

DURATION DEPENDENCE IN THE US BUSINESS CYCLE

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ABSTRACT

Durland and McCurdy (1994) investigated the issue of duration dependence in US business cycle phases using a Markov regime switching approach, introduced by Hamilton (1989) and extended to the case of variable transition parameters by Filardo (1994). In Durland and McCurdy's model duration alone was used as an explanatory of the transition probabilities. They found that recessions were duration dependent whilst expansions were not. In this paper, we explicitly incorporate the widely-accepted US business cycle phase change dates as determined by the NBER, and use a state-dependent multinomial Logit (and Probit) modelling framework. The model incorporates both duration and movements in two leading indexes - one designed to have a short lead (SLI) and the other designed to have a longer lead (LLI) - as potential explanators. We find that doing so suggests that current duration is not only a significant determinant of transition out of recessions, but that there is some evidence that it is also weakly significant in the case of expansions. Furthermore, we find that SLI has more informational content for the termination of recessions whilst LLI does so for expansions.

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1. INTRODUCTION AND BACKGROUND

The question of whether business cycle phases are duration dependent has been of interest for many decades. One widely held view is that the older an expansion is, the more likely it is to end. There was much discussion along these lines in the US in the late 1990s as that expansion approached – and eventually passed – the longest previous US expansion ever recorded (since the 1850s). On the other hand, many economists have questioned whether there is any strong underlying rationale for this belief or whether it is simply the business cycle analogue of the view that 'nothing lasts forever'.

While it is obvious that no business cycle phase has ever lasted forever – and is never likely to – the issue surrounding duration dependence is whether there exists statistical evidence that the probability of a phase change systematically increases with the length of the current phase. In the relatively recent past a number of papers have investigated the issue of business cycle phase duration dependence.

Sichel (1991) used a hazard function approach in which a specific functional form for the hazard rate was assumed, and the necessary parameters were estimated from the US business cycle chronology as determined by the National Bureau of Economic Research (NBER). Using phase lengths derived from the NBER chronology for post-WWII data Sichel found evidence supporting duration dependence in recessions but insignificant evidence for expansions. Diebold and Rudebusch (1990) also used the NBER dates but used a non-parametric methodology. They found against duration dependence for both

expansions and recessions; however, they acknowledged that, although the evidence was statistically insignificant, the data available at the time more strongly favoured recession duration dependence. They also found some evidence of whole-cycle duration dependence. They argued that their results provided some justification for Hamilton's (1989) assumption of constant, time-invariant transition probabilities in his regime switching model.

With the immediate widespread popularity of Hamilton's Markov regime switching methodology, a number of papers subsequently tested the notion of duration dependence within that framework. This framework is described in more detail in the Modelling Framework section below but basically the approach amounts to allowing for the possibility that the transition parameters - representing the probability of transitioning out of particular phases - may vary over time in accordance with some underlying determinants. The relevance of each of the underlying determinants is then tested statistically for its significance.

For example, Durland and McCurdy (1994) incorporated current phase duration as a potential explainer variable for the transition probability parameters governing phase switches. They found that, within this framework, quarterly US GNP data suggested recessions were duration dependent (the relevant coefficient was negative - as required for duration dependence - and also over four times its robust standard error) but not so for expansions (the relevant coefficient was negative but was less than its standard error).

It is important to point out that none of the afore-mentioned papers which have investigated the duration issue to date have incorporated any other variable into the analysis which might have explanatory power as far as phase changes are concerned. If other factors are important in determining phase shifts then apparent duration dependence may simply be the spurious result of omitted variables bias. We believe this to be a serious limitation of earlier work on this subject and we therefore seek to redress the issue here.

In summary, in the current paper we look again at the issue of duration dependence of US business cycle phases but employ a different approach to earlier papers. In a similar way to Sichel and Diebold and Rudebusch we explicitly recognise and incorporate into the testing model the widely known and accepted US business cycle phase chronology as determined by the NBER dating panel. In this respect the approach is also similar to the earlier work of Neftci (1982, 1984). Specifically, our methodology assumes ex post observability of regime states.

The approach can therefore be contrasted with the duration-dependent regime switching extensions of Hamilton which explicitly assume that the latent regime state is unobservable and must be inferred on the basis of the presumed influence that the state has on some variable (such as output growth rates) known to be cycle dependent. Indeed, it is precisely in circumstances where there is no clear a priori knowledge of the phase change dates that the Hamilton-type models are most useful. This is certainly the case in many non-business cycle applications and could also be the case in business cycle

analyses for countries for which there is no widely-accepted set of phase change dates. However, for the case of the US, to ignore the existence of such dates is to ignore very important relevant data.

In using these dates in the analyses, however, we do not follow Sichel and Diebold and Rudebusch in calculating test statistics using either non-parametric methods (the latter) or from estimating a parametric form of a hazard function (the former). Rather, we model business cycle phase changes as following a first-order Markov process with varying transition probabilities. We model the transition probabilities as a function of both leading indicators and cycle durations. It turns out that our model can be viewed as a regime-switching multinomial Logit (or Probit) model where the potential drivers of the observed phase changes include indexes of leading indicators as well as the current phase duration. In this respect, [our](#) approach has the flavour of Estrella and Mishkin (1998) who also use the NBER dates to define their binary [dependent variable representing recession](#). However, unlike Estrella and Mishkin, we model phase changes [rather than recessions and, significantly, we](#) allow the drivers of the phase changes to have different coefficients across phases.

We believe our approach to the issue of duration dependence in the US business cycle is preferable to the Hamilton-type approaches in that we investigate the issue directly using the NBER-determined business cycle chronology. In using GNP as their “dependent variable” Durland and McCurdy in effect used a proxy for the chronology which is known to have a somewhat different chronology to the NBER chronology. If one accepts

the NBER chronology – as most commentators and researchers seem to – why not directly use it to construct the test?

The current approach also represents an extension over earlier work by including not only the current phase duration as a possible explanatory variable for the probability of a phase shift but also other variables which could reasonably be expected to impact on the probability of a phase switch. This will mitigate any omitted variables concerns one may have over univariate models of business cycle transition probabilities. We believe the estimated model represents a more complete framework and allows for richer interpretations of the resulting estimated model. Finally, as mentioned above, the paper represents a clear extension of Estrella and Mishkin's work in that we allow for regime switching for the estimated coefficients across different phases.

In the next section we present the modelling framework and how it relates to the Hamilton approach – of which it may be regarded as a special case variant. The estimation results follow, with concluding remarks presented in Section 4.

2. THE MODELLING FRAMEWORK

In many modelling situations it is sensible to allow for the possibility that the variable of interest may come from one of several different 'states', 'phases', or 'regimes' and that whatever is the data generating mechanism driving the observed variable it may differ across regimes. For instance, it is common to conceptualise the business cycle as

consisting of two phases: expansion and recession.¹ It is widely accepted that there are a number of asymmetries across these two business cycle regimes. For instance, the average duration of expansions is much longer than for recessions, the variability of economic growth rates is different in each regime, and, to some extent, researchers have found that the dynamic properties of economic growth may differ across regimes.

A recent modelling approach which gained great popularity for studying these asymmetries is the Markov regime-switching model of Hamilton (1989). It allows for shifts from one phase into another and, in its simplest form, it assumes constant transition probabilities with the distribution of the variable under study assumed to be normal with a different mean and variance across phases.² The probability of switching from one phase into the other is characterised by a discrete first-order Markov process.³

Suppose the business cycle consists of two phases, summarized by the discrete random variable S_t ($i = 1, 2$) which takes two possible values respectively denoting expansion (1) and recession (2). The transition matrix describing the evolution of S_t is given by

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \quad (1)$$

Where p_{11} denotes the probability of remaining in phase 1 from period $t-1$ to period t , and p_{22} is the probability of staying in phase 2 from period $t-1$ to period t . Because these are

¹ Some researchers and analysts also sometimes allow for the possibility of a third “recovery” phase. See, for example, Sichel (1994) and Layton and Smith (2001).

² It is also possible to allow for autoregressive dynamics which may be the same or which may differ across phases.

³ I.e. the probability distribution of the discrete phases at time t depends only on the phase in period $t-1$.

probabilities the off diagonal elements are simply: $p_{12} = 1 - p_{11}$, the probability of changing from phase 1 to phase 2; and $p_{21} = 1 - p_{22}$, the probability of changing from phase 2 to phase 1.

Let y_t denote the business cycle indicator whose distribution depends on the business cycle phase S_t . For simplicity we will assume that y_t is normally distributed conditional on the state, or $y_t | S_t = i \sim N(\mu_i, \sigma_i^2)$, which implies the conditional density of y_t is given by:

$$f(y_t | S_t = i; \theta) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[-\frac{(y_t - \mu_i)^2}{2\sigma_i^2}\right], \quad (2)$$

with $\theta = (p_{11} \ p_{22} \ \mu_1 \ \mu_2 \ \sigma_1^2 \ \sigma_2^2)'$ the relevant parameter vector to be estimated by maximum likelihood.

A natural extension of the simple model – and one which allows for some interesting causal hypothesis tests - is to allow the matrix P to be a time-varying function of some conditioning information variables. A more general version of (1) is:

$$P_t(S_t = 1 | S_{t-1} = 1) = p_{11t}, \quad P_t(S_t = 2 | S_{t-1} = 1) = p_{12t} = 1 - p_{11t} \quad (3a)$$

$$P_t(S_t = 1 | S_{t-1} = 2) = p_{21t} = 1 - p_{22t}, \quad P_t(S_t = 2 | S_{t-1} = 2) = p_{22t}$$

Where

$$p_{iit} = 1 / (1 + \exp(-\gamma_i' X_{t-1})); \ i = 1, 2; \quad (3b)$$

and

$$X_{t-1} = (1, x_{1,t-1}, x_{2,t-1}, \dots, x_{k-1,t-1})'; \gamma_i = (\gamma_{i0}, \gamma_{i1}, \dots, \gamma_{i,k-1})'$$

and where $k-1$ is the number of determinants of the transition probabilities. The functional form, $(1+\exp\{-x\})^{-1}$, is the logistic and is one of several different specifications which could be used to ensure that the estimated transition probabilities are well-behaved, i.e. lie in the unit interval.

The above “variable transition probability” model was that used by Durland and McCurdy in their test of the duration dependence hypothesis. They used quarter-to-quarter GNP growth rates as their dependent variable (y) and used current business cycle phase duration to summarize the time-varying transition probabilities conditioning information (ie X in (3b) above consisted only of the variable, duration). Estimation of this type of model is complicated by the generally unobservability of the phase change dates and hence phase durations.

When estimating Markov-switching models by maximum likelihood it is necessary to keep track of the probabilities of different phases in past periods, define the distribution of y conditional on possible phases in past and the current period, and calculate the marginal density of y by integrating, or summing, over the joint density of y and the various possible phases. With unobserved phases, duration becomes path dependent since we must explicitly keep track of all past possible peak/trough dates and this gives rise to an exponentially expanding range of possibilities. For example, if the peak was last period, then the duration variable takes the value 1, while if the peak was six periods ago,

the duration variable would take the value 6. It could also have been 2, 3, 4 or 5 - or any other number for that matter - as well. Since it is not known with certainty exactly when the peak actually did occur it is therefore necessary to keep track of all possibilities. Clearly, estimation quickly becomes infeasible if we allow for the possibility of arbitrarily long durations. To overcome this path dependence problem Durland and McCurdy arbitrarily truncate the duration variable at a maximal value D^* . The probability of staying in phase i is simply assumed constant for durations above this upper threshold.

Parameter estimation is considerably simplified if one has certain knowledge of the phase change dates. This knowledge will also significantly increase the expected precision of the estimates of the various parameters of the model - including the duration parameters - since we avoid the noise involved in using an imperfect proxy variable to represent the business cycle chronology. In the current case, by using the available NBER dates as the US business cycle chronology, we effectively define the issue of phase duration dependence in terms of whatever apparent duration dependence is evident in these pre-defined phases. This eliminates all uncertainty as to phase switches and defines exactly the value of the duration variable at each time period.⁴

Given this simplification, we retain a Markov-type process for phase changes, define transition probabilities conditional only on the phase last period, and model these transition probabilities as functions of a list of relevant explanatory variables, namely,

⁴ This is not quite true as there will be some inevitable uncertainty surrounding the most recent observations subsequent to the most recent determination by the NBER of the last turning point but in advance of any further NBER turning point determination.

current phase duration, and readings on some leading economic indicator indexes of interest. We use conditioning information available at time $t-1$ to model the probability of staying in (and therefore of leaving) state i from period $t-1$ to period t . As mentioned, the conditioning information consists of a constant, phase duration d_{t-1} , and a vector of other relevant explanatory variables Z_{t-1} (containing the leading indexes).⁵

To ensure that the transition probabilities are well defined, we model them as

$P(S_t = i | S_{t-1} = i, \psi_{t-1}) = g(\alpha_i + \delta_i d_{t-1} + Z_{t-1} \beta_i)$ for $g : \mathcal{R} \mapsto (0,1)$ and in particular we use two different functions for g that map the real line into the unit interval: the Logistic and standard normal cumulative density function (CDF). As we discuss below, these models can be interpreted respectively as yielding multinomial LOGIT and PROBIT models.

More specifically, for the LOGIT alternative, the probability of staying in phase i ($i=1,2$) may therefore be given as

$$P(S_t = i | S_{t-1} = i, \psi_{t-1}) = (1 + \exp\{-(\alpha_i + \delta_i d_{t-1} + \beta_i^t Z_{t-1})\})^{-1} \quad (4)$$

where ψ_{t-1} represents the information set available up to period $t-1$, Z_{t-1} is a column vector of two selected leading economic indicator indexes (with β_i representing the two column vectors (one vector for each phase) of associated parameters), d_{t-1} is the duration of the current expansion or recession up to period $t-1$ (with associated parameters, δ_i)

⁵ Thus, from here, for convenience we split the vector X_{t-1} (in 3b) into our duration variable, d_{t-1} , and the vector, Z_{t-1} .

and defined as $d_{t-1} = \begin{cases} d_{t-2} + 1 & \text{if } S_{t-1} = S_{t-2} \\ 1 & \text{if } S_{t-1} \neq S_{t-2} \end{cases}$, and the use of the logistic transformation,

$(1 + \exp\{-x\})^{-1}$, maps the argument from the real line to the unit interval, guaranteeing the estimation of a properly defined probability.

The alternative PROBIT formulation (denoted, let us say, as expression (5) below) is obtained by simply replacing the RHS of (4) with $\Phi(\alpha_i + \delta_i d_{t-1} + \beta_i' Z_{t-1})$, where $\Phi(x)$ is the standard normal CDF and maps the argument from the real line to the unit interval, again guaranteeing the estimated probability lies in the unit interval. Thus,

$$P(S_t = i | S_{t-1} = i, \psi_{t-1}) = \Phi(\alpha_i + \delta_i d_{t-1} + \beta_i' Z_{t-1}). \quad (5)$$

Considering (5), at each point in time, $t-1$, only one of four possible outcomes can occur:

1. The economy can stay in expansion: $S_{t-1} = 1$ and $S_t = 1$.
2. The economy can transition from expansion to recession (a peak): $S_{t-1} = 1$
and $S_t = 2$.
3. The economy can transition from recession to expansion (a trough): $S_{t-1} = 2$
and $S_t = 1$.
4. The economy can stay in recession: $S_{t-1} = 2$ and $S_t = 2$.

To summarize these four outcomes and simplify the expression for the likelihood function, define the following four dummy variables h_t^A through h_t^D which we notionally collect into a four-element vector h_t :⁶

$$\begin{aligned}
 h_t^A &= \begin{cases} 1 & \text{if } S_t = 1 \text{ and } S_{t-1} = 1 \\ 0 & \text{otherwise} \end{cases} \\
 h_t^B &= \begin{cases} 1 & \text{if } S_t = 2 \text{ and } S_{t-1} = 1 \\ 0 & \text{otherwise} \end{cases} \\
 h_t^C &= \begin{cases} 1 & \text{if } S_t = 1 \text{ and } S_{t-1} = 2 \\ 0 & \text{otherwise} \end{cases} \\
 h_t^D &= \begin{cases} 1 & \text{if } S_t = 2 \text{ and } S_{t-1} = 2 \\ 0 & \text{otherwise} \end{cases}
 \end{aligned} \tag{6}$$

Thus, at each point in time, exactly one element of the vector h_t takes the value 1, while all the other three are zero. When using the standard normal CDF to map the conditioning variables into probabilities, the likelihood function is therefore defined as

$$\begin{aligned}
 L(h; \theta) &= \prod_{t:h_t^A=1} \Phi(\alpha_1 + \delta_1 d_{t-1} + \beta_1' Z_{t-1}) \times \\
 &\quad \prod_{t:h_t^B=1} 1 - \Phi(\alpha_1 + \delta_1 d_{t-1} + \beta_1' Z_{t-1}) \times \\
 &\quad \prod_{t:h_t^C=1} 1 - \Phi(\alpha_2 + \delta_2 d_{t-1} + \beta_2' Z_{t-1}) \times \\
 &\quad \prod_{t:h_t^D=1} \Phi(\alpha_2 + \delta_2 d_{t-1} + \beta_2' Z_{t-1}).
 \end{aligned} \tag{7}$$

Where the product operators are over each particular outcome. To save space we will limit the discussion to the standard normal CDF case. The extension to the logistic transformation is obtained simply by replacing $\Phi(x)$ with $(1 + \exp(-x))^{-1}$.

⁶ Conceptually, the four dummy variables defined in equation (6) are the “bins” of the relevant multinomial Logit or Probit model.

The likelihood function is high when we accurately pick when the business cycle stays in, or transitions out of, its phase at each date in the sample. Were we interested in the simple constant transition probabilities model alternative, and knowing the business cycle phase changes as represented by the NBER turning point dates, a simple proportional count of the observed phase changes would yield our best estimates of the unconditional phase transition probabilities.

The proposed maximum likelihood procedure can be viewed as an extension of this where the model allows for the possibility of the influence of relevant information in modeling changes in the transition probabilities over time. Note that the model density defined at each time point makes full use of the known phase of the business cycle at the last time point. This is a key departure from Estrella and Mishkin's Probit modeling approach in which the relevant conditioning variables did not include knowledge of the phase of the business cycle in the previous period: they ignored the lagged phase in their model specification. Another very important difference here is that Estrella and Mishkin did not allow for the possibility that the model's parameters could change across different phases of the business cycle.

An alternative representation of the likelihood function, (7), for the standard normal CDF functional form is:

$$L(h; \theta) = \prod_{t=1}^T \Phi(\alpha_1 + \delta_1 d_{t-1} + \beta_1' Z_{t-1})^{h_t^A} \times (1 - \Phi(\alpha_1 + \delta_1 d_{t-1} + \beta_1' Z_{t-1}))^{h_t^B} \times (1 - \Phi(\alpha_2 + \delta_2 d_{t-1} + \beta_2' Z_{t-1}))^{h_t^C} \times \Phi(\alpha_2 + \delta_2 d_{t-1} + \beta_2' Z_{t-1})^{h_t^D}.$$

with corresponding log-likelihood function:

$$LL(h; \theta) = \sum_{t=1}^T h_t^A \log(\Phi(\alpha_1 + \delta_1 d_{t-1} + \beta_1' Z_{t-1})) + h_t^B \log(1 - \Phi(\alpha_1 + \delta_1 d_{t-1} + \beta_1' Z_{t-1})) + h_t^C \log(1 - \Phi(\alpha_2 + \delta_2 d_{t-1} + \beta_2' Z_{t-1})) + h_t^D \log(\Phi(\alpha_2 + \delta_2 d_{t-1} + \beta_2' Z_{t-1})). \quad (8)$$

This then is the most convenient form for the Probit model alternative. The Logit model again obtains when the standard normal CDF in (8) is replaced by logistic transformation.

Recall that only one of the four dummy variables in h_t can take the value one at any time. Furthermore, each of the four bracketed terms in the summation, (8), is a probability so their logs are weakly negative: the maximum theoretical value is zero. The highest possible likelihood therefore obtains when the model assigns a probability of one to a phase shift at the NBER-determined turning points and assigns a probability of one to continuing in an expansion or recession at all other times—in other words the model ‘gets it exactly right’ at every date. In this case the log-likelihood will be zero. Whenever the model assigns a probability less than one to any observed phase “event”, the log likelihood becomes more negative.

Thus, in this case of observable phases, the calculated values of the log-likelihoods for the various models allows the use of a well-known and widely-used statistical test – that, under the null of no improvement, twice the difference in the log-likelihoods is

distributed as chi-squared – to directly test the relative goodness-of-fit to the NBER business cycle chronology of the various alternative models under consideration.⁷

3. THE EMPIRICAL RESULTS

3.1 Data Issues

We use monthly data for the analysis spanning the period 1/1949 – 12/2002. Data on the four dummies, h_t^A , h_t^B , h_t^C , and h_t^D and the Duration variable are defined from the monthly NBER business cycle dates - as presented in Table 1. A graph of the resulting Duration variable is provided in Figure 1.

The two other variables used in the analysis are the two US leading indexes compiled by ECRI: the short leading index (SLI) and the long leading index (LLI). The individual components of these indexes have been reported elsewhere (see Layton and Katsuura, 2001, Table 1, p409). Further information on the construction of the two indexes may be obtained by contacting ECRI directly at www.businesscycle.com. The splitting of leading indicators into those with a short lead and those with a longer lead is a little unusual. Interested readers may want to refer to Cullity and Moore ("Long-Leading and Short-Leading Indexes") in Moore (1990). An important difference between the two

⁷ Others who have used a regime switching modelling framework without explicitly using the NBER dates as dependent variable have used the quadratic probability score (QPS) to do this. The closer QPS is to zero the better the fit to the NBER dates. However, statistically comparing different QPS values for different models is problematic since its distributional properties are unknown. Smith and Layton (2001) discuss model evaluation in the context of the QPS.

indexes is that LLI explicitly contains an interest rate measure and thus can be expected, at least in part, to reflect changes in US monetary policy.

For these two variables, the index data were first converted to month-to-month growth rates. Then, for each variable, a series was constructed consisting of a moving sum of the growth rates. For SLI this moving sum spanned the most recent six months and for LLI it spanned the most recent eight months. The use of a moving sum has been used successfully in previous research (see, for example, Layton (1998)) to capture the strength and persistence of any swing in the index. The different spans reflect the different expected leads of each index in relation to business cycle phase shifts. Graphs of the two resulting variables are provided in Figures 2 and 3. These were the data used in the estimation of the various models discussed below.

3.2 A Preliminary Model for Comparison with Durland and McCurdy (1994)

As mentioned in the previous section, in their test of US business cycle duration dependence, Durland and McCurdy (1994) used only duration as a potential explanatory variable in their variable transition parameter regime switching model. We therefore first estimate our regime switching multinomial Logit and Probit models with current Duration (up to period $t-1$) as the sole explanatory variable. The results are provided in Tables 2 and 3 respectively in column 2 of each table.

The first point to note derives from comparing column 2 with column 1 in each table. Column 1 represents the estimation results for the multinomial models assuming the switching probabilities for each phase are constant through time. Column 2 allows these switching probabilities to potentially depend on the duration of the current phase. A comparison of the value of the log likelihood (LL) for the two alternatives clearly statistically rejects that the switching probabilities are invariant with respect to Duration.

Using Table 2 as an example, twice the difference in the values of the LL is 15.58 (2×7.79). The critical value – at say the 10% (or 5%) level of significance - for rejection of the null of no duration dependence derives from the chi-square with 2 degrees of freedom and is 4.61 (or 5.99). Thus the data strongly reject the null and we conclude there is evidence supporting the notion that business cycles are duration dependent. This, of course, accords with the findings of Durland and McCurdy (DM).

Furthermore, both estimated coefficients are negative which is consistent with the view that the probability of remaining in a particular business cycle phase decreases with the age of the phase. For recessions, the estimated parameter is $-.3059$ and, with a robust t-ratio of -4.31 , is highly significant. The estimated parameter for expansions is $-.0179$, clearly much less negative than that for recessions. This implies expansion duration has a weaker estimated impact on the probability of an expansion terminating than in the case of recessions. All of this is also broadly consistent with DM. However, of considerable interest here is that, contrary to DM, the robust t-ratio for this coefficient is -1.72 , implying the likelihood of this parameter being statistically significantly different from

zero is much greater than what was found by DM (with a t-ratio in their case of just $-.86$). This is most likely due, in part, to our direct use of the NBER business cycle chronology. We thereby avoid the noise generated by using some selected time series as an imprecise proxy from which the chronology is imperfectly inferred (quarterly GNP growth rates in the case of DM).

Given this new result for expansions we thought it would be of interest to repeat our estimation using the 1951 to 1984 sample period as used by DM to enhance comparability. We again found that the results were very similar for both Logit and Probit alternatives and so we only report the Logit model results. The recession duration parameter was estimated at $-.2958$ with a robust t-ratio of -3.38 . The expansion duration parameter was estimated at $-.0176$ with a robust t-ratio of -1.59 . As is evident the essential features of the results are unchanged.

Importantly, we find that, using the same sample period as DM, the expansion parameter continues to have a robust t-ratio considerably larger than that found by DM using their alternative approach. Whilst the robust t-ratio nonetheless remains less than 2 for both the shorter and the longer periods analysed, we would argue that it is sufficiently large as to suggest the possibility of the existence of at least some weak duration dependence for expansions over both time periods.

3.3 Incorporating the Leading Indexes

Of course both the above analysis and that of DM may be regarded as only partial in that the only explanatory variable included in the model is Duration. Suppose the actual determinants of observed phase durations were variations in some set of underlying economic fundamentals driving the business cycle. If these fundamental drivers were cyclically mean reverting but were omitted from the model and, in their place, observed duration was the only explainer used, then it could spuriously appear that phase changes were duration dependent.

In this sub-section we report the results of incorporating the two leading indexes described in Section 3.1 into the models. Results are also reported in Tables 2 and 3. There are a number of intermediate columns in the tables corresponding to various combinations of the explanatory variables. These are provided for the sake of completeness; however, the column of most interest is Column 8 which contains the estimation results arising from including all three explanatory variables in the model. Again, the results for the two alternative functional forms are qualitatively similar and so, for the sake of brevity, we discuss only the Logit results in Table 2. A graphical indication of how the probability of expansion (recession) changes in accordance with changes in each of the three explanatory variables is provided in Figure 4.

First, the estimated model incorporating the two leading indexes is statistically superior (as measured by the difference in LLs) to the model with Duration alone. The converse is

also true. The inclusion of Duration in addition to the two leading indexes adds significantly to the statistical explanation of the business cycle phase change dates (refer to Column 5 in comparison to Column 8).

Second, all coefficients for which we had prior expectations as to their signs had the appropriate signs except for the coefficient of LLI in recessions (ie β_2^{LLI}) which should logically be negative. However, with a robust t-ratio of less than one, it is clearly statistically insignificant, and so the estimated sign is of no concern to us.

Third, the expansion Duration parameter coefficient remains greater than its robust standard error but the t-ratio has reduced to -1.23. Its absolute magnitude has also reduced and is furthermore smaller relative to the recession Duration coefficient. The recession Duration coefficient is now larger in absolute magnitude and also continues to have a robust t-ratio of about three. All of this leads to the conclusion that phase duration is considerably more important in predicting the end of recession than it is for predicting the end of an expansion (refer also to Figure 4).

Fourth, interestingly, the results for the leading indexes point to the conclusion that the long leading index is of no value in predicting the end of recessions once Duration is incorporated into the model but that the short leading index continues to have informational content. Furthermore, whilst both indexes seem to have predictive power as far as the termination of expansions is concerned, of the two indexes, the long leading index would appear to be the stronger explanator. Its estimated coefficient is more than

twice its SE while that of SLI is not and the actual estimated value of the coefficient of LLI is also quite a bit larger than that of SLI.⁸ We would speculate that these differences stem from the inclusion of the interest rate measure in the LLI leading economic indicator index. High interest rates – perhaps as a result of a tightening of monetary policy - are widely accepted as having a greater impact in bringing about an end to an expansion than low interest rates do in stimulating an economy out of a recession.

In sum, the estimated results may be interpreted as suggesting that Duration and the SLI have significant informational content as far as predicting the probability of the imminent termination of a recession. However, once movements in the leading indexes are taken into account, Duration has little predictive information in predicting the probability of the imminent termination of an expansion. Of the two leading indexes, LLI seems to have the stronger predictive power in expansions.

In Figure 5, we provide the model-derived period-by-period probabilities of recession along with the true NBER-determined probabilities (taking values 0 or 1). As can be seen in the figure, the model incorporating Duration and the two leading indexes does very well in replicating the true probabilities. In Figures 6 – 9 we provide 3-D graphical displays of how the probabilities of staying in a recession (or expansion) vary according to duration and differing values for the two leading indexes. These are 3-D alternatives (and probably more interesting ones) to visualizing the same basic features as are evident in Figure 4, namely, that duration is a more significant determinant of the termination of

⁸ It should be noted in passing that SLI and LLI were both standardised by adjusting for their respective mean and standard deviation.

recessions than expansions, that the LLI does not have significant informational content in recessions, but that, in expansions, the LLI has stronger predictive power than the SLI. As is quite clear from the figures, despite lengthening duration, little change occurs to the probability of staying in a recessionary (expansionary) phase while the leading indicators remain strongly negative (positive). Similarly, the probability of a phase change is not significantly impacted by movements in the leading indicators when duration remains low.

4. CONCLUSIONS

In this paper we have revisited the issue of phase duration dependence in the US business cycle. We would argue there are three novelties in the analysis compared with other recent investigations into the issue. First, rather than use some alternative imprecise macroeconomic variable (like GNP) to imperfectly infer the US business cycle chronology; we use the widely accepted NBER chronology. If one is prepared to accept this chronology – as most analysts and commentators seem to – then its use avoids the issue of measurement error imprecision and bias in the modeling analysis. Second, unlike other investigators, we have incorporated other potentially relevant explanatory variables into our extended models to avoid issues of omitted variables bias. Third, in contrast to some, we have allowed our models' parameters to vary across the two different business cycle phases.

Results include the following. When only duration is included as the explanatory variable in the phase switching model our results not only support the findings of others that recessions appear to be strongly duration dependent but also represent stronger evidence than has previously been found in favour of duration dependence in expansions. This we believe is due to the explicit use of the NBER chronology. Furthermore, once other variables are introduced into the model, recessions continue to appear to be quite strongly duration dependent but the evidence for duration dependence in expansions becomes considerably weaker. Finally, the selected leading indicators introduced into the model also appear to have important informational content in predicting the probability of imminent business cycle phase shifts beyond that contained in duration alone. This is the case for both expansion and recession phases.

TABLES AND GRAPHS

Table 1: Augmented NBER Chronology - <http://www.nber.org/cycles/>.

Trough	Peak
October 1949	July 1953
May 1954	August 1957
April 1958	April 1960
February 1961	December 1969
November 1970	November 1973
March 1975	January 1980
July 1980	July 1981
November 1982	July 1990
March 1991	March 2001
November 2001	

Table 2 Parameter Estimates of Various Markov Regime Switching Multinomial Logit Models.

	1.	2.	3.	4.	5.	6.	7.	8.
α_1	4.0850	4.9666	4.2619	4.5649	4.6248	5.0121	5.3454	5.3240
	(0.3361)	(0.6748)	(0.4021)	(0.4719)	(0.5044)	(0.7283)	(0.8030)	(0.8040)
	[0.3361]	[0.7011]	[0.3557]	[0.4245]	[0.4281]	[0.7758]	[0.8840]	[0.8607]
δ_1		-0.0179				-0.0143	-0.0142	-0.0127
		(0.0100)				(0.0100)	(0.0101)	(0.0099)
		[0.0104]				[0.0103]	[0.0108]	[0.0103]
β_1^{SLI}			1.2491		0.7516	1.2806		0.7571
			(0.4219)		(0.4877)	(0.4552)		(0.5250)
			[0.2662]		[0.3886]	[0.2940]		[0.4332]
β_1^{LLI}				1.2362	1.0097		1.2868	1.0452
				(0.3588)	(0.4100)		(0.3892)	(0.4419)
				[0.2973]	[0.4011]		[0.3530]	[0.4687]
α_2	2.2304	4.6753	1.4287	1.9999	1.5458	4.2159	4.3469	4.7891
	(0.3328)	(0.9725)	(0.4817)	(0.3444)	(0.5091)	(1.0848)	(1.0504)	(1.2922)
	[0.3329]	[0.7269]	[0.5271]	[0.3472]	[0.5530]	[0.8778]	[0.7780]	[1.2361]
δ_2		-0.3059				-0.3353	-0.2759	-0.4234
		(0.0948)				(0.1057)	(0.1029)	(0.1481)
		[0.0710]				[0.0927]	[0.0744]	[0.1449]
β_2^{SLI}			-0.8062		-0.4988	-0.6355		-0.9418
			(0.4312)		(0.4599)	(0.3368)		(0.4785)
			[0.5737]		[0.5928]	[0.2863]		[0.4569]
β_2^{LLI}				-0.7032	-0.5541		-0.2644	0.4488
				(0.2978)	(0.3208)		(0.2960)	(0.4948)
				[0.2506]	[0.2556]		[0.2408]	[0.5117]
LL	-78.6601	-70.8710	-72.3067	-69.5822	67.8167	-64.9296	64.9305	61.7618

Note: The probability of staying in regime i ($i=1$ denotes expansion, $i=2$ denotes recession) is given by

$$P(S_t = i | S_{t-1} = i, \psi_{t-1}) = (1 + \exp\{-(\alpha_i + \delta_i d_{t-1} + \beta_i' Z_{t-1})\})^{-1}$$

Where Z_{t-1} is a vector of leading indicators, d_{t-1} is the duration of the current expansion

or recession and defined as $d_t = \begin{cases} d_{t-1} + 1 & \text{if } S_t = S_{t-1} \\ 1 & \text{if } S_t \neq S_{t-1} \end{cases}$. Parameter estimates are reported

for a range of models with asymptotic standard errors in parenthesis and robust standard errors in square brackets.

Table 3 Parameter Estimates of the various Markov Regime Switching Multinomial PROBIT Models.

	1.	2.	3.	4.	5.	6.	7.	8.
α_1	2.1310	2.4727	2.2112	2.3286	2.3663	2.4731	2.5897	2.6050
	(0.1328)	(0.2551)	(0.1626)	(0.1858)	(0.2044)	(0.2891)	(0.3141)	(0.3282)
	[0.1328]	[0.2582]	[0.1419]	[0.1720]	[0.1711]	[0.3021]	[0.3541]	[0.3524]
δ_1		-0.0071				-0.0052	-0.0051	-0.0046
		(0.0041)				(0.0044)	(0.0045)	(0.0046)
		[0.0041]				[0.0044]	[0.0047]	[0.0047]
β_1^{SLI}			0.5874		0.3822	0.5812		0.3804
			(0.2040)		(0.2335)	(0.2147)		(0.2425)
			[0.1396]		[0.1850]	[0.1428]		[0.1931]
β_1^{LLI}				0.5544	0.4431		0.5531	0.4449
				(0.1672)	(0.1855)		(0.1752)	(0.1921)
				[0.1492]	[0.1862]		[0.1662]	[0.2050]
α_2	1.2983	2.6646	0.9240	1.1550	0.9639	2.3489	2.4598	2.5511
	(0.1699)	(0.5074)	(0.2634)	(0.1819)	(0.2741)	(0.5522)	(0.5601)	(0.6316)
	[0.1699]	[0.3679]	[0.2930]	[0.1839]	[0.2969]	[0.4172]	[0.4009]	[0.5559]
δ_2		-0.1741				-0.1859	-0.1551	-0.2189
		(0.0532)				(0.0560)	(0.0580)	(0.0750)
		[0.0406]				[0.0474]	[0.0423]	[0.0702]
β_2^{SLI}			-0.3534		-0.2005	-0.3640		-0.4830
			(0.2029)		(0.2221)	(0.1903)		(0.2576)
			[0.2683]		[0.2761]	[0.1608]		[0.2380]
β_2^{LLI}				-0.3886	-0.3231		-0.1513	0.1762
				(0.1615)	(0.1747)		(0.1711)	(0.2533)
				[0.1359]	[0.1404]		[0.1414]	[0.2417]
LL	-78.6601	-70.5209	-72.1359	-69.1367	67.3486	-64.4447	64.5854	61.4079

Note: The probability of staying in regime i ($i=1$ denotes expansion, $i=2$ denotes recession) is given by

$$P(S_t = i | S_{t-1} = i, \psi_{t-1}) = \Phi(\alpha_i + \delta_i d_{t-1} + \beta_i' Z_{t-1})$$

Where Z_{t-1} is a vector of leading indicators, d_{t-1} is the duration of the current expansion

or recession and defined as $d_t = \begin{cases} d_{t-1} + 1 & \text{if } S_t = S_{t-1} \\ 1 & \text{if } S_t \neq S_{t-1} \end{cases}$, and $\Phi(\cdot)$ is the cumulant of the

standard normal density function. Asymptotic standard errors appear in parenthesis and robust standard errors in square brackets.

