The Becker Paradox and Type I vs. Type II Errors in the Economics of Crime

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Abstract:
Two real-world observations are not easily replicated in models of crime. First, although capital punishment is optimal in Becker's (1968) model, it is rarely observed in the real world. Second, criminal procedure and the evaluation of evidence vary across societies and historical periods, the standard of proof being sometimes very high and sometimes quite low. In this paper, we develop a general equilibrium model of judicial procedure allowing for innocent persons being convicted. We show that the median voter theorem applies to this model, making judicial procedure endogenous. So formulated, the model can replicate both empirical observations.

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1. INTRODUCTION

In the ever-growing literature on the economics of crime, there are two important issues deserving more attention. One is the Becker (1968) paradox. From that seminal paper, it follows that capital punishment is optimal for society, even for minor crimes. The reason is that if punishment is sufficiently severe, no crimes will be committed, and no punishment will be inflicted; the government will save tax money, and social optimum will be attained. The paradox lies in the fact that although this result is very robust to the model specification, we rarely observe capital punishment in the real world – in particular not for minor crimes – neither across countries nor across time in a given country. Thus, there might be something missing in the theoretical model that makes it deviate from reality, and a small literature has emerged dealing with this issue.[2] One prominent attempt to resolve the paradox was published by Stigler (1970), who argued that it is optimal to have a scale of punishment according to the nature of the crime; if there were only one type of punishment, the marginal deterrence effect would disappear, and a criminal engaged in a minor crime might as well commit a more brutal, and more profitable, crime instead. The fallacy of this argument is evident: if punishment were so severe that no crimes were committed, there would be no need for marginal deterrence.

The second issue deserving attention is that of Type II errors, i.e., the risk that an innocent person might be convicted (in the original Becker model, there is only Type I errors, i.e., the guilty criminal might escape). There is a small but growing literature in this field. Harris (1970) discussed how to extended Becker's analysis by taking Type II errors into account. He formulated a problem where the social loss of crime, crime fighting and punishment should be minimized, one of the parameters representing the degree of legal safeguards to suspects. However, his formal analysis does not discriminate between errors of Type I and errors of Type II. Png (1986) not only mentions the occurrence of Type II errors, but also analyzes their effects on optimal policy, treating the burden of proof as exogenous. The same can be said of Tullock's (1980) penetrating analysis of legal procedure. Kaplow and Shavell (1994, p. 13) go one step further by mentioning the problem of choosing an optimal burden of proof, which is at the heart of the trade-off between Type I and Type II errors, but they do not pursue that analysis. Rubinfeld and Sappington (1987) do analyze the choice of standard of proof but this does not affect incentives to commit crimes and thus not the number of criminals in society. Andreoni (1991) endogenizes the number of crimes, but does not close the model completely; he assumes that the criminal's gain from committing a crime, as well as the judge's disutility from convicting an innocent person, are exogenous parameters.[3]

These papers assume that the court has aversion to Type II errors. The origin of this aversion is, however, not explained but rather taken for granted. To analyze the trade-off between Type I and Type II errors satisfactorily, i.e., to investigate the appropriate choice of criminal procedure, it is necessary to derive the court's aversion to these errors from fundamentals. Since judges and juries could be seen as representing the population in general it is natural to employ a median-voter framework to study not only legislation per se, but also judicial procedure (i.e., the trade-off between Type I and Type II errors). In fact, such a set-up is in conformity with Becker's methodological approach to explain social phenomena from individuals' self interest, not from abstract moral principles.

In doing so, we can also point out one possible resolution to the Becker paradox: if the median voter runs a risk of being innocently convicted, he or she might be reluctant to endorsing capital punishment. This is, however, not always the case. If punishments are very severe, and consequently no crimes are committed, the median voter does not run any risk of being innocently punished. The Type I-Type II model therefore needs additional features, for instance a mechanism that leads to some crimes being committed regardless of the punishment, in order for the median voter to vote against capital punishment.

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[2] See Friedman (1999).

[3] The problem of errors of Type I and Type II also occurs in principal-agent models with testing possibilities. Nalebuff and Scharfstein (1987) show that high enough fines may result in a full-information competitive equilibrium where no low-productivity individuals mimic high-productivity ones. This is parallel to Becker´s case of capital punishment leading to a no-crime equilibrium.

2. THE MODEL

2.1 General Setup

For simplicity, we study a stationary economy. There is a continuum of agents, and each agent chooses whether to be a worker or a criminal, thereby maximizing his expected utility. The proportions of workers and criminals in society are endogenously determined in general equilibrium. If a criminal encounters a worker, a robbery takes place. If a robbery has been committed, the culprit (or an innocent bystander) may be convicted.

We assume that all individuals have identical instantaneous utility functions. However, they differ with respect to time preference (see below). The difference in time difference leads to different occupational choice; some individuals choose to be workers, some to be criminals. The instantaneous utility is a function of consumption, work effort, and pain from punishment. A person who chooses to be a criminal does not have any disutility from work effort. A person who is not convicted (regardless of whether he is guilty or not) does not suffer any pain from punishment.

A worker earns $w(1-t)$ in each period, where w is the wage rate and t the tax rate (taxes are used to finance the police force and the legal system). For simplicity, we assume (1) that the utility function is additively separable in income and work effort, (2) there is no saving either by the individual or society, and (3) the number of working hours is exogenously given and set equal to one for convenience. Denoting work effort by $\hat{a}$ and the disutility of work effort by $a(\hat{a})$, the instantaneous utility of a worker who is not robbed can, therefore, be written as $u(w(1-t))-a(\hat{a})$. For notational simplicity, we will write this as $u(w(1-t))-a(\hat{a})=u-a$. If the worker is robbed, his utility is instead $u(0)-a(\hat{a})=-a$, where we have set $u(0)=0$.

A robber, if he meets a victim, will rob the latter of his after-tax earnings; thus the successful robber consumes $w(1-t)$. A robber who does not encounter a suitable victim consumes zero. Having committed a robbery, the robber may be convicted. Let us denote the probability of Type I error, that is, the culprit is *not* convicted, by P_1. Thus, $1-P_1$ is the probability of a successful robber being convicted; in that case, a punishment $\hat{\varphi}$ is inflicted upon him. Like the disutility of work effort $a(\hat{a})$, the disutility of punishment $\varphi(\hat{\varphi})$ enters separably in the utility function. Note that $\hat{\varphi}$ should not necessarily be regarded as a monetary fine; in order to keep $\hat{\varphi}$ out of the government budget equation, we would rather interpret it as the pain costlessly inflicted upon the convicted person by the legal authorities.[4] We assume that the legal process takes one time period, that is, a crime committed in period t leads to a punishment in period t+1. A successful crime thus leads to a robber's utility $u(w(1-t)) = u$ in one time period, and (with probability $1-P_1$) a punishment $\hat{\varphi}$ in the next period. If δ is the robber's discount factor, the present value of the convicted robber's utility is thus $u(w(1-t)) - \delta\varphi(\hat{\varphi}) = u - \delta\varphi$.

The probabilities that a worker meets a robber, and that a robber meets a worker, depend on the relative numbers of workers and robbers in society. To derive these probabilities, we must specify the encounter technology of our model economy.

2.2 The Encounter Technology

For Type II errors to occur, we need an encounter technology allowing for innocent persons being convicted. The easiest way to do this is by assuming that people interact in triplets. In each point of time each individual is randomly assigned to a triplet, with two other people in it. Thereby we achieve the simplest possible configuration in society that can consist of one criminal, one victim and one innocent bystander.

Assume the proportion of criminals in the population to be c, while the proportion of workers is (1 – c). With no loss of generality, we normalize the size of the population to unity; thus c is also the number of criminals in society. Further, we assume that people are randomly allocated into triplets. A given triplet can alternatively consist of three criminals (C, C, C), two criminals and a worker (C, C, W), two workers and a criminal (W, W, C), or three workers (W, W, W). The order of the elements is immaterial to the reasoning. The probabilities of the four possible configurations are thus c^3, $3c^2(1-c)$, $3c(1-c)^2$ and $(1-c)^3$, respectively.

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[4] For a model where punishment does not only occur in the criminal's utility function, but also has a time dimension (i. e., an incarceration effect), see Persson and Siven (2006).

A robbery takes place if there is at least one worker and one criminal in a triplet. This setup is used since it provides a simple representation of *proximity*. In the real world, a crime is often triggered by the criminal being close to the victim. And an innocent person may be suspected of the crime if he is close to the victim (either in a social or a spatial sense).

In the (C, C, W) case, only one of the criminals can commit the robbery; which one is determined by tossing a fair coin. When the robbery has taken place, the police arrive. There will then be one victim (who can, by definition, not be suspected), one guilty criminal, and one innocent bystander (who also happens to be a professional robber, although innocent of this particular crime) in the triplet.

In the (W, W, C) case, the sole criminal can rob only one worker. The one to be robbed is determined by tossing a coin. When the police arrive, there will be one victim, one guilty criminal, and one innocent bystander (who in this case happens to be a worker) in the triplet.[5]

The police analyze the evidence, which is somewhat distorted, so it might be that the innocent bystander sometimes looks quite guilty. In that case, which occurs with probability P_2, a Type II error is committed.[6] In the normal case, however, the true culprit is convicted; this happens with probability $1-P_1$.

The probabilities of the two types of error depend on two things: police resources per crime, and judicial procedure. Police resources per crime, in turn, depend on the size of the tax base, the tax rate and the number of crimes committed. We will return to this issue in Section 3.1, where we discuss the government's budget constraint. Here, it suffices to note that since the tax base and the number of crimes depend on *c*, we can regard P_1 and P_2 as functions of, *inter alia*, *c*.

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[5] In the (C, C, C) and (W, W, W) cases, no robbery takes place. Consequently, nobody can be innocently suspected, and neither Type I nor Type II errors can occur.

[6] The explicit modeling of the distortion of evidence will be discussed in Section 3.2.

Judicial procedure can be characterized by society's method of evaluating evidence. If the evidence happens to be very faint compared to the stringency requirements of the legal system, both persons standing in the vicinity of the victim may be acquitted. This generally happens when society has a large aversion to Type II errors, and thus defines "beyond a reasonable doubt" so that P_2 is quite small. With such a stringent evaluation of evidence, $1-P_1$ will also be quite small and consequently, P_1 will be large. In other societies, or for other types of crimes, one may rather convict an innocent person than running the risk of releasing a criminal; thus both persons might be convicted although we know that only one of them can be guilty. For such a case, we have P_2 quite large. Which case should apply depends on technology and preferences (i.e., how dangerous to society a particular type of crime is).

2.3 Individual Utility

Let us now look at a given worker. In his triplet, the other two people will be

- (W, W) with probability $(1-c)^2$. The worker will then spend the rest of the period unharmed, consuming all his net income.
- (W, C) with probability $2(1-c)c$. If he is robbed (which occurs with a 50 percent probability) he cannot be convicted of that robbery. If he is not robbed, he will be an innocent bystander but may nevertheless be convicted of the robbery that took place in his triplet; this will happen with probability P_2.
- (C, C) with probability c^2. He will then be robbed with certainty.

The worker's expected utility can then be written

$$V_w = (1-c)^2(u-a) + 2c(1-c)\left[0.5\{(1-P_2)(u-a) + P_2(u-a-\delta\varphi)\} + 0.5\cdot(-a)\right] + \\ + c^2\cdot(-a) + \delta V_w,$$

where each of the first three terms correspond to one of the three contingencies in the encounter technology. Note that we have added the discounted value of next period's utility at the end of the expression. Rearranging the terms, the present value of the worker's expected utility in a stationary equilibrium can be written as

(1) $$V_w = \frac{1}{(1-\delta)} \cdot \left[(1-c)u - a - \delta c(1-c)P_2\varphi\right].$$

The first two terms inside the square brackets stand for the expected utility of working in a certain period, whereas the last term represents the expected discounted utility of being unjustly punished in the next period.

For a given criminal, the other two people in his triplet will be

- (W, W) with probability $(1-c)^2$. The criminal then robs one of the workers, and is convicted in the next period with probability $1-P_1$.
- (W, C) with probability $2c(1-c)$. The criminal will then have a 50 percent chance of robbing the worker, in which case he is convicted in the next period with probability $1-P_1$. With a 50 percent probability, he will instead be the innocent bystander; he then gets no booty, but is innocently convicted in the next period with probability P_2.[7]
- (C, C) with probability c^2. From our assumption of only one encounter per period, it follows that criminals cannot steal from each other (since, before they have encountered a worker, they have no money in their pockets). A criminal who meets another criminal will thus end the day empty-handed, but may find consolation in the fact that he cannot be innocently convicted.

We can thus write the criminal's utility as:

$$V_c = (1-c)^2\left[u + (1-P_1)\delta \cdot (-\varphi)\right] + 2c(1-c)\left[0.5\left(u + (1-P_1)\delta \cdot (-\varphi)\right) + 0.5P_2\delta \cdot (-\varphi)\right] + c^2 \cdot 0 + \delta V_c.$$

This can be simplified as

(2) $$V_c = \frac{1}{1-\delta}\left[(1-c)u - \delta(1-c)\varphi(1-P_1 + cP_2)\right],$$

[7] For simplicity we consequently assume that the probability of being innocently convicted is the same for a worker as for an (in this particular case innocent) criminal. The court is thus not taking the previous criminal record into account when scrutinizing evidence.

where, in analogy with (1), the first term within the square brackets represents the criminal's expected utility of his booty, whereas the second term represents his expected utility from being punished in the next period (punishment being either deserved or undeserved).

2.4 *Market Equilibrium*

It is now time to specify the heterogeneity of the population. In most crime models, agents are assumed to differ with respect to the real wage, *w*. This could easily be done also in our model, but we have chosen another route. For simplicity, we assume that people have identical wages, but differ with respect to their time preference, δ, which simplifies the analysis in two respects. First, the robber's booty does not depend on whom he robs; it will be the same for all victims. Second, we can characterize hard-core criminals in a simpler way than if people differed with respect to their wage rates. But the assumption of people differing with respect to their time preference rates is not made for simplicity only; we also think that it contains a certain element of realism. While we lack empirical studies of the rates of time preference of criminals versus non-criminals, the notion of some persons (for example, drug addicts), being more concerned about immediate satisfaction than about possible consequences for the future, does not seem too far-fetched.

Setting $V_w = V_c$, we obtain the cut-off time preference

$$(3) \qquad \hat{\delta} = \frac{a}{(1-c)\varphi(1-P_1)}$$

such that $V_w > V_c$ for all $\delta > \hat{\delta}$ and $V_w < V_c$ for all $\delta < \hat{\delta}$. Thus, everybody with a time preference rate above $\hat{\delta}$ will choose to be a worker, while everybody with a time preference rate below $\hat{\delta}$ will choose to be a criminal.[8] This is a surprisingly simple

[8] This follows from the fact that since $V_c - V_w = a - \delta(1-c)(1-P_1)\varphi$, we have that $\partial(V_c - V_w)/\partial\delta < 0$.

expression where we note that the cut-off rate $\hat{\delta}$ is independent of the utility function $u(\cdot)$.

Let us now consider the relation between c and $\hat{\delta}$. If δ were uniformly distributed on $[0, 1]$, then $c = \hat{\delta}$, and we could substitute this into the equation $V_w = V_c$ to get rid of one variable. However, in addition to assuming the distribution of δ to be rectangular, we introduce a mass point of measure σ at $\delta = 0$. This mass point corresponds to the number of "hard core" criminals who, since criminal process takes one time period, are unaffected by punishment.[9] It means that the total number of criminals in society is given by

$$(4) \qquad c = \sigma + \hat{\delta}(1-\sigma), \quad 0 < \sigma < 1.$$

We can now set V_w, as given by (1), equal to V_c, as given by (2), to yield the cut-off rate $\hat{\delta}$. Since c and $\hat{\delta}$ are connected through (3), we can choose which of these two variables we want to eliminate from the cut-off equation. It turns out to be somewhat simpler to eliminate $\hat{\delta}$. Thus, $V_w = V_c$ and (3) implies the equilibrium relationship

$$(3') \qquad \varphi = \frac{a(1-\sigma)}{(1-c)(c-\sigma)(1-P_1)}.$$

This equation gives us the equilibrium relation between the severity of punishment, φ, and the number of criminals in society, c, for given values of the technological parameters (a and σ), and a given value of the variable P_1 representing the legal system. For the rest of the paper, it is convenient to work with this equation rather than with its equivalent (3). In the Appendix (section 1) we show that for every value of φ it has at least two roots c, the larger of which is the unstable one.

2.5 Political Equilibrium

[9] The mass point serves the purpose of ruling out a zero-crime equilibrium. Another way of achieving the same goal would be to build a model where some random events occur that look like crimes even if they are not (like a worker dropping his wallet). For a discussion of such a model – the empirical relevance of which may perhaps be discussed – see footnote 18 below.

We will now demonstrate that the median voter theorem applies to this model. Let us first multiply each agent's utility (as expressed by (1) for workers and (2) for criminals) by $1-\delta$. This is just a linear transformation, which affects neither the individual's ranking of different policy alternatives nor his marginal rates of substitution. With this transformation, and substituting (3') into (1), we immediately see that worker's preferences can be written in the form $V_w = J(q) + \delta \cdot H(q)$. Here, q is a vector of policy variables common to all individuals (in this case, t, c, P_1, and P_2; note that φ has been eliminated by using the equilibrium condition (3')), while δ is the worker's individual rate of time preference. Thus, workers have so-called intermediate preferences, and if only workers were allowed to vote, the median voter theorem would immediately apply.[10]

The preferences of criminals, represented by (2) multiplied by $1-\delta$ (and taking (3') into account in order to eliminate φ), can be similarly written in the form $V_c = \tilde{J}(q) + \delta \cdot \tilde{H}(q)$. They also satisfy the condition for intermediate preferences, which means that if only criminals were allowed to vote, the median voter theorem would apply.

The problem arises since there are two groups of individuals, with different indirect utility functions, and the individual can endogenously switch between these groups depending on the policy vector q. It is well known in the literature that the existence of such groups may cause problems in political-economics models.[11] If there were, for instance, an incentive for the extremes of each group to form a coalition, the median voter theorem would not apply. In the Appendix (section 2) we show that if the median voter is a worker, such coalitions cannot occur in our model. Thus, the median voter will be the decisive voter, and in the policy analysis below, we will study the policy vector $q^m \equiv (t, \varphi, P_1, P_2)$ preferred by the median voter. Since φ and c are related via (3'), we could as well regard $q^M \equiv (t, c, P_1, P_2)$ as the policy vector; in fact, this is somewhat more convenient and will therefore be employed for the rest of the paper.

[10] Cf. Persson and Tabellini (2000, pp. 25-28).

Substituting φ, as given by (3'), into the worker's utility (1), and noting that the median voter's time preference is $\delta^m = (0.5-\sigma)/(1-\sigma)$, gives us the median voter's utility in general equilibrium:[12]

$$(5) \qquad \hat{V}_{med} = (1-c)u - a\left[\frac{c}{2}\cdot\frac{1-2\sigma}{c-\sigma}\cdot\frac{P_2}{1-P_1}+1\right].$$

In political equilibrium, expression (5) is maximized with respect to the policy variables $t, c, P_1,$ and P_2.

2.6 Will there be Capital Punishment in Political Equilibrium?

At this early stage, we can already see the answer to one of the issues raised in the Introduction: why is capital punishment rarely observed in the real world? From Becker's (1968) model, it follows that the punishment should be set at a maximum, even for minor offences. Still, we rarely observe capital punishment in the real world – not even for the most serious of crimes. The reason for this discrepancy between Becker's result and the observed reality has given rise to a minor literature in the economics of crime.[13] Our model can be used to shed new light on that issue.

By the equilibrium relation (3'), we have eliminated φ from our notation, and instead we treat c as a policy variable. Equation (3') tells us, however, what value we should assign to φ to obtain the particular value of c that maximizes (5). The question of capital punishment could therefore be phrased in terms of c instead of φ. Thus formulated, the question boils down to asking whether φ should be set so as to drive c down to σ. We see from (5) that this is not optimal for the median voter. Assume that t, P_1, and P_2 are given, and furthermore have values such that $0 < P_2/(1-P_1) < \infty$. From (5) we then see that setting $c = \sigma$ means that the median voter's utility is minus infinity. Thus the median voter does not want $c = \sigma$, and we have therefore proved that in a model with a non-zero probability of Type II errors, there will be no capital punishment in political equilibrium.

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[11] Cf. Roine (2006).

[12] For convenience, we have multiplied the utility by $1-\delta^m$. Thus, $(1-\delta^m)V^m = \hat{V}_{med}$.

[13] See, for instance, Friedman (1999).

Note that the mere *possibility* of Type II errors is not sufficient to rule out capital punishment. We also need a mechanism leading to some crimes being committed even if φ is very high. If not, it would be optimal to actually drive crime down to zero; if no crimes occur, no innocent persons could be innocently convicted. In the triplet model, this function is performed by the spike σ; if some individuals are never deterred by the prospect of a future punishment, there will always be some crime, and thus any innocent median voter runs the risk of being punished. In fact, setting $\sigma = 0$ in (5), we cannot see that driving *c* down to σ results in an infinitely negative utility for the median voter, since $\lim_{c \to 0} \frac{c}{c} = 1$.

We have here assumed that the distribution of the rate of time preference is *connected* on [0, 1]. If the distribution is not connected, i.e., if it consists of a spike of measure σ at 0 and a rectangular distribution on [*m*, 1], where *m* > 0, there is nobody with a $\delta \in (0, m)$. In such a case there is no reason to have a punishment φ higher than what is necessary to drive down crime to its absolute minimum (= σ). Thus there is no need for capital punishment, regardless of whether Type II errors have been introduced into the model or not.[14] What we have shown above is that even if the distribution of δ is connected, capital punishment will not be optimal.

Naturally, the encounter technology with triplets and a spike σ is a stylized parable, intended to make the notion of proximity analytically tractable. There are other, equally plausible, technologies leading to similar conclusions.[15] Which setup is the most suitable depends on the question asked and is ultimately an empirical question.

[14] With such a non-connected distribution for δ, the expression for the median voter's utility, i.e., the equivalent of equation (5), becomes

$$\hat{V}_{med} = (1-c)u - a\left[\frac{c}{2} \cdot \frac{1-2\sigma+m}{c-\sigma+(1-c)m} \cdot \frac{P_2}{1-P_1} + 1\right].$$

It is evident that this expression does not necessarily go to minus infinity as $c \to \sigma$.

3. THE CONVICTION TECHNOLOGY

3.1 The General Form of the $P_2(P_1)$ *Function*

Before working out the grand maximization of (5), it is necessary to take a closer look at the conviction technology, represented by probabilities P_1 and P_2. All we know so far is that there is a trade-off between these probabilities. This trade-off is illustrated in Figure 1 and has the following properties:

- P_2 is monotonically decreasing in P_1
- the trade-off becomes more favorable if the amount of police resources per crime increases, as illustrated by the dashed curve in Figure 1
- $P_2 = 0$ for $P_1 = 1$ and $P_2 = 1$ for $P_1 = 0$.

(Figure 1)

While the first two points are rather self evident,[16] the third deserves some discussion. Recall our basic encounter technology with three persons in a triplet: when the police arrive, there is one person lying on the ground, obviously being the victim, and two bystanders both claiming innocence. If there is any uncertainty as to which of the two bystanders actually committed the crime, there is only one way of avoiding Type II errors, namely to always acquit both of them. Likewise there is only one way of completely avoiding Type I errors, namely to always punish both bystanders.

The amount of police resources per crime is equal to the total tax revenue $tw(1-c)$ divided by the number of crimes. Since the population is normalized to unity, the total number of triplets is 1/3. There is one crime in every triplet of type (C, C, W) and (W, W, C), and there are $\frac{1}{3}\left[3c^2(1-c)+3c(1-c)^2\right]=c(1-c)$ of these. Thus, the amount of police resources per crime is tw/c, and if policemen (who are assumed to be hired outside the model to keep the number of professions down) have the same wage rate

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[15] Cf. the model sketched in footnote 18.
[16] For a discussion of a model where at a constant police budget the probability of detection declines when the number of criminals increases, see Sah (1991).

as workers, the number of policemen per crime is t/c. We can thus write the error function as

$$(6) \qquad P_2 = P_2(P_1, t/c)$$

with

$$(7) \qquad \frac{\partial P_2}{\partial P_1} < 0, \quad \frac{\partial P_2}{\partial t} < 0, \quad \frac{\partial P_2}{\partial c} > 0.$$

This is the function depicted in Figure 1; if P_1 is changed, we move along the curve, and if t/c is increased, the curve is bent inwards (according to the dashed curve).

The median voter will thus choose values for t and c, such that (5) is maximized.[17] Simultaneously, he chooses a value of P_1, such that the ratio $P_2/(1-P_1)$ be minimized. This latter choice is equivalent to choosing the appropriate meaning of the phrase "proved guilty beyond a reasonable doubt", where "reasonable" means trading Type I errors against Type II errors. Since this trade-off only occurs at one place in the objective function (5), we see that regardless of the other policy variables, the optimal burden of proof is always equivalent to minimizing the ratio $P_2/(1-P_1)$. Naturally, this result is model-specific, but it is somewhat more general than one may first believe.[18]

[17] While the tax rate t is a policy variable directly at the policy-maker's disposal, the number of criminals c is only indirectly so. Choosing c means, by (3'), choosing a punishment φ consistent with the desired value of c. Note that φ is a function of c, but the reverse does not hold (there may be several values of c for each value of φ).

[18] In fact, the ratio occurs in several other models. For instance, we can conceive of a model where individuals meet in duplets, and where a worker can lose his money in two ways: either by simply dropping his wallet (with a given probability λ), or by being robbed if the other person in the duplet happens to be a criminal. The police cannot discriminate between these two events and thus, the other person in the duplet may be innocently suspected of having robbed the worker. Such a model, with separable utility, also gives rise to a median-voter utility function where the error probabilities occur in the form of a ratio $P_2/(1-P_1)$. This model represents a situation where there is uncertainty of whether a crime has been committed or not – a feature not uncommon in the history of crime (c.f., for instance, the famous Bülow case).

Depending on the curvature of the $P_2(P_1)$ function we could easily get corner solutions for the judicial procedure. As seen for Figure 1, a necessary condition for an inner solution ($0 < P_1 < 1, 0 < P_2 < 1$) is that $P_2(P_1)$ contains both concave and convex segments.[19]

3.2 An Explicit Conviction Technology

So far, we have only discussed P_1 and P_2 in general terms. Behind these abstract probabilities there is, however, a specific situation, which will be analyzed within the context of a signaling problem.

When a crime has taken place in a given triplet, the police arrive to find two persons standing there, next to the victim. One of these is the culprit, and one an innocent bystander – who may be a robber by profession (if the triplet is of the (C, C, W) type), but who happens to be innocent in this particular case. Each of these two persons sends out a signal; the culprit's signal is "1", while the bystander's signal is "0". The police, however, receives only a distorted version of these signals, i.e., "$1-\varepsilon$" from the culprit, and "ε" from the bystander. The random distortion ε is drawn from a known probability distribution with a density function $f(\varepsilon)$. The court looks at the signal received and weighs the evidence according to the following decision rule: If the signal received is below a threshold *s*, the person who emitted the signal is acquitted. If the signal received is equal to or above the threshold *s*, the person is convicted.

The critical value *s* represents the strictness of legal procedure. It is to be optimally chosen in order to maximize the median voter's utility (5). Different values of *s* attach different meanings to the phrase "proved guilty beyond a reasonable doubt" and thus correspond to different points on the (P_1, P_2) curve. For a given *s*, we have

(8) $$P_1 \equiv \Pr(1-\varepsilon < s), \quad P_2 \equiv \Pr(\varepsilon \geq s).$$

[19] This is seen by drawing a half-ray from the point ($P_1 = 1, P_2 = 0$) to any point on the curve in *Figure 1.* $P_2/(1-P_1)$ is minimized when the (numerical value of the) slope of that half-ray is minimized.

A high value of s means that the court requires a very strong signal in order to convict a person; thus the probability of a Type II error is low while the probability of a Type I error is high. Similarly, a low value of s means that the court is not scrutinizing the evidence very carefully; the probability of a Type II error is thus higher, and the probability of a Type I error is correspondingly lower. This is why we could regard P_2 as a function of P_1 in Section 3.1 above (where s had not yet been introduced in the notation). Note that if, in his optimization, the median voter chooses a low value of s, it may frequently be the case that both persons in the triplet are convicted, although we know that only one could possibly be guilty of the crime. Such cases are well documented in the history of criminal procedure. Similarly, if s is relatively high, which seems like the normal case in modern Western legal systems, both persons would often be acquitted, although we know that one of them must be guilty.

A simple way of obtaining a function with both convex and concave segments, like in Figure 1, is to assume the density function to be bimodal. To analyze this feature in a simple fashion, we take as our point of departure the density function

(9) $$\tilde{f}(\varepsilon) = (1+\alpha)(1-\varepsilon)^{\alpha}, \quad \varepsilon \in [0, 1] \ .$$

This particular parametrization is not chosen for its realism, but for its simplicity. For instance, by (8), it yields very simple expressions for the Type I and Type II probabilities:

$$P_1 = s^{1+\alpha},$$
$$P_2 = (1-s)^{1+\alpha}.$$

Parameter α is an index of skewness. For $\alpha < 0$, the density function is an upward-sloping function of ε; it displays a negative skewness (note that if $\alpha < 0$, then we must impose the additional constraint that $\alpha > -1$ for $\tilde{f}$ to be a density function). For $\alpha = 0$, the density function is rectangular (skewness = 0), and for $\alpha > 0$, it displays positive skewness. For each of these cases, there is a plausible story. The case of a positive skewness corresponds to the normal case where the distorted signal $(1-\varepsilon, \varepsilon)$ received by the police is likely to be rather similar to the original signal (1, 0) sent out from the triplet. The negatively skewed density represents the case where the offender manipulates the evidence in order to make himself look more innocent, and the bystander look guiltier, in the eyes of the police. This is the typical case in crime fiction – where even the physical environment in our model (a triplet containing one victim, one culprit and one innocent bystander) is not a mere parable for more realistic circumstances, but has a stark resemblance to the scene of the crime in classical crime novels. [20]

Now, we can conceive of a case where the police receive two stories, one from the offender (who tries to manipulate the evidence) and one from the bystander (who presumably tells a story closer to the truth). Thus, the police hear two versions of what has happened in the triplet. The two versions are somewhat distorted; the bystander might for some purely random reason also display personal traits making him seem suspicious in the eyes of the police. The stories are therefore represented by two probability densities, both of the general form (9), but one with positive and the other with negative skewness. The consolidated picture the police obtain is a convex combination of the two probability densities. We denote the weight attached by the police to the story from the innocent bystander by *k* and that attached to the story from the culprit by 1 – *k*. Thus, the probability density function of ε can be written as a weighted sum of two monotone functions:

$$f(\varepsilon) = k \cdot (1+\alpha_1)(1-\varepsilon)^{\alpha_1} + (1-k) \cdot (1+\alpha_2)(1-\varepsilon)^{\alpha_2}, \quad 0 < k < 1,$$

where $\alpha_1 > 0$ and $\alpha_2 < 0$. This function yields the following expressions for the Type I and Type II probabilities:

[20] In our model, the culprits do not really have any incentive to cast blame on others; all they want is to make themselves look innocent, i.e., to make their signal $(1-\varepsilon)$ fall below the critical threshold *s*. But since we have assumed the disturbance to be the same for the two persons, such a policy automatically increases the bystander's signal, ε; thus, there is a built-in incentive to cast blame on others. There are many other, equally plausible, model formulations dealing with this issue in different ways. For instance, it may be assumed that the culprit's and the bystander's signals are subject to different distortions, and that the police will thus observe $(1-\varepsilon_1, \varepsilon_0)$, where ε_1 and ε_0 are drawn from different probability distributions. Further, the court's *decision rule* affects the agent's incentive to cast blame on others. For instance, if, instead of looking at absolute signals $(1-\varepsilon_1, \varepsilon_0)$ and comparing them to the threshold *s*, the court looks at relative magnitudes (for example, by always convicting the person with the highest signal), it can benefit the perpetrator to cast the blame on the

$$(10) \quad \begin{aligned} P_1 &= k \cdot s^{1+\alpha_1} + (1-k) \cdot s^{1+\alpha_2}, \\ P_2 &= k \cdot (1-s)^{1+\alpha_1} + (1-k) \cdot (1-s)^{1+\alpha_2}. \end{aligned}$$

Now, the shape of the density function is endogenous; it depends on the amount of police resources assigned to the investigation of a particular crime. A simple way to represent this is to assume that *k* is increasing in the amount of police resources assigned to a particular crime; with more police resources, it will be more difficult for the true culprit to cast the blame on the innocent bystander. We assume the amount of police resources to be the same for all crimes. As noted in Section 3.1, the number of policemen per crime is *t/c*. Thus, we want *k* to be increasing in *t/c*; a simple function satisfying this, as well as the condition that $0 \leq k \leq 1$, is

$$(11) \quad k(t/c) = 1 - e^{-\beta t/c},$$

where β is a productivity parameter.[21] Inserting this into (10) above closes the model; maximizing the median voter's utility (5) subject to (10) and (11) yields the optimal values of the policy parameters *c*, *t*, and *s*.

4. SOME NUMERICAL PROPERTIES

Having parameterized the model in the way outlined above, numerical solutions are easily obtained. For instance, assuming linear utility $u(w(1-t)) \equiv w(1-t)$, and choosing the parameter values $a = 1$, $w = 10$, $\sigma = 0.01$, $\beta = 1$, $\alpha_1 = 5$ and $\alpha_2 = -0.5$, yields the following solution for the endogenous variables:

Criminal procedure: $s = 0.786$

Tax rate: $t = 0.032$

Punishment: $\varphi = 471$

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bystander. The optimal choice of a decision rule is an interesting extension of the model that will not be further pursued here.

Number of criminals: $c = 0.013$

Probability of Type I Error = 0.291

Probability of Type II Error = 0.040

This particular parameter configuration thus yields a criminal procedure that seems rather similar to that of modern Western societies, with a high value of s and the probability of Type II errors correspondingly close to zero. Whether the values chosen for the exogenous parameters are realistic or not is beyond the scope of this paper; the above solution is intended as an illustration (and an existence proof) only.

Trying other configurations shows that criminal procedure is sometimes very sensitive to changes in the exogenous parameters. For one configuration we may get an s close to unity, while for a slightly different configuration we get an s close to zero. In terms of parameter values, there may thus be only a small step between the judicial systems of Western society on one hand and those of medieval Europe, or of Stalinist Russia, on the other hand.[22] An interesting question, also beyond the scope of this paper, is whether actual changes in the legal system – for instance, the increase in s and the fall in φ that we have witnessed since the Middle Ages – can be replicated in the context of the present model, by entering realistic figures for, e.g., real wage growth, police productivity, etc.

5. CONCLUDING COMMENTS

Let us first summarize our results. We have developed a general equilibrium model of crime, with a conviction technology allowing for innocent persons being convicted. In such a model, not only the severity of punishment and the amount of police resources, but also criminal procedure, can be treated as endogenous. We have also shown that although there are two types of agents in the model, with individuals endogenously switching from one type to another, the median voter theorem still applies. Solving

[21] There are many other ways of modeling this. For instance, instead of assuming k to be a function of t/c, we could assume that parameters α_1 and α_2 are functions of t/c.

[22] Granted, neither punishment in medieval Europe nor Stalinist Russia was the results of a median voter's optimization. These terms are used here only to give a local habitation and a name to theoretic properties of our model.

the model, it turns out that capital punishment is not optimal. This resolution of the Becker paradox is not only due to the possibility of errors of Type II. In addition, a hard core of unalterable criminals is required. Paradoxically, the existence of such a hard core thus makes it optimal to have more lenient punishments in society. It is therefore possible that the movement towards a more liberal policy observed in many countries over the last century is due to the existence of such a hard core being increasingly acknowledged. One would think that the introduction of more lenient punishment would be the result of policy-makers thinking that people can easily be deterred – but it might as well be the other way around.

A strategic simplification in our model lies in the particular encounter and conviction technologies used. As for the former, we used the setup with triplets as the simplest way of modeling proximity. For the conviction technology, it would be an interesting task to investigate other types of signaling mechanisms, decision rules for the courts, and technologies by which police resources affect the probabilities of the two error types. This aspect points in two different directions. First, it is not evident that police resources should be evenly spread across all crimes. Concentrating resources to a few crimes, while disregarding the others, might instead be an optimal way of trading Type I errors against Type II errors. The simplest way of doing this would be by a purely random device: the police only investigate x percent of all crimes, while writing off the rest (this is a behavior we sometimes seem to observe in the real world). A more intriguing policy would be to condition police resources on some observable feature, like the signal received ($1-\varepsilon$ for the culprit and ε for the innocent bystander). The decision rule would then be to concentrate resources to those crimes where the observed signal is particularly strong (or weak). Furthermore, punishment could also be a more general function of the signal received, and not only the simple step function $\varphi = 0$ for signal $< s$ and φ = constant for signal $> s$.

Second, it would be tempting to assume that police resources are primarily concentrated to individuals with a previous record of criminal activities. Such a formulation would, however, introduce a state variable into the model and thus make the analysis much more complicated. Accepting that additional complication opens a rich field for future research. For instance, not only could police investigations be made contingent on the individual's track record, but so could criminal procedure. In our notation, the optimal threshold s would thus be different for different persons – a feature that is not allowed in modern Western societies (at least not officially), but is well known from other historical periods and other societies.

Let us finally say a few words about empirical testing. In the real business cycle literature, the point of departure is a stylized fact observed in the real world, for instance a positive correlation between two macroeconomic variables. The issue is then whether one can construct a general equilibrium model such that, when exposed to a particular set of disturbances, it would generate a similar correlation between these variables. This would, in principle, also be possible in the economics of crime. For instance, if we observed that the severity of punishment (φ) is positively correlated with procedural strictness (s), across countries or over time, we would ask whether there is some stochastic process of disturbances in the deep parameters of our model that would generate the same correlation. Unfortunately, most of the variables used in the economics of crime are only observable with great difficulty. Although we may have a feeling for some societies employing a more rigorous standard of proof, or providing more severe punishment than others, it would be very costly indeed to provide hopeful researchers with reliable data on these variables. These costs in themselves constitute a major explanation why there is a trade-off between Type I and Type II errors – and why this trade-off, although difficult to quantify, is so important for our understanding of society.

APPENDIX

1. *Some Properties of the General Equilibrium Solution*

In this appendix we will analyze the general properties of equation (3'). First, we note that the φ function has two asymptotes where $\varphi \rightarrow \infty$, namely one for $c \rightarrow 1$ and one for $c \rightarrow \sigma$. Its general shape is depicted in Figure A1.[23] From the existence of the two asymptotes, and the fact that φ is a continuous function, we know that for any value of φ such that (3') has a solution $\sigma < c < 1$, there must be at least two solutions[24] (there could be more than two solutions, due to the P_1 function). Rearranging and completing the square terms, (3') could be written as

$$\text{(A.1)} \quad c = \frac{1+\sigma}{2} \pm \sqrt{\frac{(1-\sigma)^2}{4} - \sigma - \frac{(1-\sigma)a}{(1-P_1)\varphi}} \,,$$

where the expression under the root sign must be non-negative for a real solution to exist. Note that P_1 is a function of *c*. Thus, (A.1) is not a closed-form solution to (3'); it merely characterizes the solution. We will use this characterization to show that (disregarding the minimum point in Figure A1, where the two roots coincide) at least one of the solutions satisfies $c < (1+\sigma)/2$.

(Figure A1)

If P_1 had merely been a constant, then it would immediately have been evident from (A.1) that one root satisfies $c < (1+\sigma)/2$. But since P_1 is a function of *c*, this requires some more consideration. Note, however, that if we can show that there exists a solution associated with the minus sign in (A.1), then we have proved our conjecture.

[23] There might be some wiggles at the bottom of the curve, corresponding to additional roots due to the non-linearities caused by the $P_1(c)$ function. We have, however, performed a large number of numerical simulations of the model (cf. Section 4) without detecting any such wiggles. All additional roots, in excess of those depicted in *Figure 1*, occur outside the admissible domain $\sigma \leq c \leq 1$.

[24] The feed-back effects of general equilibrium models of crime often generate multiple solutions in the fraction of criminals; see for example Murphy et al. (1993), Huang et al. (2004) and Burdett et al (2004).

Assume that there exists a solution $c_1 > (1+\sigma)/2$. This solution must therefore be associated with the plus sign in (4') and it is then straightforward to show[25] that there cannot exist an additional root $c_2 > (1+\sigma)/2$. Since we know that (3') has at least two roots, there must be at least one root associated with the minus sign in (A.1). This proves our conjecture: there exists at least one root $c < (1+\sigma)/2$. Further, the roots must be located in the fashion depicted in Figure A1, with the minimum point of the $\varphi(c)$ curve located to the left of the $(1+\sigma)/2$ line.

The model thus has (at least) two solutions: one equilibrium with many criminals (a high *c*), and one equilibrium with few criminals (a low *c*). Since all parameter values are the same, the two equilibria differing only by the former having more criminals and a lower GDP, it is tempting to assume that such an equilibrium can never be efficient. This is not true, however; the equilibrium with a high *c* means that relatively few crimes are committed (since there are so few potential victims). Thus, the probability of being innocently convicted is relatively low in such an equilibrium, and we cannot *a priori* rule out the possibility of this dominating over the higher probability of being robbed (for a worker) or the lower probability of finding a victim (for a criminal). However implausible it may sound, we cannot therefore say that the equilibrium with a low *c* is always preferred to that with a high *c*.

Instead, we rely on a stability argument to rule out the equilibrium with a high number of criminals. We will show that if there exists a root $c > (1+\sigma)/2$, then that root must be unstable.[26] In our the paper, we therefore only consider equilibria with a low *c*, characterized by $c < (1+\sigma)/2$. Since σ, the number of hard criminals is assumed to be quite small, we will only consider solutions[27] such that $0 < c < ½$. In fact, in the numerical simulations in Section 4, *c* will be in the order of magnitude of a few

[25] The expression under the root sign in (A.1) is decreasing in P_1, while P_1, in turn, is increasing in *c* (cf. section 3.2, where we explicitly formulate the P_1 function). Thus, the right-hand side in (4') is decreasing in *c*. If a solution $c > (1+\sigma)/2$ exists, it must satisfy (A.1) with the plus sign. Increasing *c* somewhat will increase the left-hand side, while decreasing the right-hand side of (A.1), and vice versa for reducing *c*. Thus, if there is a solution $c > (1+\sigma)/2$, no other such solution can exist.

[26] For a similar result in a different model of crime and punishment, see Persson and Siven (2006).

[27] If there are multiple stable equilibria, there must be some coordination of expectations to drive the economy towards one of these.

percent.[28] This has implications for the political equilibrium to be discussed in the next section; it means that the median voter will always be a worker.

From equations (1) and (2), we see that the utility difference between being a criminal and a worker, for an individual with time preference δ, is

$$V_c(\delta) - V_w(\delta) = \frac{1}{1-\delta}\left[a - \delta(1-c)(1-P_1)\varphi\right] \equiv F(\delta).$$

Let us now look at the marginal individual, i.e., the individual with a time preference rate, $\tilde{\delta}$ such that there are exactly c individuals with $\delta \le \tilde{\delta}$ and $(1 - c)$ individuals with $\delta \ge \tilde{\delta}$. Thus, we know from (3) that $\tilde{\delta} = (c - \sigma)/(1 - \sigma)$. Note that if c is the equilibrium c, i.e., one of the roots of equation (3'), then $\tilde{\delta} = \hat{\delta}$, where $\hat{\delta}$ is the cut-off rate defined by $V_c(\hat{\delta}) = V_w(\hat{\delta})$. So far, however, c could be any number, such that $\sigma \le c \le 1$.

Assume that there is a dynamic adjustment such that if the marginal individual finds it more attractive to be a criminal than a worker, i.e., if $F(\tilde{\delta}) > 0$, then the number of criminals will increase:

$$\dot{\tilde{\delta}} = h(F(\tilde{\delta})),$$

where $h(.)$ is an increasing function. For the model to be stable, a larger number of criminals should mean a lower utility differential $F(\tilde{\delta})$ for the marginal individual. Approximating $F(\tilde{\delta})$ around the equilibrium $\hat{\delta}$ yields

$$F(\tilde{\delta}) \approx F(\hat{\delta}) + \frac{\partial F(\hat{\delta})}{\partial \tilde{\delta}}(\tilde{\delta} - \hat{\delta}).$$

Since $F(\hat{\delta}) = 0$ by the definition of $\hat{\delta}$, we have

[28] Note, however, that this refers to equilibrium values of c; for a full analysis, we have to compare out-

$$\text{sgn } \dot{\hat{\delta}} = \text{sgn } \frac{\partial F(\hat{\delta})}{\partial \tilde{\delta}} (\tilde{\delta} - \hat{\delta}) .$$

If we can show that $\partial F(\hat{\delta}) / \partial \tilde{\delta} > 0$, we then know that in a small neighborhood of $\hat{\delta}$, any $\tilde{\delta} > \hat{\delta}$ will continue to increase further and further away from $\hat{\delta}$. Thus, the root *c* associated with $\tilde{\delta}$ by the relation $c = \sigma + \tilde{\delta}(1 - \sigma)$ will be unstable.

After some calculation, and taking into account that $F(\hat{\delta}) = 0$, we have

$$\frac{\partial F(\hat{\delta})}{\partial \tilde{\delta}} = \frac{1}{1 - \hat{\delta}} \left[\varphi \cdot (1 - \sigma)^2 (1 - \hat{\delta}) \hat{\delta} \frac{\partial P_1}{\partial c} + \varphi \cdot (1 - P_1)(1 - \sigma)(2\hat{\delta} - 1) \right].$$

We know that $\partial P_1 / \partial c > 0$; thus the first term within square brackets is always positive. The second term is positive if $2\hat{\delta} > 1$. Since $\hat{\delta} = (c - \sigma)/(1 - \sigma)$, this condition is equivalent to

$$c > \frac{1 + \sigma}{2} .$$

This completes our proof: if equation (3') has a root larger than $(1 + \sigma)/2$, that root must be unstable.

2. *The Median Voter Theorem*

When proving that the median voter will be the decisive voter in this model, we will study two policy vectors: q^m, which is the policy vector preferred by the median voter, and *q*, which is an arbitrary policy vector. By equation (4'), we only deal with solutions of the model such that $c < ½$, which means that the median voter will always be a worker.

By the definition of q^m, we have

of-equilibrium values, too.

$$J(q^m) + \delta^m \cdot H(q^m) \geq J(q) + \delta^m \cdot H(q)\,,$$

where the *J* and *H* functions are defined by the worker's utility function (1), multiplied by $(1-\delta^m)$, and where δ^m is the median worker's rate of time preference. This expression can be written as

(A.2) $$J(q^m) - J(q) \geq \delta^m \left[H(q) - H(q^m)\right].$$

Let us first look at a *q*, such that $J(q^m) - J(q) \leq 0.$ This means that the right-hand side must also be non-positive. Since δ^m is a positive number, inequality (A.2) will also hold for all $\delta \geq \delta^m$ (note that everyone with a $\delta \geq \delta^m$ is also a worker). Thus, for all *q* such that $J(q^m) - J(q) \leq 0$, all individuals with a time preference $\delta \geq \delta^m$ will side with the median voter if confronted with the two policy alternatives, *q* and q^m. Since these individuals constitute 50% of the population, the median voter's preferred policy q^m will be a Condorcet winner.

Let us now turn to policies *q,* such that $J(q^m) - J(q) \geq 0$. This case is slightly more complicated than that above, and it is helpful to graphically illustrate the various combinations of time preference rates on [0, 1]. In Figure A2, we show the median voter's time preference δ^m, the time preference $\hat{\delta}_{qm}$ of an individual who is indifferent between being a worker and a criminal if the median-voter's preferred policy q^m is implemented, and the time preference $\hat{\delta}_q$ of an individual who is indifferent between being a worker and a criminal, if an arbitrary policy *q* is implemented. Since we here only deal with the case where the median worker is a worker, we have that $\hat{\delta}_{qm} < \delta^m$. We first assume that $\hat{\delta}_q < \hat{\delta}_{qm}$, and will deal with the opposite case later.

(Figure A2)

We will now study individuals in all three line segments to the left of δ^m in Figure A2. Let us first look at an arbitrary person with a δ such that $\hat{\delta}_{qm} \le \delta \le \delta^m$. This person will be a worker both at q and q^m. Whether he will side with the median voter depends on the sign of the expression

$$J(q^m) + \delta \cdot H(q^m) - [J(q) + \delta \cdot H(q)].$$

This can be written as

(A.3) $$J(q^m) - J(q) + \delta \cdot [H(q^m) - H(q)].$$

By (A.2) we know that this is non-negative for $\delta = \delta^m$. By assumption, $J(q^m) - J(q) \ge 0$. If the expression within square brackets in (A.3) is also non-negative, then the proof is complete; expression (A.3) is non-negative. If the expression within square brackets is negative, we know from (A.2) and the fact that $\delta < \delta^m$ that (A.3) must still be non-negative; thus the person with $\delta \in [\hat{\delta}_{qm}, \delta^m]$ will support the median voter's preferred policy.

Let us now look at the intermediate line segment in Figure A2. We thus deal with a person with a δ such that $\hat{\delta}_q \le \delta \le \hat{\delta}_{qm}$, i.e. who will be a worker at q, but a criminal at q^m. If such an individual is to support the median voter's preferred policy against any policy q such that $J(q^m) - J(q) \ge 0$, then the expression

(A.4) $$\tilde{J}(q^m) + \delta \cdot \tilde{H}(q^m) - [J(q) + \delta \cdot H(q)]$$

must be non-negative. Here, the $\tilde{J}$ and $\tilde{H}$ functions are defined by the criminal's utility function (2), multiplied by $(1-\delta)$. To show that (A.4) is, in fact, non-negative, we first need some properties of the J, H, $\tilde{J}$ and $\tilde{H}$ functions. Since the switch-point individual $\hat{\delta}_{qm}$ is indifferent between being a criminal and a worker at q^m, we have

$$\tilde{J}(q^m) + \hat{\delta}_{qm}\tilde{H}(q^m) = J(q^m) + \hat{\delta}_{qm}H(q^m).$$

This expression, together with the definitions of the J and $\tilde{J}$ functions by (1) and (2), gives us

(A.5) $$\tilde{J}(q^m) = J(q^m) + a, \quad \tilde{H}(q^m) = H(q^m) - \frac{a}{\hat{\delta}_{qm}}.$$

We substitute (A.5) into (A.3) to obtain

$$\left[J(q^m) - J(q) + \delta\left(H(q^m) - H(q)\right)\right] + a\left(1 - \frac{\delta}{\hat{\delta}_{qm}}\right).$$

The first term in this expression (within square brackets) is obviously non-negative by (A.2) and the fact that $\delta < \delta^m$. The second term is non-negative, since $\delta < \hat{\delta}_{qm}$. Thus, expression (A.4) is non-negative, which means that a person on the line segment $\left[\hat{\delta}_q, \hat{\delta}_{qm}\right]$ in Figure A2 will also support the median voter's preferred policy.

Finally, we look at a person with a δ, such that $0 \le \delta \le \hat{\delta}_q$. This is an individual who will be a criminal at both q and q^m. For such a person to prefer q^m to an arbitrary policy vector q such that $J(q^m) - J(q) \ge 0$, the expression

(A.6) $$\tilde{J}(q^m) + \delta \cdot \tilde{H}(q^m) - \left[\tilde{J}(q) + \delta \cdot \tilde{H}(q)\right]$$

must be non-negative. To show that this is indeed the case, we first note that for individual $\hat{\delta}_q$, who is indifferent between being a criminal and a worker at the policy vector q, the following must hold:

$$\tilde{J}(q) + \hat{\delta}_q \tilde{H}(q) = J(q) + \hat{\delta}_q H(q).$$

This, together with (1) and (2), means that

(A.7) $\tilde{J}(q) = J(q) + a, \quad \tilde{H}(q) = H(q) - \dfrac{a}{\hat{\delta}_q}.$

Inserting this into (A.6), taking account of (A.5), and rearranging terms, yields

$$\left[J(q^m) - J(q) + \delta\left(H(q^m) - H(q)\right)\right] + \delta\left(\frac{a}{\hat{\delta}_q} - \frac{a}{\hat{\delta}_{qm}}\right).$$

The first term (within square brackets) is non-negative by (A.3). The sign of the second term depends on whether $\hat{\delta}_q < \hat{\delta}_{qm}$. But we see that for $\delta = 0$, the sign of that term is irrelevant to the sign of (A.6). Further, if we let δ increase from zero to $\hat{\delta}_q$, the sign of (A.6) is not affected either, since we have proved above that a person at the switch-point $\hat{\delta}_q$ will prefer q^m to *q*. Thus, letting δ increase will not cause (A.6) to change signs. Accordingly, (A.6) is non-negative; an individual with a time preference rate of δ in the left line segment of Figure A2 will prefer q^m to any other policy *q*, such that $J(q^m) - J(q) \geq 0$.

For the parameter configuration in Figure A2, we have thus shown that the median voter's preferred policy will be a Condorcet winner over all other policies *q* such that $J(q^m) - J(q) \geq 0$. It now remains to show that this is also the case if we change the order of $\hat{\delta}_q$ and $\hat{\delta}_{qm}$, so that $\hat{\delta}_q > \hat{\delta}_{qm}$ as displayed in Figure A2. This is straightforward; for the rightmost (i.e., $\delta \geq \hat{\delta}_q$) and leftmost (i.e., $\delta \leq \hat{\delta}_{qm}$) line segments, the proof will be the same as before (note that we just showed that it is immaterial for the sign of (A.6) whether $\hat{\delta}_q > \hat{\delta}_{qm}$. For the intermediate case (i.e., $\hat{\delta}_{qm} \leq \delta \leq \hat{\delta}_q$), we want to investigate the sign of

(A.8) $J(q^m) + \delta \cdot H(q^m) - \left[\tilde{J}(q) + \delta \cdot \tilde{H}(q)\right].$

Taking account of (A.7), this can be written as

$$\left[J(q^m) - J(q) + \delta\left(H(q^m) - H(q)\right)\right] + a\left(\frac{1}{\hat{\delta}_q} - 1\right).$$

The first term is non-negative by (A.3), while the second term is non-negative since $\hat{\delta}_q \leq 1$. Thus (A.8) is non-negative; the policy vector q^m will be a Condorcet winner also in this case.

(Figure A2)

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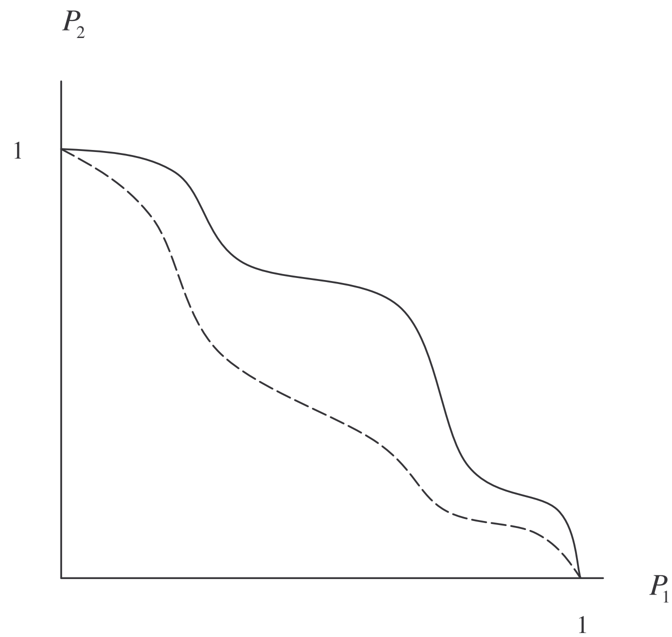

Figure 1: The general shape of the P_1, P_2 function.

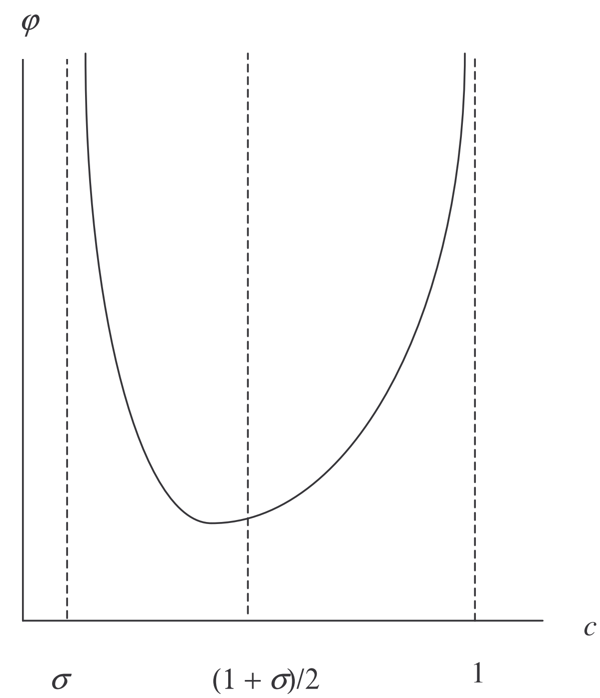

Figure A1: The image of equation (3’) in (c, φ) space.

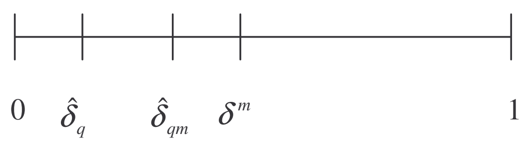

Figure A2: Various combinations of discount factors on [0, 1].