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Public and Private Expenditures on Health in the Presence of Inequality and Endogenous Mortality: A Political Economy Perspective*

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Abstract:

In this paper we study an overlapping-generations model in which agents' mortality risks, and consequently impatience, are endogenously determined by private and public investment in health care. The proportion of revenues allocated for public health care is also endogenous, determined as the outcome of a voting process. Higher substitutability between public and private health is associated with a "crowding-out" effect which leads to lower public expenditures on health care in the political equilibrium. This in turn impacts on mortality risks and impatience leading to a greater persistence in inequality and long run distributions of wealth that are bimodal.

JEL Classifications: I12; I20; O5

Keywords: health; inequality; political economy; income distribution dynamics

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1. Introduction

The dual provision of health care is an issue commonly discussed in policy circles of both developed and developing economies. Central to some of these discussions is the idea that the extent or optimality of public or private provision depends on whether these services are viewed as substitutes or complements. Also, politico-economic factors play a significant role in the determination of the public/private share in a mixed system of health care provision. Consequently, they could potentially provide an explanation for the observed diversity in public expenditures on health across countries. Furthermore, one is also interested in the long run macroeconomic implications of these issues, which have so far not been explored in the literature. For example, one is interested in how political factors, and the degree of substitutability between public and private health care, impact on the choice of public health care expenditures and eventually on inequality and growth.

The aim of this paper is to explore these issues within the framework of a dynamic general equilibrium model with overlapping generations of heterogeneous agents facing mortality risk. In our model, which is a simple extension of Chakraborty and Das (2005), (henceforth referred to as CD) , mortality risk is endogenous, and depends on the individualÊs private investment in health. In addition, we extend the CD framework by assuming that the mortality risk faced by agents is also affected by *public* investment in health care. The proportion of public revenues that are used for the public provision of health care is also endogenous, and is determined by a political process, modelled in this context as the outcome of voting by agents.

We find that the political outcome critically depends on the degree of substitutability between private and public health expenditures, and has interesting implications for economic growth and the persistence of inequality. In some cases a political outcome exists only if the voting procedure allows a result that is based on the plurality rule rather than the majority rule. Furthermore, an interesting implication of the degree of substitutability relates to the characteristic of the long run invariant distribution of income and wealth in the economy. Typically, high substitutability is associated with an ergodic

distribution that is bimodal, while a lower degree of substitutability corresponds to uni-modal distributions.

Numerical simulations of the *static* version of our model suggest that even in the case of majoritarian outcomes, the political outcome is often influenced by the preferences of the agents at the middle and top end of the wealth distribution. The political result is sometimes also characterized by the „ends against the middle‰feature observed in Epple and Romano (1996a, 1996b) and Gouveia (1997), although in these studies the modelling of the dual provision of the public good in question is associated with some agents choosing to „opt out‰of using the public good or use it as a supplement, which is not the case in our model. The only exception in our model, in terms of its *outcomes,* is a situation in which *all* agents opt out of the public good by voting in favour of distributing all of the tax revenue in the form of a lump sum transfer to agents in the economy. This type of situation occurs if public investment in health care is a perfect substitute for private investment in health care in the „health production function‰which is of the constant elasticity of substitution form. For relatively low values of the elasticity of substitution, we have another type of „corner solution‰in which agents vote in favour of tax revenues being allocated entirely to public investment in health. In this case, since public and private expenditures are somewhat complementary to each other, agents also choose to invest in private health care. [1]

For an intermediate range of values of the elasticity of substitution, and for moderately high levels of inequality, a diverse set of results emerges, with the proportion of revenues allocated to public health increasing as the elasticity of substitution decreases. The underlying intuition for these results is related to how public expenditure on health influences the mortality of agents in the economy, and the extent of „crowding out‰between private and public expenditures. This interaction between the „crowding out‰feature and the endogenous time preference aspect in this framework also influences the long run outcomes of the model. Specifically, these features also have interesting implications for the dynamics of income distributions. In the long run poverty traps may occur, and wealth distributions may be characterized by the „twin peaks‰often associated

[1] This applies to moderately high levels of inequality. For low levels of inequality, however all agents vote for revenues to be allocated entirely to health care.

with polarization of wealth in cross-sectional world income distributions (Quah 1996, 1997). Within the context of our model, there are in fact two possibilities for the evolution of wealth distributions, and relate to the value of the parameter inversely representing the elasticity of substitution in the „health production function‰ Depending on initial conditions, the political economy mechanism can either reinforce or alleviate the persistence in inequality. Regardless of the initial distributions of income and wealth, there is a decline in inequality until it reaches a point where it fluctuates around an average. However, this average level is higher in the case characterised by higher substitutability between public and private health care.

Furthermore, once distributional statistics converge, one finds that the political outcome typically converges to the welfare maximising outcome in cases where the elasticity of substitution between public and private health care expenditures is low. On the other hand, in the case characterised by high elasticity of substitution, political outcomes in the long run are typically indeterminate, in addition to being different from the social welfare maximising values of public health care expenditures.

Various strands of literature have motivational relevance for this study. The model of this paper is in the spirit of the emerging macroeconomics literature on health investment, mortality, and inequality, of which Glomm and Palumbo (1993), Ray and Streufert (1993), and Galor and Mayer (2002) are a few examples. To our knowledge, the political economy implications of such models have not been examined, and our paper is an exploratory step in this direction. Furthermore, extant political economy models that examine the public-private mix in health care provision study this issue in a static micro-theoretic context. See for example, Epple and Romano (1996) and Gouveia (1997). It is then of obvious interest to explore the implications of the political economy mechanism in a dynamic, macro-theoretic context, especially if one is seeking potential explanations for the observed diversity in the public-private mix in health care systems across countries.

A further issue of interest relates to discussions in the health economics literature on the degree of substitutability between public and private health services and its implication for the composition of health care demand. Cutler and Gruber (1997), Rask and Rask (2005), among others, comment on a „crowding out‰œeffect associated with

public health care expansions; *viz*, private expenditures tend to decline with public health care coverage. The model of this paper captures this feature in the case where public and private expenditures are substitutes in the health care production function. While it may not be appropriate to infer a political economy link between the degree of substitutability and the public-private mix in health care systems based on the empirical studies mentioned above, they do provide *indirect* evidence to speculate that such a link exists. Furthermore, discussions in policy circles suggest that the degree of substitutability or complementarity between private and public health care provision matters for the determination of public policy in this regard.[2]

Another sub-strand of literature that has a bearing on these issues relates to the notion of endogenous time preference. As mentioned above, in our model, public and private health investment have a positive impact on the agentÊs patience via an improvement in the survival probability. The model of this paper therefore falls into the class of models with variable time preference · an area of research which has been growing in recent years.[3] Endogenous time preference models are more general versions of models with a fixed rate of time preference and have often been considered worthy of exploration simply from the point of view of checking whether the results of fixed time preference models are robust to this generalisation. In addition, recent work has shown that the assumption of endogenous time preference also has implications for persistence in inequality. (Chakrabarty 2008). As discussed above, we find that the endogenous time preference aspect of our model interacts with the extent of „crowding out‰to generate a diverse set of results in relation to the dynamics of income distributions.

Remaining sections of this paper are organized as follows. Section 2 describes the model of this paper. Section 3 presents some analytical results and a discussion of their implications for the outcomes of the model. Section 4 presents the results from numerical simulations based on a parameterization of the model. Section 5 concludes.

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[2] Australian Industry Commission report on private health insurance in 1997 suggests that „the core issue is the extent to which private funding should be seen as, or in fact is replacing public funding (eg private patients in private hospitals) or topping up public funding to provide extra dimensions of service (eg doctor of choice, or private room‰(As quoted in Butler and Connely, 2007).

[3] See for example Lahiri (2002, 2007) and references therein.

2. The Economic Environment

As mentioned above, our model is a simple political-economy extension of the framework presented in Chakraborty and Das (2005), henceforth cited as CD. There are overlapping generations of agents in a small open economy who potentially live for two periods. Time is discrete and indexed by t = 0,1,2,ÿ . As in CD the agent born in any given period survives the first period with certainty, but may die before reaching old age, the probability of premature death being a function of Âhealth investmentÊ in the first period of her life.

However, we modify this construct in that we allow the agentÊs survival probability to be a function of a Âcomposite goodÊ that incorporates public health services in addition to individual private health investment. This modification also entails introducing a role for the government in this economy, particularly in relation to the financing of public health services. Specifically, in order to finance various redistributive expenditures, the government raises revenue by means of a progressive linear wealth tax τ, levied on the heterogeneous wealth endowments W_t of the young agents in the economy. Wealth endowments of the young essentially constitute intended or unintended bequests left by the previous generation. We assume that the distribution of these endowments is described by a density function $g(W)$ with support $[0,\infty)$. Tax revenue raised in any period in then given by $\tau\int_0^\infty Wg(W)dW = \tau\overline{W}$.

A proportion ψ of this revenue is used to finance the Âpublic health care systemÊ which is part of the composite good affecting the agentÊs survival probability. The remainder of revenues, i.e. $(1-\psi)\tau\overline{W}$, is used to finance a lump sum transfer to the young agents in the economy. However, the proportion ψ is endogenously determined · at the beginning of each period, before making their lifetime consumption, savings, and bequest plans, the young agents vote for the proportion allocated to the public health care system. The political outcome is then determined using the plurality rule. The equilibrium outcome is *subgame perfect* · the consumption, savings, and bequest plans made in the „second stage%after the vote on ψ has taken place are taken into account by agents during the voting process.

We first characterize the agentÊs optimization in the second stage. The agentsÊ consumption and bequest plans are denoted by c_t, c_{t+1}, b_{t+1}, and expected lifetime utility is described by

$$U_t = u(c_t) + \phi(h_t)\left[u(c_{t+1}) + \theta v(b_{t+1})\right]. \tag{1}$$

In the above u and v are twice continuously differentiable, $\phi(h_t)$ is the survival probability function where h_t represents the composite good ÂhealthÊgiven by

$$h_t = \left[\alpha(h_t^p)^{-\nu} + (1-\alpha)(h_t^g)^{-\nu}\right]^{-\frac{1}{\nu}},$$

where h_t^p and h_t^g represent private and public health expenditures and $h_t^g = \psi\tau\bar{W}_t$.[4] The agent born in t chooses her consumption, saving and bequest plans by maximizing (1) subject to the following budget constraints:

$$c_t = \bar{w} + (1-\tau)W_t + (1-\psi)\tau\bar{W}_t - h_t^p - s_t, \tag{2}$$

$$c_{t+1} = \bar{w} + Rs_t - b_{t+1}. \tag{3}$$

In equations (2) and (3), $\bar{w}$ represents income earned as a result of supplying the unit endowment of labor when young or old in a perfectly competitive market, and R is the gross world interest rate, taken as given in this small open economy. In the first period of her life the agent uses her post-tax wealth endowment, income earned in the labor market, and lump-sum transfers from the government to finance consumption, saving and private health investment. In the second period, the income endowment and returns to saving are used to finance consumption and bequests. As in CD we assume that in the event the agent does not survive to the second period the unintended bequests to the next generation equal s_t.

Assumptions regarding the survival probability function $\phi(h_t)$ are identical to those in CD. Specifically,

$$\phi(h_t) \in [0,1], \quad \phi' > 0, \quad \phi'' < 0, \quad \lim_{h\to\infty}\phi(h) \equiv \bar{\phi} \le 1.$$

Furthermore, as in CD, the functional form for $\phi(h_t)$ is described as follows:

[4] Of course, whether the choice of the CES form for the health production function here is appropriate is a debatable issue. From our point of view, this is an exploratory attempt toward examining the issue of substitutability between public and private health care. We do so by varying the parameter ν.

$$\phi(h_t) = \begin{cases} a h_t^{\varepsilon} & \text{if } h_t \in [0, \hat{h}_t] \\ \bar{\phi} & \text{otherwise.} \end{cases} \tag{4}$$

In equation (4) $\hat{h}_t = \left(\frac{\bar{\phi}}{a}\right)^{1/\varepsilon}$. Note, however, that in our model *h* is a composite good including both public and private health expenditures, while in the CD model it refers to private health investment only. In the analysis below we also consider a critical level of private health investment, which given the tax rate and other parameters, is implicitly defined by

$$\hat{h}_t = \left[\alpha\left(\hat{h}_t^{p}\right)^{-\nu} + (1-\alpha)\left(\psi\tau\bar{W}\right)^{-\nu}\right]^{-\frac{1}{\nu}} = \left(\frac{\bar{\phi}}{a}\right)^{\frac{1}{\varepsilon}}.$$

Rearranging,

$$\hat{h}_t^{p} = \left[\frac{1}{\alpha}\left(\frac{\bar{\phi}}{a}\right)^{-\frac{\nu}{\varepsilon}} - \frac{1-\alpha}{\alpha}\left(\psi\tau\bar{W}\right)^{-\nu}\right]^{-\frac{1}{\nu}}. \tag{5}$$

As is obvious from (5), the critical level of private health investment for which the survival probability function attains its maximum value is negatively related to the proportion of tax revenue used to finance the public health good, the average tax rate, and the average level of wealth in the economy.

We also assume, as in CD, the following functional forms for the period utility functions *u(c)* and *v(b)*:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad v(b) = \frac{b^{1-\sigma}}{1-\sigma}, \quad \sigma \in (0,1).$$

The reason for restricting σ to be less than unity are discussed in CD and are similar in spirit to assumptions generally required in models with variable rates of time preference.

First, we characterize the optimal solution given ψ in the range $[0, \hat{h}_t^{p}]$, or equivalently $[0, \hat{h}_t]$. The first order necessary conditions associated with $s_t, h_t^{p}, \& b_{t+1}$ are:

$$u'(c_t) = R_t \phi(h_t) u'(c_{t+1}) \quad \Rightarrow \quad c_t^{-\sigma} = R\phi(h_t)\left(c_{t+1}\right)^{-\sigma} \tag{6}$$

$$u'(c_t) = \frac{\partial\phi}{\partial h_t}\frac{\partial h_t}{\partial h_t^{p}}\left[u(c_{t+1}) + \theta v(b_{t+1})\right] \quad \Rightarrow \quad (1-\sigma)c_t^{-\sigma} = \frac{\partial\phi}{\partial h_t}\frac{\partial h_t}{\partial h_t^{p}}\left[c_{t+1}^{1-\sigma} + \theta b_{t+1}^{1-\sigma}\right] \tag{7}$$

$$u'(c_{t+1}) = \theta v'(b_{t+1}) \quad \Rightarrow \quad b_{t+1} = \beta c_{t+1}, \tag{8}$$

where $\beta = \theta^{\frac{1}{\sigma}}$. Manipulating (6), (7), (8), and the budget constraints (2) and (3) we can write the variables c_t, c_{t+1}, s_t, and b_{t+1} as functions of h_t^p:

$$c_t = \left(R^{1-\frac{1}{\sigma}} \right) \left(\phi(h_t)\right)^{-\frac{1}{\sigma}} \frac{\delta h_t}{\frac{\partial h_t}{\partial h_t^p}}, \tag{9}$$

$$c_{t+1} = \frac{\delta R h_t}{\frac{\partial h_t}{\partial h_t^p}}, \tag{10}$$

$$s_t = \frac{\delta(1+\beta)h_t}{\frac{\partial h_t}{\partial h_t^p}} - \frac{\overline{w}}{R}, \tag{11}$$

$$b_{t+1} = \frac{\beta \delta R h_t}{\frac{\partial h_t}{\partial h_t^p}}. \tag{12}$$

In the above equations $\delta = \frac{(1-\sigma)}{(1+\beta)\varepsilon}$. Derivations are shown in part A of the Appendix. It is worth noting here that the CD model has similar expressions for the above variables with the difference that in our model the term $\partial h_t / \partial h_t^p$ appears in the denominator of (9), (10), and (12), and in the denominator of the first term in (11). In the special case in which public and private health expenditures are perfect substitutes (i.e. $\nu = -1$), $\partial h_t / \partial h_t^p = \alpha$, the features of our model are likely to be more similar to the CD model.

Now, the period *t* and *t+1* budget constraints can be combined to yield

$$c_t + h_t^p + \frac{c_{t+1}}{R} + \frac{b_{t+1}}{R} = y_t, \tag{13}$$

where $y_t \equiv \overline{w} + (\overline{w}/R) + (1-\tau)W_t + \tau(1-\psi)\overline{W_t}$. Substituting for (9)-(12) in (13) we get

$$\xi(h_t^p) \equiv h_t^p + \frac{\delta h_t}{\frac{\partial h_t}{\partial h_t^p}} \left[1 + \beta + \frac{R^{1-\frac{1}{\sigma}}}{(\phi(h_t))^{1/\sigma}} \right] = y_t. \tag{14}$$

Equation (14) implicitly determines the optimal private health expenditure as a function of income $h_t^p = \eta(y_t)$ in the range $[0, h_t^p]$, given policy parameters ψ and τ.

Next, we consider the agent's optimization problem for incomes above $\hat{y}_t = \eta(\hat{h}_t^p)$. As described above, the survival probability function reaches its maximum value at $\hat{h}_t^p$, which means health investment will be maintained at the level $\hat{h}_t^p$ for income levels $y_t > \hat{y}_t$. The agent's problem then reduces to

$$\max_{c_t, b_{t+1}} \quad \frac{c_t^{1-\sigma}}{1-\sigma} + \bar{\phi}\left[\frac{c_{t+1}^{1-\sigma}}{1-\sigma} + \theta\frac{b_{t+1}^{1-\sigma}}{1-\sigma}\right]$$

subject to

$$c_t + \frac{c_{t+1}}{R} + \frac{b_{t+1}}{R} = y_t - \hat{h}_t^p .$$

Analogous to the CD framework, we can then derive closed form solutions described by:

$$c_t = \left(\frac{1}{1+\rho(1+\beta)}\right)\left(y_t - \hat{h}_t^p\right), \tag{15}$$

$$c_{t+1} = \left(\frac{\rho R}{1+\rho(1+\beta)}\right)\left(y_t - \hat{h}_t^p\right), \tag{16}$$

$$b_{t+1} = \left(\frac{\beta\rho R}{1+\rho(1+\beta)}\right)\left(y_t - \hat{h}_t^p\right), \tag{17}$$

$$s_t = \left(\frac{\rho(1+\beta)}{1+\rho(1+\beta)}\right)\left(y_t - \hat{h}_t^p\right) - \frac{\overline{w}}{R}, \tag{18}$$

where $\rho \equiv \bar{\phi}^{1/\sigma} R^{\frac{1}{\sigma}-1}$. Combining (9)-(12) and (15)-(18), we then have a complete characterization of the agent's problem in the second stage.

We now turn to the discussion of the dynamic aspects of the model. Based on the characterization of the agent's optimization problem discussed above, the intended and unintended bequests for the entire wealth distribution are given by

$$b_{t+1} = \Omega_1(W_t) \equiv \begin{cases} \dfrac{\beta(1-\sigma)R}{(1+\beta)\varepsilon}\left[\eta_0(W_t) + \left(\dfrac{1-\alpha}{\alpha}\right)\left(\psi\tau\bar{W}_t\right)^{-\nu}\left(\eta_0(W_t)\right)^{1+\nu}\right], & W_t < \hat{W}_t \\ \dfrac{\beta\rho R}{1+\rho(1+\beta)}\left[\bar{w} + \dfrac{\bar{w}}{R} + (1-\tau)W_t + \tau(1-\psi)\bar{W}_t - \hat{h}_t^p\right], & W_t \geq \hat{W}_t \end{cases} \tag{19}$$

$$s_t = \Omega_2(W_t) \equiv \begin{cases} \dfrac{(1-\sigma)}{\varepsilon}\left[\eta_0(W_t) + \left(\dfrac{1-\alpha}{\alpha}\right)\left(\psi\tau\bar{W}_t\right)^{-\nu}\left(\eta_0(W_t)\right)^{1+\nu}\right] - \dfrac{\bar{w}}{R}, & W_t < \hat{W}_t \\ \dfrac{\rho(1+\beta)}{1+\rho(1+\beta)}\left[\bar{w} + \dfrac{\bar{w}}{R} + (1-\tau)W_t + \tau(1-\psi)\bar{W}_t - \hat{h}_t^p\right], & W_t \geq \hat{W}_t \end{cases} \tag{20}$$

Given the optimal savings and bequest decisions above, the wealth dynamics for the *ith* agent in the economy are characterized by the following non-linear Markov process:

$$W_{t+1}^i = \begin{cases} \Omega_1(W_t^i) & \textit{with probability } \phi(h(W_t^i)) \\ \Omega_2(W_t^i) & \textit{otherwise} \end{cases} \tag{21}$$

{ Add remarks to link sections}

3. Analytical Results and Discussion

A. Comparative Static Results

Before discussing results in relation to the dynamics, it is useful to examine some features associated with a two-period version of the model. We focus on the characterization of the agentÊs optimal choices in the range of income levels above $\hat{y}_t = \xi(\hat{h}_t^p)$, and it is useful to examine some of the analytical results in the CD article corresponding to the income levels below this critical level, with reference our extension. Specifically, they show that the restriction $\sigma > \varepsilon$ implies that private health investment is a luxury good, as are bequests and second period consumption. This assumption also implies that first period consumption is a normal good. While analytical results of this sort are difficult to derive in our extension of the CD model, we can show that they hold in the special case of our model in which private and public health are perfect substitutes, i.e. in the case $\nu = -1$. We can also analyse the special case of $\nu = 0$; in this case the health production function is of the Cobb-Douglas form. In the latter case, however,

similar results are obtained by imposing slightly different assumptions regarding the parameters. We summarize these results in Propositions 1 and 2 below:

Proposition 1: Let $\nu = -1$ and $\sigma > \varepsilon$. Then,

(i) Private health investment is a luxury good. That is, $\frac{\partial \eta}{\partial y_t} > 0$, and $\frac{\partial^2 \eta}{\partial y_t^2} > 0$, so that the income-expansion path for private health is convex .

(ii) Old age consumption, and bequests are luxury goods.

(iii) Consumption when young is a normal good.

Proposition 2: Let $\nu = 0$ and $\sigma > \varepsilon\alpha$. Then,

(i) Private health investment is a luxury good. That is, $\frac{\partial \eta}{\partial y_t} > 0$, and $\frac{\partial^2 \eta}{\partial y_t^2} > 0$, so that the income-expansion path for private health is convex .

(ii) Old-age consumption and bequests are luxury goods.

(iii) Consumption when young is a normal good.

The proofs of the above propositions are presented in parts B and C of the appendix respectively. From the point of view of our paper, the above propositions establish that for a range of parameters considered the features of the extended model are common to that of the CD model. Therefore, studying the political economy implications of the above model is to some degree the same as studying the implications of some of the specific features of the CD model, in addition to studying the implications of the specific features of our more general framework.

Since it is hard to explicitly characterize the political outcome in the first stage, our analysis is primarily based on the numerical simulations presented in the next section. However, to extract some intuition about the political equilibrium, we now analyse how the agentsÊconsumption, saving and bequest plans are affected by changes in ψ. We also look at the implications of these changes on their indirect utility functions $V(\psi, W)$; while one cannot analytically obtain a solution for the political outcome, such an analysis identifies the tradeoffs faced by the agents while making their voting decision. In what

follows, we therefore attempt to establish some benchmark conditions under which agents prefer extreme values of ψ - i.e a value of ψ equal to 0 or 1, which would be the case if the indirect utility functions were decreasing or increasing over the entire range of $\psi \in [0,1]$. Interpreting these conditions also enables us to gain some insight about what must occur when „interior‰values of ψ are to be the preferred outcome, and makes it a little easier to interpret the results of the numerical experiments in Section 3 of the paper.

We first analyse the case in which agentsÊ incomes are above the critical level of income and wealth above which the survival probability is at the maximum possible level of $\bar{\phi}$. Note that the critical level of private health investment required to attain the maximum survival probability is decreasing in ψ, so that changes in ψ alter the number of agents in the two different groups we consider, namely, those with incomes such that their survival probability is less than $\bar{\phi}$, and those with income and wealth above the level required to attain the maximal survival probability $\bar{\phi}$. For agents with survival probability $\bar{\phi}$, we can establish some conditions under which the preferred choice of ψ will be either 0 or 1. These conditions are summarized below in the following results, proved in Appendix D.

Proposition 3: For agents with survival probability $\bar{\phi}$

(i) Consumption in both periods of life, intended bequests, and savings are decreasing in ψ *iff* $\left(\frac{1-\alpha}{\alpha}\right) < \left(\frac{h_t^g}{\hat{h}_t^p}\right)^{1+\nu}$,

(ii) Agents vote for a value of ψ equal to zero *iff* $\left(\frac{1-\alpha}{\alpha}\right) < \left(\frac{h_t^g}{\hat{h}_t^p}\right)^{1+\nu}$. (Basically, the indirect utility function is decreasing in ψ *iff* $\left(\frac{1-\alpha}{\alpha}\right) < \left(\frac{h_t^g}{\hat{h}_t^p}\right)^{1+\nu}$).

Proposition 3 implies that for agents with survival probability $\bar{\phi}$, the vote on ψ depends on (a) the share of government expenditures relative to private health expenditures in the health production function (as represented by $1-\alpha$); (b) the ratio of public health

expenditures to the survival-probability maximizing level of private health expenditure; and (c) the elasticity of substitution between private and public health expenditures in the health production function. If the inequality in (i) and (ii) of the proposition above holds, then the agents in this group will prefer $\psi = 0$. If it is reversed, on the other hand, they will prefer $\psi = 1$. A value of $\psi \in (0,1)$ is preferred if $\left(\frac{1-\alpha}{\alpha}\right) = \left(\frac{h_t^g}{\hat{h}_t^p}\right)^{1+\nu}$. Note, for example, in the case of perfect substitutes $(\nu = -1)$, the indirect utility function is decreasing in ψ *iff* $\alpha > 1/2$ - i.e if private health matters more than public health in contributing towards composite health, these agents will vote for $\psi = 0$. On the other hand, a value of $\psi = 1$ is preferred if $\alpha < 1/2$. In the Cobb-Douglas case, agents in this group vote for $\psi = 0$ if $\left(\frac{1-\alpha}{\alpha}\right) < \left(\frac{h_t^g}{\hat{h}_t^p}\right)$ and $\psi = 1$ if the inequality is reversed. A value of $\psi \in (0,1)$ is preferred if $\left(\frac{1-\alpha}{\alpha}\right) = \left(\frac{h_t^g}{\hat{h}_t^p}\right)$. The tradeoffs faced by the agents are represented by the ratios $(1-\alpha)/\alpha$ and $h_t^g / \hat{h}_t^p$ - the former may be interpreted as the relative contribution of public expenditures in determining overall health, while the latter may be interpreted as the cost of financing that contribution expressed relative to the maximum expenditure on private health. (Recall that all agents in this group spend the same amount on their health · i.e. $\hat{h}_t^p$, which is enough to attain the survival probability $\bar{\phi}$).

Next, consider agents with incomes lower than the level required to reach a survival probability $\bar{\phi}$. Again, since it is difficult to characterize their preferences over ψ analytically we resort to analysing some special cases, and then consider results based on numerical simulations in the next section. Note that since we do not have closed form solutions for the variables entering the utility function, we can only analyse how the indirect utility function changes with ψ if we can determine how private health investment and composite health of agents responds to changes in ψ. A feature of relevance to the political outcome appears to be the extent of „crowding out‰ in private

health investment that occurs as a result of these changes. This is examined in Proposition 4. We again summarize conditions in which „corner solutions‰ may emerge for the cases in which the health production function is of linear or Cobb-Douglas form.

Proposition 4: Let $\nu = -1$ and $\sigma > \varepsilon$. Then,

(i) There is a „crowding out‰ effect, viz $\frac{\partial h_t^p}{\partial \psi} < 0$.

(ii) The sign of $\partial h_t / \partial \psi$ is ambiguous.

$$(\frac{\partial h_t}{\partial \psi} < 0 \quad iff \quad 1+\delta(1-\alpha)^2\left[1+\beta+R^{1-\frac{1}{\sigma}}\left(\phi(h_t)\right)^{-1/\sigma}\right] < 2\alpha \ ).$$

(iii) Period t and t+1 consumption, intended bequests, savings are decreasing in ψ *iff* $\frac{\partial h_t}{\partial \psi} < 0$.

(iv) The indirect utility function $V(\psi, W)$ is decreasing in ψ *iff* $\frac{\partial h_t}{\partial \psi} < 0$.

(Alternatively agents vote for ψ equal to zero *iff* $\frac{\partial h_t}{\partial \psi} < 0$).

Proposition 5: Let $\nu = 0$ and $\sigma > \alpha\varepsilon$. Then,

(i) The sign of $\partial h_t^p / \partial \psi$ is ambiguous.

$$(\frac{\partial h_t^p}{\partial \psi} < 0 \quad iff \quad \frac{\delta}{\alpha}\left(\frac{h_t^p}{h_t}\right)\left(\frac{\varepsilon}{\alpha}\right)\left(\phi(h_t)\right)^{-1/\sigma} R^{1-\frac{1}{\sigma}} < 1).$$

(ii) The sign of $\frac{\partial h_t}{\partial \psi}$ is ambiguous.

(iii) Period t+1 consumption, savings and intended bequests are decreasing in ψ iff $\frac{\partial h_t^p}{\partial \psi} < 0$.

(iv) The sign of $\frac{\partial V}{\partial \psi}$ and $\frac{\partial c_t}{\partial \psi}$ is ambiguous.

Proofs are relegated to parts E and F of the appendix. The „crowding out‰ effect, which we interpret as the situation in which *private* health expenditures decrease if the proportion ψ of tax revenues devoted to health increases, seems to have a role to play in the numerical simulations discussed in the next section. In particular, we find that private health expenditures unambiguously decrease as ψ decreases in the case of perfect substitutes. Whether the agents in this group vote for a certain value of ψ depends on the extent to which composite health h_t is affected by the crowding-out effect. (As described by part (iv) of Proposition 4). Proposition 5 on the other hand, establishes that there is no clear-cut crowding-out effect in the Cobb-Douglas case, and one is more likely to get an interior solution for ψ.

B. Persistence in Inequality

Referring to equations representing the dynamics of the model, viz equations (19)-(21), it is clear that, as in CD, persistence in inequality depends on the shape of $\eta_0(W)$, which in turn determines the shape of the savings and bequest functions described above. Specifically, the nature of the long-run distribution depends on the shape of $\Omega_1(W)$ and $\Omega_2(W)$, which is in turn determined by the shape of $\eta_0(W)$.

While we cannot determine this shape for the general case of the model, we can establish the same results as in CD with reference to the special cases of the model in which ν is set equal to -1 or 0. In these cases intended and unintended bequests can be shown to be linearly related to $\eta_0(W)$, which means that their shape is similar to $\eta_0(W)$. The results in relation to the shape of $\eta_0(W)$ in these special cases are therefore summarized in propositions 6 and 7 below:

Proposition 6: Let $\nu = -1$ and $\sigma > \varepsilon$. Then, optimal private health investment $h_t^p = \eta_o(W_t)$ satisfies:

(i) $\eta_o(0) > 0 \quad if \quad \overline{w} > 0,$

(ii) $\partial \eta_o(W_t) \big/ \partial W_t > 0 \ for\ W_t \le \hat{W}_t^3$, *while* $\partial \eta_o(W_t) / \partial W_t = 0 \ for\ W_t > \hat{W}_t^3,$

(iii) $\partial^2\eta_o(W_t)/\partial W_t^2 > 0$ *for* $W_t \leq \hat{W}_t^3$, *while* $\partial^2\eta_o(W_t)/\partial W_t^2 = 0$ *for* $W_t > \hat{W}_t^3$.

Proposition 7: Let $\nu = 0$ and $\sigma > \alpha\varepsilon$. Then, optimal private health investment $h_t^p = \eta_o(W_t)$ satisfies:

(i) $\eta_o(0) > 0$ *if* $\bar{w} > 0$,

(ii) $\partial\eta_o(W_t)/\partial W_t > 0$ *for* $W_t \leq \hat{W}_t^3$, *while* $\partial\eta_o(W_t)/\partial W_t = 0$ *for* $W_t > \hat{W}_t^3$,

(iii) $\partial^2\eta_o(W_t)/\partial W_t^2 > 0$ *for* $W_t \leq \hat{W}_t^3$, *while* $\partial^2\eta_o(W_t)/\partial W_t^2 = 0$ *for* $W_t > \hat{W}_t^3$.

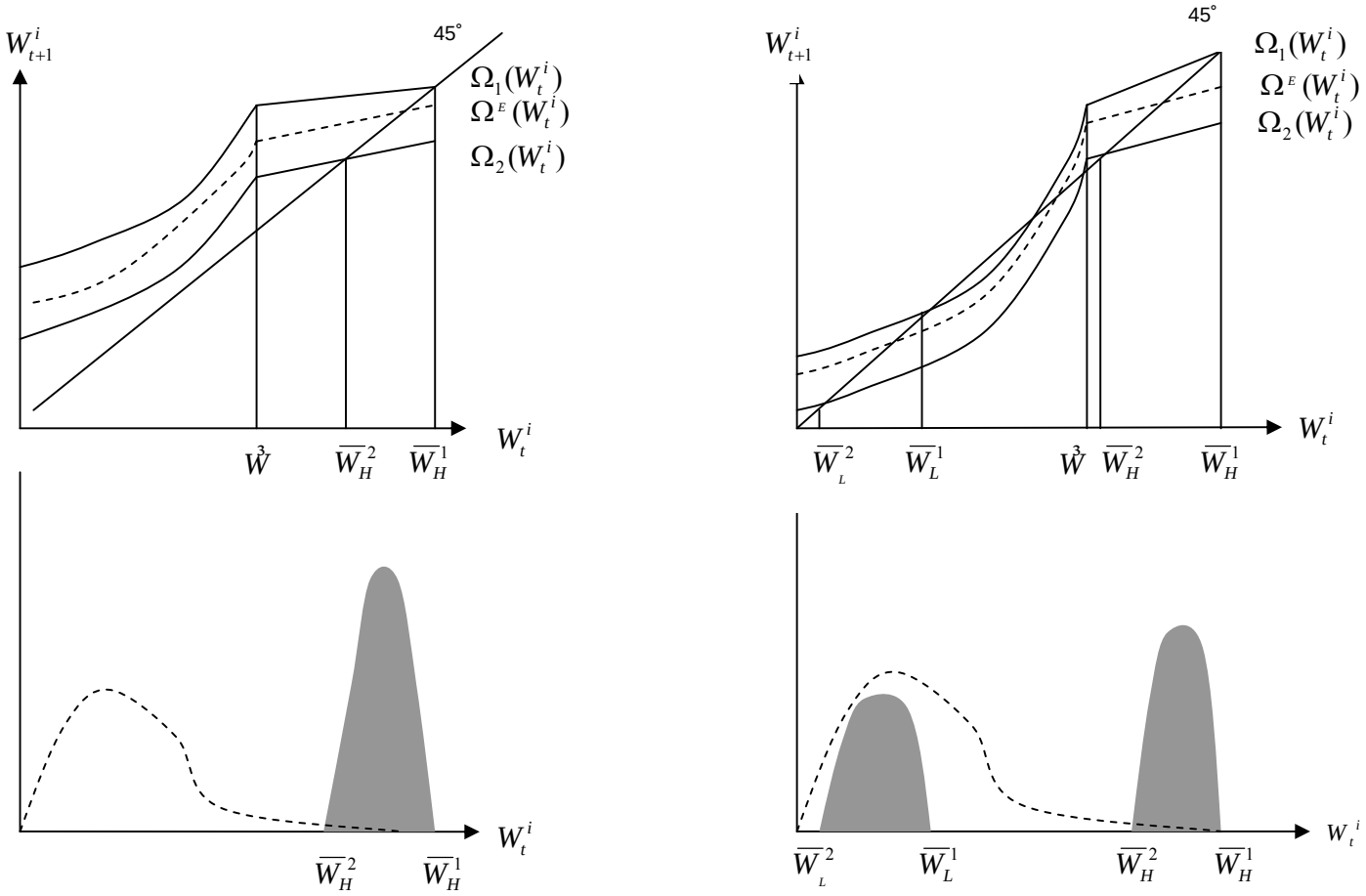
Essentially, in these special cases it can be shown that the shape of the savings and bequest functions is convex for wealth levels below $\hat{W}_t^3$ and linear for wealth levels greater than or equal to $\hat{W}_t^3$. The technical details are presented in the appendix. {Remember to add proof in appendix}. In what follows, it is convenient to reiterate the argument made in CD in relation to persistence in inequality, given that the argument applies to some degree in the special cases of our model.

Figure 1 below represents $\Omega_1(W)$ and $\Omega_2(W)$ and the expected bequest line defined by

$$\Omega^E(W_t^i) \equiv \phi(h(W_t^i))\Omega_1(W_t^i) + \left[1 - \phi(h(W_t^i))\right]\Omega_2(W_t^i)$$

Following CD, three possible scenarios in relation to the wealth dynamics of the model are presented in Figure 1 (a), (b) and (c). Referring to figure 1 (a), although the bequest and savings functions are initially convex, the intersection of these lines with the 45 degree line occurs at a relatively higher level of wealth. The intended and unintended bequest functions are however linear in the region where they intersect the 45 degree line. In this scenario, all agents converge towards a distribution with support $\left[\bar{W}_H^2, \bar{W}_H^1\right]$. No development trap is observed and all dynasties converge to a unique invariant long-run distribution, as shown in the second panel of Figure 1 a). Figure 1 (b), however, illustrates the case where $\Omega_1(W)$ and $\Omega_2(W)$ intersect with the 45 degree line in both the convex and the linear region. In this can the long run invariant distribution can be bimodal: dynasties which start out with wealth above $\bar{W}$ converge on the support

$\left[\overline{W}_H^2, \overline{W}_H^1\right]$ whereas dynasties who have wealth below this Âthreshold Ê converge to $\left[\overline{W}_L^2, \overline{W}_L^1\right]$. Therefore one observes polarisation in the distribution of wealth.

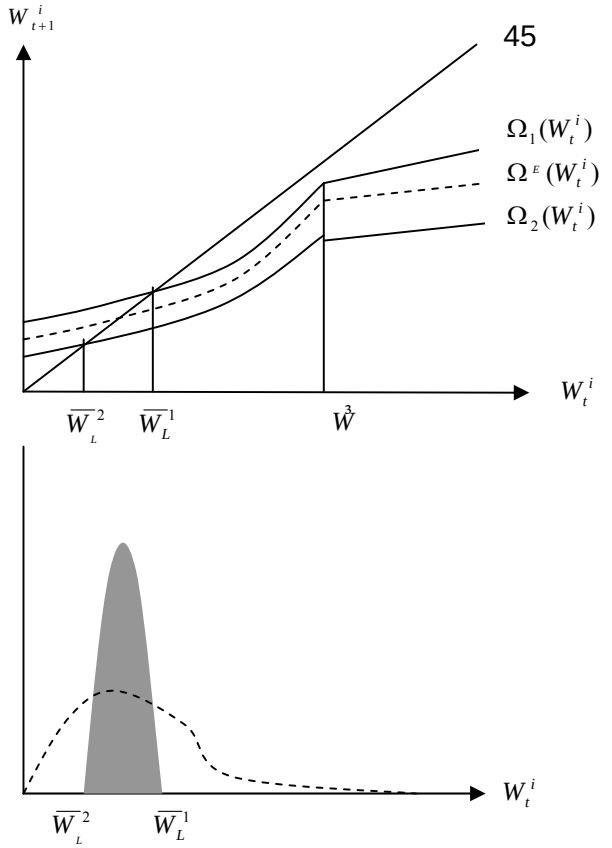


(a) Convergence **(b) Non convergence**

Figure 1. Wealth dynamics: (a) Convergence and (b) Non-convergence

A third scenario is presented in Figure 1 (c). Note here that $\Omega_1(W)$ and $\Omega_2(W)$ intersect the 45 degree line once only but at a point associated with a low level of wealth, in the

region where they are convex. Therefore, irrespective of initial wealth, all dynasties asymptotically converge to a distribution on support $\left[\overline{W}_L^2, \overline{W}_L^1\right]$. Whilst inequality is not persistent in this case, all agents converge to a low wealth distribution where everyone ends up in a „poverty trap‰ Numerical experiments in the following section indicate that bimodality occurs in the case where private and public health expenditures are substitutes rather than complements.



(c) Catching-point dynamics

Figure 1 cont. Wealth dynamics: (c) Catching-Point dynamics

Note, however, that in our model, intended and unintended bequests are also a function of $\tilde{\psi}$, the political outcome of the vote on ψ. The above discussion in relation to the

dynamics of the model, nevertheless applies in our case as well. Based on the analysis above, we can claim that in the special cases at least, the shape and curvature of the savings and bequest functions do not change · only the magnitude is altered. However, we can speculate that initial conditions with respect to the distributional statistics and parameters of the „health production function‰ will matter a great deal in determining the path that is taken by the economy during the transition to the long-run distribution. To analyse these issues further, we turn to the numerical experiments presented in the next section.

4. Results Based on Numerical Experiments

Our focus in this section is on the political results of the voting on ψ, and how it changes depending on the degree of substitutability between private and public health inputs in contributing to each agentÊs overall health. We are also interested in the extent to which the initial inequality in the distribution matters for the determination of the proportion of revenues allocated to health.

To examine the effect of changing the parameter ν, which inversely impacts on the elasticity of substitution (measured as $1/(1+\nu)$), we examine the results summarized in Table 1. The results presented in this table are based on a random sample of 501 observations drawn from a lognormal distribution with mean 3.2 and standard deviation 1.5. The associated Gini coefficient of the wealth distribution based on this sample is .7507. The parameter α represents the contribution of private expenditures in overall health. An approximate measure of this parameter would be the percentage share of private expenditure in total health expenditures. Since there is a great deal of variation in these estimates across countries, we consider different values in experiments to follow. However, for the results in Table 1 $\alpha = 0.55$, implying a relatively larger contribution to overall health, as would be the case for a transitional economy. This roughly corresponds to the private share of total health expenditures in Mexico for the year 2005. (World Bank, 2006). We set $R = 1 + \bar{r} = 1.055$, as in Heidjra and Romp, 2008. We set $\sigma = 0.8$, a value consistent with the assumption that $\sigma < 1$ described in Section 2. The parameter θ is calibrated as per the restriction suggested in Chakraborty and Das (2005). That is, to ensure that intended bequests in the model are always higher than unintended bequests we must impose $\theta > (1/\bar{r})^{\sigma}$. To that end, we set $\theta = (1/\bar{r})^{\sigma} + .01$. The parameters of the

survival probability function are set as $a = 0.06$, and $\varepsilon = 0.85$ - for an elasticity of substitution close to 1 these parameters ensure a range of survival probability that increases from 0.3 to $\bar{\phi}$, which is set at 0.96. This range roughly corresponds to estimates of cross-country survival probabilities based on the data presented in World Health Organisation, *Core Health Indicators,* 2004.

However keeping a and ε fixed while we vary ν leads to some problems in relation to interpreting the results presented in Table 1. In particular, the range of the survival probability function decreases as we increase the elasticity of substitution, so we are in effect looking at economies with different mortality risks. An alternative would be to change these parameters as we change the elasticity of substitution, such that the range of survival probabilities would be preserved across the experiments. We conducted some simulations of this nature, and the results are presented in Appendix H · in a qualitative sense at least, the results were similar to those presented in Table 1 below.

According to the experiments summarized in Table 1, decreasing the elasticity of substitution between private and public health expenditures leads to a vote in favour of higher levels of ψ - the proportion of revenues allocated to health care. In the case of higher substitutability, there is a „crowding out‰effect · higher ψ leads to a decline in private health investment that is large enough to offset the increase in public health spending, so that the survival probability is adversely affected. As shown in the previous section the decline in overall health has implications for other variables · consumption, savings, bequests and consequently utility decrease as ψ increases. For lower levels of the elasticity of substitution, however, the crowding-out effect is not that strong · private health investment falls, but overall health increases as ψ increases. The resulting increase in survival probability makes the agent more patient, so that declines in future consumption and bequests are not as large as the perfect-substitutes case, and expected lifetime utility increases as ψ increases. This interaction between the „crowding out‰ feature and the endogenous time preference aspect of the model also has interesting implications for the dynamics of the model, which will be discussed shortly.

Table 1. Political economy experiments altering the elasticity of substitution parameter ν

ν	Elasticity of Substitution	$\tilde{\psi}$	Percent in favour of $\tilde{\psi}$	Welfare maximising ψ	Desired ψ poorest agent	Desired ψ of the median agent	Desired ψ of the richest agent
-1	¥	0	100	0	0	0	0
-.95	20	.05	97.8	.05	.05	.05	.05
-.94	16.66	.05	97.6	.05	.05	.05	.05
-.93	14.28	.1	94.8	.1	.15	.1	.1
-.92	12.50	.15	91.6	.15	.2	.15	.15
-.91	11.11	.2	82.4	.2	.25	.2	.2
-.90	10.00	.25	78.2	.25	.25	.25	.25
-.89	9.09	.3	81.0	.3	.3	.3	.3
-.87	7.69	.35	62.9	.35	.4	.35	.35
-.85	6.66	.45	65.1	.5	.5	.45	.45
-.84	6.25	.45	48.7	.55	.45	.65	.45
-.83	5.88	.5	51.8	.55	.5	.65	.5
-.82	5.55	.5	42.1	.6	.55	.7	.5
-.78	4.54	.6	35.5	.7	.6	.85	.6
-.05	1.05	1	69.1	1	.85	1	.95

Another interesting feature of the results here is that for some ranges of parameters, the rich and middle-income agents in the economy prefer a higher ψ relative to poorer agents. This may simply be the result of a preference for the lump sum transfer, which serves as a better mechanism of redistribution due to its direct nature. Furthermore, it is important to note that the share of the governmentÊs contribution to overall health is relatively small.

For lower values of the elasticity of substitution, there are some cases which exhibit the „ends-against-the-middle‰feature discussed in Epple and Romano (1996). The trade/offs to the richer agents are as follows: a higher ψ may be preferred because public health expenditures are somewhat complementary to private health investment, which is increasing in wealth. A higher ψ also implies that the lump sum transfer to the richer agents is substantially smaller relative to what they pay in taxes. The poor may prefer a lower ψ because the direct lump-sum transfer is more progressive than the health transfer, given that it can be regarded as a perfect substitute for consumption.

We now turn to a discussion of the dynamic, long run features of the model. Figure 2 represents the long run invariant distribution of income and wealth for different values of the elasticity of substitution parameter ν. In each case, the initial income and wealth distribution and level of inequality is fixed, the latter characterised by a Gini coefficient of 0.7. As evident, the long run outcome varies significantly depending on the degree of substitutability between public and private health expenditures. When $\nu = -.95$, indicating a high elasticity of substitution (of 20) between public and private health expenditures, we observe a polarisation of wealth over time and the emergence of Âtwin peaksÊ Moving to a more intermediate value for the elasticity of substitution of 5.5 ($\nu = -.82$), the income and wealth distribution is also bimodal, however not as strikingly twin peaked as in the high elasticity of substitution case. Further, the support of the wealth and income distribution in this intermediate case is shifted further upwards than in an economy with a higher elasticity of substitution between public and private health care. In the case when $\nu = -.5$, (elasticity of substitution = 2), where public and private health expenditures are more complementary, the wealth and income distribution is single peaked and is shifted further to right compared to the two other cases. Thus, when public and private health care are complementary, an economy converges to a long-run invariant

distribution characterised by a higher level of average income. Also, as characterized by Figure 3, higher substitutability also implies that the economy will converge to a higher level of inequality.

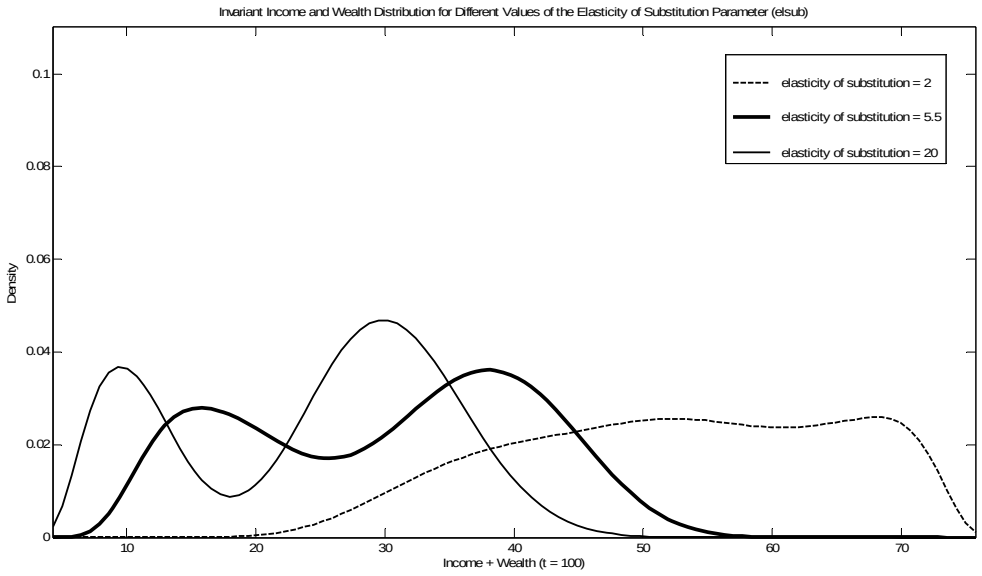


Figure 2. Income and Wealth Distribution for different values of ν

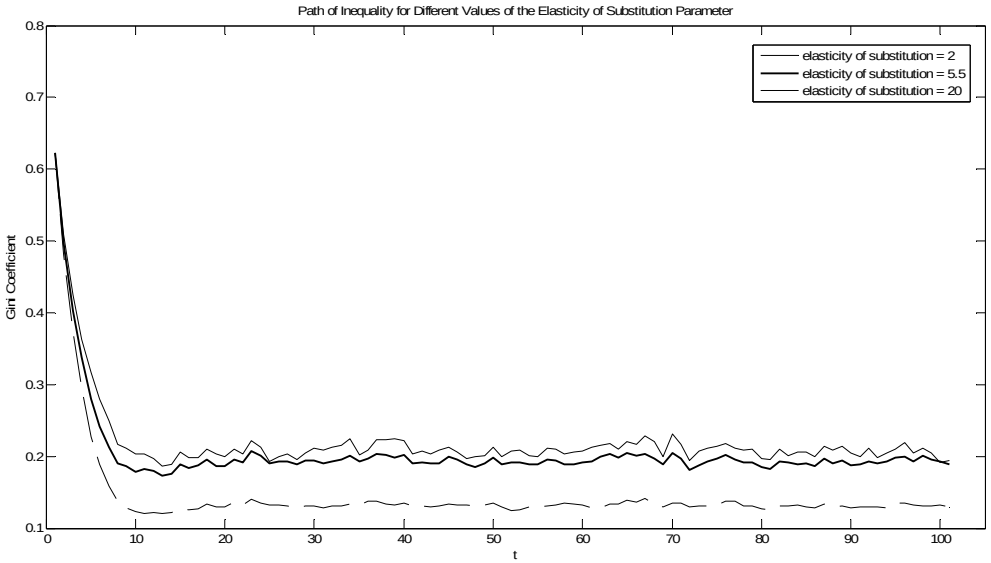


Figure 3. Inequality persistent for different values of ν.

The above result leads to an interesting proposition that is amenable to empirical testing. While it is difficult to measure „substitutability‰we know that in our model the crowding-out mechanism is much stronger in the case of higher substitutability between public and private health expenditures. The implication then is that a higher crowding-out effect could lead to a higher level of inequality in the long run. Given that in the literature there have been several estimates of the crowding out effect (Cutler and Gruber 1996; Dubay and Kenney 1997; Yazichi and Kaestner 1998; Blumberg et al 1999) an interesting direction of future research would be to examine the macroeconomic implications of this effect. Furthermore, the empirical literature in the area of health economics does not provide an explanation for the mechanism of the crowding out effect. To that end, the model above provides some useful insights.

The interaction between the „crowding out‰feature and the endogenous time preference aspect of the model provides an intuitively appealing explanation for the outcomes of our model. As in the static case, higher substitutability implies a „crowding-out‰effect which also impacts negatively on the survival probability of agents consequently leading to a lower level of revenues allocated to public health on the transition path to the stochastic steady state of the model. This feature is illustrated in Figure 4 (a). This figure compares the elected vote on ψ and the welfare maximising proportion of tax revenues allocated to health care, ψ^0, over time in the case of substitutes. As shown, the political outcome diverges significantly from the value of ψ which maximises the sum of agents utility. Eventually as inequality decreases, a higher level of ψ^* is chosen but in contrast to Figure 4 (b), which presents the case of complements, the outcomes for public health care provision are less favourable.

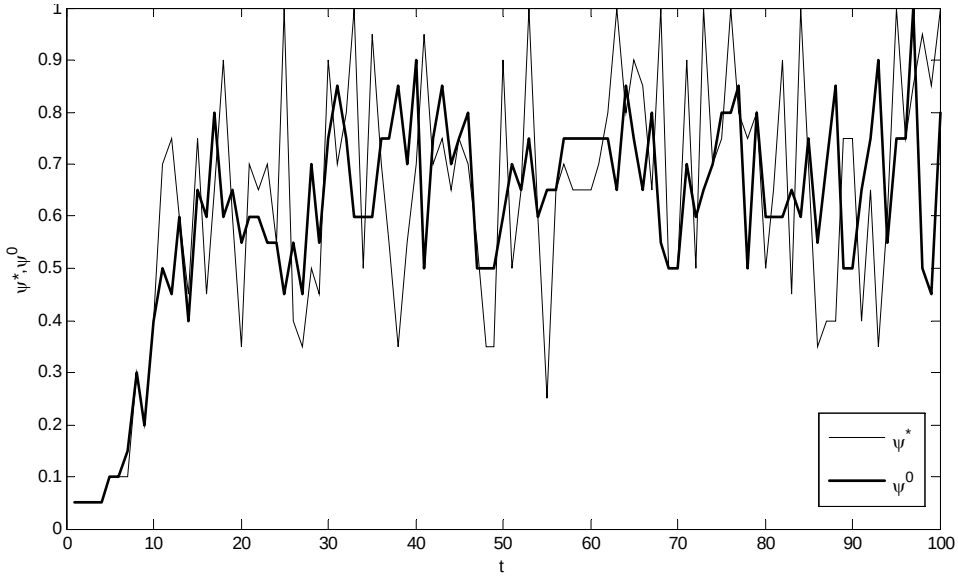


Figure 4 (a) Political economy versus welfare maximising equilibrium paths for ν =-0.95 (substitutes case)

Examining the complementary case, the elected outcome ψ^* and the welfare maximising converge ψ^0 and are at a higher level in comparison to the case presented in figure 4 (a).

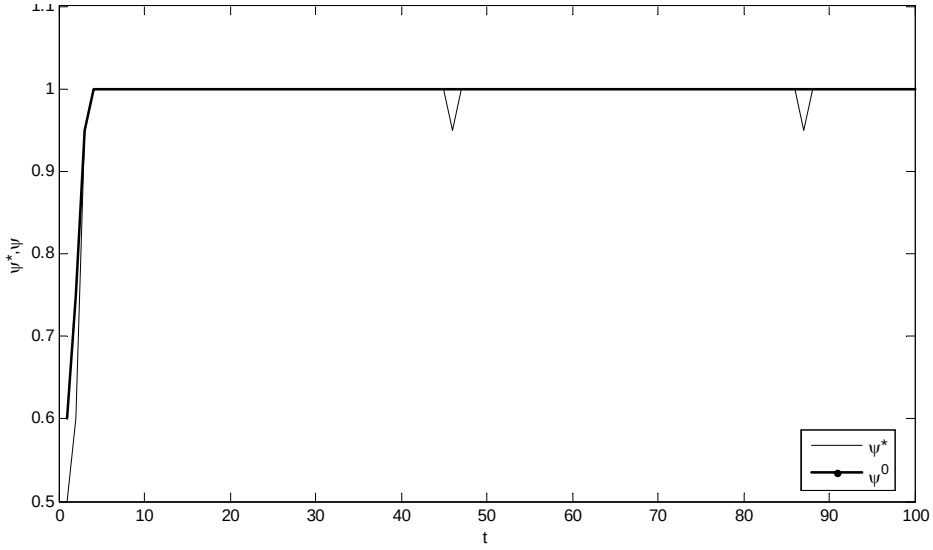


Figure 4 (b) Political economy versus welfare maximising equilibrium paths for ν =-0.82

Another aspect of these results is illustrated in figure 5 which shows the percentage of agents who voted for the elected value of ψ^* over time. Clearly, in the case of substitutes, the outcome is non-majoritarian, with the percentage of votes in favour of the winning value of ψ^* less than 50% in most cases. Therefore, when public and private health are close substitutes, the political outcome is indeterminate.

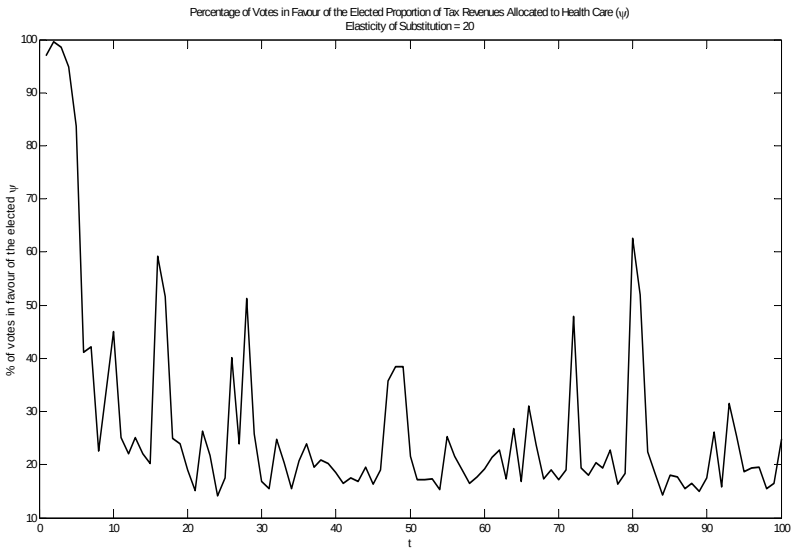


Figure 5 (a) Percentage of votes in favour of the elected ψ^* for ν =-0.95

Figure 5 (b) illustrates the case of more complementary public and private health expenditures. In this case, there is a majoritarian outcome, with the percentage of votes for the elected value of ψ^* greater than 50%. The political consensus, once the inequality converges to the level associated with the stochastic steady state favour 100% of tax revenues allocated towards public health care.

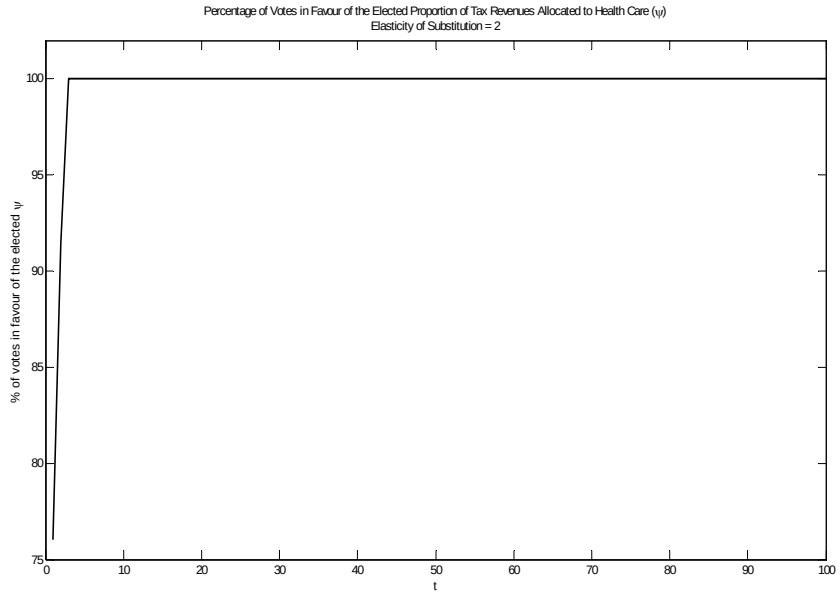


Figure 5 (b) Percentage of votes in favour of the elected ψ^* for ν =-0.82

5. Concluding Remarks

In this paper we study an overlapping-generations model in which agentsÊmortality risks, and consequently impatience, are endogenously determined by private and public investment in health care. The proportion of revenues allocated for public health care is also endogenous, determined as the outcome of a voting process. Higher substitutability between public and private health is associated with a „crowding-out‰effect which leads to lower public expenditures on health care in the political equilibrium. This in turn impacts on mortality risks and impatience leading to a greater persistence in inequality and long run distributions of wealth that are bimodal. On the other hand, when public and private health expenditures are complementary, the long run wealth distribution is typically unimodal in addition to being characterised by a lower level of inequality.

The results here suggest interesting directions for future research. Specifically, there is to our knowledge, no empirical study directly examining the link between substitutability

between public and private health care and its implications for macroeconomic outcomes. While it is difficult to measure „substitutability‰ we know that in our model the crowding-out mechanism is much stronger in the case of higher substitutability between public and private health expenditures. The implication then is that a higher crowding-out effect could lead to a higher level of inequality in the long run. Given that in the literature there have been several estimates of „crowding-out‰ an interesting direction of future research would be to examine the macroeconomic implications of this effect.

5. Appendix

A. Derivation of Equations (9)-(12)

Substituting (6) and (8) into (7) we get

$$c_{t+1} = \frac{(1-\sigma)R\phi(h_t)}{(1+\beta)\dfrac{\partial\phi(h_t)}{\partial h_t}\dfrac{\partial h_t}{\partial h_t^p}}.$$

Given the functional form for assumed in (4), note that $\dfrac{\dfrac{\partial\phi(h_t)}{\partial h_t}h_t}{\phi(h_t)} = \varepsilon$. We can then write

$$c_{t+1} = \frac{(1-\sigma)Rh_t}{(1+\beta)\varepsilon\dfrac{\partial h_t}{\partial h_t^p}}.$$

Defining $\delta = \dfrac{(1-\sigma)}{(1+\beta)\varepsilon}$, we obtain (10). It is then easy to derive (9), (11), and (12) using (6), (8), and (3).

B. Proof of Proposition 1

In the case of $\nu = -1$, note that $h_t = \alpha h_t^p + (1-\alpha)h_t^g = \alpha h_t^p + (1-\alpha)\psi\tau\overline{W}_t$, and $\dfrac{\partial h_t}{\partial h_t^p} = \alpha$.

Differentiating (14) with respect to h_t^p we get

$$\xi'(h_t^p) = 1 + \delta\left[1 + \beta + \frac{R^{1-\frac{1}{\sigma}}}{(\phi(h_t))^{1/\sigma}}\left(1 - \frac{\varepsilon}{\sigma}\right)\right] > 0 \quad if \quad \sigma > \varepsilon.$$

Also,

$$\xi''(h_t^p) = \delta\left(1 - \frac{\varepsilon}{\sigma}\right)R^{1-\frac{1}{\sigma}}\left(-\frac{1}{\sigma}\right)(\phi(h_t))^{-1/\sigma}\frac{\alpha\varepsilon}{h_t} < 0 \quad iff \quad \sigma > \varepsilon.$$

Using the inverse function rule

$$\frac{\partial\eta}{\partial y_t} = \frac{1}{\xi'} > 0 \text{ and } \frac{\partial^2\eta}{\partial y_t^2} = -(\xi')^{-2}\xi'' > 0.$$

Therefore, as in CD, we find that the income-expansion path for private health is convex, so that (i) follows, i.e. private health is a luxury good. Given that $\frac{\partial h_t}{\partial h_t^p} = \alpha$, consumption when old, and intended bequests are linearly related to private health expenditures, and consequently (ii) follows. To prove (ii) note that differentiating (9) w.r.t. y_t we get

$$\frac{\partial c_t}{\partial y_t} = R^{1-\frac{1}{\sigma}}\left(\phi(h_t)\right)^{-1/\sigma}\left(1-\frac{\varepsilon}{\sigma}\right)\frac{\partial h_t^p}{\partial y_t} > 0 \quad iff \quad \sigma > \varepsilon .$$

C. Proof of Proposition 2

In the case $\nu = 0$, $h_t = \left(h_t^p\right)^{\alpha}\left(h_t^g\right)^{1-\alpha}$. This means that

$$\frac{h_t}{\frac{\partial h_t}{\partial h_t^p}} = \frac{\left(h_t^p\right)^{\alpha}\left(h_t^g\right)^{1-\alpha}}{\alpha\left(h_t^p\right)^{\alpha-1}\left(h_t^g\right)^{1-\alpha}} = \frac{h_t^p}{\alpha} .$$

In this case

$$\xi'(h_t^p) = 1+\frac{1}{\alpha}\left[1+\beta+\frac{R^{1-\frac{1}{\sigma}}}{\left(\phi(h_t\right)^{1/\sigma}}\left(1-\frac{\varepsilon\alpha}{\sigma}\right)\right] > 0 \quad if \quad \varepsilon\alpha < \sigma .$$

Also,

$$\xi''(h_t^p) = -\frac{1}{\sigma}\left(1-\frac{\varepsilon\alpha}{\sigma}\right)R^{1-\frac{1}{\sigma}}\frac{\varepsilon}{h_t^p} < 0 \quad iff \quad \varepsilon\alpha < \sigma .$$

Using the inverse function rule

$$\frac{\partial \eta}{\partial y_t} = \frac{1}{\xi'} > 0 \text{ and } \frac{\partial^2 \eta}{\partial y_t^2} = -(\xi')^{-2}\xi'' > 0 .$$

Again, we find that the income-expansion path for private health is convex, so that (i) follows, i.e. private health is a luxury good. Given that $\frac{h_t}{\frac{\partial h_t}{\partial h_t^p}} = \frac{h_t^p}{\alpha}$, consumption when old, and intended bequests are linearly related to private health expenditures, and consequently (ii) follows. To prove (ii) note that differentiating (9) w.r.t. y_t we get

$$\frac{\partial c_t}{\partial y_t} = R^{1-\frac{1}{\sigma}}\left(\phi(h_t)\right)^{-1/\sigma}\frac{\delta}{\alpha}\left(1-\frac{\varepsilon}{\sigma}\right)\frac{\partial h_t^p}{\partial y_t} > 0 \quad iff \quad \sigma > \varepsilon .$$

D. Proof of Proposition 3

To show part (i) note that

$$\frac{\partial c_t}{\partial \psi} = \frac{1}{1+\rho(1+\beta)}\left[\frac{\partial y_t}{\partial \psi} - \frac{\partial \hat{h}_t^p}{\partial \psi}\right].$$

To obtain $\frac{\partial \hat{h}_t^p}{\partial \psi}$ we totally differentiate the following expression for $\hat{h}_t$,

$$\hat{h}_t = \left[\alpha\left(\hat{h}_t^p\right)^{-\nu} + (1-\alpha)\left(\psi\tau\bar{W}\right)^{-\nu}\right]^{-\frac{1}{\nu}} = \left(\frac{\bar{\phi}}{a}\right)^{\frac{1}{\varepsilon}}.$$

Then,

$$\frac{\partial c_t}{\partial \psi} = \frac{1}{1+\rho(1+\beta)}\left[-\tau\bar{W}_t + \frac{1-\alpha}{\alpha}\left(\frac{\hat{h}_t^p}{h_t^g}\right)^{1+\nu}\tau\bar{W}_t\right] < 0 \quad iff \quad \left(\frac{1-\alpha}{\alpha}\right) < \left(\frac{h_t^g}{\hat{h}_t^p}\right)^{1+\nu}.$$

Part (ii) follows since all other variables are linearly related to period t consumption and

$$\frac{\partial V(\psi, W)}{\partial \psi} = c_t^{-\sigma}\left[1+\bar{\phi}(\rho R)^{1-\sigma}(1+\beta)\right]\frac{\partial c_t}{\partial \psi}.$$

E. Proof of Proposition 4

Starting from (14), we can rearrange terms such that we have an implicit function of the form $\Gamma(h_t^p, \psi) = 0$. Applying the implicit function theorem we then have

$$\frac{dh_t^p}{d\psi} = -\frac{\Gamma_\psi}{\Gamma_{h_t^p}}.$$

Here $\Gamma_\psi = \frac{\delta}{\alpha}(1-\alpha)\tau\bar{W}_t\left[1+\beta+R^{1-\frac{1}{\sigma}}\left(\phi(h_t)\right)^{-1/\sigma}\left(1-\frac{\varepsilon}{\sigma}\right)\right] + \tau\bar{W}_t > 0 \; if \; \sigma > \varepsilon$, and

$\Gamma_{h_t^p} = 1+\delta\left[1+\beta+R^{1-\frac{1}{\sigma}}\phi(h_t)^{-1/\sigma}\left(1-\frac{\varepsilon}{\sigma}\right)\right] > 0 \; if \; \sigma > \varepsilon$. Therefore (i) follows. Since in the case of perfect substitutes, $h_t = \alpha h_t^p + (1-\alpha)\psi\tau\bar{W}_t$, differentiating with respect to ψ and manipulating we get (ii). To prove (iii), note that

$$\frac{\partial c_t}{\partial \psi} = \frac{\delta}{\alpha} R^{1-\frac{1}{\sigma}} \left(\phi(h_t)\right)^{-1/\sigma} \left(1 - \frac{\varepsilon}{\sigma}\right) \frac{\partial h_t}{\partial \psi}.$$

Also, intended bequests, savings, and period *t+1* consumption are linearly related to composite health given that $\frac{\partial h_t}{\partial h_t^p} = \alpha$. Furthermore, in the range of income and wealth such that $h_t^p \in \left[0, \hat{h}_t^p\right]$, we have

$$\frac{\partial V}{\partial \psi} = u'(c_t)\frac{\partial c_t}{\partial \psi} + \phi'(h_t)\left[u(c_{t+1}) + \theta v(b_{t+1})\right]\frac{\partial h_t}{\partial \psi} + \phi(h_t)\left[u'(c_{t+1})\frac{\partial c_{t+1}}{\partial \psi} + \theta v'(b_{t+1})\frac{\partial b_{t+1}}{\partial \psi}\right]$$

Given our assumptions about u, v, ϕ, and recognizing the linear relation of period t+1 consumption and bequests to overall health, the first term and the third term are negative if *h* is decreasing in ψ. The second term is also negative as we have assumed utility is positive, as is common in the endogenous time preference models of this nature. Therefore (iv) follows.

F. Proof of Proposition 5

Using the same steps as in Proposition 4, we can show that in the Cobb-Douglas case

$$\frac{\partial h_t^p}{\partial \psi} = -\frac{\tau \overline{W}_t \left(1 - \frac{\delta}{\alpha}\left(\frac{h_t^p}{h_t}\right)\left(\frac{\varepsilon}{\sigma}\right) R^{1-\frac{1}{\sigma}} \left(\phi(h_t)\right)^{-1/\sigma}\right)}{1 + \frac{\delta}{\alpha}\left[1 + \beta + R^{1-\frac{1}{\sigma}} \left(\phi(h_t)\right)^{-1/\sigma}\left(1 - \frac{\varepsilon}{\sigma}\right)\right]}.$$

Note that the denominator is positive since $\sigma > \varepsilon$. Therefore the sign of the above depends on the numerator, and (i) follows. Also, in the Cobb-Douglas case,

$$h_t = \left(h_t^p\right)^{\alpha}\left(h_t^g\right)^{1-\alpha} \Rightarrow \frac{\partial h_t}{\partial \psi} = \alpha\left(\frac{h_t^p}{h_t^g}\right)\frac{\partial h_t^p}{\partial \psi} + (1-\alpha)\left(\frac{h_t^p}{h_t^g}\right)\tau \overline{W}_t.$$

If the inequality in (i) holds and private health investment decreases as ψ increases, then overall health may be negatively or positively affected by ψ, depending on the magnitude of the second term in the above expression. Part (ii) follows from the fact that

$\frac{h_t}{\frac{\partial h_t}{\partial h_t^p}} = \frac{h_t^p}{\alpha}$ in the Cobb-Douglas case, so that c_{t+1} and b_{t+1} are linear in private health investment. It is then also difficult to determine the sign of $\frac{\partial c_t}{\partial \psi} = R^{1-\frac{1}{\sigma}}\left(\phi(h_t)\right)^{-1/\sigma}\frac{\delta}{\alpha}\left[\frac{\partial h_t^p}{\partial \psi} - \frac{\varepsilon}{\sigma h_t}\frac{\partial h_t}{\partial \psi}\right]$. Likewise, the sign of the indirect utility function is difficult to determine.

G: Proof of Propositions 6 and 7

Note that for $W_t < \hat{W}_t^3$, we know that $b_{t+1} = \frac{\beta\delta R h_t}{\frac{\partial h_t}{\partial h_t^p}}$.

Here $h_t = \left[\alpha(h_t^p)^{-\nu} + (1-\alpha)(h_t^g)^{-\nu}\right]^{-\frac{1}{\nu}}$, so that

$\frac{\partial h_t}{\partial h_t^p} = \left[\alpha(h_t^p)^{-\nu} + (1-\alpha)(\psi\tau\bar{W}_t)^{-\nu}\right]^{-\frac{1}{\nu}-1}\alpha(h_t^p)^{-\nu-1}$. This further implies

$$\frac{h_t}{\partial h_t / \partial h_t^p} = \left[h_t^p + \frac{1-\alpha}{\alpha}(\psi\tau\bar{W}_t)^{-\nu}(h_t^p)^{1+\nu}\right].$$

So, we may write: $b_{t+1} = \frac{\beta(1-\sigma)R}{(1+\beta)\varepsilon}\left[\eta_o(W_t) + \left(\frac{1-\alpha}{\alpha}\right)(\psi\tau\bar{W}_t)^{-\nu}(\eta_o(W_t))^{1+\nu}\right]$, for wealth levels below $\hat{W}_t^3$.

For the perfect substitutes case, that is $\nu = -1$,

$$b_{t+1} = \frac{\beta(1-\sigma)R}{(1+\beta)\varepsilon}\left[\eta_o(W_t) + \left(\frac{1-\alpha}{\alpha}\right)(\psi\tau\bar{W}_t)\right], \text{ for wealth levels below } \hat{W}_t^3.$$

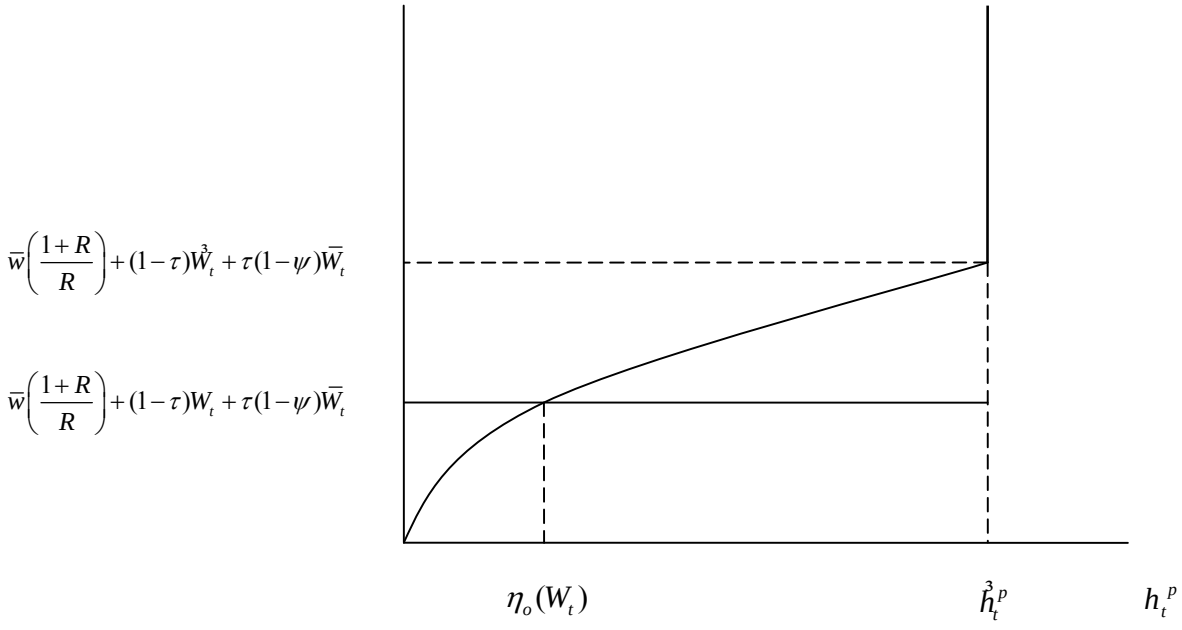
On the other hand, in the Cobb Douglas case ($\nu = 0$):

$$b_{t+1} = \frac{\beta(1-\sigma)R}{(1+\beta)\varepsilon}\left[1 + \left(\frac{1-\alpha}{\alpha}\right)(\psi\tau\bar{W}_t)\right]\eta_o(W_t), \text{ for wealth levels below } \hat{W}_t^3.$$

This means that in both cases we have a linear relationship between b_{t+1} and $h_t^p = \eta_o(W_t)$. Now $h_t^p = \eta_o(W_t)$ is implicitly defined by:

$$\xi(h_t^p) \equiv h_t^p + \frac{\delta h_t}{\frac{\partial h_t}{\partial h_t^p}}\left[1+\beta+\frac{R^{1-\frac{1}{\sigma}}}{(\phi(h_t))^{1/\sigma}}\right] = \overline{w}(\frac{1+R}{R}) + (1-\tau)W_t + \tau(1-\psi)\overline{W}_t.$$

In the perfect substitutes case, $\frac{\partial h_t}{\partial h_t^p} = \alpha$, so the LHS can be written as $\xi(h_t^p) \equiv h_t^p + \frac{\delta}{\alpha}\left[\alpha(h_t^p) + \psi\tau\overline{W}_t\right]\left[1+\beta+\frac{R^{1-\frac{1}{\sigma}}}{(\phi(h_t))^{1/\sigma}}\right]$. In the Cobb Douglas case, $\xi(h_t^p) \equiv h_t^p + \frac{\delta}{\alpha}h_t^p\left[1+\beta+\frac{R^{1-\frac{1}{\sigma}}}{(\phi(h_t))^{1/\sigma}}\right]$. We can represent the inverse function $\eta_o(W_t)$ in either case as follows [5]



[5] We have already established that for wealth levels below $\hat{W}_t^3$, this function is increasing and concave. See appendices B and C. For wealth levels above $\hat{W}_t^3$, we know private health investment is constant at h_t^p.

As we can see from the above figure, $\eta_o(0) > 0$ as long as $\bar{w}\left(\frac{1+R}{R}\right) + \tau(1-\psi)\bar{W}_t > 0$. So even in the case where the political outcome involves $\psi = 1, \eta_o(0) > 0 \; if \; \bar{w} > 0$. Furthermore, we have established in appendices B and C that $\frac{\partial \eta_o}{\partial y_t} = \frac{\partial \eta_o}{\partial W_t} > 0; \& \frac{\partial^2 \eta_o}{\partial W_t^2} > 0$ if $\sigma > \varepsilon$ in the perfect substitutes case and $\sigma > \alpha\varepsilon$ in the Cobb Douglas case. Also, since $\eta_o(W)$ takes a constant value $\hat{h}_t^p$ for wealth levels above $\hat{W}_t$, we have, in both cases $\frac{\partial \eta_o}{\partial W_t} = 0 = \frac{\partial^2 \eta_o}{\partial W_t^2}$.

H. Experiment with the range of survival probabilities preserved across simulations

ν	Elasticity of Substitution	$\tilde{\psi}$	Percent in favour of $\tilde{\psi}$	Preferred ψ of the poorest agent	Preferred ψ of the median agent	Preferred ψ of the richest agent
-0.99	100	0.01	100%	0.01	0.01	0.01
-0.97	33.33	0.01	96.008%	0.01	0.01	0.01
-0.95	20	0.05	55.%	0.05	0.05	0.20
-0.92	12.50	0.20	31.7365%	0.15	0.20	0.70
-0.90	10	0.25	24.9501%	0.25	0.30	1.00
-0.85	6.67	0.50	19.5609%	0.45	0.55	1.00
-0.80	5	1	27.9441%	0.55	0.70	1.00
-0.50	2	1	91.6168%	1	1	1

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