Decentralising public goods production*

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May 25, 2004

Abstract

Decentralised decisions, to a bureau with a given budget, about the production of public goods is analysed within a general equilibrium model with a representative agent and no pure profits. It is shown that decentralisation (i) does not necessarily imply aggregate production efficiency and (ii) need not be optimal even if all public goods are neutral. Also, cost benefit criteria are derived and the marginal cost of public funds is characterised.

JEL: H21, H41, H43.

Keywords: Cost benefit criteria, marginal cost of public funds, production efficiency, delegation, decentralisation.

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1 Introduction

In cost benefit analysis a general rule of thumb for the optimal level of public goods projects is that the sum over individuals of marginal rates of substitution between the public project and some arbitrary private numeraire commodity should equal the marginal cost of the public goods project ($\sum MRS = MC$). This rule originates from the Samuelson (1954, 1955) treatment of the Pareto efficient supply of public goods and is sometimes known as the Samuelson–rule. When the rule is valid it implies that a trade off between any two public goods should be such that the sum of the marginal rates of substitution over consumers between two public goods should equal the marginal rates of transformation between the same two public goods. Thus, first best implies that the central government could delegate to a subordinate bureau to decide about public goods following this rule of thumb subject to a given expenditure limit.

The Samuelson–rule, however, requires access to the type of differentiated lump–sum taxes that are required by the second fundamental welfare theorem. It then follows that in the presence of different types of second best tax systems this rule of thumb is only valid in special cases; as is well known in the theoretical literature as well as in applied cost benefit analysis.[1]

The reason for this is that is not only the fact that there is a dead–weight loss associated with second best tax systems used for marginal finance of public goods projects as hypothesised by Pigou (1947, p. 33f). There are in fact generally three additional effects: (1) Second best taxes are also associated with income effects which may reinforce or work opposite to the substitution effects. (2) The public good projects may be complements or substitutes with the private goods that constitute the tax base so the marginal cost of producing of the public good may not be equal to the net effect on the public budget. (3) There are general equilibrium effects on the price structure.

The deviation of the second best cost–benefit criterion from the Samuelson–rule has created two large research fields. One is concerned with the question whether the second best optimal level of the public good is larger or smaller than the first best level (the level issue).[2] The other is concerned with under which circumstances the second best criterion will take the same form as the Samuelson–rule (the rule or decentralisation issue).[3] Although the form of the rule for a second best optimal supply of public goods may seem to be of academic interest only, it has an important consequence: If the second best cost–benefit criterion

[1] See, e.g., Diamond and Mirrlees (1971a,b), Dasgupta and Stiglitz (1971), Atkinson and Stern (1974), Christiansen (1981), King (1986), Wilson (1991) and Boadway and Keen (1993).

[2] See for instance Wilson (1991), Gaube (2000) and Gronberg and Liu (2001).

[3] See, e.g., Lau et al. (1978), Christiansen (1981), Besley and Jewitt (1991), Boadway and Keen (1993).

take the same form as the Samuelson–rule then public investment decisions may be delegated and still a second best optimum can be achieved.

In this case therefore, public bureaus make the production decision about public goods with instructions to maximise the same welfare function as the central government subject to the constraint that the cost for producing public goods should not exceed a given amount of public tax revenues reserved for this purpose. The issue has in particular been analysed by Lau et al. (1978) and Besley and Jewitt (1991) who provide conditions stating that if a sufficient degree of separability between public goods and other goods is present, then the Samuelson–rule will describe a second best optimum and decentralisation is optimal.

In reality, however, actual decision making about public goods is at odds with the conclusions of the theoretical literature. Typically the decisions to produce (which is in focus in the paper) and provide public goods projects and services to households are not coordinated with the decisions regarding the tax system. The lack of coordination may arise from different forms of delegation about decisions of how much to produce or provide of public goods projects. One weak form of delegation is when both financing decisions (i.e., decisions about the structure of taxation) and decisions about the public goods mix are made sequentially by the central government. Another reason for uncoordinated decisions is when decisions about production or provision of public goods are explicitly delegated to government agencies that have no role in determining the overall design of the tax system. A third variant is when decisions about public goods are delegated to local authorities, which may or may not have the possibility to make financing decisions of their own in addition to the financing decisions made by the central government. For all these variants of delegation, the decision to delegate may include either public provision or public production or both.[4]

The present paper takes decentralisation of public goods *production* decision as given. Decentralisation or delegation is interpreted in the weak form of uncoordinated decisions about the structure of taxation and how much and how to produce different public goods or as an explicit delegation to a central government bureau with no agency problem vis-à-vi the bureau. Through its internal auditing service the central government is assumed to have full information about the cost

[4]Two recent examples where rather strong forms of delegation of public goods are the following: Boadway et al. (1999) is rather closely related to the present study but within a framework with some very specific assumptions (quasi–linear utility, different agencies serving different parts of the population and a principal–agent relationship between the central government and the agencies). In particular, the agencies seem to work as local communities. They do not address the question of production efficiency or if decentralisation is optimal. Focus is on the characterisation of the marginal cost of public funds. Besley and Coate (2003) considers the effect that centralisation and decentralisation of local public goods may have on the level goods provided under different political equilibria in a federal–like structure.

structure of its agencies. In a sense, therefore, the agencies are part of the central government, but decisions regarding the design of the tax system and the structure of public goods investments are uncoordinated. The central government then acts as a Stackelberg leader taking into account that its decisions will affect the behaviour of the subordinate bureau. The public bureau is assumed to have the same objective function as the central government. Throughout, taxes, as well as other policy instruments, are assumed to be chosen on an optimal level.

The model used is a general equilibrium model with a representative agent and no pure profits. It is essentially the Diamond and Mirrlees (1971a) model but with public production of public goods with private goods as inputs rather than public production of private goods. Therefore, the analysis in the paper is about public production rather than public provision.

This will allow us to not only address the question if optimality of decentralisation still holds under the same restrictions on preferences in general equilibrium (as claimed by Besley and Jewitt (1991)), but also to consider the issue of aggregate production efficiency under decentralisation. As the model is specified central decision making will imply aggregate production efficiency. The more general question when decentralised supply of public goods is optimal is not addressed. We will also ask the following questions: (1) What will the cost–benefit rule look like under decentralisation compared to a centralised second best optimum? (2) What is the characterisation of the cost of public funds under decentralisation?

The paper establishes two main results. First, decentralisation implies aggregate production inefficiency. The main implication of this result is that, given decentralisation, to achieve a constrained Pareto efficient solution the government should *not* let private firms, or publicly owned firms working under the same conditions as private firms, produce the public goods. Instead the optimum will be inconsistent with private cost minimisation. Second, decentralisation is not optimal even if all public goods are neutral vis-à-vi all private goods. It is not optimal because under this assumption the government could not decentralise public goods production and achieve the same solution as in the centralised second best case. This means that focus for future research should shift to the question why decentralised (or rather uncoordinated) decision making regarding public goods and the tax system occur at all. One way to answer this question would be to find new criteria under which decentralisation (or uncoordinated decisions) is optimal.

For both these results there is the same economic mechanism explaining them. Under centralised decision making changes in government control variables will cause equilibrium producer prices to change. The result is a change in tax revenues from commodity taxation (called tax rate effect in the paper) and a change in the cost of production of public goods (called an expenditure effect in the paper). These effects on the government's budget can then in the centralised regime be shown to equal the change in pure profits in the private sector. However, since pure profits are zero in a long run equilibrium the effect on the government's budget due to changes in equilibrium prices sum up to zero. This is one way of looking at the Diamond and Mirrlees (1971a) production efficiency result; there is no mechanism under constant returns to scale and centralised and coordinated decision making in the public sector through which changes in government control variables may affect social welfare via equilibrium price changes. Under decentralised decision making, however, the central budget is split up so that central government decides about taxes and aggregate expenditure on public goods whereas the bureau decides about the allocation of public goods given the structure of taxation and aggregate expenditure decided by the central government. Even if the change in pure profits is still zero, the tax rate effect will affect the central government and the expenditure effect will affect the bureau.

The paper is organised so that the most simple version of the model is discussed in section 2 and section 3. In section 2 household behaviour is described and in section 3 the centralised second best optimum. In section 4 the decentralised second best optimum, i.e., when decentralisation is a constraint, is analysed. Section 5 contains conclusions.

2 Model

The model used is in principal the representative individual version of Diamond and Mirrlees (1971a), but where the public sector produces different public goods instead of different private goods. As in the Diamond–Mirrlees case the government is restricted not to use lump sum taxation although the population is homogeneous. These assumptions then implies that all goods are taxable, that a pure profits tax is unnecessary under centralised decision making since profits are zero due to constant returns to scale and that differential taxation on inputs in the private sector is not used.

The representative individual has preferences, satisfying standard assumptions, over an $n + 1$ dimensional vector of private goods $\mathbf{x}$ and an m dimensional vector of public goods $\mathbf{y}$. These preferences are represented by the real–valued strictly quasi–concave function u defined by $u(\mathbf{x}, \mathbf{y})$ on the nonnegative orthant of $\mathbb{R}^{n+m+1}$. Facing the consumer price vector $\mathbf{q}$ the solution to the consumer's utility maximisation problem, i.e., the vector Marshallian demand functions, is

$$\mathbf{x}^*(\mathbf{q}, \mathbf{y}) := \operatorname{argmax}\Big\{u(\mathbf{x}, \mathbf{y}) : \mathbf{q}'\mathbf{x} \leq 0\Big\}. \tag{1}$$

The interpretation is then that $\mathbf{x}$ denotes the net demand/supply of the representative individual. Let the indirect utility function be $\upsilon(\mathbf{q}, \mathbf{y}) := u(\mathbf{x}^*(\mathbf{q}, \mathbf{y}), \mathbf{y})$. We can then write Roy's identity as $\frac{\partial \upsilon}{\partial q_k} = -\lambda x_k^*(\mathbf{q}, \mathbf{y})\ \forall k = 0, \ldots, n$, where λ is the marginal utility of income in the optimal point. It follows from the Envelope theorem that $\partial v/\partial y_\ell = \partial u/\partial y_i$ $\forall i = 1, \ldots, m$, where y_i is the level of the public goods project.

Private goods are produced by private firms on perfectly competitive markets with the use of a constant returns to scale production technology represented by the private sector transformation function $F(\mathbf{z}) = 0$, where $\mathbf{z}$ is the $(n+1)$ dimensional netput vector. Since firm size under constant returns to scale is indeterminate we normalise the number of firms to unity and talk about a representative firm. Facing the producer prices $\mathbf{p}$ the firm chooses its netput $\mathbf{z}$ so as to maximise its profits $\mathbf{p}'\mathbf{z}$ subject to the production constraint. Because of constant returns to scale pure profits will be zero in equilibrium. As a consequence, see, e.g., Munk (1978), one consumer price and at least on producer price must be normalised. Therefore, let the consumer and producer prices on commodity 0 be normalised so that $q_0 = p_0 = 1$. Note that this normalisation implies that the marginal utility of income equals the marginal utility of the numeraire commodity (commodity 0). The first order conditions can be written as

$$\frac{\frac{\partial F}{\partial z_i}}{\frac{\partial F}{\partial z_0}} = p_i \qquad \forall i = 1, \ldots, n \tag{2}$$

These first order conditions together with the zero profit condition $\mathbf{p}'\mathbf{z} = 0$ (i.e., a requirement that we are in a long run equilibrium) then determine the optimum netput vector $\mathbf{z}^*(\mathbf{p})$.

Public goods are produced in the public sector with use of a production technology represented by the transformation function

$$G(\mathbf{y}, -\mathbf{w}) = 0. \tag{3}$$

where $\mathbf{w}$ is an $n+1$ dimensional vector representing the public demand of private goods used to produce public goods. Of particular interest is which marginal increase in factor j that is required for a marginal increase in the production of project i (i.e., the inverse of the marginal product of factor j in the production of project i), which is defined as

$$\frac{\partial w_j}{\partial y_i} := \frac{\frac{\partial G}{\partial y_i}}{\frac{\partial G}{\partial w_j}}. \tag{4}$$

We can then define the *private marginal cost* of producing public good i using an arbitrary factor j, i.e., the marginal cost that a private firm using the same production technology as the public sector would have faced, as

$$c_{ij} := p_j \frac{\partial w_j}{\partial y_i} \qquad \forall i = 1, \ldots, m \text{ and } j = 1, \ldots, n. \tag{5}$$

Prices $\mathbf{p}$ on private goods are determined by the market clearing condition

$$\mathbf{x}^*(\mathbf{q}, \mathbf{y}) + \mathbf{w} = \mathbf{z}^*(\mathbf{p}), \tag{6}$$

where consumer prices $\mathbf{q}$ will be treated as control variables of the government together with public goods supply $\mathbf{y}$ and public demand of private goods $\mathbf{w}$. These control variables will accordingly affect the equilibrium producer prices, which therefore are written as $\mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})$, with $p_0 = 1$. Multiplying the equilibrium condition (6) with producer prices $\mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})$ we get $\mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})' (\mathbf{x}^* (\mathbf{q}, \mathbf{y}) + \mathbf{w}) = \mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})' \mathbf{z}^* (\mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w}))$, where the left hand side by definition is the profit function. As a digression note that differentiation of the zero profit condition,

$$\pi (\mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})) = \mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})' \mathbf{z}^* (\mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})) = 0 \tag{7}$$

with respect to a government control variable $\xi \in \{\mathbf{q}, \mathbf{y}, \mathbf{w}\}$, with employment of Hotelling's lemma, i.e., $\frac{\partial \pi}{\partial p_i} = z_i^* \ \forall i = 0, \ldots, n$, the market clearing condition (6) and that $\frac{\partial p_0}{\partial \xi} = 0 \ \forall \xi \in \{\mathbf{q}, \mathbf{y}, \mathbf{w}\}$, gives

$$\frac{d\pi}{d\xi} = \sum_{i=1}^{n} \frac{\partial \pi}{\partial p_i} \frac{\partial p_i}{\partial \xi} = \sum_{i=1}^{n} z_i^* \frac{\partial p_i}{\partial \xi} = \sum_{i=1}^{n} (x_i^* + w_i) \frac{\partial p_i}{\partial \xi} = 0. \tag{8}$$

Continuing, since pure profits as well as the market value of consumer net demand, the latter valued at consumer prices, both are zero, i.e., $\pi (\mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})) = \mathbf{q}'\mathbf{x}^* (\mathbf{q}, \mathbf{y}) = 0$, we have $\mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})' (\mathbf{x}^* (\mathbf{q}, \mathbf{y}) + \mathbf{w}) = \mathbf{q}'\mathbf{x}^* (\mathbf{q}, \mathbf{y})$ or

$$(\mathbf{q} - \mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w}))' \mathbf{x}^* (\mathbf{q}, \mathbf{y}) = \mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})' \mathbf{w}, \tag{9}$$

which is the government's aggregate budget constraint.[5] Note that the difference $\mathbf{q} - \mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})$ is interpreted as the specific commodity taxes used to finance the public sector; i.e., $\mathbf{t} = \mathbf{q} - \mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w}) \in \mathbb{R}^{n+1}$ with $t_0 = 0$ due to the normalisation of prices.

The aggregate budget constraint is however inappropriate for an analysis of decentralised production decisions. Therefore, let E denote aggregate government expenditure on public projects so that the budget constraint for the central government is

$$(\mathbf{q} - \mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w}))' \mathbf{x}^* (\mathbf{q}, \mathbf{y}) = E, \tag{10}$$

and the budget constraint of the bureau deciding about public production

$$E = \mathbf{p}(\mathbf{q}, \mathbf{y}, \mathbf{w})' \mathbf{w}. \tag{11}$$

[5]Notice that this implies that there no pure profits in the public sector. This follows from the assumption that public goods are financed through taxation and not through user charges.

3 Centralised investment decisions

The standard assumption in the literature is that public decision making is centralised and coordinated in the sense that decisions about the structure of taxation and the structure of expenditure are made simultaneously by the central government. In this section some of the results from the literature are recapitulated but in a form, that is we use the budget constraints (10) and (11) rather that (9), that facilitates comparison with the results for decentralised decisions presented in the subsequent sections.

When the central government controls all taxes as well as the expenditure levels on the individual projects it solves

$$\max_{\mathbf{q},E,\mathbf{y},\mathbf{w}} \quad \upsilon(\mathbf{q},\mathbf{y}) \text{ s.t. (3), (10) and (11).} \tag{12}$$

This problem has the Lagrangian function

$$\begin{aligned} M^c :=& M(\mathbf{q},E,\mathbf{y},\mu_1,\mu_2,\mu_3) = \\ =& \upsilon(\mathbf{q},\mathbf{y}) + \mu_1 \left[(\mathbf{q}' - \mathbf{p}(\mathbf{q},\mathbf{y},\mathbf{w})')\mathbf{x}^* (\mathbf{q},\mathbf{y}) - E \right] + \\ & + \mu_2 \left[E - \mathbf{p}(\mathbf{q},\mathbf{y},\mathbf{w})'\mathbf{w} \right] - \mu_3 G(\mathbf{y},-\mathbf{w}) . \end{aligned} \tag{13}$$

Let the optimal point in the centralised regime be denoted

$$(E^c, \mathbf{t}^c, \mathbf{w}^c, \mathbf{y}^c, \mu_1^c, \mu_2^c, \mu_3^c) ,$$

where, for notational simplicity, the notation for the optimal tax is used instead of the difference between consumer and producer prices evaluated in the optimal point. Using $\frac{\partial p_0}{\partial \xi} = 0 \; \forall \xi \in \{\mathbf{q},\mathbf{y},\mathbf{w}\}$ the first order conditions can be written as

$$\frac{\partial M^c}{\partial E} = -\mu_1^c + \mu_2^c = 0, \tag{14a}$$

$$\frac{\partial M^c}{\partial q_j} = -\lambda x_j^* + \mu_1^c \left[x_j^* + \sum_{i=1}^n t_i^c \frac{\partial x_i^*}{\partial q_j} - \sum_{i=1}^n x_i^* \frac{\partial p_i}{\partial q_j} \right] - \mu_2^c \sum_{i=1}^n w_i^c \frac{\partial p_i}{\partial q_j} = 0, \tag{14b}$$

$$\frac{\partial M^c}{\partial w_k} = -\mu_1^c \sum_{i=1}^n x_i^* \frac{\partial p_i}{\partial w_k} - \mu_2^c \left[p_k + \sum_{i=1}^n w_i \frac{\partial p_i}{\partial w_k} \right] + \mu_3^c \frac{\partial G}{\partial w_k} = 0, \tag{14c}$$

$$\frac{\partial M^c}{\partial y_\ell} = \frac{\partial \upsilon}{\partial y_\ell} + \mu_1^c \left[\sum_{i=1}^n t_i^c \frac{\partial x_i^*}{\partial y_\ell} - \sum_{i=1}^n x_i^* \frac{\partial p_i}{\partial y_\ell} \right] - \mu_2^c \sum_{i=1}^n w_i \frac{\partial p_i}{\partial y_\ell} - \mu_3^c \frac{\partial G}{\partial y_\ell} = 0, \tag{14d}$$

$$\frac{\partial M^c}{\partial \mu_r} = 0, \tag{14e}$$

$\forall j = 1, \ldots, n$, $k = 0, \ldots, n$, $\ell = 1, \ldots, m$ and $r = 1, 2, 3$. The Lagrange–multiplier μ_1^c is the marginal social welfare of increased tax revenue, the multiplier μ_2^c the marginal social welfare of increased public expenditure and the multiplier μ_3^c is the marginal social welfare of relaxing the production constraint.

Studying the first order conditions we notice three different effects through which government control variables *indirectly* affect the problem since consumer net demand and equilibrium producer prices are affected by these instruments; in the case of public inputs only equilibrium prices are affected: (i) *tax base effect*, (ii) *tax rate effect* since the tax rate is affected via changes in equilibrium producer prices and (iii) *expenditure effect* since expenditure on public inputs via changes in equilibrium producer prices. Effects (ii) and (iii) sum up to the change in pure profits according to (8) and the change in pure profits is zero due to constant returns to scale so their combined effect via the public budget is zero. Remains the tax base effect which allows the choice of consumer prices and the level of public goods to affect aggregate tax revenues.

The first order conditions (14a)–(14d) can now be used to describe the optimal point along different trade offs. First we have the trade–off between gross public expenditure and public tax revenues defined as $\delta := \frac{\mu_2}{\mu_1}$. From the first order condition (14a) we have

$$\delta^c = 1 \tag{15}$$

implying that in the optimal point the trade off between *gross* expenditure and tax revenues is one–to–one. That is, a marginal increase in public expenditure will require an equal marginal increase public revenues because the government assigns the same value to marginal tax revenues as to marginal aggregate expenditure.

Second, there is the trade off between public tax revenues and individual income. It is described by the concept marginal cost of public funds defined as $\gamma := \frac{\mu_1}{\lambda}$.[6] From (14b), using (8) and (15), then follows

$$\gamma^c = \frac{x_j^*}{x_j^* + \sum_{i=1}^{n} t_i^c \frac{\partial x_i^*}{\partial q_j}}, \qquad \forall j = 1, \ldots, n \tag{16}$$

The marginal cost of public funds described by (16) is the same for all commodity taxes in the optimal point. The enumerator is the individual's cost and the denominator the government's gain of an increase in the tax on commodity k; the

[6] This definition is by no means self–evident. I here follow Sandmo (1998) who advocates a definition of the marginal cost of public funds which is independent of the particular project studied. An alternative would be to define project specific marginal cost of public funds. Much of the early literature on the marginal cost of public funds was obscured because the definitions used where project specific (public goods projects or lump sum transfer projects); see Ballard and Fullerton (1992) for a discussion. This is reflected in the empirical/simulation literature where estimates of the marginal cost of public funds are project specific; see for instance Hansson (1984).

latter deviates from the former because the tax also have indirect consequences on the tax base (i.e., the fiscal externality). These indirect consequences are due to changes in individual behaviour caused by the tax change on commodity k. The utility consequences to the individual of these behavioural changes are of course zero according to the envelope theorem.

The third trade off is between the different private goods used as inputs in the production of public goods. From (14c) and using (8) we get

$$\frac{\mu_3^c}{\mu_2^c} = \frac{p_j}{\frac{\partial G}{\partial w_j}} \qquad \forall j = 0, \ldots, n. \tag{17}$$

From (17) it follows that any marginal technical rate of substitution, between any two factors, say a factor $j = 1, \ldots, n$ and factor 0 (the numeraire), in public goods production, should be equal to their relative producer price; i.e.,

$$\frac{\frac{\partial G}{\partial w_j}}{\frac{\partial G}{\partial w_0}} = p_j = \frac{\frac{\partial F}{\partial z_j}}{\frac{\partial F}{\partial z_0}} \qquad \forall j = 1, \ldots, n, \tag{18}$$

where the last equality follows from the first order condition of private firms (2). This is the classic Diamond and Mirrlees (1971a) production efficiency result; the public sector should use factors of production so that its marginal rates of transformation equal the marginal rates of transformation for private firms using the same factors of production.[7] The public sector as a producer of public goods should therefore replicate the cost minimising behaviour of the private firm if it were producing the public goods and was reimbursed (under the assumption that there is no agency problem) by the government for its production cost. The marginal cost for increasing the production of a public good should therefore be same same independent of which factor of production that is used to expand production, or else a reallocation of factors would reduce the cost for a given level of public project output. Accordingly, using (4), (5), and (17) and letting C_j denote the uniform social marginal cost and c_j the uniform private marginal cost of producing public good j we get

$$C_i^c = c_i^c = c_{ij}^c = \frac{\mu_3^c}{\mu_2^c}\frac{\partial G}{\partial y_i} \qquad \forall i = 0, \ldots, m \text{ and } j = 0, \ldots, n. \tag{19}$$

The fourth trade off is between consumption of a public good and private consumption described by the individuals private marginal benefit of a project (the

[7] Private goods are inputs or outputs in the private sector but only inputs in the public sector. The right–hand side of (18) can therefore be either a marginal rate of transformation, a marginal rate of technical substitution or a marginal product but the left–hand side is a marginal rate of technical substitution. To keep terminology simple only the term marginal rate of transformation is used however.

individual marginal willingness to pay for the project) defined as $b_j := \frac{\partial v/\partial y_j}{\lambda}$ $\forall j = 1, \ldots, m$. The first order conditions (14a) can then, using equations (4), (15), (16), and (17), be rewritten as a cost–benefit criterion,

$$b_j^c = \gamma^c \delta^c \left[c_j^c - \sum_{i=1}^{n} t_i^c \frac{\partial x_i^*}{\partial y_j} \right] \qquad \forall j = 1, \ldots, m. \tag{20}$$

In the cost–benefit criterion (20) we interpret the left–hand side as the social marginal benefit and the right–hand side is the social marginal cost, but where these are defined so that social and private marginal benefits coincide. We therefore focus on the social marginal cost which has two components. The bracketed term is the net effect on the public budget of spending one additional unit of tax revenue on project j, given that the tax system is unchanged; i.e., the optimal shadow price for the public good. This net effect is the direct marginal cost of the project minus the indirect effect on tax revenues due to the fiscal externality caused by the distortionary tax system. However, taxes are not unchanged because the additional expenditure has to be financed. But financing is in itself subject to the fiscal externality. Every net unit spent on the project therefore must be multiplied with the marginal cost of public funds in order to get the social marginal cost.

Both the tax used for financing and the project size itself may increase or decrease the tax base and therefore we generally do not know whether the social marginal cost is higher or lower than the private marginal cost. Hence, if both the project itself and the tax used on the margin to finance it increase the tax base, then the social marginal benefit is less than the private marginal cost even if the tax system is distortionary. Available simulations suggest that the combined effect the marginal cost of public funds and the shadow price of the public good under distortionary marginal financing may be in a range from about 0.8 and up wards far beyond unity depending on the type project and the marginal source of financing.[8]

The marginal cost of public funds has been emphasised as a tool in practical cost benefit analysis. This role should not be over–emphasised, however. The concept is important when one analyse the overall size of the government, either from the revenue side or from the expenditure side. To see that the marginal cost of public funds is not important for the composition of expenditure note that the cost–benefit criterion (20) describes the trade–off between consumption of a public good and private consumption. If we instead consider the, fifth and final, trade–off between two public goods projects α and β we form the marginal rate of

[8]The literature is extensive. As an example see Hansson (1984).

substitution between these two projects (i.e., the ratio b_α / b_β) and get

$$\frac{b_\alpha^c}{b_\beta^c} = \frac{c_\alpha^c - \sum_{i=1}^{n} t_i^c \frac{\partial x_i^*}{\partial y_\alpha}}{c_\beta^c - \sum_{i=1}^{n} t_i^c \frac{\partial x_i^*}{\partial y_\beta}} \quad \forall \alpha, \beta = 1, \ldots, m, \quad \alpha \neq \beta, \tag{21}$$

which is independent of the marginal cost of public funds. Instead, the trade–off between the two projects, given the overall size of the public sector, is optimal when the marginal rate of substitution equals the *social* marginal rate of transformation (i.e., the ratio between the optimal shadow prices). Equation (21) therefore illustrates the fact that generally in second best economies the private marginal rate of substitution between two public projects does not equal the private marginal rate of transformation due to potential differences in the non–neutrality vis-à-vi the tax base of different public goods projects. The reason the marginal cost of public funds does not enter into (21) is that this equation is derived under the assumption that the project mix is determined for a given public budget, whereas the marginal cost of public funds occurs in (20) where the policy policy maker by choosing the level of all individual projects is at the same time determining the overall size of the public budget.

4 Delegated investment decisions

A problem with the cost–benefit rule (20) is that a marginal variation in the project size requires a corresponding variation in some tax parameter under a balanced budget constraint. Hence, it requires coordinated decisions regarding project sizes and the design of the tax structure.[9] The main problem of this paper is to analyse how the cost benefit rule will change if the central government provides a given budget for public projects to a public bureau and also delegates to this bureau to decide how this amount should be divided on the different projects. Hence, the overall size of the government is decided directly by the central government but the project mix is determined by the bureau.

The public decision procedure is then assumed to be made in two steps (starting backwards):

Step 2: For a given policy $(\mathbf{t}, E)$ decided by the central government the public bureau solves

$$\max_{\mathbf{y},\mathbf{w}} \quad \upsilon(\mathbf{q}, \mathbf{y}) \text{ s.t. (11) and (3).} \tag{22}$$

[9] Compare for instance with Boadway and Keen (1993) who analyse the cost–benefit criterion under an optimal non–linear income tax. They concluded that coordination between tax and expenditure decisions was required, and that therefore the cost–benefit criterion they derived could not be employed in practical cost–benefit analysis where expenditure decisions are delegated.

The solution to this problem is a vector of expenditure levels on the different projects and a vector public sector factor demands, which both depend on consumer prices, individual lump sum income and the aggregate expenditure level; i.e., $\mathbf{y}^d(\mathbf{q}, E)$ and $\mathbf{w}^d(\mathbf{q}, E)$.

Step 1: The central government takes the actions of the bureau, i.e., $\mathbf{y}^d(\mathbf{q}, E)$ and $\mathbf{w}^d(\mathbf{q}, E)$, into account and solves

$$\begin{aligned} &\max_{\mathbf{q},E} \quad \upsilon(\mathbf{q}, \mathbf{y}^d(\mathbf{q}, E)) \\ \text{s.t.} \ &\left(\mathbf{q} - \mathbf{p}\left(\mathbf{q}, \mathbf{y}^d\left(\mathbf{q}, E\right), \ \ \mathbf{w}^d\left(\mathbf{q}, E\right)\right)\right)' \mathbf{x}^*(\mathbf{q}, \mathbf{y}^d(\mathbf{q}, E)) = E. \end{aligned} \tag{23}$$

We solve to the two–step problem backwards. The step 2 Lagrangian function for the decision of the bureau is

$$M_2^d := M(\mathbf{y}, \mathbf{w}, \mu_2, \mu_3) = \upsilon(\mathbf{q}, \mathbf{y}) + \mu_2 \left[E - \mathbf{p}\left(\mathbf{q}, \mathbf{y}, \mathbf{w}\right)' \mathbf{w}\right] - \mu_3 G(\mathbf{y}, -\mathbf{w}), \tag{24}$$

and the first order conditions are

$$\frac{\partial M_2^d}{\partial w_j} = -\mu_2^d \left[p_j + \sum_{i=1}^n w_i^d \frac{\partial p_i}{\partial w_j}\right] + \mu_3^d \frac{\partial G}{\partial w_j} = 0, \qquad \forall j = 0, \ldots, n, \tag{25}$$

$$\frac{\partial M_2^d}{\partial y_k} = \frac{\partial \upsilon}{\partial y_k} - \mu_2^d \sum_{i=1}^n w_i^d \frac{\partial p_i}{\partial y_k} - \mu_3^d \frac{\partial G}{\partial y_k} = 0, \qquad \forall k = 1, \ldots, m, \tag{26}$$

and $\frac{\partial M_2^d}{\partial \mu_\ell} = 0 \ \forall \ell = 2, 3$, where the super index d on the endogenous variables indicates evaluation in the optimal point.

Before we analyse the bureau's decision we consider the central government's problem. It turns out that we can simplify it by constructing the value function to the bureau's problem;

$$\hat{\upsilon}(\mathbf{q}, E) := \upsilon(\mathbf{q}, \mathbf{y}^d(\mathbf{q}, E)). \tag{27}$$

From the envelope theorem then follows that

$$\frac{d\hat{\upsilon}}{dq_i} = \frac{\partial M_2^d}{\partial q_i} = -\lambda x_i^* \quad \forall i = 1, \ldots, n \text{ and } \frac{d\hat{\upsilon}}{dE} = \frac{\partial M_2^d}{\partial E} = \mu_2^d, \tag{28}$$

where the partial derivatives of the Lagrangian function are evaluated in the optimal point.

The decentralised structure differs from the centralised in several respects. Most importantly the control variables of the bureau will depend on the control variables of the central government. Central government control variables will therefore have one additional link to affect consumer choices and equilibrium producer prices. To keep the notation as simple as possible we therefore introduce the total derivatives of the producer price p_i $\forall i = 1, \ldots, n$ with respect to the control variables $\mathbf{q}$ and E, i.e.,

$$\frac{dp_i}{dE} = \sum_{j=1}^{m} \frac{\partial p_i}{\partial y_j} \frac{\partial y_j^d}{\partial E} + \sum_{j=1}^{n} \frac{\partial p_i}{\partial w_j} \frac{\partial w_j^d}{\partial E} \qquad \text{and} \tag{29a}$$

$$\frac{dp_i}{dq_k} = \frac{\partial p_i}{\partial q_k} + \sum_{j=1}^{m} \frac{\partial p_i}{\partial y_j} \frac{\partial y_j^d}{\partial q_k} + \sum_{j=1}^{n} \frac{\partial p_i}{\partial w_j} \frac{\partial w_j^d}{\partial q_k} \qquad \forall k = 1, \ldots, n. \tag{29b}$$

We can then rewrite (8), for $\xi \in \{E, \mathbf{q}\}$, as

$$\frac{d\pi}{d\xi} = \sum_{i=1}^{n} \frac{\partial \pi}{\partial p_i} \frac{dp_i}{d\xi} = \sum_{i=1}^{n} z_i^* \frac{dp_i}{d\xi} = \sum_{i=1}^{n} (x_i^* + w_i^c) \frac{dp_i}{d\xi} = 0. \tag{30}$$

The step 1 Lagrangian function for the decision of the central government is

$$\begin{aligned} M_1^d =& M(\mathbf{q}, E, \mu_1) = \hat{v}(\mathbf{q}, E) \\ &+ \mu_1 \left[\left(\mathbf{q} - \mathbf{p}\left(\mathbf{q}, \mathbf{y}^d(\mathbf{q}, E), \mathbf{w}^d(\mathbf{q}, E) \right) \right)' \mathbf{x}^*(\mathbf{q}, \mathbf{y}^d(\mathbf{q}, E)) - E \right], \end{aligned} \tag{31}$$

and the first order conditions, using equation (28) and (29) are

$$\frac{\partial M_1^d}{\partial E} = \mu_2^d + \mu_1^d \left[\sum_{j=1}^{m} \sum_{i=1}^{n} t_i^d \frac{\partial x_i^*}{\partial y_j} \frac{\partial y_j^d}{\partial E} - \sum_{i=1}^{n} x_i^* \frac{dp_i}{dE} - 1 \right] = 0 \tag{32a}$$

$$\begin{aligned} \frac{\partial M_1^d}{\partial q_k} =& -\lambda x_k^* + \mu_1^d \Bigg[x_k^* + \sum_{i=1}^{n} t_i^d \frac{\partial x_i^*}{\partial q_k} \\ &+ \sum_{i=1}^{n} \sum_{j=1}^{m} t_i^d \frac{\partial x_i^*}{\partial y_j} \frac{\partial y_j^d}{\partial q_k} - \sum_{i=1}^{n} x_i^* \frac{dp_i}{dq_k} \Bigg] = 0, \end{aligned} \tag{32b}$$

$\forall k = 1, \ldots, n$, and $\frac{\partial M_1^d}{\partial \mu_1^d} = 0$.

Delegated decisions means that additional channels are created by which the choice of consumer prices and aggregate expenditure, i.e., the control variables of the central government, can affect the tax base (i.e., the tax rate and the expenditure effects). More important, however, is that the central government, due to the decentralisation of public goods production, will only consider effects the tax base effect and the tax rate effect and whereas the expenditure effect is only considered by the bureau. The tax rate and the expenditure effects still sum up to zero according to (30), but there is no decision maker that takes both into account. Hence, changes in equilibrium producer prices will affect the results even if pure profits are zero.

Consider now the same four trade–offs as in the centralised case, where we start with the control variables of the central government (aggregate expenditure and consumer prices). Starting with the trade of between aggregate tax revenues and public expenditure as measured by δ^d, we get from (31) that

$$\delta^d = 1 - \sum_{j=1}^{m} \sum_{i=1}^{n} t_i^d \frac{\partial x_i^*}{\partial y_j} \frac{\partial y_j^d}{\partial E} + \sum_{i=1}^{n} x_i^* \frac{dp_i}{dE} \tag{33}$$

and therefore generally will not be equal to unity. The reason is that in the general case the public goods are non–neutral so aggregate expenditure, via the public goods levels, now create a tax base effect and a tax rate effect. Even if all public goods are neutral vis-à-vi all private goods there would be a tax rate effect. This because aggregate expenditure will potentially affect the input as well as output mix in the production of public goods even if private consumption is unaffected by public goods. The bureau will not take any of this into account since it will consider itself with the expenditure effect only. As a consequence the central government and the bureau will assign different values to a marginal increase in aggregate expenditure.

The second trade off is between public tax revenues and private income as measured by the marginal cost of public funds.

$$\gamma^d = \frac{x_k^*}{x_k^* + \sum_{i=1}^{n} t_i^d \frac{\partial x_i^*}{\partial q_k} + \sum_{i=1}^{n} \sum_{j=1}^{m} t_i^d \frac{\partial x_i^*}{\partial y_j} \frac{\partial y_j^d}{\partial q_k} - \sum_{i=1}^{n} x_i^* \frac{dp_i}{dq_k}}, \qquad \forall k = 1, \ldots, n \tag{34}$$

The marginal cost of public funds also contains these two additional effects. First, a change in consumer prices now creates a tax base effect via a change in the bureau's choice of public goods mix. Second, a change in consumer prices creates a tax rate effect due to changes in equilibrium producer prices. Generally we cannot say whether in which direction these two additional effects will cause the marginal cost of public funds

Turning to the control instruments of the bureau, the third trade off is between the different inputs in public goods production. We can then state the first major result of this paper:

Proposition 1. *If public investment decisions are decentralised, with constant returns to scale in private production and an homogeneous population, then aggregate production efficiency does not hold, i.e., marginal rates of transformation between any two factors of production should not be the same in public and private production.*

Proof. Consider two factors of production, such as factor j and 0. From the first order condition (25) then follows

$$\frac{\frac{\partial G}{\partial w_j}}{\frac{\partial G}{\partial w_0}} = \frac{p_j + \sum_{i=1}^{n} w_i^d \frac{\partial p_i}{\partial w_j}}{1 + \sum_{i=1}^{n} w_i^d \frac{\partial p_i}{\partial w_0}} \neq p_j = \frac{\frac{\partial F}{\partial z_j}}{\frac{\partial F}{\partial z_0}} \qquad \forall j = 1, \ldots, n, \tag{35}$$

where the last equality follows from the private first order condition (2). □

The reason the efficiency theorem does not hold under decentralised decision making is that, in the trade off between two different inputs, the bureau will take into account only the expenditure effect but not the tax rate effect. Delegation creates new incentives for the bureau, which explains the result. Constant returns to scale is no longer a sufficient condition for aggregate production efficiency. One may also note that the proposition holds independent of whether public goods are non–neutral or not.

Does the result in Proposition 1 has any practical relevance? Whenever the project is such that at least one producer price is affected, then the theorem will hold. Only when the project is so limited and small that no producer price is affected can the problem be regarded as if supply is perfectly elastic and therefore all producer prices will be unaffected by the project.

As a consequence the marginal cost to the bureau of increasing the level of a public goods project will no longer be equal to the private marginal but to the private marginal cost plus the expenditure effect of that increase in the public good; i.e.,

$$C_j^d = \frac{\mu_3^d}{\mu_2^d}\frac{\partial G}{\partial y_j} = c_{jk}^d + \sum_{i=1}^{n} w_i^d \frac{\partial p_i}{\partial w_k}\frac{\partial w_k^d}{\partial y_j} \qquad \forall j = 1, \ldots, m \text{ and } k = 0, \ldots, n. \tag{36}$$

The bureau will therefore not behave as a cost minimising firm. Also, equalisation of the private marginal cost of production over factors of production will not longer be the case. The bureau will instead equalise the social marginal cost of production C_j as given by (36). Therefore, whenever equilibrium price effects are non–negligible, then Proposition 1 has something to say about how production should be organised in the public sector; i.e., it will increase the weight to public production.

We can then consider the fourth and last trade off between private and public consumption:

$$b_j^d = \gamma^d \delta^d \left[C_j^d + \sum_{i=1}^{n} w_i^d \frac{\partial p_i}{\partial y_j} \right]. \tag{37}$$

The cost–benefit criterion (37) resembles the cost benefit criterion under centralised decision making (20). However, the marginal cost is no longer the private marginal cost and the addition term is not how public goods affect the tax base but how public goods affect the cost of production via changes in equilibrium producer prices. As before, however, is the trade off between public goods affected by the marginal cost of public funds, i.e.,

$$\frac{b_\alpha^d}{b_\beta^d} = \frac{C_\alpha^d + \sum_{i=1}^{n} w_i^d \frac{\partial p_i}{\partial y_\alpha}}{C_\beta^d + \sum_{i=1}^{n} w_i^d \frac{\partial p_i}{\partial y_\beta}} \quad \forall \alpha, \beta = 1, \dots, m, \quad \alpha \neq \beta, \tag{38}$$

Condition (38) tells us about the optimal trade off for the bureau between two public goods. It should be compared with (21). From the previous literature on decentralisation of public goods supply we know that the decentralised solution will coincide with the centralised solution, i.e., decentralisation is optimal, if preferences satisfy the sufficient conditions provided by e.g., Lau et al. (1978, theorems 1 and 2) and Besley and Jewitt (1991, theorems 1 and 2).[10] However, these sufficient conditions are derived in an economy with fixed producer prices. Although Besley and Jewitt (1991, p. 1770) claims that none of their results depend on that assumption they do not show that this is actually the case. Lau et al. (1978, footnote 5) note that fixed producer prices implies zero pure profits which in turn is the condition for production efficiency in Diamond and Mirrlees (1971a); which Lau et al. (1978) interprets as a condition for decentralisation.

The conditions for sufficiency provided by Lau et al. (1978) and more generally by Besley and Jewitt (1991) are restrictions on preferences such that a sufficient degree of separability between the public goods and other commodities are achieved. Now make the following assumption:

Assumption 1. *All public goods are neutral, i.e.,*

$$\frac{\partial x_i^*}{\partial y_j} = 0 \qquad \forall i = 0, \dots, n \text{ and } j = 1, \dots, m. \tag{39}$$

Assumption 1 is a sufficient but not necessary condition for the criteria provided by Lau et al. (1978) and Besley and Jewitt (1991).This means that under this assumption decentralisation is optimal under fixed producer prices. However, with equilibrium changes in producer prices the following result follows:

[10]Both Lau et al. (1978) and Besley and Jewitt (1991) provide different conditions depending on whether consumer prices are optimally chosen or not. Besley and Jewitt (1991) also show that their sufficient condition also is a necessary condition.

Proposition 2. *Under constant returns to scale in private production and an homogeneous population, then all public goods being non–neutral in the sense of Assumption 1, is not a sufficient condition for decentralisation of the supply of public goods to be optimal.*

Proof. Suppose Assumption 1 holds. This means that (21) and (38) can be rewritten as

$$\frac{b^c_\alpha}{b^c_\beta} = \frac{c^c_\alpha}{c^c_\beta} \qquad \forall \alpha, \beta = 1, \ldots, m, \quad \alpha \neq \beta \text{ and,} \tag{40a}$$

$$\frac{b^d_\alpha}{b^d_\beta} = \frac{C^d_\alpha + \sum_{i=1}^{n} w^d_i \frac{\partial p_i}{\partial y_\alpha}}{C^d_\beta + \sum_{i=1}^{n} w^d_i \frac{\partial p_i}{\partial y_\beta}} \qquad \forall \alpha, \beta = 1, \ldots, m, \quad \alpha \neq \beta. \tag{40b}$$

Since producer are affected by the quantity decisions of the bureau, conditions (40a) and (40b) will not coincide. □

In the present model the public sector tax base will, under Assumption 1, be independent of the level of public expenditure and the project mix. As a consequence, equilibrium producer prices will be unchanged due to changes in public goods supply (direct effect), but increased public goods will increase public sector demand for private goods and change the equilibrium producer prices (indirect effect). Therefore Assumption 1, or the criteria provided by Lau et al. (1978) and Besley and Jewitt (1991), will not imply that decentralisation of public goods supply is optimal in the present model.

However, Besley and Jewitt (1991) explicitly claimed that their result holds under variable producer prices. Why is it that this turns out not to be the case? In their model Besley and Jewitt (1991) did not only assumed fixed producer prices but they also assumed fixed and optimally chosen marginal costs of producing public goods. Therefore, allowing producer prices to be endogenously determined in equilibrium would not in their model create an effect like the expenditure effect as in the model analysed in this paper. The reason is that the marginal cost of producing public goods would still be fixed. Changes in the equilibrium producer prices will therefore only affect the incentives of the central government, an effect which turns out to be zero due to zero pure profits, and not the incentives of the bureau.

What is the equivalent to the assumption of fixed and optimally chosen marginal costs of producing public goods in the present model? Since the problem is variable input prices rather than variable marginal costs the solution seems to lie in an assumption about fixed but optimally chosen input prices for the bureau. To replicate the second best optimum under the Besley and Jewitt (1991) separability requirement the government sets the internal prices equal to the equilibrium producer prices in the second best optimum. In the general case, however, with no restrictions on preferences, such a solution is not necessarily welfare improving. It may be an welfare improvement since it allows the policy maker to use more policy instrument, but since the incentive structure is changed as well a welfare improvement is not self–evident; some other set of prices may improve welfare.

More important is, however, if it is reasonable to assume that the policy maker can choose input prices for the bureau in such a manner? First, it would imply that the government could use two price structures; an internal price structure, which is chosen by the central government, for determining the mix of factors and public goods and an external price structure (i.e., market producer prices) to determine the actual government expenditure on public goods production. Second, the central government could furthermore make this choice of input prices coordinated with its choice of tax system. Nonetheless, the perspective employed in this paper is that the central government at the same time is restricted to coordinate the productions (i.e., quantity) decisions concerning public goods with the choice of the tax system. A natural question is that if internal prices for public goods production can be coordinated with the tax system why cannot quantity decisions be coordinated in the same way? If we relax the assumption of decentralisation of public goods production and allow for coordinated quantity decisions, then the assumption of centrally chosen internal prices seems to be much more reasonable. However, if the government can make centralised decisions regarding public goods production and coordinate these decisions with decisions regarding the tax system it does not need the to use internal prices as an additional instrument.

5 Conclusions

The purpose of this paper has been to analyse the consequences for optimal policies of decentralising public goods supply. The analysis has been performed within the same model as Diamond and Mirrlees (1971a), i.e., a representative agent and a constant returns to scale private production technology, with the alteration that the public sector produces public goods with private goods as inputs rather than produces private goods in addition to what is produced by the private sector. The two key features of the model are pure profits, which are zero in a long run equilibrium, and decentralisation of the supply of public goods to a subordinate public bureau with a given amount of tax revenues to spend on the production of public goods.

The main results of the paper are that (i) decentralisation implies that production of public goods should use private goods in proportion such that their marginal rate of transformation does not equal that in private production, i.e., aggregate production inefficiency, and (ii) not even the rather strong requirement that all public goods are neutral visa vi all private goods implies that decentralisation is optimal.

One main question not analysed in this paper is under which conditions decentralisation actually is optimal. The present analysis suggests that the conditions putting restrictions on preferences must be complemented with restriction on the cost function used by the subordinate bureau or production technology for producing public goods.

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