1$

Proof:

First

$$L^* = L \iff \left(\frac{a_D}{a_D^*}\right)^k = \frac{F_D^*}{F_D} \left(\frac{1 - \Omega^*}{1 - \Omega}\right) = 1.$$

That  $L^* = L \iff \left(\frac{a_D}{a_D^*}\right)^k > 1$  for  $\Omega^*, \Omega < 1$  now follows from  $\frac{\partial}{\partial L^*} \left(\frac{a_D}{a_D^*}\right)^k > 0$  for  $\Omega^*, \Omega < 1$ .

**6.5**  $\frac{\partial \left(\frac{a_D}{a_X}\right)}{\partial L} < 0$  for  $\Omega < 1$ .

Proof:

$$\frac{\partial}{\partial L} \left(\frac{a_D}{a_X}\right)^k = \gamma L^{\gamma-1} \frac{F_X^*}{F_D^2} \frac{(1 - \Omega^*)}{\Omega^* (1 - \Omega)} \left( (\beta - 1) \frac{\Omega \left(1 - \frac{F_D}{F_X}\right)}{(1 - \Omega)} - 1 \right). \quad (52)$$

The sign of (52) will depend on the sign of the term:

$$\Theta \equiv \left( \frac{(\beta - 1) \Omega \left(1 - \frac{F_D}{F_X}\right)}{(1 - \Omega)} - 1 \right) \quad (53)$$

The F.O.C. when maximising  $\Theta$  w.r.t.  $\phi$  is:

$$\frac{(\beta - 1) \beta \Omega \left(1 - \frac{F_D}{F_X}\right)}{\phi (1 - \Omega)} - \frac{(\beta - 1) \beta \Omega^2 \left(1 - \frac{F_D}{F_X}\right)}{\phi (1 - \Omega)^2} = 0 \iff 1 - \frac{\Omega}{(1 - \Omega)} = 0. \quad (54)$$

So the only stationary point is  $\Omega = 1$ . Furthermore,  $\Theta(\Omega = 0) = -1$  and  $\lim_{\Omega(L) \rightarrow 1} \Theta = 0$ .

Therefore, it follows that for  $\Omega \in [0, 1)$ :

$$\frac{d}{dL} \left(\frac{a_D}{a_X}\right)^k < 0.$$

## 6.6 Countries included in Figure 2.

The following countries are included in Figure 2. This is a subset of the full STAN sample but it is the only set of countries for which there is data for the full length of 1970 until 2002.

Austria  
 Belgium  
 Denmark  
 Finland  
 Italy  
 Japan  
 Korea

Mexico  
Netherlands  
Norway  
Portugal  
United Kingdom  
United States