Cost–benefit analysis and the marginal cost of public funds

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Abstract

The marginal cost of public funds defined as the ratio between the shadow price of tax revenues and the population average of the social marginal utility of income, is analysed within an explicit cost–benefit context. It is shown that for an optimal tax system the measure is always equal to one. Benefit and cost measures congruent with this definition are derived. Under optimal taxes a positive net social benefit is a necessary and sufficient condition for a project that passes the cost–benefit test. Under non–optimal taxes this is not the case: If taxes are too low a positive net social benefit is a necessary but not sufficient condition and if taxes are too high a sufficient but not necessary condition for an accepted project.

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1 Introduction

The term 'marginal cost of public funds' (MCPF) is a problematic term in the cost–benefit literature. Different economic concepts originating from different research questions have come to compete about the term. Most frequently, however, it seems that the marginal cost of public funds is the answer to the following question: *By which factor should the marginal resource cost of a public project be scaled to take into account that the project is financed through distortionary taxation?* Or in the words of Arthur Pigou:

> "The raising of an additional £ of revenue necessitates increasing the tax rates at which taxation is imposed, either now or (if resort has been had to loans) subsequently. With some sorts of taxes this inflicts indirect damage on the tax payers as a body above the loss they suffer in actual money payment. When there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure."
> Pigou (1947, p. 33–34)

Although it may seem to be a straight forward question to answer this is not the case. Ballard and Fullerton (1992) identified two different traditions; the Harberger-Pigou–Browning–tradition in which the marginal cost of public funds is always larger than unity and the Dasgupta–Stiglitz–Atkinson–Stern–tradition in which it may be larger or lower than one. In the first tradition the marginal project is a lump sum transfer to a representative consumer financed by a distortionary tax. A marginal cost of public funds greater than unity then occurs because the dead–weight loss of taxation. In the second the marginal project is arbitrarily defined. The size of the marginal cost of public funds then depends on factors such as, e.g., whether the tax system is optimal or if non–optimal which the marginal source of financing is. It may even be lower than unity because then a small perturbation of the tax system will create income effects that may increase government tax revenues (due to normality of leisure). Ballard and Fullerton (1992) also illustrated that the fact that the MCPF may never be lower than unity in the first tradition but may be lower in the second has caused considerable confusion among economists. But it is clear that the first intuition behind Pigou's statement is not generally correct.

Regardless of tradition it seems, however, that the starting point is an ideal situation where a policy maker has access to a first best tax system. If taxes are not ideal in this sense, how must the criteria for a correct decision be altered due to the presence of the distortionary tax system? If b' is the private marginal benefit

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and c' the private marginal cost of a project, then the optimal project size for under a first best tax system satisfies the condition

$$b' = c'. \tag{1}$$

If taxes are distortionary, the perspective in the literature related to marginal cost of public funds has been to rewrite this cost-benefit test, typically as

$$b' = \hat{\eta}C', \tag{2}$$

where $\hat{\eta}$ is the marginal cost of public funds and C' is the marginal shadow cost of the project. Still, the left–hand side only consists of the private marginal benefits. In a sense, therefore, the perspective is a mix between of a private decision maker and a public policy maker. This is reflected in the MCPF which captures the trade-off between the value of additional tax revenues to the policy maker versus the value to individual of additional lump sum income; i.e, the ratio between the marginal shadow price of tax revenues and the individual marginal utility of income.

This paper takes as its starting point another quotation from Pigou:

> "The government is not, therefore, simply an agent for carrying out on behalf of its citizens their several separate instructions; it cannot simply balance at the margin each man's desire to buy battleships against his desire to by clothes, in the way the individual that an individual balances his desire for clothes against his desire for coal. As the agent of its citizens collectively, it must exercise coercion upon them individually, securing the funds it needs either by a contemporary tax or by a loan associated by a subsequent tax to provide for interest and sinking fund."
> Pigou (1947, p. 33)

Then, to capture that the cost–benefit decision is made by a policy maker using coercion we let the presence of distortionary taxation affect both the benefit side and the cost side, so that the test becomes

$$B' = \eta C', \tag{3}$$

where B' is the social marginal benefit, C' is the social shadow cost of the project and where η is called *social marginal cost of public funds* (SMCPF). The SMCPF is different from the MCPF and captures the trade-off between the value of additional tax revenues to the policy maker versus the *policy maker's valuation* of additional income to the individual. Which are the benefits to society of providing

the individual with a marginal unit of lump sum income? The answer to this question is given by *social marginal utility of income* introduced by Diamond (1975). The SMPC can therefore be defined to be the ratio between the marginal shadow price of tax revenues and the social marginal utility of income to the representative individual. As consequence we have on the benefit side the social marginal benefit defined as the trade-off between the social marginal value of the project (which we assume equals the private marginal utility) versus social marginal utility of income.

This definition of the social marginal cost of public funds will then, within a representative individual framework, be identical to a measure of marginal cost of public funds given by Håkonsen (1998, equation (7)). In this paper a generalisation to an economy with a heterogeneous population is offered, so that SMPC is defined as the ratio between the marginal shadow price of tax revenues and the average social marginal utility of income in the economy. Also we analyse optimal as well as non–optimal taxes. The fact that Håkonsen (1998) analysed his measure in a representative individual framework *and* assumed optimal taxes (in a very specific and restricted sense) will make his measure have different properties compared to the the generalised measure analysed here. In this paper this definition is put into the context of an explicit cost–benefit problem and generalised to an economy with a heterogeneous population, so that SMPC is defined as the ratio between the marginal shadow price of tax revenues and the average social marginal utility of income in the economy.

Whereas the the measure analysed by Håkonsen (1998) always was larger than unity (unless the elasticity of substitution is zero and the measure equals unity), the generalised social marginal cost of public funds will always be equal to unity for any optimal tax system. This means even if a tax system is highly distortionary, the social marginal cost of public funds is equal to one if the tax system is optimal. Also, if taxes are non–optimal the measure is not restricted to be above unity. The unity result under an optimal tax system is in a sense not surprising since the social marginal cost of public funds (defined for any specific model) being equal to unity can be shown to be a necessary condition for an optimal tax system in the same model. The reason is that a necessary condition for optimality is that it should not be possible to increase the value of the policy maker's objective function by disturbing the tax system (i.e., move income from the individual to the public sector or vice versa). Since the change in the objective function of moving lump sum income on the margin from the individual to the public sector is the marginal shadow price of tax revenues minus the social marginal utility of income the result follows. The social marginal cost of public funds under any optimal tax system will therefore have the same property as the marginal cost of public funds under a first best optimal tax system, i.e., be equal to unity.

Does this mean that there is no role for the concept of marginal cost of public funds? The answer is no for the simple reason that, of course, taxes may not be optimal. Then the interpretation is that the social marginal cost of public funds measures the deviation from an optimal tax system: If the overall level of taxation is in a sense too high, then the social marginal cost of public funds is less than a critical value and the cost benefit test tend to accept marginal projects for which the social marginal benefit is less than the marginal shadow cost. If the overall level of taxation is too high the social marginal cost of public funds is higher than a critical value and the cost benefit test tend turn down marginal costs unity. Hence, the social marginal costs should be scaled down. Similarly, if the overall burden of taxation is too small in the sense that a marginal lump sum tax is desirable then social marginal costs should be scaled up. That is, if optimal taxes are out of reach for the policy maker then the public projects should be used to compensate for that.

Accordingly, under non–optimal taxes a new levels issue is introduced. Instead of the problem whether a second best optimal taxation implies a lower level of the public project compared to first best we can now ask whether a certain type of non–optimal taxes implies a higher or lower level of the public project compared to the second best optimal solution.

The paper is organised as follows: In section 2 the model is presented. The social marginal cost of public funds is defined in section 3 and the social marginal benefits and costs in section 4. The relevant criterion for cost–benefit analysis under an optimal tax system is derived in section 5. The same criterion under a non–optimal tax system is derived in section 6. Section 7 contains conclusions.

2   Model

The model used in this paper is exactly the model analysed by Sandmo (1998). In this economy there is a private good x, labour supply ℓ and a public good project g. The private good is produced by a linear production technology so that the producer prices are fixed. User charges for the public good are assumed to be unfeasible on technical or political grounds. Therefore, the public project is financed through taxation. With two taxable commodities we express the problem as if labour income is taxed. We assume that the policy maker is restricted to use a linear income tax with the lump tax parameter α and the proportional tax parameter β.

There are n individuals all who have the same preferences represented by the strictly concave utility function $u : \mathbb{R}_+^3 \to \mathbb{R}$ defined by $u^i = u(x_i, \ell_i, g)$. The policy maker is assumed to have choosen that specific cardinal transformation of the individuals' utility function which reflects the policy maker's degree of inequality

aversion. An individual then solves the problem

$$\max_{x_i, \ell_i} \quad u(x_i, \ell_i, g) \text{ s.t. } (1 - \beta)w_i\ell_i = x_i + \alpha, \tag{4}$$

where the price on private consumption is numeraire and normalised to unity and w_i is the wage rate (labour productivity) of type $i = 1, \ldots, n$. The Lagrangian function can be written as

$$L(x_i, \ell_i, \lambda_i) = u(x_i, \ell_i, g) + \lambda_i \left[(1 - \beta)w_i\ell_i - px_i - \alpha \right], \tag{5}$$

where λ_i is the Lagrange–multiplier. The first order conditions are

$$\frac{\partial L}{\partial x_i} = u_x^i(x_i^*, \ell_i^*, g) - \lambda_i^* p = 0, \tag{6a}$$

$$\frac{\partial L}{\partial \ell_i} = u_\ell^i(x_i^*, \ell_i^*, g) - \lambda_i^*(1 - \beta)w_i = 0 \text{ and} \tag{6b}$$

$$\frac{\partial L}{\partial \lambda_i} = (1 - \beta)w_i\ell_i^* - px_i^* - \alpha = 0, \tag{6c}$$

where $x_i^* = x_i(\alpha, \beta, g)$, $\ell_i^* = \ell_i^*(\alpha, \beta, g)$ and $\lambda^* = \lambda(\alpha, \beta, g)$, the latter being the marginal utility of income in the optimal point. We get the indirect utility function by inserting the optimal demand for private consumption and optimal labour supply into the direct utility function:

$$V^i(\alpha, \beta, g) = u(x_i^*, \ell_i^*, g) \tag{7}$$

Applying the Envelope theorem we have

$$V_\alpha^i = -\lambda_i^* < 0, \tag{8a}$$

$$V_\beta^i = -\lambda_i^* y_i^* < 0 \text{ and} \tag{8b}$$

$$V_g^i = \frac{\partial u^i}{\partial g} > 0, \tag{8c}$$

where $y_i^* = w_i\ell_i^*$. The Slutsky equation is $\frac{\partial \ell_i^*}{\partial \beta} = \frac{\partial \ell_i^S}{\partial \beta} - y_i^* \frac{\partial \ell_i^*}{\partial y}$.

Suppose the policy maker has choosen the policy optimally. In such cases the government chooses the tax system and the size of the public project so as to maximise the sum of individual indirect utility subject to the government's budget constraint:

$$\max_{\alpha, \beta, g} \quad \sum_{i=1}^n V^i(\alpha, \beta, g) \text{ s.t. } \sum_{i=1}^n [\alpha + \beta w_i\ell_i^*] = c(g), \tag{9}$$

where $c(g)$ the total direct cost of the public project g. That is, this is the cost that would have been incurred by a private decision maker undertaking the project.

The function $c : \mathbb{R}^n_+ \to \mathbb{R}$ is assumed to be strictly increasing in g and strictly concave. The Lagrangian function to this problem is

$$K(\alpha, \beta, g, \kappa) = \sum_{i=1}^n V^i(\alpha, \beta, g) + \kappa \left[\sum_{i=1}^n (\alpha + \beta w_i \ell_i^*) - c(g) \right], \qquad (10)$$

where κ is the Lagrange multiplier (i.e., the marginal social welfare or shadow price on additional tax revenue). The first order conditions for an interior solution are

$$\frac{\partial K}{\partial \alpha} = \sum_{i=1}^n \left[-\lambda_i^* + \kappa^* \left(1 + \beta w_i \frac{\partial \ell_i^*}{\partial \alpha} \right) \right] = 0, \qquad (11a)$$

$$\frac{\partial K}{\partial \beta} = \sum_{i=1}^n \left[-\lambda_i^* y_i^* + \kappa^* \left(y_i^* + \beta w_i \frac{\partial \ell_i^*}{\partial \beta} \right) \right] = 0, \qquad (11b)$$

$$\frac{\partial K}{\partial g} = \sum_{i=1}^n V_g^i + \kappa^* \left[\sum_{i=1}^n \beta w_i \frac{\partial \ell_i^*}{\partial g} - c' \right] = 0 \text{ and} \qquad (11c)$$

$$\frac{\partial K}{\partial \kappa} = \sum_{i=1}^n \left[\alpha + \beta w_i \ell_i^* \right] - c(g) = 0. \qquad (11d)$$

3   The social marginal cost of public funds

Generally there is a great degree of arbitrariness in the definitions of the marginal cost of public funds (MCPF). On reason is different views on how indirect effects of the public project and its financing on tax revenues should be treated. Some of the early literature included these feed–back effects into the definition of the marginal costs of public funds, making this concept depending not only on the marginal financing instrument but also the the public project that the tax increase financed; see, e.g., Hansson (1984).[1] The alternative, represented by, e.g., Sandmo (1998), is to make a distinction between the marginal cost of public funds and the social marginal cost of the project (i.e., its marginal shadow price) and include the feedback effects in the latter. Regardless of this, however, the typical question in the literature is the following: The government is changing a certain tax so that tax revenues increase with one dollar which is spent on a public project. Which is the policy maker's valuation of the additional tax revenue, when its alternative cost is measured as the *representative consumer's valuation of the margin unit of income*?

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[1]Hansson (1984) clearly derives project specific marginal cost of public funds. The above mentioned indirect effects may only, however, be present in the in the case of infrastructure investments. The extent to which that is the case is, however, undocumented.

In this paper a different question is asked: Which is the policy maker's valuation of the additional tax revenue, when its alternative cost is measured as the *policy maker's valuation of marginal unit of income to the representative consumer*? This different question will lead to a different definition of the marginal cost of public funds, called the *social marginal cost of public funds* (SMCPF) which will allow us to present the government's cost–benefit test in an intuitive way (see sections 4 and 5) as well as relating this SMCPF to the optimality properties of the tax system (see below and section 6).

To define the SMCPF we need two things, (i) the marginal social welfare of tax revenues when a specific tax instrument is used to raise the revenue and (ii) the policy maker's evaluation of the alternative use of the additional dollar of tax revenues raised. For the present model we can define the marginal social welfare of tax revenues for the two policy instruments α and β as

$$\kappa_\alpha := \frac{\sum_{i=1}^{n} \lambda_i^*}{\sum_{i=1}^{n} \left(1 + \beta w_i \frac{\partial \ell_i^*}{\partial \alpha} \right)} \text{ and} \tag{12a}$$

$$\kappa_\beta := \frac{\sum_{i=1}^{n} \lambda_i^* y_i^*}{\sum_{i=1}^{n} \left(y_i^* + \beta w_i \frac{\partial \ell_i^*}{\partial \beta} \right)}. \tag{12b}$$

In an optimal tax system the marginal social welfare of tax revenues must be same for all financing sources; else the tax system is not optimal and the policy maker could change the composition of taxes in order to increase social welfare. It then follows from (11a) and (11b) that under an optimal tax system

$$\kappa_\alpha = \kappa_\beta = \kappa^*, \tag{13}$$

where κ^* is the Lagrange multiplier in the policy maker's decision problem evaluated in the optimal point.

To a private individual the value of a unit of lump sum income is the marginal utility of income. Under a first best tax system this private valuation coincides with the policy maker's valuation. We must, however, find the value to the policy maker of this marginal tax revenue under a second best tax system. In the present model we ask what is the effect on social welfare effect of reducing α marginally? All individuals then receive a marginal unit of lump sum income. They re-optimise in response and change their labour supply via income effects and therefore also change their income tax payments. The effects of this thought experiment are captured by the *social marginal utility of income* to individual $i = 1, \ldots, n$, defined by Diamond (1975).[2] Depending on model the definition

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[2] Note that the social marginal utility of *consumption* is the policy maker's valuation of the private marginal utility of income. The social marginal utility of *income* is the social marginal

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will look different, but for the model we employ it is[3]

$$\gamma_i := \lambda_i^* - \kappa_\alpha \beta w_i \frac{\partial \ell_i^*}{\partial \alpha} \quad i = 1, \ldots, n. \tag{14}$$

If the tax system is optimal κ_α is replaced by κ^*. Then the value of the alternative use of this marginal tax revenue is given by the population average marginal social utility of income

$$E(\gamma) = \frac{1}{n} \sum_{i=1}^n \gamma_i. \tag{15}$$

Now, let Z denote the set of available tax instruments. In the present model $Z = \{\alpha, \beta\}$. We can then define the marginal cost of public funds:

Definition 1 (Social Marginal Cost of Public Funds (SMCPF)). *The social marginal cost of public funds (η_z) for a tax instrument (z) is the ratio between the marginal social welfare of tax revenue for that tax instrument (κ_z) and the population average marginal social utility of income (γ), i.e.,*

$$\eta_z := \frac{\kappa_z}{E(\gamma)} \quad \forall z \in Z.$$

For any optimal tax system the marginal social welfare of tax revenues must be the same all tax instruments. Then the SMCPF can be measured for a marginal uniform lump sum tax and it follows from equation (11a) that $\kappa^* = \gamma$ in the present model. Under Definition 1 the following result therefore holds:

Proposition 1. *The marginal cost of public funds according to Definition 1 is always equal to one under an optimal tax system, i.e., $\eta^* = 1$.*

Proof. Suppose the tax system is optimal. Then the necessary first order conditions (11a)–(11d) are satisfied. Using the definition of social marginal utility of income and Definition 1 we see that (11a) can be written as $\frac{\partial L}{\partial \alpha} = nE(\gamma)\left[\eta_\alpha - 1\right] = 0$ and implies $\eta_\alpha = 1$. Since the tax system us optimal we have $\eta_\alpha = \eta_\beta = \eta^*$, where η^* is the social marginal cost of public funds in an optimal tax system. Accordingly $\eta^* = 1$. □

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utility of consumption plus the product of the shadow price on tax revenues and the change in tax revenues due to a marginal transfer to the individual. This terminology follows Diamond (1975) and Auerbach (1985, p. 87) but one may observe that the terminology is different in, e.g., Diamond and Mirrlees (1971) and Mirrlees (1975).

[3]See for instance Bovenberg and van der Ploeg (1994, equation (8)) and Pirtälä and Toumala (1997) who analyse models with externalities.

However, in the heterogeneous population case it is well known that under 'standard' assumptions $\beta^* \in (0, 1)$.[4] Therefore the average marginal social utility of income will not coincide with the average marginal utility of income unless the labour supply of all individuals is completely inelastic with respect to changes in lump sum income.[5]

Proposition 1 holds for any optimal tax model: Regardless of how the tax system is defined, a necessary condition for an optimal tax system is that a marginal perturbation of lump sum taxation/transfers or shift in the tax system must not change social welfare.[6] Or else the tax system would not be optimal. Since this welfare difference is the marginal social welfare of additional tax revenues minus the average marginal social utility of income and that the marginal cost of public funds is the ratio between these two numbers, the result follows (in a sense) as a tautology.

We can now compare this definition with existing ones from the literature. A noteworthy overview is found in Ballard and Fullerton (1992). Most frequently a representative agent framework is used.[7] Then the two definitions of the MCPF are

$$\frac{\kappa}{\lambda^*} \text{ and } \frac{\kappa}{\gamma}. \tag{16}$$

Consider the case of $\frac{\kappa}{\lambda^*}$ and assume that taxes are non–optimal. If we assume, in the present model, that we have the representative agent case then $n = 1$. In this case, of course, $E(\gamma) = \gamma$. Then the difference between the two definitions is the secondary effects on tax revenues of a lump sum transfer to the individual. If we believe that leisure is a normal good, then $\lambda^* > \gamma$. For non–optimal tax systems, therefore, the two measures do not coincide and the SMCPF for an income tax is always larger than the first of the standard definitions of the MCPF. This difference disappears if we invoke conditions of optimality, in which case both measures are equal to unity. [8]

A representative agent framework, however, is problematic to use if one wants to analyse distortionary taxation. Then the heterogeneous population assumption seems reasonable and explicitly or implicitly, the average marginal utility of income $E(\lambda) := \frac{1}{n} \sum_{i=1}^{n} \lambda_i^*$ has been used in as a measure of the opportunity cost of

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[4] See Sheshinski (1972), Hellwig (1986) and Svensson and Weibull (1986) for different assumptions to get this result.

[5] This of course also holds in a representative agent framework. Cf. Håkonsen (1998, Figure 1) for the case where the elasticity of substitution is zero in a simulation using a CES utility function. Then the SMCPF is equal to unity even if taxes are distortionary.

[6] See for instance Christiansen (1981).

[7] In the following we drop the individual specific index when $n = 1$.

[8] We already know that the SMCPF equals unity under optimal taxes. In the representative agent case optimal taxes implies implies $\kappa^* = \gamma_i$. Using (11b) $\beta^* = 0$ follows. But then $\lambda^* = \gamma_i$ and the result follows.

marginal tax revenues. The MCPF is then written as $\frac{\kappa}{E(\lambda)}$. Normality of leisure still implies $E(\lambda) > E(\gamma)$ and the principal differences between the two measures of MCPF remain unchanged with the SMCPF larger than the MCPF. However, invoking an assumption of optimality of taxes will not restore the equality between the measures.[9]

The case of κ/γ is analysed specifically by Håkonsen (1998, equation (7)). However, he evaluates this measure under the constraint that a lump sum tax is unfeasible (in our case $\alpha = 0$) and that the proportional income tax is optimally chosen (i.e., determined by (11b) with $n = 1$ and denoted β^*).[10] Using the Slutsky equation for labour supply his measure can be written as

$$\left.\frac{\kappa_\beta}{\gamma}\right|_{\substack{\alpha=0 \\ \beta=\beta^* \\ n=1}} = \frac{\ell_i}{\ell + \beta\frac{\partial\ell^S}{\partial\beta}} > 1, \tag{17}$$

where the inequality follows from the compensated effect on labour supply in the denominator. It is the representative agent framework that is responsible for this result; i.e., there is no other effect that can counteract with the inefficiency due to the proportional income tax. However, if the lump sum tax is unfeasible and the proportional income tax is determined by (11b) but with $n > 1$ we get using the definition of the SMCPF

$$\left.\eta_\beta\right|_{\substack{\alpha=0 \\ \beta=\beta^*}} = \left.\frac{\kappa_\beta}{E(\gamma)}\right|_{\substack{\alpha=0 \\ \beta=\beta^*}} = \frac{1 + \frac{\text{cov}(y,\gamma)}{E(y)E(\gamma)}}{1 + \sum\limits_{i=1}^{n}\beta w_i\frac{\partial\ell_i^S}{\partial\beta}}. \tag{18}$$

Although we know that the denominator as in the representative agent case is less than unity (due to the compensated tax base effect) we do not know whether the enumerator is larger or smaller than unity; c.f., Dixit and Sandmo (1977, p. 421f). Therefore, given the choice of tax system with no lump sum tax and an "optimal" income tax, the result that the SMCPF is larger than unity in the representative agent case does not carry over to a heterogeneous population situation. See also the analysis below in section 6.2.

That the marginal cost of public funds is equal to one in a first best tax system has in the literature been attributed to the first best character of the problem. But taxes can be non–optimal even in a first best situation and then the standard

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[9]The heterogeneous population assumption is made by, e.g., Wilson (1991), Dahlby (1998) and Sandmo (1998). Wilson (1991), however, does not use the concept of marginal cost of public funds although he implicitly uses the social marginal cost of public funds as defined by Definition 1 in this paper and that it equals unity under an optimal lump sum tax. Also Johansson-Stenman (2001) analyses a cost–benefit problem using a heterogeneous population assumption, but he is not concerned with the marginal cost of public funds.

[10]The lump sum tax need not be zero, just not optimally chosen.

definition of the MCPF will not equal unity. In this section, however, we have redefined the MCPF into the SMCPF according to Definition 1. Since the definitions considered do not differ under first best it means that this result is due to the optimality of the tax system rather than the first best character of the problem.

4 Aggregate social marginal benefits and cost

Before we analyse the cost–benefit criterion, which is done in the next section, we here aim at identifying the aggregate social benefits and costs of the public projects. Consider first the marginal benefits of a public project. Suppose we can ask an individual i about her valuation of a marginal increase in the project size and get a truthful answer. The individual would answer that her *private marginal benefit* b'_i of an increase in the project size is

$$b'_i := \frac{V^i_g}{\lambda^*_i}; \tag{19}$$

i.e., the ratio between the private marginal utility of increasing the project size and the private marginal utility of income.

In a situation where the policy maker has access to differentiated lump sum taxes and therefore do not use distortionary taxation the policy maker would agree with the private citizen: Equation (19) gives the social marginal benefit to individual i. However, in second best economies two things happens with the policies that are employed. First, distortionary taxes tend to be used. Second, if lump sum taxes at all are available they tend to be uniform and not differentiated on individual characteristics. A policy maker will therefore value the benefit to the individual differently from how the individual makes the valuation. Taking these two constraints on policy as given we can argue that the policy maker defines the social marginal benefit to individual i as

$$B'_i := \frac{V^i_g}{E(\gamma)}; \tag{20}$$

i.e., the ratio between the private marginal utility of increasing the project size and the social marginal utility of income. Aggregating over individuals the aggregate social marginal benefit is $\sum_{i=1}^{n} B'_i$.

An alternative way to define the social marginal benefit would be to use $\mathcal{B}'_i := \frac{V^i_g}{\gamma_i}$, where the inability to target a marginal lump sum transfer to or extract a marginal lump sum tax from a specific individual is not taken into account in the definition. We can then rewrite the original definition $\sum_{i=1}^{n} B'_i$ in terms of $\sum_{i=1}^{n} \mathcal{B}'_i$,

using the covariance definition, as

$$\sum_{i=1}^{n} B'_i = \left[1 + \frac{\operatorname{cov}(\gamma, \mathcal{B}')}{E(\gamma) E(\mathcal{B}')}\right] \sum_{i=1}^{n} \mathcal{B}'_i. \tag{21}$$

The covariance term enters because of the policy maker's inability to target lump sum taxes and transfers to specific individuals. If differentiated taxes and transfers are feasible and optimally chosen the social marginal utility of income is uniform over the population and the covariance is zero. We can in a similar fashion rewrite the definition of the aggregate social marginal benefit $\sum_{i=1}^{n} \mathcal{B}'_i$ in terms of the aggregate private marginal benefit $\sum_{i=1}^{n} b'_i$:

$$\sum_{i=1}^{n} \mathcal{B}'_i = \left[1 + \frac{\operatorname{cov}\left(\frac{\lambda}{\gamma}, b'\right)}{E\left(\frac{\lambda}{\gamma}\right) E(b')}\right] E\left(\frac{\lambda}{\gamma}\right) \sum_{i=1}^{n} b'_i. \tag{22}$$

Here the covariance term enters because of the distortionary income tax system which creates a difference between the individual's and the policy maker's valuation of the value off additional lump sum income to the individual. Whenever distortionary taxes are not used then $\lambda_i = \gamma_i \ \forall i$ and the covariance term is zero. Note that this statement is made as if differentiated lump sum taxes were feasible; these, however, need not be optimally chosen. Now, combining (21)–(22) we get

$$\sum_{i=1}^{n} B'_i = \left[1 + \frac{\operatorname{cov}\left(\frac{\lambda}{\gamma}, b'\right)}{E\left(\frac{\lambda}{\gamma}\right) E(b')}\right] \left[1 + \underbrace{\frac{\operatorname{cov}(\gamma, \mathcal{B}')}{E(\gamma) E(\mathcal{B}')}}_{A1}\right] E\left(\frac{\lambda}{\gamma}\right) \sum_{i=1}^{n} b'_i. \tag{23}$$

This decomposition takes into account that taxes are distortionary and that optimal differentiated lump sum taxes are not used. We can now make two thought experiments: First, suppose that the tax system is non–distortionary in the sense that $\beta = 0$. Whenever this is the case $\lambda_i = \gamma_i \ \forall i$. Then $E\left(\frac{\lambda}{\gamma}\right) = 1$, $\operatorname{cov}\left(\frac{\lambda}{\gamma}, b'\right) = 0$ and $\operatorname{cov}(\gamma, \mathcal{B}') = \operatorname{cov}(\lambda^*, b')$. Equation (23) then reduces to

$$\sum_{i=1}^{n} B'_i = \left[1 + \underbrace{\frac{\operatorname{cov}(\lambda, b')}{E(\lambda) E(b')}}_{A2}\right] \sum_{i=1}^{n} b'_i. \tag{24}$$

Still the aggregate social marginal benefit deviates from aggregate private marginal benefit and this is because the government still only uses uniform lump sum taxation to finance the public goods project. Although there are no distortions created by the tax system, the restriction that only a uniform lump sum tax is available prevents the policy maker to choose the desired income distribution and

therefore the aggregate social marginal benefit deviates from the aggregate private marginal benefit. The right–hand side of (24) is exactly how the aggregate social marginal benefit was expressed by Sandmo (1998), although he assumed that taxes were distortionary.

Sandmo (1998) calls the normalised covariance, expression A2 in (24), the distributional characteristic of the public good. Given our definition of the social marginal benefit, however, we would call it the distributional characteristic of the public good in a non–distortionary tax system; cf. Sandmo (1998, p. 372). The general distributional characteristic of the public good with a distortionary tax system is in our model given by expression A1 in equation (23).

This, however, does not mean that (24) can be used under distortionary financing. But since $\mathrm{cov}\,(\lambda, b') = E(\lambda b') - E(\lambda)E(b')$ the aggregate social marginal benefit can be decomposed into

$$\sum_{i=1}^{n} B'_i = \left[1 + \frac{\mathrm{cov}\,(\lambda, b')}{E(\lambda)E(b')} \right] \frac{E(\lambda)}{E(\gamma)} \sum_{i=1}^{n} b'_i. \qquad (25)$$

The right–hand side of (24) therefore has to be scaled with the factor $\frac{E(\lambda)}{E(\gamma)}$ to be used under distortionary taxation.

Second, suppose that the government in addition uses optimal differentiated lump sum taxes. Hence, this is a true first best situation. This means that λ_i are uniform over the population with the consequence that $\mathrm{cov}\,(\lambda, b') = 0$. Then $B'_i = b'_i \; \forall i$. We can also see that this follows from (23): In this case $\gamma_i = \lambda_i = E(\lambda)$ $\forall i$. Then $\mathrm{cov}\,(\lambda, b') = \mathrm{cov}\,(\gamma, \mathcal{B}') = 0$ and $E\left(\frac{\lambda}{\gamma}\right) = 1$.

Considering the social marginal cost (or the marginal shadow price of the public good) is comparably easier. The private marginal cost is of course c', but the policy maker has to take into account that changing the project size will have indirect effects on the individuals. The direct welfare consequence is already considered on the benefit side but there are indirect effects through changed behaviour, in this model changed labour supply, which will change the government's tax revenues. The social marginal cost therefore is[11]

$$C' := c' - \sum_{i=1}^{n} \beta w_i \frac{\partial \ell_i^*}{\partial g}. \qquad (26)$$

If the public good project is a complement (substitute) to labour then the social marginal cost (marginal shadow price) will be lower (higher) than the private marginal cost.

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[11] See Wilson (1991) and Schöb (1994) for two alternative ways to treat the marginal cost and the feedback on tax revenues of the size of the public project. Their purposes are, however, different from ours: Wilson (1991) has the purpose to investigate the 'levels issue', that is whether second best provision of public goods implies a lower or higher level of the public good than first best and Schöb (1994) evaluates tax reforms.

5 Cost–benefit under optimal taxes

Having defined the social benefits and costs we are now ready to consider the cost–benefit criterion for an optimal project size. The first order condition (11c) for an optimal public project size can be written as

$$\sum_{i=1}^{n} V_g^i = \kappa^* \left[c' - \sum_{i=1}^{n} \beta w_i \frac{\partial \ell_i^*}{\partial g} \right] = 0. \tag{27}$$

Now, dividing both sides with $E(\gamma)$ we get

$$\sum_{i=1}^{n} \underbrace{\frac{V_g^i}{E(\gamma)}}_{B_i'} = \underbrace{\frac{\kappa^*}{E(\gamma)}}_{\eta^*} \underbrace{\left[c' - \sum_{i=1}^{n} \beta w_i \frac{\partial \ell_i^*}{\partial g} \right]}_{C'}. \tag{28}$$

Given that the tax system is optimal it follows from Proposition 1 that $\eta^* = \kappa^*/E(\gamma) = 1$. Using the definition of the aggregate social marginal benefits and costs we get[12]

$$\sum_{i=1}^{n} B_i' = C', \tag{29}$$

i.e., aggregate social marginal benefits should equal the social marginal cost at the optimal project size. In a sense, this is hardly a surprising way of representing the criterion in a second best situation: Defining social marginal benefits and costs appropriately and choosing the tax system optimally (i.e., taking into account both the use of the distortionary income tax and that only a uniform lump sum tax is available, rather than differentiated lump sum taxes) means that all there is to consider for the policy maker is considered. In particular, there is no wedge between aggregate social benefits and social costs in the form of a scaling factor like the marginal cost of public funds. We can summarise these results as follows:

Proposition 2. *If taxes are chosen optimally, then for an optimally sized marginal project marginal social benefits equal marginal social costs, i.e., the cost–benefit criterion* (29) *is satisfied.*

Using the alternative definition of the social marginal benefit $\mathcal{B}_i$ the cost benefit criterion can then, using equation (21), be written as

$$\left[1 + \frac{\text{cov}(\gamma, \mathcal{B}')}{E(\gamma) E(\mathcal{B}')} \right] \sum_{i=1}^{n} \mathcal{B}_i' = C'. \tag{30}$$

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[12]Cf. with Wilson (1991, equation (11)) who gives a condition on the form $\sum_{i=1}^{n} \left[B_i' + \beta w_i \frac{\partial \ell_i^*}{\partial g} \right] = c'$; i.e., implicitly the unity result for the SMCPF under an optimal tax system is invoked. On the proper relationship between the cost–benefit test for a public goods and the marginal cost of public funds, see also Kaplow (1996).

This alternative definition of the social marginal benefit also means that we express the cost–benefit test without the use of the social marginal cost of public funds when taxes are optimal. However, the intuitive character of (29) is lost; i.e., that the optimal level of the public project is given by the balance of aggregate social marginal benefits and costs. Instead we get benefits expressed in a form where the inability of the policy maker to target lump sum taxes and transfers to individuals is emphasised, under the the hypothetical assumption that taxes are non–distortionary. Of course, the optimal level of the public project is unchanged since this is just another way to present the first order conditions.

We can also confront these ways of presenting the cost–benefit tests with the previous practise in the literature on the marginal cost of public funds. Using (25) to replace the left–hand side in (28) we get

$$\left[1 + \frac{\text{cov}\,(\lambda, b')}{E(\lambda)E(b')}\right] \frac{E(\lambda)}{E(\gamma)} \sum_{i=1}^{n} b_i' = \frac{\kappa^*}{E(\gamma)} C'. \tag{31}$$

Eliminating $E(\gamma)$ and dividing both sides with $E(\lambda)$ we get

$$\left[1 + \frac{\text{cov}\,(\lambda, b')}{E(\lambda)E(b')}\right] \sum_{i=1}^{n} b_i' = \frac{\kappa^*}{E(\lambda)} C', \tag{32}$$

where $\frac{\kappa^*}{E(\lambda)}$ is *marginal cost of public funds* with the definition used by Sandmo (1998) and the normalised covariance is his definition of the distributional characteristic of the public good. This means that the consequences of the distortionary tax system on the aggregate social marginal benefits are captured captured by the distributional characteristic and the marginal cost of public funds which appears on the cost side. This gives a different perspective on the standard definition of the marginal cost of public funds: Even under an optimal tax system it does not appear on the right–hand side in the cost–benefit test to scale costs because of distortionary taxes. *It appears on the cost side to scale costs because the benefit side is defined as if taxes were non–distortionary in a situation where taxes are distortionary*. This should be compared with the cost–benefit test (29) which together with the decomposition (23) gives

$$\left[1 + \frac{\text{cov}\left(\frac{\lambda}{\gamma}, b'\right)}{E\left(\frac{\lambda}{\gamma}\right) E\,(b')}\right]\left[1 + \frac{\text{cov}\,(\gamma, \mathcal{B}')}{E\,(\gamma)\, E\,(\mathcal{B}')}\right] E\left(\frac{\lambda}{\gamma}\right) \sum_{i=1}^{n} b_i' = C', \tag{33}$$

where the effect of distortionary taxes is entirely on the left–hand side and divided into a uniform lump sum effect and a distortionary tax effect as discussed in the previous section.

6 Cost–benefit under non–optimal taxes

Suppose now that the tax system is not optimal and that the policy maker considers a marginal increase in the public project. That increased project size could be financed by either a non–distortionary (α) or distortionary (β) tax increase or a combination of both. Generally, the welfare differential of such a policy change is

$$dW = -\sum_{i=1}^{n} \lambda_i^* d\alpha - \sum_{i=1}^{n} \lambda_i^* y_i^* d\beta + \sum_{i=1}^{n} V_g^i dg. \tag{34}$$

Requiring a balanced budget for the policy change we get

$$\left(n + \sum_{i=1}^{n} \beta w_i \frac{\partial \ell_i}{\partial \alpha} \right) d\alpha + \left(\sum_{i=1}^{n} y_i^* + \sum_{i=1}^{n} \beta w_i \frac{\partial \ell_i}{\partial \beta} \right) d\beta + = C' dg \tag{35}$$

from a total differentiation of the budget constraint with respect to the two policy parameters. Note that use was made of the definition of the marginal social cost from equation (26).

Since the tax system is not optimal there is no reason why the two possible financing instrument should be associated with the same marginal cost of public funds. We therefore, in turn, consider the non–distortionary tax and the distortionary tax.

6.1 Non–distortionary taxation

First, setting $d\beta = 0$ and using the definition of the aggregate social marginal benefit equation (34) implies

$$dW = E(\gamma) \left[\sum_{i=1}^{n} B_i' dg - \frac{nE(\lambda)}{E(\gamma)} d\alpha \right]. \tag{36}$$

With $d\beta = 0$ we can use (35) to solve for $d\alpha$:

$$d\alpha = \frac{C'}{\left(n + \sum_{i=1}^{n} \beta w_i \frac{\partial \ell_i}{\partial \alpha} \right)} dg = \frac{\eta_\alpha C'}{nE(\lambda)}, \tag{37}$$

where the last equality follows with the use of (12a) and the definition of the SMCPF (Definition 1). Substituting for $d\alpha$ in equation (36) and re–arranging we get

$$\left. \frac{dW}{dg} \right|_{\substack{d\alpha \neq 0 \\ d\beta = 0}} = E(\gamma) \left[\sum_{i=1}^{n} B_i' - \eta_\alpha C' \right] \geq 0 \quad \Leftrightarrow \quad \frac{\sum_{i=1}^{n} B_i'}{C'} \geq \eta_\alpha. \tag{38}$$

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The strict equality above refers to the case when the project is (locally) optimally sized. Since the first order (11a) implies $\frac{\partial L}{\partial \alpha} = nE(\gamma)[\eta_\alpha - 1] = 0$ for an optimal lump sum tax it follows that

$$\eta_\alpha \gtreqless 1 \quad \Leftrightarrow \quad \begin{array}{c} \text{increase} \\ \text{decrease} \end{array} \alpha \tag{39}$$

to improve welfare.

Let us first look at the case where the marginal project is not (in a local sense) optimally sized. Suppose that $\eta_\alpha < 1$. Then the cost benefit test (38) may call for an increased project size even if $\sum_{i=1}^n B_i' < C'$; i.e., the policy maker will undertake marginal projects which are in this sense too costly because it is a means to compensate for lump sum taxes being too high. Similarly, if $\eta_\alpha > 1$, then the policy maker will not undertake some marginal projects for which $\sum_{i=1}^n B_i' > C'$. This can be interpreted as a way to compensate for too low lump sum taxation; i.e., only marginal projects for which benefits exceed costs sufficiently will be undertaken.

If we consider in local sense optimally sized projects ($dW/dg = 0$) the cost–benefit criterion is

$$\sum_{i=1}^n B_i' = \eta_\alpha C'. \tag{40}$$

The results now become a bit stronger: If $\eta_\alpha < 1$ then aggregate marginal social benefits will fall short of marginal social costs ($\sum_{i=1}^n B_i' < C'$) at the optimal project size to compensate for too low lump sum taxation. If $\eta_\alpha > 1$ then aggregate marginal social benefits exceeds marginal social costs ($\sum_{i=1}^n B_i' > C'$) at the optimal project size to compensate for too high lump sum taxation.

6.2 Distortionary taxation

A similar analysis can be applied to the case of distortionary taxation. Setting $d\alpha = 0$ and using the definition of the aggregate social marginal benefit equation (34) implies

$$dW = \gamma \left[\sum_{i=1}^n B_i' dg - \frac{nE(\lambda y)}{E(\gamma)} d\beta \right]. \tag{41}$$

With $d\alpha = 0$ we can use (35) to solve for $d\beta$:

$$d\beta = \frac{C'}{\sum_{i=1}^n \left(y_i + \beta w_i \frac{\partial \ell_i}{\partial \beta} \right)} dg = \frac{\kappa_\beta C'}{nE(\lambda y)}, \tag{42}$$

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where the last equality follows from (12b) and the definition of the SMCPF (Definition 1). Substituting for $d\beta$ in equation (36) and rearranging we get

$$\frac{dW}{dg}\bigg|_{\substack{d\alpha=0 \\ d\beta\neq0}} = E(\gamma)\left[\sum_{i=1}^{n} B_i' - \eta_\beta C'\right] \geq 0 \quad \Leftrightarrow \quad \frac{\sum_{i=1}^{n} B_i'}{C'} \geq \eta_\beta. \quad (43)$$

From, (11b) follows[13]

$$\eta_\beta \gtreqless \frac{1 + \frac{\text{cov}(y,\gamma)}{E(y)E(\gamma)}}{1 + \varepsilon_{y\beta}^s} := \tilde{\eta}_\beta \quad\quad \Leftrightarrow \quad\quad \begin{matrix}\text{increase}\\\text{decrease}\end{matrix}\ \beta \quad (44)$$

where $\varepsilon_{y\beta}^s = \frac{\beta}{\sum_{i=1}^{n} y_i^s} \sum_{i=1}^{n} w_i \frac{\partial \ell_i^s}{\partial \beta}$ is the elasticity of the compensated tax base with respect to the proportional income tax and $\tilde{\eta}_\beta$ is the critical value for the MCPF at which the distortionary tax is optimal for any given value on α.[14] From (11a) and (11b) we know that $\frac{\text{cov}(y_i,\gamma_i)}{y\gamma} = \varepsilon_{y\beta}^s$ if taxes are optimally chosen. That is, any thought experiment starting with $\tilde{\eta}_\beta = 1$ requires that also α is optimal which is not necessarily the case. This means that if we cannot generally draw the same type of conclusions in this case of marginal distortionary finance as in the case of non–distortionary finance; we need to estimate the critical value $\tilde{\eta}_\beta$ which is hard because not only the estimation of the compensated labour supply function is needed but also the exact cardinalisation of the individual utility function chosen by the policy maker.

In order to draw the same type of conclusions we can consider the special case in which we make the evaluation at a point at which the lump sum tax is actually optimal. In that case, had β also been optimal then $\tilde{\eta}_\beta$ must equal unity. Consider then a small deviation of the proportional tax when it is above its optimum level. In that case, naturally, the policy maker would like to decrease the proportional tax again. That implies $\tilde{\eta}_\beta < 1$; i.e, the equity component in the denominator of $\tilde{\eta}_\beta$ is smaller than the efficiency component in the numerator. Similarly, for a proportional tax slightly below the optimal level $\tilde{\eta}_\beta > 1$ and the policy maker would like to increase the proportional tax. That is, whenever the lump sum tax is optimal, then

$$\eta_\beta \gtreqless 1 \quad\quad\quad \Leftrightarrow \quad\quad\quad \begin{matrix}\text{increase}\\\text{decrease}\end{matrix}\ \beta \quad (45)$$

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[13]Compare with Dixit and Sandmo (1977, equation (16)) where a similar condition for an optimal value of β is derived. Note however, that (44) is derived from (11b) only without the use of (11a).

[14]Compare with equation (18). The measure of η_β in this equation need not coincide with $\tilde{\eta}_\beta$ since the former is derived under the restriction that $\alpha = 0$ and not only given at some arbitrary level. Note also that the top index s denotes compensated supply and that at the individual optimum Hicksian labour supply equals Marshallian labour supply so that $y_i^s = y_i\ \forall i = 1,\ldots,n$.

and the conclusion from the non–distortionary financing case holds: Whenever $\eta_\beta < 1$ the cost benefit test may call for an increased project size even if $\sum_{i=1}^{n} B_i' < C'$; i.e., the policy maker will undertake marginal projects which are in this sense too costly because it is a means to compensate for too high income taxation. Similarly, if $\eta_\beta > 1$, then the policy maker will not undertake some marginal projects for which $\sum_{i=1}^{n} B_i' > C'$; i.e., some beneficial projects will not be undertaken to compensate for too low income taxation.

As in the case of non–distortionary taxation the conclusions become stronger if look at optimally sized projects because then

$$\sum_{i=1}^{n} B_i' = \eta_\beta C'. \tag{46}$$

Therefore, if the lump sum tax is optimal and if $\eta_\beta < 1$, then aggregate marginal social benefits will fall short of marginal social costs ($\sum_{i=1}^{n} B_i' < C'$) at the optimal project size to compensate for too low income taxation. If the lump sum tax is optimal and if $\eta_\beta > 1$ then aggregate marginal social benefits exceeds marginal social costs ($\sum_{i=1}^{n} B_i' > C'$) at the optimal project size to compensate for too high income taxation.

7 Conclusions

The object of analysis in this paper has been a definition of the marginal cost of public where the opportunity cost of additional tax revenue is set to be the policy maker's valuation of a marginal unit of income to an individual (in a representative individual framework) or its population average in a heterogeneous population framework. This definition, called the social marginal utility of income, has the property of being equal to unity whenever the tax system is optimal. The relationship between the social marginal cost of public funds and other definitions were performed. It was argued that the standard definition of the marginal cost of public funds is used (i.e., when the individual's marginal utility of income was used as the opportunity cost of tax revenues or its population average in the heterogeneous population case) was always smaller. Measures of the social marginal benefits and costs, congruent with the social marginal cost of public funds where also derived. Cost benefit test under optimal and non–optimal taxes where analysed. It was shown that, even if taxes are optimal, the standard definition of the marginal cost of public funds appears in the cost–benefit test, not because taxes are distortionary, but because taxes are distortionary but benefits are defined as if tax are non–distortionary. Under non–optimal taxes it was shown that the interpretation of the social marginal cost of public funds was that it is a wedge forcing the cost

benefit analyst to compensate for the non–optimality of taxes. This wedge can be larger or smaller than a critical value. If larger the cost–benefit test should reject some projects for which the social marginal benefit exceeds the social marginal costs to compensate for a too low overall level of taxation. If smaller the analyst should accept some projects even if the social marginal benefit is smaller than the social marginal cost to compensate for a too high overall level of taxation.

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