# Systemic Risk and Network Formation in the Interbank Market

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## Abstract

We propose a novel mechanism to facilitate understanding of systemic risk in financial markets. The literature on systemic risk has focused on two mechanisms, common shocks and domino-like sequential default. Our approach is a formal model that provides an intellectual combination of the two by looking at how shocks propagate through a network of interconnected banks. Transmission in our model is not based on default. Instead, we provide a simple microfoundation of banks' profitability based on classic competition incentives. As competitors lending quantities change, both for closely connected ones and the whole market, banks adjust their own lending decisions as a result, generating a 'transmission' of shocks through the system. We provide a unique equilibrium characterization of a static model, and embed this model into a full dynamic model of network formation with $n$ agents. Because we have an explicit characterization of equilibrium behavior, we have a tractable way to bring the model to the data. Indeed, our measures of systemic risk capture the propagation of shocks in a wide variety of contexts; that is, it can explain the pattern of behavior both in good times as well as in crisis.

**Keywords:** Financial networks, interbank lending, interconnections, network centrality, spatial autoregressive models.

**JEL Classification:** G10, C21

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# 1 Introduction

Since the onset of the financial crisis in August of 2007, the discourse about bank safety has shifted strongly from the riskiness of financial institutions as individual firms to concerns about systemic risk. As the crisis evolved, the debate did as well, with concerns about systemic risk growing from too-big-to-fail (TBTF) considerations to too-interconnected-to-fail (TITF) ones. The spectacular collapse of Lehman Brothers in September of 2008 and the subsequent rescue of AIG brought this to the forefront of academic and policy debates.[1]

We propose a novel approach to facilitate understanding of systemic risk in financial markets. To date, the literature on systemic risk has focused principally on one of two mechanisms. The first comes from Herring and Wachter (2001), in which agents are simultaneously impacted by a shock to underlying asset prices.[2] The second is the Allen and Gale (200) mechanism in which the default of a given entity can lead to domino-like series of subsequent defaults based on exposures to the defaulting entity.

Our approach is a formal model that provides an intellectual combination of the two by looking at how external shocks propagate through the system of interconnected banks.[3] Risk in our model, however, is not a function of default. Instead, we provide a simple microfoundation of banks' profitability based on classic competition incentives. As competitors' lending quantities change, both for closely connected ones and the whole market, banks adjust their own lending decisions as a result.

These incentives can be built into a model of bank action that is dependent on the network structure at each point in time. A key feature of this model is that we are able to find a unique equilibrium outcome of bank lending behavior for any network pattern.[4] Because we have an explicit characterization of equilibrium behavior, we have a tractable way to bring the model to the data. Indeed, our measures of systemic risk capture the propagation of shocks in a wide variety of contexts; that is, it can explain the pattern of behavior both in good times as well as in crisis.

As shocks hit a system, the existing pattern of network links will evolve. As such, reduced form and/or static models of systemic risk may be insufficient for understanding the importance of interconnectedness on financial markets. With this in mind, we explicitly embed our static model into a complete dynamic model of network formation. Thus, we are able to characterize not only the equilibrium pattern of behavior at each point in time, but also how this behavior evolves over time. As banks form and break links, the structure of the network will change, and the nature of systemic risk with it. As with our static model, we provide an expansion from existing work by generalizing the network formation game to networks of $n$ institutions. This generalization allows us to directly analyze the applicability of the model to realized data patterns. As

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[1] A general perception and intuition has emerged that the interconnectedness of financial institutions is potentially as crucial as their size. Recent papers that emphasize such interconnectedness or try to explain it include Allen, Babus and Carletti (2010), Cohen-Cole, Kirilenko and Patacchini (2010), Boyson, Stahel and Stulz (2010), Adrian and Brunnermeier (2009), and Danielsson, Shin and Zigrand (2009).

[2] In a creative expansion of this theory, Allen, Babus and Carletti (2010) illustrate how the accumulation of exposure to these shocks depends on the incentives for individual banks to diversify holdings.

[3] Indeed, even prior to the crisis there is a wide range of research on the importance of interbank markets, including some that address the systemic risk inherent to these markets. Some examples include Freixas, Parigi and Rochet (2000), Iori and Jafarey (2001), Boss et al (2004), Furfine (2003), Iori, Jafarey, and Padilla (2006), Soramäki, et al (2007), Pröpper, Lelyveld and Heijmans (2008), Cocco, Gomes and Martin (2009), Mistrulli (2010), and Craig and Von Peter (2010). Each of these discuss some network properties or discuss the importance of these markets to systemic risk evaluation.

[4] Prior work has either used reduced form empirical descriptions of network patterns or analyzed equilibrium phenomena in small networks or with particular network structures.

well, because of the generality, it permits regulators or market analysts to evaluate patterns of systemic risk in a wide array of contexts.

We are, of course, not the first to discuss the use of network methods in a financial market context. In part due to availability of data, much existing work has been related to payment systems.[5] In other aspects of financial systems Bech, Chapman and Garratt (2010) discuss the importance of being central in interbank markets. Rotemberg (2010) derives the implications for liquidity needs in payment systems for a set of small networks. Others have dealt with the presence of spillovers through connected individuals in a finance context. This subsection of the literature emphasizes the role of non-market interactions that can influence prices.[6]

Based on our model, we will construct a measure of systemic risk that is a precise measure of the aggregate liquidity reduction that results from a change in lending by an individual financial institution.

We bring the model to data and show its tractability and accuracy using transaction level data from the European interbank loan market between 2002 and 2009. The interbank market plays an important role for financial institutions in smoothing daily imbalances in balance sheets. In some locations, this market was also used as a principal funding device. Indeed, the collapse of Bear Stearns is widely attributed to its reliance on interbank markets for funding it balance sheet and an unwillingness of its counterparties to provide credit.

Empirically, we find that the dynamic model matches the network pattern closely.[7] Indeed, we illustrate that the model is very precise at describing the network patterns that exist in the market over time, even as the importance of network position changes markedly with the crisis. We also show that the static model fits the data very well; using the network pattern of linkages, we are able to describe between 5% and 10% of the distribution of loans in the interbank market in each period of time. As well, we show that the our measures of systemic risk are good at characterizing the increased risk that built up in the market starting in early 2007. Finally, we provide some evidence that quantities of loans were less impacted by the crisis than prices. That is, there appears to have been greater systemic impact in price changes than in quantities.

A key innovation in our paper vis-a-vis the literature is a full characterization of the strategic game played by banks. Specifically, we embed bank's strategic decision making on interbank lending into a static model of network effects. We will separately model two games; a lending game and a borrowing game. Beginning with the lending game, banks decide at each point in time how much they want to lend. This decision is a complex one for a few reasons. The banks must take into account the structure of the network insofar as who they lend to, who else lends to the same banks, etc. Further, the quantity decision is also a function of the total quantity of loans in the market. In aggregate, quantities impact prices, and because there is a single market price, agents must make lending decisions at their point in the network cognizant that their choices impact both the competition at that point in the network as well as prices for the entire market.

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[5] See for example Freixas and Parigi (1998), Lublóy (2006) Soramäki et al, (2007), Afonso and Shin (2008), Leinonen (2009), Kyriakopoulos, et al (2009), Bech, Chapman and Garratt (2010).

[6] Among others, Kaustia and Knüpfer (2008) find evidence in IPO participation; Kaustia and Knüpfer (2009), Hong, Kubik, and Stein (2004) and Brown, Ivkovich, Smith, and Weisbenner (2008) find a link to stock market participation; Shive (2009) finds interactions in trading activity; Grinblatt, Keloharju, and Ikäheimo (2008) find interactions in automobile purchases. Cohen, Malloy and Frazzini (2008, 2010) find peer networks in a couple of contexts. Faulkender and Yang (2010) find evidence that peer networks influence executive compensation and Leary and Roberts (2009) find that peers influence corporate financing policy. John and Kadyrzhanova (2008) look at spillovers in governance.

[7] Specifically, we will show that the model's prediction for the degree distribution (number of links for each bank) matches the observed distribution very closely.

The solution to our model is a Nash equilibrium quantity of loans for each agent at each point in time that is a function of the network structure. The equilibrium of this game can be described as the set of strategies that maximizes outcomes conditional on the strategies of others. Thus, one wants ideally to capture the borrowing / lending strategies themselves and from those strategies determine whether network links influence outcomes. In our empirical work, we will do precisely this - and, as mentioned before, show that the network patterns explain between 5 and 10% of the lending decisions by banks.

Theoretically, we will show that the Nash equilibrium is the unique pure-strategy one (Ballester, Calvo-Armengol, and Zenou, 2006; Calvo-Armengol, Patacchini, Zenou, 2009). Conceptually, the Nash equilibrium of this game provides a simple characterization of equilibrium market patterns that includes a measure of network shock amplification, or systemic risk. Using this result, we estimate the model's parameters using European interbank data from January 2002 to December 2009.

A key step to understanding the importance of the static model is that banking networks evolve over time. So, to validate that the results of our static model are reasonable, we must be able to validate that the choices banks make that are conditioned on the network are reasonable given the evolution of network. Banks choose borrowers or lenders strategically. Links formed today may not be relevant to decision making tomorrow. Our static model, as most in the literature, does not allow for strategic link formation.[8] Acknowledging this fact, our dynamic model uses the framework from König, Tessone and Zenou (2010a) to embed the lending choice game from the static model and permit banks to form and destroy links over time. Both the static and dynamic model are sufficiently general to permit the analysis of any arbitrary network patterns.[9] We believe that an equilibirum model of network formation for $n$ agents is an innovation in the finance literature. Each choice that a bank makes is based on the profitability trade-off of the new network versus the status quo one. We calibrate this model to the European interbank data patterns over the same time period.

We provide a short overview of the interbank lending market in section 2. We continue in sections 3 to outline our static model of interbank lending in the context of network linkages. Within this section, we outline the structure of the unique pure strategy Nash Equilibrium. With the stage-game set, we provide the full dynamic model in section 4, before turning to empirical analysis in section 5. In the empirical section, we outline our data, discuss estimation issues in this context and provide estimation results. Section 6 provides some discussion before we conclude in section 7.

## 2   Interbank Lending

A now widely discussed feature of the banking system is the presence of an interbank lending market. Bank balance sheets are typically composed on loans on the asset side and deposits (plus equity) on the liability side. Regularly, the natural businesses of banks leads to higher loans or deposits on a given day. These structural positions can be balanced in the short term through the interbank market. For example, a bank with $1100 in loans, $900 in deposits and $100 in equity, can use the interbank market to borrow an additional $100 to fully fund its balance sheet.

When the interbank market was very liquid, some banks indeed used the market to fund most of their balance sheet. Instead of collecting deposits at all, a bank could simply issue loans and fill the liability side

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[8] Notable exceptions are Babus (2009) and Allen, Babus and Carletti (2010). However, each of these focus on a limited number of agents. Our apporach will allow us to consider any arbitrary network pattern.

[9] The tradeoff for this generality will be a simplified characterization of banking. Future research could embed more complete characterizations of bank balance sheets, external drivers and liquidity needs, etc.

of the balance sheet with interbank loans.

When the crisis arrived, a combination of credit quality fears and liquidity shortages led to difficulties in the interbank market. US and European central banks intervened at various points in time to ensure that banks would have continued access to funding. The Federal Reserve began this effort with the TAF in December of 2007. Eventually, the Federal Reserve created a wide variety of related programs and the European Central Bank moved on October 15 of 2008 to a 'full-allotment' policy in which it provided unlimited credit to banks in the euro area.

For our purposes, the key question is the degree to which the network features of the interbank market are important in determining access, profitability and liquidity. It has been widely acknowledged that the markets are not complete networks; many banks would establish relationships with other banks either through repeated transactions or through commitments to future lending. While Banks in crisis will call around to look for additional liquidity, the establish lending relationships are a primary source of funding. As we move into the model, we will address this in two ways.

With our static model, we will effectively assess the role of existing network relationship - those that are identified by prior lending activity. Our dynamic model will then allow us to assess the role of link formation on liquidity provision.

## 3 Static Model

### 3.1 Notation and model

We begin with a population of banks. We define for this population a *network* $g \in \mathcal{G}$ as a set of ex-ante identical banks $N = \{1, \ldots, n\}$ and a set of *links* between them. We assume at all times that there are least two banks, $n \geq 2$. The set of bank $i$'s direct links is: $N_i(g) = \{j \neq i \mid g_{ij} = 1\}$. Links in this context can be defined in a variety of ways. In prior work, they have represented the exchange of a futures contract (Cohen-Cole, Kirilenko and Patacchini, 2010). In the banking networks that we study, the links will represent the presence of a interbank loan.

The cardinality of this set is denoted by $n_i(g) = |N_i(g)|$.[10] The $n-$square adjacency matrix $\mathbf{G}$ of a network $g$ keeps track of the direct connections in this network. By definition, banks $i$ and $j$ are directly connected in $g$ if and only if $g_{ij} = 1$, (denoted by $ij$), and $g_{ij} = 0$ otherwise. Links are taken to be reciprocal, so that $g_{ij} = g_{ji}$ (*undirected* graphs/networks). By convention, $g_{ii} = 0$. Thus $\mathbf{G}$ is a *symmetric* $(0, 1)-$matrix. All our theoretical results hold with directed and weighted networks.

We hypothesize that these direct links produce some type of reduction in costs of the collaborating banks. For example, as the size of the loan increases, the cost per dollar of loan is reduced for both parties to the loan. It is a straightforward assumption that the operational costs of a trading floor or treasury operation decline per dollar of loan as loan size increases.

We will model the quantity choice based on competition in quantities of lending a la Cournot between $n$ banks with a single homogenous product (a loan). We will then look at quantities of borrowing on the same market.

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[10] Vectors and matrices will be denoted in bold and scalars in normal text.

We assume the following standard linear inverse market demand where the market price is given by:

$$p = \theta - \sum_{j \in N} q_j \tag{1}$$

where $\theta > 0$. The marginal cost of each bank $i \in N$ is: $c_i(g)$. The profit function of each bank $i$ in a network $g$ is therefore given by:

$$
\begin{aligned}
\pi_i(g) &= pq_i - c_i(g)q_i \\
&= \theta q_i - \sum_{j \in N} q_i q_j - c_i(g)q_i
\end{aligned}
$$

where $q_i$ is the loan quantity produced by bank $i$. We assume throughout that $\theta$ is large enough so that price and quantities are always strictly positive.

Our specification of inter-related cost functions is as follows. The cost function is assumed to be equal to:

$$c_i(g) = \phi_0 - \phi \left[ \sum_{j=1}^{n} g_{ij} q_j \right] \tag{2}$$

where $\phi_0 > 0$ represents a bank's marginal cost when it has no links while $\phi > 0$ is the cost reduction induced by each link formed by a bank. The parameter $\phi$ could be bank specific as well so that $\phi_i$, but for simplicity of notation, we do not report this case.

Equation (2) means that the marginal cost of each bank $i$ is a decreasing function of the quantities produced by all banks $j \neq i$ that have a direct link with bank $i$. This is the specification that drives the functional relationships between banks.

To ensure that we obtain a reasonable solution, we assume that $\phi_0$ is large enough so that $c_i(g) \geq 0$, $\forall i \in N$, $\forall g \in \mathcal{G}$. The profit function of each bank $i$ can thus be written as:

$$
\begin{aligned}
\pi_i(g) &= pq_i - c_i(g)q_i \\
&= \theta q_i - \sum_{j \in N} q_i q_j - \phi_0 q_i + \phi \sum_{j=1}^{n} g_{ij} q_i q_j \\
&= a q_i - q_i^2 - \sum_{j \neq i} q_i q_j + \phi \sum_{j=1}^{n} g_{ij} q_i q_j \tag{3}
\end{aligned}
$$

where $a \equiv \theta - \phi_0 > 0$.

We highlight a few features of equation (3). First, we can see that profits are a negative function of total loans. This we call *global strategic substitutability,* as the effect operates only through the market and not through the direct links that form the network. So as $q_j$ increases, $\frac{\partial \pi_i(g)}{\partial q_i}$ is reduced as demand falls.

Second, we can see that profit is increasing in the quantity of direct links, via the cost function impact. This we refer to as *local strategic complementarities* since if $j$ is linked with $i$, then if $q_j$ increases $\frac{\partial \pi_i(g)}{\partial q_i}$ is increased because of the reduction in the cost. Total profits are of course, dependent on the two jointly.

To our knowledge, the fact that we incorporate both local and global compoents is unique to the financial networks literature; by incorporating both the direct network influences as well as the system-wide effects, our model is particularly suited to the description of financial markets. These markets influenced both by prices (global) and as well by network impacts (local).

Third, we can define $\sigma_{ij}$ as the cross partial of profitability with respect to a bank's quantity change and another bank's quantity change. We have:

$$\sigma_{ij} = \frac{\partial^2 \pi_i(g)}{\partial q_i \partial q_j} = \begin{cases} \overline{\sigma} = -1 + \phi \sum_{j=1}^{n} g_{ij} q_j & \text{if} \quad g_{ij} = 1 \\ \underline{\sigma} = -1 & \text{if} \quad g_{ij} = 0 \end{cases} \tag{4}$$

so that $\sigma_{ij} \in \{\underline{\sigma}, \overline{\sigma}\}$, for all $i \neq j$ with $\underline{\sigma} \leq 0$.

Notice that the model here generates systemic risk insofar as shocks that impact a given bank, such as an exogenous decrease in capital and ability to lend, pass through to the rest of the market through a competition mechanism. The global effect of the reduction in lending by a single bank is an increase by others. The local effect, however, that passes through the network linkages, is that costs increase. As a result, loans volumes of direct network links decline as well. Once network links change their choices, their links do so as well, and so on.

The next section discuss the Nash equilibrium of this model.

## 3.2 Nash equilibrium

Consider a Cournot game in which banks chose a volume of interbank lending conditional on the actions of other banks. We expand the standard game to fit the model above. Agents have the defined profit function in (3), which implies that cost is intermediated by the network structure.

It is easily checked that the first-order condition is:

$$q_i^* = \frac{1}{2} a - \frac{1}{2} \sum_{j \neq i} q_j + \frac{1}{2} \phi \sum_{j=1}^{n} g_{ij} q_j \tag{5}$$

Formally, we show below that this game has a unique Nash equilibrium.

Let us first define a network centrality measure due to Katz (1953), and latter extended by Bonacich (1987), that proves useful to describe the equilibrium of our network model.

**The Katz-Bonacich network centrality**  The Bonacich centrality will provide a measure of direct and indirect links in the network. Effectively, a relationship between two banks is not made in isolation. If bank A lends money to bank B, and bank B already lends to bank C, the strategic decisions of bank A will depend, in part on the strategic decisions of B. Of course, B's decisions will also be a function of C's. The Bonacich measure will help keep track of these connections and, as we will see in the subsequent section, has a natural interpretation in the Nash solution.

Let $\mathbf{G}^k$ be the $k$th power of $\mathbf{G}$, with coefficients $g_{ij}^{[k]}$, where $k$ is some integer. The matrix $\mathbf{G}^k$ keeps track of the indirect connections in the network: $g_{ij}^{[k]} \geq 0$ measures the number of paths of length $k \geq 1$ in $g$ from $i$ to $j$.[11] In particular, $\mathbf{G}^0 = \mathbf{I}$.

Given a scalar $\phi \geq 0$ and a network $\mathbf{g}$, we define the following matrix:

$$\mathbf{M}(g, \phi) = [\mathbf{I} - \phi \mathbf{G}]^{-1} = \sum_{k=0}^{+\infty} \phi^k \mathbf{G}^k$$

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[11] A *path* lof length $k$ from $i$ to $j$ is a sequence $\langle i_0, ..., i_k \rangle$ of players such that $i_0 = i$, $i_k = j$, $i_p \neq i_{p+1}$, and $g_{i_p i_{p+1}} > 0$, for all $0 \leq k \leq k-1$, that is, players $i_p$ and $i_{p+1}$ are directly linked in $\mathbf{g}$. In fact, $g_{ij}^{[k]}$ accounts for the total weight of all paths of length $k$ from $i$ to $j$. When the network is un-weighted, that is, $\mathbf{G}$ is a $(0,1)-$matrix, $g_{ij}^{[k]}$ is simply the number of paths of length $k$ from $i$ to $j$.

where $\mathbf{I}$ is the identity matrix. These expressions are all well-defined for low enough values of $\phi$. It turns out that an exact strict upper bound for the scalar $\theta$ is given by the inverse of the largest eigenvalue of $\mathbf{G}$ (Debreu and Herstein, 1953).[12] The parameter $\phi$ is a decay factor that scales down the relative weight of longer paths. If $\mathbf{M}(g,\phi)$ is a non-negative matrix, its coefficients $m_{ij}(g,\phi) = \sum_{k=0}^{+\infty} \phi^k g_{ij}^{[k]}$ count the number of paths in $g$ starting from $i$ and ending at $j$, where paths of length $k$ are weighted by $\phi^k$. Observe that since $\mathbf{G}$ is symmetric then $\mathbf{M}$ is also symmetric.

**Definition 1** *Consider a network g with adjacency $n-$square matrix $\mathbf{G}$ and a scalar $\phi$ such that $\mathbf{M}(g,\phi) = [\mathbf{I}-\phi\mathbf{G}]^{-1}$ is well-defined and non-negative. Let $\mathbf{1}$ be the $n-$dimensional vector of ones. Then, the Katz-Bonacich centrality of parameter $\phi$ in g is defined as:*

$$\mathbf{b}(g,\phi) = \sum_{k=0}^{+\infty} \phi^k \mathbf{G}^k \mathbf{1} = [\mathbf{I}-\phi\mathbf{G}]^{-1}\mathbf{1} \tag{6}$$

An element $i$ of the vector $\mathbf{b}(g,\phi)$ is denoted by $b_i(g,\phi)$. For all $\mathbf{b}(g,\phi) \in \mathbb{R}^n$, $b(g,\phi) = b_1(g,\phi) + ... + b_n(g,\phi)$ is the sum of its coordinates. Observe that, by definition, the Katz-Bonacich centrality of a given node is zero when the network is empty. It is also null when $\phi = 0$, and is increasing and convex with $\phi$.

**Equilibrium behavior**

We now characterize the Nash equilibrium of the game. Denote by $\omega(\mathbf{G})$ the largest eigenvalue of $\mathbf{G}$.

**Proposition 1** *Consider a game where the profit function of each bank i is given by (3). Then this game has a unique Nash equilibrium in pure strategies if and only if $\phi\omega(\mathbf{G}) < 1$. This equilibrium $\mathbf{q}^*$ is interior and given by:*

$$\mathbf{q}^* = \frac{a}{1 + b(g,\phi)}\mathbf{b}(g,\phi) \tag{7}$$

This result is a direct application of Theorem 1 in Ballester, Calvo-Armengol, and Zenou (2006). Appendix 1 shows in more detail how the first order condition can be written as a function of Katz-Bonacich centrality. It also provides an example.

This solution is useful for a couple of reasons: One, notice that this equation provides a closed form solution to the Cournot problem with any number of banks. No matrix math is needed to calculate output, only the matrix of interconnections and the bank specific cost functions. Two, this equation provides the basis for estimation of any network linked bank decision. We explore this implication below in more detail.

We can now calculate the equilibrium profit of each bank by replacing the equilibrium value of $q_i^*$ into the profit function (3). It is easily verified that we obtain:

$$\pi_i^* = (q_i^*)^2 = \frac{a^2 b_i^2(g,\phi)}{[1 + b(g,\phi)]^2} \tag{8}$$

so that the profit function of each bank is an *increasing* function of the Bonacich centrality of each bank.

# 4 Dynamic Model

In this section, we extend the model of Section 3 to include strategic link formation amongst banks. This step is crucial in that it permits us to include in our analysis not only the quantity and price choices amongst

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[12] In a row-standardized matrix, the largest eigenvalue is 1. We will work with row-standardized $\mathbf{G}$ matrices in our empirical analysis.

banks conditional on their existing network, but also their decisions on how to change the network structure itself. The model here will show the equilibrium outcome network structure conditional on these strategic choices.

Such a model gives us the ability to validate that our static model results are reasonable insofar as they are not contradicted by strategic network formation incentives. It also allows us to investigate how strategic behavior can impact network structure and liquidity availability.

Our central modeling assumptions will be that links are formed based on the profitability tradeoff that emerges from the game in the static model. Effectively, banks know that the game will be played in the subsequent period and that all other banks are also making network formation decisions. Based on these, banks can choose whether or not to form a link; that is, to make a loan to a new customer.

We will also specify an exogenous probability of link formation, $\alpha$.[13] After describing the model below, we will calibrate the network patterns in the interbank market using this single parameter.

Let us describe the network formation process. We here follow König, Tessone and Zenou (2010a). Let time be measured at countable dates $t = 1, 2, ...$ and consider the network formation process $(G(t))_{t=0}^{\infty}$ with $G(t) = (N, L(t))$ comprising the set of banks $N = \{1, ..., n\}$ together with the set of links (i.e. loans) $L(t)$ at time $t$. We assume that initially, at time $t = 1$, the network is empty. Then every bank $i \in N$ optimally chooses its quantity $q_i \in \mathbb{R}_+$ as in the standard Cournot game with no network. Then, a bank $i \in N$ is chosen at random and with probability $\alpha \in [0, 1]$ forms a link (i.e. loan) with bank $j$ that gives her the highest payoff . We obtain the network $G(1)$. Then every bank $i \in N$ optimally chooses its quantity $q_i \in \mathbb{R}_+$, and the solution is given by (7). The profit of each bank is then given by (8) and only depends on its Bonacich centrality, that is its position in the network. At time $t = 2$, again, a bank is chosen at random and with probability $\alpha$ decides with whom she wants to form a link while with probability $1 - \alpha$ this bank has to delete a link if she has already one. Because of (8), the chosen bank will form a link with the bank that has the highest Bonacich centrality in the network. And so forth.

As stated above, the randomly chosen bank does not create or delete a link randomly. On the contrary, it calculates all the possible network configurations and chooses to form (delete) a link with the bank that gives her the highest profit (reduces the least her profit). It turns out that connecting to the bank with the highest Bonacich centrality (deleting the link with the agent that has the lowest Bonacich centrality) is a best-response function for this bank. Indeed, at each period of time the Cournot game described in Section 3 is played and it rationalizes this behavior since the equilibrium profit is increasing in her Bonacich centrality (see 8).

To summarize, the dynamics of network formation is as follows: At time $t$, a bank $i$ is chosen at random. With probability $\alpha$ bank $i$ creates a link to the most central bank while with complementary probability $1 - \alpha$ bank $i$ removes a link to the least central bank in its neighborhood.

## 4.1 Characterization of equilibrium

We would like to analyze this game and, in particular, to determine the degree distribution of the network obtained in steady-state. The degree distribution gives the percentage of banks with degree $d = 1, ..., n$ (the degree is number of links each bank has).

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[13] While we don't discuss in detail, this asssumption can be relaxed in a number of ways. For example, König, Tesson and Zenou (2010b) show that a capacity constraint, what this model would interpret as a capital constraint, generates similar network patterns.

In König, Tessone and Zenou (2010a), it is shown that, at every period, the emerging network is a *nested split graph* or a *threshold network*, whose matrix representation is stepwise. This means that agents can be rearranged by their degree *rank* and agents with degree $d$ are connected to all agents with degrees larger than $d$. Moreover, if two agents $i, j$ have degrees such that $d_i < d_j$, this implies that their neighborhoods satisfy $\mathcal{N}_i \subset \mathcal{N}_j$. Below, we will show how closely the theoretical patterns implied by this model are replicated in the data.

Given the symmetry in the adjacency matrix $\mathbf{G}$, in order to solve the dynamic evolution of the network, it is enough to solve the dynamics for the banks with degree smaller or equal than $K/2$, when there are $K$ distinct degrees in the network. Denote by $N(d, t)$ the number of agents with degree $d \leq K/2$ at time $t$. Starting from an empty network, it can be shown that the dynamic evolution is given by:

$$N(d, t' + 1) - N(d, t') = \left(\frac{1 - \alpha}{n}\right) N(d + 1, t') + \frac{\alpha}{n} N(d - 1, t') - \frac{1}{n} N(d, t') \tag{9}$$

$$N(0, t' + 1) - N(0, t) = \left(\frac{1 - 2\alpha}{n}\right) - \frac{\alpha}{n} N(0, t) + \left(\frac{1 - \alpha}{n}\right) N(1, t) \tag{10}$$

These equations mean that the probability to add nodes to the class with degree $d$ is proportional to the number of nodes with degree $d - 1$ (resp. $d + 1$) when selected for node addition (deletion). The dynamics of the adjacency matrix (and from this the complete structure of the network) can be directly recovered from the solution of these equations.

Since the complement $\bar{G}$ of a nested split graph $G$ is a nested split graph, we can derive the stationary distribution of networks for any value of $1/2 < \alpha < 1$ if we know the corresponding distribution for $1 - \alpha$. With this symmetry in mind we restrict our analysis in the following to the case of $0 < \alpha \leq 1/2$. Let $\{N(t)\}_{t=0}^{\infty}$ be the degree distribution with the $d$-th element $N_d(t)$, giving the number of nodes with degree $d$ in $G(t)$, in the $t$-th sequence $N(t) = \{N_d(t)\}_{d=0}^{n-1}$. Further, let $n_d(t) = N_d(t)/n$ denote the proportion of nodes with degree $d$ and let $n_d = \lim_{t \to \infty} \mathbb{E}(n_d(t))$ be its asymptotic expected value (as given by $\mu$). In the following proposition (König, Tessone and Zenou, 2010a), we determine the asymptotic degree distribution of the nodes in the independent sets for $n$ large enough.

**Proposition 2** *Let $0 < \alpha \leq 1/2$. Then the asymptotic expected proportion $n_d$ of nodes in the independent sets with degrees, $d = 0, 1, ..., d^*$, for large $n$ is given by*

$$n_d = \frac{1 - 2\alpha}{1 - \alpha} \left(\frac{\alpha}{1 - \alpha}\right)^d, \tag{11}$$

*where*

$$d^*(n, \alpha) = \frac{\ln\left(\frac{(1 - 2\alpha)n}{2(1 - \alpha)}\right)}{\ln\left(\frac{1 - \alpha}{\alpha}\right)}. \tag{12}$$

The structure of nested split graphs implies that if there exist nodes for all degrees between $0$ and $d^*$ (in the independent sets), then the dominating subsets with degrees larger than $d^*$ contain only a single node.

**Corollary 1** *There exists a phase transition in the asymptotic average number of independent sets, $d^*(n, \alpha)$, as $n$ becomes large such that*

$$\lim_{n \to \infty} \frac{d^*(n, \alpha)}{n} = \begin{cases} 0, & \text{if } \alpha < \frac{1}{2}, \\ \frac{1}{2}, & \text{if } \alpha = \frac{1}{2}, \\ 1, & \text{if } \alpha > \frac{1}{2}. \end{cases} \tag{13}$$

Corollary 1 implies that as $n$ grows without bound the networks in the stationary distribution $\boldsymbol{\mu}$ are either sparse or dense, depending on the value of the link creation probability $\alpha$. Moreover, from the functional form of $d(n, \alpha)$ in Equation (12) we find that there exists a sharp transition from sparse to dense networks as $\alpha$ crosses $1/2$ and the transition becomes sharper the larger is $n$.

## 4.2 Dynamic Model and Systemic Risk

Above, we permit banks, at each point in time, to calculate all possible network configurations and choose the best possible partner to match with. This choice is the one that maximizes profitability. We found that connecting to highly central counterparty is the optimal action for this bank.

This characterization of network formation is critical to understanding systemic risk in this context. We define systemic risk as the propagation of shocks through the network of connected banks (with or without actual default). In each stage game, banks internalize the shocks that hit their neighbor institutions, and re-optimize their lending decisions. This simultaneous re-optimization is what generates systemic risk. Here, we include in this re-optimization, the decision by banks to sever links or form new ones. And we find that the optimal link formation decision is a function of the centrality of other players; that is, because of agents know that systemic risk exists and that their profits are a function of the connections to other banks, they choose their partners very carefully.

## 5 Empirical analysis

With the models complete, we will proceed to illustrate their empirical salience in a few steps. Working sequentially, the first step is to show that the dynamic model well approximates the network structure of interbank markets. With this result, we use the dynamic model to help define each static 'network' for estimation. Notice that without a dynamic model, network definition can be particularly difficult in this context. Even assuming *a priori* that the structure of the network matters, it is not obvious which network of trades should be relevant. Trades occur effectively continuously, leading to the problem that any definition of a network must necessarily make an assumption about the beginning and end of time for that network. The choice of the period of time within which a network is defined is important as it contains valuable information on the resulting network structure. Indeed, with more time, more transactions are formed and the resulting interbank network can get denser. We will use our theoretical model of network formation to point us towards the choice of the appropriate time window. Once we have a network definition, we can parse the data in a sequence of networks. For each of these, we can estimate systemic risk in liquidity provision, i.e. we can test our static model. Finally, we complement the (quantity) liquidity risk measures with a similar estimation based on prices. We show that there was very high systemic risk in interbank loan prices and some risk in loan quantities. Let us begin our empirical analysis by describing our data in detail.

## 5.1 Data

Our dataset is the transaction level data from an electronic interbank market. The e-MID SPA (or e-MID) was the reference marketplace for liquidity trading in the Euro area during the time period studied. It was the first electronic marketplace for interbank deposits (loans), a market that has traditionally been conducted bilaterally. Our data includes every interbank loan transaction conducted on the e-MID during

the time period from January 2002 to December 2009. During this time period, transactions on this exchange represented about 20 percent of the Euro area market. As such, during this time period, it served as a good representation of general market activity.

The e-MID market is an open access one. All banks in the European interbank market can participate. The market opens at 8am and closes at 6pm Central European Time. Both bids and asks can be posted on the exchange along with a price and quantity. Each trader may decide to initiate a transaction with any of the counterparties present on the book. Once a trader chooses 'hits' the transaction, the two parties bilaterally negotiate the trade. The benefit of the bilateral negotiation is that it allows each party the ability to refuse the transaction, change the quantity and/or the price. Such bilateral negotiation allows banks to maintain lending limits for each specific counterparty.

Outside of e-MID banks privately negotiate lines of credit (or 'liquidity guarantees') with each other and conduct regular transactions with each other based on these lines. As a result, e-MID can support the continuation of the bilateral lending arrangements without forcing banks to accept / give loans outside their prior guidelines.

Our data includes approximately 250 institutions that participated in at least one transaction during the time period. Loans in the database range from overnight to two years in length, though about 70% of the loans are for overnight alone. Our information spans 945,566 loans of all types, of which 752,901 are overnight loans. We will focus on the overnight loans for simplicity, but will include longer loans as a robustness check in section 6, below.

The principal purpose of the interbank market is provide a mechanism for banks to re-allocate deposit imbalances. For larger shocks or gross liquidity needs, most institutions borrow directly from the ECB.

Our network maps is constructed as follows. Each loan marks a link between a lender and borrower. Our approach will be to define the network as a given number of transactions. As it will be explained below, we create a matrix of interconnections based on each consecutive 1000 loans. With this single network map, we will then estimate lending and borrowing separately. As our model discusses Cournot-style competition in quantities, we isolate two distinct 'markets'. The first is a producers market of lending and the second a borrowers market. In each, competition exists on quantities.

Table 1 reports descriptive statistics for the e-MID market. We report the average daily volume for overnight and total lending. As well, we include the proportion of lending made by the 25 largest market participants, which averages about 20% prior to the crisis and 5-10% after. In addition to total volumes dropping, the market shifts from being highly centralized to considerably less so after the crisis. This finding is consistent with our estimation of the role of centrality over time; we find below that importance of being central declines after the crisis.

[Insert Table 1 here]

We also show in Figure 1, the daily volume of lending for overnight and long-term loans. This shows the stylized patterns of lending in this market. Notice that lending volume of long-term loans drops precipitously beginning with the onset of the crisis in August of 2007. Both overnight and long-term loans decline following the beginning of the ECB full allotment policy in October of 2008.

In Figure 2 we report the daily price volatility, as the standard deviation of prices. The reverse pattern is observed here. After a long period with relatively low volatility dating back to the beginning of 2002, the onset of the crisis saw price volatility increase dramatically. Two significant increases are apparent, in August of 2007 and in October of 2008.

Each of these two figures shows the daily value as well as a two-month moving average.

[*Insert Figures* 1 *and* 2 *here*]

## 5.2   Network formation

Our theoretical model of Section 4 provides a set of predictions for network structure that depend on a single parameter. This result emerges because agents can adopt a very simple link formation rule that depends only on the Bonacich centrality of the instantaneous network structure. Because profits are greater when one links with those with higher Bonacich centrality, link formation patterns can be described in a parsimonious way.

That link formation is a function of profitability is crucial to understanding systemic risk in the context of this type of model. The incentive process produces predictable network patterns, as we discussed above. Thus, as a result, the model provides additional ability to understand and explain systemic risk, over and above what would be feasible with reduced form approaches.

The key parameter for determining network structure, is the probability of creating a link, $\alpha$. This parameter is exogenous to the dynamic model, and provides us with a way to determine the efficacy of the dynamic model is describing the observed pattern. Notice that Proposition 2 describes the precise relations between the $\alpha$ and the degree distribution on the entire network. Recall that the each agent has a degree, which is the count of number of links to other agents. Recall as well that the degree distribution is simply the distribution over agents in a particular network of their respective degrees. So, the model generates a prediction for the degree distribution that is a precise function of $\alpha$.

To establish the accuracy of the model, we begin by calculating an empirical version of $\alpha$ based on some arbitrary network definition. Effectively, we will 'guess' the right network density and then test whether the $\alpha$ and degree distribution that we observe in this network corresponds to what the dynamic model would produce.

Empirically the probability of creating a link ($\alpha$) can be estimated by considering the ratio between the number of actual links and the possible ones (in a network of given size). Different network definitions, which imply under our approach a different level of network density, are necessarily associated with different estimates of $\alpha$. We experiment by considering a network as a sequence of 250 up to 2000 consecutive transactions. This corresponds to approximately 1/2 day of trading to 5 days of trading. For each network type we then estimate $\alpha$ and compare the observed degree distribution with the theoretical one that emerges from calibrating the model at that given value of $\alpha$. Such a comparison between empirical and theoretical degree distributions will allow as to identify the relevant network definition which is consistent with our behavioral model. Figure 3 shows the theoretical degree distributions that are obtained when calibrating the model for different values of $\alpha$.

[*Insert Figure* 3 *here*]

We find that the network definition in line with our model of network formation is the one based on approximately 1000 transactions. Figure 4 shows the estimated values of $\alpha$ under this network definition over the observed period. So, for instance, the graphs shows that in networks with a total number of 1000 transactions at the beginning of the period (January 2002) the estimated link creation is about 0.4, whereas at the end of the period (December 2009) it has fallen to about 0.25. The fall reflects that the network

has become more sparse once the crisis took hold. This occurred both because the ECB began to provided unlimited access to funds and because market participants may have become less willing to lend.

[*Insert Figure 4 here*]

Figure 5 then compares the degree distribution predicted by our model against the data in three ways. Figure 5a and 5b shows the full degree distribution for $\alpha$ at two points in time. Figure 5a shows $\alpha = 0.4$, corresponding to January 2002 and Figure 5b shows $\alpha = 0.25$, corresponding to December 2009. The important feature of these two figures is that the dynamic model captures the change in degree distribution that occurs as a result of the crisis. As the networks become more sparse ($\alpha$ declining), the incentives to form new links change. We can see in these two figures that the change network formation behavior observed in these markets is closely matched by the model's predictions.

[*Insert Figures 5a and 5b here*]

Figure 5c shows that this close alignment between model and data occurs over the entire time period. We plot the 0 degree component of the degree distribution for 250 networks over the 8 year time period. A perfect prediction would yield a 45 degree line; here we find a slight divergence, but a very consistent ability of the model to predict the network structure in the data.

[*Insert Figure 5c here*]

That the model produces distributions that are empirically so close to the data supports the ability of the static model to generates estimates of systemic risk that are plausible. In particular, it allows us to claim that the results are robust to selection issues, the incentive of agents to change partners. We proceed in the next section to estimate the systemic risk spillover for our networks.

## 5.3 Empirical model and estimation issues

The estimates in this section will be based both on the static Nash equilibrium quantity choice as well as the empirical network structure at the time in question. In the prior section, we validated that these network patterns are generally consistent with the invariant network structure that emerges using the static model as a stage game.

Assume that there are $K$ networks in the economy, each of them connecting $n_\kappa$ agents, $\sum_{\kappa=1}^{K} n_\kappa = N$. The empirical counterpart of equation (5) is the following model:

$$q_{i,\kappa} = c + \phi \frac{1}{g_{i,k}} \sum_{j=1}^{n_\kappa} g_{ij,\kappa} q_{j,\kappa} + \upsilon_{i,\kappa}, \qquad \text{for } i = 1, ..., n_\kappa \; ; \; \kappa = 1, ..., K. \tag{14}$$

where $c = \frac{1}{2}a - \frac{1}{2}\sum_{j=1,j\neq i}^{n} q_j$. This equation indicates that the equilibrium quantity choice of a bank is a function of quantity choices of others in the same market. We denote as $q_{i,\kappa}$ the output of bank $i$ in the network $k$, $g_{i,k} = \sum_{j=1}^{n_\kappa} g_{ij,\kappa}$ is the number of direct links of $i$, $\frac{1}{g_{i,k}} \sum_{j=1}^{n_\kappa} g_{ij,\kappa} q_{j,\kappa}$ is a spatial lag term and $\upsilon_{i,k}$ is a random error term. This model is the so-called *spatial lag model* in the spatial econometric literature (see, e.g. Anselin 1988).[14] Because of the typical simultaneity problem in dealing with spatial

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[14]In the empirical model, we work with a row-standardized adjacency matrix, i.e. if we normalize the spatial lag term by $g_{i,\kappa} = \sum_{j=1}^{n_\kappa} g_{ij,\kappa}$, the number of direct links of $i$. Because a row-standardized matrix implies that the largest eigenvalue is 1, we present the analysis using this approach to ease the interpretation of the results. Indeed, by providing a common upper bond for $\phi$, it allows a comparison of the importance of systemic risk in different network structures, i.e. for different **G** matrices.

lag models,[15] which yields inconsistent OLS estimators, it is estimated using Maximum Likelihood (see, e.g. Anselin, 1988).

The architecture of networks allows us to get an estimate of $\phi$, while eluding the so-called "reflection problem." As many starting with Manski (1993) have observed, we cannot identify $\phi$ if the network is complete.[16] This arises from the fact that if we calculate the expected mean outcome of the group, then the expectation of $q$ will appear on both sides of the equation. This generates a particular type of endogeneity problem. That is, the expected mean outcome is perfectly collinear with the mean background of the group: how can we distinguish between bank $i's$ impact on $j$ and $j's$ impact on $i$? Effectively, we need to find an instrument: a variable that is correlated with the behavior of $i$ but not of $j$. Bramoullé, Djebbari and Fortin (2009) noted that in incomplete networks, one observes 'intransitivities.' These are connections that lead from $i$ to $j$ then to $k$, but not from $k$ to $j$. Thus, we can use the partial correlation in behavior between $i$ and $j$ as an instrument for the influence of $j$ on $k$. That is, network effects are identified if we can find two banks in the economy that differ in the average connectivity of their direct contacts. A formal proof is in Bramoullé, Djebbari and Fortin (2009). Of course, a complex trading network such as the one we are concerned with has a very rich structure and identification essentially never fails. Thus, using the architecture of networks we can obtain estimates of the relevant structural parameters

## 5.4 Estimation results

The estimation results of model (14) for each network of 1000 transactions (our preferred network definition) are contained in Table 2. We report the results for each year between 2002 and 2009.

[*Insert Table* 2 *here*]

Panel A shows the results from lending networks. Panel B shows the results from borrowing networks.

**Systemic risk**

The first row of each panel shows the estimates of our target parameter $\phi$, i.e. a measure of systemic risk. As highlighted in the theoretical model, the parameter $\phi$ captures the strength of network interactions that stems from the network architecture. That is, it measures the systemic risk in the network: the impact that a change in a single bank's liquidity has on his trading partners' liquidity.

Our findings reveal a sizeable effect. Indeed, we find that a single dollar increase in quantity leads to 2-2.5 additional dollars of liquidity by the traders' neighbors. This ratio (2 to 1 or 2.5 to 1), which measures the ratio of total network outcome over an individual level shock, is often referred to as the social multiplier; in this context, it is a measure of the average systemic risk multiplier. It is reported in the second row of each panel. To see how this number emerges, one can calculate the systemic risk multiplier as follows. In a complete network (one in which every agent is connected to every other), it can be calculated as $\psi = 1/(1-\phi)$. Since $\phi$ in our results ranges from approximately .50-.60, the average impact, $\psi$, for all agents in the network

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[15] This stems from the fact that the spatial lag term contains the dependent variable for neighboring observations, which in turn contains the spatial lag for their neighbors, and so on, leading to a nonzero correlation between the spatial lag and the error terms.

[16] A number of solutions exist to this reflection problem. Brock and Durlauf (2001) use a structural approach to identification, Glaeser, Sacerdote and Scheinkman (1996) use the variance of group average outcomes, and Cohen-Cole (2006) uses the presence of multiple group influence. Each of the latter two were later formalized further by Graham (2008) and Bramoullé, Djebbari and Fortin (2009), respectively. Topa (2001) and Conley and Topa (2002) use spatial clustering techniques to identify spillovers. A complete network is one in which every node is directly connected to every other node.

ranges from approximately 2 to 2.5. Because our market is not complete, risk is propagated through the network via realized loans. In such a context, the impact of a given bank must pass through a limited number of other agents, as described by the transaction pattern. As the effect dissipates in each successive link, the impact on directly connected agents is necessarily greater (see equation 6). One can see then that for directly connected agents, the impact of systemic risk is much larger; being 'close' to an impacted party leads to a greater risk of impact.[17] Thus, while the average impact is 2-2.5 times in the initial shock, the maximum shock a bank may face wil be many times larger.[18]

The positive and statistically significant estimates of $\psi$ point towards the existence of a cross-sectional dependence in quantities that is not explicitly mediated by the market. Looking at the $R-$squared values, one can see that such network effects explain around 5% to 10% of the variation of individual bank lending and borrowing respectively. These results are roughly consistent over time. Network structure thus appears as an important mechanism that determines a bank's liquidity.

**Systemic risk and network structure**

To understand better the link between network topology and returns, Table 2 also reports in the last row the impact of Bonacich centrality. We use our estimated $\phi$ for each network to calculate the Bonacich centrality for each bank in our networks (equation 6). This calculation will generate a distribution of individual centralities depending on the strength of network interactions and on the heterogeneity of network links (as captured by the estimate of $\phi$ and the matrix $\mathbf{G}$ in formula (6), respectively). Our results shows that the influence of network structure varies greatly over time. So, for example, a unit increase in Bonacich centrality, i.e. a better bank positioning in the network, raises lending about 17 million in 2002 and 3 million in 2009. The one-unit impact of a change in individual centrality moves significantly over time, as does the variance of individual centrality for each network. Figure 6 plots these patterns for the lending market.

[*Insert Figure* 6 *here*]

Even as we continue to capture the relevance of network structure, its role is not constant. Why? As we approach crisis in 2006, the market was very, very liquid, but the central lenders became increasingly important. The importance of centrality became very large; moving from the periphery to the center meant very large changes in liquidity provision. These changes were 4/5 times larger than in 2002/2003. With the onset of the crisis the role of the central lenders declined.

The borrowing market looked a bit different. Here the importance of central players and the variance of centrality both declined secularly over time, with a slight up-tick during the crisis. We interpret this as the converse of the lending market. As lenders became more centralized, borrowers became more dispersed, with many relying a few key lenders. As the crisis hit and lenders dispersed, we begin to see some additional concentration on the borrowing side.

Thus we view our results as illustrating that the model can explain the role of network structure consistently over time, even as the market changes along many dimensions. By being able to do so, the method has great value. In particular, one can think of a couple of dimensions of systemic risk. The first, the

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[17]One can think of this as how banks' liquidity is impacted by the systemic risk in the network. A bank will gain (lose) when the banks linked to her gain (lose). Below in this section, we look at the extent to which banks' liquidity is affected by changes in banks' centrality, which is a trasformation of the systemic risk parameter (equation 6). That is, does it help to change position in the network?

[18]Figure 6, above, shows the variance of centrality figures for networks in each of the 8 yeras in our data. As variance increases, the difference between the maximum and minimum shock rises as well.

average impact on the network of a shock, is captured in our systemic risk estimates. The second reflects the distributional impacts of a shock. As the variance of Bonacich centrality changes, it reflects changes in how shocks are absorbed by the market. A high variance suggests that a concentrated group of banks will absorb the total effect of the shock. One type of limit here is the contagious default model - successive agents bear the full cost of the default. A zero variance indicates that the shocks will be equally distributed across all agents in the network.

Taken as a whole, our evidence indicates that network centrality is a relevant (and so far unnoticed) factor that plays a role in explaining the link between the cross-sectional variation of lending and systemic risk.

**Robustness check**

Our primary results look at the overnight lending market only. We check here the robustness of our results to the inclusion of all forms of lending, including longer term loans. These markets act as partial substitutes and we report in Table 3 results from the full market. Table 3 has the same structure of Table 2. For our analysis of lending networks, our results are nearly identical across the two sets of results, suggesting that network influences are important and similar in both contexts. For borrowing networks, when we consider all forms of lending we find results that appears about 1/3 smaller in magnitude, and not significant in most cases. Our interpretation of this result is that borrowers of long term funding are more willing to deviate from prior network affiliations to obtain funding. Because their structural needs for funding may be stronger, and their elasticity of demand much lower, the network potentially plays a less important role.

[*Insert Table* 3 *here*]

## 5.5   A closer look at prices

The core of our model is a Cournot name in quantities that is intermediated by the network structure of lending relationships. Underlying this model is a price quantity relationship specified in equation 1. We diverge from this structure here momentarily to evaluate a reduced form relationship related to prices alone. That is, we look directly at the empirical links in returns that exist in the data. The benefit to doing so is that we will be able to provide some intuition in addition to that which emerged from the model above.

Effectively, we estimate the counterpart to (14) in returns.

$$r_{i,\kappa} = c' + \phi' \frac{1}{g_{i.,k}} \sum_{j=1}^{n_\kappa} g_{ij,\kappa} r_{j,\kappa} + e_{i,\kappa}, \qquad \text{for } i = 1, ..., n_\kappa \ ; \ \kappa = 1, ..., K. \tag{15}$$

where $r_{i,\kappa}$ is the idiosyncratic return of bank $i$ in the network $k$, $g_{i.,k} = \sum_{j=1}^{n_\kappa} g_{ij,\kappa}$ is the number of direct links of $i$, $\frac{1}{g_{i.,k}} \sum_{j=1}^{n_\kappa} g_{ij,\kappa} r_{j,\kappa}$ is the spatial lag term and $e_{i,k}$ is a random error term.

Specifically, we will estimate the systemic risk in returns, as measured by $\phi'$. In each period, we measure lending returns as the log change in prices of loans over the time period of the loan (one day), net of the average market price during that time. Thus a positive return is obtained by issuing a loan above the prevailing market price at the time of the loan.

We do not have a way to measure risk adjusted returns in this market as we do not have default probabilities for borrowers. While we expect that this would impact the level of returns across agents, we do not believe that this would impact estimation of systemic risk in prices. That is if prices reflect underlying risk,

we would observe banks that lend to risky banks earning higher returns, but unless these same banks trade disproportionately with each other, this will not appear in the estimation of systemic risk.

Table 4 has the same structure of Table 2 and shows the estimation results of a spatial lag model in prices.

[*Insert Table* 4 *here*]

Notice that spillovers are much stronger here and remain so until the ECB implements the full allotment policy. One interpretation is that price elasticity is much higher in these contexts. Quantity needs were apparently less elastic due to business considerations. Once the full allotment policy is in place, price elasticity necessarily falls; as a result, the impact of the network on prices becomes much lower.

Figure 7 plots the systemic risk coefficient in returns over time. The drop around the period when the ECB implements the full allotment policy is apparent.

[*Insert Figure* 7 *here*]

It is tempting to draw conclusions based on this finding about the nature of the crisis itself. While the dramatically larger risk in prices than quantities lends support to the argument that markets were not completely impaired, due to data limitations we cannot consider the impact of ECB intervention.

We view that there are two possibilities. One, the ECB's policy of providing unlimited funding at a set price would lead to a reduction in price volatility and in principle, no network price impacts. Because banks would always have an outside option, there is little reason to suspect that existing networks of lending would impact the price of loans. Two, the interbank market remained an important substitute for ECB loans as collateral requirements at the ECB would keep some lending in the private sector.

With complete information on the interbank lending markets in the Euro area, one could answer this question more definitively.

# 6 Discussion

Our approach is designed to understand the role of network structure on interbank lending. As with any model and data, there are some limitations to the exercise. We point to a few here. First, while our data is exceptional in providing comprehensive coverage of the European interbank market during most of the time period studied, e-MID particular role in the market limits the capacity of the model to explain some shocks. Because e-MID is transparent, and European banks have access to the ECB for emergency borrowing purposes, e-MID evolved as a way to balance relatively small liquidity shocks. Larger structural shocks would be dangerous to post on a public platform and access to the ECB provided an alternate outlet. As such, our results should be seen as a way to analyze shocks and network impacts on the margin. That said, this should indicate that increases in systemic risk in this market would understate the level of risk in the market as a whole.

Second, in order to keep the model parsimonious, we abstract from some institutional and financial details of banking. For example, we do not consider the full balance sheet of banks in consider loan quantity; instead, we view that each bank has unlimited lending capacity and lends only conditional on its profit maximization problem. We believe that the profit function is a reasonable approximation of bank tradeoffs, though but construction we cannot address the relationship between de-leveraging and liquidity or default cascades and liquidity.

# 7 Conclusion

We have constructed two models of the interbank loan market, a static and dynamic one. To complement these, we have provided empirical evidence of the models' accuracy in describing the data. Then, using these models, we have presented a measure of systemic risk in this market, which is a precise measure of the aggregate liquidity cost of a reduction in lending by an individual financial institution. This systemic risk measure is presented as an innovation vis-a-vis existing approaches. It is based on the foundation of a microfounded dynamic model of behavior. As well, it provides a tool to understand the transmission of shocks that extends beyond default events and generalized price shocks. In the combination of these lies our tool; the competitive responses that banks make generate the transmission of shocks in our model and provide a tractable method of measuring and understanding sytsemic risk.

There are a number of tangible benefits to the models and methods presented in this paper. The calculation of spillovers in interbank markets gives regulators an ability to gauge the market's sensitivity to shocks. We find multiples that grow as large as 2.5 times the initial reaction. When we calculate similar risk in prices, we find that a one-unit change in returns can propagate into the market, generating aggregate systemic risk multipliers that are approximately 30 in most years; that is, a change in returns of one Euro to one bank led to system-wide changes of about 30 Euros. In isolation, this suggests that the risk in this market is principally in the cost of funding; in principle, loans can be obtained for a sufficiently high price.

We view the principal limitation of the paper as also its strongest feature. The model is sufficiently general to capture a wide range of network structure and network formation patterns. The cost to this generality is, in part, that we have abstracted from a number of institutional details. An subsequent area of inquiry will be the role of credit risk in the determination of network formation and network spillovers.

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## Appendix 1

**Nash Equilibrium and Katz-Bonacich centrality**

Let us show how the first order condition can be written as a function of Katz-Bonacich centrality. For each bank $i = 1, ..., n$, maximizing (3) leads to:

$$a - q_i^* - \sum_{j=1}^{j=n} q_j + \phi \sum_{j=1}^{n} g_{ij} q_j = 0$$

This is equivalent to:

$$q_i^* = a - \sum_{j=1}^{n} q_j^* + \phi \sum_{j=1}^{n} g_{ij} q_j^* \tag{16}$$

We can write this equation in matrix form to obtain:

$$\mathbf{q}^* = a\mathbf{1} - q^*\mathbf{1} + \phi \mathbf{G} \mathbf{q}^*$$

where $\sum_{j=1}^{j=n} q_j^* = q^*$. This equation is equivalent to:

$$[\mathbf{I} - \phi \mathbf{G}] \mathbf{q}^* = (a - q^*) \mathbf{1}$$

which is equivalent to:

$$\begin{aligned} \mathbf{q}^* &= (a - q^*) [\mathbf{I} - \phi \mathbf{G}]^{-1} \mathbf{1} \\ &= (a - q^*) \mathbf{b}(g, \phi) \end{aligned}$$

Observing that $q^* = \sum_{j=1}^{j=n} q_j^* = b(g, \phi)$, we obtain:

$$\mathbf{q}^* = [a - b(g, \phi)] \mathbf{b}(g, \phi)$$

Simple algebra leads to:

$$\mathbf{q}^* = \frac{a}{1 + b(g, \phi)} \mathbf{b}(g, \phi) \tag{17}$$

which is (7).

**Example 1**

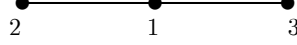Consider the network **g** in Figure 8 with three agents.



Figure 8. Three agents on a line.

The corresponding adjacency matrix is,

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

The $k$th powers of **G** are then, for $k \geq 1$:

$$\mathbf{G}^{2k} = \begin{bmatrix} 2^k & 0 & 0 \\ 0 & 2^{k-1} & 2^{k-1} \\ 0 & 2^{k-1} & 2^{k-1} \end{bmatrix} \quad \text{and} \quad \mathbf{G}^{2k+1} = \begin{bmatrix} 0 & 2^k & 2^k \\ 2^k & 0 & 0 \\ 2^k & 0 & 0 \end{bmatrix}.$$

For instance, we deduce from $\mathbf{G}^3$ that there are exactly two paths of length three between agents 1 and 2, which are $12 \rightarrow 21 \rightarrow 12$ and $12 \rightarrow 23 \rightarrow 32$.

When $\phi$ is small enough,[19]

$$\mathbf{M} = [\mathbf{I} - \phi\mathbf{G}]^{-1} = \frac{1}{1 - 2\phi^2} \begin{pmatrix} 1 & \phi & \phi \\ \phi & 1 - \phi^2 & \phi^2 \\ \phi & \phi^2 & 1 - \phi^2 \end{pmatrix}$$

and the vector of Katz-Bonacich network centralities is:

$$\mathbf{b}(g, \phi) = \begin{bmatrix} b_1(g, \phi) \\ b_2(g, \phi) \\ b_3(g, \phi) \end{bmatrix} = \frac{1}{1 - 2\phi^2} \begin{bmatrix} 1 + 2\phi \\ 1 + \phi \\ 1 + \phi \end{bmatrix}$$

Not surprisingly, the center (bank 1) is more central than the peripheral banks 2 and 3. The Nash equilibrium is then given by (using (17)):

$$\mathbf{q}^* = \begin{bmatrix} q_1^* \\ q_2^* \\ q_3^* \end{bmatrix} = \frac{a}{4(1 + \phi)(1 - 2\phi^2)} \begin{bmatrix} 1 + 2\phi \\ 1 + \phi \\ 1 + \phi \end{bmatrix}$$

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[19] Here, the largest eigenvalue of **G** is $\sqrt{2}$, and so the exact strict upper bound for $\phi$ is $1/\sqrt{2}$.

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# Table 1: Summary Statistics

| | *2002* | *2003* | *2004* | *2005* | *2006* | *2007* | *2008* | *2009* |
|---|---|---|---|---|---|---|---|---|
| **Overnight Lending Only** | | | | | | | | |
| Average Daily Volume (mm euros) | 10,046 | 9,666 | 10,458 | 9,577 | 8,912 | 7,615 | 6,028 | 3,669 |
| Daily standard dev of volume | 1,321 | 1,860 | 1,360 | 1,389 | 1,563 | 1,146 | 1,389 | 816 |
| Daily standard dev of prices | 0.12 | 0.35 | 0.09 | 0.09 | 0.38 | 0.23 | 0.53 | 0.46 |
| Number of Loans | 130,614 | 114,844 | 104,393 | 97,551 | 90,370 | 86,453 | 75,933 | 52,743 |
| largest 25 lenders | | | | | | | | |
| Total Lending (mm euros) | 1,634 | 1,913 | 2,162 | 2,033 | 2,109 | 1,326 | 346 | 181 |
| Fraction of total | 16% | 20% | 21% | 21% | 24% | 17% | 6% | 5% |
| Number of loans | 14,755 | 14,766 | 15,793 | 14,645 | 14,759 | 9,408 | 2,635 | 1,701 |
| *Panel B* | | | | | | | | |
| **All Lending** | | | | | | | | |
| Average Daily Volume (mm euros) | 17,892 | 18,369 | 21,258 | 22,412 | 24,745 | 22,835 | 13,731 | 5,516 |
| Daily standard dev of volume | 2,606 | 3,940 | 4,028 | 3,580 | 4,662 | 6,262 | 3,628 | 1,810 |
| Daily standard dev of prices | 0.11 | 0.34 | 0.08 | 0.09 | 0.38 | 0.23 | 0.53 | 0.47 |
| Number of Loans | 166,139 | 143,562 | 129,082 | 124,444 | 118,548 | 110,596 | 93,069 | 60,124 |
| largest 25 lenders | | | | | | | | |
| Total Lending (mm euros) | 2,370 | 2,975 | 2,800 | 2,946 | 2,587 | 1,697 | 1,210 | 32 |
| Fraction of total | 13% | 16% | 13% | 13% | 10% | 7% | 9% | 1% |
| Number of loans | 15,108 | 17,669 | 14,774 | 13,085 | 11,254 | 6,222 | 3,626 | 176 |

# Table 2: Interbank Network Systemic Risk

Note: Panel A shows results from the lending networks. Panel B shows results from the borrowing networks. Each of the two panels shows estimation results from model (14). We report the average estimates for each year between 2002 and 2009. Recall that model (14) estimates the relationship: $q_{i,\kappa}=c+\varphi(1/(g_{i,k}))\sum_{j=1}^{n_{\kappa}}g_{ij,\kappa}q_{j,\kappa}+\upsilon_{i,\kappa}$ , i.e. the individual loan volume on the network patterns of the loan volume of the rest of the market. The adjacency matrix of realized trades is a symmetric, non-directed matrix of 1's and 0's with 1's indicating the presence of a loan and 0 the absence. The first row shows the estimates of the parameter $\varphi$, the systemic risk measure, from the above specification. T-statistics are reported below coefficient estimates. The average systemic risk multiplier is total network impact of a one unit shock to an individual bank loan volume. Averaging across the impact for all individuals in the network produces this number, which is equal to $1/(1-\varphi)$.

## Panel A

|  | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---|---|---|---|---|---|---|---|---|
| **Lending Networks** | | | | | | | | |
| Average Systemic Risk Coefficient | 0.58 | 0.57 | 0.57 | 0.54 | 0.55 | 0.54 | 0.55 | 0.49 |
| t - statistic | 3.42 | 3.24 | 3.23 | 2.86 | 3.02 | 2.96 | 2.96 | 2.42 |
| Average Systemic Risk Multiplier | 2.39 | 2.31 | 2.30 | 2.15 | 2.23 | 2.20 | 2.21 | 1.97 |
| R-Squared | 0.07 | 0.07 | 0.07 | 0.06 | 0.07 | 0.06 | 0.07 | 0.06 |
| Impact of Bonacich Centrality | 17.37 | 8.77 | 3.55 | 4.27 | 50.03 | 27.57 | 25.52 | 3.00 |
| Variance | 52.65 | 59.80 | 280.18 | 146.66 | 249.69 | 185.17 | 79.81 | 18.05 |

## Panel B

|  | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---|---|---|---|---|---|---|---|---|
| **Borrowing Networks** | | | | | | | | |
| Average Systemic Risk Coefficient | 0.59 | 0.57 | 0.56 | 0.54 | 0.55 | 0.54 | 0.51 | 0.47 |
| t - statistic | 3.65 | 3.32 | 3.10 | 2.95 | 2.98 | 2.94 | 2.59 | 2.26 |
| R-Squared | 0.09 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.05 | 0.05 |
| Average Systemic Risk Multiplier | 2.47 | 2.34 | 2.26 | 2.18 | 2.21 | 2.18 | 2.03 | 1.89 |
| Impact of Bonacich Centrality | 30.18 | 2.46 | 3.16 | 0.57 | 0.61 | 0.59 | 2.36 | 0.16 |
| Variance | 77.99 | 15.09 | 16.69 | 7.30 | 7.40 | 7.60 | 18.90 | 6.94 |

# Table 3: Interbank Network Systemic Risk (long term and overnight lending)

Note: Panel A shows results from the lending networks. Panel B shows results from the borrowing networks. Each of the two panels shows estimation results from model (14). We report the average estimates for each year between 2002 and 2009. Recall that model (14) estimates the relationship: $q_{i,\kappa}=c+\varphi(1/(g_{i.,k}))\sum_{j=1}^{n_{\kappa}}g_{ij,\kappa}q_{j,\kappa}+\upsilon_{i,\kappa}$, i.e. the individual loan volume on the network patterns of the loan volume of the rest of the market. The adjacency matrix of realized trades is a symmetric, non-directed matrix of 1's and 0's with 1's indicating the presence of a loan and 0 the absence. The first row shows the estimates of the parameter $\varphi$, the systemic risk measure, from the above specification. T-statistics are reported below coefficient estimates. The average systemic risk multiplier is total network impact of a one unit shock to an individual bank loan volume. Averaging across the impact for all individuals in the network produces this number, which is equal to $1/(1-\varphi)$.

## Panel A

| | *2002* | *2003* | *2004* | *2005* | *2006* | *2007* | *2008* | *2009* |
|---|---|---|---|---|---|---|---|---|
| **Lending Networks** | | | | | | | | |
| Average Systemic Risk Coefficient | 0.54 | 0.51 | 0.51 | 0.52 | 0.52 | 0.51 | 0.51 | 0.47 |
| t - statistic | 3.33 | 2.96 | 3.00 | 3.04 | 3.08 | 3.03 | 2.93 | 2.58 |
| R-Squared | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.05 |
| Average Systemic Risk Multiplier | 2.15 | 2.04 | 2.05 | 2.06 | 2.08 | 2.06 | 2.02 | 1.88 |

## Panel B

| | *2002* | *2003* | *2004* | *2005* | *2006* | *2007* | *2008* | *2009* |
|---|---|---|---|---|---|---|---|---|
| **Borrowing Networks** | | | | | | | | |
| Average Systemic Risk Coefficient | 0.38 | 0.35 | 0.37 | 0.38 | 0.41 | 0.39 | 0.35 | 0.35 |
| t - statistic | 1.73 | 1.50 | 1.70 | 1.77 | 1.95 | 1.84 | 1.50 | 1.57 |
| R-Squared | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| Average Systemic Risk Multiplier | 1.61 | 1.53 | 1.60 | 1.62 | 1.68 | 1.65 | 1.53 | 1.55 |

# Table 4: Interbank Network Systemic Risk (returns)

Note: The table shows estimation results from model (15). We report the average estimates for each year between 2002 and 2009. Recall that model 15 estimates the relationship: $r_{i,\kappa}=c'+\varphi(1/(g_{i.,k}))\sum_{j=1}^{n_{\kappa}}g_{ij,\kappa}r_{j,\kappa}+e_{i,\kappa}$ , i.e. the individual returns on the network patterns of the returns of the rest of the market. The adjacency matrix of realized trades is a symmetric, non-directed matrix of 1's and 0's with 1's indicating the presence of a loan and 0 the absence. The first row shows the estimates of the parameter $\varphi$, the systemic risk measure, from the above specification. T-statistics are reported below coefficient estimates. The average systemic risk multiplier is total network impact of a one unit shock to an individual bank return. Averaging across the impact for all individuals in the network produces this number, which is equal to $1/(1-\varphi)$.

|  | *2002* | *2003* | *2004* | *2005* | *2006* | *2007* | *2008* | *2009* |
|---|---|---|---|---|---|---|---|---|
| **Full Network** | | | | | | | | |
| Average Systemic Risk Coefficient | 0.99 | 0.97 | 0.99 | 0.97 | 0.97 | 0.97 | 0.97 | 0.84 |
| t - statistic | 31775.37 | 20560.72 | 24915.21 | 25683.11 | 19956.05 | 22447.60 | 2532.05 | 305.39 |
| R-Squared | 0.97 | 0.95 | 0.97 | 0.94 | 0.96 | 0.95 | 0.92 | 0.65 |
| Average Systemic Risk Multiplier | 94.06 | 33.61 | 87.89 | 39.85 | 32.07 | 38.31 | 30.25 | 6.18 |
| Impact of Bonacich Centrality | 10.06 | 18.72 | 112.97 | 67.33 | 32.96 | 65.44 | 51.64 | 8.35 |
| Variance | 81.53 | 88.69 | 70.29 | 55.91 | 158.08 | 108.05 | 80.76 | 64.80 |

# Figure 1: Daily Lending Quantities
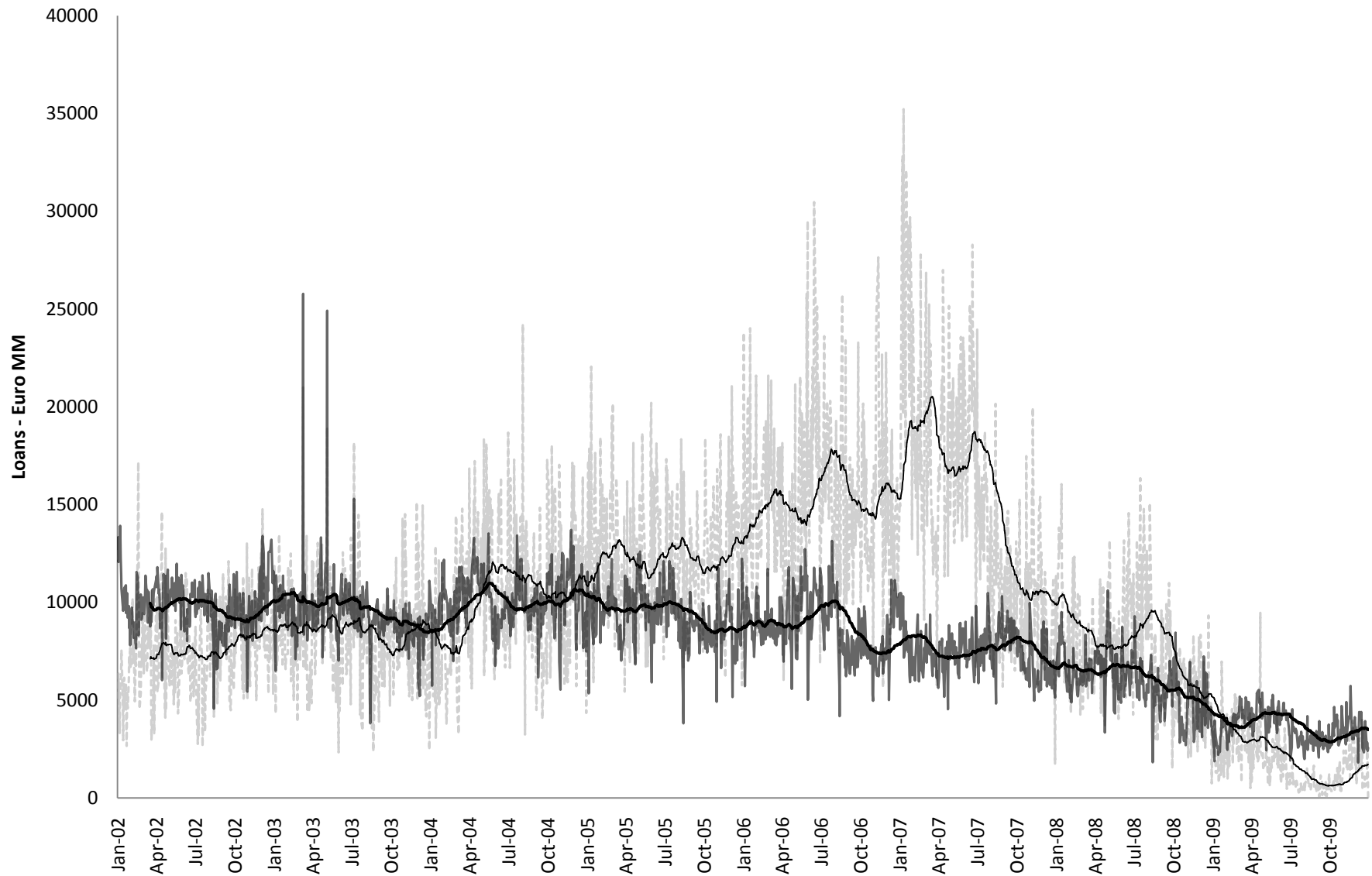


Figure 1 shows daily lending quantities over the sample for overnight and longer term lending. The black solid line reports overnight lending quantities. The grey dashed line reports all other lending. Each has a 2 month moving average trend added.
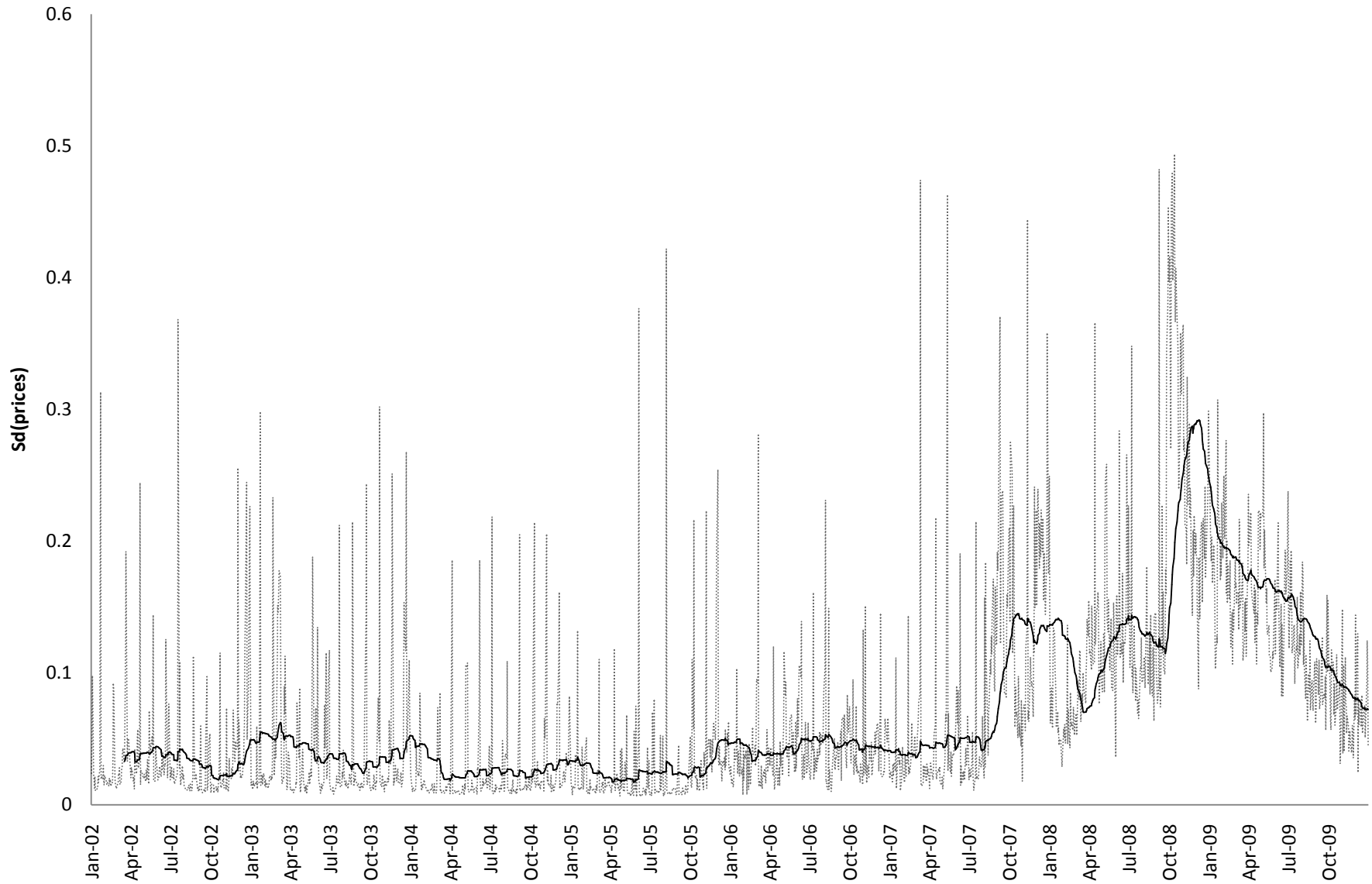
# Figure 2: Daily Price Volaility



Figure 2 reports the daily standard deviation of prices (taken over prices during the day and normalized). The price volatility itself is reported in a grey dashed line and the 2-month moving average reported in a black solid line.

# Figure 3: Dynamic Model Output: Theoretical Degree Distribution
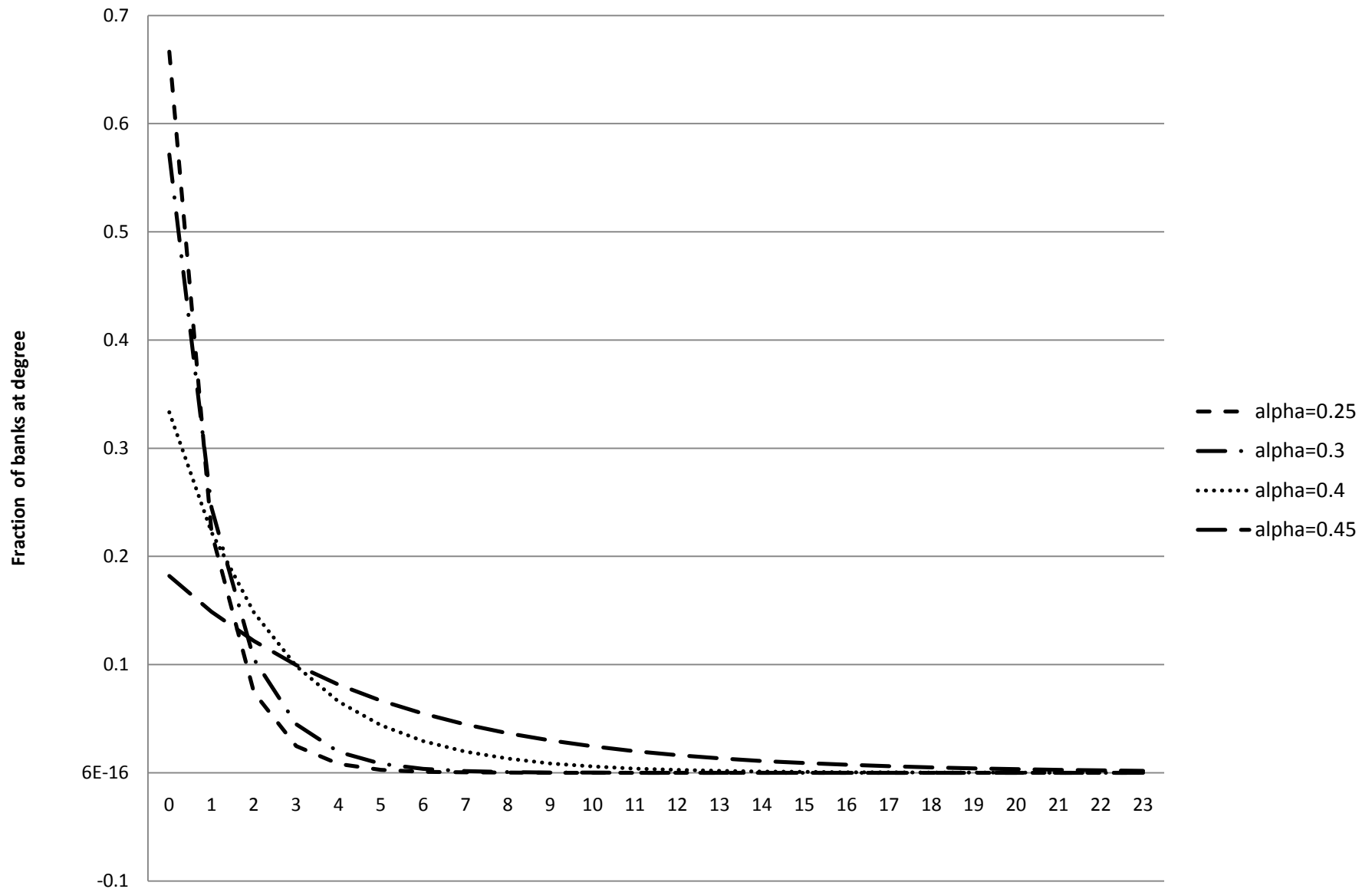


Figure 3 shows the theoretical degree distribution of the dynamic network formation model in the paper. For each level of α (link formation probability), the model general an invariant distribution of network links. Precise invariant distribution is described in the text: n(d)=((1-2α)/(1-α))((α/(1-α)))^{d}, where n is the proportion at end degree. We report these distributions for α={.25, .3, .4, .45}.

# Figure 4: Estimated Link Formation Probability (Alpha)
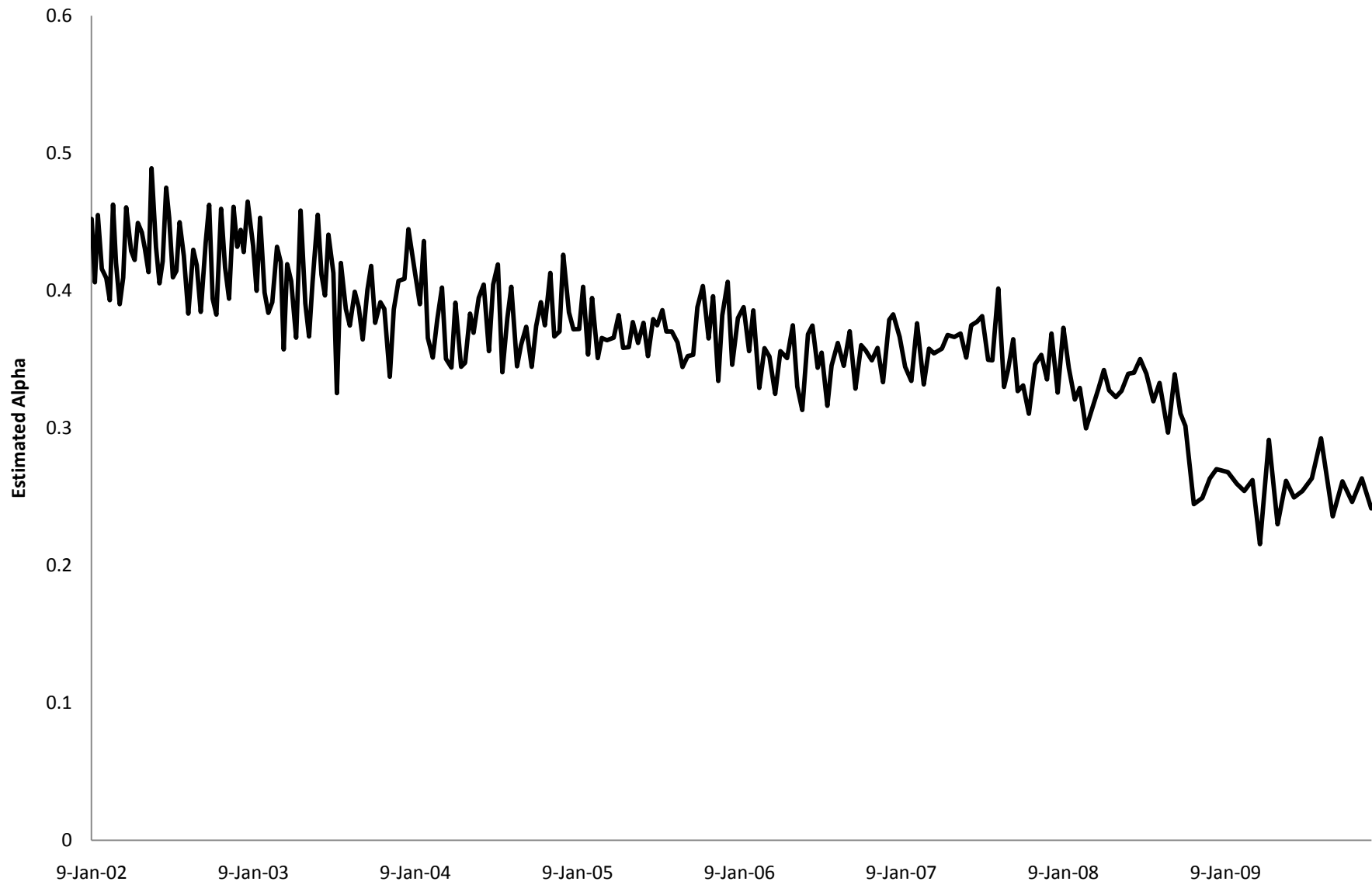


Figure 4 shows the estimated probability of link formation for each network. Recall that we use a network definition of 1000 loans as a benchmark.

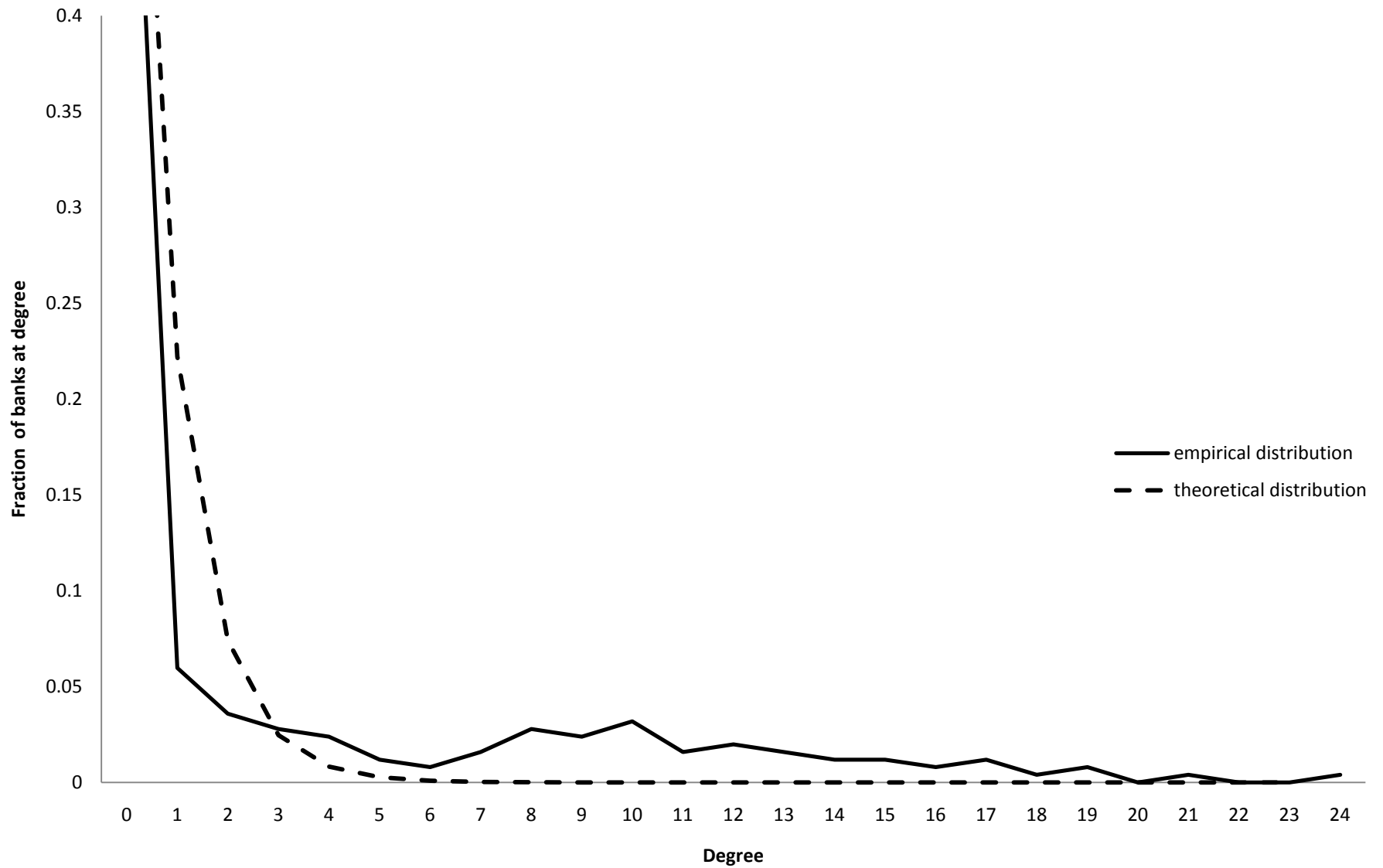**Figure 5a: Empirical vs Theoretical Degree Distribution – January 2002**
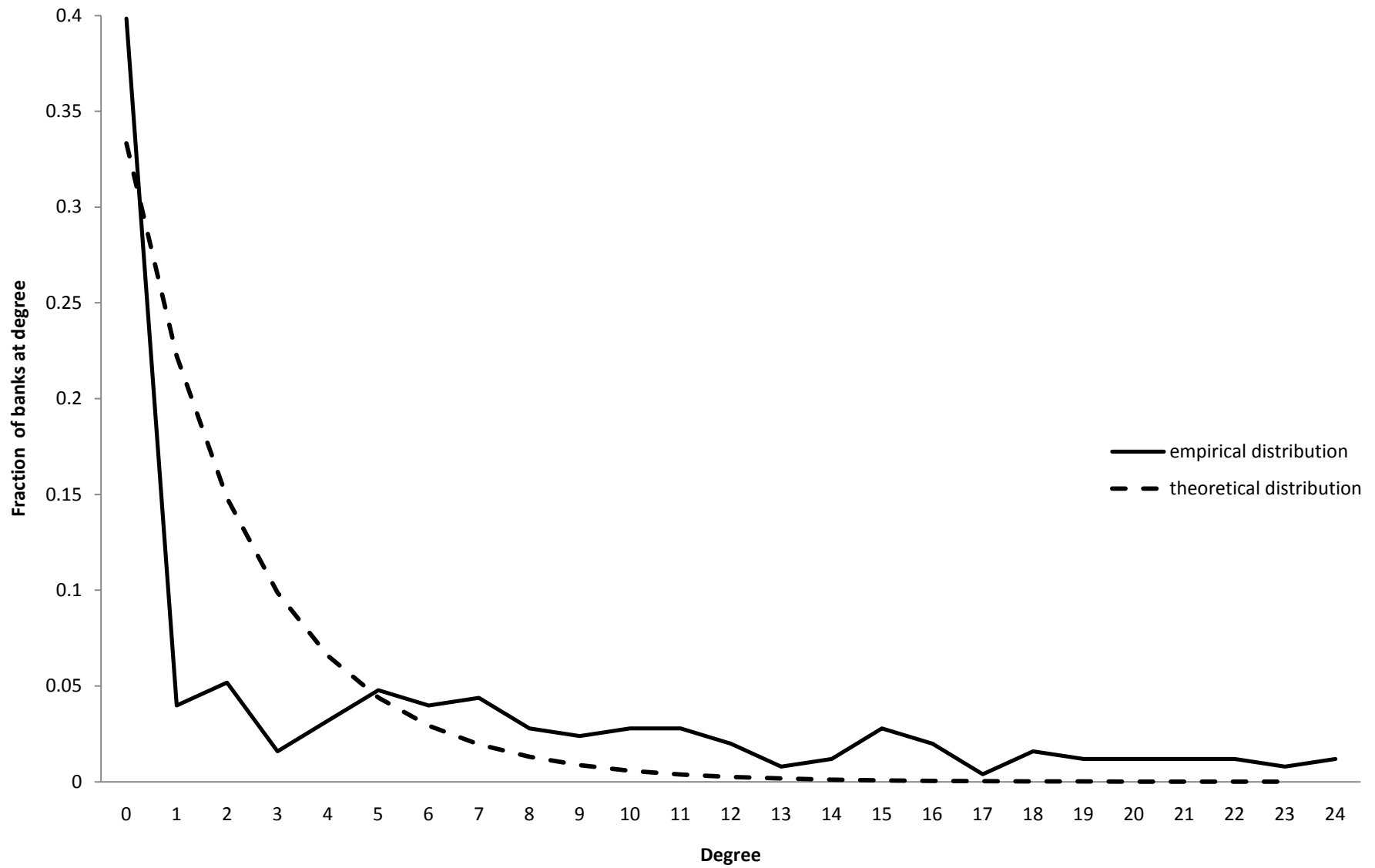
Figure 5a shows the empirical degree distribution of the network that existed on January 9, 2002. On the same figure, we plot the theoretical distribution generated by the empirical link formation probability on that day.

# Figure 5b: Empirical vs Theoretical Degree Distribution – December 2009



Figure 5b shows the empirical degree distribution of the network that existed on December 31, 2009. On the same figure, we plot the theoretical distribution generated by the empirical link formation probability on that day.
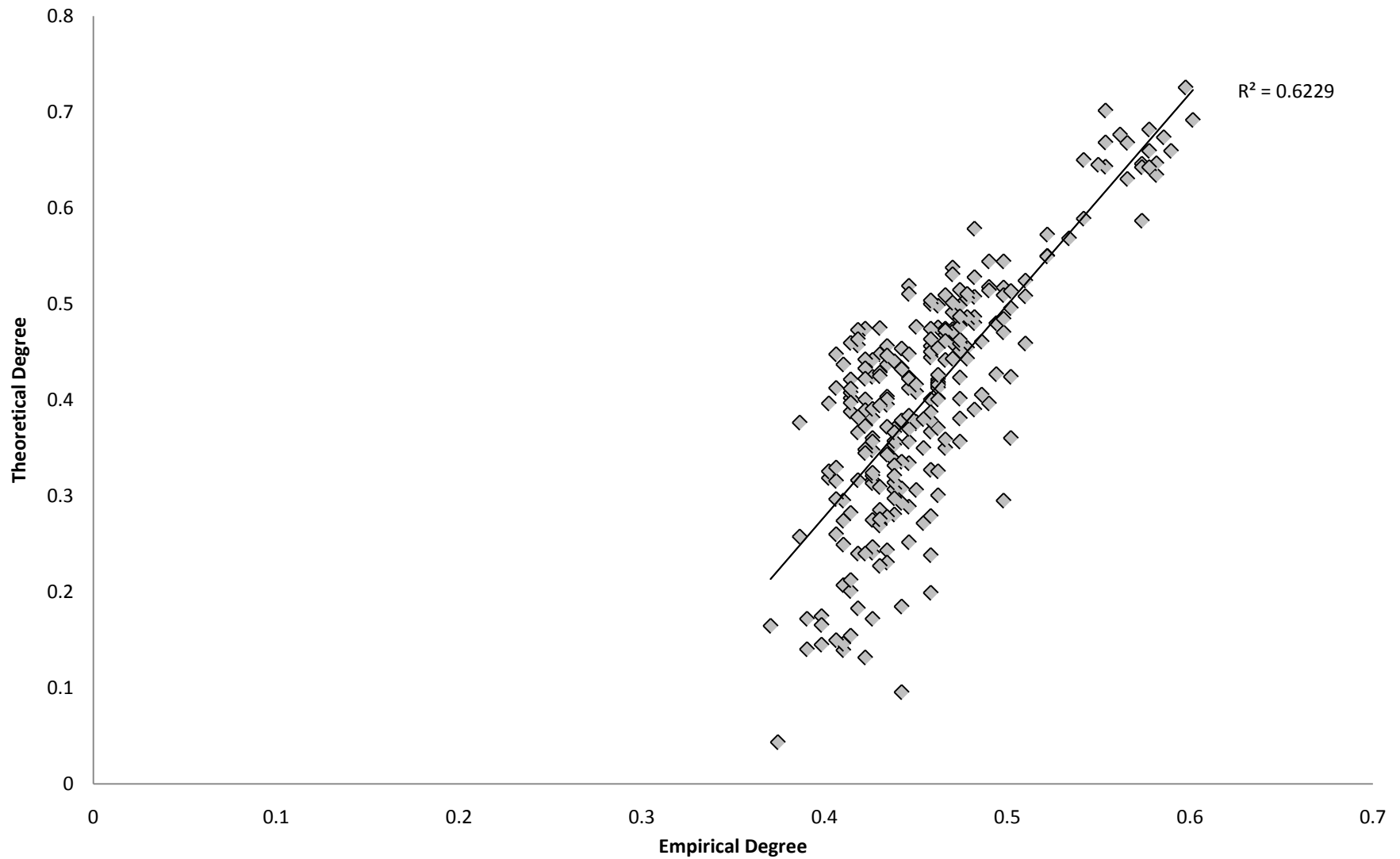
# Figure 5c: Dynamic Model: Model Fit



Figure 5c shows a scatterplot of two variables. The first (on the horizontal axis) is the fraction of zero-degree participants in network (1000 loans) in our dataset. The second (on the vertical axis) is the fraction of zero-degree participants implied by our dynamic model, conditional on the α for that given network. Recall that α is the empirical probability of link formation. A 45% line implies that the model works perfectly and a high correlation implies that the model is consistent over a wide range of network structures.
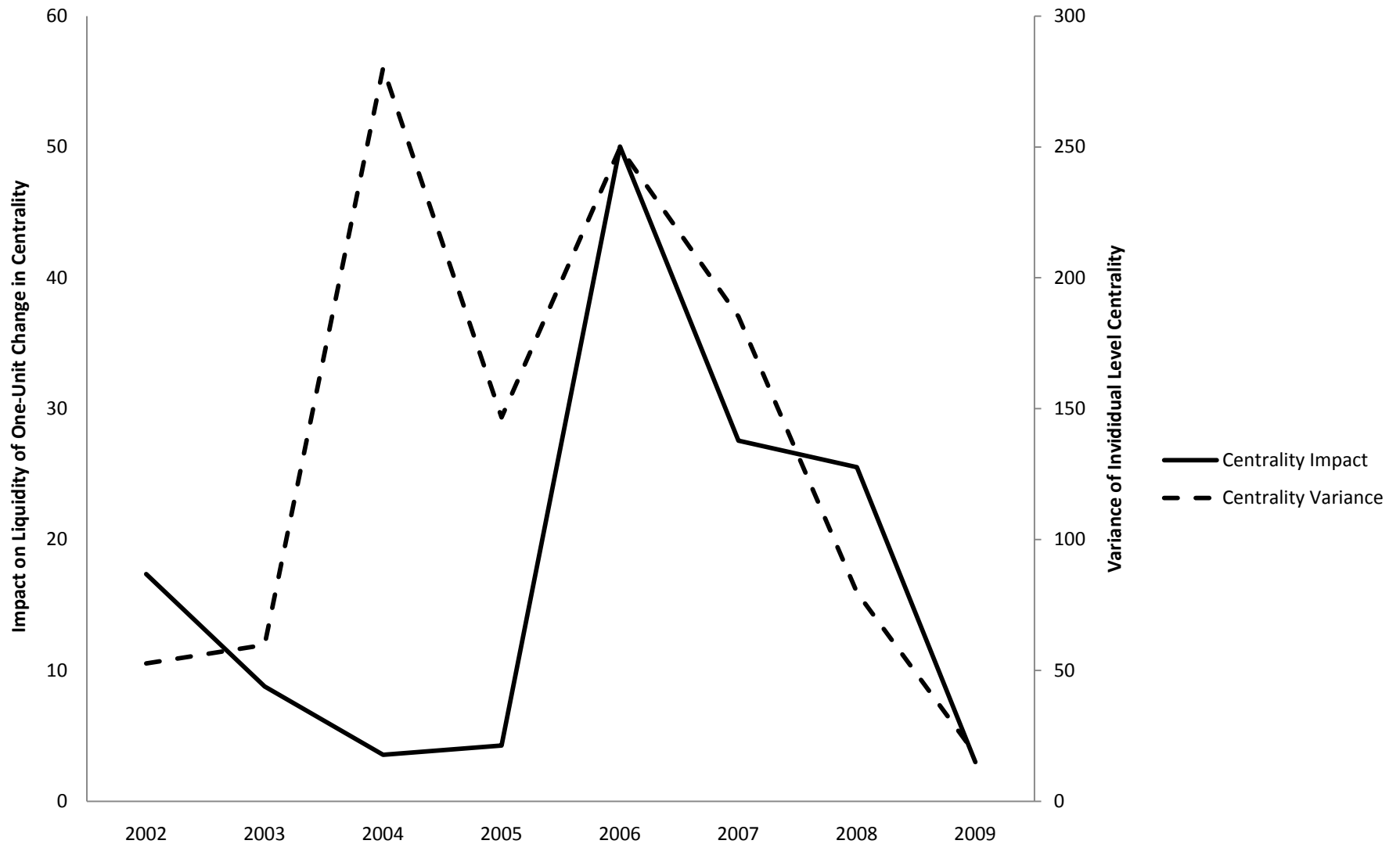
**Figure 6: Role of Network Structure**

Figure 6 shows the time series of two variables. Each reports one value for each year of data between 2002 and 2009. The first, along the left hand vertical axis, is the impact on quantity of loans (liquidity) of a one-unit change in the centrality of participants. All values of this series are positive, indicating that becoming more central is positively related to the provision of liquidity. The second, along the right hand vertical axis, is the variance of individual level centrality. This measures the distribution over the agents of network centrality (some banks with low centrality and some with high lead to high variance).
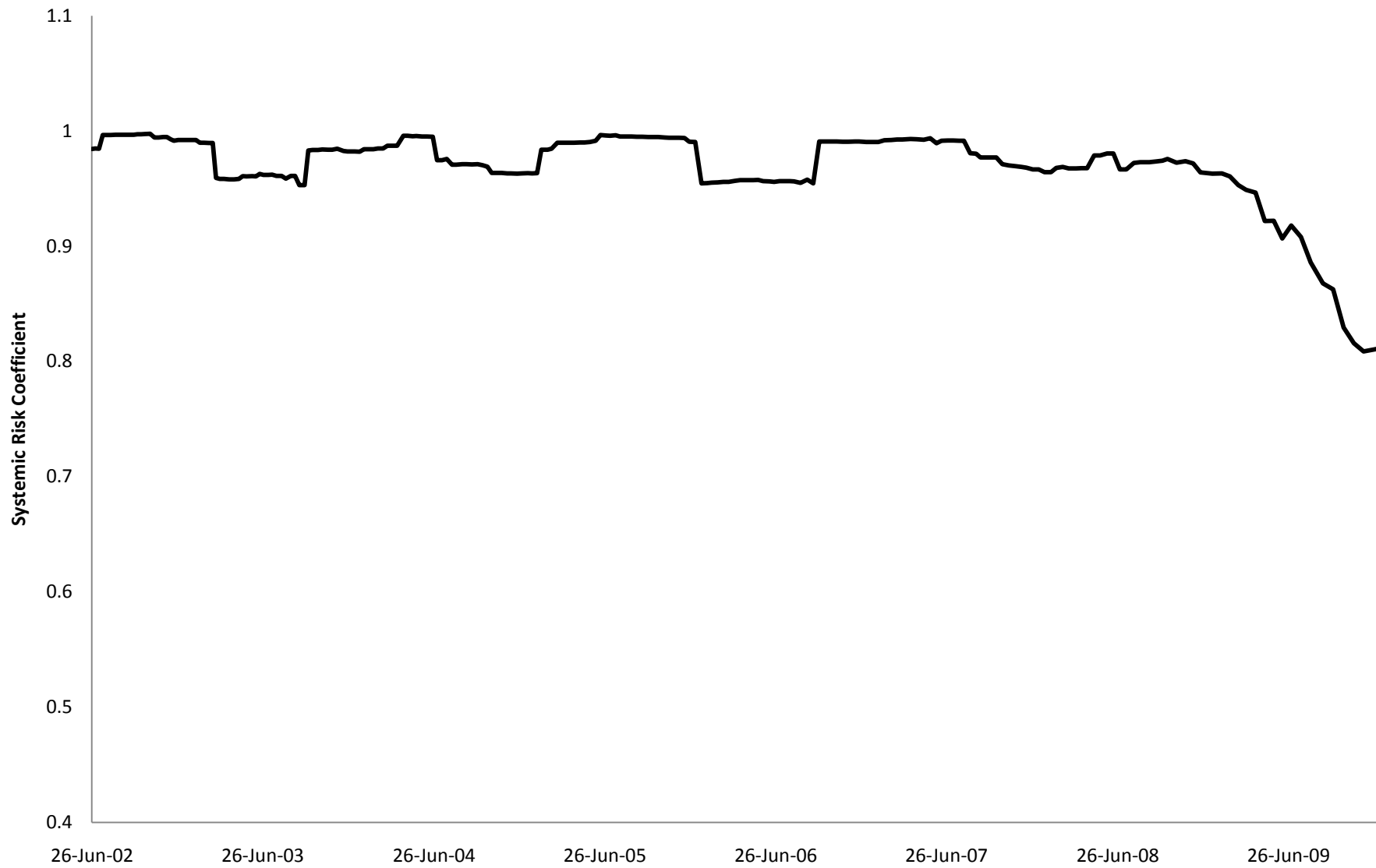
# Figure 7: Systemic Risk in Returns over Time



Figure 7 shows the coefficient from the regression $r_{i,\kappa}=\alpha+\theta(1/(W_{i.,k}))\sum_{j=1}^{n_{\kappa}}W_{ij,\kappa}r_{j,\kappa}+\upsilon_{i,\kappa}$. The results of this regression on an annual basis are reported in Table 4. Here we report the $\theta$ coefficient over time.