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Market Structure and the Competitive Effects of Vertical Integration*

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Abstract

We analyze the competitive effects of backward vertical integration in a model with oligopolistic firms that exert market power upstream and downstream. In contrast to previous literature, we show that a small degree of vertical integration is always procompetitive because efficiency effects dominate foreclosure effects. Moreover, vertical integration even to monopoly can be procompetitive. With regard to market structure, we find, somewhat surprisingly, that vertical integration is more likely to be procompetitive if the industry is more concentrated. Our model thus suggests that antitrust authorities should be particularly wary of vertical integration in relatively competitive industries. We demonstrate that the quantitative welfare effects can be substantial there.

Keywords: Vertical Integration, Market Structure, Downstream Oligopsony, Competition Policy.

JEL-Classification: D43, L41, L42

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1 Introduction

The effects of vertical integration on consumer and overall welfare are subject of ongoing debates amongst economists, antitrust lawyers, and policy makers. Over the last two decades substantial progress has been made in identifying pro- and anticompetitive effects of vertical integration. Productivity increases due to cost synergies have been advanced as a major source of efficiency gains from vertical integration while the ability of integrating parties to raise their rivals' costs has been recognized as a factor fostering foreclosure.[1] Yet an open theoretical question of substantial practical relevance is how these effects depend on the underlying market structure. In particular, is vertical integration more likely to harm consumers when the industry is otherwise highly competitive, or should antitrust authorities be more vigilant when the integrating firm's competitors exert substantial market power?

To shed light on these questions we present a model that permits us to study the competitive effects of vertical integration as a function of the underlying market structure and of the historically given degree of vertical integration, taking into account both productivity gains and incentives to raise rivals' costs. The following is a sketch of our model, which builds on Riordan (1998). There are a number of non-integrated firms and one partly vertically integrated firm. All firms exert oligopolistic market power downstream, where they compete à la Cournot, and oligopsonistic market power upstream. To produce the final good, firms need a fixed input, termed capacity, that is competitively offered on an upward sloping supply curve. The more capacity a firm buys on the market, the lower is its marginal cost of producing the final good. The vertically integrated firm owns some capacity at the outset. This is referred to as its ex ante degree of vertical integration and can be as low as zero or so large that the firm effectively monopolizes the market (or anything in between). As the ex ante degree of vertical integration increases, the marginal cost of the integrated firm decreases, and so it produces more output. Thus, our model explicitly allows for productivity gains from vertical integration due to economies of scale.[2] However, because the cost of acquiring the inframarginal units of capacity is sunk by the time the integrated firm interacts with the other firms on the input

[1]For recent surveys on the effects of vertical integration, see Church (2008), Rey and Tirole (2007) and Riordan (2008).

[2]Such productivity gains are not only commonly advanced by merging parties as motivation for their desire to vertically integrate but they are also well documented empirically (see e.g., Hortaçsu and Syverson, 2007). Church (2008) argues that one of the reasons why vertical mergers are complicated to evaluate is that the incentives to integrate often arise because of non-price efficiencies and are usually not attributable to market power effects.

market, it bids more aggressively for capacity. Therefore, increases in its ex ante degree of vertical integration lead to increases in the market price of capacity, which raises its rivals' costs and thus leads to (partial) foreclosure.

Within this setup, we obtain the following results. First, vertical integration is more likely to be procompetitive (i) the *more* concentrated is the industry, i.e., the fewer are the non-integrated competitors, and (ii) the smaller is the ex ante degree of integration.[3] While result (ii) is arguably as one would expect, result (i) is somewhat surprising.[4] However, a clear intuition for this result based on our model exists and will be provided below. The result implies that antitrust authorities should be more suspicious about vertical mergers when there are more firms in the industry. We also demonstrate that the effects from vertical integration on consumer surplus can be substantial even if the number of firms is large. Second, vertical integration is procompetitive under a fairly wide array of circumstances. In the extreme, even monopolizing the downstream market can enhance consumer welfare because the integrated firm expands its quantity by a very large extent after integrating.[5] Third, we show that as the number of competitors becomes large, vertical integration is anticompetitive irrespective of the ex ante degree of vertical integration. In the limit, our model thus yields Riordan's (1998) powerful result that vertical integration by a dominant firm who faces a competitive fringe is always anticompetitive.[6] Fourth, even if it is procompetitive, vertical integration is not necessarily welfare increasing. Procompetitive but welfare reducing mergers are possible because vertical integration changes the cost structure in the industry. Last, there exist critical thresholds for input and output market shares for an integrating firm above which further vertical integration is anticompetitive. These are useful measures for practical antitrust policy purposes.

Let us now develop the basic intuition for these results, starting with a few preliminaries. Since the downstream market is Cournot, firms with lower marginal costs produce larger quantities. This implies that firms with lower marginal costs incur larger inframarginal losses from

[3] Two remarks on terminology are in order. First, "procompetitive" ("anticompetitive") effects are a shorthand expression for saying that consumer prices fall (increase) due to increases in the ex ante degree of vertical integration. Second, when we say that something is more (less) likely we mean that it occurs for a larger (smaller) set in the parameter space.

[4] For example, Lafontaine and Slade (2007) note that most empirical studies on vertical integration are conducted for highly concentrated markets because evidence for foreclosure is thought most likely to be found there.

[5] This point is related to but distinct from Quirmbach (1986)'s observation that consumer prices fall *after* vertical integration to monopoly is complete. Our result is that consumer prices can fall along all the way towards complete foreclosure.

[6] This means also that the dominant firm model provides a good approximation to nearby market structures.

a price decrease. Therefore, a firm with a larger capacity produces at lower marginal cost but utilizes its capacity less intensively. Conversely, firms with little market power have marginal costs that are approximately equal to price—like fringe firms in the model with a dominant firm facing a competitive fringe— and utilize their capacity very intensively. By increasing its degree of vertical integration, the integrated firm reduces the capacity available to rival firms because the market clearing input price increases. Vertical integration has the strongest negative effect on consumer welfare if rival firms have little to no market power. Operating already close to the point where marginal costs equal price, their only way to adapt is to decrease their output. On the other hand, if rival firms exert market power themselves, the anticompetitive effect of reducing the capacity available to them will be partly offset because smaller capacities induce them to use capacity more intensively. In other words, market power of rival firms mitigates the anticompetitive foreclosure effect of vertical integration.

Based on these preliminary observations, rather intuitive explanations for our main results are now at hand. As the number of competitors increases, the market power of each of them decreases, which makes vertical integration more likely to be anticompetitive. This is also the reason why in the limit as the number of firms grows very large, our model encompasses the case with a dominant firm who faces a competitive fringe, in which vertical integration is always anticompetitive.

When the ex ante degree of vertical integration increases, the integrated firm bids more aggressively on the capacity market, as noted above. Thereby it increases its total capacity and decreases the capacity of the non-integrated firms. This has opposite effects on their capacity utilization, implying that output expansion of the integrated firm becomes smaller relative to output reduction of non-integrated firms. Therefore, vertical integration is more likely to be anticompetitive the larger is the integrated firm's ex ante degree of vertical integration.

There is, however, an at least partially off-setting effect. The aggregate capacity employed in the industry becomes larger as integration increases. Depending on the given competitiveness of the industry, this effect can dominate the anticompetitive effects just mentioned, so that even vertical integration up to monopoly can be procompetitive.

Vertical integration shifts capacity to the integrated firm that utilizes it less intensively. This results in higher aggregate costs of production, which may render vertical integration welfare reducing even if it is consumer welfare enhancing.

Our paper is most closely related to Riordan (1998), whose setup includes a dominant, partly integrated firm facing a competitive fringe. We extend this by allowing the integrated firm's competitors to exert market power as well. Riordan's model is a notable exception within the theoretical literature on vertical integration because it incorporates exertion of market power in both markets whereas most of this literature is concerned with the trade-off between avoidance of double marginalization and foreclosure. For example, Hart and Tirole (1990), Ordover, Saloner, and Salop (1990) and Chen and Riordan (2007) are only concerned with foreclosure motives. In Salinger (1988), Choi and Yi (2000), Chen (2001) and Inderst and Valletti (2011a), the downstream market is comprised of an oligopoly and both effects are present but downstream firms have no market power in the intermediate goods market.[7]

A different approach to vertical integration is developed by De Fontenay and Gans (2005),[8] in which there is efficient bilateral bargaining between pairs of upstream and downstream firms.[9] As Gans (2007) notes, the bargaining approach fits relatively well to an industry with few upstream and downstream firms while in our model the input is supplied competitively, which corresponds to general mass markets for inputs.

A paper that, like ours, considers a competitive upstream industry is Esö, Nocke, and White (2010). They study a model in which competing downstream firms bid for scarce upstream capacity and show that if this capacity is sufficiently large, the asymmetric downstream market structure analyzed here and in Riordan (1998) emerges endogenously.[10]

As in most of the literature, we consider the case of one-shot interaction between firms. An important exception is the paper by Nocke and White (2007),[11] who consider a dynamic model and show that vertical integration facilitates upstream collusion because it reduces the number of buyers for rival firms, which decreases their incentives to deviate from a collusive agreement.[12]

[7]Hendricks and McAfee (2010) present a model with both effects, where upstream and downstream firms exert market power in the intermediate goods market. However, when analyzing vertical mergers, they keep the downstream price fixed and suppose that the market structure consists of no vertical integration at the outset in order to keep the model tractable. Under these assumptions they show that output increases with vertical mergers. In contrast, in our model the downstream price is flexible and, as argued above, we show that a crucial variable to determine the competitive effects of vertical integration is the degree to which there is already integration.

[8]This approach is used by Gans (2007) to derive concentration measures for vertical and horizontal mergers in an oligopolistic vertical market structure.

[9]For a related analysis of vertical integration with multilateral bargaining, see Bolton and Whinston (1993).

[10]Inderst and Valletti (2011b) consider a model with take-it-or-leave-it offers of an upstream firm but without vertical integration. They allow for one buyer to be larger than the others and show that this buyer obtains a favorable deal because its outside option is higher. However, this leads to higher wholesale prices for rival buyers to the detriment of consumers.

[11]For a similar, analysis but with a different upstream pricing regime, see Normann (2009).

[12]We also concentrate on the case of a single (or marginal) vertical merger. Recently, Nocke and Whinston

Our model is also broadly consistent with recent evidence. In a comprehensive review of empirical studies on the effects of vertical integration for several highly concentrated industries, Lafontaine and Slade (2007) find that the efficiency effect dominates the foreclosure effect in almost all studies, and that, therefore, vertical integration has led to a fall in the final good price in almost all cases. In a similar vein, Hortaçsu and Syverson (2007) find that vertical integration in the cement and ready-mixed concrete industries has led to output increases and price decreases and show that these effects can be attributed to productivity increases that arise from firm size.

The remainder of the paper is organized as follows. Section 2 lays out the model and Section 3 presents the equilibrium. In Section 4 we derive the competitive effects of vertical integration and show how these effects change with the competitiveness of the industry. Section 5 analyzes the effects of vertical integration on social welfare and Section 6 concludes. All proofs are in the appendix.

2 The Model

There are two types of firms, one (partially) vertically integrated firm, which we index by I and $N \geq 1$ non-integrated firms. A typical non-integrated firm is indexed by j. All firms produce a homogenous good and compete à la Cournot on the downstream market. The inverse demand function is $P(Q)$, where $P(Q)$ is the market clearing price for the aggregate quantity $Q \equiv q_I + \sum_{j=1}^{N} q_j$ satisfying $P'(Q) < 0$. To produce the final good firms require a fixed input, referred to as capacity. The cost function of firm $j = 1, ..., N$ for production of q_j units is given by

$$c(q_j, k_j) = k_j C\left(\frac{q_j}{k_j}\right),$$

where k_j is firm j's capacity and $C'(q_j/k_j) \geq 0$ and $C''(q_j/k_j) > 0$.[13] Capacity is combined with variable inputs to produce the final good. This cost function is more general than most cost functions used in models of vertical integration since it allows a firm to vary its quantity for given capacity. In particular, it is more general than the widely used fixed proportions cost function which allows a firm to produce only a maximal quantity of output for a given quantity of input. The cost function is a good description of a firm's production technology whenever

(2010) considered the case in which multiple horizontal mergers might arise over time and showed under which conditions the optimal policy for an antitrust authority is myopic.

[13] This type of cost function was introduced by Perry (1978) and used e.g., by Perry and Porter (1985), Riordan (1998) and Hendricks and McAfee (2010).

the capacity input reduces marginal costs but does not prevent the firm from substituting this input for other, more expensive inputs. This appears to be an appropriate description of the production process in a variety of industries in which the availability of a scare input factor has the effect of reducing the marginal cost of production. Examples include landline telecommunication, the concrete industry and the steel industry, where the cost reducing input is fibre-optic cable, cement and iron ore, respectively.[14] The integrated firm I has a cost advantage of $\gamma \geq 0$ per unit of output.[15] Therefore, its cost function can be written as

$$c(q_I, k_I) = k_I C\left(\frac{q_I}{k_I}\right) - \gamma q_I.$$

As a consequence, marginal costs for all firms are increasing in the produced quantity for given capacity but $c(q_i, k_i)$ exhibits constant returns to scale in q_i and k_i, $i = I, 1, ..., N$.

Capacity is supplied competitively with an inverse supply function of $R(K)$, with $R'(K) > 0$ and $K \equiv k_I + \sum_{j=1}^{N} k_j$. Firm I is partially vertically integrated, that is, it owns $\underline{k} \geq 0$ units of capacity. We refer to $\underline{k}$ as its ex ante degree of vertical integration.

The timing of the game is as follows: In the first stage, the capacity stage, all firms i simultaneously choose their level of capacity k_i. The ex ante degree of vertical integration $\underline{k}$ is exogenously given and common knowledge. Firm I buys $k_I - \underline{k}$ units of capacity at the market price $R(K)$. Thus, the profit function of firm I at the capacity stage is given by

$$\Pi_I(q_I, k_I) = P(Q)q_I - k_I C\left(\frac{q_I}{k_I}\right) + \gamma q_I - (k_I - \underline{k})R(K), \tag{1}$$

and the one of a non-integrated firm j is $\Pi_j(q_j, k_j) = P(Q)q_j - k_j C(q_j/k_j) - k_j R(K)$. In the second stage, the quantity stage, all firms simultaneously choose their quantities after having observed all capacity levels $\mathbf{k} = (k_I, k_1, .., k_N)$. The aggregate quantity Q determines the market clearing price $P(Q)$, and payoffs are realized.

Equation (1) implies that firm I has the opportunity to supply undesired capacity to an outside market, which occurs if $k_I < \underline{k}$. Notice that the cost of acquiring $\underline{k}$, which we do not model, is sunk by the time firm I acquires k_I on the input market. Therefore, an increase in the degree of vertical integration reduces the number of inframarginal units of capacity $k_I - \underline{k}$ on which the integrated firm bears the market clearing price $R(K)$ when buying additional

[14]For a detailed description of the concrete industry, see e.g., Syverson (2008), and for an analysis of vertical mergers in the steel industry, see Mullin and Mullin (1997).

[15]One can also interpret γ as a quality advantage of the integrated firm's product. Throughout the paper we assume that γ is small.

units of capacity.[16] As we will see shortly, this reduction in the number of inframarginal units induces the integrated firm to bid more aggressively on the input market.

We focus on symmetric subgame perfect equilibria, where symmetry means that the non-integrated firms play the same strategies. To ensure interior solutions and a unique equilibrium, we make some shape assumptions on the demand, supply and cost function. We suppose that $\lim_{Q\to\infty} P(Q) = 0$, that $P''(Q)$ is not too positive and that $P'''(Q)$, $C'''(q_i/k_i)$ and $R''(K)$ are not too negative. These assumptions are relatively mild and guarantee a unique equilibrium. A special case that satisfies these assumptions is the linear-quadratic model, in which $P(Q) = \alpha - \beta Q$ for $Q \in [0, \alpha/\beta]$, $R(K) = \delta K$, $C_I(q_I/k_I) = \frac{c}{2}\left(\frac{q_I}{k_I}\right)^2 - \gamma q_I$ and $C_j(q_j/k_j) = \frac{c}{2}\left(\frac{q_j}{k_j}\right)^2$ for all $j \in \{1, ..., N\}$, where α, β, c and δ are positive constants.

3 Equilibrium

We solve the game by backward induction.

3.1 The Quantity Stage (Stage 2)

At the quantity stage, $\mathbf{k}$ is already determined. Since $\underline{k}$ has a direct effect only on k_I but not on q_I, the first-order condition for a profit maximum for each firm does not depend directly on $\underline{k}$. So, the first-order condition of a non-integrated firm $j \neq I$ in the subgame of the quantity stage is given by[17]

$$P + P'q_j = C'_j, \tag{2}$$

while the first-order condition of firm I is given by

$$P + P'q_I = C'_I - \gamma. \tag{3}$$

It is easy to see that the second-order conditions are satisfied given that P'' is not too positive, which we assumed above. Our assumptions also imply that firm i's reaction function has a negative slope greater than -1. Therefore, every quantity-stage subgame has a unique equilibrium. We denote by $q_i^*(\mathbf{k})$ the equilibrium quantity of firm i, given any vector of capacities $\mathbf{k}$. From the first-order conditions we get the following intuitive lemma.

[16] Observe also that although the integrated firms's cost of acquiring $\underline{k}$ is sunk, the social cost of producing $\underline{k}$ is taken into account in this formulation because $R(.)$ depends on K rather than only $K - \underline{k}$.

[17] To simplify notation, in the following we abbreviate $P(Q)$ by P, $C(q_i/k_i)$ by C_i and $R(K)$ by R. We do so also for all derivatives.

Lemma 1

$$\frac{dq_i^*(\mathbf{k})}{dk_i} > 0 \quad and \quad \frac{dq_i^*(\mathbf{k})}{dk_j} < 0 \quad for\ all \quad i \neq j,\ i, j = I, 1, ..., N. \tag{4}$$

Therefore, all own effects are positive and all cross effects are negative. That is, a firm's optimal quantity increases in its own capacity and falls in the capacity of its rivals, independently of the type of the firm. Next we obtain the following result:

Lemma 2 $\frac{q_i^*(\mathbf{k})}{k_i}$ *decreases in* k_i $\forall i \in \{I, 1, ..., N\}$.

The same result is obtained by Riordan (1998). As observed above, a firm with a larger capacity produces a larger quantity, but because it produces more inframarginal units, it suffers more from a fall in the final output price. As a consequence, it utilizes its capacity less intensively than firms with lower capacity. This means that q_i^*/k_i is smaller.

3.2 The Capacity Stage (Stage 1)

We now move on to the first stage of the game, the capacity choice game. Using the envelope theorem and dropping all arguments, the first-order condition of a non-integrated firm j in the capacity stage is given by

$$\frac{\partial \Pi_j}{\partial k_j} = P' \frac{dQ_{-j}^*}{dk_j} q_j^* - C_j + C_j' \frac{q_j^*}{k_j} - R - k_j R' = 0, \tag{5}$$

where Q_{-j}^* is the equilibrium quantity of all firms but firm j. The first-order condition of the integrated firm I is given by

$$\frac{\partial \Pi_I}{\partial k_I} = P' \frac{dQ_{-I}^*}{dk_I} q_I^* - C_I + C_I' \frac{q_I^*}{k_I} - R - (k_I - \underline{k}) R' = 0. \tag{6}$$

Showing that an equilibrium exists and, if it does, is unique is more involved in the capacity stage than in the quantity stage. The reason is that now a change in firm i's capacity has an effect on the equilibrium quantity of each firm in the second stage. Thus, the expression for the reaction function is more complicated than in a standard single stage game.[18] Nevertheless, the next lemma establishes that an equilibrium exists and is indeed unique.

Lemma 3 *There exists a unique symmetric equilibrium in the capacity stage. In this equilibrium,* k_I^* *and* k_j^*, $j = 1, ..., N$, *are determined by* (5) *and* (6).

[18] Moreover, the game is not an aggregative game. The reaction of a non-integrated firm is different if firm I changes its capacity than if a non-integrated firm changes its capacity because this has different effects on the overall quantity produced in the second stage.

From the two first-order conditions we can now derive the following lemma:

Lemma 4

$$\frac{dk_I^*}{d\underline{k}} > 0 \quad \textit{and} \quad \frac{dk_j^*}{d\underline{k}} < 0, \quad j = 1, ..., N.$$

This result, that k_I^* increases and k_j^* decreases in $\underline{k}$, is intuitive. If $\underline{k}$ increases, firm I owns more capacity units. Thus, the number of inframarginal units for which it has to pay the capacity price R on the upstream market decreases. As a consequence, firm I finds it optimal to increase its overall amount of capacity.[19] While $\underline{k}$ does not directly influence the optimal capacity of the non-integrated firms, the price of capacity increases because k_I^* increases. Therefore, each non-integrated firm optimally acquires less capacity as $\underline{k}$ rises. Hence, non-integrated firms become (partially) foreclosed as $\underline{k}$ increases.

It follows immediately from Lemma 4 that $k_I^* > k_j^*$ for $\underline{k} > 0$, even if $\gamma = 0$. Thus, if firm I is vertically integrated to some extent, its equilibrium capacity is larger than the one of the non-integrated firms. From Lemma 2 we know that this implies that its capacity utilization q_I^*/k_I^* is lower than for the non-integrated firms.

4 Competitive Effects of Vertical Integration

We now turn to the analysis of the effects of vertical integration on consumer surplus.

4.1 Competitive Threshold

We first analyze under which conditions vertical integration is pro - or anticompetitive, i.e., whether a marginal change in $\underline{k}$ increases or decreases the aggregate equilibrium quantity supplied in the downstream market. From above it follows that an increase in $\underline{k}$ has a direct positive effect on k_I and via that an indirect negative effect on all k_j.[20] This in turn leads to an increase in q_I and to a decrease in all q_j. Thus, vertical integration is procompetitive at the margin if and only if

$$\frac{dQ}{d\underline{k}} = \left(\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right)\frac{dk_I}{d\underline{k}} + N\left(\frac{dq_I}{dk_j} + \frac{dq_j}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right)\frac{dk_j}{d\underline{k}} > 0, \quad i \neq j,$$

or equivalently

$$\frac{\left(\frac{dk_j}{d\underline{k}}\right)}{\left(\frac{dk_I}{d\underline{k}}\right)} > -\frac{\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}}{N\left(\frac{dq_I}{dk_j} + \frac{dq_j}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right)}, \quad i \neq j. \tag{7}$$

[19] The effect is similar to the one arising from price caps (or floors) that may induce firms to behave more aggressively by shifting the balance from inframarginal losses to marginal gains.

[20] To simplify notation here and in what follows we omit the superscript $*$ on equilibrium quantities and capacities.

The left-hand side of (7) expresses the relative change of a non-integrated firm's capacity with $\underline{k}$ to the change in the integrated firm's capacity at the equilibrium. We know from Lemma 4 that this relative change is negative. The right-hand side gives a benchmark against which to compare this term. The inequality says that if the relative change is small enough in absolute terms, then vertical integration is procompetitive. Intuitively, if k_j does not fall by much after firm I becomes more integrated, the positive effect resulting from the increase in q_I dominates the negative effect that stems from the decrease in q_j of all non-integrated firms.

Inserting the respective derivatives (derived in the proof of Lemma 1) into the right-hand side of (7) and simplifying yields

$$\left(\frac{dk_j}{d\underline{k}}\right) / \left(\frac{dk_I}{d\underline{k}}\right) > -\frac{C_I'' \frac{q_I}{k_I}(C_j'' - k_j P')}{N C_j'' \frac{q_j}{k_j}(C_I'' - k_I P')}. \tag{8}$$

To gain some intuition for this formula suppose that both $\underline{k}$ and γ are zero. In this case all $N+1$ firms are the same and we have $q_I = q_j$, $k_I = k_j$ and thus $C_I'' = C_j''$. As a consequence, the right-hand side of (8) simplifies to $-1/N$, so that all firms have the same capacity utilization. Thus, to keep overall output constant, the aggregate capacity reduction of the non-integrated firms must be the same as the increase in the capacity of firm I. Since all N non-integrated firms are symmetric, each of them must lower its capacity by $1/N$ of the increase in the integrated firm's capacity.

Suppose now that $\gamma = 0$ but $\underline{k} > 0$. From the above lemmas we know that in this case $k_I > k_j$, $q_I/k_I < q_j/k_j$ and thus $C_I'' < C_j''$. Then, the right-hand side of (8) is in absolute value smaller than $1/N$. The reason is that the integrated firm uses its capacity less intensively than a non-integrated firm. As a consequence, if all non-integrated firms reduced their capacity in sum by the same amount as the capacity increase of the integrated firm, overall output would fall since capacity is shifted to the less efficient firm. Thus, to keep output constant the reduction in capacity by non-integrated firms has to be smaller and overall capacity must rise.

To characterize how vertical integration changes overall output, we begin with the case where $\underline{k}$ is small.

Proposition 1 *For any finite N there exists a competitive threshold $\underline{k}^* > 0$, such that for all $\underline{k} < \underline{k}^*$, vertical integration is procompetitive at the margin.*

Intuitively, if $\underline{k}$ is small, firm I uses its capacity more intensively than a non-integrated firm if its cost advantage γ is large enough or only slightly less intensively if γ is small. However, the aggregate reaction of the non-integrated firms to an increase in $\underline{k}$ is always smaller than the increase in k_I^*. Thus, the aggregate capacity that is used increases and overall output rises.

Next assume that $\underline{k}$ is so large that the equilibrium value of k_I^* is large enough to induce $k_j^* = 0$ for all $j \neq I$ and define $\bar{\underline{k}}$ as the ex ante degree of vertical integration at which $k_j^* = 0$. Observe that this implies $q_j^* = 0$. In words, at $\underline{k} = \bar{\underline{k}}$, only the integrated firm is active and the market is monopolized.[21] Accordingly, we refer to the case where $\underline{k}$ approaches $\bar{\underline{k}}$ as vertical integration to monopoly.

Proposition 2 *For any finite N, vertical integration to monopoly can be procompetitive at the margin.*

Thus, even marginal vertical integration that leads to a complete foreclosure of rival firms by the integrated firm is not necessarily detrimental to consumer welfare. In addition, as we will show below, vertical integration to monopoly may not only be locally procompetitive, i.e. when $\underline{k}$ is close to $\bar{\underline{k}}$, but also globally, i.e. for any $\underline{k} \in [0, \bar{\underline{k}})$. This implies that starting from any $\underline{k} \in [0, \bar{\underline{k}})$ vertical integration to $\bar{\underline{k}}$ may maximize consumer welfare. This is the case because our model explicitly takes into account efficiency gains in production beyond pure avoidance of double marginalization. If a firm acquires such a large amount of capacity that its competitors stop producing, its production costs become so low that it may produce a quantity that is larger than the oligopoly quantity without the capacity increase.

Even though according to Proposition 2 vertical integration to monopoly can be procompetitive, it need not necessarily be so. The reason is that firm I utilizes its capacity less and less intensively as $\underline{k}$ rises. Thus, for vertical integration to be procompetitive, the decrease in k_j (relative to the increase in k_I) as a reaction to the rise in $\underline{k}$ must get smaller and smaller as $\underline{k}$ rises.

We now turn to the analysis of intermediate values of $\underline{k}$, that is, values of $\underline{k} \in (\underline{k}^*, \bar{\underline{k}})$. It is of particular interest to explore if there is a unique threshold of $\underline{k}$ below which vertical integration is procompetitive and above which vertical integration is anticompetitive. Moreover, if no such threshold exists, is vertical integration procompetitive over the whole range from 0 to $\bar{\underline{k}}$? The expressions that are involved in the calculations are too complicated to allow us to answer this

[21] Such a $\bar{\underline{k}}$ necessarily exists since from Lemma 4 we know that $dk_I/d\underline{k} > 0$ and $dk_j/d\underline{k} < 0$. In addition, variable production costs $c(q_j, k_j)$ are decreasing in k_j since $C''(q_j/k_j) > 0$. Thus, both production and capacity costs are increasing in $\underline{k}$ for a non-integrated firm j, while revenue is decreasing because q_j is decreasing and q_I is increasing. So if $\underline{k}$ and therewith k_I is large enough, j's costs are too high relative to $P(Q)$, and so it is optimal for firm j to stop producing.

question in general. Nonetheless, we are able to show that the threshold, provided it exists, is indeed unique for two important subclasses of the general specification. The first class consists of models where the supply function $R(K)$ is very inelastic.[22] The second class is the widely used linear-quadratic specification introduced above.

Proposition 3 *Suppose either that (i) the supply function $R(K)$ is very inelastic, i.e., $R'(K)$ is large, or that (ii) the model is linear-quadratic. Then, for any finite N vertical integration is always procompetitive or there exists a unique $\underline{k}^* \in (0, \bar{k})$, such that vertical integration is procompetitive at the margin for all $\underline{k} < \underline{k}^*$ and anticompetitive at the margin for all $\underline{k} > \underline{k}^*$.*

The intuition for case (i) of the proposition is that if $R(K)$ is very inelastic, the capacity reaction of a non-integrated firm to a change in k_I, and therefore also to a change in $\underline{k}$, is independent of the value of $\underline{k}$. Therefore, $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ stays constant as $\underline{k}$ varies. However, the right-hand side of (8) is strictly increasing since firm I utilizes its capacity less and less intensively with further integration. Thus, there is a unique intersection point between the left-hand and the right-hand-side of (8). Case (ii) of the proposition is important because it shows that the threshold is unique (given that it exists) in the general linear-quadratic specification used in many industrial organization models. In addition, this indicates that the threshold is unique also for specifications that are close to the linear-quadratic one and suggests that the threshold may be unique even more generally.[23]

Our result that the efficiency gains of vertical integration are often larger than the foreclosure effects is in contrast to the results of the dominant firm model. In the dominant firm model, vertical integration leads to foreclosure of fringe firms. However, since fringe firms have no market power, their marginal cost is equal to the final consumer price, implying that some of them exit the market as a consequence of foreclosure. This has highly detrimental effects on the aggregate output because fringe firms utilize their capacity intensively. By contrast, in the case of an oligopoly, the non-integrated firms also exert market power and restrict their output to keep the final goods price high. As a consequence of the foreclosure through vertical

[22] A steeply increasing supply curve can be observed in many high technological industries. For example, dedicated fiber-optic cables or several semiconductor devices like customized integrated circuits that are produced in specialized plants exhibit large production costs that are steeply increasing once a plant produces close to its capacity limit. Often, a firm, which also produces downstream products, already owns some of these plants while other firms need to acquire the specialized inputs from scratch. In our terminology, the firm that owns some plants would be considered vertically integrated.

[23] It may also be worth mentioning that, although we tried several different specifications, we have not found any counterexamples, i.e., cases where the left-hand and right-hand side of (8) are the same for different values of $\underline{k}$.

integration, a rival firm lowers its quantity to a smaller extent than the exit of a fringe firm reduces final output in the dominant firm model. Moreover, as vertical integration increases, each rival firm in the oligopoly case buys a smaller amount of capacity and produces at a higher marginal costs, so that its capacity utilization increases. As a result, in the dominant firm model the output contraction of fringe firms after foreclosure is larger than the reaction of rival firms under oligopoly.

4.2 Anticompetitive Integration in Competitive Industries

We now consider the effect of a change in the number of firms on the competitive effects of vertical integration. Understanding how these effects depend on the competitive structure of the industry is particularly relevant for antitrust policy implications. We start by looking at the case in which the number of downstream firms becomes large. This case is also of interest from a theoretical perspective because this limit corresponds to the model Riordan (1998) analyzes.

Proposition 4 *If $N \to \infty$, then vertical integration is anticompetitive for all $\underline{k} \in [0, \bar{\underline{k}}]$.*

Hence, if the downstream market becomes perfectly competitive, vertical integration is always anticompetitive. Intuitively, the aggregate reaction of the non-integrated firms to an increase in $\underline{k}$ is larger, the more firms are in the market. Therefore, the aggregate capacity reduction and, hence, the quantity reduction of the non-integrated firms increases in their number. As N goes to infinity this effect dominates any cost advantage of the integrated firm. Thus in the limit, as the market power of the non-integrated firms vanishes, we obtain the result of Riordan (1998). As the integrated firm has no first-mover advantage in our model, but has one in Riordan's, Proposition 4 also shows that his strong result stems genuinely from the dominant firm's market power rather than from the first-mover advantage.[24]

We now turn to the case of N being finite, and analyze how $\underline{k}^*$ changes with N. From the previous subsection we know that a unique threshold $\underline{k}^*$ exists in the linear-quadratic specification and if the supply function is steep. For tractability reasons we therefore restrict ourselves to these cases. In the following, we denote the threshold as $\underline{k}^*(N)$ to explicitly account for its dependence on N.

We start with the case of $R'(K)$ being large. Here we obtain the following result:

[24] That Riordan's result extends is not trivial and has been an open question hitherto.

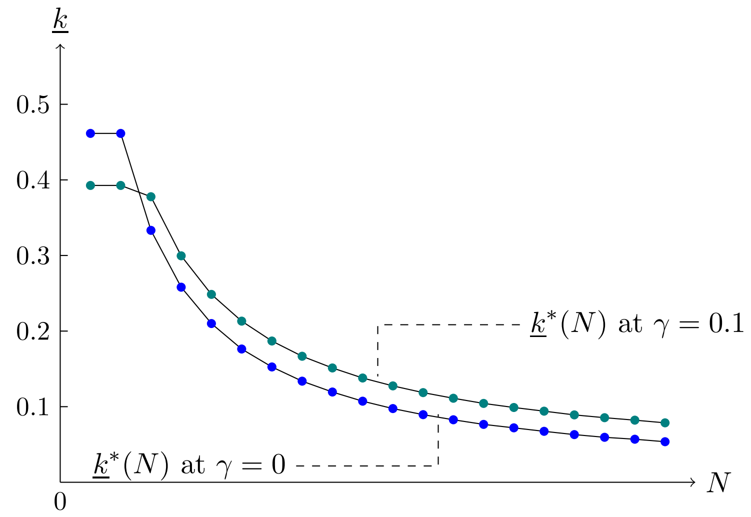

Figure 1: The competitive threshold $\underline{k}^*(N)$ in the linear-quadratic model.

Proposition 5 *Suppose that $R'(K)$ is large. If $C(\cdot)'''$ is relatively small and $C''_j \approx C''_I$, then $\underline{k}^*(N)$ is strictly decreasing in N.*

Although Proposition 5 is restricted by the assumptions that $C(\cdot)'''$ is relatively small and $C''_j \approx C''_I$, our general insight does not seem to be confined to this case. In fact, it is easy to demonstrate numerically that the result also holds if the above assumptions are relaxed.

Proposition 5 shows that the more competitive the industry gets, i.e., the larger is the number of firms, the more likely it is that vertical integration reduces consumer welfare. While the result may come as a surprise at first glance, the intuition behind it is relatively simple. If the number of firms is larger, each non-integrated firm becomes smaller and utilizes its capacity more intensively. Since the non-integrated firms are foreclosed through integration, overall capacity utilization in the industry falls. This effect is more likely to dominate the fact that integration leads to an increase in the overall capacity if the industry is more competitive.

Numerical computations also demonstrate that the threshold $\underline{k}^*(N)$ decreases in N for the linear-quadratic specification. This is illustrated in Figure 1.[25] The figure also shows that $\underline{k}^*(N)$ is larger for larger values of γ, i.e., vertical integration is more likely to be procompetitive the larger is the cost advantage of the integrated firm. This result is intuitive since vertical integration shifts capacity to the firm that produces more efficiently. When this efficiency

[25] Figure 6 in Appendix B[not for publication] also shows that qualitatively the results do not vary with γ. The numerical computations are based on the parameterization $\alpha = \beta = c = \delta = 1$. For $N = 1$ and $N = 2$, vertical integration is procompetitive for all $\underline{k}$. So for these values of N the curves do not depict $\underline{k}^*$ but rather $\bar{k}$.

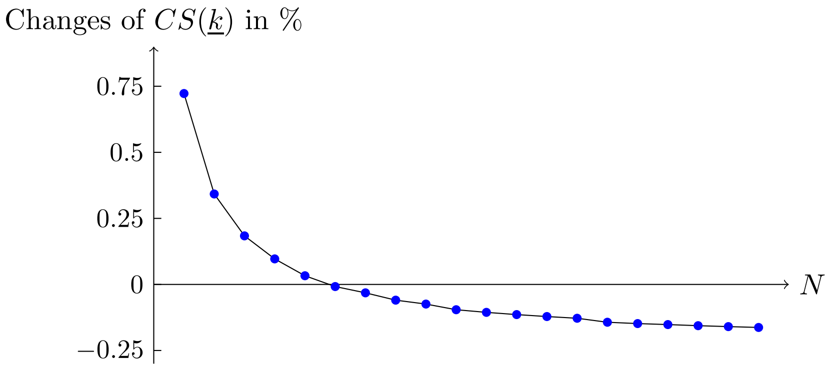

Figure 2: Changes in $CS(\underline{k})$ when $\underline{k}$ increases marginally in % of $CS(\underline{k})$ when I's downstream market share is kept at 50%.

difference is larger, the output increase of the integrated firm is also larger.

4.3 Quantifying the Effects of Vertical Integration

So far we have been focusing on the direction or sign of output changes upon vertical integration. This leaves open the question how important these effects are quantitatively, in particular, because one might guess that the effects are relatively small in competitive industries. We now demonstrate that this is not necessarily the case.

There is obviously a multitude of ways of presenting these quantitative effects. One way that we find insightful is to consider, as a function of N, the percentage change in consumer surplus when the integrated firm's degree of vertical integration increases marginally, keeping the integrated firm's downstream market share fixed at some given level. Figure 2 displays the change in consumer surplus for the linear-quadratic model when the degree of vertical integration increases marginally. This is expressed as a percentage of consumer surplus before the increase, with $\underline{k}$ chosen such that for any given N, the vertical integrated firm's downstream market share is 50%.[26,27] The results displayed in Figure 2 show that the marginal effect of vertical integration is positive and large when N is small and negative yet still sizeable in absolute terms when N is large. Additionally, vertical integration is often not a continuous process but involves acquiring a non-negligible fraction of the intermediate good market. Thus, the computation shows that even in higly competitive industries, the absolute effect of a discrete vertical merger is sizeable.

[26]Put formally, Figure 2 displays $100(CS(\underline{k}+0.01)-CS(\underline{k}))/CS(\underline{k})$ at the point where $\underline{k}$ is such that $q_I^*/Q^* = 1/2$. Here, 0.01 is the smallest increment for changes in $\underline{k}$ that we used in our simulations.

[27]This exercise is also insightful as it captures the way in which many antitrust authorities may think about evaluating the competitive effects of vertical integration.

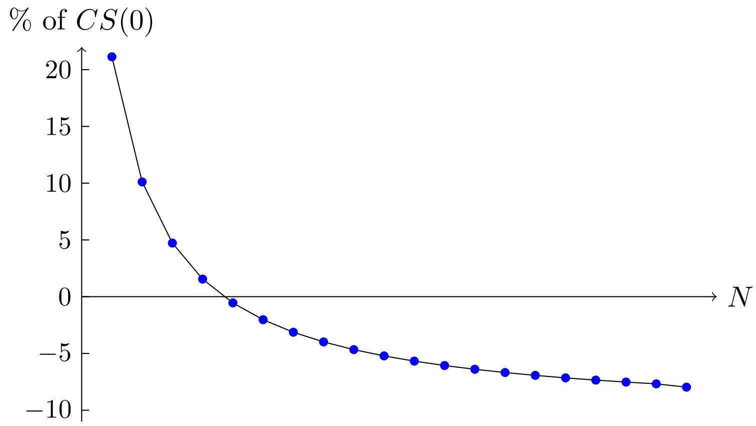

Figure 3: The difference in consumer surplus between vertical integration to monopoly and no vertical integration, i.e., $CS(\bar{\underline{k}}) - CS(0)$, in % of $CS(0)$.

Another important feature of our model is that vertical integration up to complete monopolization of the markets can enhance consumer surplus. Figure 3 illustrates the order of magnitude of these effects in the linear-quadratic model for $\gamma = 0$. It depicts the difference in consumer surplus between vertical integration to monopoly and no vertical integration, i.e., $CS(\bar{\underline{k}}) - CS(0)$, as percentage of $CS(0)$ as a function of N. If the only objective were to maximize consumer surplus and if the ex ante degree of vertical integration were 0, then vertical integration that would lead to monopoly should be permitted when the number of competitors is small absent vertical integration to monopoly but not when it is large.

At first glance, it seems like a very intuitive proposition that vertical integration is most harmful to consumers when competition is low but has little effect when the industry is otherwise highly competitive. This leads to the policy recommendation of prohibiting vertical integration when N is small but not when it is large. Our numerical results show that such intuitive, but in our model ultimately misguided policy recommendations can lead to mistakes with substantial costs to consumers at both ends of the spectrum of market structures.

4.4 Partial Integration of all Firms

We now briefly consider the case in which all firms with whom firm I competes have, at the outset, the same degree of ex ante vertical integration $\underline{k}_n \geq 0$ whereas firm I's ex ante degree of vertical integration is as before denoted $\underline{k}$. We assume $\underline{k} \geq \underline{k}_n$ and analyze further integration by firm I.[28] It is not hard to show that none of our previous results is affected qualitatively by

[28]So firms other than I are now more aptly called 'non-integrating' rather than 'non-integrated'.

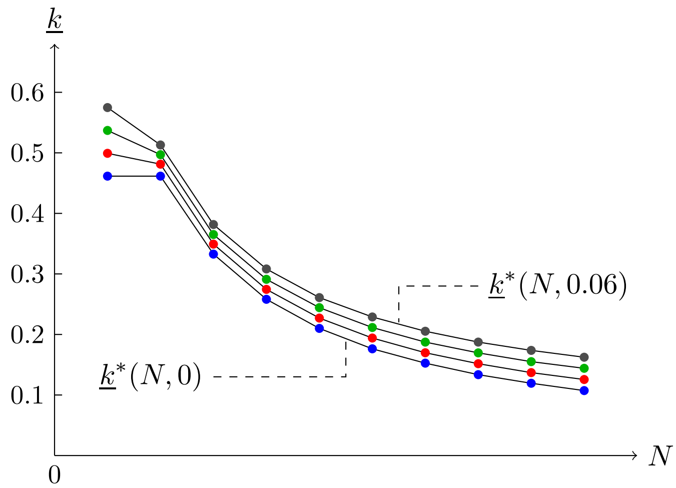

Figure 4: The competitive threshold $\underline{k}^*(N, \underline{k}_n)$ with partially integrated competitors at $\gamma = 0$ as a function of N for $\underline{k}_n \in \{0, 0.02, 0.04, 0.06\}$.

this change in assumptions.[29] In particular, marginal vertical integration starting at $\underline{k} = \underline{k}_n$ is always procompetitive. Moreover, vertical integration to monopoly can still be procompetititve, in which case firm I's rivals now sell their capacities to firm I or to an outside market upstream.

It is also informative to analyze how the competitive threshold, now denoted $\underline{k}^*(N, \underline{k}_n)$ to explicitly account for its dependence on $\underline{k}_n$, varies with N and $\underline{k}_n$. This threshold is such that for any smaller ex ante degree of vertical integration for firm I, i.e., for any $\underline{k} < \underline{k}^*(N, \underline{k}_n)$, vertical integration is procompetitive at the the margin. We do this analysis numerically for the linear-quadratic specification with $\gamma = 0$, for which the results can easily be computed. Figure 4 depicts $\underline{k}^*(N, \underline{k}_n)$ for four values of $\underline{k}_n$. As one would expect based on the model with $\underline{k}_n = 0$, $\underline{k}^*(N, \underline{k}_n)$ decreases in N. Interestingly, $\underline{k}^*(N, \underline{k}_n)$ increases in $\underline{k}_n$, which re-emphasizes a theme that has emerged from this paper: Antitrust authorities should be less wary of vertical integration the more market power the integrating firm's competitors have. The intuition behind this result is clear: When all firms are vertically integrated to some extent, the capacity reduction of non-integrating firms following an increase in $\underline{k}$ is smaller than at $\underline{k}_n = 0$. These firms do not bear the market price, $R(K)$, on their $\underline{k}_n$ inframarginal units, which makes them less sensitive to increases in the input price. Thus, the foreclosure effect from integration is smaller when all firms are integrated to a positive degree at the outset. So we obtain again the result that in an industry with several large firms, the efficiency effect of vertical integration dominates the foreclosure effects.

[29] A sketch of the proof can be found in Appendix B [not intended for publication].

5 Welfare Effects

So far we have only looked at the competitive effects of vertical integration, i.e., if vertical integration leads to an increase in overall quantity and thereby to an increase in consumer surplus. Since competition authorities both in the U.S. and in Europe base their decisions mainly on the effects on consumer surplus, this analysis is most relevant for competition policy. Yet, it is of equal importance to analyze the implications of vertical integration on social welfare, which can be expressed as

$$W = \int_0^Q P(x)dx - k_I C\left(\frac{q_I}{k_I}\right) + \gamma q_I - N k_j C\left(\frac{q_j}{k_j}\right) - \int_0^K R(y)dy.$$

The first term is gross consumer surplus, the second and third term are the variable cost of the integrated firm while the fourth term represents the variable cost of all non-integrated firms. The last term is the opportunity cost of capacity. Differentiating this expression with respect to $\underline{k}$ (and dropping arguments) yields that welfare is increasing in $\underline{k}$ if and only if

$$P\frac{dQ}{d\underline{k}} - NC_j\frac{dk_j}{d\underline{k}} - Nk_jC_j'\left(\frac{1}{k_j}\frac{dq_j}{d\underline{k}} - \frac{q_j}{k_j^2}\frac{dk_j}{d\underline{k}}\right) - C_I\frac{dk_I}{d\underline{k}} - k_IC_I'\left(\frac{1}{k_I}\frac{dq_I}{d\underline{k}} - \frac{q_I}{k_I^2}\frac{dk_I}{d\underline{k}}\right) + \gamma\frac{dq_I}{d\underline{k}} - R\frac{dK}{d\underline{k}} > 0. \tag{9}$$

We can now solve the first-order conditions of the quantity stage, (2) and (3), for C_j' and C_I', and insert them into (9). Similarly, inserting C_j' and C_I' from (2) and (3) into the first-order conditions from the capacity stage, (5) and (6), and solving them for C_j and C_I, we can replace C_j and C_I in (9). After rearranging we obtain

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} > -\frac{-P'\left(q_I\frac{dQ}{dk_I} + q_j\frac{dQ_{-I}}{dk_I}\right) + R'(k_I - \underline{k})}{N\left[-P'\left(q_j\frac{dQ}{dk_j} + (N-1)q_j\frac{dq_i}{dk_j} + q_I\frac{dq_I}{dk_j}\right) + R'k_j\right]}. \tag{10}$$

This inequality has a similar structure as (7). The left-hand side is again the equilibrium ratio of the response of k_j to a change in $\underline{k}$ over the response of k_I. The right-hand side is now different because when considering social welfare we have to take into account that the cost structure and therefore the absolute value of overall costs changes as $\underline{k}$ varies. Nevertheless, one can show that for any finite N there exists a $\underline{k}_W^* > 0$ such that for all $\underline{k} < \underline{k}_W^*$ vertical integration is welfare increasing at the margin. It is also possible that vertical integration to monopoly increases overall welfare.[30]

[30] A formal statement and a sketch of the proof are in Appendix B [not intended for publication].

The intuition is similar to the one for Propositions 1 and 2. If the ex ante degree of vertical integration is low, further vertical integration increases final output and has the effect of shifting production to the more efficient firm. Therefore, it is welfare increasing. On the other hand, if $\underline{k}$ is already very large, the overall quantity may decrease and, in addition, the less efficient firm produces more, which raises production costs even for a given quantity.

We can also show a result that is akin to Proposition 3: If $R'(K)$ is large or if the model is linear-quadratic, then for any finite N there either exists a unique $\underline{k}_W^* \in (0, \bar{\underline{k}})$ so that vertical integration is welfare enhancing at the margin for all $\underline{k} < \underline{k}_W^*$ and welfare reducing at the margin for all $\underline{k} > \underline{k}_W^*$, or vertical integration is always welfare enhancing.[31]

The analysis so far resembles the one of the previous section. However, the threshold value of $\underline{k}$ obtained in the welfare analysis is different from the one obtained for consumer surplus because, as mentioned, the variable costs of production and the opportunity costs of capacity change with an increase in $\underline{k}$. Since the rise in k_I caused by an increase in $\underline{k}$ is larger than the fall in aggregate capacity of non-integrated firms, K is increasing in $\underline{k}$ and so capacity costs are increasing. If, in addition, firm I utilizes its capacity less intensively than a non-integrated firm, we know that overall production costs must increase. In this case the set of $\underline{k}$ for which vertical integration is welfare enhancing is smaller than the one for which it is procompetitive. The next proposition confirms that for the linear-quadratic specification such a case can indeed occur.

Proposition 6 *In the linear-quadratic case, there either exists a unique $\hat{\gamma}$ such that $\underline{k}_W^* < \underline{k}^*$ for all $\gamma < \hat{\gamma}$ and $\underline{k}_W^* > \underline{k}^*$ for all $\gamma > \hat{\gamma}$, or $\underline{k}_W^* < \underline{k}^*$ for all γ.*

This result implies that if the cost advantage of the integrated firm is small, i.e., $\gamma < \hat{\gamma}$, and the ex ante degree of integration is between $\underline{k}_W^*$ and $\underline{k}^*$, vertical integration benefits consumers but lowers social welfare. The intuition is that for small γ, firm I utilizes its capacity less intensively than a non-integrated firm at $\underline{k}^*$. As a consequence, vertical integration increases overall production costs at $\underline{k}^*$ for constant aggregate quantity. Thus, even if aggregate quantity increases slightly, the effect of increased production costs dominates and welfare falls. The result is interesting since it seems natural to conjecture that procompetitive vertical integration also improves welfare because firms' profits should rise as the industry becomes more integrated. However, what is missing in this reasoning is that vertical integration shifts production costs

[31] The sketch of the proof is in Appendix B [not intended for publication].

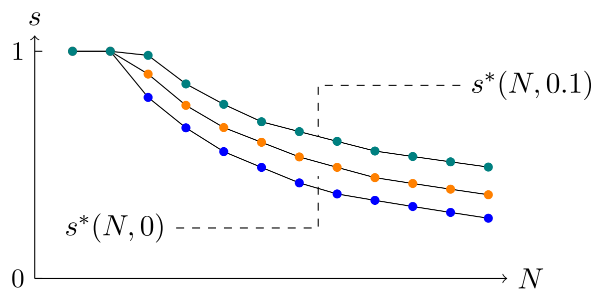

Figure 5: The critical output market share of the integrated firm $s^*(N, \gamma)$ for $\gamma = 0$, 0.05 and $\gamma = 0.1$ in the linear-quadratic model.

between firms. Proposition 6 shows that this effect can be so large that procompetitive but welfare reducing mergers are possible.

On the other hand, if the cost advantage of the integrated firm is sufficiently large, vertical integration may shift production to the more efficient firm. In this case, anticompetitive but welfare enhancing mergers occur for $\underline{k} \in (\underline{k}^*, \underline{k}_W^*)$, provided that $\underline{k}^* < \underline{k}_W^*$. Although overall quantity decreases, this smaller quantity is now produced more efficiently. This result is also consistent with Riordan's (1998) finding that welfare increasing but anticompetitive vertical integration is possible if the cost advantage of the dominant firm is large. However, procompetitive but welfare reducing mergers cannot occur in the dominant firm model.

Another important issue for practical application is to derive conclusions about the welfare effects of vertical integration that are based on observable market conditions.[32] For the linear-quadratic specification, one can numerically compute the critical input or output market shares of the integrated firm, given the thresholds $\underline{k}^*$ and $\underline{k}_W^*$, beyond which further vertical integration reduces consumer surplus or social welfare. Figure 5 plots the threshold market shares $s^*(N, \gamma)$ for the integrated firm's downstream market share beyond which vertical integration becomes anticompetitive on the margin as a function of N for different values of γ. In line with our previous results we obtain that the critical input and output market shares fall in the number of firms. In addition, these critical market shares are almost identical, which implies that it is enough for competition authorities to look at only one of these shares.

[32]A particularly nice feature of Riordan's (1998) dominant firm model is that it establishes an indicator about the welfare effects of vertical integration that holds for general functions and is based on the ratio of input to output market shares.

6 Conclusion

We have analyzed a model in which the effects of vertical integration on consumer and overall welfare depend on the underlying market structure. We have shown that, perhaps surprisingly, vertical integration is more likely to be procompetitive exactly when the market structure is more concentrated. More generally, in our model vertical integration is procompetitive under fairly wide circumstances since efficiency effects tend to dominate foreclosure effects. Because of this, even vertical integration to monopoly can be procompetitive. However, vertical integration can also be consumer welfare increasing but total welfare reducing at the same time because final output may be produced at higher costs after integration. Our numerical results also indicate that—within the confines of our model—the effects of seemingly intuitive but ultimately misguided policy recommendations can be sizeable.

Assuming that firms produce differentiated products in the downstream market is an interesting avenue for future research. Downstream market interaction between firms is smaller in this case, which suggests that the quantity reduction by non-integrated firms following vertical integration will be smaller as well. However, this leaves open the question of how sensitive the reaction of non-integrated firms is to increases in the price of capacity. In general, it seems plausible that the theme that emerges from our analysis—that vertical integration tends to be procompetitive if the market exhibits little competition and anticompetitive if the market is otherwise highly competitive—will be echoed in this case. Another interesting avenue for future work is to endogenize the market structure by additionally allowing non-integrated firms to enter and exit.

Appendix

A Proofs

A.1 Proof of Lemma 1

Let $j \neq h, j \neq I$ and $h \neq I$. Totally differentiating (2) with respect to k_j yields[33]

$$P' \frac{dQ}{dk_j} + P' \frac{dq_j}{dk_j} + P'' q_j \frac{dQ}{dk_j} = -C''_j \frac{q_j}{k_j^2} + C''_j \frac{1}{k_j} \frac{dq_j}{dk_j}. \tag{11}$$

We can write dQ/dk_j as $dQ/dk_j = dq_I/dk_j + \sum_{h \neq j} dq_h/dk_j + dq_j/dk_j$, which under the symmetry assumption that $k_h = k_j$ for all $h, j \in \{1, ..., N\}$, becomes

$$\frac{dQ}{dk_j} = \frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j} + \frac{dq_j}{dk_j}.$$

Therefore, (11) can be written as an equation that depends on the three variables dq_h/dk_j, dq_j/dk_j and dq_I/dk_j, which we wish to determine.

Totally differentiating the first-order condition of firm h, which is analogous to (2), with respect to k_j yields

$$P' \frac{dQ}{dk_j} + P' \frac{dq_h}{dk_j} + P'' q_h \frac{dQ}{dk_j} = C''_h \frac{1}{k_h} \frac{dq_h}{dk_j}, \tag{12}$$

and differentiating the first-order condition for I, equation (3), with respect to k_j yields

$$P' \frac{dQ}{dk_j} + P' \frac{dq_I}{dk_j} + P'' q_I \frac{dQ}{dk_j} = C''_I \frac{1}{k_I} \frac{dq_I}{dk_j}. \tag{13}$$

The system of the three equations (11), (12) and (13) is linear in the three unknowns dq_h/dk_j, dq_j/dk_j and dq_I/dk_j. Its unique solution, after imposing symmetry, i.e. $q_h = q_j$, $k_h = k_j$ and $C''_h = C''_j$, is

$$\frac{dq_I}{dk_j} = \frac{C''_j q_j k_I (P' + P'' q_I)}{\eta k_j} < 0 \quad \text{for} \quad j \neq I, \tag{14}$$

$$\frac{dq_h}{dk_j} = \frac{C''_j q_j (C''_I - P' k_I)(P' + P'' q_j)}{\eta (C''_j - P' k_j)} < 0 \quad \text{for} \quad j \neq h \tag{15}$$

and

$$\begin{aligned} \frac{dq_j}{dk_j} &= \frac{C''_j q_j [(P')^2 k_j k_I (N+1) + P'(P'' k_j k_I (q_I + (N-1) q_j) - 2C''_j k_I - C''_I k_j N)]}{\eta k_j (C''_j - P' k_j)} \\ &+ \frac{C''_j q_j [C''_j C''_I - P''(C''_j k_I q_I + (N-1) C''_I k_j q_j)]}{\eta k_j (C''_j - P' k_j)} > 0, \end{aligned} \tag{16}$$

[33] To simplify notation, we omit the superscript $*$ on equilibrium quantities and equilibrium capacities throughout this appendix.

where $\eta \equiv (P')^2(N+2)k_Ik_j + P'[P''k_jk_I(q_I + Nq_j) - C_I'k_j(N+1) - 2k_IC_j''] + C_I''C_j'' - P''(C_j''q_Ik_I + C_I''q_jk_jN) > 0$. The inequality sign follows from P'' being negative or not too positive.

Totally differentiating the first-order conditions of firm I and j with respect to k_I yields

$$P'\frac{dQ}{dk_I} + P'\frac{dq_I}{dk_I} + P''q_I\frac{dQ}{dk_I} = -C_I''\frac{q_I}{k_I^2} + C_I''\frac{1}{k_I}\frac{dq_I}{dk_I} \tag{17}$$

and

$$P'\frac{dQ}{dk_I} + P'\frac{dq_j}{dk_I} + P''q_j\frac{dQ}{dk_I} = C_j''\frac{1}{k_j}\frac{dq_j}{dk_I}, \tag{18}$$

respectively, where under symmetry $dQ/dk_I = dq_I/dk_I + Ndq_j/dk_I$. Using the last equation to replace dQ/dk_I in (17) and (18) yields a system of two linear equations in the two unknowns dq_I/dk_I and dq_j/dk_I. The solution is

$$\frac{dq_j}{dk_I} = \frac{C_I''q_Ik_j(P''q_j + P')}{k_I\eta} < 0 \quad \text{and} \quad \frac{dq_I}{dk_I} = -\frac{C_I''q_I[k_j(P'(N+1) + P''Nq_j) - C_j'']}{k_I\eta} > 0. \tag{19}$$

Again, the inequality sign follows from P'' not being too positive. ■

A.2 Proof of Lemma 2

From Lemma 1 we know that $q_i(\hat{k}_i, \mathbf{k}_{-i}) > q_i(k_i, \mathbf{k}_{-i}) \Leftrightarrow \hat{k}_i > k_i$. Now suppose to the contrary of the claim in the lemma that $q_i(\hat{k}_i, \mathbf{k}_{-i})/\hat{k}_i \geq q_i(k_i, \mathbf{k}_{-i})/k_i$. Since $C_i'' > 0$, this is equivalent to the right-hand sides of (2) and (3) being weakly greater for $\hat{k}_i$ than for k_i.

Now we can turn to the left-hand side of (2) and (3), respectively. From (1) we can calculate dQ/dk_j and dQ/dk_I to get

$$\frac{dQ}{dk_j} = \frac{q_jC_j''(C_I'' - k_IP')}{k_j\eta} > 0 \quad \text{and} \quad \frac{dQ}{dk_I} = \frac{q_IC_I''(C_j'' - k_jP')}{k_I\eta} > 0.$$

Since $P' < 0$, the first term of the left-hand side of (2) and (3) is smaller for $\hat{k}_i$ than for k_i. Also, since $q_i(\hat{k}_i, \mathbf{k}_{-i}) > q_i(k_i, \mathbf{k}_{-i})$, $P' < 0$ and P'' is negative or not too positive, the second term on the left-hand side of (2) and (3) is either smaller for $\hat{k}_i$ than for k_i or only slightly bigger. Therefore, the left-hand sides of (2), and (3), are strictly smaller for $\hat{k}_i$ than k_i, which is the desired contradiction. ■

A.3 Proof of Lemma 3

Differentiating (5) with respect to k_j and (6) with respect to k_I yields the second-order conditions

$$\frac{\partial^2\Pi_j}{\partial k_j^2} = P'\frac{dq_j}{dk_j}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + P'q_j\left[\frac{d^2q_I}{dk_j^2} + (N-1)\frac{d^2q_h}{dk_j^2}\right] + \tag{20}$$

$$+P''q_j\left[\frac{dq_I}{dk_j}+(N-1)\frac{dq_h}{dk_j}\right]\left[\frac{dq_j}{dk_j}+\frac{dq_I}{dk_j}+(N-1)\frac{dq_h}{dk_j}\right]+C_j''\frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j}-\frac{q_j}{k_j}\right)-2R'-k_jR''<0$$

and

$$\frac{\partial^2\Pi_I}{\partial k_I^2}=P'\frac{dq_I}{dk_I}N\frac{dq_j}{dk_I}+P'q_IN\frac{d^2q_j}{dk_I^2}+ \tag{21}$$

$$+P''q_IN\frac{dq_j}{dk_I}\left[\frac{dq_I}{dk_I}+N\frac{dq_j}{dk_I}\right]+C_I''\frac{q_I}{k_I^2}\left(\frac{dq_I}{dk_I}-\frac{q_I}{k_I}\right)-2R'-(k_I-\underline{k})R''<0,$$

with $h\neq j$, $h,j=1,...,N$. In the following we show that (20) is indeed fulfilled when the first-order conditions are satisfied. The second-order condition for the integrated firm can then be shown to be fulfilled in exactly the same way.

In the proof of Lemma 1 we determined the equilibrium expressions for dq_i/dk_j, $i = I,1,...,N$, that appear in (20). To determine the sign of $\partial^2\Pi_j/\partial k_j^2$ we still have to determine d^2q_I/dk_j^2 and d^2q_h/dk_j^2. To that end we now state the expressions for dq_I/dk_j and dq_h/dk_j without imposing symmetry, i.e. explicitly distinguishing between non-integrated firm h and j, that is between q_h and q_j, k_h and k_j and C_h'' and C_j''. This gives us

$$\frac{dq_I}{dk_j}=\frac{C_j''q_jk_I(P'+q_IP'')(C_h''-P'k_h)}{k_j\nu}\quad\text{and}\quad\frac{C_I''q_jk_h(P'+q_hP'')(C_I''-P'k_I)}{k_j\nu}, \tag{22}$$

with

$$\nu=-k_Ik_jk_h(N+2)(P')^3+(3C_h''k_jk_I+k_Ik_j(N+1)C_j''+k_hk_j(N+1)C_I''-P''k_Ik_hk_j((N-1)q_h+q_I+q_j))(P')^2+$$

$$((C_j''k_hk_I(q_I+(N-1)q_h)+C_h''k_hk_j(q_j+(N-1)q_h)+C_h''k_Ik_j(q_j+q_I))P''-Nk_hC_I''C_j''-2k_IC_j''C_h''-2k_jC_I''C_h'')P'$$

$$-((N-1)C_I''C_j''q_hk_h+C_h''(q_jk_jC_j''+q_Ik_IC_j''))P''+C_h''C_I''C_j''.$$[34]

Differentiating both equations of (22) with respect to k_j, using dq_h/dk_j, dq_j/dk_j and dq_I/dk_j from the proof of Lemma 1, and inserting the resulting expressions into the second-order condition yields

$$\frac{\partial^2\Pi_j}{\partial k_j^2}=-\frac{q_j^2\left(\sum_{s=1}^{9}(P')^s(\sum_{h=1}^{3}\kappa_{sh}(P'')^h+\kappa_{s4}P'''+\kappa_{s5}C_j'''+\kappa_{s6}C_I'''+\kappa_{s7})\right)}{k_j^2(C_j''-k_jP')^3\eta^3}-2R'-k_jR'', \tag{23}$$

where we have used that in equilibrium $q_h=q_j$, $k_h=k_j$ and $C_h''=C_j''$. In equation (23) $\kappa_{sh}=\kappa_{sh}(q_j,k_j,q_I,k_I,C_j'',C_I'',P',P'',N)$, $s\in\{1,...,9\}$ and $h\in\{1,...,7\}$. We do not specify the exact expressions for κ_{sh} here since they stand for rather complex expressions consisting of several terms. Yet, in each case the sign of these expressions is easy to determine and this

[34] One can easily check that if $q_h=q_j$, $k_h=k_j$ and, therefore, $C_h''=C_j''$ (which is the case in equilibrium), these formulas yield the expressions in (19).

is the only point of relevance for our purpose. These signs are the following: For $h = \{1, 2, 3\}$ $\kappa_{sh} \geq 0$, if both s and h are either even or odd and $\kappa_{sh} \leq 0$ if one is even and the other one is odd. κ_{s4}, κ_{s5}, $\kappa_{s6} \geq 0$ for s even and κ_{s4}, κ_{s5}, $\kappa_{s6} \leq 0$ for s odd. $\kappa_{s7} > 0$ for s even and $\kappa_{s7} < 0$ for s odd. Thus, the numerator in the fraction is positive because P'' is not too positive and P''' and C''' are not too negative. Since $\eta > 0$, the denominator is positive as well. Therefore, the first term in (23) is negative. Since R'' is not too negative as well, we get that $\partial^2\Pi_j/\partial k_j^2 < 0$. In exactly the same way we can show that the second-order condition for firm I is satisfied. Thus, the profit function of each firm is quasiconcave in its own capacity and we have an interior equilibrium.

We now turn to the question of uniqueness. From Kolstad and Mathiesen (1987) and Vives (1999) we know that the equilibrium is unique if and only if the Jacobian determinant of minus the marginal profits is positive. In our case this determinant is given by

$$|J| = \begin{vmatrix} -\frac{\partial^2\Pi_j}{\partial k_j^2} & -\frac{\partial^2\Pi_j}{\partial k_j \partial k_h} & \cdots & -\frac{\partial^2\Pi_j}{\partial k_j \partial k_I} \\ -\frac{\partial^2\Pi_h}{\partial k_h \partial k_j} & -\frac{\partial^2\Pi_h}{\partial k_h^2} & \cdots & -\frac{\partial^2\Pi_h}{\partial k_h \partial k_I} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial^2\Pi_I}{\partial k_I \partial k_j} & -\frac{\partial^2\Pi_I}{\partial k_I \partial k_h} & \cdots & -\frac{\partial^2\Pi_I}{\partial k_I^2} \end{vmatrix}, \tag{24}$$

with $h \neq j$, $h, j = 1, ..., N$. The terms that determine this determinant are given by the second-order conditions, (20) and (21), and the terms $\partial^2\Pi_j/(\partial k_j \partial k_I)$, $\partial^2\Pi_j/(\partial k_j \partial k_h)$, $\partial^2\Pi_h/(\partial k_h \partial k_j)$, $\partial^2\Pi_I/(\partial k_I \partial k_j)$ and $\partial^2\Pi_I/(\partial k_I \partial k_h)$. Because of symmetry we know that in equilibrium $\partial^2\Pi_h/(\partial k_h \partial k_j) = \partial^2\Pi_j/(\partial k_j \partial k_h)$ and $\partial^2\Pi_I/(\partial k_I \partial k_h) = \partial^2\Pi_I/(\partial k_I \partial k_h)$. The remaining terms can be written as

$$\begin{aligned}
\frac{\partial^2\Pi_j}{\partial k_j \partial k_I} &= P'\frac{dq_j}{dk_I}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + P'q_j\left[\frac{d^2q_I}{dk_j dk_I} + (N-1)\frac{d^2q_h}{dk_j dk_I}\right] \qquad (25)\\
&+ P''q_j\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right]\left[\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right] + C_j''\frac{q_j}{k_j^2}\frac{dq_j}{dk_I} - R' - k_jR'', \\
\frac{\partial^2\Pi_j}{\partial k_j \partial k_h} &= P'\frac{dq_j}{dk_h}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + P'q_j\left[\frac{d^2q_I}{dk_j dk_h} + (N-2)\frac{d^2q_k}{dk_j dk_h} + \frac{d^2q_h}{dk_j dk_h}\right] \qquad (26)\\
&+ P''q_j\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right]\left[\frac{dq_h}{dk_h} + \frac{dq_I}{dk_h} + (N-1)\frac{dq_j}{dk_h}\right] + C_j''\frac{q_j}{k_j^2}\frac{dq_j}{dk_h} - R' - k_jR'', \\
\frac{\partial^2\Pi_I}{\partial k_I \partial k_j} &= P'\frac{dq_I}{dk_j}N\frac{dq_j}{dk_I} + P'q_I\left[\frac{d^2q_j}{dk_j dk_I} + (N-1)\frac{d^2q_h}{dk_j dk_I}\right] \qquad (27)\\
&+ P''q_IN\frac{dq_I}{dk_j}\left[\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right] + C_I''\frac{q_I}{k_I^2}\frac{dq_I}{dk_I} - R' - (k_I - \underline{k})R'',
\end{aligned}$$

The second derivatives that appear in these expressions can be derived in the same way as above where we checked that the second-order conditions are satisfied.

Proceeding in a similar way as Kolstad and Mathiesen (1987), i.e. subtracting the first column in (24) from the other columns, and then dividing the i-th row by $\partial^2\Pi_i/\partial k_i\partial k_j - \partial^2\Pi_i/\partial k_i^2$, with $i = I, 1, ..., N$, yields

$$|J| = \begin{vmatrix} -\dfrac{\frac{\partial^2\Pi_j}{\partial k_j^2}}{\frac{\partial^2\Pi_j}{\partial k_j\partial k_h}-\frac{\partial^2\Pi_j}{\partial k_j^2}} & -1 & -1 & \cdots & & -\dfrac{-\frac{\partial^2\Pi_j}{\partial k_j\partial k_I}+\frac{\partial^2\Pi_j}{\partial k_j^2}}{\frac{\partial^2\Pi_j}{\partial k_j\partial k_h}-\frac{\partial^2\Pi_j}{\partial k_j^2}} \\ -\dfrac{\frac{\partial^2\Pi_h}{\partial k_h\partial k_j}}{\frac{\partial^2\Pi_h}{\partial k_h\partial k_j}-\frac{\partial^2\Pi_h}{\partial k_h^2}} & 1 & 0 & 0 & \cdots & -\dfrac{-\frac{\partial^2\Pi_h}{\partial k_h\partial k_I}+\frac{\partial^2\Pi_h}{\partial k_h\partial k_j}}{\frac{\partial^2\Pi_h}{\partial k_h\partial k_j}-\frac{\partial^2\Pi_h}{\partial k_h^2}} \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ -\dfrac{\frac{\partial^2\Pi_I}{\partial k_I\partial k_j}}{\frac{\partial^2\Pi_I}{\partial k_I\partial k_j}-\frac{\partial^2\Pi_I}{\partial k_I^2}} & 0 & 0 & \cdots & & 1 \end{vmatrix}.$$

We can then calculate the determinant in a relatively straightforward way. Cumbersome but otherwise routine manipulations show that this determinant is unambiguously positive and, therefore, that the equilibrium of the capacity stage is unique. ■

A.4 Proof of Lemma 4

Differentiating (5) and (6) with respect to $\underline{k}$ yields

$$\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{dk_j}{d\underline{k}} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j\partial k_h}\frac{dk_h}{d\underline{k}} + \frac{\partial^2\Pi_j}{\partial k_j\partial k_I}\frac{dk_I}{d\underline{k}} = 0$$

and

$$\frac{\partial^2\Pi_I}{\partial k_I^2}\frac{dk_I}{d\underline{k}} + N\frac{\partial^2\Pi_I}{\partial k_I\partial k_j}\frac{dk_j}{d\underline{k}} + \frac{\partial^2\Pi_I}{\partial k_I\partial\underline{k}} = 0.$$

Using the fact that in equilibrium $dk_h/d\underline{k} = dk_j/d\underline{k}$ for $h, j \neq I$ we get

$$\frac{dk_j}{d\underline{k}} = \frac{\frac{\partial^2\Pi_j}{\partial k_j\partial k_I}\frac{\partial^2\Pi_I}{\partial k_I\partial\underline{k}}}{\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{\partial^2\Pi_I}{\partial k_I^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j\partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2} - N\frac{\partial^2\Pi_j}{\partial k_j\partial k_I}\frac{\partial^2\Pi_I}{\partial k_I\partial k_j}} \tag{28}$$

and

$$\frac{dk_I}{d\underline{k}} = -\frac{\frac{\partial^2\Pi_I}{\partial k_I\partial\underline{k}}\left(\frac{\partial^2\Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j\partial k_h}\right)}{\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{\partial^2\Pi_I}{\partial k_I^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j\partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2} - N\frac{\partial^2\Pi_j}{\partial k_j\partial k_I}\frac{\partial^2\Pi_I}{\partial k_I\partial k_j}}. \tag{29}$$

The terms that appear in these expressions are given by (20), (21), (25), (26), (27) and by

$$\frac{\partial^2\Pi_I}{\partial k_I\partial\underline{k}} = R' > 0.$$

Tedious but routine calculations then show that all terms in (25), (26) and (27) have a negative sign. Thus, the numerators of the fractions on the right-hand side of (28) and (29) are both negative. The denominator in these fractions is the same in both equations. It is easy to show that $|\partial^2\Pi_j/\partial k_j^2| > |\partial^2\Pi_j/(\partial k_j \partial k_h)|$ which implies that

$$\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{\partial^2\Pi_I}{\partial k_I^2} > \frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2}. \tag{30}$$

In addition one can also easily show that $|\partial^2\Pi_I/\partial k_I^2| > |\partial^2\Pi_I/(\partial k_I \partial k_j)|$. Tedious calculations then reveal that

$$\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2} > \frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial k_j}. \tag{31}$$

The inequalities in (30) and (31) then imply that the denominator is positive. As a consequence, we get that $dk_j/d\underline{k} < 0$ and $dk_I/d\underline{k} > 0$. ■

A.5 Proof of Proposition 1

We start with the right-hand side of (8). Suppose first that $\gamma = 0$. As mentioned in the main text, if $\underline{k} = 0$, the right-hand side of (8) simplifies to $-1/N$.

We now turn to the left-hand side of (8). From equations (28) and (29) we obtain that it is given by

$$\frac{\left(\frac{dk_j^*}{d\underline{k}}\right)}{\left(\frac{dk_I^*}{d\underline{k}}\right)} = -\frac{\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}}{\frac{\partial^2\Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}} < 0, \quad h \neq j,\ h, j = 1, ..., N. \tag{32}$$

At $\gamma = 0$ and $\underline{k} = 0$, we know that there is no difference between firm I and any of the non-integrated firms. This implies that $\partial^2\Pi_j/(\partial k_j \partial k_h) = \partial^2\Pi_j/(\partial k_j \partial k_I)$. Now, since all the second derivatives appearing in (32) are known to be negative (from the proof of Lemma 4), $(dk_j/d\underline{k}) \,/\, (dk_I/d\underline{k}) > -1/N$ is equivalent to $\partial^2\Pi_j/\partial k_j^2 - \partial^2\Pi_j/\partial k_j \partial k_I < 0$. Since at $\underline{k} = 0$ and $\gamma = 0$ all firms are symmetric, subtracting (25) from (20) yields

$$\frac{\partial^2\Pi_j}{\partial k_j^2} - \frac{\partial^2\Pi_j}{\partial k_j \partial k_I} = P' N \frac{dq_h}{dk_j}\left[\frac{dq_j}{dk_j} - \frac{dq_h}{dk_j}\right]$$

$$+P' q_j \left[N\frac{d^2 q_h}{dk_j^2} - \frac{d^2 q_I}{dk_j dk_I} - (N-1)\frac{d^2 q_h}{dk_j dk_I}\right] + C_j'' \frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j} - \frac{dq_h}{dk_j}\right) - R'.$$

In the proof of Lemma 1 we determined dq_j/dk_j and dq_h/dk_j. Evaluating these expressions at $q_I = q_j$ and $k_I = k_j$ we can determine $dq_j/dk_j - dq_h/dk_j$ and $dq_j/dk_j - q_j/k_j - dq_h/dk_j$ to get

$$\frac{dq_j}{dk_j} - \frac{dq_h}{dk_j} = \frac{q_j C_j''}{k_j(C_j'' - P'k_j)} \quad \text{and} \quad \left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j} - \frac{dq_h}{dk_j}\right) = \frac{q_j P'}{C_j'' - P'k_j}. \tag{33}$$

Determining the second derivatives $(d^2q_h)/(dk_j^2)$, $(d^2q_I)/(dk_jdk_I)$ and $(d^2q_h)/(dk_jdk_I)$ and using (33) we obtain, after simplifying,

$$\frac{\partial^2\Pi_j}{\partial k_j^2} - \frac{\partial^2\Pi_j}{\partial k_j \partial k_I} = -\frac{q_j^2 P'\xi}{k_j^2(C_j'' - (2+N)k_jP' - (1+N)k_jq_jP'')(C_j'' - P'k_j)^3} - R' < 0,$$

where

$$\xi = k_j^2 \left(q_jC_j'''N + k_jC_j''(3N+2)\right)(P')^3 +$$
$$\left(q_jk_j^2(k_j(1+3N)C_j'' + q_jC_j'''N)P'' - k_jC_j''(3k_j(N+2)C_j'' + q_jC_j''')\right)(P')^2 +$$
$$\left(-q_jk_jC_j''(k_jC_j''(3N+4) + q_jC_j''')P'' + 5(C_j'')^3k_j\right)P' - (C_j''')^3(C_j'' - 3q_jP''k_j) < 0.$$

That is, $\partial^2\Pi_j/\partial k_j^2$ is larger in absolute terms than $(\partial^2\Pi_j)/(\partial k_j\partial k_I)$. As a consequence, $(dk_j/d\underline{k})\,/\,(dk_I/d\underline{k}) > -1/N$, which implies that the left-hand side of (8) is larger than the right-hand side. Thus, at $\gamma = 0$ and $\underline{k} = 0$ vertical integration is procompetitive at the margin.

We now turn to the case $\gamma > 0$. From (5) and (6) we know that if $q_I = q_j$, we have $k_I = k_j$ at $\underline{k} = 0$. But since $\gamma > 0$, equations (2) and (3) imply $q_I > q_j$ at $k_I = k_j$. Together with (5) and (6) this in turn implies that $k_I > k_j$. But one can show that nevertheless $q_I/k_I > q_j/k_j$ because $q_I > q_j$ is a first-order effect. Thus, at $\underline{k} = 0$ and $\gamma > 0$, firm I utilizes capacity more efficiently. This implies that a shift in capacity to firm I is also procompetitive for $\gamma > 0$. By continuity it follows that vertical integration is procompetitive at the margin for all $\underline{k}$ below a certain, positive threshold denoted by $\underline{k}^*$. ■

A.6 Proof of Proposition 2

We show that for any finite N there either exists a $\underline{k}^{**} < \bar{k}$, such that vertical integration is anticompetitive at the margin for all $\underline{k} > \underline{k}^{**}$, or it is procompetitive at the margin for all $\underline{k}$ close to $\bar{k}$.

Let $\underline{k} = \bar{k}$, so that $k_j = 0$ for all $j \neq I$. We first have to determine q_j/k_j in this case. Because C_j is strictly convex, C_j' is invertible and equation (2) can be written as $q_j = k_jC_j'^{-1}\left(P(Q) + P'(Q)q_j\right)$. It follows directly that if $k_j = 0$ we also have $q_j = 0$.

Observe that the inverse $C_j'^{-1}(.)$ is strictly increasing and that it is zero if and only if its argument is zero. By using the rule of L'Hôpital we get $q_j/k_j = C_j'^{-1}\left(P(q_I)\right) > 0$, if $q_j = 0$ and $k_j = 0$. To simplify notation in the following we denote $\rho \equiv C_j'^{-1}(P(q_I))$.

We now turn to (8). The right-hand side of (8) in the case of $\underline{k} = \bar{k}$ can be written as

$$-\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_IP')N}.$$

The left-hand side of (8) in case of $q_j = k_j = 0$ can be calculated from (28) and (29). To do so we first have to determine the second derivatives in (20), (21), (25), (26) and (27) at $q_j = k_j = 0$. From the right-hand side of (20) we know that $\partial^2\Pi_j/\partial k_j^2$ at $q_j = k_j = 0$ is given by

$$\frac{\partial^2\Pi_j}{\partial k_j^2} = P'\frac{dq_j}{dk_j}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + C_j''\frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j}\right) - 2R'. \tag{34}$$

We can then calculate dq_I/dk_j, dq_h/dk_j and dq_j/dk_j at $q_j = k_j = 0$ from (15) and (16). Taking into account that $q_j/k_j = \rho$ we get, by using the rule of L'Hôpital, that

$$\frac{dq_I}{dk_j} = -\frac{k_I\rho(P' + q_IP'')}{2k_IP' + q_Ik_IP'' - C_I''}, \quad \frac{dq_h}{dk_j} = 0 \quad \text{and} \quad \frac{dq_j}{dk_j} = \rho.$$

Calculating the second term of the right-hand side in (34) at $q_j = k_j = 0$ gives us, again by using L'Hôpital's rule, that

$$C_j''\frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j}\right) = \frac{\rho^2P'(3k_IP' + q_Ik_IP'' - 2C_I'')}{2k_IP' + q_Ik_IP'' - C_I''}.$$

Inserting these terms into (34) and simplifying then yields

$$\frac{\partial^2\Pi_j}{\partial k_j^2} = 2\frac{\rho^2P'(k_IP' - 2C_I'')}{2k_IP' + q_Ik_IP'' - C_I''} - 2R'. \tag{35}$$

In the same way we can determine the expressions for $\partial^2\Pi_I/\partial k_I^2$, $\partial^2\Pi_j/(\partial k_j\partial k_I)$, $\partial^2\Pi_j/(\partial k_j\partial k_h)$ and $\partial^2\Pi_I/(\partial k_I\partial k_j)$ at $q_j = k_j = 0$. Inserting them in (28) and (29) and simplifying we obtain that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{C_I''P'\rho\frac{q_I}{k_I} + \sigma}{(N+1)\left(\rho^2P'(C_I'' - k_IP') + \sigma\right)}, \tag{36}$$

with $\sigma \equiv R'k_I\left(2P'k_I + P''k_Iq_I - C_I''\right) < 0$.

It follows that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_IP')N}$$

if and only if

$$-\left(\frac{N}{1+N}\right)\left(\frac{C_I''P'\rho\frac{q_I}{k_I} + \sigma}{\rho^2P'(C_I'' - k_IP') + \sigma}\right) < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_IP')}. \tag{37}$$

But the left-hand side of (37) can either be larger or smaller than the right-hand side. To see this suppose first that σ is small in absolute terms. In this case, the second term of the left-hand side is approximately the same as the right-hand side. But since $-N/(1+N) > -1$, the left-hand side is larger. On the other hand, suppose that N is very large. In this case,

$N/(1+N)$ is close to 1. We then have that vertical integration is anticompetitive at the margin if

$$-\frac{C_I'' P' \rho \frac{q_I}{k_I} + \sigma}{\rho^2 P'(C_I'' - k_I P') + \sigma} < -\frac{C_I'' \frac{q_I}{k_I}}{\rho(C_I'' - k_I P')}. \tag{38}$$

Obviously the left-hand side equals the right-hand side if $\sigma = 0$. But since $\sigma < 0$ and $\rho^2 P'(C_I'' - k_I P') < P' C_I'' \rho \frac{q_I}{k_I} < 0$, the inequality in (38) is fulfilled. By continuity, vertical integration can be procompetitive at the margin for all $\underline{k}$ close to $\bar{\underline{k}}$. ■

A.7 Proof of Proposition 3

Case (i): We first look at the right-hand side of (8). Differentiating it with respect to $\underline{k}$ reveals that this derivative has the same sign as

$$\begin{aligned} &-C_j'' C_I'' (C_I'' - k_I P')(C_j'' - k_j P') \left(\frac{d\,(q_I/k_I)}{d\underline{k}} \frac{q_j}{k_j} - \frac{d\,(q_j/k_j)}{d\underline{k}} \frac{q_I}{k_I} \right) \\ &-P' C_j'' C_I'' \frac{q_j}{k_j} \frac{q_I}{k_I} \left((C_j'' - k_j P') \frac{dk_I}{d\underline{k}} - (C_I'' - k_I P') \frac{dk_j}{d\underline{k}} \right) \\ -P' \frac{q_j}{k_j} \frac{q_I}{k_I} &\left(\frac{dC_j''}{d\underline{k}} C_I'' k_j (C_I'' - k_I P') - \frac{dC_I''}{d\underline{k}} C_j'' k_I (C_j'' - k_j P') \right) + P'' \frac{dQ}{d\underline{k}} C_j'' C_I'' \frac{q_j}{k_j} \frac{q_I}{k_I} (k_j C_I'' - k_I C_j''). \end{aligned} \tag{39}$$

From Lemma 4 we know that $dk_I/d\underline{k} > 0$ and $dk_j/d\underline{k} < 0$. Because of Lemma 2 this implies that $d\,(q_I/k_I)/d\underline{k} < 0$ and $d\,(q_j/k_j)/d\underline{k} > 0$. Since $q_j/k_j > q_I/k_I$, the first term in (39) is positive. Also, since $dk_I/d\underline{k} > 0$ and $dk_j/d\underline{k} < 0$, the second term is positive as well.

Now let us turn to the third term. Since C''' is positive or not very negative, we get that $dC_j''/d\underline{k}$ is also positive or not very negative while $dC_I''/d\underline{k}$ is negative or not very positive. Therefore, the third term is either positive, or, if it is negative, then only slightly so. As a consequence, the sum of the first three terms in (39) is positive.

Now let us look at the fourth term. Since $k_j < k_I$ and $C_I'' < C_j''$ the last term in brackets is negative. Since P'' is negative or not too positive we have that for $dQ/d\underline{k} \geq 0$ the fourth term is positive or only slightly negative.

But in sum this implies that (39) is positive and thus the right-hand side of (8) is strictly increasing in $\underline{k}$ if $dQ/d\underline{k} \geq 0$.

Now we turn to the left-hand side of (8) which is given by $(dk_j/d\underline{k})/(dk_I/d\underline{k})$. If R' is relatively large, we get, after inserting (20), (21) and (25) into (32), that $(dk_j/d\underline{k})/(dk_I/d\underline{k}) = -1/(N+1)$. Thus, the left-hand side of (8) does not vary with $\underline{k}$. Since we know that the right-hand side is smaller than the left-hand side at $\underline{k} = 0$ and since the right-hand side is strictly increasing at any point of intersection, there can at most be one such point.

Case (ii): We first solve for the equilibrium in the linear-quadratic case. The profit function of the integrated firm in this case can be written as

$$\Pi_I = \left[\alpha - \beta q_I - \beta \sum_{j=1}^{N} q_j\right] q_I - \frac{cq_I^2}{2k_I} + \gamma q_I - \delta(k_I - \underline{k})(k_I + \sum_{j=1}^{N} k_j), \tag{40}$$

and the one of a non-integrated firm j as

$$\Pi_j = \left[\alpha - \beta q_j - \beta q_I - \beta \sum_{h=1, h\neq j}^{N} q_h\right] q_j - \frac{cq_j^2}{2k_j} - \delta k_j(k_j + k_I + \sum_{h=1, h\neq j}^{N} k_h). \tag{41}$$

Differentiating with respect to q_I and q_j and solving for the equilibrium quantities yields

$$q_I = \frac{(\beta(\alpha + (N+1)\gamma) k_j + c(\gamma + \alpha)) k_I}{\beta(\beta k_j(N+2) + 2c)k_I + c^2 + k_j \beta c(N+1)} \quad \text{and} \quad q_j = \frac{(\beta k_I(\alpha - \gamma) + c\alpha) k_j}{\beta(\beta k_j(N+2) + 2c)k_I + c^2 + k_j \beta c(N+1)}.$$

After substituting these quantities into the respective profit functions, we can take derivatives of Π_I with respect to k_I and of Π_j with respect to k_j.[35] The equilibrium capacities k_I is then implicitly defined by

$$\begin{aligned} &\left(c^2(c + k_j(1+N)\beta)\right)\left(c^2(\gamma+\alpha)^2 + 2c((\gamma+\alpha)\beta(\alpha + \gamma(1+N)) - N\delta c^2)k_j\right. \\ &\left.+\beta((\alpha + \gamma(1+N))^2\beta - 4N(1+N)\delta c^2)k_j^2 - 2N\delta c\beta^2(1+N)^2 k_j^3\right) = \sum_{t=1}^{4} k_I^t \theta_t - \theta_0 \underline{k}, \end{aligned} \tag{42}$$

with

$$\begin{aligned}
\theta_0 &= 2\delta(\beta^2(2+N)k_j k_I + \beta(N+1)k_j c + 2\beta c k_I + c^2)^3, \\
\theta_1 &= (6\beta^4\delta N c(2+N)(1+N)^2 k_j^4 - \beta^3((2+3N)(\alpha + (N+1)\gamma)^2\beta - 4\delta c^2(1+N)(7N^2 + 11N + 1))k_j^3 \\
&\quad -2\beta^2 c((\alpha + (N+1)\gamma)(3\alpha(N+1) + \gamma(3+4N))\beta - 3\delta c^2(7N^2 + 10N + 2))k_j^2 \\
&\quad -\beta c^2((\gamma+\alpha)((7N+6)\gamma + 3\alpha(N+2))\beta - 12\delta c^2(2N+1))k_j - 2c^3(\beta(\alpha+\gamma)^2 - 2\delta c^2)), \\
\theta_2 &= (6\beta^5\delta N c(1+N)(2+N)^2 k_j^4 + 6\beta^4\delta c^2(2+N)(N^2 + 10N + 2)k_j^3 \\
&\quad +24\beta^3\delta c^3(2N+3)(2N+1)k_j^2 + 12\beta^2\delta c^4(7N+6)k_j + 24\beta\delta c^5), \\
\theta_3 &= 2\beta^2\delta(k_j^2\beta^2 N(2+N) + 2\beta k_j c(4N+3) + 6c^2)(k_j\beta(N+2) + 2c)^2, \\
\theta_4 &= 4\beta^3\delta(k_j\beta(N+2) + 2c)^3.
\end{aligned}$$

while the equilibrium capacity q_j is implicitly defined by

$$c^3(2k_I\beta + c)(\beta(-8c^2\delta + \beta(\alpha-\gamma)^2)k_I^2 + 2c(\beta\alpha(\alpha-\gamma) - c^2\delta)k_1 + c^2\alpha^2 - 8\beta^2 c\delta k_I^3)$$

[35] As before, we have to distinguish between k_j and k_h $h \neq j$, $h, j = 1, ..., N$. Of course, in equilibrium we will have $k_h = k_j$.

$$
\begin{aligned}
&= \Big((8\beta^4 c\delta(8+3N)k_I^4 - \beta^3((\alpha-\gamma)^2(6+N)\beta - 16c^2\delta(7+4N))k_I^3 - \\
&\quad -\beta^2 c((\alpha-\gamma)((14+3N)\alpha - \gamma(N+2))\beta - 18c^2\delta(3N+4))k_I^2 + \\
&+c^2(k_I\alpha(-(3N+10)\alpha - 2(N+2)\gamma)\beta^2 + c(2(9N+10)ck_I\delta - \alpha^2(N+2))\beta + 2c^3\delta(N+1))) + \sum_{t=1}^{4} k_j^t\tau_t\Big)k_j,
\end{aligned}
\tag{43}
$$

with

$$
\begin{aligned}
\tau_1 &= c\beta(12\beta^4 c\delta(N+4)(N+2)k_I^4 + \beta^3(-(\alpha-\gamma)^2(2+3N)\beta + 2c^2\delta(116N+104+27N^2))k_I^3 \\
&\quad +\beta^2 c((\alpha-\gamma)((3N-1)\gamma - 3(3+N)\alpha)\beta + 6c^2\delta(39N+28+12N^2))k_I^2 \\
&\quad +\beta c^2(\alpha(2(3N-1)\gamma - 9N\alpha)\beta + 12c^2\delta(3N+5)(N+1))k_I + c^3(((1-3N)\alpha^2)\beta + 2c^2\delta(3N+4)(N+1) \\
\tau_2 &= 2\beta^2\delta c((2+N)k_I\beta + (N+1)c)(\beta^3(N+8)(N+2)k_I^3 + c\beta^2(8N^2+45N+40)k_I^2 \\
&\quad +2c^2\beta(5N+14)(N+1)k_I + 3c^3(N+2)(N+1)), \\
\tau_3 &= 2\beta^3\delta((2\beta^2(1+N))k_I^2 + c\beta(N+9)(N+1)k_I + c^2(N+4)(N+1))((2+N)k_I\beta + (1+N)c)^2 k_j^4, \\
\tau_4 &= 2\beta^4\delta(N+1)((N+2)k_I\beta + (N+1)c)^3.
\end{aligned}
$$

We now turn to the competitive effects of a change in $\underline{k}$. Since $Q = q_I + Nq_j$, we can insert the above explicit solutions for the quantities and differentiate Q with respect to $\underline{k}$. From this we get that $dQ/d\underline{k} > 0$ if and only if

$$
\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} > -\frac{(k_j\beta + c)(\beta(\gamma(N+1)+\alpha)k_j + c(\gamma+\alpha))}{N(k_I\beta + c)((\beta(\alpha-\gamma))k_I + c\alpha)}. \tag{44}
$$

Via differentiating (42) and (43) with respect to k_j, k_I and $\underline{k}$ and solving for $dk_j/d\underline{k}$ and $dk_I/d\underline{k}$, we can calculate the left-hand side of (44). Subtracting the right-hand side from the left-hand side yields an expression that has the following structure:

$$
\sum_{u=0}^{6}\sum_{z=0}^{5} k_j^u k_I^z \upsilon_{uz}, \tag{45}
$$

where $\upsilon_{uz} = \upsilon_{uz}(\alpha,\beta,\gamma,\delta,c,N)$. We do not spell out the exact expressions for υ_{uz}, $u \in \{1,...,6\}$, $z \in \{1,...,5\}$ because they are rather complicated. As will become clear, we are mainly interested in determining their signs and compare them, which can be done relatively easily.

Differentiating (45) with respect to $\underline{k}$ we get

$$
\sum_{u=1}^{6}\sum_{z=1}^{5}\upsilon_{uz}\left(zk_j^u k_I^{z-1}\frac{dk_I}{d\underline{k}} + uk_j^{u-1}k_I^z\frac{dk_j}{d\underline{k}}\right) + \sum_{z=1}^{5}\upsilon_{0z}zk_I^{z-1}\frac{dk_I}{d\underline{k}} + \sum_{u=1}^{6}\upsilon_{u0}uk_j^{u-1}\frac{dk_j}{d\underline{k}},
$$

where, from Lemma 4, $dk_j/d\underline{k} < 0$ and $dk_I/d\underline{k} > 0$.

First, one can show that all $\upsilon_{uz} > 0$ if $u > z$. Thus, the expressions $\upsilon_{uz}(zk_j^u k_I^{z-1}(dk_I/d\underline{k}) + uk_j^{u-1}k_I^z(dk_j/d\underline{k}))$ for $u > z$ are all negative. The expressions for υ_{uz} with $u < z$ can have different signs. So let us first take each term $\upsilon_{uz}(zk_j^u k_I^{z-1}(dk_I/d\underline{k}) + uk_j^{u-1}k_I^z(dk_j/d\underline{k}))$, where $z = z_a > u_a = u$. Now we compare it with the corresponding expression where $u = z_a$ and $z = u_a$. One can then show that the latter expression is larger than the former in absolute values in any comparison. Therefore, the sum of each of the comparisons is negative. Finally, we have to look at terms with $u = z$. Again, υ_{uz} can be positive or negative, i.e. $\upsilon_{uz} > 0$ for $u = z = 1, 2, 3$, $\upsilon_{uz} < 0$ for $u = z = 4$ and $\upsilon_{uz} = 0$ for $u = z = 5$. Now for any of these expressions $\upsilon_{uz}(zk_j^u k_I^{z-1}(dk_I/d\underline{k}) + uk_j^{u-1}k_I^z(dk_j/d\underline{k}))$ with $u = z$ we can find a previous comparison, to which we can add the expression and the resulting sum still stays negative. Thus, equation (45) is strictly decreasing in $\underline{k}$. Since at $\underline{k} = 0$, the left-hand side of (44) is larger than the right-hand side, we know that there exists either a unique intersection or no intersection between the terms on the two sides. ■

A.8 Proof of Proposition 4

We first show that $q_j \to 0$ and $k_j \to 0$, $j \neq I$, as $N \to \infty$. Suppose to the contrary that $q_j > 0$. But since $Q = q_I + Nq_j$ and $P(Q) \leq 0$, as $N \to \infty$, the first-order condition for firm j given by (2) cannot be satisfied if $q_j > 0$, since the right-hand side is positive while the left-hand side would be negative. Therefore, $q_j \to 0$, as $N \to \infty$. Given this, suppose now that $k_j > 0$. But then in the first-order condition of the capacity stage, (5), the left-hand side would be negative while the right-hand side is zero. In order to fulfill this condition we must have $k_j \to 0$. Therefore, as $N \to \infty$, $q_j \to 0$ and $k_j \to 0$.

In the proof of Proposition 2 we already calculated the case of $q_j \to 0$ and $k_j \to 0$. Taking in addition that $N \to \infty$ we get from (37) that vertical integration is anticompetitive if

$$-\frac{C_I''P'\rho\frac{q_I}{k_I} + \sigma}{\rho^2 P'(C_I'' - k_I P') + \sigma} < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_I P')},$$

where ρ and σ are defined in the proof of Proposition 2. But we already showed in this proof that the inequality is fulfilled. Therefore, vertical integration is anticompetitive if $N \to \infty$. ■

A.9 Proof of Proposition 5

From the proof of Proposition 3 we know that the left-hand side of (8), $(dk_j/d\underline{k})/(dk_I/d\underline{k})$, equals $-1/(N+1)$ if R' is relatively large. Multiplying (8) by N, the left-hand side is given

$-N/(N+1)$. Taking the derivative with respect to N, we obtain $-1/(N+1)^2 < 0$, implying that the left-hand side is decreasing with N.

We now turn to the right-hand side of (8), multiplied by N, which is given by

$$-\frac{C_I'' q_I k_j (C_j'' - k_j P')}{C_j'' q_j k_I (C_I'' - k_I P')}. \tag{46}$$

To take the derivative of (46) we need to determine $\partial k_j/\partial N$ from (5) and $\partial k_I/\partial N$ from (6) and use these derivatives to calculate $\partial q_j/\partial N$ from (2) and $\partial q_I/\partial N$ from (3), taking into account that R' is large. Doing so yields

$$\frac{\partial k_j}{\partial N} = \frac{\partial k_I}{\partial N} = -\frac{k_j}{N+2} < 0,$$

$$\frac{\partial q_j}{\partial N} = -\frac{N q_j C_j'' \left(C_I'' - k_I P'\right)}{\Omega} < 0 \quad \text{and} \quad \frac{\partial q_j}{\partial N} = -\frac{N q_I C_I'' \left(C_j'' - k_j P'\right)}{\Omega} < 0,$$

with $\Omega = (N+2)((P')^2 k_j k_I (N+2) + P'(k_j k_I (N q_j + q_I) P'' - k_j C_I''(N+1) - 2 k_I C_j'') - P''(q_I k_I C_j'' + N q_j k_j C_I'') + C_I'' C_j'')$.

Taking the derivative of (46) and using the above expressions, we obtain, under the additional assumptions (i) C''' is relatively small and (ii) $C_j'' \approx C_I''$, that this derivative is given by

$$\begin{aligned}
&-(k_I - k_j)(-2(P')^4 k_j^2 k_I^2 (N+2)\left(C_j''\right)^2 + \left(C_j''\right)^4 P''(k_I - k_j)(q_I - q_j) \\
&-(P')^2 \left(C_j''\right)^3 \left(C_j''(k_j(k_j + 2k_I N) + k_I(k_I + 4k_j)) - 2P'' k_j k_I (q_j N(3k_j + k_I) + q_I(k_j + 3k_I))\right) \\
&+P' \left(C_j''\right)^4 \left(C_j''(k_j N + k_I) - P''(q_j N(k_I - k_j)^2 + q_I(k_I^2 + k_j(2k_I - k_j)))\right) \\
&+(P')^3 k_j k_I \left(C_j''\right)^2 \left(C_j''(3k_j(N+1) + k_I(N+5)) - 2P'' k_j k_I (q_I + N q_j)\right).
\end{aligned}$$

But since P'' is negative or not too positive, the derivative is positive, implying that (46) is falling in N.

From the previous analysis we know that $\underline{k}^*$ is given by the intersection of the left-hand side and the right-hand side of (8). As shown in the proof of Proposition 3, if R' is very large, the left-hand side is constant in k while the right-hand side is increasing at the point of intersection. Now we just showed that the left-hand side is lower if N is larger while the right-hand side is larger if N is larger. But this implies that the two sides cross each other at a lower value of k if N is larger. Since $\underline{k}^*(N)$ is defined as the point of interestion, it follows that it is decreasing in N. ■

A.10 Proof of Proposition 6

We know that welfare is increasing in $\underline{k}$ if and only if (9) holds. The first term on the left-hand side of (9), $PdQ/d\underline{k}$, has the same sign as the condition for pro- or anticompetitive vertical integration. Therefore, we know that it is zero at $\underline{k}^*$. As a consequence, if the rest of the left-hand side is negative at $\underline{k}^*$, this implies that $\underline{k}^*_{WF} < \underline{k}^*$.

We start with the case of $\gamma = 0$. In this case $\gamma(dq_I/d\underline{k}) = 0$. The term $-R(dK/d\underline{k})$ is negative since overall capacity is increasing in $\underline{k}$. Thus, if the terms

$$-NC_j\frac{dk_j}{d\underline{k}} - Nk_jC_j'\left(\frac{1}{k_j}\frac{dq_j}{d\underline{k}} - \frac{q_j}{k_j^2}\frac{dk_j}{d\underline{k}}\right) - C_I\frac{dk_I}{d\underline{k}} - k_IC_I'\left(\frac{1}{k_I}\frac{dq_I}{d\underline{k}} - \frac{q_I}{k_I^2}\frac{dk_I}{d\underline{k}}\right) \tag{47}$$

are negative at $\underline{k}^*$, we have established that $\underline{k}^*_{WF} < \underline{k}^*$ at $\gamma = 0$. We can now use the respective expressions for the cost functions and the equilibrium values of q_j and q_I in the linear-quadratic case that we calculated in the proof of Proposition 3, case (ii). Inserting them into (47) and simplifying reveals that the sign of this expression is the same as the sign of

$$\begin{aligned}&-\Big[N\alpha^2(c+\beta k_I)\left(k_I^2\beta^2(2c-\beta k_j(N+2)) + k_Ic\beta(c-\beta k_j(2N+5)) + c^2(c-\beta k_j(N+1))\right)\Big]\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} - \\ &-\alpha^2\left[c+k_j\beta\right]\Big[k_j^2\left(\beta^2c(N+1) - k_I\beta^3(N+2)\right) + k_jc\beta\left(c(2-N) - k_I\beta(3N+4)\right) - 2k_Ic^2\beta + c^3\Big].\end{aligned} \tag{48}$$

From (44) we know that $dQ/d\underline{k} = 0$ at $\gamma = 0$ if $\underline{k}$ implies equilibrium values of k_I and k_j such that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{(k_j\beta + c)^2}{N(k_I\beta + c)^2}.$$

Inserting the last equation into (48) and simplifying gives

$$-\frac{2\alpha c\beta(k_I - k_j)(c+\beta k_j)\left(k_I\beta(2c + k_j\beta(N+2)) + c(c + k_j\beta(N+1))\right)}{(c+\beta k_I)},$$

which is negative because $k_I > k_j$ at $\underline{k}^*$. Thus, we have shown that $\underline{k}^*_{WF} < \underline{k}^*$ at $\gamma = 0$.

Now we turn to the case $\gamma > 0$. Writing (9) under the linear-quadratic specification for the case of $\underline{k} = \underline{k}^*$, i.e. when $dQ/d\underline{k} = 0$, we get

$$\begin{aligned}&-\frac{k_j^2k_I^2\varrho}{\left(k_I\beta(2c + k_j\beta(N+2)) + c(c + k_j\beta(N+1))\right)^3} \\ &+c\gamma\frac{(c+\beta k_j(N+1))(c(\alpha+\gamma) + \alpha\beta k_j + \beta\gamma k_j(N+1)) - k_I\beta N(\alpha c + \beta k_I(\alpha-\gamma))\left(\frac{dk_j}{d\underline{k}}/\frac{dk_I}{d\underline{k}}\right)}{\left(k_I\beta(2c + k_j\beta(N+2)) + c(c + k_j\beta(N+1))\right)^2}\end{aligned} \tag{49}$$

$$-\delta(k_I + Nk_j)(N\left(\frac{dk_j}{d\underline{k}}\Big/\frac{dk_I}{d\underline{k}}\right) + 1),$$

with

$$\varrho \equiv \Big[N(\alpha c + \beta k_I(\alpha-\gamma))(k_I^2\beta^2(\alpha-\gamma)(2c - \beta k_j(N+2))$$

$$+k_I c\beta(c(\alpha - 3\gamma) - \beta k_j(\alpha(2N+5) + \gamma(N+1))) + c^2\alpha(c - \beta k_j(N+1))\Big]\left(\frac{dk_j}{d\underline{k}}\Big/\frac{dk_I}{d\underline{k}}\right)$$

$$+\left[c(\alpha+\gamma) + k_j\alpha\beta + \beta\gamma k_j(N+1)\right]\Big[k_j^2(\alpha + \gamma(N+1))\left(\beta^2 c(N+1) - k_I\beta^3(N+2)\right)$$

$$+k_j c\beta\left(c(2(\alpha+\gamma) - N(\alpha - 2\gamma)) - k_I\beta(\alpha(3N+4) + \gamma(N+4))\right) - 2k_I c^2\beta(\alpha+\gamma) + c^3(\alpha+\gamma)\Big].$$

From (44) we have that $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ at $\underline{k} = \underline{k}^*$ is given by

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{(k_j\beta + c)(\beta(\gamma(N+1)+\alpha)k_j + c(\gamma+\alpha))}{N(k_I\beta + c)((\beta(\alpha-\gamma))k_I + c\alpha)}.$$

Inserting this into (49), differentiating the resulting expression with respect to γ and using the fact that $dk_I/d\gamma > 0$ and $dk_j/d\gamma < 0$ reveals that the expression is strictly increasing in γ. But from the first part of the proof we know that (49) evaluated at $\underline{k} = \underline{k}^*$ is negative at $\gamma = 0$ which implies that $\underline{k}^*_{WF} < \underline{k}^*$. Therefore, we have shown there exists either a unique value of γ denoted by $\hat{\gamma}$ such that $\underline{k}^*_{WF} < \underline{k}^*$ for all $\gamma < \hat{\gamma}$ and $\underline{k}^*_{WF} > \underline{k}^*$ for all $\gamma > \hat{\gamma}$, or no such value exists because (49) turns positive only at such high values of γ at which the non-integrated firms are not active. In the latter case $\underline{k}^*_{WF} < \underline{k}^*$ for all γ. ■

B Additional Material (not intended for publication)

In this appendix (which is not intended for publication) we provide additional results based on numerical calculations for the linear-quadratic model.

B.1 $\underline{k}^*$ as a function of N and γ

Figure 6 summarizes the results from the numerical computations. It plots the competitive threshold, here denoted $\underline{k}^*(N,\gamma)$, as a function of N and γ for the linear quadratic model for $\gamma \in \{0, 0.05, 0.1, 0.15, 0.2\}$.[36] It complements Figure 1 by adding results for values of $\gamma > 0$. Since $\underline{k}^*$ decreases in N, these results also show that Riordan's (1998) dominant firm model becomes an increasingly better approximation as the downstream industry becomes more and more competitive.

[36] All simulations were done in Python and are available upon request. In Figure 6 we set the parameters α, β, c and δ equal to one.

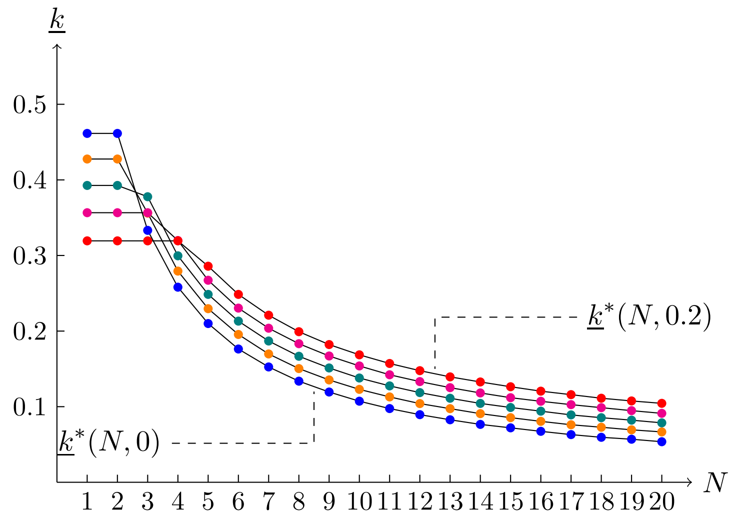

Figure 6: The competitive threshold $\underline{k}^*(N, \gamma)$ for $\gamma \in \{0, 0.05, 0.1, 0.15, 0.2\}$.

Figure 6 also reveals that vertical integration is procompetitive for a larger set of $\underline{k}$ the larger is γ because increases in γ result in upward shifts of $\underline{k}^*$.[37] The intuition is that the integrated firm utilizes its capacity to a larger degree if its cost advantage is bigger. Therefore, capacity is shifted to the more efficient firm which makes vertical integration more likely to be procompetitive.

B.2 Sketch of the proof of Subsection 4.4

The competitive effects of vertical integration can still be evaluated using (8). If all firms are integrated, the right-hand side of (8) is the same but the left-hand side may differ.

As in the proof of Proposition 1, if all firms including firm I are vertically integrated to the same extent, (8) can be evaluated by determining $\partial^2\Pi_j/\partial k_j^2 - \partial^2\Pi_j/\partial k_j \partial k_I$. These two expressions are given by (20) and (25) but with the difference that in both expressions the last term is now given by $(k_j - \underline{k}_n)R''$ instead of $k_j R''$. However, since this change is the same in both expressions, the difference between the two is still the same as in our main analysis. Thus, the proof of Proposition 1 goes through in exactly the same way.

It is easy to check that $\underline{k}_n$ plays no role in the proof of Propostion 2 since $k_j = 0$ there, that is, all (partially) integrated firms $1, ..., n$ no sell their quantity to firm I or to an outside

[37]Each curve $\underline{k}^*(N, \gamma)$ also exhibits a flat segment initially. This flat part corresponds to the smallest value of $\underline{k}$ such that the non-integrated competitors stop production (in our notation $\bar{\underline{k}}$), at which we stopped our simulations. For any $\underline{k} > \bar{\underline{k}}$, vertical integration is procompetitive simply because it reduces the cost of the only active firm. The fact that the curves $\underline{k}^*(N, \gamma)$ intersect for small values of N does therefore not conflict with the statement that vertical integration is procompetitive for a larger set of $\underline{k}$ the larger is γ.

market. The ame holds for the proof of Proposition 4.

Tedious but standard calculations that closely follow those of the proof of Proposition 3 show that the arguments used there also hold if all firms are vertically integrated. This is the case because the proofs of cases (i) and (ii) of Proposition 3 depend on the equilibrium capacities and quantities and not on the degree of integration. Although the degree of integration affecs the equilibrium, the calculations are very similar.

B.3 Omitted Propositions and Proofs of Section 5

The next proposition presents the result that similar statements as the ones given in Propositions 1 and 2 hold for the welfare analysis:

Proposition 7 *For any finite N there exists a $\underline{k}^*_W > 0$ such that for all $\underline{k} < \underline{k}^*_W$ vertical integration is welfare increasing at the margin. There also either exists a $\underline{k}^{**}_W < \bar{k}$ such that for all $\underline{k} > \underline{k}^{**}_W$ vertical integration is welfare decreasing at the margin, or it is welfare increasing at the margin for any $\underline{k}$ close to $\bar{k}$.*

Sketch of the proof We start with the case where $\underline{k} = 0$ and $\gamma = 0$. In the proof of Proposition 1 we calculated the left-hand side of (10). To determine the right-hand side of (10) we first insert $dQ/dk_I = dq_I/dk_I + Ndq_j/dk_I$, $dQ_{-I}/dk_I = Ndq_j/dk_I$ and $dQ/dk_j = dq_I/dk_j + dq_j/dk_j + (N-1)dq_h/dk_j$ into the right-hand side and then use equations (14), (15), (16) and (19) from the proof of Lemma 1, i.e., the derivatives of q_i with respect to k_j, $i, j = I, 1, ..., N$. Knowing that at $\underline{k} = 0$ and $\gamma = 0$ we have $q_I = q_j$, $k_I = k_j$ and $C''_I = C''_j$, the right-hand side simplifies to $-1/N$. But from the proof of Proposition 1 we know that the left-hand side is larger than $-1/N$ at $\underline{k} = 0$ and $\gamma = 0$. Therefore, marginal vertical integration is welfare increasing at this point. In the same way as in the proof of Proposition 1 we can show that it is also welfare increasing for $\gamma > 0$. By continuity there exists a threshold $\underline{k}^*_W$ such that vertical integration is welfare enhancing at the margin for all $\underline{k} < \underline{k}^*_W$.

Now we turn to the case where $\underline{k} = \bar{k}$. From the proof of Proposition 2 we know that in this case

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{C''_I P' \rho \frac{q_I}{k_I} + \sigma}{(N+1)\left(\rho^2 P'(C''_I - k_I P') + \sigma\right)}.$$

Proceeding in the same way as above to determine the right-hand side of (10) but now inserting

$\underline{k} = \bar{k}$, $q_j = k_j = 0$ yields

$$-\frac{P' q_I^2 C_I'' - k_I R' \left(C_I'' - k_I(2P' + q_I P'')\right)(k_I - \underline{\bar{k}})}{P' \rho q_I k_I^2 N(P' + q_I P'')} = -\frac{P' q_I^2 C_I'' + \sigma(k_I - \underline{\bar{k}})}{N P' \rho q_I k_I^2 (P' + q_I P'')}, \tag{50}$$

where the equality sign is due to $\sigma \equiv R' k_I (2P' k_I + P'' k_I q_I - C_I'')$ as defined in the proof of Proposition 2. Subtracting (50) from $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ then reveals that this difference has the same sign as

$$-k_I(1+N)\sigma^2 - \sigma P' \left[C_I''(N+1)(k_I \rho^2 + q_I^2) - k_I^2 \rho \left(P'(\rho(N+1) + q_I N) + P'' N q_I^2\right)\right]$$
$$-C_I'' q_I^2 \rho^2 (P')^2 \left[C_I''(N+1) - k_I P'(2N+1) - k_I q_I N P''\right] + (1+N)\underline{\bar{k}}\sigma \left[\sigma + \rho^2 P'(C_I'' - k_I P')\right].$$

The first three terms in this expression are negative while the last term is positive. Therefore, if the ex ante capacity that is needed to induce the non-integrated firms to stop producing, $\underline{\bar{k}}$, is small, the fourth term is small as well. In this case the expression is negative and welfare is decreasing at $\underline{k} = \underline{\bar{k}}$. By continuity there then exists a $\underline{k}_W^{**}$ such that for all $\underline{k} > \underline{k}_W^{**}$ vertical integration is welfare reducing at the margin. If instead $\underline{\bar{k}}$ is relatively large, the fourth term dominates the first three terms. The expression is then positive and vertical integration is welfare enhancing at the margin. ■

The next result is akin to Proposition 3:

Proposition 8 *Suppose either that (i) $R(K)$ is very inelastic or that (ii) the model is linear-quadratic. Then, for any finite N there either exists a unique $\underline{k}_W^* \in (0, \underline{\bar{k}})$ such that vertical integration is welfare enhancing at the margin for all $\underline{k} < \underline{k}_W^*$ and welfare reducing at the margin for all $\underline{k} > \underline{k}_W^*$, or vertical integration is always welfare enhancing.*

Sketch of the proof Case (i): From (10) we know that vertical integration enhances welfare if

$$N\left[-P'\left(q_j \frac{dQ}{dk_j} + (N-1)q_j \frac{dq_h}{dk_j} + q_I \frac{dq_I}{dk_j}\right) + R' k_j\right]\left(\frac{dk_j}{d\underline{k}}\right) + \tag{51}$$
$$+\left(-P'\left(q_I \frac{dQ}{dk_I} + q_j \frac{dQ_{-I}}{dk_I}\right) + R'(k_I - \underline{k})\right)\left(\frac{dk_I}{d\underline{k}}\right) > 0.$$

If R' is dominating all other derivatives, we can calculate $dk_j/d\underline{k}$ and $dk_I/d\underline{k}$ from (28) and (29) to get

$$\frac{dk_j}{d\underline{k}} = -\frac{1}{N+2} \quad \text{and} \quad \frac{dk_I}{d\underline{k}} = \frac{N+1}{N+2}. \tag{52}$$

Inserting this into the last expression and using the fact that R' is dominating all other derivatives yields $R'(-Nk_j+(N+1)(k_I-\underline{k}))/(N+2) > 0$. Differentiating the left-hand side of the last equation with respect to $\underline{k}$ and using (52) yields $d\left(R'(-Nk_j+(N+1)(k_I-\underline{k}))/(N+2)\right)/d\underline{k} = -R'/(N+2)^2 < 0$. Therefore, the term that determines the sign of (51) is strictly decreasing in $\underline{k}$. Since welfare is increasing in $\underline{k}$ at $\underline{k} = 0$, there is either a unique intersection point or none.

The proof for case (ii) proceeds along the same lines as the proof of case (ii) in Proposition 3 and is therefore omitted. ■

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