Service Outsourcing and Specialization: A Theory on Endogeneous Task Scope*

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VERY PRELIMINARY

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Abstract

We develop a model of outsourcing and trade in service inputs where the scope of tasks produced by both manufacturing firms and service providers is endogeneous. Manufacturing firms have to perform a fixed set of tasks in order to produce their final good but can decide to outsource some of these tasks to service providers, which, contrary to manufacturers, have the possibility to sell tasks to different manufacturers and thereby benefit from economies of scale in their task production. The key assumption is that the marginal cost of a firm (manufacturer or service provider) increases in the scope of tasks performed inside the firm: a firm which specializes in a narrow scope of tasks is more productive. Working against this incentive to produce as few tasks as possible "inhouse" is a fixed cost paid by each firm. The model yields several new predictions about trade liberalization and welfare as measured by aggregate productivity. An increase in the size of an economy raises the scale of all firms, facilitates greater specialization and therefore raises each firm's productivity. The model therefore generates gains from trade or larger market size through a "specialization effect" as opposed to the classical "variety effect" usually generated by models building on Dixit Stiglitz utility structures. Welfare increases due to adjustments in task scope allowed by the emergence of specialized service firms. Detailed Swedish data on what tasks (or occupations) are performed by workers is used to test this prediction. Indeed, we find that manufacturing firms in larger cities (controlling for firm size) perform fewer tasks inhouse than firms in smaller cities.

Keywords: service outsourcing, division of labour, productivity, specialization.
JEL codes: F10, F43, L24.

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1 Introduction

> "The greatest improvement in the productive powers of labour, and the greater part of the skill, dexterity, and judgment with which it is any where directed, or applied, seem to have been the effects of the division of labour."
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> "To take an example, therefore, from a very trifling manufacture; but one in which the division of labour has been very often taken notice of, the trade of the pin-maker. [.....] Each person, therefore, making a tenth part of forty-eight thousand pins, might be considered as making four thousand eight hundred pins in a day. But if they had all wrought separately and independently, [.....] they certainly could not each of them have made twenty, perhaps not one pin in a day."

(Adam Smith, "An Inquiry into the Nature and Causes of the Wealth of Nations", Book I, Chapter I.)

The phenomenon of outsourcing has generated a great deal of attention in recent years. Recent declines in information and communication costs have made it increasingly possible for firms to source their material and service inputs from suppliers outside the firm, be they located in the domestic economy or abroad. However, the nature of outsourcing appears to be changing in some important aspects. First, while the globalization of production networks has long involved the outsourcing of manufactured inputs, it increasingly concerns outsourcing of services, though still at quantitatively lower levels, see Amiti and Wei (2005). Second, this shift towards service outsourcing goes hand in hand with the emergence of firms which specialize in providing one or a few particular services. A typical example of domestic service outsourcing is a manufacturing company which outsources part of its need for human resources tasks to a company specializing in recruiting staff, the canteen to a catering firm and the cleaning of factories and offices to a cleaning company. Examples of foreign outsourcing could involve the after sales services to call centres abroad or the development of software to IT engineers abroad. One important implication of this new trend is that, contrary to material inputs outsourcing which mainly involves trade in intermediate goods, service outsourcing mainly involves trade in tasks. And this outsourcing of tasks has the potential to raise aggregate productivity in countries by allowing firms to focus on the tasks involving their core competencies. Moreover, it appears as if larger economic areas sustain more specialized service providers. There is plenty of anecdotal evidence of the existence of extremely specialized service providers in very large cities, something which is discussed in detail by Chinitz (1961).

We develop in this paper an analytically solvable outsourcing model which we believe focuses more specifically on the outsourcing of services rather than manufacturing inputs.[1] In our model, the scope of tasks produced by both manufacturing firms and service providers is endogenous. We assume that the task inputs required in the production of a manufacturing firm's good are arranged along a line.[2] However, manufacturers have the possibility to outsource some tasks in order to reduce the scope of tasks they produce.[3] Indeed, the narrower the range of tasks performed "inhouse" is, the more efficient the firm will be in the performance of these tasks or, equivalently, the lower marginal cost the firm will have. One reason could be that management and supervision becomes more efficient the less different types of tasks it has to monitor. Downward pressure on the optimal task scope of manufacturing firms is therefore due to this benefit of specialization but is costly since service providers will share the surplus generated by production. Service providers, similarly, produce a range of tasks and are also more efficient the more narrow is their task scope. However, contrary to manufacturers, service providers do not have to produce the full range of tasks and can therefore specialize in a more narrow range of tasks and sell these tasks to manufacturers. They therefore benefit from economies of scale from specialization. The gains from specialization for service providers is bounded from below by the presence of fixed costs. The model yields several new predictions. First, larger markets consist of more specialized firms (both manufacturers and service providers). This is due to economies of scale in the service industry; more service providers can survive given the higher demand and it becomes more profitable for manufacturers to outsource more tasks. Second, aggregate productivity rises as producers become more efficient. The model therefore generates gains from trade or larger market size through a "specialization effect" as opposed to the classical "variety effect" usually generated by

[1] In our view, one key difference between outsourcing of services versus intermediate inputs is that the outsourcing of services involves the explicit outsourcing of certain tasks. The service acquired from an external supplier involves more or less only that service and not much more. Acquiring intermediate inputs, however, involves the implicit outsourcing of a wider range of tasks or services which are embodied in the production of the intermediate good.

[2] This is similar to Grossman and Rossi-Hansberg (2008b).

[3] An important literature has already explored the determinants of the *make* or *buy* decision and highlights the importance of incomplete contracts theory, see in particular Grossman and Helpman (2002). While these papers focus on the extensive margin of outsourcing, we are interested in the intensive margin of offshoring and the causes and effects of the firm's decision about what tasks to produce inhouse and which it should outsource.

models building on Dixit Stiglitz utility structures such as in Krugman (1980). Welfare increases due to adjustments in task scope allowed by the emergence of specialized service firms. Finally, trade liberalization (as proxied by larger population size), increases the level of specialization and aggregate productivity through the same mechanisms. We focus most closely on the autarky equilibrium in the model and use country size as an indirect proxy for trade integration but also analyse an open economy version of the model and specifically the two cases of (i) trade in final goods and (ii) trade in tasks.

We subsequently use detailed Swedish data which links employees to plants. We know the occupation code of each individual in the data set and can therefore use the occupation codes as a proxy for how many tasks are performed at each plant. We find that among manufacturers, plants in smaller cities tend to perform more tasks inhouse than plants in larger cities when we control for plant size. It therefore seems as if larger cities are characterized by more specialization as predicted by the model and which is in line with anecdotal evidence of the presence of more specialized firms in larger cities.

Moreover, the model is very close to one of the core arguments for trade and economic integration in the "Wealth of Nations" by Adam Smith. Smith argues that the production becomes more efficient when labour is divided such that workers focus on specific tasks instead of each worker doing the same thing.[4] One example, described above, he mentions is that of a pin factory with ten workers.[5] One worker in the factory can probably make about ten pins per day. However, if the workers divide the eighteen steps involved among them, the total output by all workers of pins in one day could reach as high as 48,000 pins because they become more efficient when focusing on a more narrow range of tasks. This is exactly the driving force in our model and, in fact, the only force (since we have a fixed set of varieties) that drive gains from trade and larger market size. When the market grows, manufacturing firms decrease the scope of tasks produced inhouse but instead expand output per good which spreads the fixed cost over more output units. Specialists act in the same way and also become more specialized. Ultimately, aggregate productivity increases as all firms become more specialized (by producing fewer tasks inhouse) and therefore also more efficient.

[4] This is, obviously, an issue that has received much focus by economists, see, for example, also Stigler (1951). However, to our knowledge, the issue has not been approached from this perspective previously and especially not from an open economy perspective.

[5] Smith (1776), bk. V, ch. 1.

We therefore also see our model as complementary to the recent literature of productivity gains through the reallocation of production within industries, as in Melitz (2003). While the firm heterogeneity literature focuses on reallocation of production across firms (which have constant productivity) and productivity gains which are *external* to the firm, our model generates productivity gains that are *internal* to the firm.

Some important mechanisms of the model are in line with existing empirical evidence regarding service offshoring. First, there is by now some empirical evidence that most developed economies are not only big insourcers of service offshoring, they are also net exporters of services, see Amiti and Wei (2005). Therefore, service outsourcing cannot be explained only by the possibility to source some tasks from low wage countries. In our model, service outsourcing emerges from firms' need to focus on a narrow range of tasks; the marginal cost of producing each task decreases for manufacturers and service providers. This model is thus consistent with the existence of service outsourcing between similar countries. Second, empirical finding suggest that service offshoring does not appear to affect employment at home in a significantly negative way, see Amiti and Wei (2005) and Ekholm and Hakkala (2006). One common argument is that while an offshored job can be a job loss, efficiency gains stemming from service offshoring allow the firm to expand which in turns has a positive effect on productivity, see Amiti and Wei (2009).

In terms of theoretical work on the issue of task outsourcing and offshoring, several important contributions have already been made. Grossman and Rossi-Hansberg (2008b) develop in a Heckscher Ohlin setting a model of north-south trade in tasks where tasks differ in how easy they are to offshore. Most importantly, they identify a productivity effect at the industry level which raises the return to the factors that are more easily moved offshore. This model is similar to ours in the sense that manufacturing production requires task inputs and that these tasks consist of a fixed set arranged along a line. Our model, however, does not differentiate between tasks and focuses instead on the case when firms become less efficient the more different in nature the tasks that are produced inhouse are. Grossman and Rossi-Hansberg (2008a) also develop a model of tasks offshoring between similar countries but focus on economies of scale which are external to the firm. In our model, we propose a rather different story. The outsourcing of service tasks by manufacturing firms to service providers raises aggregate productivity by adjustments in tasks scope which lead to productivity gains which are internal to the firm.

We believe, however, that the two theoretical papers closest to ours in terms of modelling and focus are those by Grossman and Helpman (2002) and Antràs and Helpman (2004). We view both of these papers as complementary to our approach and we have tried to abstract from the issues analysed in these papers and thereby focus explicitly on the channel which is unique for our model: the fact that the set of tasks needed in production is fixed and that the task scope of each firm is endogenously determined. More specifically, the former develop a model with outsourcing where manufacturing firms decide whether to outsource all production or integrate vertically. In this model, manufacturers search for suppliers with which to match and this gives rise to interesting interactions between the relative mass of manufacturers versus suppliers. Our model, in contrast, focuses more directly on the issue of specialization since the focus is on the *intensive* scope margin rather than the *extensive* margin as is the point of analysis in Grossman and Helpman (2002). In our setting, a narrower task scope is the equivalent of greater specialization. Antràs and Helpman (2004) develop, instead, a model in which manufacturers buy intermediate inputs and where the set of intermediates purchased is endogenously determined. However, we believe that since the set of inputs is not fixed, their model is more applicable for manufacturing intermediate inputs rather than trade in tasks.

Finally, we believe that our model can explain two phenomena and be viewed upon from two angles: (i) a model of trade in service inputs between service suppliers and final good firms (where trade liberalization is either proxied by population size or by changes in intermediate trade costs); or (ii) a model explaining the economic geography of the supply of services, such as different degrees of specialization, aggregate productivity and the extent of outsourcing in different sectors or locations.

Section 2 describes the model we have developed. We then proceed to analyse our setting in autarky which generates our most important results. Section 3 uses Swedish data linking employee with employers to test the main implications of our model. Section 4 concludes. In the Appendix, we also analyse a more complex open economy setting where we allow for trade in (i) goods or (ii) tasks.

2 Model

2.1 Setup in autarky

The model depicts an economy with a primary production factor labour, L, which is used in all sectors. Production includes three sectors. The agricultural sector is a Walrasian, homogenous-goods sector with costless trade. The manufacturing sector is characterized by increasing returns, Dixit-Stiglitz monopolistic competition and iceberg trade costs. Finally, the service sector does not produce any consumer good on its own, but instead produces tasks that it sells to manufacturing firms. The last category consists of the firms to which manufacturing firms can outsource some of the tasks needed in manufacturing production.

Consumers have two-tier utility functions with the upper tier (Cobb-Douglas) determining the consumer's division of expenditure among the sectors and the second tier (CES) dictating the consumer's preferences over the various differentiated varieties within the manufacturing sector.

More specifically, individuals have the following utility function

$$U = C_M^{\mu} C_A^{1-\mu}, \tag{1}$$

where $\mu \in (0, 1)$, and C_A is the consumption of the homogenous good. Manufactures enter the utility function through the index C_M, defined by

$$C_M = \left[\int_0^N c_i^{\frac{(\sigma-1)}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

N being the mass of varieties consumed, c_i the amount of variety i consumed and $\sigma > 1$ the elasticity of substitution between manufacturing goods.

Each consumer spends a share μ of his income on manufactures, and demand for a variety i is therefore

$$q_i = \frac{p_i^{-\sigma}}{P_j^{1-\sigma}} \mu Y, \tag{3}$$

where p_i is the consumer price of variety i, Y is income and $P \equiv \left(\int_0^N p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$

the price index of manufacturing goods. We will rewrite this as

$$q_i = A p_i^{-\sigma}$$

where $A \equiv \frac{\mu Y}{P_j^{1-\sigma}}$ and is taken as given by each manufacturing firm.

The unit factor requirement of the homogeneous good is one unit of labour. This good is freely traded and since it is chosen as the numeraire

$$p_A = w = 1; \tag{4}$$

w being the nominal wage of workers. This also means that

$$Y = L.$$

Turning to production, manufacturing firms require a range of tasks, indicated by i and ranging from 0 to 1, to be performed in its production. This range of tasks are arranged along a line and our intuition here is that tasks located close to each other are more similar in nature. Along the same argument, the further away two tasks are, the more different they are. To clarify this point, we take again the example of the pin factory described by Adam Smith. Consider the following three tasks: (i) hammering the pins so that they are completely straight, (ii) hammering the end of the pin so that it has a flat end, (iii) cleaning the factory floor and, (iv) disposing of waste created in the manufacturing process. This may be very stylized but tasks (i) and (ii) are most likely much more similar in nature than say (i) and (iii). So in our case, (i) and (ii) would be close to each other on the line and some distance away would be (iii) and (iv) close to each other. When we refer to "similarity of tasks", we therefore essentially mean how close tasks are to each other in the task scope.

Manufacturing firms face a marginal production cost per task j, $\widetilde{\varphi}_M(j)$, which will depend on the task scope, and a fixed cost, f_M. The cost of performing $q(j)$ units of a task is

$$C(j) = wq(j)\,\widetilde{\varphi}_M(j). \tag{5}$$

The manufacturing firm's total cost is therefore:

$$TC_i = f_M + \overbrace{w \int_0^1 \gamma(j)\, q(j)\, \widetilde{\varphi}_M(j)\, dj}^{\text{Goods produced inhouse}} + \overbrace{\int_0^1 (1-\gamma(j))\, q(j)\, p_S(j)\, dj}^{\text{Outsourced goods}} \tag{6}$$

where $\gamma(j) \in \{0, 1\}$. It is 1 if the firm performs the task and 0 if it is outsourced. If firms do not perform the good inhouse it can procure the service from a specializing firm at the price $p_S(j)$. In other words, a manufacturing firm can either perform a task and pay $w\widetilde{\varphi}_M(j)$ per unit or procure it from a specializing firm at the price $p_S(j)$ per unit.[6] Finally, one key assumption is that *it is more costly to produce multiple tasks if they lie far from each other in the task scope.* The marginal cost, $\widetilde{\varphi}_M(j)$, of task j increases in the average distance of task j from all other tasks performed within the firm

$$\widetilde{\varphi}_M(j) = \frac{\int_0^1 (j' - j)^\delta \gamma(j')\, dj'}{\int_0^1 \gamma(j')\, dj'}. \tag{7}$$

where $\delta > 1$ to attain a convex relationship between task scope and marginal cost. Moreover, δ is restricted to the set of *even* numbers. This key assumption is founded upon our belief that it is more costly for firms to perform tasks of very different characteristics within the same firm than to specialize in a more narrow range of tasks. This is essentially how we capture Smith's description of the pin factory. A firm that specialize in just a few tasks that are close to each other in nature will be more productive than a firm that does all the tasks within its boundaries. A more modern justification for this could also be an assumption of management supervision as a scarce resource. A manager will find it easier to focus his or her time on a narrow set of tasks rather than supervising a very broad range of tasks which differ greatly in nature.

The parameter δ is therefore very important in our model. It is a measure of how much more expensive it is to produce multiple tasks when they lie far from each other; or, equivalently, how expensive it is for a firm to operate a wide task scope.

We also assume that all tasks have to be performed in the same level to produce output q_i and more specifically that the production function is

$$q_i = \min_{j \in [0,1]} (q(j)). \tag{8}$$

Due to the Leontief nature of the production function, cost minimization means that the demand for each task, given the output level will be

$$q(j') = q(j) = q_M \tag{9}$$

[6] In our model, outsourcing will not be modelled as paying a price p but rather as sharing some of the revenues, but this will be clarified shortly.

where $j \neq j'$. What this means is that the level of each task input will equal the output of the final good in equilibrium.

Now, we introduce service providers which are firms that produce no final goods but instead a set of tasks which they then sell to the final good producers. Specializing firms (service providers) need to pay two fixed costs. First, it pays $f_S < f_M$. This is lower than the fixed cost for manufacturers since specializing firms do not have to cover distribution and retails costs. However, as regards its other costs it faces the same setup as manufacturers with a marginal cost per task, $\widetilde{\varphi}_S(j)$, that increases in its average distance from all other tasks performed within the specializing firm. The total cost of the service provider l is therefore:

$$TC_l = f_S + w \int_0^1 \gamma(j)\, q(j)\, \widetilde{\varphi}_l(j)\, dj \tag{10}$$

where

$$\widetilde{\varphi}_l(j) = \frac{\int_0^1 (j' - j)^2 \gamma(j')\, dj'}{\int_0^1 \gamma(j')\, dj'}. \tag{11}$$

Especially important to note is that service providers (unlike manufacturing firms) do not have to produce all tasks along the line from 0 to 1. Instead, they choose a set of tasks, produce these tasks and then sell these tasks to manufacturers. We believe that this feature is a key difference between the supply of services versus intermediate inputs. An accounting firm, for example, focuses on a very narrow range of tasks in its production while a, for example, car tyre producer still needs a wide range of tasks in order to produce. The former would then be a typical service provider and the latter a typical provider of an intermediate good.

To sum up, manufacturing is characterized by two types of firms: manufacturers and service providers. Manufacturers produce final goods and produce tasks inhouse or procure them from service providers. Service providers, on the other hand, only produce tasks and sell these to manufacturing firms.

All three sectors have free entry and firms therefore make zero profit in equilibrium.

Finally, we have to make assumptions about the competition structure between, first, manufacturers and service providers and, second, between service providers. Grossman and Helpman (2002) analyse in detail the case where the matching of these types of firms is characterized by search frictions and we therefore wish to abstract from such frictions in order not to duplicate their results and instead focus on our specific mechanism. In our model, the sequence of events will be as follows: (i) manufacturing firms enter and decide on their task scope. This process continues until their expected profits are driven to zero. Since it will not be profitable for manufacturers to produce different tasks in our model (manufacturers are homogenous), we simply assume that they always produce task 0 inhouse and then all tasks from 0 and onwards until some optimal level of $t_M < 1$. (ii) Service providers enter one by one and position themselves along the task scope (over the range of tasks that are outsourced) until their expected profits are zero. This means that they divide the scope of tasks not produced by manufacturers, $j \in (t_M, 1]$, between themselves. Since service providers are identical it means that they will each produce a set of tasks t_S such that $(1 - t_M) = n_S t_S$ where n_S denotes the number of service providers. This setup is illustrated in figure 1. (iii) Manufacturers make contracts with task providers. This means, for example, that task providers install their operations in the manufacturing firm's plants. The key assumption here is that once the manufacturer and task provider have set up their joint operations, the manufacturer cannot renege on the contract. Specifically, we assume that a task provider can produce tasks of different quality and while the quantity can be verifiable by a court of law, the quality of the tasks cannot.[7] Due to the Leontief production function of the manufacturer, this gives the task provider full bargaining power over the manufacturer since it can stop the manufacturer's production completely. (iv) Manufacturers and service providers bargain over the revenues generated by production. As mentioned, the service provider has full bargaining power and can therefore, as in Grossman and Helpman (2002), determine the level of output of the manufacturer (by deciding which quality of tasks to deliver, which indirectly determines the quantity). It subsequently receives all the revenues generated by the specific tasks it delivers.[8] (v) Production and consumption of manufacturing goods, agricultural goods and service inputs take place.

Another important point to clarify is that while manufacturers optimize their task scope, t_M, to maximize profits, service suppliers' task scope is determined by free entry in their sector. We believe that this is reasonable given the fact that if service providers can optimize their task scope, they would make positive

[7] This assumption is similar to that in Grossman and Helpman (2002).

[8] Grossman and Helpman (2002) assumes some exogenous bargaining power parameter of the suppliers, ω, but we assume here that service providers have full bargaining power over the revenues generated by their specific tasks. Our assumption is based on the fact that service providers have the power to completely stop production in the manufacturer's plant.

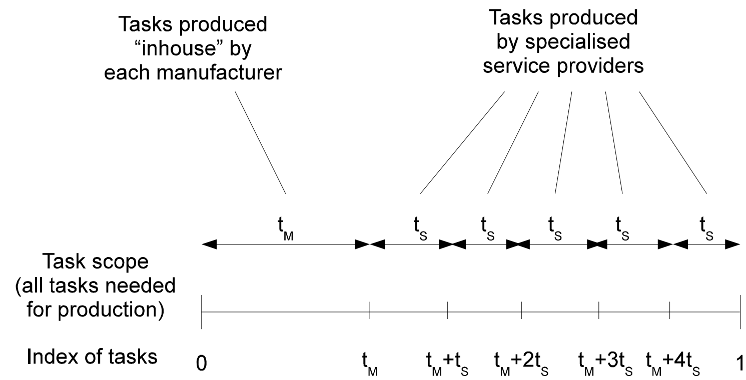

Figure 1: The structure of task production.

profits in equilibrium and this cannot be the case if there is free entry for service providers.

Figure 1 describes in detail how different tasks in the economy are produced. First, the tasks from 0 to t_M are produced by *each* manufacturer inhouse. The range of tasks from t_M to 1 are then produced by service providers to which manufacturers outsource. Since service providers are homogeneous, they divide the range $1-t_M$ among themselves and each service providers therefore produces $t_S = \frac{1-t_M}{n_S}$ tasks.

To sum up, the conditions used to close the model are:

1. Consumers maximize utility through consumption given their income.

2. Manufacturers maximize their profits by determining their task scope (they take output as given since this is determined by the service providers as explained).

3. Service providers maximize their profits by determining what quantity of each task to produce (thereby also deciding on the output of the manufacturing firm).

4. Zero profits for manufacturers due to free entry.

5. Zero profits for task providers due to free entry.

Together, these equations will determine the output level of each final good and each task, q, the price of each final good, p, the task scope of manufacturers, t_M, the task scope of service providers, t_S, the number of manufacturers, n_M, the number of service providers, n_S, aggregate prices for manufacturing goods,

P, and aggregate utility, UL.

Moreover, we restrict our analysis to equilibria with outsourcing taking place to some extent. The analysis of the binary cases of complete outsourcing versus full vertical integration is already thoroughly analysed by Grossman and Helpman (2002).

The following section will analyse the equilibrium of the model.

2.2 The autarky equilibrium

First, the total profit of a manufacturing firm is:

$$\Pi_i = R_i - C_i \tag{12}$$
$$= p_i q_i - f_M - \int_0^1 \gamma_i(j)\, q_i(j)\, \widetilde{\varphi}_i(j)\, dj - \int_0^1 (1-\gamma_i(j))\, q_i(j)\, p_S(j)\, dj. \tag{13}$$

The way the marginal cost increases in the difference between tasks means that all firms will produce tasks that are adjacent to each other. If the tasks are ordered on a line between 0 and 1, we can, for simplicity, assume that the tasks manufacturing firms produce inhouse are in the range between 0 and t_M, where t_M is determined endogenously. Moreover, due to the homogeneity of firms, we now drop the index i. With this setup, the marginal cost of a specific task j will look like

$$\widetilde{\varphi}(j) = \frac{\int_0^1 (j'-j)^{\delta}\, \gamma(j)\, dj'}{\int_0^1 \gamma(j)\, dj} \tag{14}$$
$$= \frac{\int_0^{t_M} (j'-j)^{\delta}\, dj'}{\int_0^{t_M} dj} \tag{15}$$
$$= \frac{1}{t_M} \int_0^{t_M} (j'-j)^{\delta}\, dj' \tag{16}$$
$$= \frac{1}{t_M(\delta+1)} \left((t_M - j)^{\gamma+1} - (-j)^{\delta+1} \right). \tag{17}$$

This means that the average marginal cost for a manufacturing firm, $\varphi_M(t_M)$

will be a function of t_M:

$$\varphi_M(t_M) = \frac{1}{t_M}\int_0^{t_M} \widetilde{\varphi}_M(j)\,dj \tag{18}$$

$$= \frac{1}{t_M^2(\delta+1)}\int_0^{t_M}\left((t_M-j)^{\delta+1} - (-j)^{\delta+1}\right)dj \tag{19}$$

$$= \lambda_1 t_M^{\delta} \tag{20}$$

where $\lambda_1 \equiv \frac{2}{(\delta+1)(\delta+2)}$. We note that *the marginal cost of tasks produced inhouse strictly increases in the scope of tasks that are produced inhouse.*

Service providers face a similar structure such that

$$\varphi_S(t_S) = \lambda_1 t_S^{\delta}. \tag{21}$$

Now, the demand of a manufacturing good (under a CES demand structure) can be denoted as:

$$q_M = A p_M^{-\sigma} \tag{22}$$

where $A \equiv \frac{\mu Y}{P^{1-\sigma}} = \frac{\mu L}{P^{1-\sigma}}$ and P is the Dixit Stiglitz ideal price index.

The manufacturer produces a share t_M of all tasks needed for production inhouse. Due to the competition structure described above, it is the case that manufacturers pay *all* the revenues generated by outsourced tasks to the service providers from which they buy the task. They therefore pay

$$\frac{(1-t_M)}{n_S} p_M q_M \tag{23}$$

to *each* supplier where $n_S = \frac{1-t_M}{t_S}$ is the mass of service providers in the economy. The total cost of the manufacturer for buying service inputs is therefore:

$$(1-t_M)\,p_M q_M.$$

The profits of a manufacturer can then be rewritten as:

$$\pi_M = p_M q_M - t_M \varphi_M(t_M)\,q_M - (1-t_M)\,p_M q_M - f_M \tag{24}$$

$$= t_M q_M\left(p_M - \varphi_M(t_M)\right) - f_M. \tag{25}$$

A supplier produces t_S tasks taking demand $q = A p_M^{-\sigma}$ and the scope of manufacturers t_M as given. At this point, it also takes its own scope, t_S, as given. Since the supplier will have the same output of a task as the manufacturer will have of its final good, due to the Leontief production function in (8), we denote output of both items simply as q where $q = q_M = q_S$.

Per manufacturer customer, it gets paid $(1 - t_M) \frac{t_S}{1-t_M} p_M q = t_S p_M q$. Its costs are $f_S + t_S \varphi_S (t_S) q$. Therefore its operating profit (denoted by $\overline{\pi}_S$) from dealing with one manufacturer is:

$$\overline{\pi}_S = t_S q (p_M - \varphi_S (t_S)). \tag{26}$$

It faces the problem of how much intermediate inputs to produce (knowing that it can completely control the output of the manufacturer):

$$\max_q \pi_S = t_S p_M q - t_S \varphi_S (t_S) q \tag{27}$$

$$= t_S A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} - t_S \varphi_S (t_S) q \tag{28}$$

which gives the following FOC (note that $\varphi_S (t_S) = \lambda_1 t_s^{\delta}$):

$$q = \left(\frac{1}{\widetilde{\sigma} \lambda_1 t_S^{\delta}} \right)^{\sigma} A \tag{29}$$

where $\widetilde{\sigma} \equiv \frac{\sigma}{\sigma-1}$.

This gives the optimal output for a supplier given its own scope and the manufacturer's scope. Note that our assumption is that the manufacturer (due to the holdup problem) *has* to accept the output volume chosen by the suppliers if it decides to outsource task production. We also note that the service supplier produces more if demand is high (high A) and less if its task scope is large (high t_S) since the price of its good in this case will be higher due to its lower productivity (higher marginal cost). As expected, a high elasticity of substitution makes output more sensitive to marginal costs.

The manufacturer faces the following profit function:

$$\pi_M = p_M q - t_M \varphi_M (t_M) q - (1 - t_M) p_M q - f_M \tag{30}$$

$$= A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} t_M - \lambda_1 t_M^{\delta+1} q - f_M. \tag{31}$$

It takes q as given so it will use t_M to maximize profits,

$$\max_{t_M} \pi_M = A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} t_M - \lambda_1 t_M^{\delta+1} q - f_M, \tag{32}$$

which gives the following FOC:

$$t_M = \left(\frac{1}{\lambda_1 (\delta + 1)} A^{\frac{1}{\sigma}} q_S^{-\frac{1}{\sigma}} \right)^{\frac{1}{\delta}}. \tag{33}$$

The solution for t_M above gives the solution for a manufacturer's optimal scope given the quantity produced by each intermediate supplier. We know the output of a supplier from (29) and t_M can therefore be written as:

$$t_M = t_S \left(\frac{\widetilde{\sigma}}{\delta + 1} \right)^{\frac{1}{\delta}} \tag{34}$$

where $\widetilde{\sigma} \equiv \frac{\sigma}{\sigma.1}$. This means that there is a monotonic and linear relationship between the task scope of a manufacturer and that of the service suppliers. The reason for this is simply that a higher task scope of service providers, t_S, make them less specialized and therefore less efficient. Less efficient service providers are not as attractive for the manufacturer to use for outsourcing and the manufacturer then prefers to perform relatively more production inhouse and raises t_M.

Now, we turn to the free entry condition in the manufacturing sector and the fact that they earn zero profits in equilibrium:

$$\pi_M = 0 \tag{35}$$

$$t_M q \left(p_M - \varphi_M \left(t_M \right) \right) = f_M \tag{36}$$

$$A = t_S^{\delta(\sigma-1)-1} \lambda_1^{\sigma-1} \widetilde{\sigma}^{\sigma-1} \left(\frac{\delta + 1}{\widetilde{\sigma}} \right)^{\frac{1}{\delta}} \left(\frac{\delta + 1}{\delta} \right) f_M \tag{37}$$

which returns a relationship between A, the demand per firm in the economy, and t_S, the task scope of suppliers, and indirectly that of manufacturers too from (34), in the economy.

There is free entry for service providers too and this will drive down their task scope, t_S, such that they earn zero profits in equilibrium:

$$\pi_S = 0 \tag{38}$$

$$n_M t_S q \left(p_M - \varphi_S \right) = f_S \tag{39}$$

$$n_M = \frac{f_S}{f_M} \frac{\widetilde{\sigma}}{\widetilde{\sigma} - 1} \frac{\delta}{\delta + 1} \left(\frac{\widetilde{\sigma}}{\delta + 1} \right)^{\frac{1}{\delta}}. \tag{40}$$

The free entry conditions yield a solution for the number of manufacturing firms, n_M, and this consists only of exogenous parameters. This is an important conclusion, especially that this variable is independent of population size which is otherwise the case with CES preferences, and we will see later that this means that there is no "variety effect" from trade. The reason for n_M being fixed, however, stems from the fact that all surplus profits coming from an expansion in market size are passed on to the service suppliers and do not stay with the manufacturing firms. This result is similar to what is found in Grossman and Helpman (2002).

Knowing this, we can use the expression for A to find the other variables:

$$\frac{\mu L}{P^{1-\sigma}} = t_S^{\delta(\sigma-1)-1} \lambda_1^{\sigma-1} \widetilde{\sigma}^{\sigma-1} \left(\frac{\delta+1}{\widetilde{\sigma}} \right)^{\frac{1}{\delta}} \left(\frac{\delta+1}{\delta} \right) f_M \tag{41}$$

$$t_S = \frac{f_S}{\mu L} \frac{\widetilde{\sigma}}{\widetilde{\sigma}-1} \tag{42}$$

where we note one of our key findings, that larger economies are more specialized. This is due to the fact that as population increases, the demand per each manufacturing variety increases. This leads to an expansion of output of each manufacturing variety since the number of varieties is fixed as was observed in (40). This also means an increase in the output of each task raising the profits of each service provider which leads to an inflow of new service firms in the economy. This entry process drives down the task scope of each service provider. Ultimately, this means that a larger economy consists of more specialized firms.

This intuition can be seen in the following equations using the result from (42):

$$t_M = t_S \left(\frac{\widetilde{\sigma}}{(\delta+1)} \right)^{\frac{1}{\delta}} \tag{43}$$

$$= \frac{f_S}{\mu L} \frac{\widetilde{\sigma}}{\widetilde{\sigma}-1} \left(\frac{\widetilde{\sigma}}{(\delta+1)} \right)^{\frac{1}{\delta}}. \tag{44}$$

$$A = \left(\frac{1}{\mu L}\right)^{\delta(\sigma-1)-1} f_S^{\delta(\sigma-1)} \left(\frac{\widetilde{\sigma}}{\widetilde{\sigma}-1}\right)^{\delta(\sigma-1)-1} \lambda_1^{\sigma-1}\widetilde{\sigma}^{\sigma-1} \left(\frac{\delta+1}{\widetilde{\sigma}}\right)^{\frac{1}{\delta}} \left(\frac{\delta+1}{\delta}\right) \frac{f_M}{f_S}. \tag{45}$$

$$P = \left(\frac{1}{\mu L}\right)^{\delta} f_S^{\delta} \left(\frac{\widetilde{\sigma}}{\widetilde{\sigma}-1}\right)^{\delta-\frac{1}{\sigma-1}} \lambda_1\widetilde{\sigma} \left(\frac{\delta+1}{\widetilde{\sigma}}\right)^{\frac{1}{\delta(\sigma-1)}} \left(\frac{\delta+1}{\delta}\right)^{\frac{1}{\sigma-1}} \left(\frac{f_M}{f_S}\right)^{\frac{1}{\sigma-1}}. \tag{46}$$

The quantity per manufacturer is:

$$q = \mu L^{(\delta+1)} f_S^{-\delta} \left(\frac{\widetilde{\sigma}-1}{\widetilde{\sigma}}\right)^{(\delta+1)} \lambda_1^{-1}\widetilde{\sigma}^{\sigma-1} \left(\frac{\delta+1}{\widetilde{\sigma}}\right)^{\frac{1}{\delta}} \left(\frac{\delta+1}{\delta}\right) \frac{f_M}{f_S}\frac{1}{\widetilde{\sigma}^{\sigma}}. \tag{47}$$

which gives the following price per manufacturing good:

$$p = \lambda_1\widetilde{\sigma} \left(\frac{f_S}{\mu L}\frac{\widetilde{\sigma}}{\widetilde{\sigma}-1}\right)^{\delta}. \tag{48}$$

The number of suppliers can be found by using that:

$$n_S = \frac{1-t_M}{t_S} \tag{49}$$

$$= \frac{L-\lambda_2\lambda_3}{\lambda_3} \tag{50}$$

which increases linearly in L. We let $\lambda_2 \equiv \left(\frac{\widetilde{\sigma}}{(\delta+1)}\right)^{\frac{1}{\delta}}$ and $\lambda_3 \equiv \frac{\delta f_S}{\mu}\frac{\widetilde{\sigma}}{\widetilde{\sigma}-1}$.

We focus especially on the result in (46) which gives the expression for the price index of the manufacturing goods. This is important since welfare can be expressed by:

$$W = P^{-\mu} \tag{51}$$

since the agricultural good is the numeraire and (51) therefore gives the real wage in the economy. The exponent for L in the expression for P is, in this model, no longer $\frac{1}{1-\sigma}$ like in Krugman (1980) or other similar models building on CES preferences, but instead $-\delta$ so it is still negative but this comes through *a specialization effect rather than a variety effect*; the elasticity is δ rather than $\frac{1}{\sigma-1}$. This outcome is a direct consequence of our new mechanism in this model. We therefore have a new margin of how welfare increases in market size. This margin is stronger the more difficult it is for firms to manage many tasks because this makes specialization relatively more important.

$$\text{Dixit and Stiglitz (1977)}: -\frac{dP}{dL}\frac{L}{P} = \overbrace{\frac{1}{\sigma-1}}^{\text{“Variety effect”}} \tag{52}$$

$$\text{Our model}: -\frac{dP}{dL}\frac{L}{P} = \overbrace{\delta}^{\text{“Specialization effect”}} . \tag{53}$$

Moreover, the “market size per firm”, A, changes with country size by the elasticity $\delta(\sigma-1)-1$ which can be either positive or negative. It increases with population size if specialization is important or if there is strong competition between manufacturing goods (higher elasticity of substitution).

Theorem 1 *Larger economies are associated with more specialization, higher aggregate productivity, lower prices and higher welfare. The elasticity of these relationships is greater the higher is the cost for firms to engage in many tasks simultaneously.*

Theorem 2 *Trade liberalization leads to: a) increased specialization of firms and b) higher welfare due to greater efficiency in production.*

Theorem 2 follows from Theorem 1 if it is assumed that trade liberalization can be proxied by an increase in population.

Smith (1776) argued that a division of labour could generate a great increase in production. One example he used was the making of pins. One worker could probably make only twenty pins per day. However, if ten people divided up the eighteen steps required to make a pin, they could make a combined amount of 48,000 pins in one day.

The exact equivalent in our model would be a decrease in t_M and t_S which makes firms more productive at what they do. Working against this mechanism is the presence of fixed costs (in Smith's example this would be a fixed cost per step in the specialization process). The presence of fixed costs makes it possible for larger economies to engage in more specialization (because consumption is larger and we have a fixed set of varieties which causes larger economies to consume more of each variety). This also, interestingly, translates into higher aggregate productivity in larger economies since the marginal cost per output decreases. To summarize, the equivalent in our model of Smith's division of labour in the pin factory is the division of production into more specialized service firms. In Smith's economy, greater size made it possible for workers to specialize more, in our model it makes it possible for service providers to specialize more. The final outcome is equivalent for the two settings: larger economies specialize more and therefore have higher aggregate productivity and higher welfare.

3 Empirical analysis

3.1 Main prediction

The most important testable prediction of our model is Theorem 1: larger economies are characterized by more specialization and a more narrow task scope performed inside the firm's boundaries. A convincing empirical test would therefore require information on firms and which tasks they decide to perform inhouse. We therefore use Swedish data which links employees to plants. Among the observables in the dataset on individuals is what occupation each individual has. This is the variable which we use as a proxy of a task. We link employees to plants and by calculating how many different occupations the employees of each plant have, we get a proxy of how many tasks are performed inside the organization.[9] For each plant, we know its main business (sector by NACE codes) and its location (by city) and can therefore test whether plants in larger cities tend to perform fewer tasks inhouse.

3.2 Data

As stated, we use Swedish data (all data is from Statistics Sweden) from the year of 2005. For our purposes, we believe that a crossectional approach is superior to using a panel estimation and therefore choose to use data only from a single year. 2,563,771 individuals are included in the dataset and this comprises all individuals employed in the private sector in Sweden in 2005. 413,387 plants are included and these are operated by 381,087 firms indicating that the vast majority of firms only operate one plant. For individuals, we use their occupation code and which plant they are employed at. For plants, we use information on how many employees they have, their location (by city/commune of which there are 290 in Sweden) and their sector (by five digit NACE codes).

[9] We do not have specific information on traditional task measures but instead occupations of workers. We believe, however, that the type of outsourcing described in the model is more related to occupations rather than specific tasks.

3.3 Method and results

As already described, we intend to test the hypothesis that plants in larger cities are more specialized (that they have a more narrow task scope). We will do this at the plant level so that we can exploit all the variation that we have and control for sector fixed effects. Moreover, larger plants tend to be more diversified so we control for plant size as well. Plant size is here proxied for by how many employees each plant has. The main specification that we run is therefore:

$$\log t_{ijl} = \beta_0 + \beta_1 \log Pop_j + \beta_2 \log Size_{ijl} + f_l + \varepsilon_{ijl} \tag{54}$$

where t_{ijl} denotes how many occupations employees have at plant i in city j in sector l, Pop_j denotes the population of city j and $Size_{ijl}$ denotes how many employees plant i has. Due to the logarithmic functions used, we can interpret coefficients β_1 and β_2 as elasticities. We control for sectors at the two or three digit NACE level as shown here by f_l. We cluster for standard errors at the city level in all regressions.

Table 1 shows the results. The first column estimates the regression in (54) using all manufacturing plants in Sweden in 2005. We find that the elasticity with respect to city size is indeed negative and significant: smaller city size tends to make plants hire a wider scope of occupations. This confirms the main prediction of our model, that larger cities are associated with a greater degree of specialization.

We then run a series of robustness tests for this specification. First, we note that many plants have very few employees and we therefore retain only the plants with more employees than the median plant (the median plant has 6 employees) in column (2). Then, in (3), we remove all plants that have less than the median number of occupations inhouse. This is equivalent to retaining only the plants with the most number of occupations inhouse. In column (4) we focus on the fact that some sectors could be substantially concentrated geographically. Our method here is then to retain in the sample only those sectors which are above the median sector as regards geographical coverage (as measured by the number of cities in which the sector is active). Finally, we use fixed effect at a more detailed sectoral level (three digit NACE instead of two digit). None of the robustness tests removes the conclusion from column (1): that firms in larger cities are associated with fewer occupations inhouse.

It should also be noted that our prior on task scope and plant size is highly significant and has a relatively high elasticity.

We conclude that this main empirical specification largely confirms the main prediction of the model and most importantly it does so for the manufacturing sector which is the sector we mostly had in mind when building the model and most likely the sector with most variation in task scope.

4 Conclusions

We develop a model of service outsourcing in which firms can choose how many tasks they wish to perform inhouse and how many to source from an external firm. Our key assumption is that it is more costly for firms to perform a wider range of tasks inhouse and that there therefore are benefits from specialization in a narrower range of tasks. Specifically, we allow service providers to focus on a narrow range of tasks and then sell these tasks to producers of final goods (manufacturers). The narrower is the scope of a service provider, the more productive it is. The same applies for manufacturers, the fewer tasks it produces inhouse, the more productive it is. We assume that there is a contracting friction due to a lack of legal verifiability of the quality of tasks provided by service firms and a holdup problem since a manufacturing firm cannot switch service provider once a contract has been made. This causes manufacturing firms to share their revenues with service providers.

The model generates analytical solutions for all variables and, most importantly, dynamics relating to market size: larger economies can sustain a greater degree of specialization since larger demand can make more firms afford the fixed costs involved in production while operating a narrower task scope. If trade liberalization is proxied by an increase in population size, the model generates benefits from trade through a rise in specialization rather than an increase in the number of varieties (despite the fact that we use Dixit Stiglitz preferences).

We use detailed data from Sweden which links employees to plants. We find that, controlling for plant size, manufacturing plants in larger cities tend to employ fewer occupations. It therefore seems as if manufacturing firms are more specialized in larger economies, possibly due to larger benefits from outsourcing.

We also model two cases of incremental trade liberalization where we develop a two country model with iceberg trade costs for (i) goods and (ii) tasks. Goods trade liberalization does not affect specialization patterns while task trade lib-

Table 1: Task scope and city size in Sweden

Dependent Variable: Log Task scope at the plant level (Manufacturing sample)					
	All	Emp > Mean	Task scope > Mean	City coverage	FE
	(1)	(2)	(3)	(4)	(5)
City Size (log population)	-0.0026*	-0.010***	-0.008**	-0.009*	-0.0096*
	(0.001)	(0.002)	(0.003)	(0.004)	(0.004)
Plant Size (log employment)	0.589***	0.488***	0.475***	0.524***	0.523***
	(0.001)	(0.004)	(0.004)	(0.005)	(0.005)
Industry fixed effect 2-digits	yes	yes	yes	yes	no
Industry fixed effects 3-digits	no	no	no	no	yes
Cluster (city level)	yes	yes	yes	yes	yes
Observations	38757	12886	12737	6558	6558
R^2	0.883	0.637	0.658	0.672	0.678

eralization actually lowers the degree of specialization. We view, however, using the autarky model and proxying trade liberalization by an increase in the population as the cleanest way of modelling trade liberalization.

Finally, we believe that our model lies close to the heart of Adam Smith's theory of the benefits of the "division of labour". In Smith's theory, a larger economy, or firm, could divide tasks to different workers who raised their productivity substantially when specializing in these tasks. The equivalent in our model of service trade is that, in larger markets, manufacturing firms outsource more tasks to service providers which specialize in a more narrow range of tasks and become more productive in these tasks. The outcome is that larger markets, or markets engaging in trade liberalization, experience a rise in specialization which translates into higher aggregate productivity and welfare.

5 Appendix: Open economy

This final section of the paper explores the open economy case in a slightly different way. In Section 2 we analysed the autarky setting and then proxied trade liberalization by an increase in population size. While this gives the analytically most robust, clearest and most intuitive results, we also wish to examine what happens in the case of incremental trade liberalization. We do this in this section.

We will explore two types of open economy settings. First, we will allow for trade in manufactured goods but not in tasks. This can be seen as a world with trade in final goods but where service trade is impossible due to too high trade costs for services. Trade costs are represented by an iceberg trade cost of $\tau > 1$. Second, we open, instead, trade in tasks which can be seen as an analysis of the recent rapid increase of service offshoring. The friction in task trade is represented by an iceberg trade cost of $\beta > 1$. As the notation suggests, the way we model is similar to Grossman and Rossi-Hansberg (2008b). We will, for now, maintain the assumption of equal wages across countries. Moreover, we will demonstrate our results in an economy consisting of two countries, Home and Foreign, where the latter is indicated by an asterix "*".

In order to maintain tractability and illustrate our main points, we assume symmetric country size. We are therefore abstracting from effects relating to economic geography and differences in relative country size.

5.1 Trade in goods

When there is trade in goods, the demand faced by each manufacturing firm in country i selling to country j is instead:

$$q_{ij} = A_j \left(\tau_{ij} p_{ij}\right)^{-\sigma} \tag{55}$$

where

$$A_j \equiv \frac{\mu L_j}{P_j^{1-\sigma}} \tag{56}$$

and where the price indices are

$$P_j = \left(\sum_{i=\Lambda} n_i \left(\tau_{ij} p_{ij}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{57}$$

where Λ denotes the set of countries in the world. For now, we have only two countries and therefore

$$P = \left(n_M p_M^{1-\sigma} + n_M^* \left(\tau p_M^* \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{58}$$

$$P^* = \left(n_M \left(\tau p_M \right)^{1-\sigma} + n_M^* p_M^{*1-\sigma} \right)^{\frac{1}{1-\sigma}} . \tag{59}$$

Now, the profits of a manufacturer in Home are:

$$\begin{aligned} \pi_M &= pq + p^* q^* - t_M \varphi_M \left(q + \tau q^* \right) - \left(1 - t_M \right) \left(pq + \tau p^* q^* \right) - f_M &(60) \\ &= \left(A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} + A^{*\frac{1}{\sigma}} q^{*\frac{\sigma-1}{\sigma}} \right) t_M - t_M \varphi_M \left(q + \tau q^* \right) - f_M &(61) \end{aligned}$$

where it should be noted that, for the foreign market, the manufacturing firm has to produce τq^* in order to sell q^* on the foreign market.

A supplier decides how much to produce for the manufacturing firm, $q + \tau q^*$, by maximizing its operating profit $\overline{\pi}_S$ per manufacturing firm:

$$\begin{aligned} \overline{\pi}_S &= t_S \left(pq + p^* q^* \right) - t_S \varphi_S \left(t_S \right) \left(q + \tau q^* \right) &(62) \\ &= t_S \left(A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} + A^{*\frac{1}{\sigma}} q^{*\frac{\sigma-1}{\sigma}} \right) - t_S \varphi_S \left(t_S \right) \left(q + \tau q^* \right) . &(63) \end{aligned}$$

The FOCs from maximizing with respect to q and q^* yield:

$$q = \left(\frac{1}{\widetilde{\sigma} \lambda_1 t_S^{\delta}} \right)^{\sigma} A \tag{64}$$

$$q^* = \left(\frac{1}{\tau \widetilde{\sigma} \lambda_1 t_S^{\delta}} \right)^{\sigma} A^* \tag{65}$$

which means that

$$q + \tau q^* = \left(\frac{1}{\widetilde{\sigma} \lambda_1 t_S^{\delta}} \right)^{\sigma} \left(A + \phi A^* \right) \tag{66}$$

where $\phi \equiv \tau^{1-\sigma} \in (0, 1]$ is an index of "globalization" and takes the value 0 in autarky and 1 at free trade.

The manufacturer takes this as given and decides on its task scope t_M:

$$\max_{t_M} \pi_M = \left(A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} + A^{*\frac{1}{\sigma}} q^{*\frac{\sigma-1}{\sigma}} \right) t_M - t_M \varphi_M \left(q + \tau q^* \right) - f_M \tag{67}$$

and maximizing with respect to t_M yields the following FOC:

$$A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} + A^{*\frac{1}{\sigma}} q^{*\frac{\sigma-1}{\sigma}} = (q + \tau q^*)(\delta+1)\lambda_1 t_M^{\delta} \tag{68}$$

$$\widetilde{\sigma}\lambda_1 t_S^{\delta} = (\delta+1)\lambda_1 t_M^{\delta} \tag{69}$$

$$t_M = \left(\frac{\widetilde{\sigma}}{\delta+1}\right)^{\frac{1}{\delta}} t_S \tag{70}$$

which is the same relationship between t_M and t_S as in the autarky model.

Now, we turn to the zero profit condition for manufacturers:

$$\pi_M = 0 \tag{71}$$

$$A + \phi A^* = f_M (\widetilde{\sigma}\lambda_1)^{\sigma-1} t_S^{-1+\delta(\sigma-1)} \left(\frac{\delta+1}{\widetilde{\sigma}}\right)^{\frac{1}{\delta}} \left(\frac{\delta+1}{\delta}\right) \tag{72}$$

$$= \lambda_4 t_S^{-1+\delta(\sigma-1)} \tag{73}$$

where $\lambda_4 \equiv f_M (\widetilde{\sigma}\lambda_1)^{\sigma-1} \left(\frac{\delta+1}{\widetilde{\sigma}}\right)^{\frac{1}{\delta}} \left(\frac{\delta+1}{\delta}\right)$.

This expression for the market size can be used in the zero profit condition for service suppliers:

$$\pi_S = 0 \tag{74}$$

$$n_M = \frac{f_S}{f_M} \frac{\widetilde{\sigma}}{\widetilde{\sigma}-1} \frac{\delta}{\delta+1} \left(\frac{\widetilde{\sigma}}{\delta+1}\right)^{\frac{1}{\delta}} \tag{75}$$

where we see that the mass of manufacturers is fixed in each country (as in autarky) and is independent of trade costs. Therefore we know that:

$$n_M = n_M^* = \frac{f_S}{f_M} \frac{\widetilde{\sigma}}{\widetilde{\sigma}-1} \frac{\delta}{\delta+1} \left(\frac{\widetilde{\sigma}}{\delta+1}\right)^{\frac{1}{\delta}}. \tag{76}$$

The price indices are:

$$P = \left(\int p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{77}$$

$$= n_M^{\frac{1}{1-\sigma}} \widetilde{\sigma}\lambda_1 t_S^{*\delta} \left(\left(\frac{t_S^*}{t_S}\right)^{\delta(\sigma-1)} + \phi\right)^{\frac{1}{1-\sigma}} \tag{78}$$

$$P^* = n_M^{\frac{1}{1-\sigma}} \widetilde{\sigma}\lambda_1 t_S^{*\delta} \left(\phi\left(\frac{t_S^*}{t_S}\right)^{\delta(\sigma-1)} + 1\right)^{\frac{1}{1-\sigma}}. \tag{79}$$

This gives:

$$A = \frac{\lambda_4}{1-\phi^2} t_S^{-1+\delta(\sigma-1)} \left(1 - \phi \left(\frac{t_S^*}{t_S}\right)^{-1+\delta(\sigma-1)}\right). \tag{80}$$

$$A^* = \frac{\lambda_4}{1-\phi^2} t_S^{-1+\delta(\sigma-1)} \left(\left(\frac{t_S^*}{t_S}\right)^{-1+\delta(\sigma-1)} - \phi\right). \tag{81}$$

Dividing the two gives:

$$\frac{A}{A^*} = \frac{L}{L^*} \frac{P^{*1-\sigma}}{P^{1-\sigma}} \tag{82}$$

$$\frac{\left(1 - \phi \left(\frac{t_S^*}{t_S}\right)^{-1+\delta(\sigma-1)}\right)}{\left(\left(\frac{t_S^*}{t_S}\right)^{-1+\delta(\sigma-1)} - \phi\right)} \frac{\left(\left(\frac{t_S^*}{t_S}\right)^{\delta(\sigma-1)} + \phi\right)}{\left(\phi\left(\frac{t_S^*}{t_S}\right)^{\delta(\sigma-1)} + 1\right)} = \frac{L}{L^*}. \tag{83}$$

Imposing size symmetry, $\frac{L}{L^*} = 1$ or $L = L^* = \overline{L}$, yields:

$$\frac{\left(1 - \phi \left(\frac{t_S^*}{t_S}\right)^{-1+\delta(\sigma-1)}\right)}{\left(\left(\frac{t_S^*}{t_S}\right)^{-1+\delta(\sigma-1)} - \phi\right)} \frac{\left(\left(\frac{t_S^*}{t_S}\right)^{\delta(\sigma-1)} + \phi\right)}{\left(\phi\left(\frac{t_S^*}{t_S}\right)^{\delta(\sigma-1)} + 1\right)} = 1 \tag{84}$$

where we note that $t = t^*$ is a solution.

Moreover,

$$A = A^* = \frac{\lambda_4}{1-\phi^2} t_S^{-1+\delta(\sigma-1)} (1-\phi) \tag{85}$$

$$= \frac{\lambda_4}{1+\phi} t_S^{-1+\delta(\sigma-1)}. \tag{86}$$

when country size is the same. This also means that the prices are:

$$P = P^* = n_M^{\frac{1}{1-\sigma}} \widetilde{\sigma} \lambda_1 t_S^{\delta} (1+\phi)^{\frac{1}{1-\sigma}}. \tag{87}$$

Using this information yields:

$$\begin{aligned}
\frac{\mu\overline{L}}{P^{1-\sigma}} &= \frac{\lambda_4}{1+\phi} t_S^{-1+\delta(\sigma-1)} \\
\frac{\mu\overline{L}}{\left(n_M^{\frac{1}{1-\sigma}} \widetilde{\sigma}\lambda_1 t_S^{\delta} (1+\phi)^{\frac{1}{1-\sigma}}\right)^{1-\sigma}} &= \frac{\lambda_4}{1+\phi} t_S^{-1+\delta(\sigma-1)} \\
t_S &= \frac{f_S}{\mu\overline{L}} \frac{\widetilde{\sigma}}{\widetilde{\sigma}-1}.
\end{aligned}$$

Note that with no difference in country size, the level of specialization of both service providers and manufacturing firms is exactly the same as in autarky and independent of trade costs.

As in autarky, the mass of service providers increases in country size:

$$n_S = \frac{1-t_M}{t_S} \tag{88}$$

$$= \frac{L-\lambda_2\lambda_3}{\lambda_3}. \tag{89}$$

Finally, note that:

$$A = A^* = \frac{\lambda_4}{1+\phi} t_S^{-1+\delta(\sigma-1)} \tag{90}$$

$$= \frac{\lambda_4}{1+\phi}\left(\frac{f_S}{\mu\overline{L}} \frac{\widetilde{\sigma}}{\widetilde{\sigma}-1}\right)^{-1+\delta(\sigma-1)} \tag{91}$$

and that

$$P = P^* = n_M^{\frac{1}{1-\sigma}} \widetilde{\sigma}\lambda_1 t_S^{\delta} (1+\phi)^{\frac{1}{1-\sigma}} \tag{92}$$

$$= (1+\phi)^{\frac{1}{1-\sigma}} \left(\frac{f_S}{\mu\overline{L}} \frac{\widetilde{\sigma}}{\widetilde{\sigma}-1}\right)^{\delta} \left(\frac{f_S}{f_M}\frac{\widetilde{\sigma}}{\widetilde{\sigma}-1}\frac{\delta}{\delta+1}\left(\frac{\widetilde{\sigma}}{\delta+1}\right)^{\frac{1}{\delta}}\right)^{\frac{1}{1-\sigma}} \widetilde{\sigma}\lambda_1. \tag{93}$$

So all the welfare effects from trade liberalization comes from the fact that more varieties become available and that their price falls as trade becomes less costly, captured by the fall in the term $(1+\phi)^{\frac{1}{1-\sigma}}$ when ϕ increases.

Theorem 3 *Trade liberalization in goods trade does not affect the degree of specialization or the mass of firms. It does generate welfare effects, however, through the decrease in price of import goods.*

Moreover, all the effects from country size (L and L^*) are identical to the autarky case and do not change with trade in goods.

5.2 Trade in tasks

Now we turn instead to the case when tasks can be traded internationally. Here, we face some difficult decisions about what assumptions to use. This depends on the fact that, in autarky, all outsourced tasks are produced in both countries by local service providers. In perfectly free trade, however, no service provider will produce the same task, regardless of where it is located. Therefore, we have decided to analyse the case of some intermediate trade cost where service providers are specializing internationally (meaning that only one service provider in the world will produce each task which is outsourced). Moreover, we will analyse the symmetric country case where $L = L^* = \overline{L}$ to ensure that $t_M = t_M^*$ because of the difficulties in dealing with how the tasks that are between t_M and t_M^* are produced (only a subset of service providers would be affected in this case and behave differently).

To model trade frictions in tasks, we assume a standard iceberg cost of $\beta > 1$ where for one unit of a task to arrive in the foreign economy, β units have to be produced by the service provider.

The profits of a manufacturer are unchanged with respect to our baseline model:

$$\pi_M = pq - t_M \varphi_M (q) - (1 - t_M) pq - f_M. \tag{94}$$

The profit of a service provider has, however, changed. When it is selling to a domestic manufacturer, the problem is still the same but consider the service provider's profits for exporting its task:

$$\begin{aligned} \pi_S^X &= t_S p^* q^* - \beta t_S \varphi_S (t_S) q^* & (95) \\ &= t_S q^* (p^* - \beta \varphi_S (t_S)) & (96) \\ &= t_S q^* \left(A^{*\frac{1}{\sigma}} q^{*-\frac{1}{\sigma}} - \beta \varphi_S (t_S) \right). & (97) \end{aligned}$$

The FOC for the service provider's profit maximization with respect to q^*

yields:

$$\frac{\sigma-1}{\sigma} t_S A^{*\frac{1}{\sigma}} q^{*-\frac{1}{\sigma}} - t_S \beta \varphi_S(t_S) = 0 \tag{98}$$

$$q^* = \left(\frac{1}{\widetilde{\sigma}\beta\lambda_1 t_S^\delta}\right)^\sigma A^*. \tag{99}$$

Since manufacturers now buy from both domestic and foreign suppliers, it means that they will have less production than before by a factor of $\beta^{-\sigma} < 1$ (also domestic service provider will produce this lower output for manufacturers due to the Leontief structure of the manufacturers' production function).

The manufacturer's problem is now:

$$\pi_M = pq - t_M \varphi_M(q) - (1 - t_M) pq - f_M \tag{100}$$

$$= t_M \left(A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} - \lambda_1 t_M^\delta\right) - f_M. \tag{101}$$

The FOC for profit maximization with respect to t_M yields:

$$\left(A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} - \lambda_1 t_M^\delta\right) - \lambda_1 \delta t_M^\delta q = 0 \tag{102}$$

$$t_M = \left(\frac{1}{\lambda_1(1+\delta)} A^{\frac{1}{\sigma}} q^{-\frac{1}{\sigma}}\right)^{\frac{1}{\delta}}. \tag{103}$$

Using the knowledge of q from before, we can rewrite this to:

$$t_M = \left(\beta \frac{\widetilde{\sigma}}{(1+\delta)}\right)^{\frac{1}{\delta}} t_S^* \tag{104}$$

which is the same as in the autarky except for the presence of β. This is because service providers now become more expensive to use due to the cost of β to ship some tasks. Here, we also assume that it is the specialization level of foreign service providers which will bound q because the foreign service providers are the ones who face the iceberg trade cost β of shipping tasks.

Now, we use the free entry condition for manufacturers:

$$pq - t_M \varphi_M(q) q - (1 - t_M) pq = f_M \tag{105}$$

$$t_M q \left(A^{\frac{1}{\sigma}} q^{-\frac{1}{\sigma}} - \lambda_1 t_M^\delta\right) = f_M \tag{106}$$

$$t_S^{*\delta(\sigma-1)-1} f_M (\beta\widetilde{\sigma})^{(\sigma-1)-\frac{1}{\delta}} \lambda_1^{\sigma-1} (1+\delta)^{\frac{1}{\delta}} \frac{1+\delta}{\delta} = A. \tag{107}$$

The free entry condition for service providers in Home yields:

$$\pi_S = 0 \tag{108}$$

$$t_S\left(n_M pq + n_M^* p^* q^*\right) - t_S \varphi_S\left(t_S\right)\left(n_M q + n_M^* \beta q^*\right) = f_S \tag{109}$$

$$\begin{rcases} t_S^{1+\delta(1-\sigma)} \lambda_1^{1-\sigma} \left(\frac{1}{\widetilde{\sigma}\beta}\right)^{\sigma-1} \left(n_M A \left(\frac{t_S^*}{t_S}\right)^{\delta(1-\sigma)} + n_M^* A^*\right) \\ -\lambda_1^{1-\sigma} t_S^{1+\delta(1-\sigma)} \left(\frac{1}{\widetilde{\sigma}\beta}\right)^{\sigma} \left(n_M \left(\frac{t_S^*}{t_S}\right)^{-\delta\sigma} A + n_M^* \beta A^*\right) \end{rcases} = f_S. \tag{110}$$

Comparing this solution to the foreign equivalent shows that there exists a symmetric solution where:

$$t_S = t_S^*$$
$$A = A^*$$
$$n_M = n_M^*$$

and we proceed to analyse this solution. Equation (110) simplifies to:

$$f_S = t_S^{1+\delta(1-\sigma)} \lambda_1^{1-\sigma} \left(\frac{1}{\widetilde{\sigma}\beta}\right)^{\sigma} n_M A \left(2\widetilde{\sigma}\beta - (1+\beta)\right) \tag{111}$$

$$A = f_S t_S^{\delta(\sigma-1)-1} \lambda_1^{\sigma-1} \left(\widetilde{\sigma}\beta\right)^{\sigma} n_M^{-1} \frac{1}{2\widetilde{\sigma}\beta - (1+\beta)}. \tag{112}$$

where we note that $2\widetilde{\sigma}\beta - (1+\beta)$ is always positive since $\widetilde{\sigma} > 1$.

This solution can be equaled to the solution in (107):

$$t_S^{*\delta(\sigma-1)-1} f_M \left(\beta\widetilde{\sigma}\right)^{(\sigma-1)-\frac{1}{\delta}} \lambda_1^{\sigma-1} (1+\delta)^{\frac{1}{\delta}} \frac{1+\delta}{\delta} = f_S t_S^{\delta(\sigma-1)-1} \lambda_1^{\sigma-1} \left(\widetilde{\sigma}\beta\right)^{\sigma} n_M^{-1} \frac{1}{2\widetilde{\sigma}\beta - (1+\beta)} \tag{113}$$

$$n_M = \frac{f_S}{f_M} \widetilde{\sigma}^{\frac{1+\delta}{\delta}} \beta^{\frac{1}{\delta}} \frac{\beta}{2\widetilde{\sigma}\beta - (1+\beta)} \frac{\delta}{1+\delta} (1+\delta)^{-\frac{1}{\delta}}. \tag{114}$$

It can be noted that

$$\frac{\partial}{\partial}\left(\frac{\beta}{2\widetilde{\sigma}\beta - (1+\beta)}\right) = \frac{2\widetilde{\sigma}\beta - (1+\beta) - \beta\left(2\widetilde{\sigma} - 1\right)}{\left(2\widetilde{\sigma}\beta - (1+\beta)\right)^2} \tag{115}$$

$$= -\frac{1}{\left(2\widetilde{\sigma}\beta - (1+\beta)\right)^2} < 0 \tag{116}$$

but $\frac{\partial}{\partial\beta}\left(\beta^{\frac{1}{\beta}}\right) > 0$ so the effect of β on the mass of manufacturing firms is not clear.

$$\frac{\partial}{\partial}\left(\frac{\beta^{\frac{1+\delta}{\delta}}}{2\widetilde{\sigma}\beta-(1+\beta)}\right)=\frac{\frac{1+\delta}{\delta}\beta^{\frac{1}{\delta}}\left(2\widetilde{\sigma}\beta-(1+\beta)\right)-\beta^{\frac{1+\delta}{\delta}}\left(2\widetilde{\sigma}-1\right)}{\left(2\widetilde{\sigma}\beta-(1+\beta)\right)^{2}} \tag{117}$$

$$=\beta^{\frac{1}{\delta}}\frac{\frac{1}{\delta}\left(2\widetilde{\sigma}\beta-(1+\beta)\right)-1}{\left(2\widetilde{\sigma}\beta-(1+\beta)\right)^{2}}. \tag{118}$$

This is negative if

$$\frac{1}{\delta}\left(2\widetilde{\sigma}\beta-(1+\beta)\right)-1<0 \tag{119}$$

$$\beta<\frac{\sigma-1}{\sigma+1}\left(1+\delta\right). \tag{120}$$

To find the remaining variables, we use the expression for n_M and the definition of $A \equiv \frac{\mu L}{P^{1-\sigma}}$ in equation (112) and find:

$$t_S=\frac{2^{1-\sigma}}{\mu L}f_S\widetilde{\sigma}\beta^{-\frac{\sigma}{\delta}}\left(\frac{\beta}{2\widetilde{\sigma}\beta-(1+\beta)}\right)^{1-\sigma}\left(\frac{f_S}{f_M}\widetilde{\sigma}^{\frac{1+\delta}{\delta}}\frac{\delta}{1+\delta}\left(1+\delta\right)^{-\frac{1}{\delta}}\right)^{-\sigma}.$$

Moreover, knowing A and t_S yield a solution also for the price index:

$$\frac{\mu L}{P^{1-\sigma}}=f_St_S^{\delta(\sigma-1)-1}\lambda_1^{\sigma-1}\left(\widetilde{\sigma}\beta\right)^{\sigma}n_M^{-1}\frac{1}{2\widetilde{\sigma}\beta-(1+\beta)}$$

$$t_S=\frac{1}{\mu L}f_S\widetilde{\sigma}2\frac{\beta}{2\widetilde{\sigma}\beta-(1+\beta)}.$$

This shows that specialization decreases when task trade is liberalized ($\frac{\partial t_S}{\partial\beta} < 0$) which can be seen in (115).

Note also that:

$$t_M=\frac{\lambda_4}{L}\beta^{\frac{1}{\delta}}\frac{\beta}{2\widetilde{\sigma}\beta-(1+\beta)}.$$

where $\lambda_4 \equiv \frac{1}{\mu}\left(\frac{\widetilde{\sigma}}{(1+\delta)}\right)^{\frac{1}{\delta}} f_S\widetilde{\sigma}2$. We note that t_M increases with trade liberalization ($d\beta < 0$) because t_S increases but also decreases (due to the extra term $\beta^{\frac{1}{\delta}}$) because now the quantity supplied by foreign service providers increases.

The price index becomes:

$$P = \lambda_5 \left(\frac{1}{L}\right)^{\delta} \beta^{1-\frac{1}{\delta(\sigma-1)}} \left(\frac{\beta}{2\widetilde{\sigma}\beta - (1+\beta)}\right)^{\delta - \frac{1}{\sigma-1}}.$$

where $\lambda_5 \equiv \mu^{-\delta}\lambda_1 (f_S 2)^{\delta-\frac{1}{\sigma-1}} \widetilde{\sigma}^{(1+\delta)} \left(\frac{1}{f_M}\widetilde{\sigma}^{\frac{1+\delta}{\delta}} \frac{\delta}{1+\delta} (1+\delta)^{-\frac{1}{\delta}}\right)^{-\frac{1}{\sigma-1}}$.

The elasticity of the price index with respect to country size is the same as before, δ. The net effect of β, the cost of task trade, is, however, uncertain. This is most likely due to the two main channels through which β affects these variables: (i) a lower β increases specialization which lowers the price index and increases welfare but (ii) a lower β increases the output of each manufacturer which lowers the range of varieties available in the economy. The net effect ultimately depends on the relative size of δ and σ or whether the preference for variety (σ) is stronger than the need for specialization (δ).

Theorem 4 *Trade liberalization in task trade decreases the level of specialization among both service providers and manufacturing firms. The net effect on the range of manufacturing varieties and welfare is, however, uncertain.*

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