

# Central-Bank Independence, Economic Behavior, and Optimal Term Lengths: Comment

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First version: September 1997

## Abstract

Waller and Walsh (1996) argue that the optimal term length of the central banker can exceed one period when the central bank is conservative enough. However, the optimal conservativeness is unlikely to be exogenous. In this note we show how the optimal conservativeness and the optimal term length are determined simultaneously in the framework of Waller and Walsh. Furthermore, we extend the study to the inflation contract and the inflation target regimes. Under both regimes, the optimal parameter of conservativeness is independent of the term length and is always 1. Moreover, it is possible to have an optimal multi-term central banker under both the state-contingent inflation contract regime and the state-contingent inflation target regime.

Keywords: central bank independence, inflation target, and optimal term length.

JEL Classification Number: E52, E58.

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In an article recently published in the American Economic Review, Waller and Walsh (1996) (referred to henceforth as W&W) offer a unified framework for analyzing both the optimal term length and the independence (conservativeness) of the central bank. They conclude that when there is political uncertainty and persistent shifts in public preferences, the appointment of a conservative central banker is able to increase the optimal term length and therefore lead to lower average inflation, without necessarily increasing the volatility of output. The (necessary) condition for achieving the optimal multi-term length is that the parameter of conservativeness should exceed a constant critical value. They treat the parameter of the conservativeness as a given constant throughout their paper. However, as suggested by many other studies, the degree of the conservativeness should be optimally determined. By introducing a determination of the optimal parameter of the conservativeness into the W&W model, we obtain a sequence of optimal parameters associated with the term length instead of a single constant. Furthermore, from our point of view, the optimal term length and the optimal conservativeness should be determined simultaneously. Therefore, the longest optimal term length might not be the optimal solution for welfare maximization.

In this note, we show how the optimal conservativeness and the optimal term length are determined simultaneously on the basis of the W&W model. We also give an example to demonstrate the possibility of a multi-equilibrium solution. Furthermore, we study the case of an inflation contract and an inflation target, which are mentioned by W&W without any clear conclusion. We conclude that the parameter of optimal conservativeness is always 1 under both the inflation contract regime and the inflation target regime. Under both the constant inflation contract and the constant inflation target regimes, the optimal term length is always one-term. Moreover, it is possible to have an optimal multi-term central banker under the state-contingent inflation contract and the state-contingent inflation target regimes.

## 1 Simplified W&W Model and Solution

We first simplify the model established by W&W, since many factors in the original paper do not seem to be important for the issue of determining the optimal term length  $T$  and the optimal conservativeness  $\bar{\alpha}$ . Ignoring the money market (the velocity shock

and the monetary shock) and the supply shocks specific to individual sectors would have no consequences for the discussion in this note. We here assume that the central bank is able to control inflation completely and precisely.

The (log) sectorial output  $y_{it}$  is determined as follows:

$$y_{it} = y^N + \ln(\pi_t / \pi^e) + u_t; \quad (1)$$

(Log)  $y^N$  is the natural rate of output and is a constant across the sectors.  $\pi_t$  is the inflation rate and  $\pi^e$  is the expected inflation rate based on the information available at time  $t-1$ : The economy-wide supply shock  $u_t$  is a zero-mean process with a constant variance  $\sigma_u^2$ :

The desired inflation rate  $\pi_{M;t}^*$  preferred by the median voter at time  $t$  randomly varies around the mean  $\bar{\pi}_{M;t}^*$ :

$$\pi_{M;t}^* = \bar{\pi}_{M;t}^* + \epsilon_t; \quad (2)$$

The mean  $\bar{\pi}_{M;t}^*$  allows for persistent but infrequent shifts:

$$\bar{\pi}_{M;t+1}^* = \begin{cases} \bar{\pi}_{M;t}^* & \text{with probability } p \\ \bar{\pi}_{M;t}^* + \phi_{t+1} & \text{with probability } 1-p; \end{cases} \quad (3)$$

where  $\phi$  is a zero-mean process with a constant variance  $\sigma_\phi^2$ . The inflation rate preferred by the central bank depends on the degree to which it is subject to partisan interests:

$$\begin{aligned} \pi_{cb;t}^* &= \bar{\pi}_{M;t}^* + \mu(\pi_{M;t}^* - \bar{\pi}_{M;t}^*) \\ &= \bar{\pi}_{M;t}^* + \mu \epsilon_t; \end{aligned} \quad (4)$$

The parameter  $\mu$ ; which is between 0 and 1; reflects the degree of partisanship in the appointment process. If  $\mu = 0$ ; the central banker merely follows the mean.

The loss functions for each individual sector are given by

$$L_{it} = \frac{1}{2}[(y_{it} - y^N - k)^2 + \omega(\pi_t - \pi_i^*)^2]; \quad (5)$$

where  $\pi_i^*$  is the inflation rate desired by sector  $i$  and is assumed to be distributed uniformly over the range  $[L; U]$  with a constant variance  $\sigma_{\pi_i^*}^2$ . The central bank has its own loss function:

$$L_t^{CB} = \frac{1}{2}[(y_t^{CB} - y^N - k)^2 + \omega^-(\pi_t^{CB} - \pi_{cb;t}^*)^2]; \quad (6)$$

where  $\omega^- \leq 1$ ; indicating that the central bank can be more conservative than the society.

We suppose that the monetary policy is carried out by an instrument independent central bank. In other words, the central bank can carry out the monetary policy independently in order to minimize its own loss function (6). A party is able to influence monetary policy via the appointment of a central banker with a certain degree of conservativeness  $\bar{\omega}$  and term length  $T$ : An influence can also be exerted by other factors, such as the parameter  $\mu$ ; as indicated in (3).

The inflation rate and the output in equilibrium under the objective function of the central bank (6) can be expressed by

$$\pi_t^{CB} = \frac{\bar{\omega}}{\bar{\omega} - 1} k + E_{t-1}(\pi_{cb;t}) + \frac{\bar{\omega} - \mu}{2 + \bar{\omega}} [\pi_{M;t} - E_{t-1}(\pi_{M;t})] - \frac{\bar{\omega}}{2 + \bar{\omega}} u_t; \quad (7)$$

and

$$y_{it}^{CB} = y^N + \frac{\bar{\omega} - \mu}{2 + \bar{\omega}} [\pi_{M;t} - E_{t-1}(\pi_{M;t})] + \frac{\bar{\omega}}{2 + \bar{\omega}} u_t; \quad (8)$$

respectively. The socially expected average loss can be obtained by substituting (7) and (8) into (5). The average expected loss under a multi-term ( $T$ ) central banker is then

$$L^T = \frac{1}{2(1 \pm)} \left[ \left(1 + \frac{2}{\bar{\omega} - 2}\right) k^2 + \frac{\bar{\omega}(\bar{\omega}^2 + \bar{\omega} - 2)}{(\bar{\omega}^2 + \bar{\omega} - 1)^2} \pi_u^2 + \bar{\omega} \pi_{M^*}^2 \right] + \frac{\bar{\omega}(1 \pm p)}{2(1 \pm)^2} \pi_{\phi}^2 + \frac{\bar{\omega}}{2(1 \pm^T)} f \left[ \frac{\bar{\omega} - 2(\bar{\omega}^2 + \bar{\omega})}{(\bar{\omega}^2 + \bar{\omega} - 1)^2} + \frac{\pm(1 \pm^T i - 1)}{1 \pm} \right] \mu^2 \pi_{\phi}^2 - \frac{T \pm^T i - 1}{2(1 \pm)} \pm(1 \pm p) \pi_{\phi}^2 ] g; \quad (9)$$

Following W&W, we can find the relationship between the optimal term length  $T$  and conservativeness  $\bar{\omega}$ :

$$\frac{\partial L^T}{\partial T} = \frac{\bar{\omega}^T (1 \pm p) \pi_{\phi}^2}{2(1 \pm)(1 \pm^T)^2} f \pm^T i - T \ln \pm i - 1 + \left[ \frac{\bar{\omega} - 2(\bar{\omega}^2 + \bar{\omega})}{(\bar{\omega}^2 + \bar{\omega} - 1)^2} i - 1 \right] A g;$$

where  $A = \frac{(1 \pm) \ln \pm \mu^2 \pi_{\phi}^2}{(1 \pm p) \pi_{\phi}^2} < 0$ : When the conservativeness  $\bar{\omega}$  is given, we can find out whether the optimal term length exceeds one period by using the necessary condition stated by W&W. If  $\bar{\omega} < H$ ; where  $H = 1 + (1 + \bar{\omega}^2 - \bar{\omega})^{1/2} > 1$ ; the optimal term length  $T$  should be one period. If  $\bar{\omega} > H$ ; it is possible that  $T > 1$  which fulfills the following condition

$$\pm^T i - T \ln \pm i - 1 + \left[ 1 + \frac{\bar{\omega} - 2(\bar{\omega}^2 + \bar{\omega})}{(\bar{\omega}^2 + \bar{\omega} - 1)^2} \right] A g; \quad (10)$$

By ignoring the integer constraints of the first order condition,  $\frac{\partial L^T}{\partial T} = 0$ , we can find the relationship between the optimal term length and the conservativeness:

$$\bar{\omega} = \frac{2B}{1 \pm \bar{\omega} B}; \quad (11)$$

where  $B = \frac{1}{2} \left[ \frac{A + T_j T \ln \pm 1}{A^{\otimes} (\sigma^2 + \otimes)} \right] > 0$ . Figures 1 and 2 show the simulation result of the first order condition (the solid lines in Figure 1 and the dashed line in Figure 2). An increase of the conservativeness  $\bar{\pi}$  would lead to an increase in the optimal term length  $T$ . However, the optimal term length  $T$  can only approach its upper limit asymptotically (for instance  $T = 23$  in Figure 1).

## 2 Conservativeness

W&W treat the parameter  $\bar{\pi}$  as a given constant. However, many other studies, for instance the studies of Rogo<sup>®</sup> (1985) and Alesina and Gatti (1995), have suggested that the parameter of conservativeness  $\bar{\pi}$  should be optimally determined and therefore should be determined in conjunction with the optimal term length. We first consider the relationship between the optimal conservativeness and the term length. If the central bank in the study is completely partisan, the decision is based on the incumbent's objective function. Therefore we have to use the median voter's loss function as an indirect objective function to identify the optimal parameters. However, according to the setting by W&W, the only difference among the sectors is the desired rate of inflation  $\pi_i^d$ ; which has been treated as a random variable. A term such as  $(\pi_{it}^d - \pi_M^d)^2$  is a known constant from the perspective of an agent in sector  $i$ ; its expectation from the perspective of sectorial anonymity is  $\frac{1}{4} \pi_i^d$ . Moreover, it will be clear later that  $\frac{1}{4} \pi_i^d$  has no power to influence the decision on the optimal parameter  $\bar{\pi}$  and the optimal term length  $T$ . Hence it is not important who is to appoint the central banker, since the optimal decision could be based on the average expected social loss function (9).

In the absence of political uncertainty, i.e.,  $\pi_{it}^d = \pi_u^d = 0$ ; the average loss function is reduced to

$$L^R = \frac{1}{2(1 + \sigma)} \left[ \left(1 + \frac{\sigma^2}{\otimes^{-2}}\right) k^2 + \frac{\otimes (\sigma^2 + \otimes^{-2})}{(\sigma^2 + \otimes^{-2})^2} \frac{1}{4} \pi_u^d + \otimes \frac{1}{4} \pi_i^d \right];$$

The first order condition,

$$\frac{\partial L^R}{\partial \bar{\pi}} = \frac{1}{1 + \sigma} \left[ \frac{\sigma^2}{\otimes^{-\pi} 3} k^2 + \frac{\otimes^2 (\sigma^2 + \otimes^{-2})}{(\sigma^2 + \otimes^{-\pi})^3} \frac{1}{4} \pi_u^d \right] = 0;$$

then indicates that the optimal parameter  $\bar{\pi}$  has to be larger than 1: Thus, as shown by Rogo<sup>®</sup> (1985), a more conservative central bank ( $\bar{\pi} > 1$ ) is an optimal choice.

When political uncertainty is introduced into the model, the optimal conservativeness would rely on the size of the shocks. By considering the case that  $T = 1$ ; the first order condition,

$$\frac{\partial L^1}{\partial \tau} = \frac{1}{1+i} \left[ i \frac{\sigma^2}{\tau^3} k^2 + \frac{\sigma^2 (-i-1)}{(\sigma^2 + \tau)^3} \frac{3}{4} u^2 + \frac{\tau^2 \mu^2 (\sigma^2 + \tau)^{-\tau}}{(\sigma^2 + \tau)^3} \frac{3}{4} u^2 \right] = 0;$$

suggests that the optimal conservativeness  $\tau^1$  would be smaller than that in the absence of political uncertainty  $\tau^0$ : (Like most other studies, we here ignore the multi-solution.) An important fact is that a more conservative central bank may not be the optimal choice if the political uncertainty  $\frac{3}{4} u^2$  is large enough. Considering the constraint condition,  $\tau \geq 1$ ; the optimal parameter  $\tau^1$  could reduce to 1: This is because within the framework proposed by W&W, the political uncertainty makes no contribution to the level of the mean, or of the inflation bias. In other words, the inflation bias and the expected inflation rate would not be affected by the political uncertainty. The uncertainty has effects merely on the second moment of the variables.

In the case of a multi-term central banker, the first order condition with respect to the parameter  $\tau$  indicates

$$\frac{\partial L^T}{\partial \tau} = \frac{1}{1+i} \left[ i \frac{\sigma^2}{\tau^{3T}} k^2 + \frac{\sigma^2 (-i-1)}{(\sigma^2 + \tau)^{3T}} \frac{3}{4} u^2 + \frac{1+i}{1+i} \frac{\tau^2 \mu^2 (\sigma^2 + \tau)^{-\tau T}}{(\sigma^2 + \tau)^{3T}} \frac{3}{4} u^2 \right] = 0; \quad (12)$$

where  $T \geq 1$ : Thus we have a sequence of optimal parameters of conservativeness  $\tau^{-\tau T}$  corresponding to the term length  $T$ : By neglecting the integer constraint,  $\tau^{-\tau T}$  may be regarded as a function of  $T$ : The simulation result in Figure 2 (the solid line) shows that  $\tau^{-\tau T}$  increases monotonically corresponding to the increase of  $T$ : This can be easily identified by rewriting (12):

$$i \frac{\sigma^2 k^2}{\tau^{3T}} (\sigma^2 + \tau)^3 + \sigma^2 (-i-1) \frac{3}{4} u^2 + \frac{(1+i) \tau^2 \mu^2 (\sigma^2 + \tau)^{-\tau T} \frac{3}{4} u^2}{1+i} = 0;$$

We then have

$$\frac{d \tau^{-\tau T}}{dT} = \frac{i (1+i) \tau^2 \mu^2 (\sigma^2 + \tau)^{-\tau T} \frac{3}{4} u^2 \ln(\sigma^2 + \tau)}{3 \sigma^4 k^2 (\sigma^2 + \tau)^{2-\tau T} + \sigma^2 \frac{3}{4} u^2 + (1+i) \tau^2 \mu^2 (\sigma^2 + \tau)^{\frac{3}{4} \tau T} (1+i)} > 0;$$

We can therefore conclude that  $\tau^0 > \tau^{-\tau T} > \tau^1$ : So an increase of the term length  $T$  would lead to a higher optimal conservativeness  $\tau^{-\tau T}$ .

When either  $k$  is small or the shock  $\frac{3}{4} u^2$  is large enough,  $\tau^1$  could be smaller than  $H$ . However, it is also possible for the parameter  $\tau^{-\tau T}$  to exceed  $H$  after some periods, as shown in Figure 2.

If we put the two first order conditions (11) and (12) together, we can obtain the optimal conservativeness  $\bar{\alpha}$  and the optimal term length  $T$ : As indicated in Figure 1, the optimal term length could reach 23 terms, but the optimal term length associated with the optimal conservativeness is approximately 4 terms (equilibrium II in Figure 2). State I in Figure 2 is not an equilibrium. However, for  $\bar{\alpha} < \bar{H}$  we have another equilibrium, that is,  $\bar{\alpha}^1$  and  $T^*$  equal to 1: So we have a multi-equilibrium solution.

### 3 Inflation Contracts and Optimal Inflation Targets

W&W have mentioned that a study of the optimal term length with the inflation contract is of importance, but they have not given an explicit discussion. Furthermore, many countries have adopted inflation targets as a means to reduce inflation in reality, so it is worth carrying out a study of the target regime. In this note, we study the optimal term length associated with the inflation contract regime suggested by Walsh (1995) and Persson and Tabellini (1993) and the optimal term length associated with the inflation target regime suggested by Svensson (1997). Since the desired inflation rate is a random variable, the contract and the target could rely on the state of the realized shock  $\epsilon_t$ . Therefore, two cases are considered in our study, the constant inflation contract and target and the state-contingent inflation contract and target.

#### 3.1 Inflation Contracts

We first consider the constant inflation contract. We suppose that the median voter can add an inflation contract,  $c \epsilon_t (\pi_t^{CB} - \pi_{cb,t}^*)$ ; which is proportional to the deviation of the inflation rate from the desired level, to the central bank's loss function (6):

$$L_t^{CBIC} = \frac{1}{2}[(y_t^{CB} - y_t^N - k)^2 + \beta^{-1}(\pi_t^{CB} - \pi_{cb,t}^*)^2] + c \epsilon_t (\pi_t^{CB} - \pi_{cb,t}^*); \quad (13)$$

where  $c$  is a constant, that is,  $c = a$ . The economic intuition underlying (13) is that the inflation contract puts more weight on inflation stability. The first order condition for minimizing (13) is

$$\frac{\partial L_t^{CBIC}}{\partial \pi_t^{CB}} = \beta^{-1}[\pi_t^{CBIC} - E_{t-1}(\pi_t^{CBIC})] + \epsilon_t u_t - \epsilon_t k + \beta^{-1}(\pi_t^{CBIC} - \pi_{cb,t}^*) + c = 0; \quad (14)$$

Taking the expectation on both sides of the first order condition (14) based on the information available at period  $t - 1$ , we obtain

$$E_{t-1}(\pi_t^{CBIC}) = \frac{\alpha k}{\beta} + E_{t-1}(\pi_{cb,t}) - \frac{c}{\beta}$$

Supposing the central bank to achieve its own desired inflation rate  $\pi_{cb,t}$ ; the rational expectation about the inflation rate  $\pi_t^{CBIC}$  by the society at time  $t - 1$  is then  $E_{t-1}(\pi_t^{CBIC}) = E_{t-1}(\pi_{cb,t})$ : This yields that  $c = \alpha k$ : Thus the inflation rate becomes

$$\pi_t^{CBIC} = E_{t-1}(\pi_{cb,t}) + \frac{\beta^{-1} \mu}{\beta^2 + \beta^{-1}} \pi_{t-1} - \frac{\alpha}{\beta^2 + \beta^{-1}} u_t \quad (15)$$

The output under the constant inflation contract regime is unchanged, i.e., it is the same as in (8). By comparing (7) and (15), we observe that the inflation bias  $\frac{\alpha k}{\beta}$  has been completely eliminated, but the political uncertainty is untouched. The average loss function under a  $T$  term central banker evaluated at time  $t - 1$  can then be expressed as

$$L^{TIC} = \frac{1}{2(1 \pm \beta)} [k^2 + \frac{\beta(\alpha^2 + \beta^{-2})}{(\alpha^2 + \beta^{-2})^2} \beta u^2 + \beta \frac{1}{4} \pi^2] + \frac{\beta \pm (1 - \beta)}{2(1 \pm \beta)^2} \beta \pi^2 + \frac{\beta}{2(1 \pm \beta)^T} f[\frac{\beta^{-2}(\alpha^2 + \beta)}{(\alpha^2 + \beta^{-2})^2} + \frac{\pm(1 \pm \beta^{T-1})}{1 \pm \beta}] \mu^2 \beta u^2 + \frac{T \pm T - 1}{2(1 \pm \beta)^T} \pm (1 - \beta) \frac{1}{4} \pi^2 g \quad (16)$$

The first order condition for the parameter  $\beta$  is

$$\frac{\partial L^{TIC}}{\partial \beta} = \frac{1}{1 \pm \beta} [\frac{\beta^2 \alpha^2 (\beta^{-T} - 1)}{(\alpha^2 + \beta^{-2})^3} \beta u^2 + \frac{1 - \beta \pm \beta}{1 \pm \beta} \frac{\mu^2 (\alpha^2 + \beta)^{-T}}{(\alpha^2 + \beta^{-2})^3} \beta u^2] = 0$$

Thus,  $\beta^{-T} = \frac{\beta^2 \alpha^2 \beta u^2}{\beta^2 \alpha^2 \beta u^2 + \frac{1 - \beta \pm \beta}{1 \pm \beta} \mu^2 (\alpha^2 + \beta) \beta u^2} < 1$  for all  $T \geq 1$ : Considering the constraint condition that  $\beta^{-T} \geq 1$ ; the optimal parameter  $\beta$  is always 1 for all  $T \geq 1$ : This indicates that a more conservative central bank is not an optimal choice under the constant inflation contract regime. Furthermore, taking this fact and condition (10) into account, the optimal term length  $T$  is always 1:

The constant inflation target is however not plausible, because the median voter (the government) would assign the inflation contract according to the realization of the shock  $u$ : In other words, since the inflation contract is assigned after the election, it could be contingent on the state of the shock  $u$ ; namely,  $c = a - \beta u$ : The rational expectation of the inflation rate should be based on this information. As a result, the constant part  $a$  is the same as that under the constant inflation contract regime, that is,  $a = \alpha k$ : Taking the expectation on both sides of the first order conditions (14), based on the information after the election and considering the fact that  $a = \alpha k$ ; we have

$$\alpha^2 [E_t(\pi_t^{CBIC}) - E_{t-1}(\pi_t^{CBIC})] + \beta^{-1} [E_t(\pi_t^{CBIC}) - E_t(\pi_{cb,t})] + \beta u_t = 0$$



b is determined according to  $E_t(\pi_t^{CBIC}) = E_t(\pi_{cb,t}^a)$ : Therefore  $E_t(\pi_t^{CBIC}) = E_{t-1}(\pi_t^{CBIC}) + \mu_t$ : Thus the optimal value of parameter b is  $\frac{\mu}{2 + \theta}$ : The corresponding inflation and output are then

$$\pi_t^{CBIC} = E_{t-1}(\pi_{cb,t}^a) + \mu_t \frac{\theta}{2 + \theta} u_t; \quad (17)$$

and

$$y_t^{CBIC} = y^N + \mu_t + \frac{\theta}{2 + \theta} u_t; \quad (18)$$

respectively. By comparing (17) and (18) with (7) and (8), respectively, we notice that the average inflation bias  $\frac{k}{\theta}$  has been completely eliminated under both the constant and the state-contingent inflation target regimes, but the political uncertainties in both the inflation rate and the output have been increased under the state-contingent inflation contract regime. The economic intuition is fairly simple, that is the median voter (the government) tries to increase its partisan influence on monetary policy. This influence works via the central bank's desired inflation rate. The state-contingent inflation contract enables the central bank to carry out the monetary policy so as to approach its desired level even more closely.

The present valued average loss for a multi-term central banker evaluated at time  $t-1$  becomes

$$L^{TIC} = \frac{1}{2(1 \pm \theta)} [k^2 + \frac{\theta(\mu^2 + \theta^{-2})}{(\mu^2 + \theta^{-2})^2} \frac{1}{4} u^2 + \theta \frac{1}{4} \mu^2 + \mu^2 \frac{1}{4} \theta^2] + \frac{\theta \pm (1 \pm \rho)}{2(1 \pm \theta)^2} \frac{1}{4} \sigma_\pi^2 + \frac{\theta}{2(1 \pm \theta)} [\frac{1}{2} \mu^2 \frac{1}{4} \theta^2 + \frac{1 \pm \rho}{1 \pm \theta} \frac{1}{4} \sigma_\pi^2]; \quad (19)$$

The first order condition for minimizing the loss function (19) with respect to the parameter  $\theta$  is

$$\frac{\partial L^{TIC}}{\partial \theta} = \frac{1}{1 \pm \theta} \frac{\theta^2 (1 \pm \rho)}{(\mu^2 + \theta^{-2})^3} \frac{1}{4} u^2 = 0;$$

Thus the optimal conservativeness is 1: The differential of  $L^{TIC}$  with respect to the term-length T is

$$\frac{\partial L^{TIC}}{\partial T} = \frac{\theta \pm (1 \pm \rho) \frac{1}{4} \sigma_\pi^2}{2(1 \pm \theta)(1 \pm \theta^T)^2} [\pm^T + T \ln \pm + 1 + \frac{1}{\theta} A];$$

Since A is negative, it is always quite possible to have a negative  $\frac{\partial L^{TIC}}{\partial T}$  when  $T = 1$ : So a further increase in the term length would lead to a decrease in the expected loss. In other words, an optimal multi-term central banker is possible. The economic intuition for a multi-term central banker under the state-contingent inflation contract is straightforward. The contingent inflation contract eliminates the inflation bias but at the cost of higher

political risk. This can be interpreted as adding a new cost to the social loss function, if we take the constant inflation contract regime as a benchmark of which the optimal term length is one period. This cost can be partly removed by a multi-term central banker.

### 3.2 Inflation Target

We now suppose that the society has adopted an inflation target regime. Thus, the parameter of conservativeness  $\bar{\omega}$  and the term length  $T$  are delegated together with an inflation target  $\mu_t^{IF}$ ; according to the median voter's preference. We also suppose that the implicit output target is  $y^N + k$  for all sectors. The new objective function of the central bank under the inflation target regime is therefore

$$L_t^{CBIF} = \frac{1}{2}[(y_t^{CB} - y^N - k)^2 + \bar{\omega}^{-1}(\mu_t^{CB} - \mu_t^{IF})^2]; \quad (20)$$

The first order condition for minimizing the new loss function (20) is

$$\lambda_t \left[ \mu_t^{CBIF} - E_{t-1}(\mu_t^{CBIF}) \right] + u_t (y_t^{CBIF} - k) + \bar{\omega}^{-1}(\mu_t^{CBIF} - \mu_t^{IF}) = 0; \quad (21)$$

We first consider the constant inflation target  $\mu_t^{IF} = h$ : By taking the expectation on both sides of (21) and assuming that the inflation would fulfill the condition  $E_{t-1}(\mu_t^{CBIF}) = E_{t-1}(\mu_{cb,t}^a)$ , the inflation target should be valued as

$$h (= \mu_t^{IF}) = E_{t-1}(\mu_{cb,t}^a) - \frac{\lambda_t k}{\bar{\omega}^{-1}}; \quad (22)$$

Accordingly, the inflation and output become

$$\mu_t^{CBIF} = E_{t-1}(\mu_{cb,t}^a) - \frac{\lambda_t}{\lambda_t^2 + \bar{\omega}^{-1}} u_t; \quad (23)$$

and

$$y_t^{CBIF} = y^N + \frac{\bar{\omega}^{-1}}{\lambda_t^2 + \bar{\omega}^{-1}} u_t; \quad (24)$$

respectively. Therefore, both the inflation bias and the political uncertainty have been completely eliminated from the economy. This leads to a better situation than that under the inflation contract regime.

The optimal conservativeness and the term length can be determined by minimizing the average expected loss function with a  $T$ -term central banker:

$$L^{TIF} = \frac{1}{2(1 - \beta)} \left[ k^2 + \frac{\bar{\omega}^{-1}(\lambda^2 + \bar{\omega}^{-2})\mu_u^2}{(\lambda^2 + \bar{\omega}^{-1})^2} + \bar{\omega}^{-1}\mu_c^2 \right] + \frac{\bar{\omega}^{-1}(1 - \beta)\mu_c^2}{2(1 - \beta)^2} - \frac{\bar{\omega}^{-1}T(1 - \beta)\mu_c^2}{2(1 - \beta)(1 - \beta)};$$

As a result, we have,

$$\bar{\pi} = 1 \text{ and } T^{\pi} = 1:$$

$T^{\pi} = 1$  can be identified as the following:

$$\frac{\partial L^{TIF}}{\partial T} = \frac{\pi^T (1 - \rho) \frac{1}{2}}{2(1 - \pi)(1 - \pi^T)^2} (\pi^T - T \ln \pi - 1) \leq 0; \text{ for all } T \geq 1:$$

Like the constant inflation contract regime, the constant inflation target could be replaced by a state-contingent inflation target, that is,  $\pi_t^I = h + j \pi_t$ ; where  $h$  and  $j$  are constants. This is because the constant inflation target is less plausible even though the society benefits from it. The state-contingent inflation target is imposed by the median voter (the government), which would at least preserve its influence on the monetary policy decision. Furthermore, the central bank is also willing to allow an inflation target to be contingent on the state of the shock  $\pi_t$ ; since its own desired inflation rate (4) is state-contingent.

Rewriting the first order condition, we have

$$\beta \pi_t^I [y_t^{CBIF} - E_{t-1}(y_t^{CBIF})] + u_t - \beta \pi_t^I (h + j \pi_t) = 0: \quad (25)$$

$h$  and  $j$  can be optimally determined according to (25) and the median voter's preference,  $E_{t-1}(y_t^{CBIF}) = E_{t-1}(y_{cb,t}^{\pi})$ ; and  $E_{\pi}(y_t^{CBIF}) = E_{\pi}(y_{cb,t}^{\pi})$ . As a result, the constant part  $h$  is the same as that in (22), and

$$j = \left( \frac{\beta}{\pi} + 1 \right) \mu: \quad (26)$$

By substituting (22) and (26) into the first order condition (25), we obtain the inflation rate and the output under the state-contingent inflation target regime. We have noticed that both these are the same as under the state-contingent inflation contract regime, that is,  $\pi_t^{CBIF} = \pi_t^{CBIC}$  and  $y_t^{CBIF} = y_t^{CBIC}$ : Therefore the equilibria under two state-contingent regimes are equivalent. Thus, the optimal parameter of conservativeness under the state-contingent inflation target regime is always 1 and the optimal term length could be more than one period.

Our main conclusions are summarized in Table 1. Even though the parameter  $\beta$  can be determined independently of the term length  $T$ ; it is possible to have an optimal multi-term central banker under both state-contingent regimes.

Our discussion in this note focuses on the optimal term length. Even if the permanent shifts in long-run median voter's preferences and the optimal term length are disregarded,

our results are still helpful in understanding the inflation target regime. The constant inflation target can improve the economy to the best possible situation, but it is less plausible. In other words, it reallocates the credibility problem rather than solving it. On the other hand, it is plausible to have a state-contingent inflation target that allows the inflation surprise policy to exert its effect to the maximum extent. Thus, there could be a trade-off between the inflation bias and the politically induced variability. This is consistent with the finding in Lin (1997).

## References

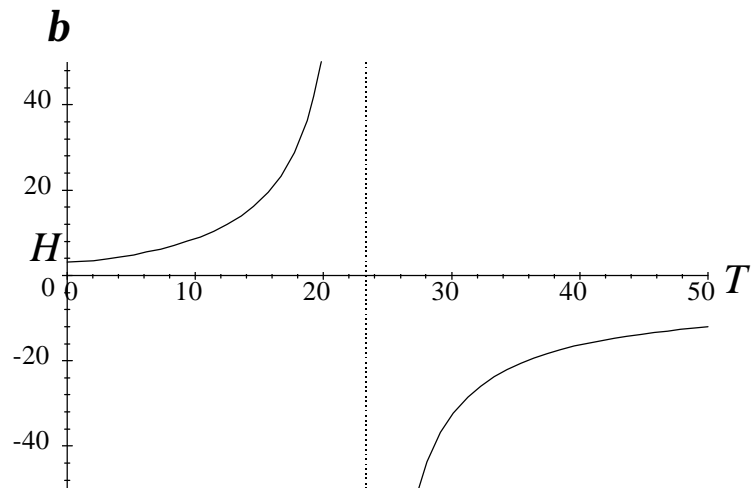
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Table 1

	Optimal Conservativeness	Optimal Term Length
Constant Inflation Contract	$\bar{\pi} = 1^a$	$T = 1$
state-contingent Inflation Contract	$\bar{\pi} = 1$	$T > 1$
Constant Inflation Target	$\bar{\pi} = 1^a$	$T = 1$
state-contingent Inflation Target	$\bar{\pi} = 1$	$T > 1$

<sup>a</sup> obtained by the constraint condition  $\bar{\pi} > 1$ :

Figure 1

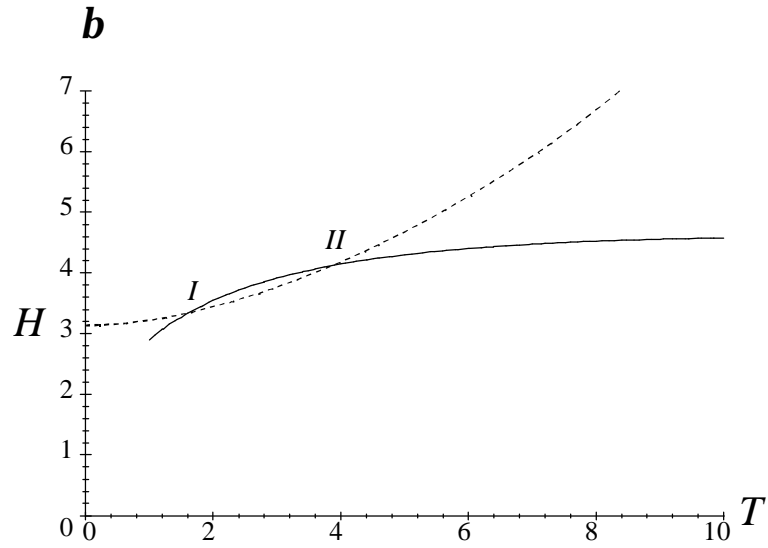


The dot line represents the asymptotic line.

Values of parameters:

$$\textcircled{R} = 2.5; \textcircled{s} = 3; \textcircled{\pm} = 0.7; \mu = 0.9; \textcircled{p} = .9; k^2 = 2.5; \textcircled{\frac{3}{4}}_u^2 = .5; \textcircled{\frac{3}{4}}_n^2 = 30; \textcircled{\frac{3}{4}}_c^2 = .9;$$

Figure 2



Values of parameters are the same as those in Figure 1.