Effects of redistribution policies - who gains and who loses?

Camilo von Greiff *

Abstract

The paper combines optimal taxation theory with human capital theory and develops a theoretical model with endogenous wages and education decision, in which redistributive policy experiments are carried out and assessed. It is argued that general equilibrium effects of labor income taxation on wages may counteract fiscal redistribution. It is also shown that education subsidies may only benefit skilled workers, suggesting that this subsidy can merely be viewed as a redistribution from unskilled to skilled individuals. Therefore, optimal policy involves a lump-sum education tax in the form of a negative education subsidy.

Keywords: Income Redistribution; Education Subsidies

JEL Classification: H21; H23

---

*Department of Economics, Stockholm University, S-106 91 Stockholm, e-mail camilo.vongreiff@ne.su.se. I would like to thank my supervisor Hans Wijkander for great guidance and support and Ann-Sofie Kolm and workshop participants at the Department of Economics, Stockholm University, for valuable suggestions and comments. I am also indebted to Mårten Sundberg for assistance with the Matlab calculations. Financial support from Finanspolitiska Forskningsinstitutet is also gratefully acknowledged.

1 Introduction

To which extent should labor income be redistributed? And what are the consequences for the economy? These questions have been on the research agenda ever since the minimum sacrifice theory was introduced by Adam Smith and John Stuart Mill, and later on by Edgeworth and Pigou[1]. The theory suggested that after-tax incomes should be completely equalized, if marginal utility of consumption is decreasing. This reasoning does not take into account the disincentive effect of taxation on labor supply decisions, which creates a trade-off between equity and efficiency considerations. Starting with the Nobel Prize winning paper by Mirrlees (1971) and later on Sheshinski (1972), the recognition of these distortive effects has been a common assumption. In those analyses, the production side of the economy only involves one type of labor. This implies that the redistribution achieved goes entirely through the fiscal system. Later contributions, initiated by Feldstein (1973) and Allen (1981), explore the situation with two complementary production factors, skilled and unskilled labor. Wages are endogenously determined in the model and depend on the relative labor supply of the two types[2].

Redistributive linear income taxation will in many cases reduce labor supply less for skilled workers than for unskilled workers since the income effects go in different directions[3]. Feldstein (1973) showed that this increase in the relative labor supply of skilled workers tends to wage convergence between the two types. Therefore, the general equilibrium effects of labor supplies on wage rates reinforce the redistribution via the fiscal system. This

[1]See Atkinson (1973) for an overview.

[2]In Fair (1971) and Atkinson (1973), the distribution of skill types does not influence the individuals' wage rates although there is a continuum of skill types.

[3]This is due to the fact that skilled individuals (who are also assumed to be high income earners) being net contributors to the tax and transfer system, whereas unskilled individuals (low income earners) are net recipients. Thus, their income effects should be of opposite signs, implying the total effect on their labor supply to generally differ.

result was scrutinized by Allen (1981) in a more flexible model. He found that under specific assumptions on the production function and the labor supply elasticities[4], redistribution via the fiscal system and the general equilibrium effects on wages may also be counteracting.

Instead of relying on assumptions on the production and labor supply function that are empirically doubtful, I argue that the wage effect reinforcing the fiscal redistribution in Feldstein (1973) may be counteracted by distortive effects of redistribution on education decisions. The reason is that redistribution makes it more attractive to remain an unskilled worker and less attractive to become a skilled worker. Thus, the share of skilled workers decreases. This tends to wage divergence between skilled and unskilled workers, since skilled labor is then relatively more scarce. The effect of redistribution on the wage rates thus depends on the relative strength of the wage *converging* effect caused by the change in relative labor supply *per individual type*, and the wage *diverging* effect caused by a lower share of skilled workers. Therefore, it is not clear whether fiscal redistribution is reinforced or counteracted by the different general equilibrium effects on wage rates. These two effects were also considered in Wilson (1982), but only one at a time and not, as in this paper, in combination with each other. Moreover, that paper focuses on optimal public employment policy and not on the effect of taxation on wage rates.

Facing the disincentive effect on education investments induced by redistribution, many countries strongly subsidize education in order to make education investments more attractive[5]. Storesletten and Zilibotti (2000) suggest that this is the reason why several Western countries with a high degree of redistribution show a surprisingly good performance when it comes

[4]In particular, that the elasticity of substitution between the two types of labor is less than one and either or both of the labor supply elasticities are negative. Note that these conditions are only necessary and not sufficient

[5]Dur and Teulings (2003) report that education expenditures constitute on average 6 % of GDP in Western countries.

to enrollment rates to higher education. This is also the theoretical reasoning in both Bovenberg and Jacobs (2005) and van Ewijk and Tang (2001).

This paper links together optimal taxation theory and human capital theory by combining the tax and transfer system and education subsidies. This was also done in Blankenau (1999) but in that paper, labor supply was fixed and the policies imposed by the government were assessed one at a time and not, like in this paper, in combination with each other. Bovenberg and Jacobs (2005) also consider both education subsidies and transfers, but in a different kind of model. They assume a specific form of the production function for human capital and a continuum of skill types. Their result is that the subsidy rate should equal the tax rate, since this yields efficiency in human capital formation. However, a prerequisite for this result is the assumption of a continuum of education choices. This makes it possible for the increase of marginal costs of education induced by taxation to exactly cancel the effect of higher marginal benefits of education induced by education subsidies. Moreover, they assume wages not to depend on the relative labor supply of different types, as they do in the model in this paper. Instead, the wage rate for an individual only depends on her innate ability and her *own* education time.

Another related model is Dur and Teulings (2003). They argue that education subsidies boost education investments and thereby the supply of skilled workers, which decreases the return to education and increases the relative wage of unskilled workers. Wages depend (negatively) on the *mean* level of human capital among workers and (positively) on their own human capital, not on the relative labor supply of different skill types as in the model in this paper. They seem to have empirical support for a negative correlation between the return to education and the mean level of human capital. However, a closer look at the data suggests that the US does not fit into the overall picture, with its high mean human capital level and relatively high returns to education. A plausible explanation for this might be that education subsidies in the US are targeted to highly educated individuals, thus increasing the mean level of human capital while maintaining a relatively high return to education investments. Like Bovenberg and Jacobs (2005, they also make assumptions about the wage function with an unclear relationship to the aggregate production function. Brett and Weymark (2003) and Cremer et al. (2005) also consider the combination of optimal taxation theory and human capital theory but in models very different from that in this paper.

This paper takes its starting point in first principles. The general equilibrium model with linear income taxation generalizes Feldstein (1973) in two ways. First, I use general utility and production functions in the theoretical part. Second, I endogenize the education decision by the individuals and hence, also the resulting shares of skilled and unskilled workers. This is done by letting individuals, heterogeneous in ability, choose between two different education levels, with two different wage rates. As explained above, endogenizing the skill distribution in the work force will have an additional effect of government intervention on the wage rates, besides the effect via labor supply reactions *per* individual of each type. By assuming that the education cost decreases with ability, there will be a threshold ability level, which separates skilled from unskilled workers. In the theoretical part, I explore qualitative effects of different policy mixes. This is followed by simulations under realistic assumptions of the production function and labor supply behavior and assessment of the optimal policy mix. Although interesting, the model abstracts from the possibility of complementarity between ability and education[6], and the linkage between incentives to work and invest in human capital[7].

The rest of the paper is organized as follows. In Section 2, I present the model and characterize the laissez-faire equilibrium. This is followed by performing comparative statics on the laissez-faire equilibrium in Section 3.

[6]See, for example, Angrist and Krueger (1991) and Dur and Teulings (2003).

[7]See Trostel and Walker (2000).

Then, in Section 4, I carry out simulations and present the main results. A brief discussion and conclusions are offered in Section 5.

2 The Model

2.1 The laissez-faire economy

Consider a model with a continuum of individuals uniformly distributed over an *in*ability endowment $X \in [0, 1]$[8], where a low value of X indicates high ability and a high value indicates low ability. Population size is normalized to one. There are two types of jobs in the economy, one advanced, which requires skilled workers and one less advanced where unskilled and high-skilled workers are equally productive. The individuals are all unskilled *ex ante* education, and simultaneously decide whether to get an education and thereby become skilled workers or remain unskilled[9]. The individuals possess no endowments but a unit of time to be allocated between labor and leisure. Education is assumed not to be time consuming. All individuals are capable of getting an education and becoming a skilled worker, but they differ in how costly this investment is for them in terms of effort required. Costs such as foregone labor income and tuition fees are not included in the education cost. Hence, this cost could be viewed as a pure utility loss of effort. The cost function is $c = c(X)$, with properties $c^{'} > 0$ and $c(0) = 0$. Let u and s indicate unskilled and skilled workers, respectively. The wage rates, which are assumed to be equal to the marginal product, are w_u and w_s, with $w_u < w_s$. Labor supplies are $L_u(w_u)$ and $L_s(w_s)$, which are increasing in the wage rate. The aggregate production function is continuously differentiable, concave in

[8]This setting allows the threshold value of X to indicate the share of high-skilled workers, on which the discussion in the paper is highly centered.

[9]Hence, this is in contrast with Spence (1973), where the skill distribution *ex ante* and *ex post* education was identical and the purpose of education was merely to signal high ability. Thus, my approach shares the more optimistic view that education is, in fact, skill improving.

both arguments and exhibits constant returns to scale. It is written

$$Y = Y(xL_s, (1-x)L_u) \tag{2.1}$$

with x indicating the share of skilled workers[10] in equilibrium. Due to the constant returns to scale assumption, wages can be written

$$w_u = \frac{\partial Y}{\partial((1-x)L_u)} = a\left(\frac{xL_s}{(1-x)L_u}\right), a^{'} > 0 \tag{2.2}$$

for unskilled workers and

$$w_s = \frac{\partial Y}{\partial(xL_s)} = b\left(\frac{xL_s}{(1-x)L_u}\right), b^{'} < 0 \tag{2.3}$$

for skilled workers[11].

The maximization problem includes two stages. First, individuals decide whether to remain an unskilled worker or to get an education and thereby become a skilled worker. After having made the education decision, both types of workers maximize their utility by choosing the labor supply maximizing a continuously differentiable and concave utility function

[10] Since inability is uniformly distributed, the individual with X such that she is indifferent between becoming a skilled or an unskilled worker also indicates the share of skilled workers, whereas $1 - X$ is the share of unskilled workers.

[11] See any standard textbook for the derivations of the second equalities in equations (2.2) and (2.3), for example Romer (1996).

$$U_u = u(C_u, F_u) \tag{2.4}$$

for unskilled individuals and

$$U_s = u(C_s, F_s) - c(X) \tag{2.5}$$

for skilled individuals, where $C_u = w_u L_u$ is consumption[12] for an individual of type u, and $F_u = 1 - L_u$, is leisure time for type u. Analogous notation holds for workers of type s. I assume separability between the consumption-leisure complex and the education cost in the utility function, which implies that the education cost does not affect the labor-leisure decision[13]. Assuming an interior solution, the first-order conditions for a maximum are

$$\frac{dU_u}{dL_u} = 0, \tag{2.6}$$

and

[12]The price of the composite consumption good is normalized to 1.

[13]This facilitates the computations considerably when performing comparative statics on the equilibrium in Section 3, since labor supply would otherwise be different for every skilled worker. In the appendix, simulations are made assuming that the education cost enters the individuals' budget constraint. The main results remain unaltered from the simulation results in Section 4.

$$\frac{dU_s}{dL_s} = 0 \tag{2.7}$$

2.2 Characterizing the laissez-faire equilibrium

An equilibrium of the model is an allocation $V = \{w_u, w_s, L_u, L_s, x\}$ such that equations (2.2), (2.3), (2.6), (2.7) and

$$u(C_s, F_s) - c(X) = u(C_u, F_u) \tag{2.8}$$

hold for the indifferent individual. Furthermore,

1. For all individuals with $X < x$,
$u(C_s, F_s) - c(X) > u(C_u, F_u)$,
so these individuals optimally decide to get educated and then choose labor supply L_s and receive wage rate w_s.

2. For all individuals with $X > x$,
$u(C_s, F_s) - c(X) < u(C_u, F_u)$,
so these individuals optimally decide not to get educated and then choose labor supply L_u and receive wage rate w_u.

The equilibrium is found by solving the typically nonlinear system of equations $E = \{2.2, 2.3, 2.6, 2.7, 2.8\}$ for the vector of variables V.

Proposition 1
If $c(1)$ is such that $u(C_s, F_s) - c(1) < u(C_u, F_u)$, there exists a $X = x \in (0, 1)$ such that E has a solution. Furthermore, the equilibrium is unique.

Proof: Follows from the property that $c(X)$ is monotonically increasing and that $c(0) = 0$.■

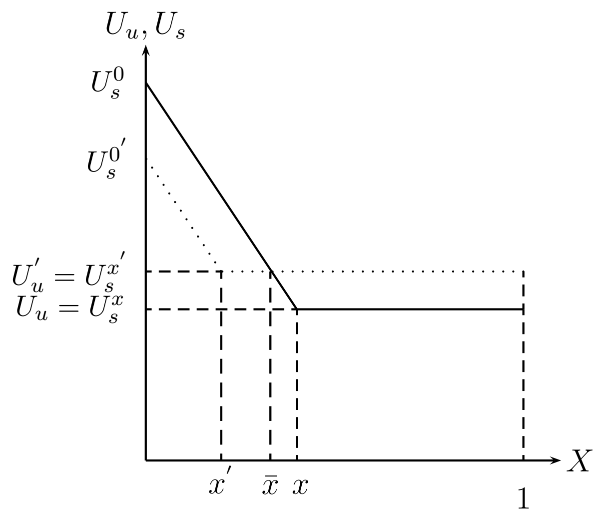
3 Comparative statics

If, for equity reasons, the government is not satisfied with the outcome of the laissez-faire economy, it can alter the equilibrium using its policy tools. That is, it can collect money using a linear tax[14] on labor income and use the revenues to distribute lump-sum transfers and subsidize education. The analysis will first be done assuming exogenous wage rates, so that fiscal redistribution could not be reinforced/counteracted by changes in the wage rates. Thereafter, endogenous wages are incorporated in the model and its influence on the equilibrium is assessed.

Before performing the policy experiments outlined above, the utility level of the two types needs to be clarified. Whereas unskilled workers have the same utility, the different education cost implies the skilled workers to have different utility levels. Therefore, if a policy change increases the utility for unskilled workers and decreases that of skilled workers, this is not true for all switchers, i.e. the individuals that would have chosen to become skilled workers *ex ante* the policy change but who choose to remain unskilled *ex post*. In this group, there are both individuals that are better off from the policy change and those who are worse off. The situation can be illustrated by figure 1:

[14] The analysis could also be done with lump-sum taxation, but this would make it more difficult to relate the results to earlier studies.

Figure 1: Utility change for skilled and unskilled workers



The continuous line represents the utility levels for all individuals $X \in [0, 1]$ *ex ante* the policy change. The individual with the highest ability, that is $X = 0$, gets utility U_s^0 and the one with $X = x$ is the indifferent individual with utility $U_s^x = U_u$. *Ex post* the policy change, the utility levels are represented by the dotted line. The share of skilled workers has decreased to $x^{'}$, the utility for the highest ability individual to $U_s^{0'}$ and that for the indifferent individual has increased to $U_u^{'} = U_s^{x'}$. Note that the indifferent individual is not the same as *ex ante* the policy change. When comparing the *ex post* and *ex ante* utility levels, it is clear that 1) the individuals that are skilled (unskilled) both *ex ante* and *ex post* the policy change are worse (better) off by the policy change and 2) in the switcher group, there are some individuals that are worse off ($X \in [x^{'}, \bar{x}]$), and some that are better off ($X \in [\bar{x}, x]$). For this reason, it is not completely correct to assess the impact of a policy change in terms of differences in utilities for the two types exclusively, as will be done in the rest of the paper. However, it seems to be the best way of proceeding in order to keep clearness and lucidness.

3.1 Exogenous wage rates

Introducing a linear labor income tax t used to finance equally large transfers T to all individuals and a subsidy S[15] to those that choose to get an education, the consumption of the two types now becomes $C_u = \omega_u L_u + T$ and $C_s = \omega_s L_s + T + S$, where $\omega_u = (1-t)w_u$ and $\omega_s = (1-t)w_s$ are the after tax wage rates. The equilibrium equations are the following:

$$T + xS = t(xw_s L_s + (1-x)w_u L_u) \tag{3.1}$$

$$L_u = f(\omega_u, T), f_1^{se} > 0, f_1^{ie} = f_2 < 0 \tag{3.2}$$

$$L_s = g(\omega_s, T+S), g_1^{se} > 0, g_1^{ie} = g_2 < 0 \tag{3.3}$$

$$c(x) = u(\omega_s L_s + T + S, 1 - L_s) - u(\omega_u L_u + T, 1 - L_u) \tag{3.4}$$

where se and ie indicate substitution effect and income effect, respectively, and the income effects are assumed to be negative. In this equation system, T, L_u L_s and x are the endogenous variables, w_u and w_s are exogenous and t and S are the policy variables that the government possesses. Equation (3.1) is the government budget constraint, with expenditures on the left-hand side and revenues on the right-hand side. Labor supplies (3.2) and (3.3) are affected by the after tax wage rate and the transfer size in three ways. The after tax wage rate has a substitution effect on unskilled workers $d\omega_u f_1^{se}$ and an income effect $\frac{\partial C_u}{\partial \omega_u} d\omega_u f_1^{ie}$ that increases the individuals' labor supply when a tax is introduced. The transfer itself only involves an income effect $\frac{\partial C_u}{\partial T} dT f_2$. These terms represent the Slutsky decomposition of the total effect of the tax on labor supply. Analogous effects are assumed for skilled workers. Note that

[15] The main results remain unaltered if the subsidy is instead assumed to be a share of the total education cost.

Equation (3.4) is (2.8) rewritten. Differentiating equations (3.1)-(3.4) using the fact that $d\omega_u = -w_u dt$[16], $d\omega_s = -w_s dt$ and evaluating the equations at $t = S = 0$[17], the system of equations can now be written in matrix form as:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -f_2 & 1 & 0 & 0 \\ -g_2 & 0 & 1 & 0 \\ u_{C_u} - u_{C_s} & \omega_u u_{C_u} - u_{F_u} & u_{F_s} - \omega_s u_{C_s} & c' \end{pmatrix} \begin{pmatrix} dT \\ dL_u \\ dL_s \\ dx \end{pmatrix} = \begin{pmatrix} x w_s L_s + (1-x) w_u L_u & -x \\ -f_1^{se} w_u - f_1^{ie} w_u & 0 \\ -g_1^{se} w_s - g_1^{ie} w_s & 0 \\ u_{C_s} w_s L_s - u_{C_u} w_u L_u & u_{C_s} \end{pmatrix} \begin{pmatrix} dt \\ dS \end{pmatrix} \tag{3.5}$$

Using the envelope conditions $u_{F_u} = \omega_u u_{C_u}$ and $u_{F_s} = \omega_s u_{C_s}$ and solving the matrix equation for the vector $\{dT, dL_u, dL_s, dx\}$ yields:

$$dT = x w_s L_s dt + (1-x) w_u L_u dt - x dS \tag{3.6}$$

$$dL_u = -f_1^{se} w_u dt - f_2 x dS + f_2 x (w_s L_s - w_u L_u) dt \tag{3.7}$$

$$dL_s = -g_1^{se} w_s dt + g_2 (1-x) dS + g_2 (x-1)(w_s L_s - w_u L_u) dt \tag{3.8}$$

$$dx = \frac{u_{C_s}((x-1)(w_s L_s - w_u L_u)dt + (1-x)dS) - u_{C_u}(x(w_s L_s - w_u L_u)dt - x dS)}{c'} \tag{3.9}$$

$$= \frac{(w_s L_s - w_u L_u)(u_{C_s}(x-1) - u_{C_u} x)dt + (u_{C_s}(1-x) + u_{C_u} x)dS}{c'} \tag{3.10}$$

[16] $d\omega_u = d((1-t)w_u) = dw_u - d(t w_u) = dw_u - t dw_u - w_u dt = -w_u dt$ where the last equality follows from the assumption of exogenous wages. An analogous result is obtained for type s.

[17] This is done for expositional reasons and in order to get easily interpretable solutions. In the simulation section, t and S are allowed to take discrete steps.

Starting with equation (3.10), the right-hand side shows that dx can have either sign. At the margin, the share of skilled workers is negatively affected by taxation but positively affected by education subsidies. Equation (3.9) establishes the connection between the change in the share of skilled workers and government policy. Policies that benefit skilled (unskilled) workers increase (decrease) x whereas those that make skilled (unskilled) workers worse off decrease (increase) x. The denominators of equations (3.9) and (3.10) show that the change in x is smaller at the margin, the steeper is the cost function. In the case of $dx > 0$, this is due to the fact that fewer individuals gain by getting an education for a given change in the utility difference between the two types. In the case of $dx < 0$, the reason is that the education cost decreases relatively fast with the ability of the individuals. The subsidy only involves an income effect that decreases the income of unskilled workers and increases that for skilled workers. Hence, it tends to increase unskilled labor supply and decrease skilled labor supply. The transfer change dT is positively affected by taxation and negatively affected by the degree of education subsidies.

The signs of dL_u in (3.7) and dL_s in (3.8) are ambiguous. However, the third term is negative in (3.7) and positive in (3.8). These expressions represent the *total* income effect, that is the sum of the income effects from the tax and the transfer. The different signs on the total income effects for the two types follow from the fact that unskilled workers are net recipients and skilled workers net contributors to the government. Thus, if $dS = 0$, $dL_u < dL_s$ if the substitution effect is not considerably more negative for skilled workers[18]. It is natural to assume that the labor supply in the laissez-faire economy is higher for skilled than for unskilled workers, since the former

[18]In the simulations in Section 4, I show that the substitution effects do not outweigh the difference in income effects between the two types. Hence, unskilled workers decrease their labor supply more than skilled workers when the tax is introduced.

type has a higher wage rate than the latter; i.e., $L_u < L_s$. If this is the case, the decrease in labor supply is relatively stronger for unskilled workers, that is $dL_u/L_u < dL_s/L_s$. This is the result that Feldstein (1973) used to argue for converging wages when income is taxed. However, when $dS = 0$, equation (3.10) shows that $dx < 0$. This makes skilled labor more scarce and therefore has a wage diverging effect when wages are not fixed. These effects on the wage rates are discussed in the following section.

3.2 Endogenous wage rates

With endogenous wage rates, the equilibrium equations are now the following:

$$T + xS = t(xw_sL_s + (1-x)w_uL_u) \tag{3.11}$$

$$L_u = f(\omega_u, T), f_1^{se} > 0, f_1^{ie} = f_2 < 0 \tag{3.12}$$

$$L_s = g(\omega_s, T+S), g_1^{se} > 0, g_1^{ie} = g_2 < 0 \tag{3.13}$$

$$w_u = a\left(\frac{xL_s}{(1-x)L_u}\right), a' > 0 \tag{3.14}$$

$$w_s = b\left(\frac{xL_s}{(1-x)L_u}\right), b' < 0 \tag{3.15}$$

$$c(x) = u(C_s, F_s) - u(C_u, F_u) \tag{3.16}$$

Equations (3.14-15) are the new equations as compared to the previous case with exogenous wages. The wage rates for the two skill types are inversely related to the total relative labor supply of their own type. In principle, the system of equations (3.11-16) can be solved for the endogenous variables in the same way as in the case with exogenous wages, but the solution seems difficult to interpret. Therefore, in the following, will only discuss the main effects of endogenizing the wages. The sign and magnitudes of these effects are then discussed along with the simulations in the next section. Differentiating equations (3.14-15) yields:

$$dw_u = a^{'} \frac{x}{1-x} \frac{L_s}{L_u} \left[\frac{dL_s}{L_s} - \frac{dL_u}{L_u} + \frac{dx}{x(1-x)} \right] \tag{3.17}$$

$$dw_s = b^{'} \frac{x}{1-x} \frac{L_s}{L_u} \left[\frac{dL_s}{L_s} - \frac{dL_u}{L_u} + \frac{dx}{x(1-x)} \right] \tag{3.18}$$

The differentiated wage equations (3.17-18) show what should be expected. The wage rate of the own worker type decreases with the labor supply per worker of the same type, increases with that of workers of the other type, and decreases with the share of workers of the same type.

It is interesting to relate wage equations (3.17-18) to the wage equation in Dur and Teulings (2003). In their paper, the wage equation is $w(s,\mu) = w_o(\mu) + e^{(-\gamma\mu)s}$, where s is the individual's human capital level and μ is the mean level of human capital in the economy. If labor supply were fixed, the corresponding variable to μ in my paper would be x, that is the share of skilled workers. The more general formulation of the wage rate in my paper allows wages to depend not only on the distribution of skill types, but also on the supply of labor for different skill types.

Two main effects for how endogenizing wages affect the equilibrium can be distinguished. First, labor supplies react when t is changed, which affects the wage rates and thus, utilities and the shares of the types are also affected. Second, for a given change in x, wage rates are affected since they are functions of x. These effects are always counteracting the initial change in x. For example, when $dx < 0$, skilled workers become relatively more scarce. This will have a positive effect on skilled workers' wage rate and a negative effect on the wage rate of unskilled workers. This makes it more attractive to become a skilled worker and thus, the drop in x is reduced. It was not possible to explore these effects in Feldstein (1973) and Allen (1982), since the shares of skilled and unskilled workers were fixed in their models. As could be seen in sections 3.1 and 3.2, redistribution through the tax and transfer system implies two different effects on wage rates. First, the relative labor supply per individual type increases, which has a wage *converging* effect. Second, the share of skilled workers decreases, which has a wage *diverging* effect. Education subsidies have the opposite effect on wages, since they make relative labor supply decrease and the share of skilled workers increase. The net effects of this labor supply effect on the one hand and the share effect on the other hand, can naturally not be assessed analytically. Therefore, they are now assessed in the next section, using simulations.

4 Simulations

Section 3 examined and described the partial effects on the endogenous variables that may be generated by government intervention in a laissez-faire economy. In this section, I use specific functional forms in order to simulate the signs and magnitudes of the net effects on these variables. The calculations were made in *Mathematica*.

4.1 Simulation Model

$$
\begin{aligned}
u(C_u, F_u) =& [\phi((1-t)w_u L_u + T)^\beta + (1-\phi)(1-L_u)^\beta]^{\frac{1}{\beta}} && (4.1)\\
u(C_s, F_s) =& [\phi((1-t)w_s L_s + T + S)^\beta + (1-\phi)(1-L_s)^\beta]^{\frac{1}{\beta}} && (4.2)\\
Y =& [\mu((1-x)L_u)^\gamma + (1-\mu)(xL_s)^\gamma]^{\frac{1}{\gamma}} && (4.3)\\
T =& t(xw_s L_s + (1-x)w_u L_u) - xS && (4.4)\\
w_u =& \frac{\partial Y}{\partial((1-x)L_u)} && (4.5)\\
w_s =& \frac{\partial Y}{\partial(xL_s)} && (4.6)\\
\frac{\partial U_u}{\partial L_u} =& 0 && (4.7)\\
\frac{\partial U_s}{\partial L_s} =& 0 && (4.8)\\
u(C_u, F_u) =& u(C_s, F_s) - \theta x^\xi && (4.9)
\end{aligned}
$$

Equations (4.1-3) assume a CES form of the utility functions for both types of workers and the aggregate production function. Equation (4.4) is the government budget constraint. Equation (4.9) is the condition for the individual that is indifferent between remaining unskilled or becoming skilled. I assume that the cost function enters linearly in that equation, that is $\xi = 1$. This assumption is relaxed in the sensitivity analysis. The other parameter values are chosen in the following way: $\phi = 0.29$, $\mu = 0.4$, $\gamma = 1/3$ and $\beta = 0.12$, as in Lundholm and Wijkander (2002). This gives labor supplies in the magnitude of 40 hours per week and around 50 per cent higher wages for skilled than for unskilled workers in the case with no government intervention. θ is set to 0.0866 for the share of skilled workers to be 50 per cent in the laissez-faire equilibrium. This makes the comparisons with the simulation results in Feldstein (1973) more transparent.

4.2 Results

Table 1: Government policy, wage rates, labor supplies, transfer size and share of skilled workers

t	S	w_u	w_s	L_u	L_s	T	x
0	0	0.4063	0.5938	0.2423	0.2519	0	0.5000
0.1	0	0.3991	0.6009	0.2155	0.2339	0.0112	0.4781
0.1	0.003	0.4037	0.5964	0.2314	0.232269	0.0098	0.4919

Table 1 shows the equilibrium values of the endogenous variables, for a given government policy. The laissez-faire equilibrium shows labor supply to be somewhat larger for skilled workers, and their wage rate to be around 50 per cent higher than that for unskilled workers.

When the income tax is imposed, labor supplies decrease for both types. However, it decreases proportionally more for the unskilled type, which is in line with the theoretical predictions in Section 3. Wages tend to *diverge* when the tax is imposed, thus counteracting the fiscal redistribution. Thus, it seems as if the wage *converging* effect of different labor supply responses is more than outweighed by the wage *diverging* effect of a lower share of skilled workers.

Switching a portion of the governments' tax revenue from financing transfers to financing an education subsidy yields wage compression between the two types. Once more, there are two effects working in opposite directions.

First, the subsidy attracts more individuals to become skilled workers, thus implying a wage *converging* effect since unskilled labor becomes more scarce. Second, the subsidy reduces the labor supply for skilled workers since they receive higher total transfers than before, and increases that for unskilled workers since they receive lower total transfers than before. This constitutes a wage *diverging* effect between skilled and unskilled workers. Thus, the wage *converging* effect of a higher share of skilled workers seems to more than outweigh the wage *diverging* effect of different labor supply responses.

Table 2: Government policy and utilities

t	S	U_u	U_s^0
0	0	4406	4839
0.1	0	4411	4825
0.1	0.003	4404	4830

Table 2 shows the equilibrium values of the utilities, for the same government policies as in table 1. The utilities are computed using the equilibrium values of the variables in equations (4.1-2)[19]. Apparently, utility for the unskilled type increases and utility for the skilled worker with highest ability, U_s^0, decreases, when the tax is introduced. The education subsidy increases the utility for the skilled type and decreases that for the unskilled type. In this equilibrium, utility is lower for both types as compared to the laissez-faire case.

[19]For expositional reasons, the values have been monotonically transformed by multiplying them with 10^4.

4.3 Comparison with Feldstein (1973)

As was presented above, the general equilibrium effects of redistribution on the wage rates counteract fiscal redistribution. This is due to the fact that the counteracting effect of a lower share of skilled workers, which was neglected in Feldstein (1973), seems to be stronger than the reinforcing effect of labor supply responses. To highlight the comparison with Feldstein (1973), I have redone the simulations with the setup of that paper. That is, using a CES utility function and a Cobb-Douglas production function and the same parameter values as he used, and introduced heterogeneity in ability and an endogenous education decision as in my model . It turns out that his results of wage compression as a consequence of redistribution vanish in most cases. On the contrary, wages typically *diverge* with the tax rate, as they do in the simulations in section 4.2. The setup of Feldstein (1973) was the following:

$$u(C_u, F_u) = [\phi(1-t)w_u L_u + T)^{-\beta} + (1-\phi)(1-L_u)^{-\beta}]^{-\frac{1}{\beta}} \tag{4.10}$$

$$u(C_s, F_s) = [\phi(1-t)w_s L_s + T)^{-\beta} + (1-\phi)(1-L_s)^{-\beta}]^{-\frac{1}{\beta}} \tag{4.11}$$

$$Y = k_0 L_s^{\gamma} L_u^{1-\gamma} \tag{4.12}$$

$$T = 0.5\, t(w_s L_s + w_u L_u) \tag{4.13}$$

$$\frac{\partial Y}{\partial L_u} = w_u \tag{4.14}$$

$$\frac{\partial Y}{\partial L_s} = w_s \tag{4.15}$$

$$\frac{\partial U_u}{\partial L_u} = 0 \tag{4.16}$$

$$\frac{\partial U_s}{\partial L_s} = 0 \tag{4.17}$$

$$\tag{4.18}$$

The parameter values were $\phi = 0.5$, $\gamma = 0.67$, $k_0 = 1$ and β was set at values 1, 0.01 and -0.5, giving the elasticity of substitution ϵ values 0.5, 0.99 and 2.00, respectively. The following results were obtained[20].

Table 3: Effects of taxes on labor supplies and gross wage rates with fixed $x = 0.5$

ϵ	t	L_s	L_u	w_s	w_u	$\frac{w_s}{w_u}$
0.50	0	0.54	0.65	0.71	0.29	2.44
	0.24	0.53	0.58	0.69	0.31	2.22
	0.48	0.50	0.50	0.67	0.33	2.03
	0.72	0.45	0.39	0.64	0.36	1.77
0.99	0	0.50	0.50	0.67	0.33	2.03
	0.24	0.45	0.41	0.65	0.35	1.85
	0.48	0.37	0.30	0.62	0.38	1.63
	0.72	0.26	0.17	0.59	0.43	1.37
2.00	0	0.38	0.29	0.61	0.40	1.52
	0.24	0.27	0.18	0.59	0.43	1.37
	0.48	0.15	0.19	0.57	0.46	1.23
	0.72	0.05	0.03	0.55	0.49	1.12

Clearly, wages converge as the tax is increased for all three choices of the elasticity of substitution between labor and leisure. Now, if the share of skilled and unskilled workers is endogenized in the same way as in my model,

[20]This is table 1 in Feldstein (1973).

the setup becomes:

$$u(C_u, F_u) = [\phi(1-t)w_u L_u + T)^{-\beta} + (1-\phi)(1-L_u)^{-\beta}]^{-\frac{1}{\beta}} \tag{4.19}$$

$$u(C_s, F_s) = [\phi(1-t)w_s L_s + T)^{-\beta} + (1-\phi)(1-L_s)^{-\beta}]^{-\frac{1}{\beta}} \tag{4.20}$$

$$Y = k_0(xL_s)^{\gamma}((1-x)L_u)^{1-\gamma} \tag{4.21}$$

$$T = t(xw_s L_s + (1-x)w_u L_u) \tag{4.22}$$

$$w_u = \frac{\partial Y}{\partial((1-x)L_u)} \tag{4.23}$$

$$w_s = \frac{\partial Y}{\partial(xL_s)} \tag{4.24}$$

$$\frac{\partial U_u}{\partial L_u} = 0 \tag{4.25}$$

$$\frac{\partial U_s}{\partial L_s} = 0 \tag{4.26}$$

$$u(C_u, F_u) = u(C_s, F_s) - \theta x^{\xi} \tag{4.27}$$

Using the same parameter values as Feldstein (1973), and letting $\theta = 0.0866$ and $\xi = 1$ as in my simulations, the following results were obtained:

Table 4: Effects of taxes on labor supplies and gross wage rates with endogenized x

ϵ	t	L_s	L_u	w_s	w_u	$\frac{w_s}{w_u}$
0.50	0	0.54	0.65	0.71	0.29	2.45
	0.24	0.52	0.59	0.74	0.27	2.74
	0.48	0.51	0.51	0.79	0.24	3.29
	0.72	0.50	0.39	0.86	0.20	4.30
0.99	0	0.50	0.50	0.72	0.29	2.48
	0.24	0.46	0.39	0.74	0.27	2.74
	0.48	0.42	0.25	0.77	0.25	3.08
	0.72	0.38	0.12	0.78	0.25	3.12
2.00	0	0.42	0.22	0.71	0.29	2.44
	0.24	0.33	0.11	0.72	0.28	2.57
	0.48	0.24	0.04	0.72	0.29	2.48
	0.72	0.14	0.01	0.71	0.29	2.45

Table 4 shows that with an endogenized share of skilled and unskilled workers, wages between the two types typically *diverge* with a higher tax rate. Only for a very high elasticity of substitution between consumption and leisure, $\epsilon = 2$, will Feldstein's results go through for tax changes at some intervals. Overall, and especially for values of the elasticity of substitution with empirical relevance, endogenizing the shares of skill types contradicts the results in Feldstein (1973) and supports the simulation results in section 4.2 in this paper.

4.4 Sensitivity analysis

4.4.1 Parameter values, the cost function and the tax rate

A sensitivity analysis was made by combining different sets of parameter values. Although such operations can never be conclusive, the main results of the benchmark case seem to hold even with very different assumptions on the parameter values. Wages typically *diverge* as an income tax is introduced, counteracting the redistributive fiscal effect. Introducing the education subsidy increases the share of skilled workers and also has a converging effect on wages. Labor supplies are decreasing in the tax rate, with the supply of unskilled labor reacting more strongly than that of skilled labor. The results also seem stable to different forms of the cost function, for example if the ability enters the cost function exponentially instead of linearly, that is $\xi > 1$.

The form of the cost function is essential. The steeper it is around the equilibrium point of the share of skilled workers, the less will redistribution affect this share. This is because a relatively steep cost function implies that additional switchers into education face a relatively high education cost. The "share effect" on wages turned out to outweigh the labor supply effect in the previous section. Hence, if the cost function is made sufficiently steep, this will no longer be the case. As dx becomes sufficiently small, the labor supply effect will outweigh the "share effect". As an experiment, I have increased ξ and adjusted θ for every value such that the shares of workers are always 0.5, as in the main simulations. In that way, the cost function becomes steeper pivoting around the point $c(0.5)$ when $\xi = 1$. It turns out that the labor supply effect dominates the "share effect" when $\xi > 6.5$[21].

In contrast to the simulation results, in many cases the utility for *both* types of workers decreases when income is taxed and this distortion increases with the tax rate. This is true for letting $t > 0.11$ in the benchmark case. The reason for this seems to be the following. The income tax distorts

[21] The exact number cannot be obtained, but the threshold is somewhere between 6.0 and 6.5.

the labor-leisure choice in such a way that labor supply is reduced. This increases leisure time, which has a positive effect on utility. At the same time, consumption is reduced, which has a negative effect on utility. If the tax change is infinitely small, as is the case in the theoretical section, these effects on utility cancel out exactly by its envelope conditions. However, if the tax increase is discrete and consumption and leisure are complements in the utility function, this is no longer the case. The negative effect on utility induced by the decrease in consumption more than outweighs the positive effect induced by a higher leisure time. When the tax rate is set sufficiently high, this distortive effect is so strong that it even outweighs the positive effect on utility for unskilled individuals induced by the redistribution from skilled individuals. Thus, utility for both types decreases above this tax level.

4.4.2 Education cost as a money cost

Naturally, the assumption of separability of the education cost in the utility function for skilled workers, equation (2.5), is unrealistic. As described earlier, it neglects the fact that the education cost includes foregone income, tuition fees etc. Therefore, as a sensitivity analysis, the separability assumption is dropped in the appendix, where the education cost is now a pure money cost instead of an effort cost. The main results remain unaltered.

4.5 Optimal policy

This section assesses the question of which policy mix is optimal from society's point of view. The social welfare function SWF is assumed to be

$$SWF(x, U_u, \overline{U_s}) = \left(x\overline{U_s}^{\psi} + (1-x)Uu^{\psi}\right)^{\frac{1}{\psi}} \tag{4.28}$$

where $\overline{U_s} = \frac{U_u + U_s^0}{2}$ is the average utility for skilled workers[22] and ψ governs the efficiency-equity trade-off. Note that this function only considers equity between, and not within, the two groups. Feldstein (1973) also used a CES social welfare function to compute optimal taxes but in his study, there were fixed shares of skill groups and all individuals had the same utility within their skill group. I evaluate equation (4.28) for tax rates between -99% and 95%[23] using Matlab. The inclusion of negative tax rates allows for the possibility that the optimal policy may be to subsidize labor and finance it by negative lump-sum transfers. I used the same values on the egalitarian preference parameter ψ as Feldstein (1973). The rest of the parameter values are the same as in Section 4.1. For the case with no education subsidies, that is $S = 0$, the results for each value of ψ are summarized in table 5 below.

Table 5: Optimal tax rates and transfers.

ψ	1	0.5	0.01	-1	-10	-50
$t(\%)$	1.56	1.63	1.69	1.84	3.15	7.59
T	0.0019	0.0020	0.0020	0.0022	0.0038	0.0087

As can be seen from table 5, the optimal tax rates are remarkably low for all values of ψ, compared to for example Feldstein (1973). The reason is that the utility for unskilled workers peaks already at $t = 0.11$, given the chosen parameter values, whereas that for skilled workers decreases monotonically with the tax rate for all values of t.

[22]The average utility for the skilled group follows from linearity of the cost function. See figure 1 for a graphic interpretation.

[23]For $t = -1$ and $t \in (0, 95, 1]$, there are values of t for which there does not seem to exist any real solution to the system of equations.

In the case where not only the tax and transfer system but also education subsidies are in the policy maker's possession, the optimal policy mix is calculated combining $t \in [-0.99, 0.45]$ and $S \in [-0.2, 0.2]$[24]. The results for each value of ψ are summarized in table 6 below.

Table 6: Optimal tax rates, transfers and subsidies.

ψ	1	0.5	0.01	-1	-10	-50
$t(\%)$	0.1	-0.1	0	0	0.1	0
S	-0.009	-0.010	-0.010	-0.011	-0.017	-0.033
T	0.0042	0.0043	0.0045	0.0049	0.0073	0.0121

As can be seen from table 6, the optimal tax rates are practically zero in the case when also education subsidies can be used since in this case, the government possesses two lump-sum instruments. Thus, it can redistribute money by collecting money through a negative education subsidy and spending money by a positive general transfer. This was not possible in the case without the education subsidy. The table also shows that the negative education subsidy decreases with the desire for equity, represented by a lower value of ψ. As this lump-sum transfer from the educated individuals increases, so does the general transfer that benefits the unskilled workers.

[24]The values are restricted for the same reason as in note 21.

5 Discussion and conclusions

This paper has investigated the effects of a linear income tax and education subsidies on wages, labor supplies, transfers, the share of skilled workers and utility levels. It was shown that endogenizing wages and the share of skilled and unskilled workers may affect each other differently and also introduces additional channels of interdependencies to other important economic variables. The main result of the paper was that, contrary to Feldstein (1973), general equilibrium effects on wages may counteract fiscal redistribution when the share of skilled and unskilled workers is endogenous. It was also shown that 1) introducing education subsidies tends to wage convergence and recuperates some incentives to higher education that may otherwise be weakened by redistribution, 2) education subsidies can be viewed as a transfer from unskilled to skilled workers and the resulting wage compression is not sufficient to make unskilled workers better off and 3) the optimal policy mix may include negative subsidies and a low (or zero) income tax.

The model used is necessarily simple and abstracts from almost all heterogeneity in individuals' preferences, characteristics and behavior. Nevertheless, a few policy relevant arguments could be advanced. First, redistribution may be counteracted by diverging wages between unskilled and skilled workers. However, the adjustment to the new equilibrium may be faster regarding individuals' labor supply than for the share of educated workers. This is because, in principle, all individuals can immediately adjust their labor supply, whereas the stock of educated people adjusts slowly due to the fact that a large share of it has already chosen which type of worker to become. Therefore, in the short run, the wage converging effect of different labor supply responses to a tax change may reinforce redistribution by wage convergence while in the long run, redistribution is counteracted by wage divergence. Second, education subsidies may merely work as a redistribution from unskilled to skilled workers. Thus, with distortive taxation, a government with redistributive ambitions may do better by having relatively modest (or no) *general* education subsidies, taxes and transfers than by implementing relatively high taxes, transfers and subsidies. Naturally, in the view of imperfect capital markets, there may still be good reasons for the government to direct education subsidies to disadvantaged individuals or give them advantageous loan conditions. However, if the share of skilled workers has a positive externality effect on the growth rate of the economy, such as suggested by parts of the endogenous growth literature, general education subsidies may benefit both skilled and unskilled workers in the long run.

References

Allen, F. (1982): Optimal Linear Income Taxation With General Equilibrium Effects on Wages. *Journal of Public Economics 17(2)*, 135-143.

Angrist, J.D. and Krueger, A.B. (1991): Does Compulsary School Attendance Affect Schooling and Earnings?. *Quarterly Journal od Economics 106(4)*, 979-1014.

Atkinson, A.B. (1973): How Progressive Should the Income Tax be?, in: *M. Parkin, ed., Essays on modern economics (Longman).*

Blankenau, W. (1999): A Welfare Analysis of Policy Responses to the Skilled Wage Premium. *Review of Economic Dynamics 2(4)*, 820-849.

Bovenberg, A.L. and Jacobs, B. (2005): Redistribution and Education Subsidies are Siamese Twins. *Journal of Public Economics 87*, 2005-2035.

Brett, C. and Weymark, J.A. (2003): Financing Education Using Optimal Redistributive Taxation. emphJournal of Public Economics 87.

Cremer, H., Pestieau, P., Thibault, E. and Vidal, J.-P.(2005): Optimal Tax and Education Policy When Agents Differ in Altruism and Productivity. *Annals of Economics and Finance 6.*

Dur, R.A.J. and Teulings, C.N. (2003): Are Education Subsidies an Efficient Redistributive Device?. *Tinbergen Institute Working Paper No. 2003-024/3.* Data posted at http://papers.ssrn.com.

Fair. R.C. (1971): The optimal distribution of income. *The Quarterly Journal of Economics 85(4)*, 551-579.

Feldstein, M. (1973): On the Optimal Progressivity of the Income Tax. *Journal of Public Economics 2(4)*, 357-376.

Lundholm, M. and Wijkander, H. (2002): Public Ownership and Income Redistribution. *Working Paper 2003:2*, http://ne.su.se.

Mirrlees, J.A. (1971): An Exploration in the Theory of Optimal Income Taxation. *Review of Economic Studies 38*, 179-208.

Romer, D. (1996): *Advanced Macroeconomics*, McGraw-Hill.

Sheshinski, E. (1972): The Optimal Linear Income Tax. *The Review of Economic Studies 39(3)*, 297-302.

Spence, M. (1973): Job Market Signaling. *The Quarterly Journal of Economics 87(3)*, 355-374.

Storesletten, K. and Zilibotti, F. (2000): Education, Educational Policy and Growth *Swedish Economic Policy Review 7*, 39-70.

Trostel, P. and Walker, I. (2000): Education and Work. *The Warwick Economics Research Paper Series 554.*

van Ewijk, C. and Tang, P.J. (2001): Efficient Progressive Taxes and Education Subsidies. *Tinbergen Institute Discussion Paper, TI 2001-002/2.*

Wilson, J.D. (1982): The Optimal Public Employment Policy. *Journal of Public Economics 17*, 241-258.

Appendix

Consumption for skilled workers is now assumed to be $C_s^X = (1-t)w_sL_s^X + T + S - \theta X$. Consumption differs among skilled workers and the labor supply will consequently also differ among these workers. Let the labor supply for the indifferent individual be denoted L_s^x and that for the one with highest ability be L_s^0. The corresponding utilities are U_s^x and U_s^0, respectively.The total labor supply of skilled workers is $\overline{L_s}$. The following benchmark is now assumed:

$$U_u = [\phi(1-t)w_uL_u + T)^\beta + (1-\phi)(1-L_u)^\beta]^{\frac{1}{\beta}} \tag{5.1}$$

$$U_s^x = [\phi(1-t)w_sL_s^x + T + S - \theta x)^\beta + (1-\phi)(1-L_s^x)^\beta]^{\frac{1}{\beta}} \tag{5.2}$$

$$U_s^0 = [\phi(1-t)w_sL_s^0 + T + S)^\beta + (1-\phi)(1-L_s^0)^\beta]^{\frac{1}{\beta}} \tag{5.3}$$

$$Y = [\mu((1-x)L_u)^\gamma + (1-\mu)\overline{L_s}^\gamma]^{\frac{1}{\gamma}} \tag{5.4}$$

$$T = t(w_s\overline{L_s} + (1-x)w_uL_u) - xS \tag{5.5}$$

$$\overline{L_s} = \int_0^x L_s^X dL_s = x\frac{L_s^x + L_0}{2} \tag{5.6}$$

$$w_u = \frac{\partial Y}{\partial((1-x)L_u)} \tag{5.7}$$

$$w_s = \frac{\partial Y}{\partial \overline{L_s}} \tag{5.8}$$

$$\frac{\partial U_u}{\partial L_u} = 0 \tag{5.9}$$

$$\frac{\partial U_s^x}{\partial L_s^x} = 0 \tag{5.10}$$

$$\frac{\partial U_s^0}{\partial L_s^0} = 0 \tag{5.11}$$

$$U_u = U_s^x \tag{5.12}$$

The second equality in equation (5.6) shows the average labor supply of skilled workers to be just the average of L_s^x and L_0. This stems from the fact that with a CES utility function, labor supply is proportional to the education cost. Using the same parameter values as in the simulations in Section 4, the results in table 7 and 8 were obtained. As can be seen, the main results are qualitatively the same as in Section 4.

Table 7: Government policy, wage rates and labor supplies

t	S	w_u	w_s	L_u	L_s^0	L_s^x	$\overline{L_s}$
0	0	0.395680	0.604379	0.157564	0.182356	0.239973	0.0889048
0.1	0	0.383750	0.617120	0.131117	0.165750	0.222420	0.0734817
0.1	0.003	0.388516	0.611912	0.134568	0.162207	0.221781	0.0757906

Table 8: Government policy, transfer size, share of skilled workers and utilities

t	S	T	x	U_u	U_s^x	U_s^0
0	0	0	0.420199	0.476366	0.476366	0.511960
0.1	0	0.00766491	0.377895	0.476864	0.476864	0.511170
0.1	0.003	0.00662380	0.394023	0.476231	0.476231	0.512251