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UK macroeconomic volatility and the term structure of interest rates.

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Abstract

This paper uses a macro-finance model to examine the ability of the gilt market to predict fluctuations in macroeconomic volatility. The econometric model is a development of the standard 'square root' volatility model, but unlike the conventional term structure specification it allows for separate volatility and inflation trends. It finds that although volatility and inflation trends move independently in the short run, they are cointegrated. Bond yields provide useful information about macroeconomic volatility, but a better indicator can be developed by combining this with macroeconomic information.

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I. Introduction

Since the second world war, the world's developed economies have experienced marked fluctuations in macroeconomic volatility. These swings include the Great Inflation or Great Acceleration of the 1970s as well as the more recent Great Moderation, which were particularly pronounced in the UK. This phenomenon has attracted a great deal of attention from economists and policymakers, encouraging the development of Bayesian methods for the estimation of stochastic volatility models. Recent examples of this fast growing literature include **?**, **?**, **?**, **?** and **?** for the US. **?** and **?** for the UK.

Instead of using this approach, this paper develops a macro-finance (MF) model of the UK economy and Treasury bond market to help throw light on this experience and in particular the **?** and **?** conjecture that macroeconomic volatility is linked to the underlying rate of inflation[1]. This approach is potentially useful for analyzing swings in macroeconomic volatility because these should in principle be reflected in bond yields.[2] Term structure researchers are well aware of the importance of volatility for bond pricing. Indeed, in contrast to the MF literature, which invariably uses homoscedastic (or constant variance) models, the mainstream finance literature stems from the CIR (1985) model in which the volatility of the nominal interest or inflation rate driving the yield curve depends upon the square root of this rate, a mechanism that links the first and second moments of the system. As **?** note, this literature 'posits a short-rate process with a single stochastic central tendency and volatility'. These models do not attempt to disentangle the movements in first and

[1] Okun (1971) was the first to note the cross-country correlation between the level of inflation and its volatility in the OECD economies. In his Nobel prize speech, Milton Friedman (1977) noted the times series correlation between secular inflation and macroeconomic volatility. **?**, **?** and many others look at the empirical evidence, suggesting that it is an important international as well as a UK phenomenon. Devereux (1989), **?** and **?** provide possible theoretical explanations.

[2] Because bond prices are convex in interest rates and other macro variables, volatility increases the expected returns on these instruments and their prices (depressing yields) in the same way that it increases those of derivatives that have convex payoff structures.

second moments.

In this paper I begin by developing a homoskedastic benchmark MF model (M0). This handles the unit root seen empirically in nominal variables like inflation and interest rates using a common stochastic trend. A second latent variable models fluctuations in the equilibrium real interest rate.[3] This is mean reverting, suggesting that inflation, interest rates and bond yields are cointegrated. I then follow the mainstream finance approach by developing a model (M1) in which the stochastic trend driving the underlying inflation rate also drives macroeconomic volatility, as conjectured by Okun and Friedman. Spencer (2008) compares the performance of these models using US data. However this paper develops two more models that use a third latent factor to disentangle the inflation and volatility trends. These models have the same general structure (known as $EA_1(N)$ in the term structure literature) as the model of **?**, allowing me to draw upon the formulae derived there. In model M2, volatility depends upon one factor and underlying inflation upon another separate factor. Finally, M3 is an encompassing model which allows the inflation rate to depend upon both the volatility factor and the third factor, which acts as a 'wedge' between volatility and inflation. I find that empirically all of the other models are rejected in favour of M3. However, the inflation wedge factor in M3 is strongly mean reverting, meaning that although inflation, interest rates and volatility can move independently in the short run they are cointegrated.

These latent factors are estimated by the Extended Kalman Filter (EKF) which updates them appropriately in line with surprises in any macro or yield variables that they affect. The volatility factor can be distinguished from the inflation factor because it affects the level of bond yields in a heteroskedastic pricing framework, allowing revisions in this factor to be inferred from yield curve surprises that are

[3] This is a homoscedastic trend, which allows for negative real interest rates.

not correlated with macro surprises. Empirically, this factor also influences future macroeconomic volatility, which distinguishes it from a simple financial factor (or latent variable that affects the yield curve but not the macroeconomy). My model M2 restricts M3 by assuming that the volatility trend does not affect inflation. Consequently, this trend is informed by yield but not inflation or other macro surprises, providing a bond market based indicator of volatility. However M2 is also rejected against M3, in which the volatility factor is informed by both yield and macro surprises. This result suggests that although the bond market can anticipate movements in volatility, combining this with macroeconomic information gives a better indicator. M3 explains the rise in macroeconomic volatility seen since 2008 in terms of a rising volatility trend associated with a rise in the underlying rate of inflation.

The paper is set out as follows. Section II first describes the preliminary econometric tests used to inform the design of the macroeconomic model and its stochastic structure. It then outlines the bond pricing framework developed in **?** and used in this study. This section is supported by the two appendices. The empirical results are discussed in Section III, then Section IV offers a brief conclusion.

II The econometric specification

The model of the macroeconomy is based on the 'central bank model' (**?** and others). This represents the behavior of the macroeconomy in terms of the output gap, inflation and the short term interest rate. I represent inflation by the annual percentage change in the Retail Price Index excluding mortgage interest payments (RPIX, π_t)[4]. The three month Treasury Bill rate is used to represent the policy (or spot) rate ($r_{1,t}$). Both of these series were supplied by Datastream[5]. The GDP

[4] This was the policy objective (with a target rate of 2.5 %) between November 1992 and April 2004 when it was replaced by the Consumer Price Index (with a target rate of 2%). The CPI series is only available since 1996.

[5] These are annual rates in percentages. In the empirical model these were appropriately converted to quarterly decimal fractions by dividing by 400.

output gap (g_t) is the OECD measure, based on a trend filtering approach. The bond yield data were taken from the Bank of England's website and are derived using the methodology discussed in **?**. Their data set starts in January 1979, which determines the beginning of my estimation period. To represent this curve I use 1,2,3,5,7,10 and 15 year maturities, the longest one for which a continuous series is available. The macroeconomic data dictated a quarterly time frame. These macroeconomic variables are shown alongside the 15 year yield in Figure 3.

Table 1 shows the means, standard deviations and first order autocorrelation coefficients of the data over the estimation period: 1979Q1-2010Q2. The table also reports the ADF and KPSS (Kwiatowski *et. al.*, 1992) test results. The latter reject the hypothesis of stationarity for the levels of inflation and interest rates. The significance of this rejection is higher for the bond yields and tends to increase with maturity. Additional tests show that these rates and yields are cointegrated. Alternatively, these results could be the consequence of common structural shifts. Indeed, it is not difficult to identify dates at which these could have occurred *a priori*. The UK experienced two deep recessions over this period, which could be regime-shifting events. Changes of government can have similar effects. The evidence of a change in regime following the election of the Thatcher government in May 1979 now seems overwhelming (**?**), **?**). The UK monetary policy regime changed from monetary to exchange rate targets during the late 1980s and then to inflation targets in October 1992, culminating in the new monetary arrangements announced by the incoming Labour government in May 1997.

Preliminary tests using the Schwarz criterion on a stand-alone heteroskedastic macro VAR model suggested that a second order lag was appropriate. **?** tests also confirmed that the nominal data were conditionally heteroskedastic. The first stage of this test is to regress a variable such as the inflation or interest rate on its lagged values and the lagged 15 year yield, used as a proxy for the underlying inflation rate. The second stage takes the squared regression residuals as a measure of volatility and regresses them on the lagged 15 year yield. The t-statistics for the squared inflation and interest rate residuals were 6.74 and 3.25 respectively. These tests offer prima facie evidence of the bond market's ability to predict macroeconomic volatility.

Finally, I conducted a series of Chow tests, looking for structural breaks in the macro VAR model at dates when there was a change of policy regime. The shift from monetary to exchange rate targets was hard to pinpoint *a priori*, but the best results were obtained dating this as 1987Q3, consistent with the view that Mr. Lawson began to 'shadow' the ERM at about that time. The initial move to inflation targets was fixed at 1992Q3 (as they were announced that November). Both of these breaks were statistically significant at the 95% level. Conditional on these two shifts, the move to Bank of England independence following the Labour election victory in 1997Q2 was not significant. This gave a three regime macro model: 'monetary targets' 1979Q1-1987Q2; 'exchange rate targets' 1987Q3-1997Q2 and 'inflation targets' 1992Q3-2010Q2.

The macroeconomic framework

The structure of the econometric model is a Kalman vector autoregression or KVAR: an unrestricted difference equation system including Kalman filters and observable macro variables:

$$z_t = K + \Phi_0 y_t + \Sigma_{l-1}^{L} \Phi_l z_{t-l} + G\eta_t, \tag{1}$$

where: z_t is an $n-$vector of macroeconomic variables and y_t a $k-$vector of latent variables that allow for gradual changes in the equation intercepts or regression constants. G is an n^2 lower triangular matrix and η_t is an $n-$vector of $i.i.d$ orthogonal errors. In the general framework of model M3: $k = 3$, $n = 3$, $y_t = \{y_{v^*,t}, y_{\pi^*,t}, y_{\mathbf{r}^*,t}\}'$ and $z_t = \{\pi_t, g_t, r_{1,t}\}'$.The latent factors follow the first order process:

$$y_t = \theta + \Xi y_{t-1} + \varepsilon_t, \tag{2}$$

where $\theta = \{\theta_{v^*}, \theta_{\pi^*}, \theta_{\mathbf{r}^*}\}'$ and $\Xi = Diag\{\xi_{v^*}, \xi_{\pi^*}, \xi_{\mathbf{r}^*}\}$[6] and $\varepsilon_t = \{\varepsilon_{v^*,t}, \varepsilon_{\pi^*,t}, \varepsilon_{\mathbf{r}^*,t}\}'$ is a vector of orthogonal white noise error terms.

In model M3, $y_{v^*,t}$ is a martingale ($\xi_{v^*} = 1$). This drives macroeconomic volatility and is also linked through the cointegration coefficient ϕ to the inflation asymptote π_t^* by the relationship: $\pi_t^* = \phi y_{v^*,t} + y_{\pi^*,t} + \varphi_{\pi^*}$[7], where $y_{\pi^*,t}$ is the second latent variable (the inflation 'wedge') and φ_{π^*} is a shift constant. Thus if $\xi_{\pi^*} = 1$, $y_{\pi^*,t}$ is a random walk and the volatility and inflation trends diverge. Alternatively, if as the preliminary tests indicate inflation and volatility share a common trend, then these divergences are mean reverting ($|\xi_{\pi^*}| < 1$) and volatility and inflation trends are cointegrated. The third latent variable $y_{\mathbf{r}^*,t}$ allows for variations in the central tendency of the real interest rate $\mathbf{r}$, which is represented by $y_{\mathbf{r}^*,t} + \varphi_{\mathbf{r}^*}$ where $\varphi_{\mathbf{r}^*}$ is another shift constant. Since $y_{\pi^*,t}$ and $y_{\mathbf{r}^*,t}$ are Gaussian latent variables the effects of φ_{π^*} and $\varphi_{\mathbf{r}^*}$ cannot be distinguished from those of θ_{π^*} and $\theta_{\mathbf{r}^*}$ and the model is identified by setting $\theta_{\pi^*} = \theta_{\mathbf{r}^*} = 0$.

The central tendency of the nominal interest rate is thus $r_{1,t}^* = \phi y_{v^*,t} + y_{\pi^*,t} + y_{\mathbf{r}^*,t} + \varphi_{\pi^*} + \varphi_{\mathbf{r}^*}$. If both $y_{\mathbf{r}^*,t}$ and $y_{\pi^*,t}$ are mean reverting, this reverts to the asymptote implied by the cointegrating relationship: $r_{1,t}^{**} = \phi y_{v^*,t} + \varphi_{\pi^*} + \varphi_{\mathbf{r}^*}$. The output gap is assumed to be a zero-mean-reverting variable: $g_t^* = 0$. These equilibrium

[6] In this paper, $Diag\{\delta\}$ represents a matrix with the elements of the row vector δ in the main diagonal and zeros elsewhere. 0_a is the $(a \times 1)$ zero vector, 1_a is the $(a \times 1)$ summation vector, $0_{a,b}$ the $(a \times b)$ zero matrix; and I_a the a^2 identity matrix.

[7] Latent variables are scale-free but this relationship normalises the first two of these.

conditions are enforced by imposing a set of restrictions on (1):

$$\Phi_0 = (I - \Sigma_{l-1}^{L}\Phi_l)R,\ K = \Phi_0\varphi, \tag{3}$$

$$\text{where}:\quad \varphi' = \{0, \varphi_{\pi^*}, \varphi_{\mathbf{r}^*}\},\ R = \begin{bmatrix} \phi & 1 & 0 \\ 0 & 0 & 0 \\ \phi & 1 & 1 \end{bmatrix}$$

to give the equilibrium relationships: $z_t^* = \{\pi_t^*, g_t^*, r_{1,t}^*\}' = (I - \Sigma_{l-1}^{L}\Phi_l)^{-1}\Phi_0(y_t + \varphi) = R(y_t + \varphi)$.

Since the latent vector is not directly observable it has to be estimated using the Extended Kalman Filter (EKF) as described in **?**. The use of this filter is now standard practice in US MF models and Stanton (1999) argues that this performs well compared to alternative approaches. This system can be consolidated by defining $x_t = \{y_t', z_t'\}'; v_t = \{\varepsilon_t', \eta_t'\}'$ and combining (1) and (2) to get an $L-$th order difference system. This can then be arranged as a first order difference system called the state space form (**?**):

$$X_t = \Theta + \Phi X_{t-1} + W_t \tag{4}$$

where $X_t = \{y_t', z_t', ..., z_{t-l}'\}'$ is the state vector, $W_t = C.\{\varepsilon_t', \eta_t', 0_{1,N-k-n}\}'$ and Θ, Φ and C are defined in appendix A. X_t has dimension $N = k + nl$.

The stochastic structure

Macro-econometric models conventionally assume that the dynamic structure is linear and the error structure is Gaussian: $W_t \sim N(0_N, \Sigma(t))$. The variance structure is either assumed to be homoscedastic (*i.e.* with a fixed Σ) or to exhibit Auto-regressive Conditional Heteroscedasticity (ARCH). However, these specifications are restrictive because any specification of the error probability distribution that has a log-linear Moment Generating Function (MGF) generates a linear model, not just

Gaussian ones (?). This paper uses the ? square root conditional heteroskedasticity (SRCH) specification with its non-central χ^2 distribution to generate a set of SRCH macroeconometric models that allow the moments of the system to be linear in the stochastic trend $y_{v^*,t}$. The conventional Gaussian specification is called model M0 and is derived as a special case of the encompassing model: M3.

CIR (1985) describe the short term interest rate as a diffusion in continuous time and show that in discrete time, this has a non-central χ^2 distribution[8]. I use this distribution to model the stochastic trend. This makes its conditional MGF a log-linear function of $y_{v^*,t}$ [9]:

$$E[\exp[\nu.y_{v^*,t+1}|y_{v^*,t}]] = \exp[\frac{\nu\xi_{v^*}y_{v^*,t}}{1-\nu/c} - c\theta_{v^*}\ln[1-\frac{\nu}{c}]] \tag{5}$$

provided that: $\nu < c$, ?. E and V denote mean and variance and ν the Laplace parameter. Differentiating any MGF with respect to this parameter n times and then setting the parameter to zero gives the n-th moment of the associated distribution. Thus differentiating (5) with respect to ν once, twice and then setting ν to zero, returns the conditional mean and variance:

$$E[y_{v^*,t+1}|y_{v^*,t}] = \theta_{v^*} + \xi_{v^*}y_{v^*,t}, \; V[y_{v^*,t+1}|y_{v^*,t}] = \delta_{01} + \delta_{11}y_{v^*,t}; \tag{6}$$

$$\text{where}: \quad \delta_{01} = \theta_{v^*}/c, \delta_{11} = 2\xi_{v^*}/c.$$

Two special cases are of interest here. The first is the conventional MF Gaussian model M0, which is the limit of (5) in which the scale factor $(2c)$ becomes infinitely

[8] If we normalize the time interval $(s-t)$ in their equation (18) as unity and replace r by y_{π^*} then : $y_{\pi^*,t+1} \sim \chi^2[2cy_{\pi^*,t+1}; 2c\mu; 2\xi_{\pi^*}cy_{\pi^*,t}]$.

[9] Parameters c, ξ_{π^*} and θ_{π^*} are related to the scale factor $(2c)$, the non-centrality parameter $(2\xi_{\pi^*}cy_{\pi^*,t})$ and the degrees of freedom $(2c\theta_{\pi^*})$.

large. This gives the familiar log-linear formula for the MGF of a normal variable:

$$E[\exp[\nu.y_{v^*,t+1}|y_{v^*,t}]] = \exp[\nu(\theta_{v^*} + \xi_{v^*} y_{v^*,t}) + \frac{1}{2}\nu^2 \delta_{01}]. \tag{7}$$

with the conditional mean $E[y_{v^*,t+1}|y_{v^*,t}] = \theta_{v^*} + \xi_{v^*} y_{v^*,t}$ and variance $V[y_{v^*,t+1}|y_{v^*,t}] = \delta_{01}$. In this limit $\delta_{11} = 2\xi_{v^*}/c$ vanishes and the variance is constant. The second is the case of a unit root, in which $\xi_{v^*} = 1$, $\delta_{01} = \theta_{v^*} = 0$, $\delta_{11} = 2/c$, and (6) simplifies to[10]:

$$E[y_{v^*,t+1}|y_{v^*,t}] = y_{v^*,t},\ V[y_{v^*,t+1}|y_{v^*,t}] = \delta_{11} y_{v^*,t}. \tag{8}$$

This process is a martingale: the expectation of any future value is equal to the current value. However, unlike the random walk model, the error variance is also proportional to this value. All of these models can be represented in the linear form (2).

In model M3, this stochastic trend also conditions the volatility of the other variables. It is ordered as $x_{1,t} = y_{v^*,t}$, the first variable in the $(k+n)$ vector x_t. The other contemporaneous variables are put into an $(k+n-1)$ vector $x_{2,t}$, so that: $x_t = \{x_{1,t}, x'_{2,t}\}'$ and conformably: $v_t = \{w_{1,t}, v'_{2,t}\}'$ and $w_t = \{w_{1,t}, w'_{2,t}\}'$, where $w_{1,t} = \varepsilon_{\pi^*,t}$. Similarly, writing $X_t = \{x_{1,t}, X'_{2,t}\}'$ and partitioning W_t, Θ, Φ, C conformably (see appendix A), (4) becomes:

$$\begin{bmatrix} x_{1,t+1} \\ X_{2,t+1} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \Theta_2 \end{bmatrix} + \begin{bmatrix} \xi_1 & 0'_{N-1} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} x_{1,t} \\ X_{2,t} \end{bmatrix} + \begin{bmatrix} w_{1,t+1} \\ W_{2,t+1} \end{bmatrix} \tag{9}$$

where $\theta_1 = \theta_{\pi^*}$, $\xi_1 = \xi_{\pi^*}$ and $w_{1,t+1} = \varepsilon_{\pi^*,t+1}$. In this paper, subscripts 1 and

[10] This model is studied by **?** and his basic results are reported in Chapter 29 of **?**. Important results have also been obtained for this case by **?**.

2 denote partitions of N (or $k+n$) dimensional vectors and matrices into 1 and $(N-1)=(k+n-1)$. The stochastic structure for (9) is described in appendix A. The distribution of $x_{2,t}$ and $X_{2,t}$ conditional upon $x_{1,t-1}$ is assumed to be Gaussian. The conditional covariance of $X_{2,t}$ is $\Sigma_0+\Sigma_1 x_{1,t-1}$, where: $\Sigma_i = C_{22}\Delta_i C'_{22}$ and $\Delta_i = Diag\{\{\delta_{i2},...,\delta_{i(k+n)}\},0'_{N-k-n}\}$, $i=0,1$. C_{22} is a lower triangular and $\Delta_i; i=0,1$ are deficient diagonal $(N-1)^2$ matrices.

The yield curve framework

Bond yields are monitored by central banks because they reflect the market's view about future developments in the economy. In this paper, an econometric model of the yield curve is used to extract estimates of the inflation, volatility and real interest rate trends. It employs the arbitrage-free pricing approach, which assumes that riskless profit opportunities are eliminated but allows for risk premia (risky profits) using the risk-neutral probability measure Q. This adjusts the state probabilities using a state-dependent utility weight N_{t+1}[11] so that bond (and other asset) prices are discounted expectations of future payoffs under this measure:

$$P_{\tau,t} = \exp[-r_{1,t}]E^{\mathcal{Q}}[P_{\tau-1,t+1}|\ X_t],\ \tau=1,...,M. \tag{10}$$

where $E^{\mathcal{Q}}[P_{\tau-1,t+1}|\ X_t] = E[N_{t+1}P_{\tau-1,t+1}|\ X_t]$ is the risk neutral expectation (**?**, **?**)[12]. For the yield curve specification to be linear in the state variables and error terms the discount bond price $P_{\tau,t}$ must be loglinear:

$$P_{\tau,t} = \exp[-\gamma_\tau - \Psi'_\tau\ X_t], \tau=1,...,M. \tag{11}$$

[11] This is related to the equivalent Stochastic Discount Factor, which is $\exp[-r_{1,t}]N_{t+1}$.

[12] The Expectations Hypothesis employed in many economic models is the special case in which $E^{\mathcal{Q}}=E$.

where τ denotes maturity. It can be shown that N_{t+1} must also be loglinear:

$$-n_{t+1} = -\ln N_{t+1} = \omega_t + \lambda_{1,t} x_{1,t+1} + \lambda'_{2,t} u_{2,t+1} \tag{12}$$

where $u_{2,t+1}$ is a $(k+n-1) \times 1$ vector of $N(0,1)$ $i.i.d$ error terms driving $x_{2,t+1}$, conditional upon $x_{1,t}$ (appendix A). $\lambda_{1,t}$ is a scalar; $\lambda_{2,t}$ and is a $(k+n-1) \times 1$ vector. These multipliers determine the risk premia and are known as 'price of risk' variables. They must also be linear in the state variables (**?**). For example, $\lambda_{1,t}$, which plays an important role in this analysis, is specified as:

$$\lambda_{1,t} = \lambda_{10} + \lambda_{11} x_{1,t}. \tag{13}$$

If $\lambda_{1,t} = 0$, then a portfolio that is constructed so that it is only exposed to shocks in x_1 (i.e. $y_{v^*,t}$) has a zero risk premium (*i.e.* is expected to earn the spot rate). If it is constant ($\lambda_{11} = 0$), then variations in this risk premium depend only upon variations in volatility, such as those induced by $y_{v^*,t}$ in the SRCH models: (M1, M2 and M3). The associated volatility parameter δ_{11} plays the key role in the associated yield curves. Alternatively if $\lambda_{11} \neq 0$ then the trend can influence the risk premia thorough variations in the price of risk, even if volatility is fixed as it is in the M0: λ_{11} plays the key role in that model.

Similarly, $\lambda_{2,t}$ is a vector of coefficients related to the prices of risk associated with shocks to $x_{2,t+1}$. It is specified in appendix B, which also reports the relationships defining the coefficients of (11) that are derived in appendix B of **?**. It is convenient to partition these coefficients conformably with (9) as $\Psi_\tau = \{\psi_{1,\tau}, \Psi'_{2,\tau}\}'$. These coefficients are recursive in the sense that $\Psi_{2,\tau}$ affects $\psi_{1,\tau}$ (but not vice versa) while both affect γ_τ. They are also recursive in maturity. Since $-p_{\tau,1} = r_{1,t}$ they have the starting values: $\gamma_1 = \psi_{1,1} = 0$, and $\Psi_{2,1} = J_{2,r}$, where $J_{2,r}$ is the selection vector such that: $r_{1,t} = J'_{2,r}X_{2,t}$. In the homoscedastic model (M0) the recursions for the slope coefficients $\psi_{1,\tau}$ and $\Psi_{2,\tau}$ are both linear, while those for the intercepts γ_τ are quadratic. The recursion formula for $\Psi_{2,\tau}$ is common to all models, but the SRCH models differ from M0 because the recursion $\psi_{1,\tau}$ include log-linear terms.

The natural logarithm of the price $P_{\tau,t}$ is denoted by $p_{\tau,t}$ and under these assumptions this is linear in X_t (11). Reversing sign and dividing by maturity τ gives the discount yields: $r_{\tau,t} = -p_{\tau,t}/\tau = \alpha_\tau + \beta'_\tau X_t$, where: $\alpha_\tau = \gamma_\tau/\tau$, $\beta_\tau = \Psi_\tau/\tau$ (using (11)). These slope coefficients β_τ are known as 'factor loadings' and are given in appendix A. In the empirical models reported in the next section, I stack the yield equations for $\tau = 4, 8, 12, 20, 28, 40$ and $M = 60$ quarters, to get the vector $r_t = \{r_{4,t}, r_{8,t}, r_{12,t}, r_{20,t}, r_{28,t}, r_{40,t}, r_{60,t}\}'$. Adding a conformable *i.i.d.* Gaussian measurement error vector e_t gives the multivariate regression model:

$$r_t = \alpha + B'X_t + e_t = \alpha + B'_0 y_t + \Sigma^L_{l-1} B'_l z_{t+1-l} + e_t \tag{14}$$

$$\text{where} : e_t \sim N(0, \bar{P}); \bar{P} = Diag\{\rho_1, \rho_2, ..., \rho_7\}.$$

Special cases

This structure (M3) encompasses several important special cases. The first is the conventional homoskedastic model: (M0). Volatility is constant in this specification ($\Sigma_1 = 0$), so $y_{v^*,t}$ is redundant and eliminated from the model ($k = 2$). The variable inflation asymptote π^*_t is just driven by $y_{\pi^*,t}$. The second is the conventional square root volatility model (M1), which assumes that the volatility and inflation trends are identical. In this case we eliminate instead the inflation 'wedge' factor $y_{\pi^*,t}$ and use $y_{v^*,t}$ to drive the inflation & interest rate asymptotes as well as volatility. These two specifications are described in more detail in Spencer (2008).

Another special case (M2) results if we specify separate volatility and inflation trends. In this case $y_{v^*,t}$ drives volatility without affecting inflation (or interest) rates, which only depends upon $y_{\pi^*,t}$ (or $y_{r^*,t}$). This model has the same form as the encompassing model M3 but sets the cointegrating coefficient ϕ in the first column of the long run response matrix R in (3) to zero, making the asymptotes of the macro variables independent of $y_{v^*,t}$, which only influences their variance structure. It is only possible to separate volatility and inflation trends like this in a yield model if they are separately priced in the bond market. In the specification of this paper, the effects of the volatility factor $y_{v^*,t}$ on bond yields are shown by the first column of B_0' in (14) (which contain the coefficients $\psi_{1,\tau}$ in (20)). Appendix B shows that they depend upon Σ_1 (which shows the impact of $y_{v^*,t}$ on the volatility structure) and λ_{10} (the price of volatility risk). In M2, revisions in the volatility factor are inferred from surprises in the yield curve that are not explained by surprises in the other state variables. If the bond market can sense changes in macroeconomic volatility then these will be reflected in $y_{v^*,t}$ and hence future macroeconomic volatility. Otherwise the estimation procedure would return $\Sigma_1 \simeq 0$ and we would interpret $y_{v^*,t}$ as a pure 'financial factor' that in M2 did not affect the macroeconomy at all. In this specification, $y_{v^*,t}$ provides a pure bond market assessment of volatility.

In the M1 model $y_{v^*,t}$ doubles up as the volatility trend and the central tendency of inflation. Consequently, the first filter is then informed by both yield and macro surprises, so it is no longer a purely yield-based measure. This is also true of the general specification M3, which allows for temporary divergences between the trend and the central tendency of inflation. We now consider the differences in the empirical performance of these various models and address the various issues raised in the introduction.

III. The empirical model

The empirical framework consists of three equations describing the macroeconomic variables (1) and seven equations describing the representative yields (14). These furnish the 'measurement equations' of the Kalman filter used to extract the latent variables, while the 'transition equations' are provided by (4). Spencer (2008) describes the Kalman learning model and the resulting likelihood function for this model framework as well as the quasi-maximum likelihood method used to estimate it. These models were estimated and tested using the Nelder-Mead Simplex and numerical gradient algorithms *fminsearch* and *fminum* on $Matlab$. Table 2 reports the likelihood statistics for the various models and table 3 the parameter values for the encompassing model: M3.

I started by estimating the homoskedastic model (M0), specified in line with the preliminary results, with three monetary policy regimes. This was estimated in two stages, first as a stand-alone macro KVAR model consisting of (4) and then after adding the yield equations (14). The macro-dynamic parameters (table 3(a)) are largely determined at the first stage and they shift very little at the second stage, which essentially fits the yields using the variance (table 3(b)) and price of risk (table 3(c)) parameters. The SRCH models (M1, M2 and M3) were then estimated by adding the extra variance parameters and in the case of M2 and M3, increasing the number of latent variables from $k = 2$ to 3.

The encompassing model M3 uses 105 parameters[13] and has a loglikelihood L(2)=8986.3 as shown in table 2. M2 separates the volatility and inflation trends, saving one degree of freedom by setting $\phi = 0$. This specification has a lower likelihood (8970.1) than M3, indicating that although the bond market provides a

[13] There are 3 sets of dynamic parameters ξ_{π^*},$\xi_{\mathbf{r}^*}$,φ_{π^*},$\varphi_{\mathbf{r}^*}$ and $\Phi(18)$, giving a total of 66. In M3 we add the cointegration coefficient ϕ, the variance parameters δ_{01}, $\boldsymbol{\Delta}_0(5)$, $\boldsymbol{\Delta}_1(5)$ and $G(3)$ and the risk parameters λ_{10}, $H_1(4)$, $\Upsilon_1(3)$ and $\Lambda_{22}(16)$. It was found that although $\lambda_{22,\mathbf{r}^*\mathbf{r}^*}$ was significant (table 3(c)) the remaining elements of the first two rows of Λ_{22} and Υ_1 were poorly determined and could be eliminated without significantly reducing the likelihood. The structural parameters μ and c can be obtained from (6) given ξ_1, δ_{01} and δ_{11}.

good volatility indicator (in M2) we can improve upon this significantly by informing $y_{v^*,t}$ using surprises in inflation and other macro variables as well as surprises in bond yields (as in M3). Table 3(a) shows that the autoregressive coefficient ξ_{π^*} associated with the inflation wedge is well below unity in all three target regimes, suggesting that volatility, inflation and interest rates can move independently in the short run in M3, but are cointegrated. Model M1 specializes M3 by assuming a common volatility and inflation factor as in the mainstream term structure model, saving 6 degrees of freedom[14]. These restrictions are rejected by the data: the $\chi^2(6)$ likelihood ratio test gives an acceptance value of $p \simeq 0$. M0 is also nested in M3, employing 10 restrictions[15]. However, its loglikelihood of L(0)=8601.9 is much lower than for the SRCH models: square root volatility is very significant in the UK economy. This result parallels my US findings (**?**) and the results of **?** and others using the conventional yield-factor approach to the term structure.

Finally, a single policy regime version of M3 was estimated. This saves 44 degrees of freedom by using a single set of dynamic parameters (ξ_{π^*},$\xi_{\mathbf{r}^*}$,φ_{π^*},$\varphi_{\mathbf{r}^*}$ & Φ) and has a loglikelihood L(2)=8946.1. Comparing this with M3, the likelihood ratio test statistic of 80.4 is higher than the 95% $\chi^2(44)$ critical value of 60.1, suggesting that structural change is significant in this framework. The next section discuses the features of my preferred 3-regime model: M3.

The empirical macro-model (M3)

At the core of model M3 there is an autoregressive macro model. The novelty here is that volatility and the inflation and interest rate asymptotes are variable, driven by the stochastic trend $y_{v^*,t}$ shown in the top panel of Figure 2. In the short term, inflation and interest rates are affected by two additional but mean reverting

[14] This model saves three parameters by reducing the dimensionality of the vectors ξ, $\mathbf{\Delta}_0$ and $\mathbf{\Delta}_1$ by one, and another three by eliminating the first column of Λ_{22}.

[15] This model eliminates ξ_{v*}, δ_{01}, $\Delta_1(5)$ and the first column of $\Lambda_{22}(3)$.

factors shown in the other panels. Most of the variance in this model is explained by $y_{v^*,t}$. This factor is estimated by a Kalman filter and reflects cumulative residuals in both macro and yield variables, which are largely negative until the mid 1990s when volatility appeared to stabilize. The Kalman gain matrix (which shows the way revisions in the latent variables respond to the forecast errors) reveals that this is largely determined by the inflation, 3 and 7 year yield residuals, which depress the volatility/inflation trend during the recessions of the early 1980s and early 1990s. However, the most dramatic fall appears to have occurred in 1997-98 following the independence of the Bank of England, pushing it close to zero.

Figures 3 to 6 show how $y_{v^*,t}$ influences the variances of the system (as well as the mean of the nominal variables). The narrowing of the confidence intervals during the Great Moderation is particularly marked, consistent with the relatively small shocks experienced over this interval. However, the confidence bands do not encompass all of the shocks occurring at the beginning and the end of the estimation period. In particular, Figures 4 and 5 show that the large negative output shocks that were seen post Lehman in 2008Q3-2009Q1 and the large negative interest rate shock of 2008Q4 are outside their 95% confidence intervals. In model M3, the volatility and inflation factors can move independently in the short term, allowing it more flexibility than model M2 over this period. For example the rise in inflation associated with the hike in world commodity prices in 2007-08 was initially reflected in the inflation wedge (shown in Figure 2(b)) rather than the volatility trend (2(a)), which remains relatively low until the summer of 2009.

The impulse responses show the dynamic effects of innovations in the macroeconomic variables on the system. Because these innovations are correlated empirically, I use the orthogonalized innovations obtained from the triangular factorization defined in (1). The orthogonalized impulse responses show the effect on the macroeconomic system of increasing each of these shocks by one percentage point for just one period using the Wold representation of the system as described for example in **?**. This arrangement is affected by the ordering of the macroeconomic variables in the vector x_t, making it important to order the variables in terms of their likely degree of exogeneity or sensitivity to contemporaneous shocks. The factors y_t are assumed to reflect exogenous expectational influences and are ordered first in the sequence. Next, I follow **?** and use the ordering: output, inflation then interest rates.

Figure 7 reports the results of this exercise. The model gives a plausible description of the macroeconomic dynamics, in line with prior expectations, in contrast to many VAR-type results (**?**). Its use of Kalman filters to pick up the effect of unobservable expectational influences seems to solve the notorious price puzzle - the tendency for increases in policy interest rates to anticipate inflationary developments and apparently cause inflation. The filter $y_{v^*,t}$ dictates the long run equilibrium of the macroeconomy (and its volatility). These effects are persistent, but the other responses are transitory. The pattern of responses is broadly similar in the three policy regimes, although interest rates appear to have been much less sensitive to inflation and output shocks in regime (b), when the UK was a targeting the exchange rate.

The empirical yield model

The behavior of the yield curve is dictated by the factor loadings (β_τ). These are depicted in Figure 8, and show the effect on different maturities (expressed in quarters) of increasing the each of the driving variables in turn by one percentage point compared to its historical value. The left hand side panel for each target regime shows the loadings on $y_{v^*,t}$, $y_{\pi,t}$ and $y_{r,t}$ while the right hand side panel shows those on π, g and r. The spot rate provides the link between the macroeconomic model and the term structure. Since it *is* the 3 month yield, this variable has a unit coefficient at a maturity of one quarter and other factors have a zero loading. This variable determines the 'slope' of the yield curve. Medium maturity yields are influenced by the behavior of the output gap, reflecting the effect of the business cycle. The loading on this factor then fades gradually over the longer maturities, allowing this to act as a 'curvature' factor. In contrast, the persistence of $y_{v^*,t}$ means that its loadings moves up towards unity over the 2 to 15 year maturity range, so that it broadly acts as a 'level' factor. The other loadings are relatively small.

IV. Conclusion

The econometric model used in this paper is a development of the 'square root volatility model' that was originally used by Cox et al (1985) to model the term structure of interest rates. However, the use of a MF specification allows me to relate the volatility trend indicated by the bond market to fluctuations in the macroeconomy. It also allows the volatility and inflation trends to be distinguished, showing that although they can move independently in the short run they share a common long run trend, providing support for the Okun-Friedman conjecture. The linearity of this structure allows a straightforward comparison with linear macro-finance models such as **?** that assume a homoscedastic variance structure. This comparison provides overwhelming evidence of conditional heteroscedasticity in the UK. As in the stochastic volatility literature cited in the introduction, I find that fluctuations in volatility are persistent, exhibiting a unit root. Mathematically, it generates linear structures that lend themselves not just to research on the term structure but to optimal control and similar intertemporal optimization problems (as shown by **?**).

Because it allows for square root volatility, my yield curve model resembles the traditional latent factor term structure model. However, like any macro-finance model, it can use a relatively large number of parameters because these are informed by macroeconomic as well as yield data. I find that the behavior of the yield curve is largely dictated by three factors: the inflation factor, the business cycle and the spot rate. The model is consistent with the traditional three latent factor specification in this respect, but links these factors into the behavior of the macroeconomy.

Appendix A: The state-space representation of the model

Stacking (1) and (2) puts the system into state space form (4), where $X_t = \{y_t', z_t', ..., z_{t-l}'\}'$, $W_t = C.\{\varepsilon_t', \eta_t', 0_{1,N-k-n}\}'$ and:

$$\Theta = \{\theta, K' + \theta'\Phi_0', 0_{1,N-k-n}\}'; \tag{15}$$

$$\Phi = \begin{bmatrix} \Xi & 0_{k,n} & ... & 0_{k,n} & 0_{k,n} \\ \Phi_0\Xi & \Phi_1 & ... & \Phi_{l-1} & \Phi_l \\ 0_{n,k} & I_n & .. & 0_{n,n} & 0_{n,n} \\ & & & & \\ 0_{n,k} & 0_{n,n} & ... & I_n & 0_{n,n} \end{bmatrix} = \begin{bmatrix} \xi_1 & 0_{N-1}' \\ \Phi_{21} & \Phi_{22} \end{bmatrix}.$$

The second matrix repartitions Φ conformably with (9), so that Φ_{21} is $(N-1)\times 1$ and Φ_{22} is $(N-1)^2$. Similarly:

$$C = \begin{bmatrix} I_k & 0_{k,n} & 0_{k,(N-k-n)} \\ \Phi_0 & G & 0_{n,(N-k-n)} \\ 0_{(N-k-n),k} & 0_{(N-k-n),n} & 0_{(N-k-n),(N-k-n)} \end{bmatrix} = \begin{bmatrix} 1 & 0_{N-1}' \\ C_{21} & C_{22} \end{bmatrix}.$$

where: C_{21} is $(N-1)\times 1$ and C_{22} is $(N-1)^2$. Conditional on the error term in the stochastic trend $w_{1,t+1}$, the error structure of (9) is assumed to be Gaussian:

$$W_{2,t+1} = C_{21}w_{1,t+1} + C_{22}S_tU_{2,t+1} \tag{16}$$

$$U_{2,t+1} \sim N(0_{N-1}, D)$$

where: $U_{2,t+1} = \{(u'_{2,t+1}, 0'_{N-k-n}\}'$, $S_t = Diag\{\{(\delta_{02} + \delta_{12}x_{1,t})^{\frac{1}{2}}, ..., (\delta_{0(k+n)} + \delta_{1(k+n)}x_{1,t})^{\frac{1}{2}}\}, 0'_{N-k-n}\}$, $D = Diag\{1'_{k+n-1}, 0'_{N-k-n}\}$, so that $S_tD = S_t$ and $E[u_{2,t+1}w_{1,t+1}] = 0_{(k+n-1)}; u_{2,t+1} \sim N[0_{(k+n-1)}, I_{(k+n-1)}]$.

Appendix B: The yield specification

This appendix reports the formulae for the coefficients of (11) derived in **?** appendix B, using the assumptions of sections 2.2 and 2.3. Since the m period bond is the discounted expectation (under $\mathcal{Q}$) of the $(m-1)$ period bond in the next period, these are generated by recursion relationships.

I also need to specify the prices of risk associated with shocks to $x_{2,t+1}$. I follow **?** and stack these in a $(N-1) \times 1$ deficient vector $\Lambda_{2,t} = [\lambda'_{2,t}, 0'_{N-(k+n)}]'$ which is conformable with the equation system for $X_{2,t+1}$ in (9). This is modelled as:

$$\Lambda_{2,t} = S_t C'_{22}\Lambda_{20} + S_t^{-1}C_{22}^{-1}\Lambda_{21}x_{1,t} + S_t^{-1}C_{22}^{-1}\Lambda_{22}X_{2,t} \tag{17}$$

$\Lambda_{22} = \{\Lambda_{22,ij}\}$ is an $(N-1)^2$ matrix that has the effect of adjusting $\Phi_{22}^{\mathcal{Q}} = \Phi_{22} - \Lambda_{22}$ in (18) below to allow for the effect of the 5 variables of $x_{2,t}$ on their associated prices of risk. $\Lambda_{22,ij}; i, j = 1, ..., 5$ are parameters (estimates are reported in table 3(c)) but the other elements are zero. Similarly the first 5 elements in the vectors $\Lambda'_{12}, \Lambda_{20}$ are parameters (table 3(c)) and the other elements are zero.

Because the shocks in (16) are Gaussian (conditional upon $w_{1,t-1}$) they are common to all of the yield models M0-M3 and have a standard linear recursive structure[16]:

$$\begin{aligned}\Psi_{2,\tau} &= (\Phi_{22}^{\mathcal{Q}})'\Psi_{2,\tau-1} + J_{2,r} \\ &= (I - (\Phi_{22}^{\mathcal{Q}})')^{-1}(I - ((\Phi_{22}^{\mathcal{Q}})')^{\tau})J_{2,r}.\end{aligned} \tag{18}$$

[16] I assume that the roots of this sub-system are stable under $\mathcal{Q}$, so this has the asymptote: $\Psi_2^* = lim_{\tau\to\infty}\Psi_{2,\tau} = (I - (\Phi_{22}^{\mathcal{Q}})')^{-1}J_{2,r}$.

This gives the slope coefficients of the homoskedastic model M0 and is derived as equation (16) in Spencer (2008). The intercept follows a quadratic recursion in this case:

$$\gamma_\tau = \gamma_{\tau-1} + (\Theta_2 - \Sigma_0\Lambda_{20})'\Psi_{2,\tau-1} - \frac{1}{2}\Psi'_{2,\tau-1}\Sigma_0\Psi_{2,\tau-1}. \qquad (19)$$

In the SRCH models, the response of the yields to the volatility trend is also given by a quadratic recursion relationship:

$$\psi_{1,\tau} = \frac{[\psi_{1,\tau-1} + \lambda_{10} + \Psi'_{2,\tau-1}C_{21}]}{1 + [\psi_{1,\tau-1} + \lambda_{10} + \Psi'_{2,\tau-1}C_{21}]/c} - \frac{\lambda_{10}}{1 + \lambda_{10}/c} - \Psi'_{2,\tau-1}\Upsilon_1 - \frac{1}{2}\Psi'_{2,\tau-1}\Sigma_1\Psi_{2,\tau-1}. \qquad (20)$$

((18) in Spencer (2008), with $\xi_1^{\mathcal{B}} \equiv \xi_1^{\mathcal{Q}} = 1$), where: $\Upsilon_1 = \Lambda_{21} + \Sigma_1\Lambda_{20}$. Recall that $c = 2/\delta_{11}$. As noted in section II, the effect of the volatility factor on macro volatility via Υ_1 and Σ_1 plays the key role here. If these objects are zero then the trend does not affect volatility. If $y_{v^*,t}$ does not affect inflation (M2) then it does not affect the macroeconomy and acts as a pure financial factor (provided that Λ_{21} and λ_{10} are non-zero so that it has an effect under $\mathcal{Q}$).

The intercept recursion is non-linear ((20) in Spencer (2008), where $\mu \equiv \theta_1$):

$$\gamma_\tau = \gamma_{\tau-1} + (\Theta_2 - C_{21}\theta_1 - \Sigma_0\Lambda_{20})'\Psi_{2,\tau-1} - \frac{1}{2}\Psi'_{2,\tau-1}\Sigma_0\Psi_{2,\tau-1} + c\theta_1 \ln[\frac{c + \psi_{1,\tau-1} + \lambda_{10} + \Psi'_{2,\tau-1}C_{21}}{c + \lambda_{10}}]. \qquad (21)$$

TABLE 1

Data Summary Statistics: 1979Q1-2010Q2

	g	π	r_1	r_4	r_8	r_{12}	r_{20}	r_{28}	r_{40}	r_{60}
Mean	0.0119	-0.0008	0.0194	0.0189	0.0191	0.0193	0.0196	0.0198	0.0199	0.0195
Std.	0.0090	0.0062	0.0096	0.0088	0.0084	0.0082	0.0080	0.0080	0.0079	0.0074
Skew.	2.1675	-0.4667	0.3350	0.1389	0.1410	0.1816	0.2358	0.2687	0.3128	0.3662
Kurt.	4.7207	0.2186	-0.6555	-0.8022	-0.8614	-0.8875	-0.9668	-1.0361	-1.0384	-0.9174
Min	0.0033	-0.0173	0.0010	0.0015	0.0023	0.0035	0.0055	0.0071	0.0083	0.0096
Max	0.0513	0.0125	0.0407	0.0374	0.0378	0.0382	0.0389	0.0386	0.0386	0.0372
Auto.	0.9429	0.9750	0.9635	0.9734	0.9717	0.9699	0.9691	0.9700	0.9702	0.9656
KPSS	0.7602	0.2668	0.9400	1.0016	1.1037	1.1269	1.1390	1.1410	1.1375	1.1161
ADF	-3.79**	-2.215	-1.849	-1.906	-2.082	-2.018	-1.957	-1.934	-1.935	-1.963

Output gap (g) is from OECD; RPIX inflation (π) and 3 month Treasury bill rate (r) are from Datastream. Yield data are discount bond equivalent data compiled by the Bank of England and published on their website. *Mean* denotes sample arithmetic mean expressed as percentage p.a.; *Std.* standard deviation and *Skew.*& *Kurt.*are standard measures of skewness (third moment) and excess kurtosis (fourth moment). *Auto* is the first order autocorrelation coefficient. *KPSS* is the Kwiatowski et al (1992) statistic for the null hypothesis of level stationarity and *ADF* is the Adjusted Dickey-Fuller statistic for the null of non-stationarity. The 5% significance levels are 0.463 and (-) 2.877 respectively.

TABLE 2
Model evaluation

Model	Specification	Parameters k(M)	Loglikelihood L(M)	Testing against M3 k(3)-k(M)	2x(L(3)-L(M))
M0		94	8601.9	11	768.80
					19.68
					0.00
M1		99	8657.2	6	658.20
					12.59
					0.00
M2		104	8070.1	1	32.40
					3.84
					0.00
M3					
		105	8986.3		

M0 represents the standard macro finance model. It has a unit root ($\xi_1 = 1$), a homoscedastic macro model ($\delta_{11} = 0, \Delta_1 = 0_4$) and Vasicek (1979)-type yield specification. M1 has a unit root but allows for square root volatility in the macro model and uses the CIR (1985)-type yield specification. M2 is similar to M1 but has a separate inflation factor. M3 is the encompassing specification.

TABLE 3a

Dynamic model structure (M3)

(asymptotic t-values in parentheses)

	Parameter	t-value				
ϕ	1.137	2.93				
Target:	(i) Monetary		(ii) Exchange rate		(iii) Inflation	
Φ_1	Parameter	t-value	Parameter	t-value	Parameter	t-value
$\phi_{1,\pi\pi}$	1.2019	10.34	1.1774	7.58	1.2228	12.92
$\phi_{1,\pi g}$	0.1607	3.66	0.1657	3.48	0.1601	5.66
$\phi_{1,\pi r}$	0.0007	0.01	0.0009	0.01	0.0000	0.00
$\phi_{1,g\pi}$	0.0403	0.24	0.0019	0.01	0.0365	0.26
$\phi_{1,gg}$	1.1283	8.52	1.1277	5.31	1.1544	9.79
$\phi_{1,gr}$	0.0015	0.01	0.0000	0.00	-0.0002	0.00
$\phi_{1,r\pi}$	0.1306	2.22	0.0907	1.48	0.1257	3.07
$\phi_{1,rg}$	0.0014	0.10	0.0027	0.17	0.0012	0.31
$\phi_{1,rr}$	0.8438	12.16	0.9175	11.87	0.8974	15.12
Φ_2						
$\phi_{2,\pi\pi}$	-0.2071	-1.96	-0.2197	-1.46	-0.2401	-2.60
$\phi_{2,\pi g}$	-0.1086	-2.22	-0.1110	-2.36	-0.1146	-3.86
$\phi_{2,\pi r}$	-0.0021	-0.02	-0.0080	-0.07	-0.0056	-0.08
$\phi_{2,g\pi}$	-0.0238	-0.16	-0.0151	-0.07	-0.0431	-0.31
$\phi_{2,gg}$	-0.1860	-1.24	-0.2259	-1.12	-0.1904	-1.41
$\phi_{2,gr}$	-0.0081	-0.06	-0.0086	-0.05	-0.0031	-0.02
$\phi_{2,r\pi}$	-0.0364	-0.60	-0.0670	-1.01	-0.0590	-1.47
$\phi_{2,rg}$	0.0408	2.50	0.0412	2.36	0.0410	8.82
$\phi_{2,rr}$	0.1073	1.81	0.0199	0.28	0.0370	0.66
Ξ						
$(1-\xi_{\pi^*})$	0.7379	6.89	0.4212	5.73	0.3593	6.98
$(1-\xi_{\mathbf{r}^*})$	0.3060	3.78	0.4336	5.00	0.5286	6.35
φ						
φ_{π^*}	-0.0607	-0.16	0.0120	2.21	-0.0028	-0.44
$\varphi_{\mathbf{r}^*}$	-0.0566	-0.16	0.0096	2.35	0.0027	0.98

TABLE 3b

Variance structure (M3)

(asymptotic t-values in parentheses)

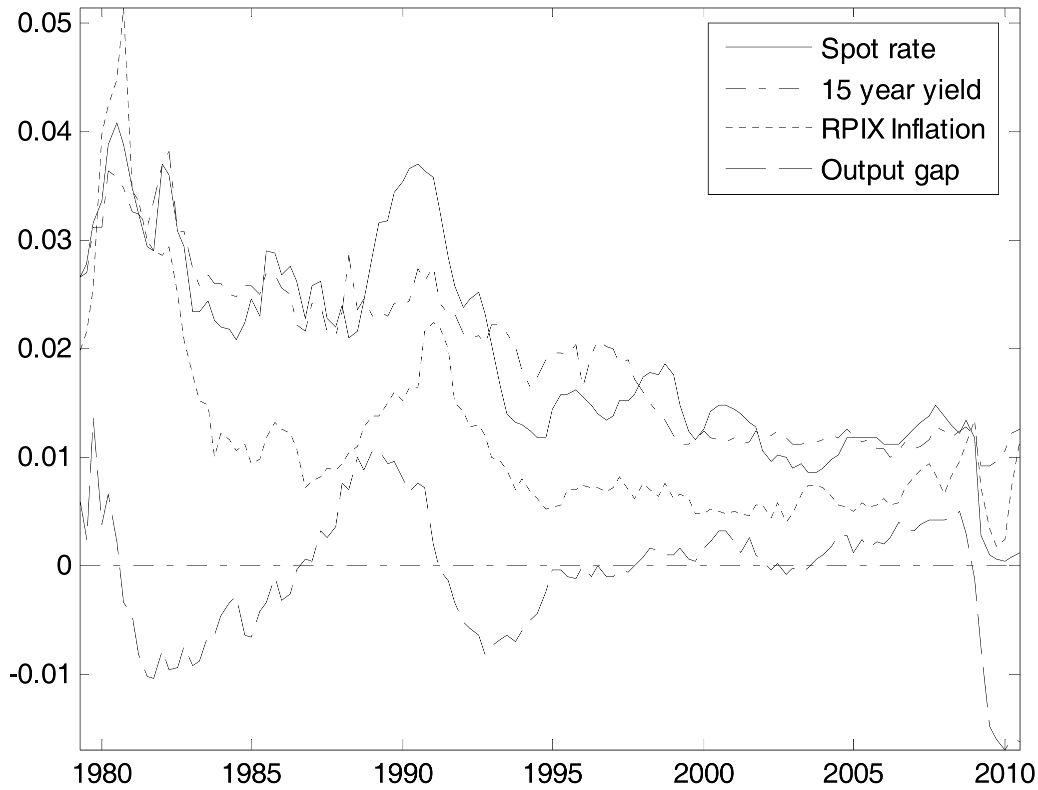
$\mathbf{\Delta}_0$	Parameter	t-value	$\mathbf{\Delta}_1$	Parameter	t-value	$\mathbf{G}$	Parameter	t-value
δ_{01}	(-)	(-)	δ_{11}	0.0212	8.54			
$\delta_{0\boldsymbol{\pi}^*}$	1.9800×10^{-5}	0.01	$\delta_{1\boldsymbol{\pi}^*}$	0.2069	1.95			
$\delta_{0\mathbf{r}^*}$	6.1088×10^{-3}	2.54	$\delta_{1\mathbf{r}^*}$	-0.0756	-2.19	$g_{g\pi}$	-0.0837	-0.90
$\delta_{0\pi}$	1.7010×10^{-4}	0.03	$\delta_{1\pi}$	0.0228	9.32	$g_{r\pi}$	0.2922	2.85
δ_{0g}	1.0001×10^{-3}	6.40	δ_{1g}	-0.0171	-6.15	g_{rg}	0.3723	12.46
δ_{0r}	4.1800×10^{-5}	0.04	δ_{1r}	0.0242	9.30			

Table 3c: Risk adjustment structures (M3)

(asymptotic t-values in parentheses.)

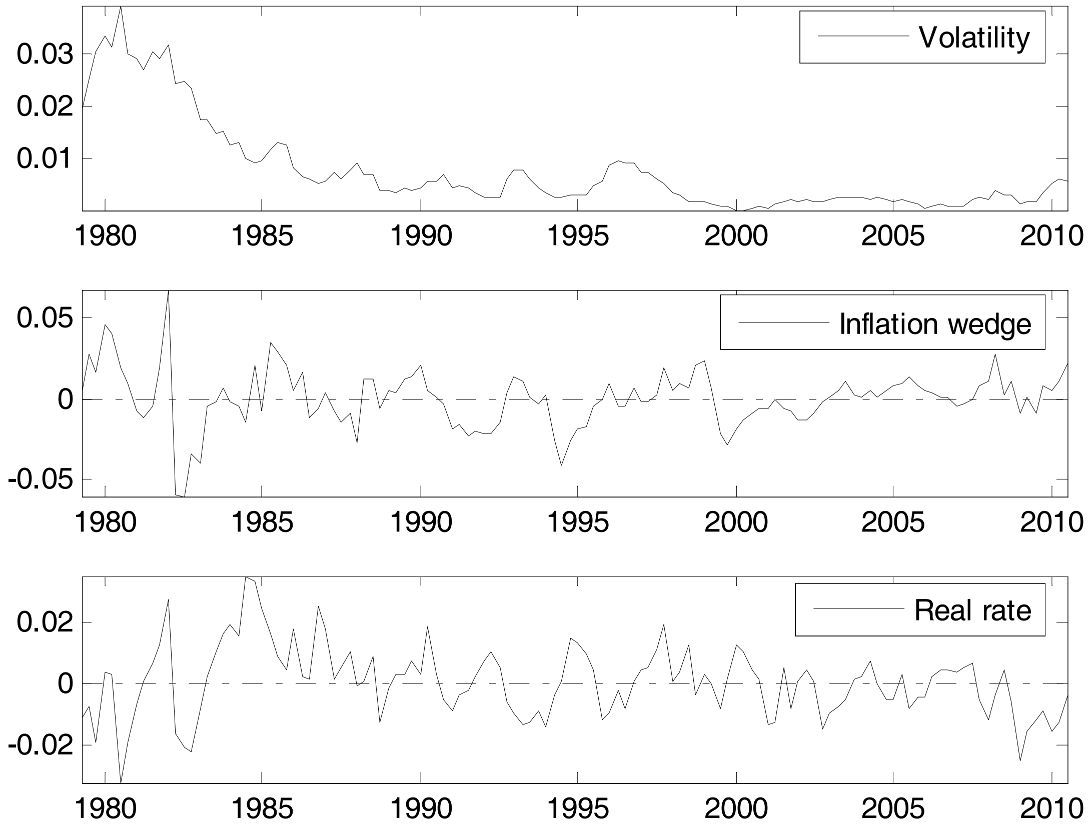
$\mathbf{H}_1$	Parameter	t-value	$\boldsymbol{\Upsilon}_1$	Parameter	t-value
$\eta_{\mathbf{r}^*}$	-0.0014	-1.96	$\upsilon_{\mathbf{r}^*}$	(-)	(-)
η_π	-0.0024	-4.06	υ_π	-0.0125	-0.18
η_g	0.0010	0.27	υ_g	-0.9735	-1.99
η_r	-0.0011	-9.21	υ_r	-0.1721	-4.77
Λ_0	Parameter	t-value	Λ_{22}	Parameter	t-value
λ_{10}	(-)	(-)	$\lambda_{22,r^*\boldsymbol{\pi}^*}$	(-)	(-)
			$\lambda_{22,\pi\boldsymbol{\pi}^*}$	0.03174	0.6032
Λ_1			$\lambda_{22,g\boldsymbol{\pi}^*}$	-0.18217	-0.3169
λ_{10}	-52.3500	-4.51	$\lambda_{22,r\boldsymbol{\pi}^*}$	-0.11016	-3.4492
Λ_{22}	Parameter	t-value	Λ_{22}	Parameter	t-value
$\lambda_{22,\mathbf{r}^*\mathbf{r}^*}$	-0.1396	-1.78	$\lambda_{22,\mathbf{r}^*\pi}$		
$\lambda_{22,\pi\mathbf{r}^*}$	0.0048	0.10	$\lambda_{22,\pi\pi}$	0.0530	10.45
$\lambda_{22,g\mathbf{r}^*}$	-0.7997	-1.77	$\lambda_{22,g\pi}$	-0.1838	-5.71
$\lambda_{22,r\mathbf{r}^*}$	-0.0713	-2.62	$\lambda_{22,r\pi}$	0.0345	94.23
Λ_{22}	Parameter	t-value	Λ_{22}	Parameter	t-value
$\lambda_{22,\mathbf{r}^*r}$	(-)	(-)	$\lambda_{22,\mathbf{r}^*g}$	(-)	(-)
$\lambda_{22,\pi r}$	0.1213	3.78	$\lambda_{22,\pi g}$	-0.0696	-0.78
$\lambda_{22,gr}$	-0.0665	-0.30	$\lambda_{22,gg}$	1.5176	2.32
$\lambda_{22,rr}$	0.0465	2.35	$\lambda_{22,rg}$	0.1564	4.78

Figure 1: Macroeconomic variables and the 15 year discount yield



Output gap (g) is from OECD; RPIX inflation (π) and 3 month Treasury bill rate (r) are from Datastream. Yield data are discount bond equivalent data compiled by the Bank of England and published on their website. Inflation and interest rates are expressed as quarterly fractions.

Figure 2: Model M3 latent factors



The volatility factor exhibits square root volatility and has a unit root. The inflation 'wedge' and real interest rate factors have a constant variance and are stationary. In model M3 the inflation rate depends upon both the volatility factor and the 'wedge' factor. Nominal interest rates and yields also depend upon the real interest rate factor. Consequently, volatility trends, inflation and interest rates & yields have unit roots but are cointegrated.

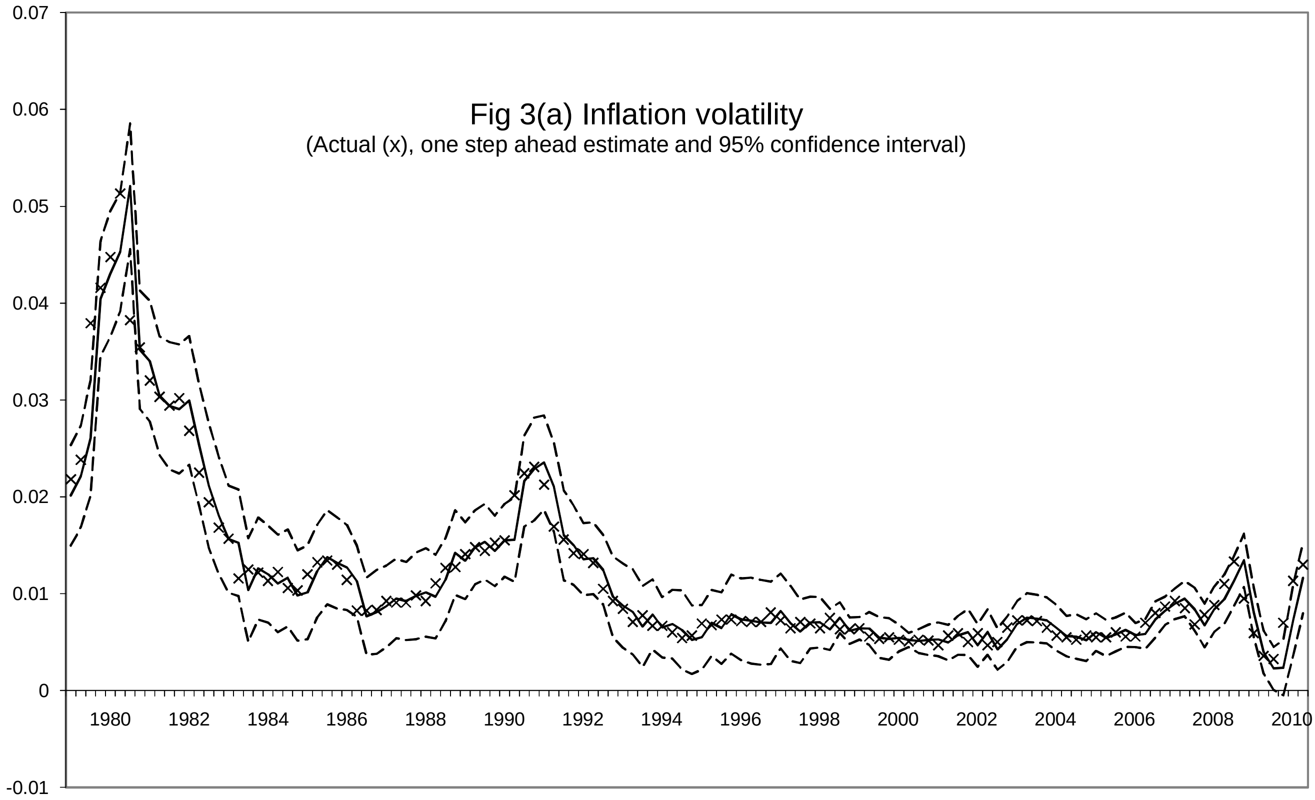


Fig 3(a) Inflation volatility
(Actual (x), one step ahead estimate and 95% confidence interval)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010

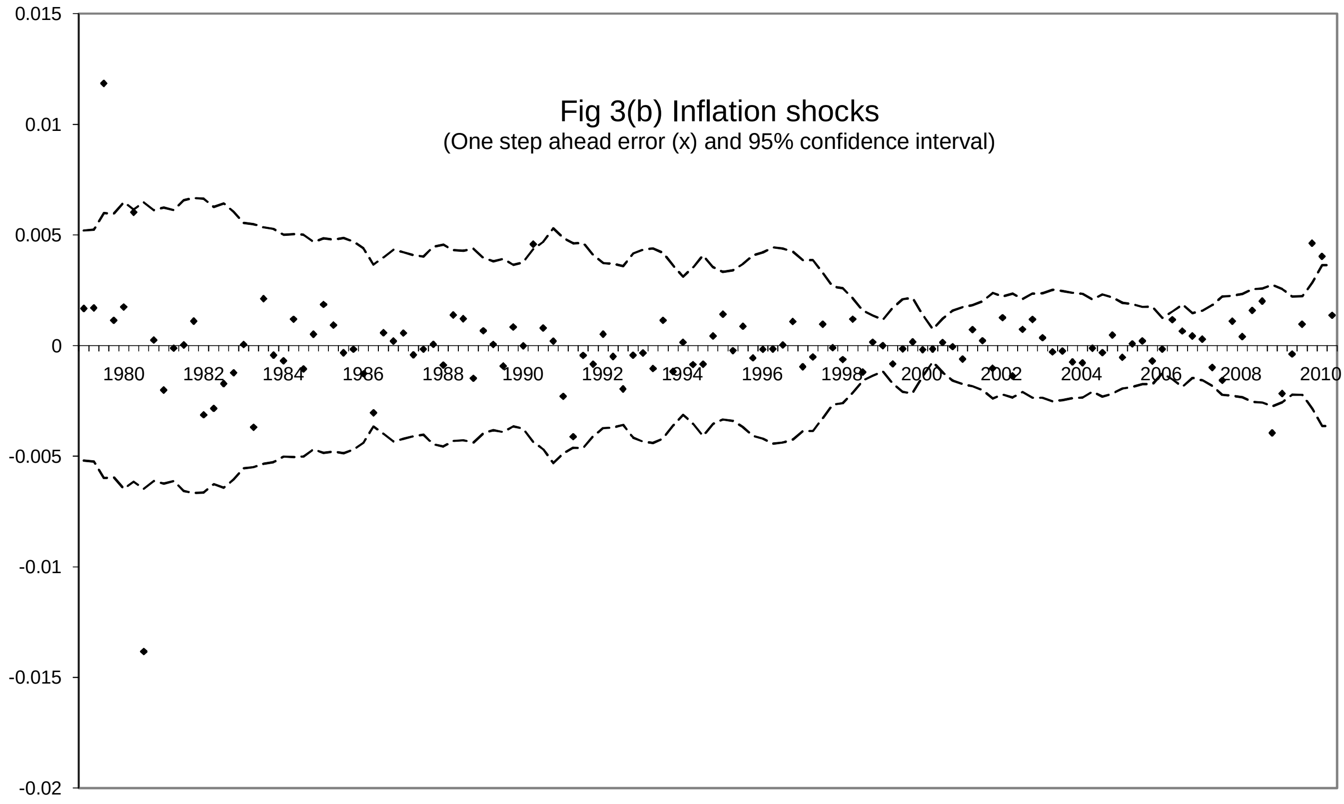


Fig 3(b) Inflation shocks
(One step ahead error (x) and 95% confidence interval)
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010

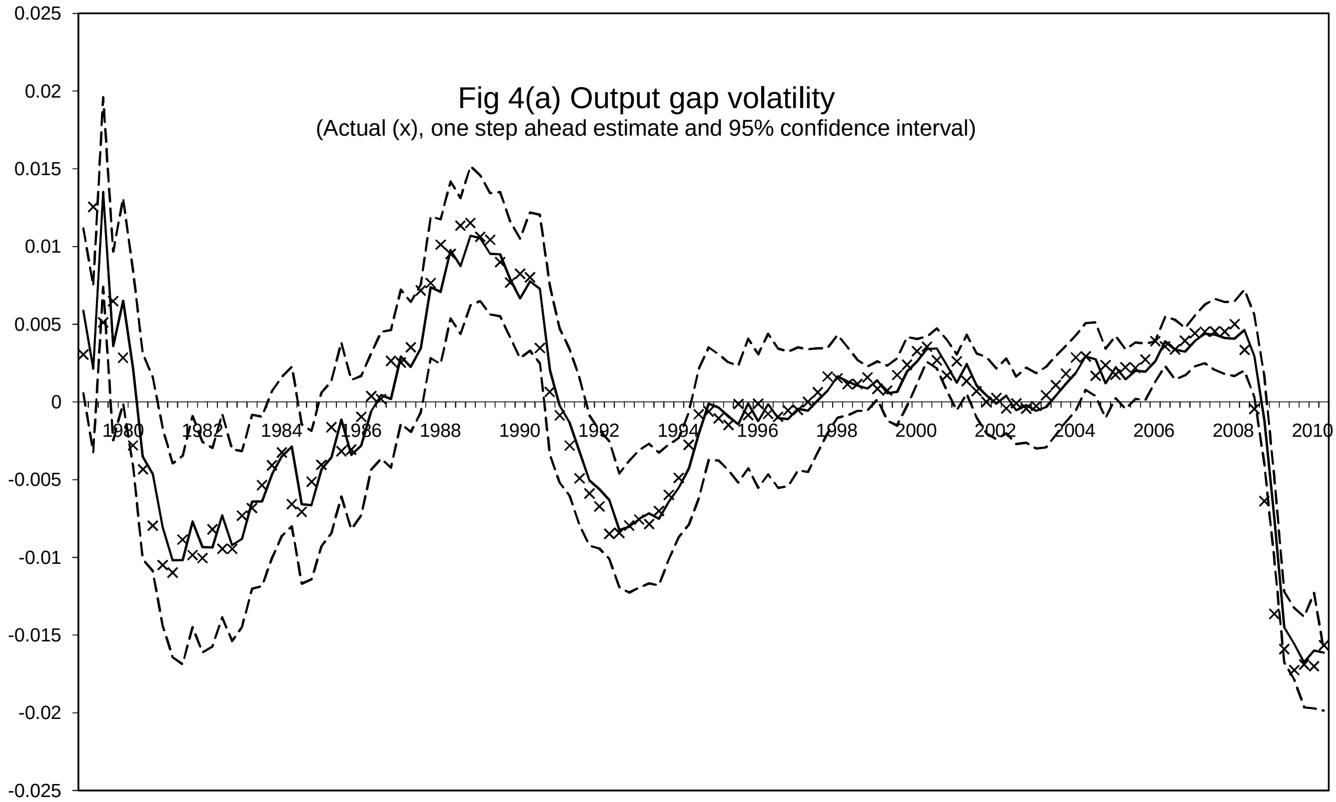


Fig 4(a) Output gap volatility
(Actual (x), one step ahead estimate and 95% confidence interval)
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010

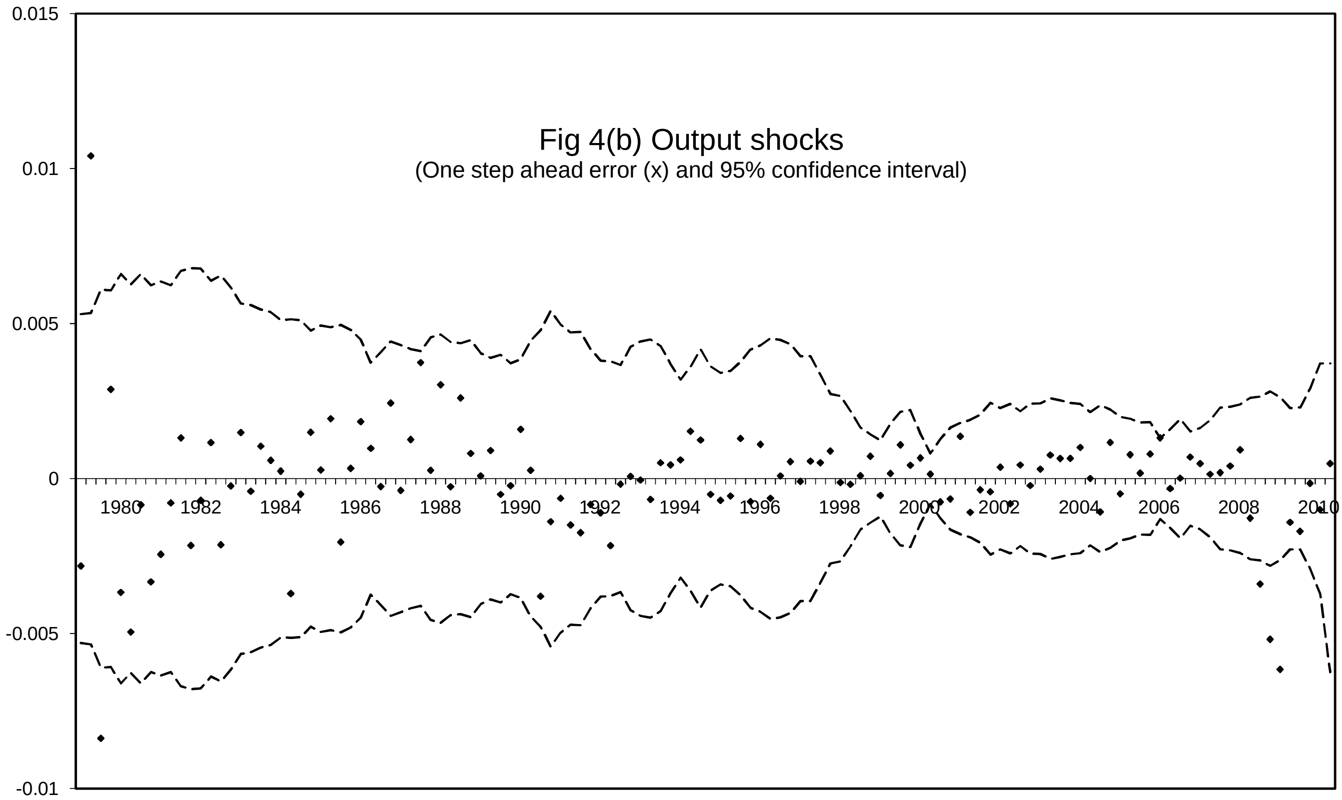


Fig 4(b) Output shocks
(One step ahead error (x) and 95% confidence interval)
0.015
0.01
0.005
0
-0.005
-0.01
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010

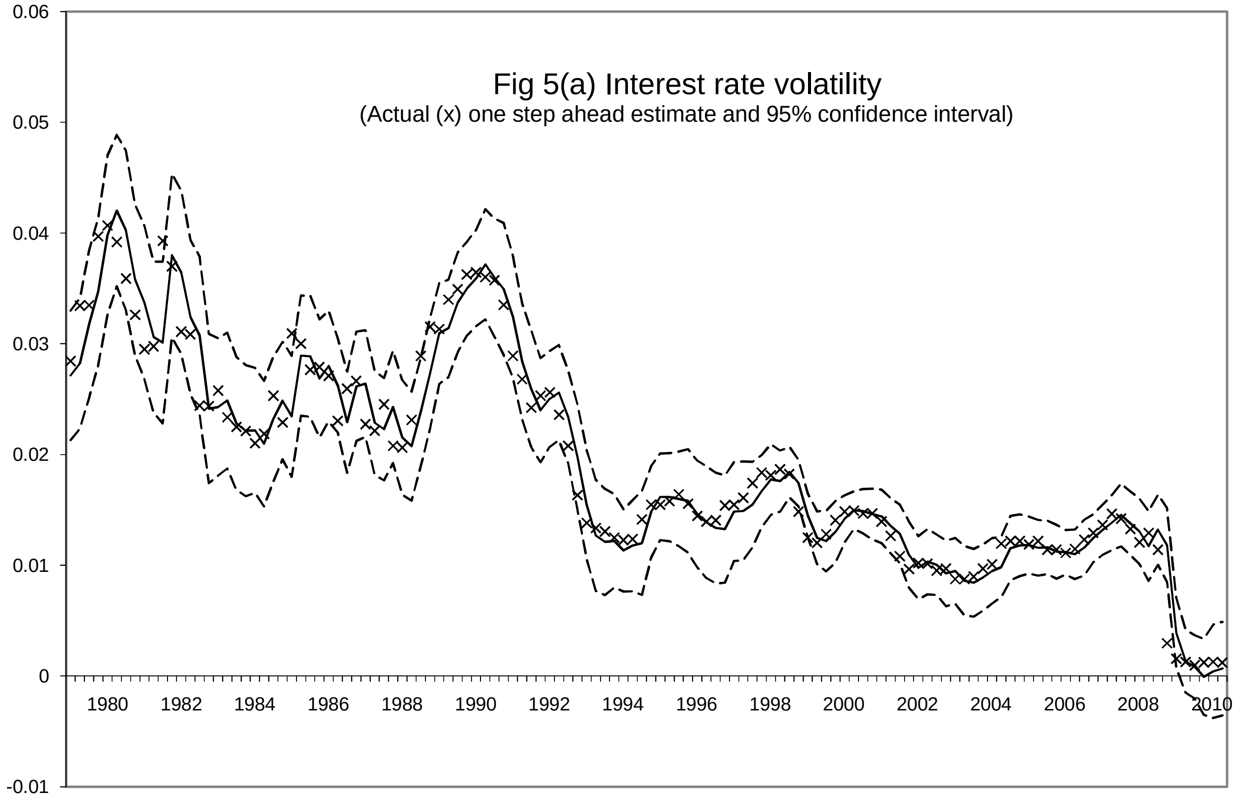


Fig 5(a) Interest rate volatility
(Actual (x) one step ahead estimate and 95% confidence interval)
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010

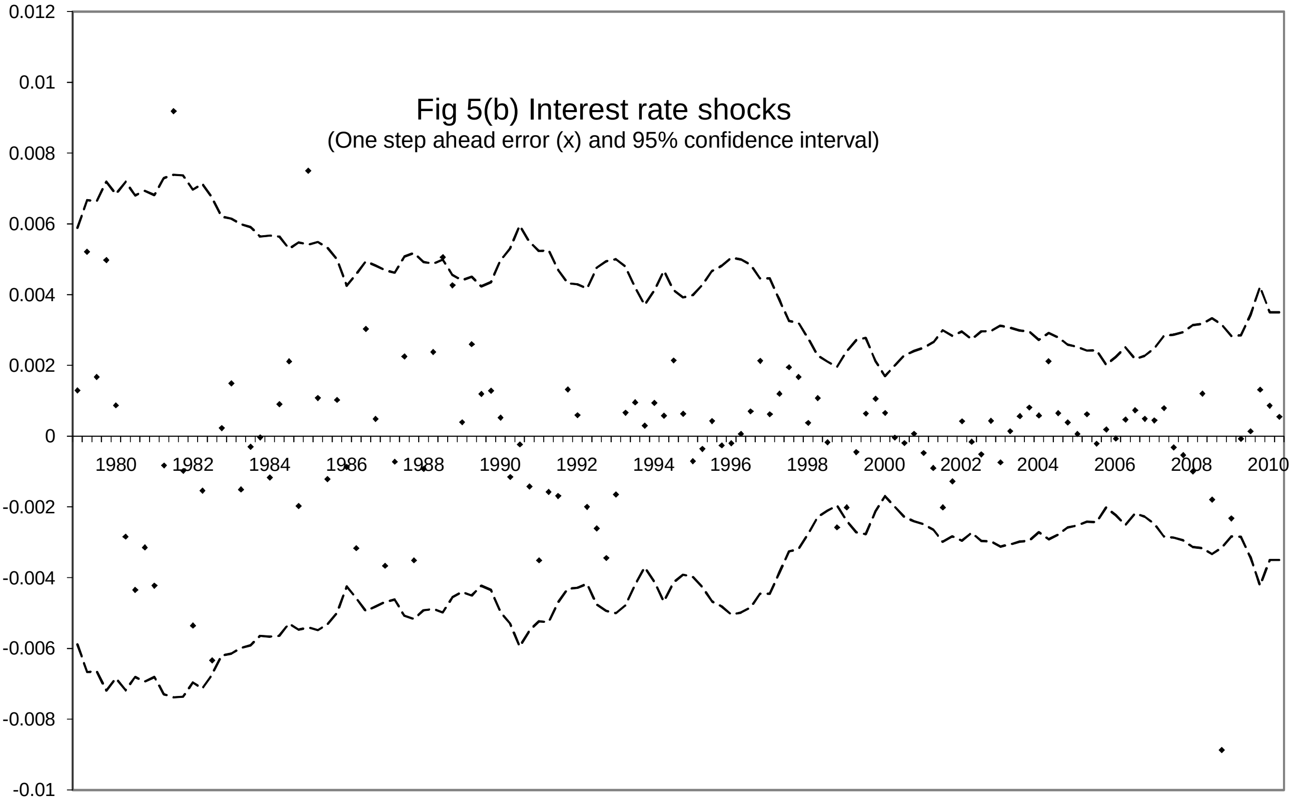


Fig 5(b) Interest rate shocks
(One step ahead error (x) and 95% confidence interval)
0.012
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010

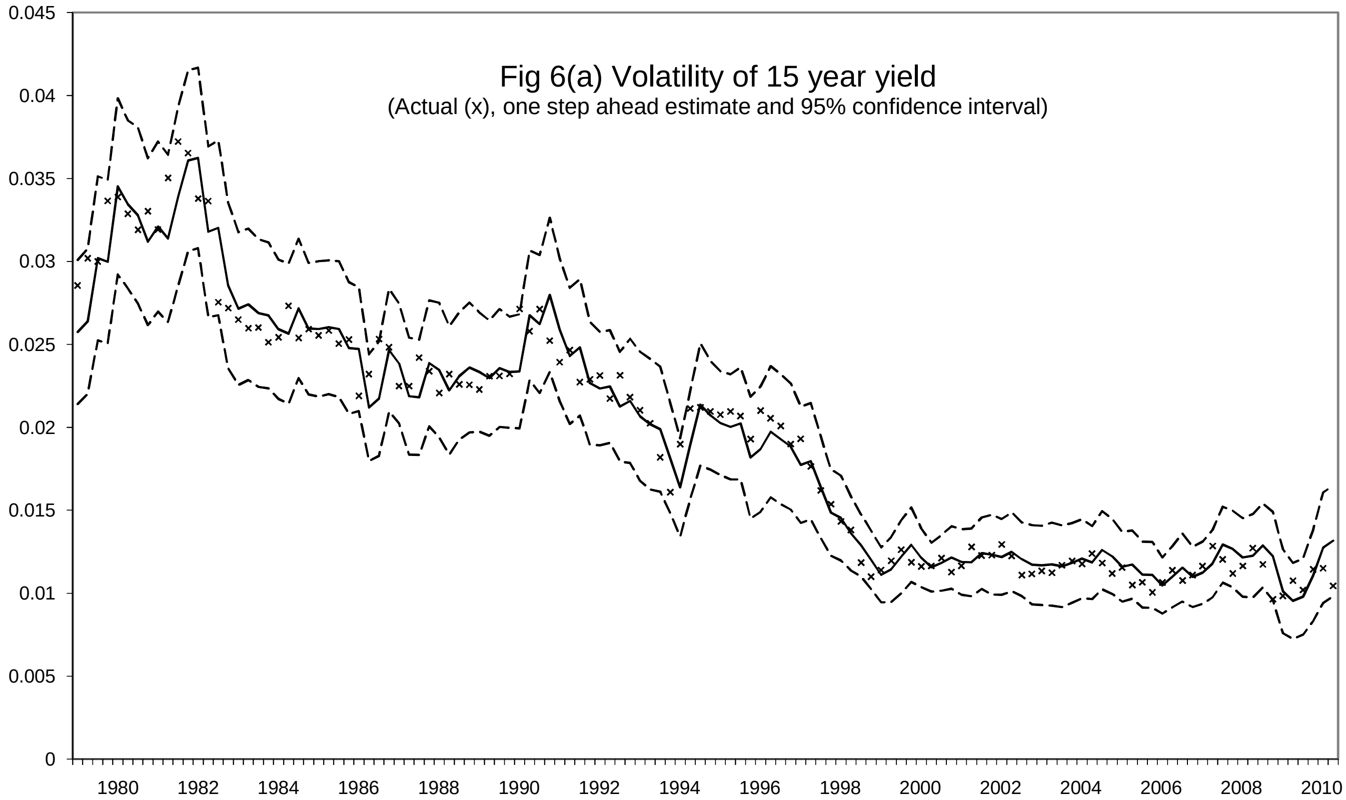


Fig 6(a) Volatility of 15 year yield
(Actual (x), one step ahead estimate and 95% confidence interval)
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010

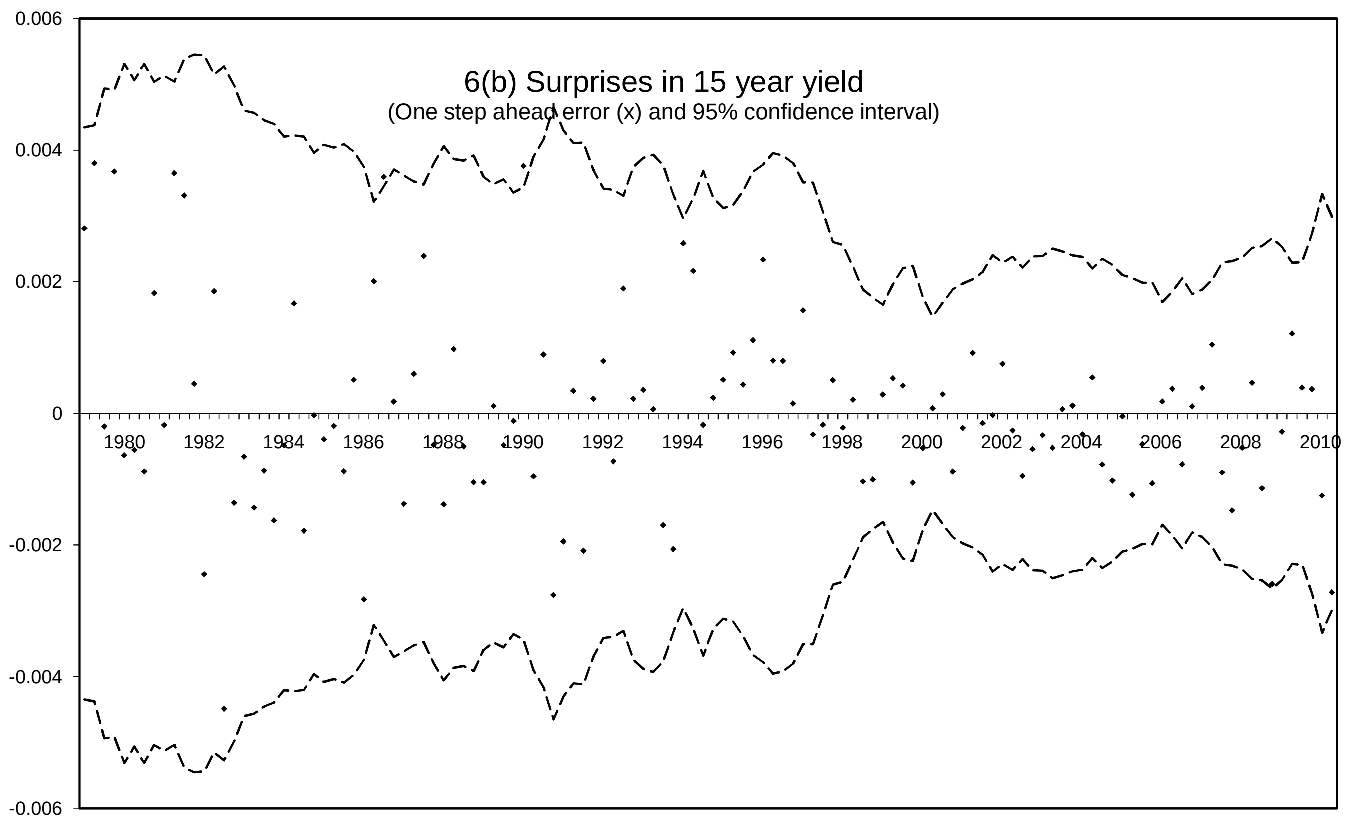
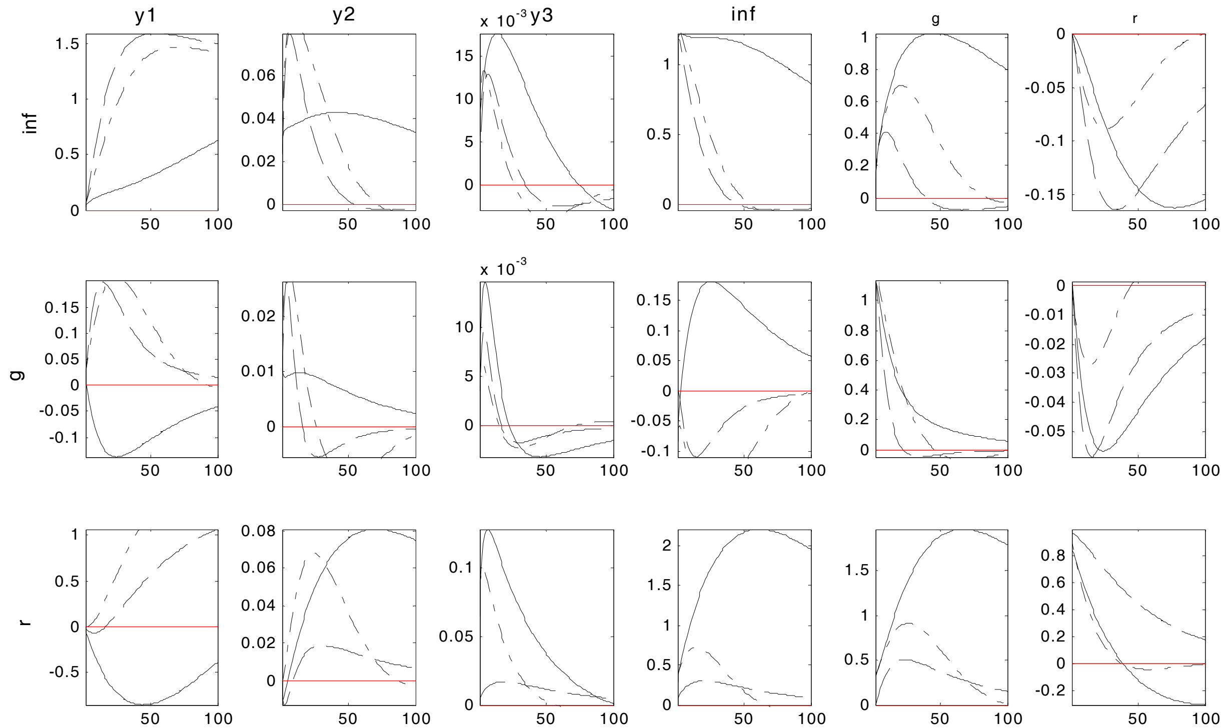
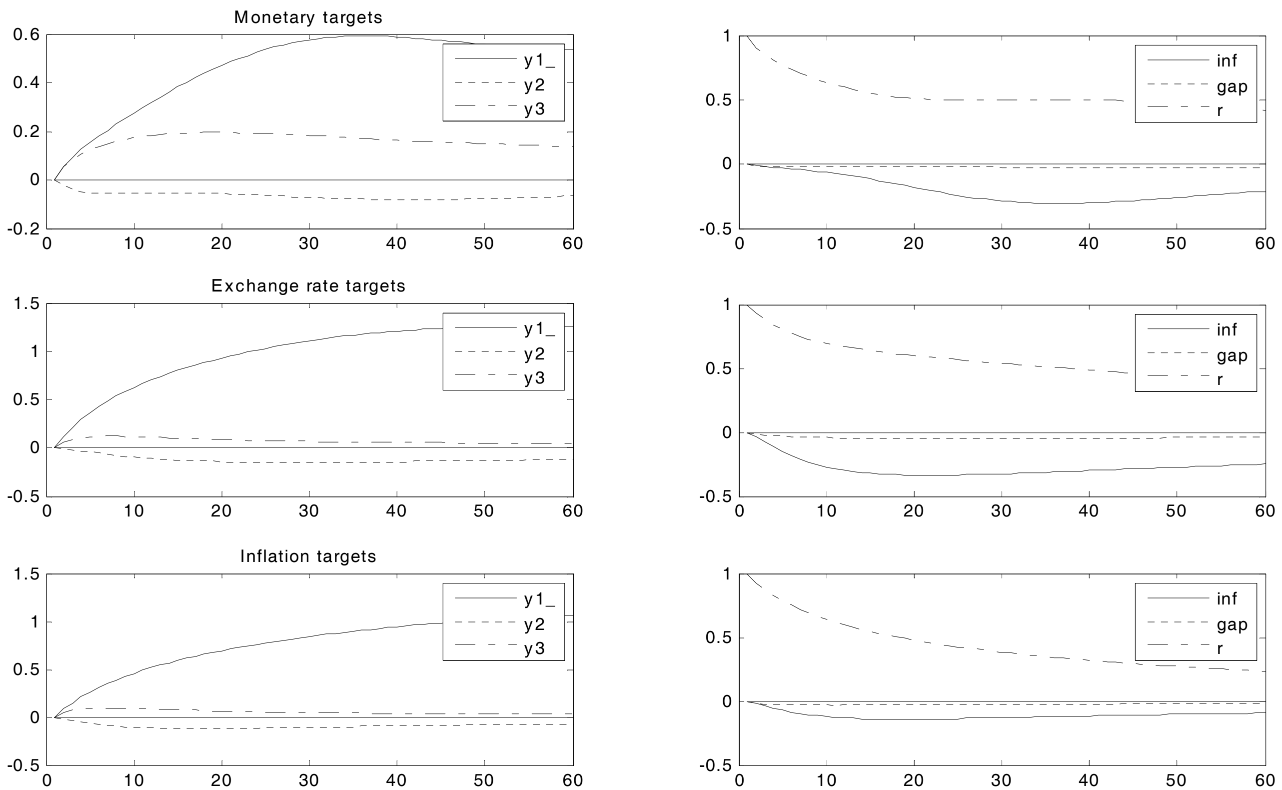

6(b) Surprises in 15 year yield
(One step ahead error (x) and 95% confidence interval)
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010

Figure 7: Model M3 macroeconomic impulse responses



Each column shows the effect of a shock to one the six orthogonal innovations (ε, η) shown in (1) and (2). These shocks increase the each of the five driving variables in turn by one percentage point compared to its historical value for just one period. Since y_{v^*} is a martingale, the first shock (ε_1) has a permanent effect on inflation and interest rates, while other shocks are transient. The continuous line shows the effect under money supply; the broken line under exchange rate and the dot-dash line under inflation target regimes. Elapsed time is measured in quarters.

Figure 8: Model M3 factor loadings



Each column shows the effect of a shock to one the six orthogonal innovations (ε, η) shown in (1) and (2). These shocks increase the each of the driving variables in turn by one percentage point compared to its historical value. The loading on the spot rate is initially unity, but then decays with maturity. Since y_{v^*} is a martingale, its loading increases with maturity over this range. Maturity is measured in quarters.