

# **THE LEVERAGE CYCLE**

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# The Leverage Cycle

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## Abstract

Equilibrium determines leverage, not just interest rates. Variations in leverage cause fluctuations in asset prices. This leverage cycle can be damaging to the economy, and should be regulated.

Key Words: Leverage, Collateral, Cycle, Crisis, Regulation

JEL: E3, E32, G01, G12

## 1 Introduction to the Leverage Cycle

At least since the time of Irving Fisher, economists, as well as the general public, have regarded the interest rate as the most important variable in the economy. But in times of crisis, collateral rates (equivalently margins or leverage) are far more important. Despite the cries of newspapers to lower the interest rates, the Fed would sometimes do much better to attend to the economy-wide leverage and leave the interest rate alone.

When a homeowner (or hedge fund or a big investment bank) takes out a loan using say a house as collateral, he must negotiate not just the interest rate, but how much he can borrow. If the house costs \$100 and he borrows \$80 and pays \$20 in cash, we say that the margin or haircut is 20%, the loan to value is  $\$80/\$100 = 80\%$ , and the collateral rate is  $\$100/\$80 = 125\%$ . The leverage is the reciprocal of the margin, namely the ratio of the asset value to the cash needed to purchase it, or  $\$100/\$20 = 5$ . These ratios are all synonymous.

In standard economic theory, the equilibrium of supply and demand determines the interest rate on loans. It would seem impossible that one equation could determine two variables, the interest rate and the margin. But in my theory, supply and demand do determine both the equilibrium leverage (or margin) and the interest rate.

It is apparent from everyday life that the laws of supply and demand can determine both the interest rate and leverage of a loan: the more impatient borrowers are, the higher the interest rate; the more nervous the lenders become, the higher the collateral they demand. But standard economic theory fails to properly capture these effects, struggling to see how a single supply-equals-demand equation for a loan could

determine two variables: the interest rate and the leverage. The theory typically ignores the possibility of default (and thus the need for collateral), or else fixes the leverage as a constant, allowing the equation to predict the interest rate.

Yet variation in leverage has a huge impact on the price of assets, contributing to economic bubbles and busts. This is because for many assets there is a class of buyer for whom the asset is more valuable than it is for the rest of the public (standard economic theory, in contrast, assumes that asset prices reflect some fundamental value). These buyers are willing to pay more, perhaps because they are more sophisticated and know better how to hedge their exposure to the assets, or they are more risk tolerant, or they simply like the assets more. If they can get their hands on more money through more highly leveraged borrowing (that is, getting a loan with less collateral), they will spend it on the assets and drive those prices up. If they lose wealth, or lose the ability to borrow, they will buy less, so the asset will fall into more pessimistic hands and be valued less.

In the absence of intervention, leverage becomes too high in boom times, and too low in bad times. As a result, in boom times asset prices are too high, and in crisis times they are too low. This is the leverage cycle.

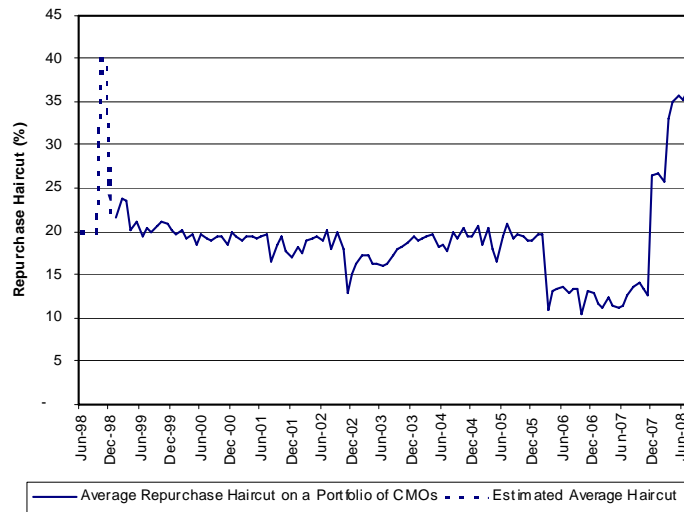
Leverage dramatically increased in the United States and globally from 1999 to 2006. A bank that in 2006 wanted to buy a AAA-rated mortgage security could borrow 98.4% of the purchase price, using the security as collateral, and pay only 1.6% in cash. The leverage was thus 100 to 1.6, or about 60 to 1. The average leverage in 2006 across all of the US\$2.5 trillion of so-called 'toxic' mortgage securities was about 16 to 1, meaning that the buyers paid down only \$150 billion and borrowed the other \$2.35 trillion. Home buyers could get a mortgage leveraged 20 to 1, a 5% down payment. Security and house prices soared.

Today leverage has been drastically curtailed by nervous lenders wanting more collateral for every dollar loaned. Those toxic mortgage securities are now leveraged on average only about 1.2 to 1. Home buyers can now only leverage themselves 5 to 1 if they can get a government loan, and less if they need a private loan. De-leveraging is the main reason the prices of both securities and homes are still falling.

The leverage cycle is a recurring phenomenon. The financial derivatives crisis in 1994 that bankrupted Orange County in California was the tail end of a leverage cycle. So was the emerging markets mortgage crisis of 1998, which brought the Connecticut-based hedge fund Long-Term Capital Management to its knees, prompting an emergency rescue by other financial institutions. The crash of 1987 also seems to be at the tail end of a leverage cycle. In the following diagram the average margin set for all securities purchased at the hedge fund Ellington Capital is plotted against time. One sees that the margin was around 20% and then spiked dramatically in 1998 to 40% for a few months, then fell back to 20% again. In late 2005 through 2007 the margins fell to around 10%, but then in the crisis of late 2007 they jumped to over 40% again, and kept rising for over a year.

In ebullient times competition drives leverage higher and higher. An investor

All CMO margins at Ellington



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comes to a hedge fund and says the fund down the block is getting higher returns. The fund manager says the other guy is just leveraging more. The investor responds, well whatever he's doing, he's getting higher returns. Pretty soon both funds are leveraging more. During a crisis, leverage can fall by 50% overnight, and by more over a few days or months. A homeowner who bought his house last year by taking out a subprime mortgage with only 5% down cannot take out a similar loan today without putting down 30% (unless he qualifies for one of the government rescue programs). The odds are great that he wouldn't have the cash to do it, and reducing the interest rate by 1 or 2% won't change his ability to act.

The policy implication of my theory of equilibrium leverage is that the fed should manage system wide leverage, curtailing leverage in normal or ebullient times, and propping up leverage in anxious times.

If agents extrapolate blindly, assuming from past rising prices that they can safely set very small margin requirements, or that falling prices means that it is necessary to demand absurd collateral levels, then the cycle will get much worse. But a crucial part of my leverage cycle story is that every agent is acting perfectly rationally from his own individual point of view. People are not deceived into following illusory trends. They do not ignore danger signs. They do not panic. They look forward, not backward. But under certain circumstances the cycle spirals into a crash anyway. The lesson is that even if people remember this leverage cycle, there will be more leverage cycles in the future, unless the Fed acts to stop them.

The crash always involves the same three elements. First is scary bad news that increases uncertainty. This leads to tighter margins as lenders get more nervous. This in turn leads to falling prices and huge losses by the most optimistic, leveraged buyers. All three elements feed back on each other; the redistribution of wealth from optimists to pessimists further erodes prices, causing more losses for optimists, and steeper price declines, which rational lenders anticipate, leading then to demand more collateral, and so on.

The best way to stop a crash is to act long before it occurs, by restricting leverage in ebullient times.

To reverse the crash once it has happened requires reversing the three causes. In today's environment, that means first of all stopping foreclosures and the free fall of housing prices. As we shall see, the only reliable way to do that is to write down principal. Second, leverage must be restored to sane, intermediate levels. The Fed must step around the banks and lend directly to investors, at more generous collateral levels than the private markets are willing to provide. And third, the Treasury must inject optimistic capital to make up for the lost buying power of the bankrupt leveraged optimists. This might also entail bailing out various crucial players.

My theory is of course not completely original. Over 400 years ago in the Merchant of Venice, Shakespeare explained that to take out a loan one had to negotiate both the interest rate and the collateral level. It is clear which of the two Shakespeare thought was the more important. Who can remember the interest rate Shylock charged Antonio? (It was zero percent.) But everybody remembers the pound of flesh that Shylock and Antonio agreed on as collateral. The upshot of the play, moreover, is that the regulatory authority (the court) decides that the collateral Shylock and Antonio freely agreed upon was socially suboptimal, and the court decrees a different collateral: a pound of flesh but not a drop of blood. The Fed too should sometimes decree different collateral rates.

In more recent times there has been pioneering work on collateral by Shleifer and Vishny SV (1992), Bernanke, Gertler, Gilchrist BGG (1996, 1999), and Holmstrom and Tirole (1997). This work emphasized the asymmetric information between borrower and lender, leading to a principal agent problem. For example, in SV (1992), the debt structure of short vs long loans must be arranged to discourage the firm management from undertaking negative present value investments with personal perks in the good state. But in the bad state this forces the firm to liquidate, just when other similar firms are liquidating, causing a price crash. The BGG (1999) model, adapted from their earlier work, is cast in an environment with costly state verification. It is closely related to the second example I give below, with utility from housing and foreclosure costs, taken from Geanakoplos (1997). But an important difference is that I do not invoke any asymmetric information. I believe that it is important to note that endogenous leverage need not be based on asymmetric information. Of course the asymmetric information revolution in economics was a tremendous advance, and asymmetric information plays a critical role in many lender-borrower relationships;

sometimes, however, the profession becomes obsessed with it. In my model the only thing backing the loan is the physical collateral. Because the loans are no-recourse, there is no need to learn anything about the borrower. All that matters is the collateral. Repo loans, and mortgages in many states, are literally no-recourse. In the rest of the states lenders rarely come after borrowers for more money beyond taking the house. And for subprime borrowers, the hit to the credit rating is becoming less and less tangible. In looking for determinants of (changes in) leverage, one should start with the distribution of collateral payoffs, and not the level of asymmetric information.

Another important paper on collateral is Kiyotaki and Moore (1997). Like BGG (1996), this paper emphasized the feedback from the fall in collateral prices to a fall in borrowing capacity, assuming a constant loan to value ratio. By contrast, my work defining collateral equilibrium, which was first published in 1997, contemporaneously with Kiyotaki and Moore, focused on what determines the *ratios* (LTV, margin, or leverage) and why they change. In practice, I believe the change in ratios has been far bigger and more important than the change in levels. This possibility is latent in the BGG models, but not emphasized by them. In 2003 I published my leverage cycle theory on the anatomy of crashes and margins (it was an invited address at the 2000 World Econometric Society meetings), arguing that in normal times leverage gets too high, and in bad times leverage is too low. In 2008 I published a paper in the AER with Ana Fostel on leverage cycles and the anxious economy. There we noted that margins do not move in lock step across asset classes, and that a leverage cycle in one asset class might spread to other unrelated asset classes. In Geanakoplos-Zame (1997, 2002, 2009) we describe the general properties of collateral equilibrium. In Geanakoplos-Kubler (2005), we show that managing collateral levels can lead to Pareto improvements.<sup>1</sup>

The recent crisis has stimulated a new generation of important papers on leverage and the economy. Notable among these are Brunnermeier and Pedersen (2008), anticipated partly by Gromb and Vayanos (2002), and Adrian and Shin (2009). Adrian and Shin have developed a remarkable series of empirical studies of leverage.

It is very important to note that leverage in my paper is defined by a ratio of collateral values to the downpayment that must be made to buy them. Those numbers are hard to get historically. I provided an aggregate of them for one hedge fund, but as far as I know they have not been systematically kept. One absolutely essential innovation would be for the Fed to gather these numbers and periodically report leverage numbers across different asset classes. It is much easier to get (debt + equity)/equity values for firms. But these numbers can be very misleading. When the economy goes badly, and the true leverage is sharply declining, many firms will find their equity wiped out, and it will appear as though their leverage has gone up, instead of down. This reversal may explain why some macroeconomists have

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<sup>1</sup>For Pareto improving interventions in credit markets, see also Gromb-Vayanos (2002) and Lorenzoni (2008).

underestimated the role leverage plays in the economy.

Perhaps the most important lesson from this work (and the current crisis) is that the macroeconomy is strongly influenced by financial variables beyond prices. This of course was the theme of much of the work of Minsky, and of James Tobin (who in (1998) explicitly defined leverage and stated that it should be determined in equilibrium, alongside interest rates), and also of Bernanke, Gertler, and Gilchrist.

## 1.1 Why was this leverage cycle worse than previous cycles?

There are a number of elements that played into the leverage cycle crisis of 2007-9 that had not appeared before, which explain why it has been so bad. I will gradually incorporate them into the model. The first I have already mentioned, namely that leverage got higher than ever before, and then margins got tighter than ever before.

The second is the invention of the credit default swap. The buyer of "CDS insurance" gets a dollar for every dollar of defaulted principal on some bond. But he is not limited to buying as much insurance as he owns bonds. In fact, he very likely is buying the CDS nowadays because he thinks the bonds are bad and does not want to own them at all. CDS are, despite their names, not insurance, but a vehicle for pessimists to leverage their views. Conventional leverage allows optimists to push the price of assets unduly high; CDS allows pessimists to push asset prices unduly low. The standardization of CDS for mortgages in late 2005 led to their trades in large quantities in 2006 at the very peak of the cycle. This I believe was one of the precipitators of the downturn.

Third, this leverage cycle was really a combination of two leverage cycles, in mortgage securities and in housing. The two reinforce each other. The tightening margins in securities led to lower security prices, which made it harder to issue new mortgages, which made it harder for homeowners to refinance, which made them more likely to default, which raised required downpayments on housing, which made housing prices fall, which made securities riskier, which made their margins get tighter and so on.

Fourth, and perhaps most important, when promises exceed collateral values, as when housing is "under water" or upside down", there are typically large losses in turning over the collateral, partly because of vandalism and so on. Today subprime bondholders expect only 25% of the loan amount back when they foreclose on a home. A huge number of homes are expected to be foreclosed (some say 8 million). The point will be that even if borrowers and lenders foresee that the loan amount is so large then there will be circumstances in which the collateral is under water, and therefore will cause deadweight losses, they will not be able to prevent themselves from agreeing on such levels.

Fifth, the leverage cycle potentially has a major impact on productive activities. High asset prices means strong incentives for production, and a boon to real construction. The fall in asset prices has a blighting effect on new real activity. This is

the essence of Tobin's Q. And it is the real reason why the crisis stage of the leverage cycle is so alarming.

## 1.2 Outline

I first present the basic model of the leverage cycle drawing on my 2003 paper, in which a continuum of investors vary in their optimism. I explain how equilibrium can determine a unique leverage ratio, and I show how dynamic variations in equilibrium leverage arise from changing tail payoff distributions, and in turn produce dramatic effects on asset prices beyond any agent's view of changing fundamentals. I describe five aspects of the leverage cycle that might motivate a regulator to smooth it out. On the other hand, I also describe how a similar sort of asset price cycle could arise with complete markets, where it might be argued that no policy intervention is warranted. Next I move to a second model, drawn from my 1997 paper, in which probabilities are objectively given, and heterogeneity arises from differences in utility for holding the collateral, as with housing. Once again leverage is endogenously determined, but now the asset prices are much more volatile than they would be with complete markets. Endogenous default appears in equilibrium. So does endogenous incomplete markets. I give three more reasons why we might worry about excessive leverage. Finally, I combine the two previous approaches to explain the double leverage cycle, in housing and in securities, which is an essential element of our current crisis. Here all eight drawbacks to excessive leverage appear at once.

## 1.3 Crises

Crises always start with bad news; there are no pure coordination failures. But not all bad news lead to crises, even when the news is very bad.

Bad news in my view must be of a special kind to cause an adverse move in the leverage cycle. The special bad news must not only lower expectations (as by definition all bad news does), but it must create more uncertainty, and more disagreement. On average news reduces uncertainty, so I have in mind a special, but by no means unusual, kind of news. The idea is that at the beginning, everyone thinks the chances of ultimate failure require too many things to go wrong to be of any substantial probability. There is little uncertainty, and therefore little room for disagreement. Once enough things go wrong to raise the spectre of real trouble, the uncertainty goes way up in everyone's mind, and so does the possibility of disagreement.

An example occurs when output is 1 unless two things go wrong, in which case output becomes .2. If an optimist thinks the chance of each thing going wrong is independent and equal to .1, then it is easy to see that he thinks the chance of ultimate breakdown is  $.01 = (.1)(.1)$ . Expected output for him is .992. In his view ex ante, the variance of final output is  $.99(.01)(1 - .2) = .0079$ . After the first piece of bad news, his expected output drops to .92. But the variance jumps to  $.9(.1)(1 - .2) = .072$ ,



a tenfold increase.

A less optimistic agent who believes the probability of each piece of bad news is independent and equal to .8 originally thinks the probability of ultimate breakdown is  $.04 = (.2)(.2)$ . Expected output for him is .968. In his view ex ante, the variance of final output is  $.96(.04)(1 - .2) = .031$ . After the first piece of bad news, his expected output drops to .84. But the variance jumps to  $.8(.2)(1 - .2) = .128$ . Note that the expectations differed originally by  $.992 - .968 = .024$ , but after the bad news the disagreement more than triples to  $.92 - .84 = .08$ .

I call the kind of bad news that increases uncertainty and disagreement *scary* news.

The news in the last 18 months has indeed been of this kind. When agency mortgage default losses were less than 1/4%, there was not much uncertainty and not much disagreement. Even if they tripled, they would still be small enough not to matter. Similarly, when subprime mortgage losses (that is losses incurred after homeowners failed to pay, were thrown out of their homes, and the house was sold for less than the loan amount) were 3%, they were so far under the rated bond cushion of 8% that there was not much uncertainty or disagreement about whether the bonds would suffer losses, especially the higher rated bonds (with cushions of 15% or more). By 2007, however, forecasts on subprime losses ranged from 30% to 80%.

## 1.4 Anatomy of a Crash

I use my theory of the equilibrium leverage to outline the anatomy of market crashes after the kind of scary news I just described.

- i) Assets go down in value on scary bad news.
- ii) This causes a big drop in the wealth of the natural buyers (optimists) who were leveraged. Leveraged buyers are forced to sell to meet their margin requirements.
- iii) This leads to further loss in asset value, and in wealth for the natural buyers.
- iv) Then just as the crisis seems to be coming under control, margin requirements are tightened because of increased uncertainty and disagreement.
- v) This causes huge losses in asset values via forced sales.
- vi) Many optimists will lose all their wealth and go out of business
- vii) There may be spillovers if optimists in one asset hit by bad news are led to sell other assets for which they are also optimists.
- viii) Investors who survive have a great opportunity.

## 1.5 Natural Buyers

A crucial part of my story is heterogeneity between investors. The natural buyers want the asset more than the general public. This could be for many reasons. The natural buyers could be less risk averse. Or they could have access to hedging techniques the general public does not that make the assets less dangerous for them. Or they could

get more utility out of holding the assets. Or they could have access to a production technology that uses the assets more efficiently than the general public. Or they could have special information based on local knowledge. Or they could simply be more optimistic. I have tried nearly all these possibilities at various times in my models. In the real world, the natural buyers are probably made up of a mixture of these categories. But for modeling purposes, the simplest is the last, namely that the natural buyers are more optimistic by nature. They have different priors from the pessimists. I note simply that this perspective is not really so different from the others. Differences in risk aversion in the end just mean different risk adjusted probabilities.

A crucial part of the story then is the heterogeneity between optimists and pessimists. A loss for the optimists is much more important to prices than a loss for the pessimists, because it is the optimists who will be holding the assets and bidding their prices up. The loss of access to capital by the optimists (and the subsequent moving of assets from optimists to pessimists) creates the crash.

Current events have certainly borne out this heterogeneity hypothesis. When the big banks (who are the classic natural buyers) lost lots of capital through their blunders in the CDO market, that had a profound effect on new investments. Some of that capital was restored by international investments from Singapore and so on, but it was not enough, and it quickly dried up when the initial investments lost money.

Macroeconomists have often ignored the natural buyers hypothesis. For example, some macroeconomists compute the marginal propensity to consume out of wealth, and find it very low. The loss of \$250 billion dollars of wealth could not possibly matter much they said, because the stock market has fallen many times by much more and economic activity hardly changed. But that ignores who lost the money.

The natural buyers hypothesis is not original with me. (See for example Harrison and Kreps (1979), Allen and Gale (1994), Shleifer and Vishny (1997).<sup>2</sup>) The innovation is in combining it with equilibrium leverage.

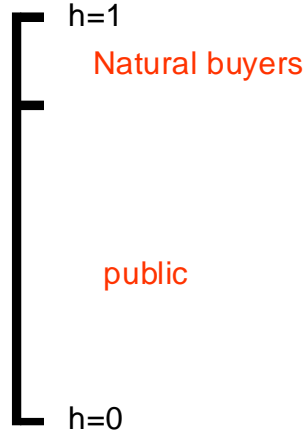
I do not presume a cut and dried distinction between natural buyers and the public. I imagine a continuum of agents uniformly arrayed between 0 and 1. Agent  $h$  on that continuum thinks the probability of good news (Up) is  $\gamma_U^h = h$ , and the probability of bad news (Down) is  $\gamma_D^h = 1 - h$ . The higher the  $h$ , the more optimistic the agent.

The more optimistic an agent, the more natural a buyer he is. By having a continuum I avoid a rigid categorization of agents. The agents will choose whether to be borrowers and buyers of risky assets, or lenders and sellers of risky assets. There will be some break point  $b$  such that those more optimistic with  $h > b$  are on one side of the market and those less optimistic, with  $h < b$ , are on the other side. But this break point  $b$  will be endogenous. See Diagram 1.

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<sup>2</sup>See also Caballero-Krishnamurthy (2001) and Fostel-Geanakoplos (2008).

## Natural Buyers-Margins Theory of Crashes



## 2 Borrowing and Asset Pricing

Consider a simple example with one consumption good  $C$ , one asset  $Y$ , two time periods 0, 1, and two states of nature  $U$  and  $D$  in the last period. Suppose that each unit of  $Y$  pays either 1 or .2 of the consumption good, in the two states  $U$  or  $D$ , respectively. Imagine the asset as a mortgage that either pays in full or defaults with recovery .2. (All mortgages will either default together or pay off together). But it could also be an oil well that might be a gusher or small. Or a house with good or bad resale value next period. Let every agent own one unit of the asset at time 0 and also one unit of the consumption good at time 0. For simplicity we think of the consumption good as something that can be used up immediately as consumption  $c$ , or costlessly warehoused (stored) in a quantity denoted by  $w$ . Think of oil or cigarettes or canned food or simply gold (that can be used as fillings) or money. The agents  $h \in H$  only care about the total expected consumption they get, no matter when they get it. They are not impatient. The difference between the agents is only in the probabilities  $\gamma_U^h, \gamma_D^h = 1 - \gamma_U^h$  each attaches to a good outcome vs bad.

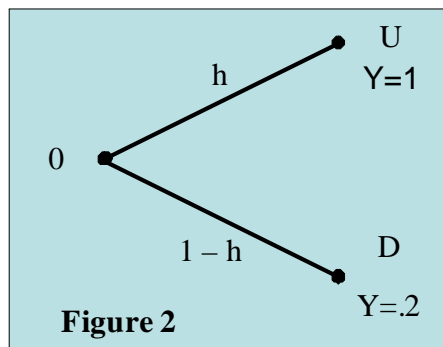
To start with, let us imagine the agents arranged uniformly on a continuum, with agent  $h \in H = [0, 1]$  assigning probability  $\gamma_U^h = h$  to the good outcome.

See diagram 2.

More formally, denoting the amount of consumption of  $C$  in state  $s$  by  $c_s$ , and the holding in state  $s$  of  $Y$  by  $y_s$ , and the warehousing of the consumption good at time

## Endogenous Collateral with Heterogeneous Beliefs: A Simple Example

Let each agent  $h \in H \subset [0,1]$  assign probability  $h$  to  $s = U$  and probability  $1 - h$  to  $s = D$ . Agents with  $h$  near 1 are optimists, agents with  $h$  near 0 are pessimists.



Suppose that 1 unit of  $Y$  gives \$1 unit in state  $U$  and .2 units in  $D$ .

0 by  $w_0$  we have

$$u^h(c_0, y_0, w_0, c_U, c_D) = c_0 + \gamma_U^h c_U + \gamma_D^h c_D = c_0 + h c_U + (1 - h) c_D$$

$$e^h = (e_{C_0}^h, e_{Y_0}^h, e_{C_U}^h, e_{C_D}^h) = (1, 1, 0, 0)$$

Storing goods and holding assets provide no direct utility, they just increase income in the future.

Suppose the price of the asset per unit at time 0 is  $p$ , somewhere between 0 and 1. The agents  $h$  who believe that

$$h1 + (1 - h).2 > p$$

will want to buy the asset, since by paying  $p$  now they get something with expected payoff next period greater than  $p$  and they are not impatient. Those who think

$$h1 + (1 - h).2 < p$$

will want to sell their share of the asset. I suppose there is no short selling, but I will allow for borrowing. In the real world it is impossible to short sell many assets other than stocks. Even when it is possible, only a few agents know how, and those typically are the optimistic agents who are most likely to want to buy. So the assumption of no short selling is quite realistic. But we shall reconsider this point shortly.

If borrowing were not allowed, then the asset would have to be held by a large part of the population. The price of the asset would be .677 or about .68. Agent  $h = .60$  values the asset at  $.68 = .60(1) + .40(.2)$ . So all those  $h$  below .60 will sell all they have, or  $.60(1) = .60$  in aggregate. Every agent above .60 will buy as much as he can afford. Each of these agents has just enough wealth to buy  $1/.68 \approx 1.5$  more units, hence  $.40(1.5) = .60$  units in aggregate. Since the market for assets clears at time 0, this is the equilibrium with no borrowing.

More formally, taking the price of the consumption good in each period to be 1 and the price of  $Y$  to be  $p$ , we can write the budget set without borrowing for each agent as

$$\begin{aligned} B_N^h(p) &= \{(c_0, y_0, w_0, c_1, c_2) \in \mathbb{R}_+^5 : c_0 + w_0 + p(y_0 - 1) = 1 \\ &\quad c_U = w_0 + y_0 \\ &\quad c_D = w_0 + (.2)y_0\}. \end{aligned}$$

Given the price  $p$ , each agent chooses the consumption plan  $(c_0^h, y_0^h, w_0^h, c_1^h, c_2^h)$  in  $B_N^h(p)$  that maximizes his utility  $u^h$  defined above. In equilibrium all markets must clear

$$\begin{aligned} \int_0^1 (c_0^h + w_0^h) dh &= 1 \\ \int_0^1 y_0^h dh &= 1 \\ \int_0^1 c_U^h dh &= 1 + \int_0^1 w_0^h dh \\ \int_0^1 c_D^h dh &= .2 + \int_0^1 w_0^h dh \end{aligned}$$

In this equilibrium agents are indifferent to storing or consuming right away, so we can describe equilibrium as if everyone warehoused and postponed consumption by taking

$$\begin{aligned} p &= .68 \\ (c_0^h, y_0^h, w_0^h, c_1^h, c_2^h) &= (0, 2.5, 0, 2.5, .5) \text{ for } h \geq .60 \\ (c_0^h, y_0^h, w_0^h, c_U^h, c_D^h) &= (0, 0, 1.68, 1.68, 1.68) \text{ for } h < .60. \end{aligned}$$

When loan markets are created, a smaller group of less than 40% of the agents will be able to buy and hold the entire stock of the asset. If borrowing were unlimited, at an interest rate of 0, the single agent at the top would borrow so much that he would buy up all the assets by himself. And then the price of the asset would be 1, since at any price  $p$  lower than 1 the agents  $h$  just below 1 would snatch the asset away from  $h = 1$ . But this agent would default, and so the interest rate would not be zero, and the equilibrium allocation needs to be more delicately calculated.

## 2.1 Incomplete Markets

We shall restrict attention to loans that are non-contingent, that is that involve promises of the same amount  $\varphi$  in both states. It is evident that the equilibrium allocation under this restriction will in general not be Pareto efficient. For example, in the no borrowing equilibrium, everyone would gain from the transfer of  $\varepsilon > 0$  units of consumption in state U from each  $h < .60$  to each agent with  $h > .60$ , and the transfer of  $3\varepsilon/2$  units of consumption in state D from each  $h > .60$  to each agent with  $h < .60$ . The reason this has not been done in the equilibrium is that there is no asset that can be traded that moves money from U to D or vice versa. We say that the asset markets are incomplete. We shall assume this incompleteness for a long time, until we consider Credit Default Swaps.

## 2.2 Collateral

We have not yet determined how much people can borrow or lend. In conventional economics they can do as much of either as they like, at the going interest rate. But in real life lenders worry about default. Suppose we imagine that the only way to enforce deliveries is through collateral. A borrower can use the asset itself as collateral, so that if he defaults the collateral can be seized. Of course a lender realizes that if the promise is  $\varphi$  in both states, then with no-recourse collateral he will only receive

$$\begin{aligned} &\min(\varphi, 1) \text{ if good news} \\ &\min(\varphi, .2) \text{ if bad news} \end{aligned}$$

The introduction of collateralized loan markets introduces two more parameters: how much can be promised  $\varphi$ , and at what interest rate  $r$ ?

Suppose that borrowing were arbitrarily limited to  $\varphi \leq .2y_0$ , that is suppose agents were allowed to promise at most .2 units of consumption per unit of the collateral Y they put up. That is a natural limit, since it is the biggest promise that is sure to be covered by the collateral. It also greatly simplifies our notation, because then there would be no need to worry about default. The previous equilibrium without borrowing could be reinterpreted as a situation of extraordinarily tight leverage, where we have the constraint  $\varphi \leq 0y_0$ .

Leveraging, that is, using collateral to borrow, gives the most optimistic agents a chance to spend more. And this will push up the price of the asset. But since they can borrow strictly less than the value of the collateral, optimistic spending will still be limited. Each time an agent buys a house, he has to put some of his own money down in addition to the loan amount he can obtain from the collateral just purchased. He will eventually run out of capital.

We can describe the budget set formally with our extra variables.

$$\begin{aligned}
B_2^h(p, r) &= \{(c_0, y_0, \varphi_0, w_0, c_U, c_D) \in \mathbb{R}_+^6 : \\
c_0 + w_0 + p(y_0 - 1) &= 1 + \frac{1}{1+r}\varphi_0 \\
\varphi_0 &\leq .2y_0 \\
c_U &= w_0 + y_0 - \varphi_0 \\
c_D &= w_0 + (.2)y_0 - \varphi_0\}.
\end{aligned}$$

We use the subscript .2 on the budget set to remind ourselves that we have arbitrarily fixed the maximum promise that can be made on a unit of collateral. At this point we could imagine that was a parameter set by government regulators.

Note that in the definition of the budget set,  $\varphi_0 > 0$  means that the agent is making promises in order to borrow money to spend more at time 0. Similarly,  $\varphi_0 < 0$  means the agent is buying promises which will reduce his expenditures on consumption and assets in period 0, but enable him to consume more in the future states U and D. Equilibrium is defined by the price and interest rate  $(p, r)$  and agent choices  $(c_0^h, y_0^h, \varphi_0^h, w_0^h, c_U^h, c_D^h)$  in  $B_2^h(p, r)$  that maximizes his utility  $u^h$  defined above. In equilibrium all markets must clear

$$\begin{aligned}
\int_0^1 (c_0^h + w_0^h) dh &= 1 \\
\int_0^1 y_0^h dh &= 1 \\
\int_0^1 \varphi_0^h dh &= 0 \\
\int_0^1 c_U^h dh &= 1 + \int_0^1 w_0^h dh \\
\int_0^1 c_D^h dh &= .2 + \int_0^1 w_0^h dh
\end{aligned}$$

Clearly the no borrowing equilibrium is a special case of the collateral equilibrium, once the limit .2 on promises is replaced by 0.

One can calculate that the equilibrium price of the asset is now .75. At that price agent  $h = .69$  is just indifferent to buying. Those  $h < .69$  will sell all they have, and those  $h > .69$  will buy all they can with their cash and with the money they can borrow. One can check that the top 31% of agents will indeed demand exactly what the bottom 69% are selling.

Who would be doing the borrowing and lending? The top 31% is borrowing to the max, in order to get their hands on what they believe are cheap assets. The bottom 69% do not need the money for buying the asset, so they are willing to lend it. And what interest rate would they get? 0% interest, because they are not lending

all they have in cash. (They are lending  $.2/.69 = .29 < 1$  per person). Since they are not impatient and they have plenty of cash left, they are indifferent to lending at 0%. Competition among these lenders will drive the interest rate to 0%.

More formally, letting the marginal buyer be denoted by  $h = b$ , we can define the equilibrium equations as

$$p = \gamma_U^b 1 + (1 - \gamma_U^b)(.2) = b1 + (1 - b)(.2)$$

$$p = \frac{(1 - b)(1) + .2}{b}$$

The first equation says that the marginal buyer  $b$  is indifferent to buying the asset. The second equation says that the price of Y is equal to the amount of money the agents above  $b$  spend buying it, divided by the amount of the asset sold. The numerator is then all the top group's consumption endowment,  $(1 - b)(1)$ , plus all they can borrow after they get their hands on all of Y, namely  $(1)(.2)/(1 + r) = .2$ . The denominator is comprised of all the sales of one unit of Y each by the agents below  $b$ .

We must also take into account buying on margin. An agent who buys the asset while simultaneously selling as many promises as he can will only have to pay down  $p - .2$ . His return will be nothing in the down state, because then he will have to turn over all the collateral to pay back his loan. But in the up state he will make a profit of  $1 - .2$ . Any agent like  $b$  who is indifferent to borrowing or lending and also indifferent to buying or selling the asset, will be indifferent to buying the asset with leverage because

$$p - .2 = \gamma_U^b(1 - .2) = b(1 - .2)$$

Clearly this equation is automatically satisfied as long as  $p$  is set to satisfy the first equation above; simply subtract  $.2$  from both sides. Agents  $h > b$  will strictly prefer to buy the asset, and strictly prefer to buy the asset with as much leverage as possible (since they are risk neutral).

As we said, the large supply of durable consumption good, no impatience, and no default implies that the equilibrium interest rate must be 0. Solving the two equations above and plugging these into the agent optimization gives equilibrium

$$b = .69$$

$$(p, r) = (.75, 0),$$

$$(c_0^h, y_0^h, \varphi_0^h, w_0^h, c_1^h, c_2^h) = (0, 3.2, .64, 0, 2.6, 0) \text{ for } h \geq .69$$

$$(c_0^h, y_0^h, \varphi_0^h, w_0^h, c_1^h, c_2^h) = (0, 0, -.3, 1.45, 1.75, 1.75) \text{ for } h < .69.$$

Compared to the previous equilibrium with no leverage, the price rises modestly, from  $.68$  to  $.75$ , because there is a modest amount of borrowing. Notice also that even at the higher price, fewer agents hold all the assets (because they can afford to buy on borrowed money).



The lesson here is that the looser the collateral requirement, the higher will be the prices of assets. Had we defined another equilibrium by arbitrarily specifying the collateral limit of  $\varphi \leq .1y_0$ , we would have found an equilibrium price intermediate between .68 and .75. This has not been properly understood by economists. The conventional view is that the lower is the interest rate, then the higher will asset prices be, because their cash flows will be discounted less. But in the example I just described, where agents are patient, the interest rate will be zero regardless of the collateral restrictions (up to .2). The fundamentals do not change, but because of a change in lending standards, asset prices rise. Clearly there is something wrong with conventional asset pricing formulas. The problem is that to compute fundamental value, one has to use probabilities. But whose probabilities?

The recent run up in asset prices has been attributed to irrational exuberance because conventional pricing formulas based on fundamental values failed to explain it. But the explanation I propose is that collateral requirements got looser and looser. We shall return to this momentarily, after we endogenize the collateral limits.

Before turning to the next section, let us be more precise about our numerical measure of leverage

$$\text{leverage} = \frac{.75}{(.75 - .2)} = 1.4.$$

The loan to value is  $.2/.75 = 27\%$ , the margin or haircut is  $.55/.75 = 73\%$ . In the no borrowing equilibrium, leverage was obviously 1.

But leverage cannot yet be said to be endogenous, since we have exogenously fixed the maximal promise at .2. Why wouldn't the most optimistic buyers be willing to borrow more, defaulting in the bad state of course, but compensating the lenders by paying a higher interest rate? Or equivalently, why should leverage be so low?

## 2.3 Equilibrium Leverage

Before 1997 there had been virtually no work on equilibrium margins. Collateral was discussed almost exclusively in models without uncertainty. Even now the few writers who try to make collateral endogenous do so by taking an ad hoc measure of risk, like volatility or value at risk, and assume that the margin is some arbitrary function of the riskiness of the repayment.

It is not surprising that economists have had trouble modeling equilibrium haircuts or leverage. We have been taught that the only equilibrating variables are prices. It seems impossible that the demand equals supply equation for loans could determine two variables.

The key is to think of many loans, not one loan. Irving Fisher and then Ken Arrow taught us to index commodities by their location, or their time period, or by the state of nature, so that the same quality apple in different places or different periods might have different prices. So we must index each promise by its collateral. A promise of .2 backed by a house is different from a promise of .2 backed by 2/3 of

a house. The former will deliver .2 in both states, but the latter will deliver .2 in the good state and only 1.33 in the bad state. The collateral matters.

Conceptually we must replace the notion of contracts as promises with the notion of contracts as ordered pairs of promises and collateral. Each ordered pair-contract will trade in a separate market, with its own price.

$$Contract_j = (Promise_j, Collateral_j) = (A_j, C_j)$$

The ordered pairs are homogeneous of degree one. A promise of .2 backed by 2/3 of a house is simply 2/3 of a promise of .3 backed by a full house. So without loss of generality, we can always normalize the collateral. In our example we shall focus on contracts in which the collateral  $C_j$  is simply one unit of Y.

So let us denote by  $j$  the promise of  $j$  in both states in the future, backed by the collateral of one unit of Y. We take an arbitrarily large set  $J$  of such assets, but include  $j=.2$ .

The  $j = .2$  promise will deliver .2 in both states, the  $j = .3$  promise will deliver .3 after good news, but only .2 after bad news, because it will default there. The promises would sell for different prices, and different prices per unit promised.

Our definition of equilibrium must now incorporate these new promises  $j \in J$  and prices  $\pi_j$ . When the collateral is so big that there is no default,  $\pi_j = j/(1+r)$ , where  $r$  is the riskless rate of interest. But when there is default, the price cannot be derived from the riskless interest rate alone. Given the price  $\pi_j$ , and given that the promises are all non-contingent, we can always compute the implied nominal interest rate as  $1 + r_j = j/\pi_j$ .

We must distinguish between sales  $\varphi_j > 0$  of these promises (that is borrowing) from purchases of these promises  $\varphi_j < 0$ . The two differ more than in their sign. A sale of a promise obliges the seller to put up the collateral, whereas the buyer of the promise does not bear that burden. The marginal utility of buying a promise will often be much less than the marginal disutility of selling the same promise, at least if the agent does not otherwise want to hold the collateral.

We can describe the budget set formally with our extra variables.

$$\begin{aligned}
B^h(p, \pi) &= \{(c_0, y_0, (\varphi_j)_{j \in J}, w_0, c_1, c_2) \in \mathbb{R}_+^2 \times \mathbb{R}^J \times \mathbb{R}_+^3 : \\
c_0 + w_0 + p(y_0 - 1) &= 1 + \sum_{j=1}^J \varphi_j \pi_j \\
\sum_{j=1}^J \max(\varphi_j, 0) &\leq y_0 \\
c_U &= w_0 + y_0 - \sum_{j=1}^J \varphi_j \min(1, j) \\
c_D &= w_0 + (.2)y_0 - \sum_{j=1}^J \varphi_j \min(.2, j)\}.
\end{aligned}$$

Observe that in the final two equations we see that we are describing no-recourse collateral. Every agent delivers the same, namely the promise or the collateral, whichever is worth less. The loan market is thus completely anonymous; there is no role for asymmetric information about the agents because every agent delivers the same way. Lenders need only worry about the collateral, not about the identity of the borrowers. Observe that  $\varphi_j$  can be positive (making a promise) or negative (buying a promise), and that either way the deliveries or receipts are given by the same formula.

The middle inequality describes the crucial collateral or leverage constraint. Each promise must be backed by collateral, and so the sum of the collateral requirements across all the promises must be met by the  $Y$  on hand.

Equilibrium is defined exactly as before, except that now we must have market clearing for all the contracts  $j \in J$ .

$$\begin{aligned}
\int_0^1 (c_0^h + w_0^h) dh &= 1 \\
\int_0^1 y_0^h dh &= 1 \\
\int_0^1 \varphi_j^h dh &= 0, \forall j \in J \\
\int_0^1 c_U^h dh &= 1 + \int_0^1 w_0^h dh \\
\int_0^1 c_D^h dh &= .2 + \int_0^1 w_0^h dh
\end{aligned}$$

It turns out that the equilibrium is exactly as before. The only asset that is traded is  $((.2, .2), 1)$ , namely  $j = .2$ . All the other contracts are priced, but in equilibrium neither bought nor sold. Their prices can be computed by the value the marginal

buyer  $b = .69$  attributes to them. So the price  $\pi_3$  of the .3 promise is .27, much more than the price of the .2 promise but less per dollar promised. Similarly the price of a promise of .4 is given below

$$\begin{aligned}\pi_2 &= .69(.2) + .31(.2) = .2 \\ 1 + r_2 &= .2/.2 = 1.00 \\ \pi_3 &= .69(.3) + .31(.2) = .269 \\ 1 + r_3 &= .3/.269 = 1.12 \\ \pi_4 &= .69(.4) + .31(.2) = .337 \\ 1 + r_3 &= .4/.337 = 1.19\end{aligned}$$

Thus an agent who wants to borrow .2 using one house as collateral can do so at 0% interest. An agent who wants to borrow .269 with the same collateral can do so by promising 12% interest. An agent who wants to borrow .337 can do so by promising 19% interest. The puzzle of one equation determining both a collateral rate and an interest rate is resolved; each collateral rate corresponds to a different interest rate. It is quite sensible that less secure loans with higher defaults will require higher rates of interest.

What then do we make of my claim about "the" equilibrium margin? The surprise is that in this kind of example, with only one dimension of risk and one dimension of disagreement, only one margin will be traded! Everybody will voluntarily trade only the .2 loan, even though they could all borrow or lend different amounts at any other rate.

How can this be? Mr  $h = 1$  thinks for every .75 he pays on the asset, he can get 1 for sure. Wouldn't he love to be able to borrow more, even at a slightly higher interest rate? The answer is no! In order to borrow more, he has to substitute say a .4 loan for a .2 loan. He pays the same amount in the bad state, but pays more in the good state, in exchange for getting more at the beginning. But that is not rational for him. He is the one convinced the good state will occur, so he definitely does not want to pay more just where he values money the most.<sup>3</sup>

The lenders are people with  $h < .69$  who do not want to buy the asset. They are lending instead of buying the asset because they think there is a substantial chance of bad news. It should be no surprise that they do not want to make risky loans, even if they can get a 19% rate instead of a 0% rate, because the risk of default is too high for them. Indeed the risky loan is perfectly correlated with the asset which they have already shown they do not want. Why should they give up more money

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<sup>3</sup>More precisely, buying Y while simultaneously using it as collateral to sell any non-contingent promise of at least .2 is tantamount to buying up Arrow securities at a price of  $b$  per unit of net payoff in state U. So  $h > b$  is indifferent to trading on any of the loan markets promising at least .2. By promising .4 per unit of Y instead of .2 he simply is buying fewer of the up Arrow securities per contract (because he must deliver more in the up state), but he can buy more contracts (since he is receiving more money at date 0). He can accomplish exactly the same thing selling less .2 promises.

at time 0 to get more money in a state U they do not think will occur? If anything, these pessimists would now prefer to take the loan rather than to give it. But they cannot take the loan, because that would force them to hold the collateral to back their promises, which they do not want to do.<sup>4</sup>

Thus the only loans that get traded in equilibrium involve margins just tight enough to rule out default. That depends of course on the special assumption of only two outcomes. But often the outcomes lenders have in mind are just two. And typically they do set haircuts in a way that makes defaults very unlikely. Recall that in the 1994 and 1998 leverage crises, not a single lender lost money on repo trades. Of course in more general models, one would imagine more than one margin and more than one interest rate emerging in equilibrium.

To summarize, in the usual theory a supply equals demand equation determines the interest rate on loans. In my theory equilibrium often determines the equilibrium leverage (or margin) as well. It seems surprising that one equation could determine two variables, and to the best of my knowledge I was the first to make the observation (in 1997 and again in 2003) that leverage could be uniquely determined in equilibrium. I showed that the right way to think about the problem of endogenous collateral is to consider a different market for each loan depending on the amount of collateral put up, and thus a different interest rate for each level of collateral. A loan with a lot of collateral will clear in equilibrium at a low interest rate, and a loan with little collateral will clear at a high interest rate. A loan market is thus determined by a pair (promise, collateral), and each pair has its own market clearing price. The question of a unique collateral level for a loan reduces to the less paradoxical sounding, but still surprising, assertion that in equilibrium everybody will choose to trade in the same collateral level for each kind of promise. I proved that this must be the case when there are only two successor states to each state in the tree of uncertainty, with risk neutral agents differing in their beliefs, but with a common discount rate. More generally I conjecture that the number of collateral rates traded endogenously will not be unique, but it will robustly be much less than the dimension of the state space, or the dimension of agent types.

### 2.3.1 Upshot of equilibrium leverage

We have shown that in the simple two state context, equilibrium leverage transforms the purchase of the collateral into the buying of the up Arrow security: the buyer of the collateral will simultaneously sell the promise of the entire down payoff of the collateral, so on net he is just buying the up Arrow security.

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<sup>4</sup>More precisely, agents with  $h < b$  will want to trade their wealth for as much consumption as they can get in the down state. But on account of the incompleteness of markets, no combination of buying, selling, borrowing on margin and so on can get them more in the down state than in the up state. So they strictly prefer making the .2 loan to lending, or borrowing with collateral, any loan promising more than .2 per unit of  $Y$ .

### 2.3.2 Endogenous leverage: reinforcer or dampener?

One can imagine many shocks to the economy that affect asset prices. These shocks will also typically change equilibrium leverage. Will the change in equilibrium leverage multiply the effect on asset prices, or dampen the effect? For most shocks, endogenous leverage will act as a dampener.

For example, suppose that agents become more optimistic, so that we now have

$$\begin{aligned}\gamma_D^h &= (1 - h)^2 < 1 - h \\ \gamma_U^h &= 1 - (1 - h)^2 > h\end{aligned}$$

for all  $h \in (0, 1)$ . Substituting these new values for the beliefs into the utility function, we can recompute equilibrium, and we find that the price of Y rises to .89. But the equilibrium promise remains .2, and the equilibrium interest rate remains 0. Hence leverage falls to  $1.29 = .89 / (.89 - .2)$ . The marginal buyer  $b = .63$  is lower than before. In short, the positive news has been dampened by the tightening of leverage.

A similar situation prevails if agents see an increase in their endowment of the consumption good. The extra wealth induces them to demand more Y, the price of Y rises, but not as far as it would otherwise, because equilibrium leverage goes down.

The only shock that is reinforced by the endogenous movement in leverage is a shock to the tail of the distribution of Y payoffs. If the tail payoff .2 is increased to .3, that will have a positive effect on the expected payoff of Y, but the effect on the price of Y will be reinforced by the expansion of equilibrium leverage. Negative tail events will also be multiplied, as we shall see later.

## 2.4 Fundamental Asset Pricing? Failure of Law of One Price

We have already seen enough to realize that assets are not priced by fundamentals in collateral equilibrium. We can make this more concrete by supposing, as in my 2003 paper, that we have two identical assets, blue Y and red Y, where blue Y can be used as collateral but red Y cannot. Suppose that every agent begins with  $\beta$  units of blue Y and  $(1 - \beta)$  units of red Y, in addition to one unit of the consumption good. Will the law of one price hold in equilibrium?

Will the two assets, which are perfect substitutes, both delivering 1 or .2 in the two states U and D, sell for the same price? Why would anyone pay more to get the same thing? The answer is that the collateralizable assets will indeed sell for a significant premium, even though no agent will pay more for the same thing. The most optimistic buyers will exclusively buy the blue asset by leveraging, and the mildly optimistic middle group will exclusively buy the red asset without leverage. The rest of the population will sell their assets and lend to the biggest optimists.

Will the scarcity of collateral tend to boost the blue asset prices above the asset prices we saw in the last section? What effect does the presence of leverage for the blue assets have on the red asset prices?

We answer these questions in the next section.

## 2.5 Legacy Assets vs New Assets

These questions bear on an important policy choice that is being made at the writing of this paper. As a result of the leverage crunch of 2007-2009, asset prices plummeted. One critical effect was that it became very difficult to support asset prices for new ventures that would allow for new activity. Who would buy a new mortgage (or new credit card loan, or new car loan) at 100 when virtually the same old asset could be purchased on the secondary market at 65?

Suppose the government wants to prop up the price of new assets, by providing leverage beyond what the market will provide. Given a fixed upper bound in (expected) defaults, would the government do better to provide lots of leverage on just the new assets, or by providing moderate leverage on all the assets, new and legacy? At the time of this writing, the government appears to have adopted the strategy of leveraging only the new assets. Yet all the asset prices are rising.

I considered these very questions in my 2003 paper, anticipating the current debate, by examining the effect on asset prices of adjusting the fraction  $\beta$  of blue assets. If the new assets represent say  $\nu = 5\%$  of the total, then taking  $\beta = \nu = 5\%$  corresponds to a policy of leveraging just the new assets. Taking  $\beta = 100\%$  corresponds to leveraging the legacy assets as well.

To keep the notation simple, let us assume that using a blue asset as collateral, one can sell a promise  $j$  of .2, but that the red asset cannot serve as collateral for any promises.

The definition of equilibrium now consists of  $(r, p_B, p_R, (c_0^h, y_{0B}^h, y_{0R}^h, \varphi_0^h, w_0^h, c_1^h, c_2^h)_{h \in H})$  such that the individual choices are optimal in the budget sets

$$\begin{aligned}
 B_{.2,B}^h(p, r) &= \{(c_0, y_{0B}, y_{0R}, \varphi_0, w_0, c_1, c_2) \in \mathbb{R}_+^7 : \\
 c_0 + w_0 + p_B(y_{0B} - \beta) + p_R(y_{0R} - (1 - \beta)) &= 1 + \frac{1}{1+r}\varphi_0 \\
 \varphi_0 &\leq .2y_{0B} \\
 c_U &= w_0 + y_{0B} + y_{0R} - \varphi_0 \\
 c_D &= w_0 + (.2)(y_{0B} + y_{0R}) - \varphi_0\}.
 \end{aligned}$$

and markets clear

$$\begin{aligned}
\int_0^1 (c_0^h + w_0^h) dh &= 1 \\
\int_0^1 y_{0B}^h dh &= \beta \\
\int_0^1 y_{0R}^h dh &= 1 - \beta \\
\int_0^1 \varphi_0^h dh &= 0 \\
\int_0^1 c_U^h dh &= 1 + \int_0^1 w_0^h dh \\
\int_0^1 c_D^h dh &= .2 + \int_0^1 w_0^h dh
\end{aligned}$$

A moment's thought will reveal that there will be an agent  $a$  indifferent between buying blue assets with leverage at a high price, and red assets without leverage at a low price. Similarly there will be an agent  $b < a$  who will be indifferent between buying red assets and selling all his assets. The optimistic agents with  $h > a$  will exclusively buy blue assets, the agents with  $b < h < a$  will exclusively hold red assets, and the agents with  $h < b$  will hold no assets and lend.

The equilibrium equations become

$$\begin{aligned}
p_R &= b1 + (1 - b)(.2) \\
p_R &= \frac{(a - b) + p_B \beta (a - b)}{(1 - \beta)(1 - (a - b))} \\
\frac{a(1 - .2)}{p_B - .2} &= \frac{a1 + (1 - a).2}{p_R} \\
p_B &= \frac{(1 - a) + p_R(1 - \beta)(1 - a) + .2\beta}{\beta a}
\end{aligned}$$

The first equation says that agent  $b$  is indifferent between buying red or not buying at all. The second equation says that the agents between  $a$  and  $b$  can just afford to buy all of the red Y that is being sold by the other  $(1 - (a - b))$  agents, noting that their expenditure consists of the one unit of the consumption good and the revenue they get from selling off their blue Y. The third equation says that  $a$  is indifferent between buying blue with leverage, and red without. The last equation says that the top  $1 - a$  agents can just afford to buy all the blue assets, by spending their endowment of the consumption good plus the revenue from selling their red Y plus the amount they can borrow using the blue Y as collateral.

In the table below we describe equilibrium for various values of  $\beta$



fraction blue	1	0.5	0.05
a	0.6861407	0.841775	0.983891
b	0.6861407	0.636558	0.600066
pred	0.7489125	0.709246	0.680053
pblue	0.7489125	0.74684	0.742279

Suppose we begin with the situation more or less prevailing six months ago, with  $\beta = 0\%$  and no leverage, just as in our very first example where we found the assets priced at .677. By setting  $\beta = 5\%$  and thereby leveraging the 5% new assets (that is turning them into blue assets) the government can raise their price from .677 to .74. Interestingly this also raises the price of the red assets which remain without leverage, from .677 to .680. Providing the same leverage for more assets, by extending  $\beta$  to .5 or 1 and thereby leveraging some of the legacy assets, raises the value of all the assets! Thus if one wanted to raise the price of just the 5% new assets, the government should leverage all the assets, new and legacy. By holding promises down to .2, there would be no defaults.

This analysis holds some lessons for the current discussion about TALF, the government program designed to inject leverage into the economy in 2009. The introduction of leverage for new assets did raise the price of new assets substantially. It also raised the price of old assets that were not leveraged (although part of that might be due to the expectation that the government lending facility will be extended to old assets as well). One might think that the best way to raise new asset prices is to give them scarcity value as the only leveraged assets in town. But on the contrary, the analysis shows that the price of the new assets could be boosted further by extending leverage to all the legacy assets, without increasing the amount of default.

The reason for this paradoxical conclusion is that optimistic buyers always have the option of buying the legacy assets at low prices. There must be substantial leverage in the new assets to coax them into buying if the new asset prices are much higher. By leveraging the legacy assets as well and thus raising the price of those assets, the government can undercut the returns from the alternative and increase demand for the new assets.

This analysis also has implications for spillovers from shocks across markets, a subject we return to later. The loss of leverage in one asset class can depress prices in another asset class whose leverage remains the same.

## 2.6 Complete Markets

Suppose there were complete markets. The distinctions between red and blue assets would be irrelevant. The equilibrium would simply be  $((p_U, p_D), (x_0^h, w_0^h, x_U^h, x_D^h))$  such that  $p_U + p_D = 1$  (so that the constant returns to scale storage earns zero profit,

assuming the price of  $c_0$  is 1) and

$$\int_0^1 (c_0^h + w_0^h) dh = 1$$

$$\int_0^1 c_U^h dh = 1 + \int_0^1 w_0^h dh$$

$$\int_0^1 c_D^h dh = .2 + \int_0^1 w_0^h dh$$

$$(x_0^h, w_0^h, x_U^h, x_D^h) \in B^h(p) = \{(x_0, w_0, x_U, x_D) :$$

$$x_0 + w_0 + p_U x_U + p_D x_D \leq 1 + p_U 1 + p_D (.2)\}$$

$$(x_0, w_0, x_U, x_D) \in B^h(p) \Rightarrow u^h(x_0, x_U, x_D) \leq u^h(x_0^h, x_U^h, x_D^h)$$

It is easy to calculate that complete markets equilibrium occurs where  $(p_U, p_D) = (.44, .56)$  and agents  $h > .44$  spend all of their wealth of 1.55 buying 3.5 units of consumption each in state U and nothing else, giving total demand of  $(1 - .44)3.5 = 2.0$  and the bottom .44 agents spend all their wealth buying 2.78 units of  $x_D$  each, giving total demand of  $.44(2.78) = 1.2$  in total.

The price of Y with complete markets is therefore  $p_U 1 + p_D (.2) = .55$ , much lower than the incomplete markets, leverage price of .75. Thus leverage can boost asset prices well above their "efficient" levels.

## 2.7 CDS and the Repo Market

The collateralized loan markets we have studied so far are similar to the Repo markets that have played an important role on Wall Street for decades. In these markets borrowers take their collateral to a dealer and use that to borrow money via non-contingent promises due one day later. The CDS is a much more recent contract.

The invention of the CDS or credit default swap moved the markets closer to complete. In our two state example with plenty of collateral, their introduction actually does lead to the complete markets solution, despite the need of collateral. In general, with more perishable goods, and goods in the future that are not tradable now, the introduction of CDS does not complete the markets.

A CDS is a promise to pay the principal default on a bond. Thus thinking of the asset as paying 1, or .2 if it defaults, the credit default swap would pay .8 in the down state, and nothing anywhere else. In other words, the CDS is tantamount to trading the down Arrow security.

The credit default swap needs to be collateralised. There are only two possible collaterals for it, the security, or the gold. A collateralisable contract promising an Arrow security is particularly simple, because it is obvious we need only consider versions in which the collateral exactly covers the promise. So choosing the normalizations in the most convenient way, there are essentially two CDS contracts to

consider, a CDS promising .2 in state D and nothing else, collateralized by the security, or a CDS promising 1 in state D, and nothing else, collateralized by a piece of the durable consumption good gold. So we must add these two contracts to the Repo contracts we considered earlier.

It is a simple matter to show that the complete markets equilibrium can be implemented via the two CDS contracts. The agents  $h > .44$  buy all the security Y and all the gold, and sell the maximal amount of CDS against all that collateral. Since all the goods are durable, this just works out.

### 2.7.1 The upshot of CDS

In this simple model, the CDS is the mirror image of the Repo. By purchasing an asset using the maximal leverage on the Repo market, the optimist is synthetically buying an up Arrow security (on net it pays a positive amount in the up state, and on net it pays nothing in the down state). The CDS is a down Arrow security. It is tantamount therefore, to letting the pessimists leverage. That is why the price of the asset goes down once the CDS is introduced.

Another interesting consequence is that the CDS kills the repo market. Buyers of the asset switch from selling repo contracts against the asset to selling CDS. It is true that since the introduction of CDS in late 2005 into the mortgage market, the repo contracts have steadily declined.

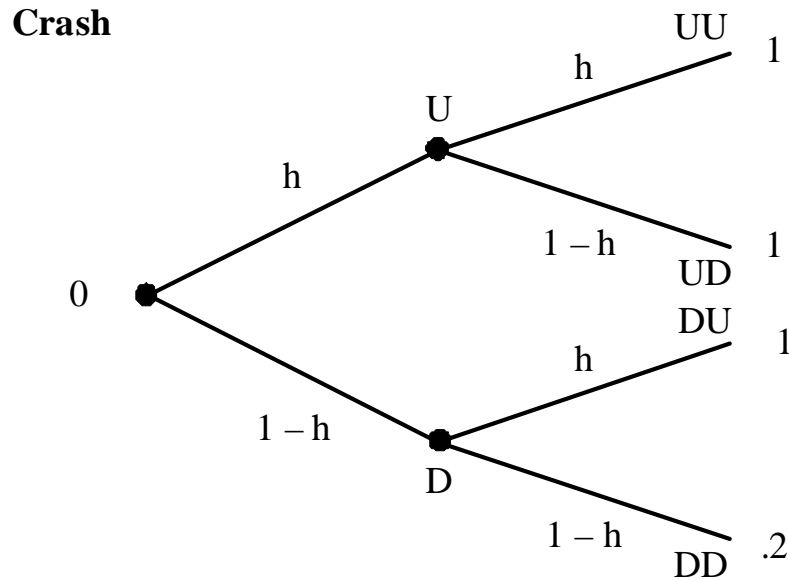
In the next section we ignore CDS and reexamine the repo contracts in a dynamic setting. Then we return to CDS.

## 3 The Leverage Cycle

If in the example bad news occurs and the value plummets to .2, there will be a crash. This is a crash in the fundamentals. There is nothing the government can do to avoid it. But the economy is far from the crash in period 0. It hasn't happened yet. The marginal buyer thinks the chances of a fundamentals crash are only 31%. The average buyer thinks the fundamentals crash will occur with just 15% probability.

The point of the leverage cycle is that excess leverage followed by excessive deleveraging will cause a crash even before there has been a crash in the fundamentals, and even if there is no subsequent crash in the fundamentals. When the price crashes everybody will say it has fallen more than their view of the fundamentals warranted. The asset price is excessively high in the initial or over-leveraged normal economy, and after deleveraging, the price is even lower than it would have been at those tough margin levels had there never been the over-leveraging in the first place.

So consider the same example but with three periods instead of two. Suppose, as before, that each agent begins in state  $s = 0$  with one unit of money and one unit of the asset, and that both are perfectly durable. But now suppose the asset Y pays off after two periods instead of one period. After good news in either period, the asset



pays 1 at the end. Only with two pieces of bad news does the asset pay .2. The state space is now  $S = \{0, U, D, UU, UD, DU, DD\}$ . We use the notation  $s^*$  to denote the immediate predecessor of  $s$ . Denote by  $\gamma_s^h$  the probability  $h$  assigns to nature moving from  $s^*$  to  $s$ . For simplicity we assume that every investor regards the U vs D move from period 0 to period 1 as independent and identically distributed to the U vs D move of nature from period 1 to period 2, and more particularly  $\gamma_U^h = \gamma_{DU}^h = h$ .

Diagram 3 here

This is the situation described in the introduction, in which two things must go wrong (i.e. two down moves) before there is a crash in fundamentals. Investors differ in their probability beliefs over the odds that either bad event happens. The move of nature from 0 to D lowers the expected payoff of the asset Y in every agent's eyes, and also increases every agent's view of the variance of the payoff of asset Y. The news creates more uncertainty, and more disagreement.

Suppose agents again have no impatience, but care only about their expected consumption of dollars. Formally, letting  $c_s$  be consumption in state  $s$ , and letting  $e_s^h$  be the initial endowment of the consumption good in state  $s$ , and letting  $y_{0^*}^h$  be the

initial endowment of the asset Y before time begins, we have for all  $h \in [0, 1]$

$$\begin{aligned}
& u^h(c_0, c_U, c_D, c_{UU}, c_{UD}, c_{DU}, c_{DD}) \\
&= c_0 + \gamma_U^h c_U + \gamma_D^h c_D + \gamma_U^h \gamma_{UU}^h c_{UU} + \gamma_U^h \gamma_{UD}^h c_{UD} + \gamma_D^h \gamma_{DU}^h c_{DU} + \gamma_D^h \gamma_{DD}^h c_{DD} \\
&= c_0 + h c_U + (1-h) c_D + h^2 c_{UU} + h(1-h) c_{UD} + (1-h) h c_{DU} + (1-h)^2 c_{DD} \\
& (e_0^h, y_0^{h*}, e_U^h, e_D^h, e_{UU}^h, e_{UD}^h, e_{DU}^h, e_{DD}^h) \\
&= (1, 1, 0, 0, 0, 0, 0, 0)
\end{aligned}$$

We define the dividend of the asset by  $d_{UU} = d_{UD} = d_{DU} = 1$ , and  $d_{DD} = .2$ , and  $d_0 = d_U = d_D = 0$ .

The agents are now more optimistic than before, since agent  $h$  assigns only a probability of  $(1-h)^2$  to reaching the only state, DD, where the asset pays off .2. The marginal buyer from before,  $b = .69$ , for example, thinks the chances of DD are only  $(.31)^2 = .09$ . Agent  $h = .87$  thinks the chances of DD are only  $(.13)^2 = 1.69\%$ . But more importantly, if buyers can borrow short term, their loan at 0 will come due before the catastrophe can happen. It is thus much safer than a loan at D.

Assume that repo loans are one-period loans, so that loan  $sj$  promises  $j$  in states  $sU$  and  $sD$ , and requires one unit of Y as collateral. The budget set can now be written iteratively, for each state  $s$ .

$$\begin{aligned}
B^h(p, \pi) &= \{(c_s, y_s, (\varphi_{sj})_{j \in J}, w_s)_{s \in S} \in (\mathbb{R}_+^2 \times \mathbb{R}^J \times \mathbb{R}_+)^{1+S} : \forall s \\
(c_s + w_s - e_s^h) + p_s(y_s - y_{s^*}) &= y_{s^*} d_s + \sum_{j=1}^J \varphi_{sj} \pi_{sj} - \sum_{j=1}^J \varphi_{s^*j} \min(p_s + d_s, j) \\
\sum_{j=1}^J \max(\varphi_{sj}, 0) &\leq y_s\}
\end{aligned}$$

In each state  $s$  the price of consumption is normalized to 1, and the price of the asset is  $p_s$  and the price of loan  $sj$  promising  $j$  in states  $sU$  and  $sD$  is  $\pi_{sj}$ . Agent  $h$  spends if he consumes and stores more than his endowment or if he increases his holdings of the asset. His income is his dividends from last period's holdings (by convention dividends in state  $s$  go to the asset owner in  $s^*$ ) plus his sales revenue from selling promises, less the payments he must make on previous loans he took out. Collateral is as always no recourse, so he can walk away from a loan payment if he is willing to give up his collateral instead. The agents who borrow (taking  $\phi_{sj} > 0$ ) must hold the required collateral.

The crucial question again is how much leverage will the market allow at each state  $s$ ? By the logic we described in the previous section, it can be shown that in every state  $s$ , the only promise that will be actively traded is the one that makes the maximal promise on which there will be no default. Since there will be no default on this contract, it trades at the riskless rate of interest  $r_s$  per dollar promised. Using this insight we can drastically simplify our notation (as in Fostel-Geanakoplos 2008)

by redefining  $\varphi_s$  as the amount of the consumption good promised at state  $s$  for delivery in the next period, in states  $sU$  and  $sD$ . The budget set then becomes

$$B^h(p, r) = \{(c_s, y_s, \varphi_s, w_s)_{s \in S} \in \mathbb{R}_+^{4(1+S)} : \forall s$$

$$(c_s + w_s - e_s^h) + p_s(y_s - y_{s^*}) = y_{s^*}d_s + \sum_{j=1}^J \varphi_s \frac{1}{1+r_s} - \varphi_{s^*}$$

$$\varphi_s \leq y_s \min(p_{sU} + d_{sU}, p_{sD} + d_{sD})\}$$

Equilibrium occurs at prices  $(p, r)$  such that when everyone optimizes in his budget set by choosing  $(c_s^h, y_s^h, \varphi_s^h, w_s^h)_{s \in S}$  the markets clear in each state  $s$

$$\int_0^1 (c_s^h + w_s^h) dh = \int_0^1 e_s^h dh + d_s \int_0^1 y_{s^*}^h dh$$

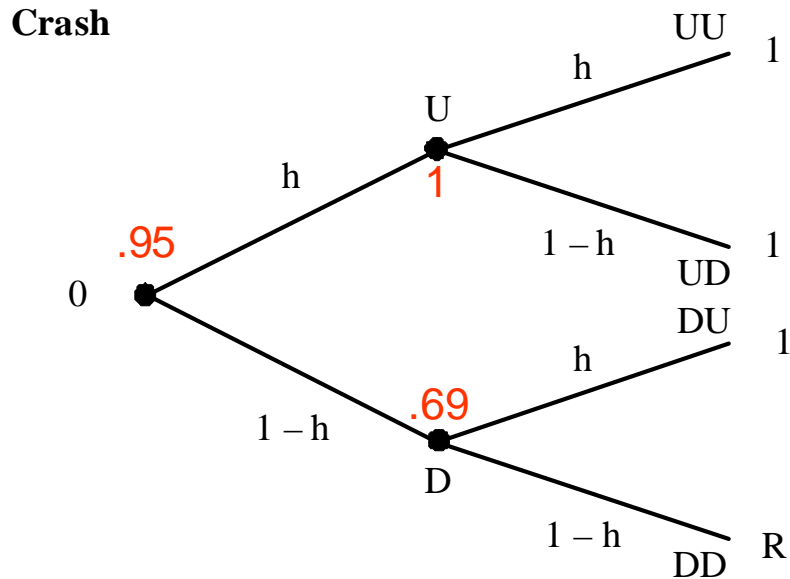
$$\int_0^1 y_s^h dh = \int_0^1 y_{s^*}^h dh$$

$$\int_0^1 \varphi_s^h dh = 0$$

It will turn out in equilibrium that the interest rate is zero in every state. Thus at time 0, agents can borrow the minimum of the price of  $Y$  at  $U$  and at  $D$ , for every unit of  $Y$  they hold at 0. At  $U$  agents can borrow 1 unit of the consumption good, for every unit of  $Y$  they hold at  $U$ . At  $D$  they can borrow only .2 units of the consumption good, for every unit of  $Y$  they hold at  $D$ . In normal times, at 0, there is not very much bad that can happen in the short run. Lenders are therefore willing to lend much more on the same collateral, and leverage can be quite high. Solving the example gives the following prices. See Diagram 4.

The price of  $Y$  at time 0 of .95 occurs because the marginal buyer is  $h = .87$ . Assuming the price of  $Y$  is .69 at  $D$  and 1 at  $U$ , the most that can be promised at 0 using  $Y$  as collateral is .69. With an interest rate  $r_0 = 0$ , that means .69 can be borrowed at 0 using  $Y$  as collateral. Hence the top 13% of buyers at time 0 can collectively borrow .69 (since they will own all the assets), and by adding their own .13 of money they can spend .82 on buying the .87 units that are sold by the bottom 87%. The price is  $.95 \approx .82/.87$ .

Why is there a crash from 0 to  $D$ ? Well first there is bad news. But the bad news is not nearly as bad as the fall in prices. The marginal buyer of the asset at time 0,  $h = .87$ , thinks there is only a  $(.13)^2 = 1.69\%$  chance of ultimate default, and when he gets to  $D$  after the first piece of bad news he thinks there is a 13% chance for ultimate default. The news for him is bad, accounting for a drop in price of about 11 points, but it does not explain a fall in price from .95 to .69 of 26 points. In fact, no agent  $h$  thinks the loss in value is nearly as much as 26 points. The biggest optimist  $h = 1$  thinks the value is 1 at 0 and still 1 at  $D$ . The biggest pessimist  $h = 0$  thinks the value is .2 at 0 and still .2 at  $D$ . The biggest loss attributable to the bad news of



arriving at D is felt by  $h=.5$ , who thought the value was .8 at 0 and thinks it is .6 at D. But that drop of .2 is still less than the drop of 26 points in equilibrium.

The second factor is that the leveraged buyers at time 0 all go bankrupt at D. They spent all their cash plus all they could borrow at time 0, and at time D their collateral is confiscated and used to pay off their debts: they owe .69 and their collateral is worth .69. Without the most optimistic buyers, the price is naturally lower.

Finally, and most importantly, the margins jump from  $(.95 - .69)/.95 = 27\%$  at U to  $(.69 - .2)/.69 = 71\%$  at D. In other words, leverage plummets from  $3.6 = .95/ (.95 - .69)$  to  $1.4 = .69/ (.69 - .2)$ .

All three of these factors working together explain the fall in price.

To see how to find this equilibrium, let  $b$  be the marginal buyer in state  $D$  and

let  $a$  be the marginal buyer in state 0. Then we must have

$$\begin{aligned}
p_D &= \gamma_{DU}^b 1 + \gamma_{DD}^b (.2) \equiv b1 + (1 - b)(.2) \\
p_D &= \frac{(1/a)(a - b) + .2}{(1/a)b} = \frac{1.2a - b}{b} \\
a &= \frac{b(1 + p_D)}{1.2} \\
p_0 &= \frac{(1 - a) + p_D}{a} \\
\frac{a(1 - p_D)}{p_0 - p_D} &\equiv \frac{\gamma_U^a(1 - p_D)}{p_0 - p_D} = \gamma_U^a 1 + \gamma_D^a \frac{\gamma_{DU}^a}{\gamma_{DU}^b} \equiv a1 + (1 - a)\frac{a}{b} \\
\frac{a(1 - p_D)}{p_0 - p_D} &\equiv \frac{\gamma_U^a(1 - p_D)}{p_0 - p_D} = \frac{\gamma_U^a 1 + \gamma_D^a p_D \frac{\gamma_{DU}^a}{\gamma_{DU}^b}}{p_0} \equiv \frac{a1 + (1 - a)p_D \frac{a}{b}}{p_0}
\end{aligned}$$

The first equation says that the price at D is equal to the valuation of the marginal buyer  $b$  at D. Because he is also indifferent to borrowing, he will then also be indifferent to buying on the margin, as we saw in the collateral section. The second equation says that the price at D is equal to the ratio of all the money spent on Y at D, divided by the units sold at D. The top  $a$  investors are all out of business at D, so they cannot buy anything. They have spent all their money, and sold all their assets in order to pay off their loans at D. Thus the remaining  $1 - a$  agents must hold all the consumption goods and Y between them, in equal amounts (since they all lent the same amount at 0). Hence at D the remaining investors in the interval  $[0, a)$  each own  $1/a$  units of Y and have inventoried or collected  $1/a$  dollars. At D the new optimistic buyers in the interval  $[b, a)$  spend all they have, which is  $(1/a)(a - b)$  dollars plus the  $.2(1)$  they can borrow on the entire stock of Y. The amount of Y sold at D is  $(1/a)b$ . This explains the second equation. The third equation just rearranges the terms in the second equation.

The fourth equation is similar to the second. It explains the price of Y at 0 by the amount spent divided by the amount sold. Notice that at 0 it is possible to borrow  $p_D$  using each unit of Y as collateral. So the top  $(1 - a)$  agents have  $(1 - a) + p_D$  to spend on the  $a$  units of Y for sale at 0.

The fifth equation equates the marginal utility at 0 to  $a$  of one dollar, on the right, with the marginal utility of putting one dollar of cash down on a leveraged purchase of Y, on the left. The marginal utility of leveraging a dollar by buying Y on margin at time 0 can easily be seen. With  $p_0 - p_D$  dollars as downpayment, one gets a payoff of  $(1 - p_D)$  dollars in state U, to which  $a$  assigns probability  $\gamma_U^a \equiv a$ , and nets nothing at D, explaining the left hand side of the equation.

To see where the right hand side of equation 5 comes from, observe first that agent  $a$  can do better by inventorying the dollar (i.e. warehousing the consumption good by taking  $w_0 > 0$ ) at time 0 rather than consuming it. With probability  $\gamma_U^a \equiv a$ , U will be reached and this dollar will be worth one utile. With probability  $\gamma_D^a \equiv 1 - a$ ,



D will be reached and  $a$  will want to leverage the dollar into as big a purchase of Y as possible. This will result in a gain at D of

$$\frac{a(1 - .2)}{p_D - .2} = \frac{a(1 - .2)}{b1 + (1 - b)(.2) - .2} = \frac{a}{b}$$

Hence the marginal utility of a dollar at time 0 is  $a1 + (1 - a)\frac{a}{b}$ , explaining the right hand side of equation 5.

The last equation says that  $a$  is indifferent to buying Y on margin at 0 or buying it for cash. The right hand side shows that by spending  $p_0$  dollars to buy Y at 0, agent  $a$  can get a payoff of 1 with probability  $a$ , and with probability  $(1 - a)$  a payoff of  $P_D$  dollars at D, which is worth  $p_D\frac{a}{b}$  to  $a$ . The last equation is a tautological consequence of the previous equation. To see this, note that by rewriting the second to last equation and using the identity  $\frac{\alpha}{\beta} = \frac{\gamma}{\delta}$  implies  $\frac{\alpha}{\beta} = \frac{\alpha+\gamma}{\beta+\delta}$  we get

$$\begin{aligned} \frac{a(1 - p_D)}{p_0 - p_D} &= \frac{p_D[a1 + (1 - a)\frac{a}{b}]}{p_D} \\ &= \frac{a(1 - p_D) + p_D[a1 + (1 - a)\frac{a}{b}]}{p_0 - p_D + p_D} \\ &= \frac{a1 + (1 - a)p_D\frac{a}{b}}{p_0} \end{aligned}$$

which is the last equation.

By guessing a value of  $b$ , and then iterating through all the equations, one ends up with all the variables specified, and a new value of  $b$ . By searching for a fixed point in  $b$ , one quickly comes to the solution just described, with the crash from .95 to .69.

In recent times there has been bad news, but according to most modelers the price of assets today is much lower than would be warranted by the news. There have been numerous bankruptcies of mortgage companies, and even of great investment banks. And the drop in leverage has been enormous.

These kind of events had occurred before in 1994 and 1998. The cycle is more severe this time because the leverage was higher, and the bad news was worse.

In my 2003 paper I had considered exactly the same example, but with the more optimistic beliefs  $\gamma_{sD}^h \equiv (1 - h)^2 < (1 - h)$  for all  $s = 0, U, D$ .

### 3.0.2 Quantifying the contributions of bad news, deleveraging, and bankruptcy of the optimists

Of the three symptoms of the leverage cycle collapse, which is playing the biggest role in our example? This is an easy calculation to make, because we can introduce each of the three effects on its own into the model and then see how much the price .95 declines.

The bad news has the effect of increasing the probability each agent  $h$  assigns to the low payoff of .2 at DD from  $(1 - h)^2$  to  $(1 - h)$ . So we can recalculate equilibrium in the same tree, but with  $\gamma_{sD}^h \equiv \sqrt{(1 - h)} > (1 - h)$  for all  $s = 0, U, D$ . The result is that at node 0 the price is now .79. Thus roughly 60% of the drop in value from .95 to .69 comes from the bad news itself.

But that still leaves 40% of the drop explainable only by non-fundamentals (or technicals as they are sometimes called). We can decompose this 40% into the part that comes from the bankruptcy and disappearance of the most optimistic buyers, and the rest due to the deleveraging.

In the main example, the most optimistic 13% went bankrupt at D. We can isolate this effect simply by beginning with an economy without these agents. Replacing the set of traders  $[0, 1]$  with  $[0, .87]$ , and therefore the value 1 with .87 in the appropriate equations 2,3,4, one can repeat the calculation and find that the price at the original node is .89, a drop of 6 points from the original .95, and roughly 20% of the original 26 point drop in the example from 0 to D.

In the main example the deleveraging occurred at D when the maximal promise was reduced to .2. We can simulate the deleveraging effect alone by reducing our tree to the old one-period model, but replacing the probability of down of  $1 - h$  with  $(1 - h)^2$ . In that new model the equilibrium promise at node 0 will be just .2, but investors will still assign the .2 payoff probability  $(1 - h)^2$ . This gives an initial price for the asset of .89. Thus deleveraging also explains about 20% of the price crash.

The roughly linear decomposition of the three factors is due to the linearity of the beliefs  $\gamma_{sU}^h = h, \gamma_{sD}^h = 1 - h$  in  $h$ . In my 2003 paper I wanted to avoid this linearity, and also to illustrate a smaller crash consistent with the minor leverage cycle crash of 1998. I assumed  $\gamma_{sU}^h = 1 - (1 - h)^2, \gamma_{sD}^h = (1 - h)^2$ , giving probability  $(1 - h)^4$  of reaching DD from 0. In that specification there are many investors with  $\gamma_{sU}^h$  near to 1, but once  $h$  moves far from 1, the decline in optimism happens faster and faster. In that model the price at 0 is  $p_{0Y} = .99$  and the price falls only 12 points to  $p_{0D} = .87$  at D. Only the top 6% of investors buy at 0, since they can leverage so much, and thus go bankrupt at D. Without them from the beginning, the price would still be .99, hence the loss of the top tier itself contributes very little. Bad news alone in that model reduces to the example I have been considering here, which has a starting price of  $p_{0Y} = .95$ .<sup>5</sup> Deleveraging alone in that example results in a starting price of  $p_{0Y} = .98$ . Hence the three factors independently add up to much less than the total drop. In that example it was the feedback between the three causes that explained much of the drop. In the example in this paper, the total drop is very close to the sum of the parts.

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<sup>5</sup>There was an error in my 2003 paper where I reported that the starting price would be .99 in the example of this paper (instead of .95).

### 3.0.3 Conservative optimists

It is very important, and very characteristic of the leverage cycle, that after the crash, returns are much higher than usual. Survivors of the crash always have great opportunities. One might well wonder why investors in the example do not foresee that there might a crash, and keep their powder dry in cash (or in assets but without leverage) at 0, waiting to make a killing if the economy goes to D. The answer is that many of them do exactly that.

The marginal buyer at 0 is  $h = .87$ . He assigns probability  $1.69\% = (.13)^2$  to reaching DD. So he values the asset at 0 at more than .98, yet he is not rushing to buy at the price of .95. The reason is that he is precisely looking toward the future. These calculations are embodied in equation 5 of this section. The marginal utility to  $a$  of reaching the down state with a dollar of dry powder is not  $(1 - a)$ , but  $(1 - a)(a/b)$  precisely because  $a$  anticipates that he will have a spectacular gross expected return of  $a/b$  at D.

In fact all the investors between .87 and .74 are refraining from buying what they regard as an underpriced asset at 0, in order to keep their powder dry for the killing at D. If there were only more of them, of course, there would be no crash at D. But as their numbers rise, so does the price at D, and so their temptation to wait ebbs. It is after all a rare bird who thinks the returns at D are so great, yet thinks D is sufficiently likely to be worth waiting for. This is owing to my assumption that investors who think the first piece of bad news is relatively unlikely (high  $h$ ) also think the second piece of bad news is relatively unlikely (high  $h$  again) even after they see the first piece of bad news. This assumption corresponds to my experience that hedge fund managers generally are the ones saying things are not that bad, even after they start going bad.

### 3.0.4 Endogenous Maturity Mismatch

Many authors have lamented the dangers of short term borrowing on long term assets, as we have in this example. It is important to observe that the short term loans I described in the three period model arise endogenously. If long, two period non-contingent loans were also available, then by the previous arguments, since there are only two outcomes even in the final period, the only potentially traded long term loan would promise .2 in every state. But the borrowers would much prefer to borrow .69 on the short term loan. So the long term loans would not be traded.

This preference for short term loans is an important feature of real markets. Lenders know that much less can go wrong in a day than in a year, and so they are willing to lend much more for a day on the same collateral than they would for a year. Eager borrowers choose the larger quantity of short term loans, and presto, we have an endogenous maturity mismatch. Endogenous collateral can resolve the puzzle of what causes maturity mismatch.

### 3.0.5 CDS

In my view an important trigger for the collapse of 2007-2009 was the introduction of CDS contracts into the mortgage market in late 2005, at the height of the market. CDS on corporate bonds had been traded for years, but until 2005 there had been no standardized mortgage CDS contract. I do not know the impetus for this standardization; perhaps more people wanted to short the market once it got so high. But the implication was that afterward the pessimists, as well as the optimists, had an opportunity to leverage. This was bound to depress mortgage security prices. This in turn made it impossible for homeowners to refinance their mortgages, forcing many to default, which then began to depress home prices, which then made it even harder to sell new mortgages and so on. I believe the introduction of CDS trading on a grand scale in mortgages is a critical, overlooked factor in the crisis. Until now people have assumed it all began when home prices started to fall in 2006. But why home prices should begin to fall then has remained a mystery.

To see the effect of introducing a CDS market midstream, suppose in our model that everyone anticipated correctly that the CDS market would get introduced in the middle period. Computing equilibrium with repo markets at time 0 and complete markets from time 1 onwards, we get not just a 26 point drop, but a bigger crash of  $p_{oY} = .85$  to  $p_{DY} = .51$ . The drop becomes an astonishing 34 points, or 40%. If the introduction of the CDS market occurred in the middle period, but was unanticipated, the crash would be even worse. The sudden introduction of CDS in 2005 probably played a bigger role than people realize.

Of course if CDS were introduced from the beginning, prices would never have gotten so high. But they were only introduced after the market was at its peak.

### 3.0.6 Complete Markets

The introduction of CDS from the beginning moves the markets close to complete. It is easy to compute the complete markets equilibrium. Nobody would consume until the final period, when all the information had been revealed. So we need only find four prices of consumption at UU,UD,DU, and DD. The supplies of goods are respectively 2,2,2,1.2, and the most optimistic people will exclusively consume good UU, the next most optimistic will exclusively consume UD and so on. The prices turn out to be  $p_{UU} = .29$ ,  $p_{UD} = .16$ ,  $p_{DU} = .16$ ,  $p_{DD} = .39$ . This gives a drop of Y from  $p_{oY} = .68$  to  $p_{DY} = .43$ .

The complete markets prices are systematically lower than the collateral equilibrium, because effectively complete markets amounts to adding the CDS, which means the pessimists can leverage as well.

With complete markets there is high volatility as well. Indeed the drop in prices from 0 to D is as big as before. With complete markets, the optimists bet on U, selling their wealth at D. The price at D therefore reflects the opinions of more pessimistic people than at U or 0, and thus we get a big drop in prices at D even with complete

markets.

The phenomenon of bigger price drops than anybody thinks is justified is thus consistent with complete markets as well. But I feel it is more likely with incomplete markets. For example, suppose that we change the beliefs of the agents so that agent  $h$  thinks the probability of up is never less than .6, that is suppose

$$\gamma_{sU}^h = \max(h, .6) \text{ for all } h \text{ and } s = 0, D, U$$

The collateral equilibrium described in the leverage cycle, in which the price of Y dropped from  $p_{0Y} = .95$  to  $p_{DY} = .69$ , is absolutely unchanged, since the lowest marginal buyer was  $b = .61$  at D. The opinions of  $h < .61$  never mattered. On the other hand, the complete markets prices are now  $p_{0Y} = .87 = (1 - (1 - .6)^2)1 + (1 - .6)^2(.2)$  and  $p_{DY} = .68 = .6(1) + .4(.2)$ . With complete markets the price drop is only 2/3 the size of the collateral equilibrium price drop, and it is completely explained by the bad news as seen by every agent with  $h \leq .6$ .

### 3.1 Upshot of the Leverage Cycle so far

The wild gyrations in asset prices as equilibrium leverage ebbs and flows is alarming in and of itself. But behind the volatility there are five more serious problems.

First, very high leverage means that the asset prices are set by a small group of investors. If agent beliefs are heterogeneous, why should the prices be determined entirely by the highest outliers? In the model above just the top 13% determined the price of the asset at date 0. In my 2003 paper, it was just the top 6% who determined the price. So few people should not have so much power to determine crucial prices. Leverage allows the few to wield great influence.

Second, if we add production to the economy, especially of the irreversible kind, then we would find a huge wave of overbuilding. The asset price at 0 is well above the complete markets price, because of the expectation by the leveraged few, that good times were coming. In the bad state that overbuilding would need to be dismantled at great cost.

Third, asset prices can have a profound effect on economic activity. As Tobin argued with his concept of Q, when the prices of old assets are high, new productive activity, which often involves issuing financial assets that are close substitutes for the old assets, is stimulated. When asset prices are low, new activity might grind to a halt. If we added another group of small business people to the model who did not participate in financial markets generally, we would find that they could easily sell loans at time 0, but would have a hard time at D. Government policy may well have the goal of protecting these people by smoothing out the leverage cycle.

Fourth, the large fluctuations in asset prices over the leverage cycle lead to massive redistributions of wealth and changes in inequality. At the beginning everybody has an equal share of wealth. In the ebullient stage, the optimists become 30% richer

than the pessimists, while in the intermediate down state, the optimists go broke. Inequality becomes extreme in both states.

This brings us to the fifth potential cost of too much leverage. Instead of regarding the optimists as crazy, let us think of them as indispensable to the economy. That is probably what is meant by the modern term "too big to fail". Geanakoplos-Kubler (2005) shows that if their marginal contribution to society is bigger than what they are paid, then their bankruptcy results in an externality, since they internalize only their private loss in calculating how much leverage to take on. If in addition the bankruptcy of one optimist makes it more likely in the short run that other optimists will go bankrupt, then the externality can become so big that simply curtailing leverage can make everybody better off.

It can easily be checked that if a regulation were passed that limited promises at date 0 to say .4 (instead of .69), then prices at time 0 would fall from .95 to .91, and prices at D would rise from .69 to .70.

In the next section we drop CDS and return to repo markets, but we analyze the more conventional case of common priors and diminishing marginal utility. We find that if some agents get more utility out of holding the collateral than others, then the endogenous equilibrium leverage may well involve default. Default can give rise to further inefficiencies, giving us more reasons to monitor and regulate leverage.

## 4 Heterogeneity based on Utility for Collateral: Endogenous Default

So far we have assumed a continuum of risk neutral agents with identical time discounting, who differed in their taste for the collateral on account of their different priors about the collateral payoffs. In this section we introduce an alternative difference, namely that some agents simply enjoy a higher utility from holding the collateral, such as when they use their houses as collateral. We assume common priors. We find once again that there is a unique leverage chosen in equilibrium. But this time the leverage is not the maximal amount short of default. On the contrary, we now find that the market will select a promise in which there is a great deal of default in the bad state. Worse still, even if both the borrowers and lenders realize there is a substantial foreclosure cost (to seizing the collateral in case of default), the free market will still choose promises which allow for a great deal of default.

In this model, if we introduce CDS to complete the markets, it turns out that nobody will trade them. Thus the markets are endogenously incomplete: even if every contract can be written, the market will only choose a few because of the need for collateral.

Because we have only two types of agents in this model, equilibrium will not necessarily entail a marginal agent indifferent to buying the asset. We shall find that every buyer of the collateral strictly prefers to buy on margin (i.e. leveraged) to

buying outright. In fact, no agent is willing to buy the collateral with 100% cash down.

## 4.1 Example: Borrowing Across Time

We consider an example taken from Geanakoplos (1997) with two kinds of agents  $H = \{A, B\}$ , two time periods, and two goods  $F$  (food) and  $H$  (housing) in each period. For now we shall suppose that there is only one state of nature in the last period.

We suppose that food is completely perishable, while housing is perfectly durable.

We suppose that agent  $B$  likes living in a house much more than agent  $A$ ,

$$u^A(x_{0F}, x_{0H}, x_{1F}, x_{1H}) = x_{0F} + x_{0H} + x_{1F} + x_{1H} ,$$

$$u^B(x_{0F}, x_{0H}, x_{1F}, x_{1H}) = 9x_{0F} - 2x_{0F}^2 + 15x_{0H} + x_{1F} + 15x_{1H} .$$

Furthermore, we suppose that the endowments are such that agent  $B$  is very poor in the early period, but wealthy later, while agent  $A$  owns the housing stock

$$e^A = (e_{0F}^A, e_{0H}^A, e_{1F}^A, e_{1H}^A) = (20, 1, 20, 0) ,$$

$$e^B = (e_{0F}^B, e_{0H}^B, e_{1F}^B, e_{1H}^B) = (4, 0, 50, 0) .$$

We suppose that there are contracts  $(A_j, C_j)$  with  $A_j = \binom{j}{0}$ , promising  $j$  units of food in period 1, and no housing, each collateralized by one house  $C_j = (0, 1)$  as before. We introduce a new piece of notation  $D_{1j}$  to denote the value of actual deliveries of asset  $j$  at time 1. Given our no recourse collateral, we know  $D_{1j} = \min(jp_{1F}, p_{1H})$ .

### 4.1.1 Arrow–Debreu Equilibrium

If in addition we had a complete set of Arrow securities with infinite default penalties and no collateral requirements, then it is easy to see that there would be a unique equilibrium (in prices and utility payoffs):

$$p = (p_{0F}, p_{0H}, p_{1F}, p_{1H}) = (1, 30, 1, 15) ,$$

$$x^A = (x_{0F}^A, x_{0H}^A, x_{1F}^A, x_{1H}^A) = (22, 0, 48, 0) ,$$

$$x^B = (x_{0F}^B, x_{0H}^B, x_{1F}^B, x_{1H}^B) = (2, 1, 22, 1) ,$$

$$u^A = 70 ; \quad u^B = 62 .$$

Assuming that  $A$  consumes food in both periods, the price of food would need to be the same in both periods, since  $A$ 's marginal utility for food is the same in both periods. We might as well take those prices to be 1. Assuming that  $B$  consumes food in the last period, the price of every good that  $B$  consumes must then be equal to  $B$ 's marginal utility for that good. With complete markets, the  $B$  agents would be able to borrow as much as they wanted, and they would then have the resources to bid the price of housing up to 30 in period 0 and 15 in period 1.

### 4.1.2 No Collateral– No contracts Equilibrium

Without the sophisticated financial arrangements involved with collateral or default penalties, there would be nothing to induce agents to keep their promises. Recognizing this, the market would set a price  $\pi_j = 0$  for the assets. Agents would therefore not be able to borrow any money. Thus agents of type  $B$ , despite their great desire to live in housing, and great wealth in period 1, would not be able to purchase much housing in the initial period. Again it is easy to calculate the unique equilibrium:

$$\begin{aligned}\pi_j &= 0 \\ p &= (p_{0F}, p_{0H}, p_{1F}, p_{1H}) = (1, 16, 1, 15) , \\ x^A &= (x_{0F}^A, x_{0H}^A, x_{1F}^A, x_{1H}^A) = \left( 20 + \frac{71}{32}, 1 - \frac{71}{32 \cdot 16}, 35 - \frac{71 \cdot 15}{32 \cdot 16}, 0 \right) , \\ x^B &= (x_{0F}^B, x_{0H}^B, x_{1F}^B, x_{1H}^B) = \left( \frac{57}{32}, \frac{71}{32 \cdot 16}, 35 + \frac{71 \cdot 15}{32 \cdot 16}, 1 \right) , \\ u^A &= 56 ; \quad u^B \approx 64 .\end{aligned}$$

In the final period 1, agents of type  $B$  are rich and they will bid the house price up to their marginal utility of 15. Agent  $A$ , realizing that he can sell the house for 15 in period 1, is effectively paying only  $16 - 15 = 1$  to have a house in period 0, and is therefore indifferent to how much housing he consumes in period 0. Agents of type  $B$ , on the other hand, spend their available wealth at time 0 on housing until their marginal utility of consumption of  $x_{0F}$  rises to  $\frac{30}{16}$ , which is the marginal utility of owning an extra dollar's worth of housing stock at time 0. That occurs when  $9 - 4x_{0F}^B = \frac{30}{16}$ , that is, when  $x_{0F}^B = \frac{57}{32}$ .

### 4.1.3 Collateral Equilibrium

We now introduce the possibility of collateral, i.e. we suppose the state apparatus is such that the house is confiscated if payments are not made. The unique equilibrium is then:

$$\begin{aligned}D_j &= \min(j, 15), \pi_j = \min(j, 15) , \\ p &= (p_{0F}, p_{0H}, p_{1F}, p_{1H}) = (1, 18, 1, 15) , \\ x^A &= (x_{0F}^A, x_{0H}^A, x_{1F}^A, x_{1H}^A) = (23, 0, 35, 0) , \\ \varphi_{15}^A &= -15 ; \quad \varphi_j^A = 0 \text{ for } j \neq 15 , \\ x^B &= (x_{0F}^B, x_{0H}^B, x_{1F}^B, x_{1H}^B) = (1, 1, 35, 1) , \\ \varphi_{15}^B &= 15 ; \quad \varphi_j^B = 0 \text{ for } j \neq 15 ; \\ u^A &= 58 ; \quad u^B = 72 .\end{aligned}$$

The only contract traded is the one  $j = 15$  that maximizes the promise that will not be broken. Its price  $\pi_{15} = 15$  is given by its marginal utility to its buyer  $A$ . Agent



$B$  sells the contract, thereby borrowing 15 units of  $x_{0F}$ , and uses the 15 units of  $x_{0F}$  plus 3 he owns himself to buy 1 unit of the house  $x_{0H}$ , at a price of  $p_{0H} = 18$ . He uses the house as collateral on the loan, paying off in full the 15 units of  $x_{1F}$  in period 1. Since, as borrower, agent  $B$  gets to consume the housing services while the house is being used as collateral, he gets final utility of 72. Agent  $A$  sells all his housing stock, since the best he can do after buying it is to live in it for one year, and then sell it at a price of 15 the next year, giving him marginal utility of 16, less than the price of 18 (expressed in terms of good  $F$ ).

The most interesting aspect of the collateral equilibrium is the first order condition for the buyer of collateral. The purpose of collateral is to enable people like  $B$ , who desperately want housing but cannot afford much (for example in the contract less economy), to buy the housing and live in it by borrowing against the future, using the house as collateral. To the extent that collateral is not a perfect device for borrowing, one might expect that  $B$  does not quite get all the housing he needs, and that the marginal utility of housing might end up greater to  $B$  than the marginal utility of food. In fact, the opposite is true.

In collateral equilibrium, the marginal utility of a dollar of housing is substantially less than the marginal utility of a dollar of food

$$\frac{MU_{x_{0H}}^B}{p_{0H}} = \frac{30}{18} < \frac{5}{1} = \frac{MU_{x_{0F}}^B}{p_{0F}}$$

So why does  $B$  buy housing at all? Because he can buy on margin, i.e. with leverage. He needs to pay only  $3 = 18 - 15$  of cash down for the house, getting 15 utiles in period 0, and then he can give the house up in period 1 to repay his loan. This leveraged purchase brings 5 utiles per dollar. This is exactly equal to the marginal utility of food per dollar.

This is a completely general phenomenon. The leveraged purchase brings more marginal utility than the straight cash purchase to any buyer with diminishing marginal utility. We now discuss why.

#### 4.1.4 Liquidity Wedge and Collateral Value

To the extent that collateral is not perfect in solving the borrowing problem, borrowers will be constrained from borrowing as much as they would like. The upshot is that the marginal utility today of the price of the contracts the borrowers are selling is much higher than the marginal utility to them of the deliveries they have to make: that is what it means for them to be constrained in their selling of loans, i.e., constrained in their borrowing. In Fostel-Geanakoplos 2008 we called this the liquidity wedge.

In the above example, contract  $j = 15$  sells for a price of 15, which gives  $B$  marginal utility at time 0 of  $(9 - 4x_{0F})15 = 5(15) = 75$ . The marginal utility of the deliveries of 15 that  $B$  must make at time 1 is  $(1)(15) = 15$ . This surplus  $B$  gains by borrowing explains why he will choose to sell only the contract  $j = 15$  that maximizes

the amount of money he raises. Selling a contract with  $j < 15$  is silly. It deprives B of the opportunity to earn more liquidity surplus. Selling contract 16 would not bring any more cash, because contract 16 sells for the same price as contract 15 even though it promises more.

The collateral has a price of 18 relative to food, which is much too high to be explained by its utility relative to food. But as explained in Fostel-Geanakoplos, the price is equal to the payoff value plus the collateral value. Housing does double duty. It enables B agents to get utility by living there, but it also enables B agents to borrow more and to gain more liquidity surplus.

$$\begin{aligned}
 p_{0H} &= \text{payoff value} + \text{collateral value} \\
 \text{payoff value} &= (MU_{x_{0H}}^B + MU_{x_{1H}}^B) / \frac{MU_{x_{0F}}^B}{p_{0F}} = (15 + 15) / 5 = 6 \\
 \text{collateral value} &= (MU_{x_{0F}}^B \pi_{15} - MU_{x_{1F}}^B D_{15}) / \frac{MU_{x_{0F}}^B}{p_{0F}} = (5 \cdot 15 - 1 \cdot 15) / 5 = 12 \\
 p_{0H} &= 6 + 12 = 18
 \end{aligned}$$

#### 4.1.5 The Failure of "Efficient Markets"

The efficient markets hypothesis essentially says that prices are priced fairly by the market, and that even an uninformed agent should not be afraid to trade, because the prices already incorporate the information acquired by more sophisticated agents. That is true in collateral equilibrium for the contracts, but it is not true of the assets that can be used as collateral. An unsophisticated buyer who did not know how to use leverage would find that he grossly overpaid for housing.

#### 4.1.6 Optimal Collateral Levels?

What would happen if the government simply refused to let borrowers leverage so much, say by prohibiting the trade in contracts for  $j > 14$ ? Although every type B agent wants to leverage up, using  $j = 15$ , when all the other type B agents are doing the same, he is actually much better off if leverage is limited by government fiat. Then everybody will borrow using asset  $j = 14$ , and with less buying power, the price of housing will fall. In fact  $p_{0H}$  will fall to 17.05, and the downpayment of 3.05 needed to buy the house is therefore barely more than before. (The consumption of the B types in period 0 is then a bit smaller than it was, raising the marginal utility of consumption in period 0. The net utility of buying the house after repaying the loan is now increased from 15 to  $15+1=16$ , so the marginal utility condition continues to hold.) The big difference is that agent B will only have to deliver 14 in period 1 instead of 15. B gains about .7 utiles and A loses about .94 utiles. In short the limit on leverage works out as a transfer from A to B.

### 4.1.7 Why did Housing Prices Rise so much from 1996-2006?

We can put our last observation more directly. Limits on leverage will reduce collateral goods prices, and an expansion of leverage will increase their prices. The remarkable run-up in housing prices in the middle 1990s to the middle 2000s is in my mind less a matter of irrational exuberance than of leverage.

We now consider a more complicated variation of our basic example in which there is uncertainty and default. Now a higher collateral requirement would mean strictly less default, but also lower housing prices. So it is interesting to see which collateral requirement best suits the sellers/lenders.

## 4.2 Example: Borrowing Across States of Nature, with Default

We consider almost the same economy as before, with two agents  $A$  and  $B$ , and two goods  $F$  (food) and  $H$  (housing) in each period. But now we suppose that there are two states of nature  $s = 1$  and  $2$  in period 1, occurring with objective probabilities  $(1 - \varepsilon)$  and  $\varepsilon$ , respectively.

As before, we suppose that food is completely perishable and housing is perfectly durable.

We assume

$$\begin{aligned} u^A(x_{0F}, x_{0H}, x_{1F}, x_{1H}, x_{2F}, x_{2H}) &= x_{0F} + x_{0H} + (1-\varepsilon)(x_{1F} + x_{1H}) + \varepsilon(x_{2F} + x_{2H}) , \\ u^B(x_{0F}, x_{0H}, x_{1F}, x_{1H}, x_{2F}, x_{2H}) &= 9x_{0F} - 2x_{0F}^2 + 15x_{0H} + (1-\varepsilon)(x_{1F} + 15x_{1H}) + \varepsilon(x_{2F} + 15x_{2H}) . \end{aligned}$$

Furthermore, we suppose that

$$\begin{aligned} e^A &= (e_{0F}^A, e_{0H}^A, (e_{1F}^A, e_{1H}^A), (e_{2F}^A, e_{2H}^A)) = (20, 1, (20, 0), (20, 0)) , \\ e^B &= (e_{0F}^B, e_{0H}^B, (e_{1F}^B, e_{1H}^B), (e_{2F}^B, e_{2H}^B)) = (4, 0, (50, 0), (9, 0)) . \end{aligned}$$

To complete the model, we suppose as before that there are assets  $A_j$  with  $A_{sj} = \binom{j}{0}$ ,  $\forall s \in S$  promising  $j$  units of good  $F$  in every state  $s = 1$  and  $2$ , and no housing. We suppose that the collateral requirement for each contract is one house  $C_j = \binom{0}{1}$ , as before.

The only difference between this model and the certainty case we had before is that  $B$  is poorer in state 2, and so the housing price must drop in state 2. The first question is how leveraged will the market allow  $B$  to become? Will it allow  $B$  to default?

It turns out that it is very easy to calculate the Arrow–Debreu equilibrium and the collateral equilibrium for arbitrary  $\varepsilon$ , such as  $\varepsilon = 1/4$ . But the no collateral equilibrium is given by a very messy formula, so we content ourselves for that case with an approximation when  $\varepsilon \approx 0$ .

### 4.2.1 Arrow–Debreu Equilibrium

The unique (in utility payoffs) Arrow–Debreu equilibrium is:

$$\begin{aligned}
p &= ((p_{0F}, p_{0H}), (p_{1F}, p_{1H}), (p_{2F}, p_{2H})) = ((1, 30), ((1-\varepsilon)(1, 15), \varepsilon(1, 15)) , \\
x^A &= ((x_{0F}^A, x_{0H}^A), (x_{1F}^A, x_{1H}^A), (x_{2F}^A, x_{2H}^A)) = \left( (22, 0), \left( 20 + \frac{28}{(1-\varepsilon)}, 0 \right), (20, 0) \right) , \\
x^B &= ((x_{0F}^B, x_{0H}^B), (x_{1F}^B, x_{1H}^B), (x_{2F}^B, x_{2H}^B)) = \left( (2, 1), \left( 50 - \frac{28}{1-\varepsilon}, 1 \right), (9, 1) \right) , \\
u^A &= 70; \quad u^B = 62 - 41\varepsilon .
\end{aligned}$$

Since agent B is so rich in state 1, he sells off enough wealth from there in exchange for period 0 wealth to bid the price up to his marginal utility of 30. Notice that agent B transfers wealth from state 1 back to period 0, and by holding the house he also transfers wealth from state 0 to state 2. With complete markets there is no collapse in housing prices in state 2, despite the hit the demanders take to their income, because those B agents effectively buy insurance against that state.

### 4.2.2 No-Collateral Equilibrium

When  $\varepsilon > 0$  is very small, we can easily give an approximation to the unique equilibrium with no collateral by starting from the equilibrium in which  $\varepsilon = 0$ .

$$\begin{aligned}
\pi_j &= 0 , \\
p &= ((p_{0F}, p_{0H}), (p_{1F}, p_{1H}), (p_{2F}, p_{2H})) = \left( (1, 16), (1, 15), \left( 1, \frac{9}{1 - \frac{71}{32 \cdot 16}} \right) \right) \\
&\approx ((1, 16), (1, 15), (1, 10.4)) , \\
x^A &= ((x_{0F}^A, x_{0H}^A), (x_{1F}^A, x_{1H}^A), (x_{2F}^A, x_{2H}^A)) \approx \left( \left( 20 + \frac{71}{32}, 1 - \frac{71}{32 \cdot 16} \right), \left( 35 - \frac{15 \cdot 71}{32 \cdot 16}, 0 \right), (29, 0) \right) , \\
x^B &= ((x_{0F}^B, x_{0H}^B), (x_{1F}^B, x_{1H}^B), (x_{2F}^B, x_{2H}^B)) \approx \left( \left( \frac{57}{32}, \frac{71}{32 \cdot 16} \right), \left( 35 + \frac{15 \cdot 71}{32 \cdot 16}, 1 \right), (0, 1) \right) , \\
u^A &\approx 56 ; \quad u^B \approx 64 .
\end{aligned}$$

### 4.2.3 Collateral Equilibrium

We can exactly calculate the unique collateral equilibrium by noting that if B promises more in state 2 than the house is worth, then he will default and the house will be confiscated. But after all the agents of type B default in state 2, they will spend all of their endowment  $e_{2F}^B$  on good 2H, giving a price  $p_{2H} = 9$ . Perhaps surprisingly the equilibrium described below confirms that the B agents do choose to promise more than they can pay in state 2, and the A agents knowingly buy those promises. Indeed the same contract  $j = 15$  is traded as when there was certainty and no default. Its price is  $\pi_{15} = (1-\varepsilon)15 + \varepsilon 9$  because the rational A agents pay less, anticipating the

default in state 2.

$$\begin{aligned}
D_{1j} &= \min(j, 15); \quad D_{2j} = \min(j, 9); \quad \pi_j = (1-\varepsilon)D_{1j} + \varepsilon D_{2j}, \\
((p_{0F}, p_{0H}), (p_{1F}, p_{1H}), (p_{2F}, p_{2H})) &= ((1, 3 + \pi_{15}), (1, 15), (1, 9)), \\
x^A &= ((x_{0F}^A, x_{0H}^A), (x_{1F}^A, x_{1H}^A), (x_{2F}^A, x_{2H}^A)) = ((23, 0), (35, 0), (29, 0)), \\
\varphi_{15}^A &= -15; \quad \varphi_j^A = 0 \text{ for } j \neq 15, \\
\varphi_{15}^B &= 15; \quad \varphi_j^B = 0 \text{ for } j \neq 15, \\
x^B &= (x_{0F}^B, x_{0H}^B, (x_{1F}^B, x_{1H}^B), (x_{2F}^B, x_{2H}^B)) = ((1, 1), (35, 1), (0, 1))
\end{aligned}$$

At the equilibrium prices, each agent of type  $A$  is just indifferent to buying or not buying any contract. At these prices any agent of type  $B$  reasons exactly as before. Since money is so much more valuable to him at time 0 than it is in the future, he will borrow as much as he can, even if it leads to default in state 2. He will only trade contract  $j = 15$ . Notice that the amount of default in the bad state, and the equilibrium downpayment of 3 on the house, do not depend on the probability  $1 - \varepsilon$  of the good state.

Thus we see that the free market will not choose levels of collateral which eliminate default. We are left to wonder whether the collateral levels are in any sense optimal for the economy: does the free market arrange for the optimal amount of default?

#### 4.2.4 Excess Volatility

Since the 1929 stock market crash it has been widely argued that low margin requirements can increase the volatility of stock prices. The argument is usually of the following kind: when there is bad news about the stocks, margins are called and the agents who borrowed against the stocks are forced to put them on the market, which lowers their prices still further.

The trouble with this argument is that it does not quite go far enough. In general equilibrium theory, every asset and commodity is for sale at every moment. Hence the crucial step in which the borrowers are forced to put the collateral up for sale has by itself no bite. On the other hand the argument is exactly on the right track.

We argued that using houses or stocks, or mortgage derivatives as collateral for loans (i.e., allowing them to be bought on margin) makes their prices more volatile. The reason is that those agents with the most optimistic view of the assets' future values, or simply the highest marginal utility for their services, will be enabled by buying on margin to hold a larger fraction of them than they could have afforded otherwise. But with bad news for the asset, there is a redistribution of wealth away from the optimists and toward the pessimists who did not buy on margin. The marginal buyer of the stock is therefore likely to be someone less optimistic or less rich than would have been the case had the stock not been purchased on margin, and the income redistribution not been so severe. Thus the fall in price is likely to be more severe than if the stock could not have been purchased on margin.

Our story is borne out vividly in the example when differences stem not from optimism but from heterogeneous tastes for housing. When the housing stock can be purchased on margin (i.e., used as collateral), agents of type  $B$  are enabled to purchase the entire housing stock, raising its price from 16 (where it would have been without collateral) to 18. In the bad state these agents default and all their holdings of the housing stock are seized. Although they end up buying back the entire housing stock, their wealth is so depleted that they can only bid up the housing prices to 9.

When there is no collateral the agents of type  $B$  can afford to purchase only a fraction  $\alpha = 71/(32)(16)$  of the housing stock at time 0 (if  $\varepsilon$  is very small). But they own that share free and clear of any debts. Thus when bad news comes, they do not lose anything. They can apply their wealth to purchasing the remaining  $1-\alpha$  of the housing stock, which forces the price up to approximately 10.4. Thus when there is no leverage, the housing prices are never as high nor never as low as when the housing stock can be used as collateral.

#### 4.2.5 Endogenous Incomplete Markets

Until now we have assumed that markets were incomplete, restricting contracts to promises that were *non-contingent*, and then finding the endogenous leverage. Suppose a *contingent* contract were offered that paid only in the down state, using the house as collateral. (Or paid only in the up state). It is evident that if such a contract could be traded, and if delivery were enforced by harsh penalties, then there would be Pareto gains to be made. Either contingent contract, together with the riskless promise, would create full spanning. But if contracts could only be enforced with collateral, would either contingent contract be traded? The answer is no! Such a contract wastes collateral, which is in very short supply, because it does not use the collateral value of the house in the other state. A moment's reflection should convince the reader that no matter what contract is offered, the only ones that will be traded would be those that promised more than the full value of the house in every state.

Of course the situation would be quite different if the same house could back multiple promises, one paying off exclusively in the first state and the other paying off exclusively in the second state. That kind of tranching would lead to spanning. (But not necessarily to the complete markets solution, since the amount of promises would still be limited by the collateral.) In practice one piece of collateral rarely has several anti-correlated loans written on it; tranching like that occurs only on big pools of assets. One house might have two mortgages written on it, but in the good states they both deliver in full, and in the bad states they are both compromised.

What is interesting here is that the scarcity of collateral does not ration trade equally in all contracts, say reducing trade in each Arrow security by 40%. Instead it shuts down trade altogether in many contracts (here in both Arrow securities) and concentrates it all in a different, less felicitous, but more collateral economizing, contract.

#### 4.2.6 Inefficiency and Government Intervention

Inefficiency arises in these models from sources apart from over-leveraging. First, promises only come in limited forms (like the non-contingent promises we have mostly assumed), preventing some kinds of insurance from being traded. In our example (or in a slight modification of it such as we shall shortly consider), agents of type B might want to buy insurance that pays off in state 2 from agents of type A. But if all contracts are non-contingent, that insurance is not available. Second, even if the promises could come in any form, their quantity and form are limited by the scarcity of collateral, as we saw in the last section. The scarcity of collateral will often shut down many financial markets altogether.

The government can always find some intervention to compensate for the missing markets or the collateral constraints to make everybody better off. For example, a transfer to every agent at time 0 who begins the period without a house, coupled with a transfer to every agent who begins a state in period 1 without a house, would make everyone better off. These transfers together simply amount to a loan from A to B: the private sector cannot manage to reproduce these additional transfers because there is not enough collateral.

But in keeping with the subject of this paper, I shall ignore these interventions and confine my attention to the efficacy of regulating leverage. In the section on heterogeneous priors we found five reasons why there might be excessive leverage. But in that model there was no default. When there is heterogeneous utility for holding collateral, we found that default naturally arises in equilibrium, if markets are incomplete. This gives rise to another three dangers from excessive leverage.

One source is *debt overhang* (see Myers 1977). In our example, agents of type B and A will agree to trade loans that promise 15 in both states in period 1, as we saw. Imagine now that B had an opportunity to invest  $\delta$  units of food-equivalent at time 0 to increase the size of his house by  $\Delta\%$  at time 1. The expected revenue this brings is  $(1 - \varepsilon)15\Delta\% + \varepsilon 9\Delta\%$ . However, if B tries to raise this money by issuing new debt that is junior to the debt already issued, he can only deliver  $(1 - \varepsilon)15\Delta\%$  because the revenue in the second state will go to the old bondholders. Even if the new debt is of equal seniority to the old debt, it will be heavily diluted in the second state. So if  $(1 - \varepsilon)15\Delta\% + \varepsilon 9\Delta\% > \delta > (1 - \varepsilon)15\Delta\%$ , an efficient investment will not be made. There is no investment in the current model, but we return to this problem in two sections.

Another important source of inefficiency is the cost of seizing collateral, which until now we have taken to be zero. We discuss this in detail in the next section.

I conclude with an eighth problem that that also could sometimes be helped by limiting leverage, whether or not there is default. When markets are incomplete, Geanakoplos-Polemarchakis (1986) showed that generically there is an intervention at time 0 alone that can lead to a Pareto improvement by changing the asset ownership structure. When markets reclear in period 1 the new distribution of assets leads to a change in prices which itself redistributes wealth across states in a way that was

not spanned by the asset payoffs. In the example of this Section, the type B agents would like to buy insurance for state 2, but cannot.

In the example as it stands, curtailing leverage does not help, because it does not change prices in period 1. In particular, prohibiting contracts  $j > 14$  would not be Pareto improving. In the new equilibrium only contract  $j = 14$  would be traded, but the price of housing in states 1 and 2 would remain at 15 and 9, respectively.

But one could imagine a variant of the example, obtained partly by making the utilities strictly concave, such that a limit on leverage would change prices in a helpful direction. With less borrowing, B would be richer in period 1. This would turn prices against B in state 1 where B was already rich and buying. But it would turn prices in B's favor in state 2 where he had been a forced seller that now can sell less. These price changes have the same effect as the missing insurance contract, which transfers wealth from B to A in state 1, and from A to B in state 2. That is the key idea to the Geanakoplos-Kubler (2005) model.

Typically the intervention will need to be on several policy dimensions, especially if there are many types of agents; curtailing leverage alone is very unlikely to lead to Pareto improvements, unless some of the previous seven elements are present.

#### 4.2.7 Under Water Collateral and Foreclosure Costs

Let us change our model in a simple way to account for the fact that foreclosure is a very expensive operation. (This is a non-pecuniary externality). Suppose that for each dollar the loan exceeds the market price of the collateral, the confiscator of the property must pay a dollar to repair the house and restore it to a condition at which it can be sold at the market price. This means that a house for which the LTV is 160% (the loan is 60% above the market value of the house) would require 60% of its market value be squandered in repairs from the damages caused by foreclosure. The lender would thus recoup only  $40\%/160\% = 25\%$  of his loan when seizing the house. These numbers are completely consistent with recovery rates on foreclosures today in the subprime housing market.<sup>6</sup>

The question is: if borrowers and lenders are aware of these terrible foreclosure losses, will they nevertheless trade loans which they foresee will create substantial deadweight losses?

In the economy with foreclosure costs, we can compute that indeed the equilibrium leverage will be just as big! No matter what the value of  $\varepsilon < 1$ , the only traded

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<sup>6</sup>Another way to explain the increasing loss with greater loans is to suppose that the values of the houses in the second state are actually not certain, but are distributed over some interval. The greater the debt, the higher the fraction of houses that must be confiscated, so with a constant foreclosure cost (or cost proportional to the sales price), the higher the debt, the greater will be the foreclosure losses.



contract will be  $j = 15$ . In equilibrium we find that

$$\begin{aligned}\pi_{15} &= (1 - \varepsilon)15 + \varepsilon(9 - (15 - 9)) \\ p_{0H} &= 3 + \pi_{15}\end{aligned}$$

The rest of the equilibrium can be guessed as before.

Now we can ask our question again: what would happen if the government regulated leverage in period 0 by prohibiting any contracts with  $j > 15 - \eta$ ?

It is easy to check that only the contract  $j = 15 - \eta$  would be traded, and that we would have

$$\begin{aligned}\pi_{15-\eta} &= (1 - \varepsilon)(15 - \eta) + \varepsilon(9 - (15 - \eta - 9)) \\ p_{0H} &\approx 3 + \pi_{15-\eta}\end{aligned}$$

The regulated curtailment of leverage would have the effect of reducing housing prices by a little less than  $(1 - \varepsilon)\eta - \varepsilon\eta = (1 - 2\varepsilon)\eta$ , lowering the utility of A by the same amount. The utility of B would rise by  $(1 - \varepsilon)\eta$ , this time giving an increase in the sum of utilities.

There is a limit on how big  $\eta$  can be, however, because if  $3 + \pi_{15-\eta}$  falls below  $1 + (1 - \varepsilon)15 + \varepsilon 9$ , then the type A agents will buy the house at time 0.

In the next section, however, we see that there is lots of room to curtail leverage.

## 5 The Double Leverage Cycle

By combining the two main models from the last sections, I build a model of the double leverage cycle that allows us to see all eight of the potential pitfalls of leverage. One of the main causes of the severity of the current leverage cycle is that there are two of them, in the housing market and in the mortgage securities market, and the two reinforce each other in a destructive feedback. Houses back mortgage securities, hence a crash in housing prices has ramifications for the securities market. But a crash in the price of mortgage securities affects the loans homeowners can get, which in turn affects the housing market. One minor twist to the models is that I assume houses must be constructed.

So consider now a population made up of the type B homeowners from the second model, who get utility from living in houses as well as from consumption, and the investors  $h \in [0, 1]$  from the first model who only get utility from consumption. The homeowners will issue long maturity mortgages in order to borrow the money to build their houses. As in the second model, these mortgages will be endogenously chosen in equilibrium at levels that lead to default when the houses lose too much value. We suppose as in the last section that there is a substantial foreclosure loss. It will turn out that the mortgage has payoffs exactly like the payoffs from the Y security in our first model. These mortgages will be packaged and sold to the optimists  $h \in [.87, 1]$

who figure the state is very likely to be a good state in which the houses are valuable and the mortgages pay off in full. The optimists will borrow the money to buy the mortgages from the pessimists  $h \in [0, .87)$  in the short maturity repo market, using the mortgages as collateral for their loans. The houses thus serve twice as collateral, first backing the homeowner mortgage loans, and then backing the mortgage securities which back the optimists' repo borrowing.

More concretely, let us consider an economy with 3 time periods 0,1,2, and states of the world 0,U,D,UU,UD,DU,DD, as in our first model. Let there be a continuum of type B agents who each begin with 1 unit of canned food and 3.15 units of labor, and no houses at time 0, and 50 units of canned food in UU,UD,DU, and 9 units of canned food in DD, and no other endowments. This is exactly like our second model, except that we stretch out the model to three periods by inserting another period in the middle. There is also an additional labor good to enable the building of the houses. Canned food can be eaten at any time, but is durable. We still use the letter F to describe it. We use the letter L to denote leisure. We shall suppose the marginal utility of leisure (that is, the marginal disutility of labor) is denoted by  $c$ .

Suppose the type B agents each own a production technology that can take labor at time 0 and transform 18.15 units into a house at time 0 (that will then be perfectly durable). Let type B agents assign probability  $(1 - \varepsilon)$  to nature moving up at any state, and let their utility be

$$\begin{aligned}
& u^B(x_{0F}, x_{0H}, x_{0L}, x_{UUF}, x_{UUH}, x_{UDF}, x_{UDH}, x_{DUF}, x_{DUH}, x_{DDF}, x_{DDH},) \\
& = 9x_{0F}^2 - 2x_{0F}^2 + 15x_{0H} + cx_{0L} + (1 - \varepsilon)^2(x_{UUF} + 15x_{UUH}) + \\
& (1 - \varepsilon)\varepsilon(x_{UDF} + 15x_{UDH}) + \varepsilon(1 - \varepsilon)(x_{DUF} + 15x_{DUH}) + \varepsilon^2(x_{DDF} + 15x_{DDH}) \\
& ((e_{0F}^B, e_{0H}^B, e_{0L}^B), (e_{UUF}^B, e_{UDF}^B, e_{DUF}^B, e_{DDF}^B)) \\
& = ((1, 0, 3.15), (50, 50, 50, 9))
\end{aligned}$$

Note that if the marginal utility of leisure is small enough,  $c < 30\frac{3.15}{18.15}$ , then the B agents will work their full 3.15 hours.

Suppose also there is a continuum of agents  $h \in [0, 1]$  who are exactly like the agents in our first model, except that instead of owning one unit of X and Y at time 0, they each own 15 units of food and 15 hours of labor at time 0. Agent  $h \in [0, 1]$  has utility and endowments

$$\begin{aligned}
& u^h(x_{0F}, x_{0L}, x_{0L}, x_{UF}, x_{DF}, x_{UUF}, x_{UDF}, x_{DUF}, x_{DDF}) \\
& = x_{0F} + \bar{c}x_{0L} + hx_{UF} + (1 - h)x_{DF} \\
& + h^2x_{UUF} + h(1 - h)x_{UDF} + (1 - h)hx_{DUF} + (1 - h)^2x_{DDF} \\
& (e_{0F}^h, e_{0H}^h, e_{0L}^h) = (15, 0, 15)
\end{aligned}$$

These agents also have a disutility of work  $\bar{c}$  which will keep them from working if the ratio of wages to the price of food falls below  $\bar{c}$ .

The contracts in the economy are of two types, depending on the collateral. In one kind of contract  $j$ , called a mortgage loan of type  $j$ , agents at time 0 can make a long term promise of the fixed amount  $j$  of good  $F$  in every one of the last states  $UU, UD, DU, DD$ , using one house as collateral. In the other type of contract  $(s, k, j)$  called a Repo, agents at node  $s$  can make a short term promise of  $k$  units of good  $F$  in every immediate successor state of node  $s$ , using mortgage contract  $j$  as collateral.

In our economy, the type B agents will borrow money at time 0 by issuing the mortgage  $j=15$ , using the house they will be constructing at the same time as collateral. The most optimistic agents in  $[0,1]$  will buy those mortgages, thereby lending the B agents the money. Since the mortgages will default in state  $DD$  (but not until then), they will be risky. Hence the pessimists will not want to buy those mortgages, and the optimists don't have enough money to buy them all. So they will borrow money from the pessimists by selling repo loans against the mortgages they hold. These safer repo loans will be held by the pessimists. The Repos are one-period loans, unlike the mortgage, which is a two-period loan.<sup>7</sup>

Let us make the hypothesis that if the house is underwater by  $y$  dollars when the loan comes due, then  $y$  dollars must be wasted in order to restore the house to mint condition to sell on a par with the other houses on the market. In all the terminal states except  $DD$ , the houses will not be underwater, and the house will sell for 15. But in  $DD$  the house will only sell for 9, which means it will be underwater with a mortgage promise of 15, so the mortgage will only deliver  $9-(15-9)=3$  to the mortgage holder after he confiscates the house and sells it, net of the restoration costs.

We see that the terminal payoffs of the mortgage security are  $(15,15,15,3)$ , which is tantamount to 15 units of the security  $Y$  from our first example. The agents of type  $h$  each own 15 units of the canned food, exactly 15 times what they owned of the durable consumption good before, and their labor income is exactly enough to buy one mortgage security, again 15 times as valuable as the security in our first example. Hence this equilibrium we are computing is just the one in our first example scaled up by a factor of 15.

In view of our earlier analysis, equilibrium is easy to describe. We normalize by taking the price of canned food to be 1 in every state. In equilibrium the price of labor in period 0 will be .95. The income of the B agents at time 0 is then  $1 + (3.15)(.95) = 4$ . By constant returns to scale, the price of the house will then be  $(18.15)(.95) = (3.15 + 15)(.95) = 3 + 14.25 = 17.25$ . The B agents will each buy a house by putting 3 dollars down and borrowing the remaining 14.25 by issuing a mortgage  $j=15$  promising 15 in every state in the last period.

In state  $D$  the mortgage will be worth  $15(.69)=14.25$  dollars. In state 0, the top 13% of agents  $h$  will buy the mortgages by issuing repos promising  $k=15(.69)$ . In

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<sup>7</sup>It can be shown that if short term mortgages were offered, they would not be traded. Since there is a large foreclosure loss from default, and since the equilibrium mortgage involves default, it is not in the interest of borrower and lender to have short term mortgages. Thus the mortgages are endogenously long term loans, and the repos are endogenously short term loans

state D these optimists will be wiped out, and the mortgages will fall into the hands of more pessimistic investors. Agents  $h \in [.61, .87)$  will buy the mortgage securities at D, issuing contract (D,k,15), where  $k=3$ , in the repo market to borrow money to help them buy the mortgages. It is easy to check that every agent is optimizing, provided  $\bar{c} < .95$ .

Part of the reason the price of mortgages is so low at D is that the payoffs are so bad at DD, which reduces not just the value of the mortgages at D but the leverage that can be obtained on that reduced value.

## 5.1 What's so bad about the leverage cycle?

In the leverage cycle asset prices shoot up and shoot down as leverage changes. This drastic change is unsettling in any real economy, and I would argue is a danger in and of itself. But why is this so bad in welfare terms? In our double leverage model we can see how all eight problems with excessive leverage could arise at once, as they have in reality.

First, note that the equilibrium mortgage prices, and therefore the equilibrium housing prices, depends on the probability beliefs of just 13% of one class of investors at 0, and 39% at D. The beliefs of the continuum of B buyers is irrelevant, as are the beliefs of the bottom 61% of the heterogeneous class.

Second, the wages of housing labor at 0 of .95 is determined by the housing prices. If the marginal disutility of labor for the heterogeneous class were say  $\bar{c} = .9$ , then we can see how the great housing boom at time 0 is fueled by the optimistic beliefs of the top 13%. Lower their beliefs a bit and mortgage prices and thus housing prices will fall, and then wages might fall below .9, which would shrink the building boom at 0.

Third, with the optimists fueling the leverage cycle, asset prices collapse at D, and new activity plunges as well. Had we allowed for new construction at D, we would find lower wages and very little construction.

Fourth, at U the top 13% of the heterogeneous agents get rich; at D they go bankrupt. Inequality rises. Fifth, their absence is one reason so little new construction would take place at D.

Sixth, not only would new homes be less likely to be built at D because of lower mortgage prices (higher mortgage rates), but existing homeowners would be less likely to spend money on repairing their houses. The homeowners are all underwater there, with a nominal debt of 15 but the price of housing only 15(.69). The debt overhang eliminates much of the incentive to repair, since increases in the value of the house at DD will not help the homeowner since the house will be foreclosed anyway.

Seventh, the large mortgages homeowners and lenders agreed upon at 0 lead to huge foreclosure losses at DD. These losses are foreseen and taken into account in the terms of the contracts at date 0, yet they still arise.

Eighth, a key externality that borrowers and lenders on both the mortgage and

repo markets at time 0 do not recognize is that if leverage were curtailed, prices would be much higher at D. For example, foreclosure costs at DD would be less, which would raise the expected payoff of the mortgage and also makes it easier to leverage at D, both of which would raise the price at D. This in effect would provide insurance for investors at D who we could imagine need to sell promises in order to start new building, but who are unable to buy the insurance directly because of the missing markets.

## 6 Contagion

The crisis of 2007-9 spread from the subprime mortgage market across the global economy. This shocked most analysts, who did not see how the losses of \$400 billion or so in one market could set off losses of \$50 trillion or more in other global markets.

Fostel-Geanakoplos (2008) gave one possible explanation for contagion. We argued that if the same investors were the leveraged optimists in many markets (called crossover investors), then bad news in just one sector could cause price drops in other markets with totally independent payoffs. Once the scale of leverage is recognized, it becomes apparent that the pool of risk taking capital is small compared to the size of global asset markets; once it shrinks, and once de-leveraging starts, prices fall in unrelated sectors.

In the first model of this paper, the price of Y falls at D because there is bad news about Y, because leverage on Y falls, and because the most optimistic buyers are wiped out. Suppose we added another asset Z for which the move to D provided no information: for instance, suppose the payoffs of Z were 1 at UU and .3 at UD, but also 1 at DU and .3 at DD. Fostel-Geanakoplos (2008) argues that the price of Z would fall as well at D, provided that the optimists about Y were also relatively more optimistic about Z. The reason is that they would be poorer at D and so less able to hold assets in general, and also more risk averse; they would be able to borrow less at D in total (because leverage of Y falls); and they would see a greater opportunity in the Y market as a result of the price decline and withdraw money from other markets like Z to invest in Y.

Fostel-Geanakoplos (2008) also argued that the assets which could serve as good collateral would fall least in value – we called this phenomenon flight to collateral, as opposed to the standard flight to quality. Finally, we also argued that if there were asymmetric information about the quality of the collateral, then agents who knew they had good collateral would reduce their borrowing by more than agents with bad collateral. We would expect debt market closures in less bad economies before we saw it in the worst economies.

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