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Unemployment Dynamics and Cyclical Fluctuations in the Icelandic Labour Market*

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Abstract

This paper studies business cycle dynamics in the Icelandic labour market with the focus on two separate but related dimensions. First, which margin for adjustment of labour input, the extensive margin or the intensive margin, accounts for more variation in total working hours? It finds that both margins are important. Variation in employment accounts for 56% of the overall variation in total hours while variation in hours per worker contributes 44% to variation in total hours. Second, which of the two unemployment transition rates, the separation rate or the job-finding rate, drives the observed fluctuations in unemployment, and how do these transition rates move over the business cycle? The results show that fluctuations in the separation rate explain 70% of the total variation in the unemployment rate. Both transition rates are highly cyclical. The procyclical job finding rate moves roughly contemporaneously with the cycle, while the countercyclical separation rate is found to lead the cycle.

Keywords: Labour adjustment, Unemployment dynamics, Worker flows

JEL Classification: E32, J63, J64

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1 Introduction

Over the business cycle, there are significant fluctuations in the labour market at various margins. During expansion, firms increase output and their demand for labour input in response to changes in aggregate demand. In recessions, however, firms' output decreases and their demand for labour input falls. The cyclical variations in labour input may be attributed to firms' two margins for adjusting labour input: number of hours per worker and number of workers employed. The former is referred to as the *intensive* margin and the latter as the *extensive* margin. Furthermore, changes in the relative rates at which labour input is adjusted through the extensive margin drive the dynamics in the unemployment rate. Even though changes in labour-market conditions are observed on the surface, these dynamics underneath remain somewhat opaque.

The objective of this paper is to explore and establish key facts about unemployment dynamics and cyclical fluctuations in the Icelandic labour market. Such facts are of fundamental importance, both for macroeconomic modelling and monetary policy. In recent years, New Keynesian (NK) models have been used to explain business cycle fluctuations. Equipped with New Keynesian features, including nominal rigidities and imperfect competition, these models have become a popular tool for analysing macroeconomic issues. However, in the benchmark NK model, all adjustment in labour input takes place along the intensive margin. This means that the model does not incorporate involuntary unemployment. Armed with evidence of involuntary unemployment, economists have incorporated unemployment into NK models by introducing search frictions along the lines of Mortensen and Pissarides (1994) (see Gertler and Trigari, 2009; Gertler, Sala and Trigari, 2008; Blanchard and Gali, 2010).¹ However, the standard search and matching model adjustst labour input exclusively along the extensive margin.² In order to construct a macroeconomic model that includes labour market dynamics, it is essential to possess information on (i) how labour input is adjusted over the business cycle and (ii) what drives unemployment dynamics.

Decomposing the variance in total hours, I find that 45% of the overall variation is due to variation in hours per worker, while 55% is due to variation in employment. Both

¹For a presentation of the building blocks of the search and matching model see Pissarides (2000).

²See *e.g.* Trigari (2006) for a search and matching model with labour adjustment along both margins.

the extensive and the intensive margins are therefore important for adjustment of labour input, the extensive margin being responsible for slightly more of the overall variation. Furthermore, hours per worker and employment are found to be positively correlated. This indicates that over the business cycle, firms adjust labour input in the same direction along both the intensive and the extensive margin. This evidence highlights the importance of modelling both workers' decision to participate in the labour market and the choice of number of working hours in order to give a good approximation of the adjustment processes in the labour market.

In small open economies such as Iceland, labour supply can vary significantly due to migration of native and foreign workers. Workers move between countries in a search for higher wages and better employment opportunities. Recently, international migration has become a more important factor in determining the evolution of the labour force in Iceland. I find evidence of cyclical migration patterns for both foreign and native workers; as unemployment rises, more native workers move from the country and fewer foreign workers choose to immigrate. As a business cycle indicator, this relationship between unemployment and migration leads to fluctuations in labour supply. As a result, a labour market with international migration would be a desirable component in a NK model for small open economies such as Iceland.

In Section 3 the attention is turned to exploring and estimating the relative importance of the drivers of unemployment dynamics. Over time, the unemployment rate is determined by two factors: inflow of workers into the pool of unemployed and outflow of workers out of unemployment. If more unemployed workers find a job each month than workers who become unemployed, then the unemployment rate falls, and *vice versa*. The flow approach to modelling unemployment dynamics, *e.g.* Mortensen and Pissarides (1994) and Pissarides (2000), relates firms' hiring, a costly and time-consuming process of opening vacancies and searching for new workers, to unemployed (and employed) workers' engagement in a time-consuming job search. The search and matching paradigm aims at capturing these processes, describing unemployment as an equilibrium phenomenon; because of search frictions, there is always a positive fraction of the labour force that is unemployed. However, unemployment is not constant and varies substantially over the business cycle. A key question is whether business cycle dynamics in unemployment are

driven by variation in the job finding rate or in the separation rate.

Even though a clear view of unemployment dynamics is crucial for understanding and predicting changes in unemployment, there is still lack of consensus on the relative importance of those driving forces. The conventional view, based on Darby, Haltiwanger and Plant (1985, 1986) and Blanchard and Diamond (1990), was that job separations are more cyclical than job findings, implying that inflows into unemployment are the driving force of unemployment dynamics. This view emphasises that in order to understand unemployment dynamics, one must understand the destruction of jobs and workers' separation from jobs. However, in recent studies Hall (2005b) and Shimer (2007) find that job separations in the US are acyclical and mostly constant over the business cycle while job-finding is strongly procyclical. Contradicting conventional wisdom, this evidence suggests that unemployment dynamics are driven by variation in the creation of new matches rather than variation in separations. A substantial literature has addressed this matter since, using different methodologies and datasets. Fujita and Ramey (2009) find that variation in the separation rate contributes between 40% and 70% of the overall variation in unemployment in the US. Furthermore, Elsby, Michaels and Solon (2009) find that during recessions, countercyclical inflow rates are important for understanding unemployment dynamics in the US. Using dynamic decomposition, Smith (2011) finds that for the UK the contribution of the two transition rates varies over the business cycle, and that the separation rate contributes more to variation in unemployment during recessions. Elsby, Hobijn and Sahin (2009) provide a decomposition for fourteen OECD countries. They find that among Anglo-Saxon economies, the variation in inflows into unemployment accounts for only one-fifth of the overall variation in unemployment, while the relative contribution of inflows and outflows is almost even for Continental European and Nordic countries.

Following the literature, I derive the two unemployment hazard rates: the job finding rate and separation rate. I then solve for those transition rates using monthly data on worker flows from the Directorate of Labour for the period 2000-2010. Building on the methodology pioneered in Shimer (2007), the total variation in the unemployment rate is decomposed into variation due to fluctuations in inflows (the separation rate) and variation due to fluctuations in outflows (the job finding rate). This allows for assessment of the

relative importance of the two different state transitions.³ I find that variation in inflows into unemployment contribute 70% of the overall variation in unemployment. This is found true both for the whole sample period and when excluding the 2008-2010 recession, which was characterized by a sudden, rapid and unusually large increase in unemployment.⁴ Having evaluated the relative importance of the driving forces of unemployment, I assess the cyclical nature of the two transition rates to determine how these driving forces evolve over the business cycle. Both transition rates are highly cyclical. The job finding rate is procyclical and moves contemporaneously with the cycle. The job separation rate is, however, countercyclical and leads the cycle. Furthermore, changes in the separation rate are found to lead changes in the job finding rate.

2 Cyclical Fluctuation and Labour Adjustment

In an economy with fully flexible wages, evolution of total hours worked would follow the evolution in the population as all shocks affecting the labour market would be absorbed through changes in the wage level. In reality, as found in Sigurdardottir and Sigurdsson (2011), wages are sticky and adjusted infrequently. As a result of rigid wages, total hours vary significantly over the business cycle, increasing in booms, but decreasing in recessions. However, what remains unclear is whether a fall in total hours is caused by employed workers working fewer hours or because more workers becoming unemployed or exiting the labour force? I now quantitatively evaluate the importance of the adjustment margins at business cycle frequencies.

2.1 Adjustment of Total Hours

The data used are from Statistics Iceland's Labour Force Survey (LFS), which has been executed with quarterly frequency since 2003, but on a semiannual basis between 1991 and 2002. Quarterly series for the period 1991-2002 are obtained by disaggregating the semi-

³Due to data limitations, I am not able to explore movements in and out of unemployment that are also entries and exits from the labour market. However, in Section 2 I explore the contribution of variation in labour force participation on variation in total hours. I find that participation fluctuates less than employment and explains less of the variation in total hours.

⁴For our sample period, January 2000 to December 2010, registered unemployment was 3.1% on average. In the 2008-2010 recession, registered unemployment peaked at 9.3% in March 2010.

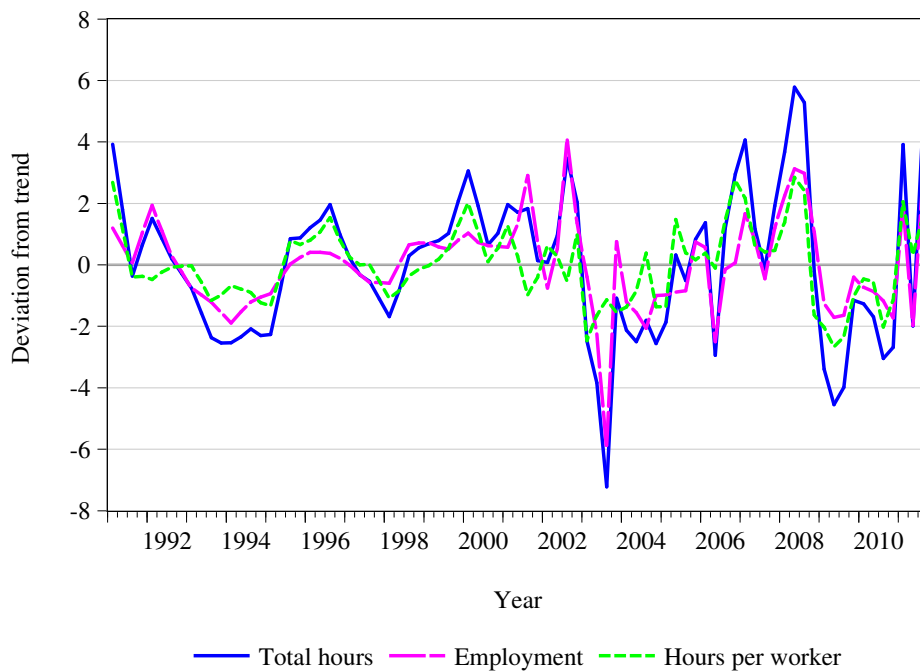


Figure 1: Cyclical Fluctuation in Total Hours and its Components

annual series using *Ecotrim*.⁵ In exploring the relative importance of labour adjustment along the intensive and extensive margins, a quarterly series of total hours is constructed. From the LFS we have data on the number of employed workers and the average number of hours they worked.⁶ Using these data, a series for total hours, TH_t , is constructed by multiplying average hours worked and the number of persons at work and dividing by the size of the labour force. I choose to transform the series using natural logarithm. Logarithmic transformations of total hours and their components are denoted by lower case letters. Total hours worked, th_t , are defined as follows in logarithmic terms:

$$th_t = h_t + n_t \quad (1)$$

where h_t is the average number of hours worked per worker, and n_t is the number of people

⁵*Ecotrim* is a program for temporal disaggregation of time series, developed by Eurostat. For disaggregation I use the flow AR(1) method with Max Log Par: -0.99 to 0.99. As a reference series for employment I use $1 - unemp_t$, where $unemp_t$ is a quarterly average of registered monthly unemployment at the Directorate of Labour.

⁶In the LFS there are two definitions for employed workers: those who are employed (and not necessarily at work in the reference week), and those who are employed and work at least one hour in the reference week. I use the latter. The series for working hours I use are hours worked, not regular working hours.

Table 1: Business Cycle Variation in Labour Input

Standard deviations of relative deviations from trend		
Total hours (th)	Hours (h)	Employment (n)
2.36	1.22	1.47
Correlation		
(th, h)	(th, n)	(n, h)
0.84	0.90	0.51

Notes: Data are quarterly time series for the period Q1/1991 - Q3/2011. Total hours, th , are defined as the average hours multiplied by by number of persons employed and divided by the labour force, and transformed using natural logarithm. Standard deviations are in percentage terms.

employed in *per capita* terms, *i.e.* divided by the size of the labour force. All series are seasonally adjusted using the Census Bureau's X12 ARIMA procedure. Since the focus is on the cyclical fluctuations of hours and employment, the time series are detrended using Hodrick- Prescott (HP) filter with a standard smoothing parameter for quarterly data. The series th_t , h_t and n_t are therefore presented as deviations from trend.

Figure 1 plots the cyclical fluctuations in total hours and its components. Total hours worked fluctuate significantly over the business cycle. The same is true for the two components, employment and hours per worker, which both display co-movement with total hours. During the upswing, 2005-2008, total hours were above trend; both employment and hours per worker contributed to this deviation. In two contraction periods, 1993-1994 and 2003, employment was the more important factor, while in 2008-2010, hours per worker contributed more to the deviation from trend. The fact that adjustment along the intensive margin is as important as observed highlights the flexibility of the Icelandic labour market, since the process of hiring new workers is generally time-consuming and adjustment of labour input through the intensive margin is an efficient channel for responding to variation in demand.

Table 1 presents standard deviations of total hours, hours per worker, and employment, as well as correlations between them. The series are deviations from trend.⁷ Since

⁷One might be concerned about the variation created by generating quarterly series for the period 1991-2002. I excluded that period and carried out the same analysis. This results in slightly larger standard deviations and somewhat stronger correlation between the components, especially employment and hours, the correlation coefficient for which is measured at 0.63.

the series are in natural logarithms the standard deviations can be interpreted as mean percentage deviations from trend. The mean deviation in total hours is 2.4 percent, the deviation in hours per worker is 1.2 percent, and the mean deviation in employment is 1.5 percent. As Figure 1 and Table 1 show, both hours per worker and employment are highly positively correlated with total hours, the correlation being 0.84 and 0.90 respectively. Furthermore, the correlation between employment and hours per worker is 0.51. These results indicate that firms adjust labour input in the same direction over the business cycle.

In order to obtain a statistical measure of the relative importance of adjustment along the intensive margin and the extensive margin over the business cycle, a decomposition of variation in total hours is proposed. The variance of total hours worked, th_t , is defined as follows in terms of the variation in its two components:

$$var(th_t) = var(h_t) + var(n_t) + 2cov(h_t, n_t) \quad (2)$$

Using definitions of variance and covariance, the variation of th_t can be written as:

$$\begin{aligned} var(th_t) &= \mathbb{E}\{(h_t + n_t - \mathbb{E}(h_t + n_t))(th_t - \mathbb{E}(th_t))\} \\ &= \mathbb{E}\{(h_t - \mathbb{E}(h_t))(th_t - \mathbb{E}(th_t)) + (n_t - \mathbb{E}(n_t))(th_t - \mathbb{E}(th_t))\} \\ &= cov(h_t, th_t) + cov(n_t, th_t) \end{aligned} \quad (3)$$

The first term on the right-hand-side of (3), $cov(h_t, th_t)$, is the amount of variation in th_t that is contributed both directly from h_t and its correlation with n_t . $cov(n_t, th_t)$ is similarly the variation in th_t that derives from variation in n_t and its correlation with h_t . Dividing through equation (3) with $var(th_t)$ gives the following:

$$1 = \gamma^h + \gamma^n \quad (4)$$

where γ^h and γ^n are given by the following equations:

$$\gamma^h = \frac{cov(h_t, th_t)}{var(th_t)} \quad (5)$$

$$\gamma^n = \frac{\text{cov}(n_t, th_t)}{\text{var}(th_t)} \quad (6)$$

Hence, (5) and (6) are the *relative* contributions of variance in h_t and n_t to the total variation in th_t . Table 2 reports the values for γ^h and γ^n for both the whole sample period and the subperiod 2003-2011.

Table 2: Decomposition of variance in th_t

	1991 – 2011	2003 – 2011
Contribution of hours per worker, γ^h	0.44	0.46
Contribution of employment, γ^n	0.56	0.54

Note: Quarterly time series for the period Q1/1991 - Q4/2002 are generated by disaggregation of semiannual data using *Ecotrim*.

I find that both margins of labour adjustment are of almost equal importance. For the whole sample period, 45% of variation in total hours is due to variation in hours per worker and 55% due to variation in employment. For the period 2003-2011 the results are very similar. This evidence indicates that over the business cycle, Icelandic firms adjust labour input in the same direction both along the intensive and extensive margin, but not evenly. Furthermore, hours per worker and employment can be interpreted as close substitutes when firms adjust total hours.

2.2 Labour Force Participation

Variation in the size of the labour force provides another dimension of flexibility in the labour market. In recessions, firms can adjust labour input by decreasing hours worked per worker or the number of employed workers. But when workers separate from firms they do not necessarily enter the pool of unemployed, as they may choose to exit the labour force. Furthermore, when firms increase labour input by hiring workers, they may hire workers that were previously outside the labour force. Over the business cycle there may therefore be a group of workers who enter and exit the labour force depending on the state of the economy. As a result the size of the labour force, as well as employment and hours per worker, may move in tandem with the business cycle, providing an additional

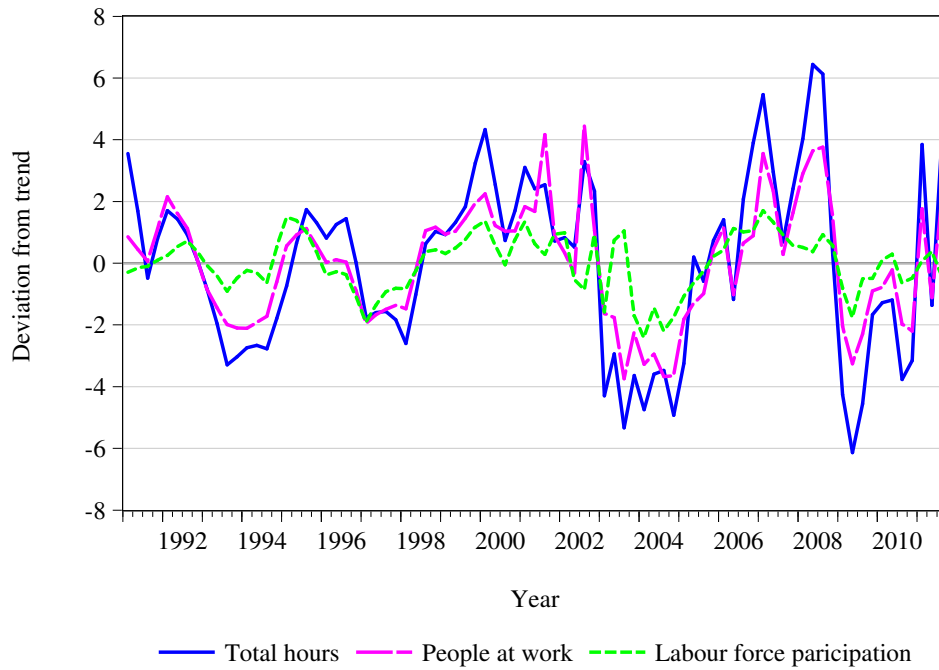


Figure 2: Cyclical Fluctuation in Labour Force Participation

dimension of labour market flexibility.

Figure 2 plots deviation from trend in total hours, people at work, and labour force participation. Data are reported on quarterly frequency and scaled with the population at working age, not the labour force as before. All series are seasonally adjusted using the X12 ARIMA procedure, transformed using natural logarithms, and detrended using an HP filter with the conventional smoothing parameter for quarterly data.

As depicted in Figure 2, participation varies over the cycle as people enter and exit the labour force. However, the variation in the number of people at work is much greater and co-moves more strongly with total hours. Standard deviations of the relative deviations from trend are reported in Table 3. The standard deviation of detrended series for people at work is 1.9 percent, more than twice the standard deviation of the detrended labour force which has a standard deviation of 0.9 percent. Furthermore, the correlation between the labour force and total hours is 0.66 which is much weaker than the correlation of either hours per worker or people at work with total hours. This indicates that the variation in labour force participation is a secondary factor in explaining cyclical fluctuations in the labour market.

Table 3: Cyclical Fluctuation in the Labour Market

Standard deviations of relative deviations from trend		
Total hours (th^*)	People at work (ep)	Labour Force (lf)
2.79	1.89	0.92
Correlation		
(th^*, ep)	(ep, lf)	(th^*, lf)
0.93	0.66	0.63
(th^*, h)	(ep, h)	(lf, h)
0.84	0.59	0.43

Notes: Data are quarterly time series for the period Q1/1991 - Q3/2011. Different from Table 1, total hours, th^* , are defined as the average hours multiplied by number of persons employed divided by the number of people in working age, and transformed using natural logarithm. Standard deviations are in percentage terms.

Some of the stylised facts presented here are similar to what studies for other countries have shown, but others are somewhat different. Similar to the findings in the current paper are the results in Krause and Lubik (2010) for the U.S., which show that variation in hours per worker accounts for 33% to 50% of variation in total hours. In an earlier study on the U.S. labour market, Cho and Cooley (1994) find that only 25% of variation in total hours can be attributed to variation in hours per worker and the remainder is assigned to variation of employment. Rogerson and Shimer (2010) conclude that variation in the size of the labour force is a secondary factor at business cycle frequencies in the U.S., agreeing with previous evidence in Lilien and Hall (1986). When comparing the U.S. to 17 OECD countries, Rogerson and Shimer (2010) find that the fraction of people at work is strongly correlated with total hours in both the U.S. and OECD countries. The correlation is 0.95 in the U.S. and 0.87 on average in the OECD countries. On the other hand they find that correlation between total hours and hours per worker is strong in the U.S. but weak in the OECD on average. Moreover, the correlation between the fraction of people at work and hours per worker is 0.68 in the U.S. but only 0.05 in the OECD on average.⁸ The results presented in the current paper indicate that the Icelandic labour market, where

⁸Rogerson and Shimer (2010) find that in both France and Japan the correlation between total hours and hours per worker is stronger than the correlation between total hours and employment, unlike in other OECD countries and the U.S.

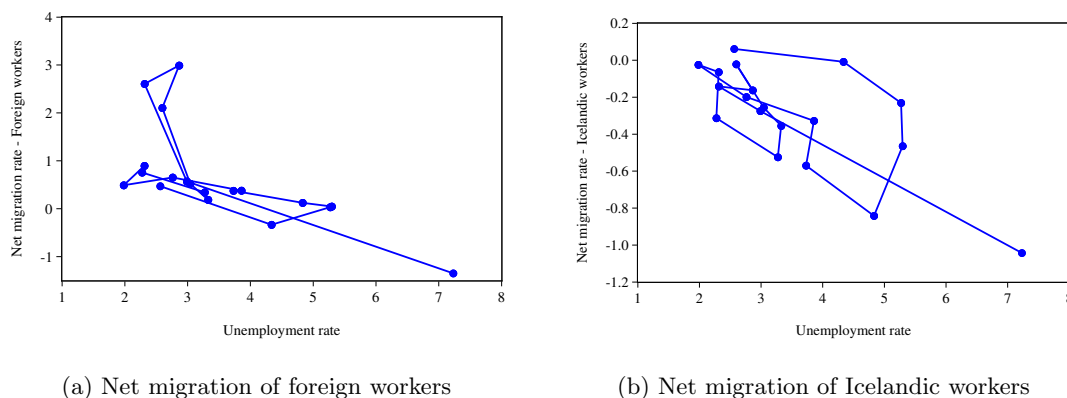


Figure 3: Labour Migration in Iceland: 1991-2010

adjustment takes place along both margins and in the same direction, is more like the U.S. labour market than the labour markets in other OECD countries. Flexible hours per worker in Iceland and the U.S. give rise to more flexible labour input than in other OECD countries since adjustment may in general be thought to be more rigid along the extensive margin.

2.3 International Migration

In addition to changes in labour force participation, the supply of labour in open economies adjusts to changes in aggregate demand through migration of workers. During expansion periods, demand for labour increases and mobile workers in foreign countries may choose to migrate in order to find employment or higher wages. In a recession, workers may respond to unemployment or lower purchasing power of wages by migrating. Since labour migration may be an important margin for labour supply, I end this section by briefly looking at the cyclicity in labour migration. Figure 3 shows the relationship between the net migration rate of Icelandic and foreign nationals and unemployment. The rate is defined as the net number of migrants at working age divided by the labour force. For foreign nationals, low unemployment, and thus high labour demand, translates into increasing migration of foreign workers to Iceland.⁹ Figure 3b shows the relationship between unemployment and net migration of Icelandic nationals; increased unemployment

⁹The increase in immigration in 2005-2007 was mainly due to large construction projects domestically and a boom that created excess demand for labour in the construction industry, but also due to enlargement of the EU in 2004 that decreased the cost of workers from Central and Eastern Europe in migrating to EU and EEA member states like Iceland.

increases emigration. Furthermore, a comparison of Figures 3a and 3b reveals two different mechanisms that affect labour supply in Iceland. The net migration of Icelandic nationals is always negative, but it is stronger at times of distressed labour market conditions. Net migration of foreign nationals, however, is negative when unemployment is above 4%, but positive when unemployment is lower. Over the last business cycle, the latter mechanism has become an increasingly important margin for adjustment of labour supply. In the years 2005-2007 – the three observations in the North-West corner of Figure 3a – migration was especially high, 2.5% of labour force on average. During the recession in 2008-2010, net migration of foreign workers peaked at 1.3% of the labour force in 2009. Putting these numbers into perspective, in 2008 foreign nationals were 11% of the labour force.

3 Unemployment Dynamics

In this section, I explore the relative importance of inflows into and outflows out of unemployment for explaining unemployment dynamics. The methodology used in the paper follows closely that of Shimer (2007) and Petrongolo and Pissarides (2008), and the dataset used is administrative data from the Directorate of Labour (DoL). I have information on the number of both unemployment registrations and deregistrations of claimants at the DoL in a given month. As the data are biased towards workers who come from employment, rather than individuals who have been outside the labour force, I follow Petrongolo and Pissarides (2008) and model unemployment dynamics in an environment where transition is only between two states: unemployment and employment. Also, it should be noted that flows in and out of unemployment take place continuously while data on unemployment flows is discrete. In order to correct for the time-aggregation bias that can arise due to this fact, I use a method pioneered by Shimer (2007).

3.1 Analytical Framework

Two states are defined for workers in the labour market: employed or unemployed. Time is denoted with $t \in \{0, 1, 2, \dots\}$, where the interval $[t, t + 1)$ is referred to as *period* t and $\tau \in [0, 1]$ is the time elapsed since beginning of the current time period. Unemployed workers find job according to a Poisson process, which gives the following probability of

finding a job:

$$\mathcal{P}(F_t) = 1 - e^{-f_t} \quad (7)$$

where the job finding rate, *i.e.* the arrival rate of jobs, is $f_t = -\ln(1 - \mathcal{P}(F_t))$. The total unemployment outflow during period t is then given by:

$$\Gamma_t = (1 - e^{-f_t})U_t + \int_0^1 [1 - e^{-f_t(1-\tau)}]\Sigma_{t+\tau} d\tau \quad (8)$$

where $\Sigma_{t+\tau}$ is unemployment inflow at $t + \tau$, and U_t is unemployment at the beginning of period t . Equation (8) is in two parts. The first part of the equation represents the number of workers who were unemployed at the beginning of period t but find a new job during the period. The second part represents the number of workers who were employed at the beginning of period t , that get separated from their job during the period but find a new job before the end of period t . The second part of equation (8) therefore accounts for time aggregation bias following Shimer (2007). In a similar fashion workers separate from jobs according to a Poisson process. The probability of separation can be written as:

$$\mathcal{P}(S_t) = 1 - e^{-s_t} \quad (9)$$

and the separation rate is therefore $s_t = -\ln(1 - \mathcal{P}(S_t))$. The total unemployment inflow during period t is then:

$$\Sigma_t = (1 - e^{-s_t})E_t + \int_0^1 [1 - e^{-s_t(1-\tau)}]\Gamma_{t+\tau} d\tau \quad (10)$$

where E_t denotes employment at the beginning of period t and $\Gamma_{t+\tau}$ is the unemployment outflow at time $t + \tau$. Assuming that the unemployment inflows and outflows are uniformly distributed over the time period t , gives the following expression for total unemployment outflow:

$$\Gamma_t = (1 - e^{-f_t})U_t + \left(1 - \frac{1 - e^{-f_t}}{f_t}\right)\Sigma_t \quad (11)$$

and analogously for unemployment inflow:

$$\Sigma_t = (1 - e^{-s_t})E_t + \left(1 - \frac{1 - e^{-s_t}}{s_t}\right)\Gamma_t \quad (12)$$

Using monthly data for Γ_t , Σ_t , U_t and E_t , equations (11) and (12) can be solved for the continuous-time transition rates f_t and s_t using a conventional numerical solver.

My aim is to examine the cyclical variation in job separation and job finding and to estimate the contribution of the two flow rates to the variance in unemployment. Denoting the unemployment rate as u_t , the evolution of the unemployment rate can be described in terms of the continuous-time transition rates:

$$\frac{du}{dt} \equiv \dot{u}_t = (1 - u_t)s_t - u_t f_t \quad (13)$$

Equation (13) describes the evolution of unemployment accurately when all inflows into unemployment are from the pool of employed workers and all flows out of unemployment are workers who find jobs.¹⁰ In steady state the unemployment rate is, by definition, changing at a rate of zero, $\dot{u}_t = 0$, which gives:

$$(1 - u_t)s_t = u_t f_t \quad (14)$$

Rearranging the above gives an expression for the flow steady state unemployment:

$$\tilde{u}_t = \frac{s_t}{s_t + f_t} \quad (15)$$

This is a key equation of the search and matching model that describe a unique equilibrium unemployment rate – a flow steady state – in terms of the transition rates f_t and s_t . If the flow steady state unemployment is a good approximation of the unemployment rate, a valid decomposition of the unemployment rate can be derived using the job finding and separation rates. More precisely, the unemployment dynamics can be decomposed into two components: the variation due to changes in the job finding rate f_t , and the variation due to changes in the job separation rate s_t . By first-differencing the steady state unemployment, $\Delta\tilde{u}_t \equiv \tilde{u}_t - \tilde{u}_{t-1}$, the following expression is obtained:

¹⁰Because of data limitations, unemployment can not be described more accurately transitions more accurately by taking account to flows in and out of the labour force.

$$\begin{aligned}\Delta \tilde{u}_t &= \frac{s_t}{s_t + f_t} - \frac{s_{t-1}}{s_{t-1} + f_{t-1}} \\ &= (1 - \tilde{u}_t)\tilde{u}_{t-1} \frac{\Delta s_t}{s_{t-1}} - (1 - \tilde{u}_{t-1})\tilde{u}_t \frac{\Delta f_t}{f_{t-1}}\end{aligned}\tag{16}$$

Under the approximation $\tilde{u}_t \approx \tilde{u}_{t-1}$, (16) can be written as a decomposition of the percentage change in the steady-state unemployment rate:

$$\frac{\Delta \tilde{u}_t}{\tilde{u}_{t-1}} = (1 - \tilde{u}_t) \frac{\Delta s_t}{s_{t-1}} - (1 - \tilde{u}_{t-1}) \frac{\Delta f_t}{f_{t-1}}\tag{17}$$

Elsby *et al.* (2009) and Fujita and Ramey (2009) use a decomposition in logarithms. Since $\frac{\Delta \tilde{u}_t}{\tilde{u}_{t-1}} \approx \Delta \ln(\tilde{u}_t)$ for small changes, the decomposition (17) can be written in logarithmic terms as:

$$\Delta \ln \tilde{u}_t = \underbrace{(1 - \tilde{u}_t) \Delta \ln s_t}_{\Pi_t^s} - \underbrace{(1 - \tilde{u}_{t-1}) \Delta \ln f_t}_{\Pi_t^f}\tag{18}$$

where Π_t^s and Π_t^f are the contribution of changes in the inflow rate and the outflow rate to the total variation in the unemployment rate, respectively. Equation (18) is a key equation describing the dynamic evolution of unemployment. Using the decomposition, I can calculate each components' contribution to the variance in steady state unemployment as:

$$\tilde{\beta}^s = \frac{cov(\Delta \ln \tilde{u}_t, \Pi_t^s)}{var(\Delta \ln \tilde{u}_t)}\tag{19}$$

$$\tilde{\beta}^f = \frac{cov(\Delta \ln \tilde{u}_t, \Pi_t^f)}{var(\Delta \ln \tilde{u}_t)}\tag{20}$$

$\tilde{\beta}^s$ and $\tilde{\beta}^f$ represent the relative contribution of the two flow components to the total variation in unemployment. Therefore, by definition, $\tilde{\beta}^s + \tilde{\beta}^f \approx 1$, where any difference from unity is due to approximation.

3.2 The Relative Importance of Ins and Outs

Figure 4 plots monthly worker flow data on unemployment, inflows and outflows for the period 2000-2010. From month to month there is a considerable variation in all series. Numerous workers enter and exit the pool of the unemployed each month, causing its size to vary. However, no clear pattern emerges from the figure. I therefore turn to the decomposition method presented above to gain a better understanding of the dynamics of unemployment and its elements.

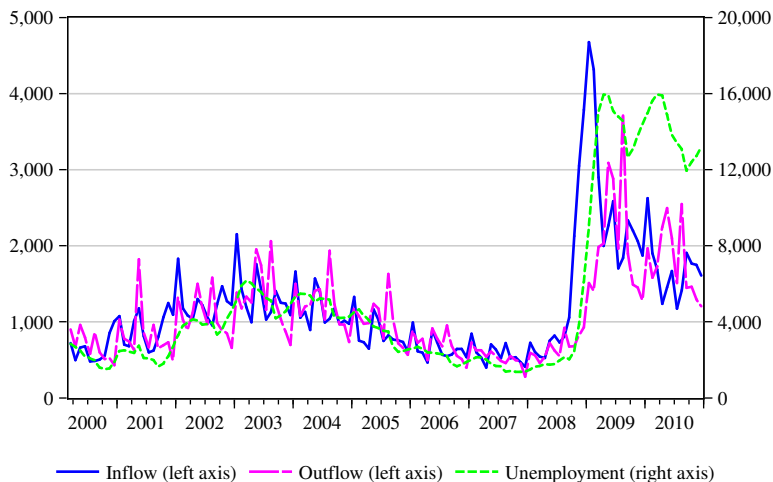


Figure 4: Unemployment, Inflow and Outflow

Monthly DoL data for outflow from unemployment (I_t), inflow into unemployment (Σ_t), registered unemployment (U_t), and employment (E_t), are used to solve equations (11) and (12) for the continuous-time transition rates f_t and s_t . The series are seasonally adjusted using the X-12 ARIMA procedure, and quarterly series are generated by averaging in order to remove excess volatility. The contribution of variation in inflows and outflows to the overall variation in unemployment, $\tilde{\beta}^s$ and $\tilde{\beta}^f$, is then calculated using the quarterly series.

The results for the decomposition of unemployment variance are presented in Table 4. I find that variation in the inflow rate explains a larger fraction of the overall variation in the unemployment rate; increased unemployment is driven by increased rate of separation. For the whole sample period, 2000-2010, changes in the separation rate explain 70% of the total variation in unemployment. During the recession starting in the autumn of

Table 4: Decomposition of Unemployment Variance

Contribution to variation in \tilde{u}_t		
	2000-2010	2000-2007
Job finding rate (β^f)	0.29	0.31
Job separation rate (β^s)	0.70	0.66
Residual	0.01	0.03

Notes: β^f and β^s are the relative contributions of variation in outflow and inflow into unemployment to variation in steady state unemployment. The contribution of the two transition rates does not sum to unity due to approximation.

2008, unemployment rose rapidly as the previously expanding construction sector and the financial sector collapsed. To check the robustness of these results the period 2008-2010 is excluded and the beta's for the period 2000-2007 are calculated. The results are almost identical; changes in the separation rate contribute two thirds of the overall variation in the unemployment rate.

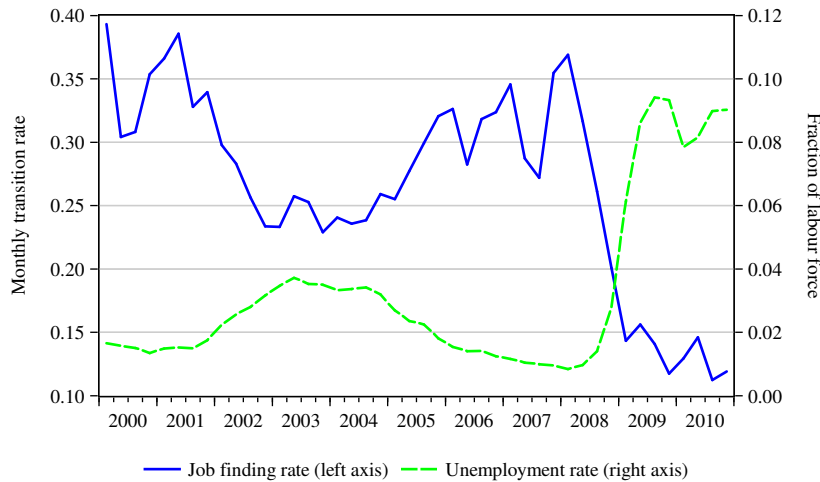


Figure 5: Job Finding Rate (f_t) and Actual Unemployment ($\frac{U_t}{U_t+E_t}$)

Figure 5 plots the job finding rate and the actual unemployment rate. The mean job finding rate is 26.5% but varies over the sample period (st.dev. 7.7%). During the economic slowdown in 2002 and 2003 there was a decrease in the job finding rate and a simultaneous increase in actual unemployment. In the recession starting in late 2008, there was a sharp fall in job finding at the same time as unemployment rose sharply.

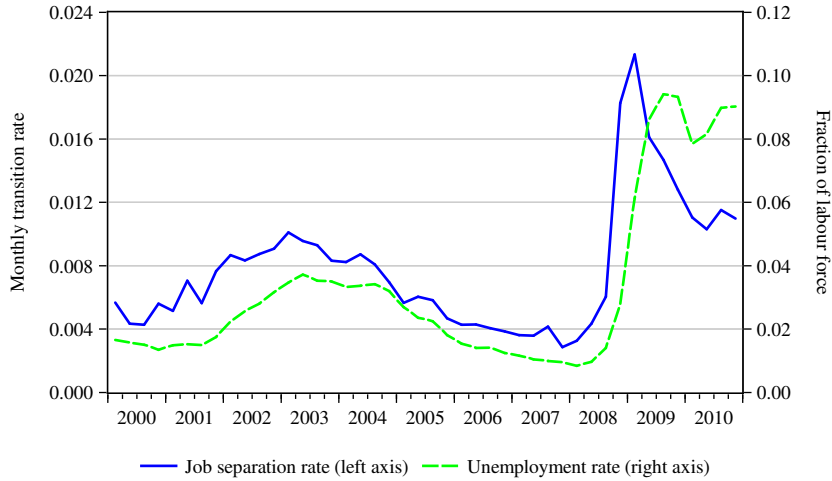


Figure 6: Job Separation Rate (s_t) and Actual Unemployment ($\frac{U_t}{U_t+E_t}$)

However, as depicted in Figure 6, the separation rate also co-moves strongly with actual unemployment. The mean monthly separation rate is 0.8%, but it varies substantially over the period (st.dev. 0.4%). This behaviour of the separation rate during recessions is somewhat similar to what Elsby, Michaels and Solon (2009) find using U.S. data. They find that even though outflow from unemployment is the most important driving force of unemployment variation in the U.S., recessions are characterised with a sudden rise in the inflow rate at the start of the recession and a parallel increase in unemployment. Furthermore, they note that in recessions the job separation rate for job leavers falls while the job separation rate rises for job losers. The evidence presented for the Icelandic labour market, as the evidence in Elsby, Michaels and Solon (2009), therefore indicates that increased unemployment in recessions is caused by the destruction of jobs and an increased number of unemployment spells rather than by a lower job finding rate and longer unemployment spells.

One of the main assumptions behind the decomposition of unemployment variance is that the actual unemployment rate is well approximated by the flow steady-state, $\frac{s_t}{s_t+f_t}$. Figure 7 plots the actual unemployment rate from the DoL data and the flow steady state unemployment rate. It is clear that although the two rates move closely together they are not identical. The contemporaneous correlation between the two rates is 0.90, but the correlation peaks at a lag of one quarter at 0.93. As a matter of fact, the unemployment

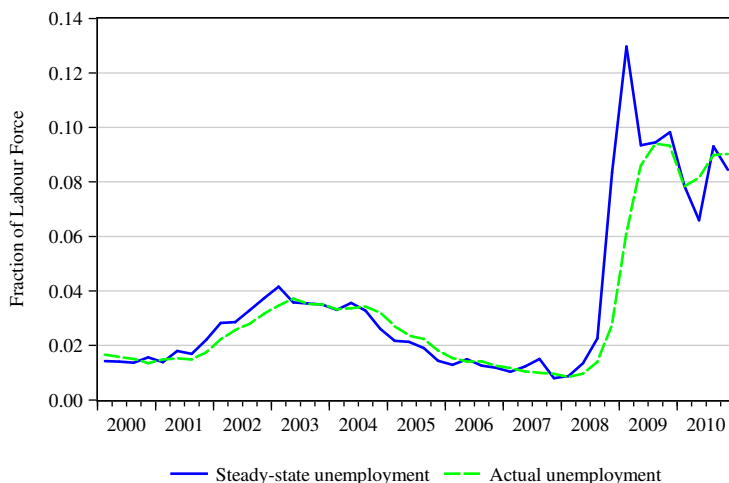


Figure 7: Steady-state Unemployment ($\frac{s_t}{s_t+f_t}$) and Actual Unemployment ($\frac{U_t}{U_t+E_t}$)

rate is moving over time towards flow steady state. This adjustment depends on the aggregate dynamics of flows in and out of unemployment. By rearranging equation (13) and dividing on both sides with $s_t + f_t$, it can be seen that unemployment consistently trails its flow steady state:

$$\begin{aligned} \frac{\dot{u}_t}{s_t + f_t} &= \frac{s_t}{s_t + f_t} - u_t \\ &= \tilde{u}_t - u_t \end{aligned} \quad (21)$$

Equation (21) provides valuable insight into the role of turnover dynamics in the labour market.¹¹ First, it shows that if the steady-state unemployment is above actual unemployment, *i.e.* the right-hand-side of equation (21) is positive, actual unemployment is rising, and *vice versa*. Second, the left-hand-side of equation (21) states that convergence of unemployment to the flow steady-state is determined by the turnover; the convergence is faster the more fluid the unemployment dynamics. In other words, as the more frequent the transitions in and out of unemployment, the closer unemployment is to its flow steady state.

¹¹The importance of turnover dynamics for unemployment has been discussed in the literature, see Hall (2005a) and Smith (2011). Hall (2005a) argues that turnover dynamics do not play a role in unemployment dynamics in the U.S. because of high transition rates. Smith (2011) finds that since aggregate transitions in the U.K. are low, turnover dynamics do matter.

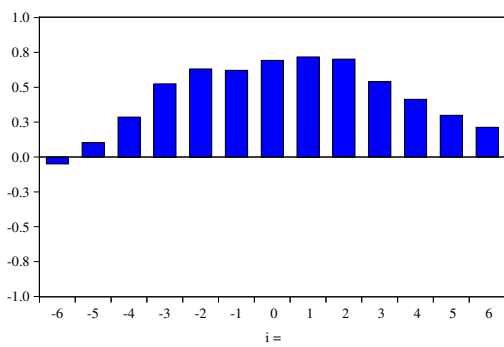
Over the pre-crisis period, both transition rates were high and the monthly outflow rate ranged from 25% to 40%. In times of such fluid dynamics, the unemployment is closely approximated with the flow steady-state. However, when the unemployment rate is changing fast, as in 2002 and 2008, due to an increase in separations and a decrease in job findings, the unemployment and the flow steady state deviates. As emphasized in Elsby and Smith (2010), when unemployment dynamics are less fluid, the flow steady state becomes a *leading indicator* of the evolution in actual unemployment in the short run. Figure 7 shows that *e.g.* from 2002 to 2003, the rise in steady-state unemployment led the rise in actual unemployment. Because of fluid dynamics, however, at other times the steady-state unemployment rate only leads the actual unemployment rate during rapid changes in the latter.

Some authors have argued that because of deviation in unemployment from equilibrium, the flow steady state does not provide a good reference point for the decomposition of unemployment. Smith (2011) argues that if the half-life of a deviation from the flow steady state is far longer than one month, *i.e.* if the sum of the transition rates is much below 50% on a monthly basis, the flow steady state does not provide a good approximation of the actual unemployment rate. Finding very low transition rates for the U.K., Smith (2011) provides a decomposition that does not build on the flow steady-state. Elsby, Hobijn and Sahin (2009) propose a decomposition in a two state environment that allows for deviations of the actual unemployment from its steady state value and present results for fourteen OECD economies. They find that for countries with unemployment transition rates similar to that in Iceland and for which the difference between the sum of the two beta values and unity is low, *e.g.* Australia, Canada and New Zealand, the difference between the steady-state decomposition and the non-steady-state decomposition is limited.

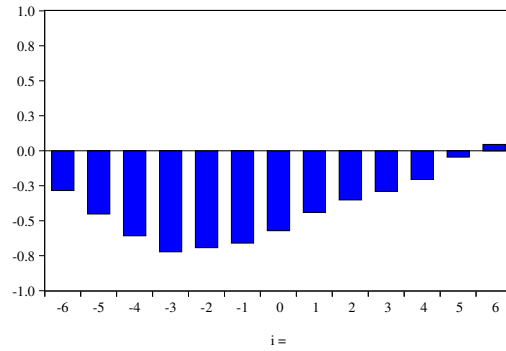
Following Elsby, Hobijn and Sahin (2009), I use a decomposition of unemployment dynamics that accounts for deviations from steady state. Mathematical details are provided in the appendix. Similar to (19) and (20), where steady-state unemployment rate is decomposed into two components, the unemployment rate can be decomposed into the cumulated contributions of contemporaneous and past variation in the job finding rate and separation rate as well as initial deviation from steady state:

$$\beta^s = \frac{\text{cov}(\Delta \ln u_t, \Psi_t^s)}{\text{var}(\Delta \ln u_t)}, \quad \beta^f = \frac{\text{cov}(\Delta \ln u_t, \Psi_t^f)}{\text{var}(\Delta \ln u_t)}, \quad \beta^0 = \frac{\text{cov}(\Delta \ln u_t, \Psi_t^0)}{\text{var}(\Delta \ln u_t)}$$

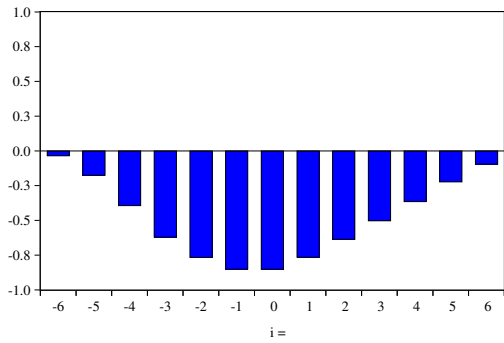
Using this dynamic decomposition I find $\beta^s = 0.64$ and $\beta^f = 0.31$, where the difference of the sum from unity is a residual. These results, which account for deviations of the unemployment rate from the steady-state, are consistent with the earlier results in the paper; approximately two-thirds of the overall variation in the unemployment rate is due to variation in the separation rate and one-third due to variation in the job finding rate.



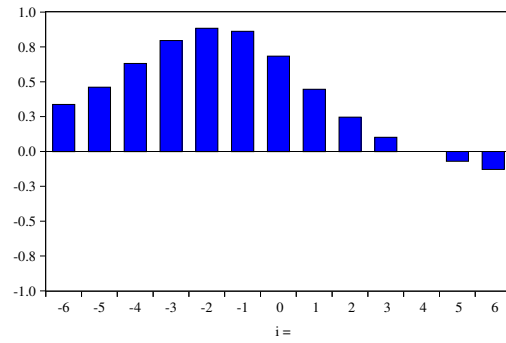
(a) Correlation between $PROD_t$ and f_{t+i}



(b) Correlation between $PROD_t$ and s_{t+i}



(c) Correlation between u_t and f_{t+i}



(d) Correlation between u_t and s_{t+i}

Figure 8: Cross-Correlograms

4 Inflows and Outflows Over the Business Cycle

Assessment of the relative importance of the two driving forces of unemployment, job finding and separation rates, is essential for understanding unemployment dynamics. However, in order to understand how unemployment evolves over time it is necessary to study the

dynamic co-movement of the transition rates with the business cycle.

I evaluate the dynamic relationships cross-correlating the job finding rate, f_t , and the separation rate, s_t , with two measures of economic activity (unemployment, u_t , and productivity, $PROD_t$) at various leads and lags.¹² The cyclical series are extracted using a HP filter with a smoothing parameter of 10^5 . The cross-correlations between productivity and the transition rates are shown in Figures 8a and 8b. For productivity and the job finding rate the correlation peaks at 0.72 at a lead of one and the cross correlation is close to being symmetrical around a lead of one. The correlation between productivity and the separation rate peaks -0.72 but at a lag of three quarters. Figures 8c and 8d further assess the cyclicity of the transition rates using the unemployment rate as a measure of economic activity. The results are roughly similar. The correlation between unemployment and job finding is -0.85 at lags of both one and zero quarters while the correlation with the separation rate peaks at 0.88 at a lag of two quarters. Figure 9 shows the cross-correlation between the job finding rate and the separation rate. The transition rates are highly negatively correlated at lags of zero to three quarters, with the correlation peaking at -0.78 at lag of one quarter. Changes in the job finding rate are therefore preceded by changes in the separation rate one period earlier.

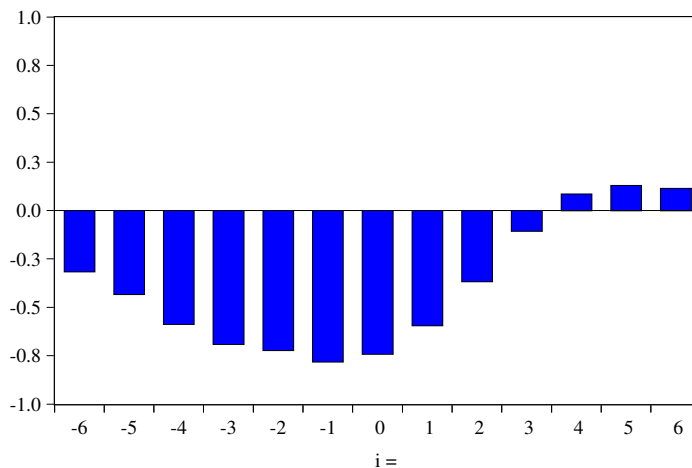


Figure 9: Cross-Correlogram of f_t and s_{t+i}

The evidence presented in Figures 8 and 9 above provides valuable insight into how unemployment evolves over the business cycle. First, the very strong correlation with

¹²Productivity, $PROD_t$, is measured as GDP divided by the level of employment in man-years.

business cycle indicators implies that unemployment transition rates are highly cyclical. The job finding rate is highly procyclical and moves contemporaneously with the cycle. While it too is highly cyclical, the job separation moves inversely to the business cycle and leads the cycle.

5 Concluding Remarks

The aim of this paper has been to study business cycle dynamics in the Icelandic labour market and establish facts about adjustment of labour input and the dynamics of unemployment. I set out to answer two main questions. First, I explored which of the two adjustment margins, the intensive or the extensive margin, is more important for adjustment of labour input in Iceland. According to the results, firms adjust labour input both by adjusting the number of workers employed and the number of hours worked per worker. Variation in employment accounts for 56% of the overall variation in total hours while variation in hours per worker contributes 44%. Furthermore, there is a positive correlation between employment and hours per worker, indicating that labour input is adjusted in the same direction along both margins. I find that even though participation varies over the business cycle, it is a secondary factor in explaining fluctuations in total hours. I also presented evidence suggesting that international migration is an important factor for labour supply.

Second, I explored whether the observed variation in unemployment is driven by the rate at which workers separate from firms or by the rate at which workers find jobs, and how these transition rates move over the business cycle. According to the evidence presented, the main driving force of unemployment is variation in the separation rate, which accounts for 70% of the overall variation in unemployment. Therefore, increased unemployment during recessions is caused by an increased number of unemployment spells rather than by longer spells. Both transition rates co-move strongly with the business cycle. The job finding rate is found to move contemporaneously with the cycle while the separation rate leads the cycle.

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A Non-Steady-State Decomposition of the Unemployment Rate

In this section a decomposition of the unemployment is derived that holds when unemployment is out of its flow steady state. Using this decomposition method, I can approximate the relative contribution of the transition rates to the actual unemployment rate rather than the steady-state unemployment rate. The non-steady-state approximation is a dynamic decomposition, as it takes into account both current and previous steady-state values and therefore accounts for deviations of the unemployment rate that arise because of slow transitions from one flow steady state to the next. Results are presented and discussed in the main text.

The unemployment rate evolves according to:

$$\dot{u}_t = (1 - u_t)s_t - u_t f_t \quad (22)$$

A solution to the differential equation (22) yields the following after some rearrangement:

$$u_t = u_{t-1}(1 - \lambda_t) + \tilde{u}_t \lambda_t \quad (23)$$

where

$$\lambda_t = 1 - e^{-(s_t + f_t)} \quad (24)$$

is the periodic convergence to flow steady-state unemployment, \tilde{u}_t . A log-linearisation of (23) with a first-order Taylor series expansion around $u_{t-1} = \tilde{u}_{t-1}$, $s_t = s_{t-1}$, and $f_t = f_{t-1}$, gives:

$$\ln u_t \approx \ln \tilde{u}_{t-1} + \lambda_{t-1}(\ln \tilde{u}_t - \ln \tilde{u}_{t-1}) + (1 - \lambda_{t-1})(\ln u_t - \ln \tilde{u}_{t-1}) \quad (25)$$

which further breaks down to a more familiar form:

$$\ln u_t \approx \ln \tilde{u}_{t-1} + \lambda_{t-1}(1 - \tilde{u}_{t-1})(\Delta \ln s_t - \Delta \ln f_t) + (1 - \lambda_{t-1})(\ln u_t - \ln \tilde{u}_{t-1}) \quad (26)$$

If unemployment does not deviate from its flow steady state, the periodic convergence λ_t equals unity and (26) reduces to a steady-state decomposition.

Rearranging (25) yields:

$$\ln u_t - \ln u_{t-1} = \lambda_{t-1}(\ln \tilde{u}_t - \ln \tilde{u}_{t-1}) - \lambda_{t-1}(\ln u_{t-1} - \ln \tilde{u}_{t-1}) \quad (27)$$

$$= -\lambda_{t-1}(\ln u_{t-1} - \ln \tilde{u}_{t-1}) + \lambda_{t-1}(\ln u_t - \ln u_{t-1}) \quad (28)$$

which implies:

$$(\ln u_t - \ln \tilde{u}_t) = -\frac{1 - \lambda_{t-1}}{\lambda_{t-1}} \Delta \ln u_t \quad (29)$$

or, for the last component in (25):

$$(\ln u_{t-1} - \ln \tilde{u}_{t-1}) = -\frac{1 - \lambda_{t-2}}{\lambda_{t-2}} \Delta \ln u_{t-1} \quad (30)$$

Substituting (30) into (25) gives:

$$\ln u_t \approx \lambda_{t-1}[(1 - \tilde{u}_{t-1})(\Delta \ln s_t - \Delta \ln f_t) + \frac{1 - \lambda_{t-2}}{\lambda_{t-2}} \Delta \ln u_{t-1}] \quad (31)$$

This is a dynamic decomposition that allows for deviation of unemployment from flow steady state. The first part gives the contribution of the current job finding and separation rates to variation in the actual unemployment rate, while the second part incorporates the impact on the unemployment rate of deviations from steady state due to past changes in the flow rates.

In order to assess the relative importance of inflows and outflows, taking past deviations from steady-state into account, one can calculate the following in a similar manner to (19) and (20):

$$\beta^s = \frac{\text{cov}(\Delta \ln u_t, \Psi_t^s)}{\text{var}(\Delta \ln u_t)} \quad (32)$$

$$\beta^f = \frac{\text{cov}(\Delta \ln u_t, \Psi_t^f)}{\text{var}(\Delta \ln u_t)} \quad (33)$$

$$\beta^0 = \frac{\text{cov}(\Delta \ln u_t, \Psi_t^0)}{\text{var}(\Delta \ln u_t)} \quad (34)$$

where β^s and β^f are, respectively, the cumulative contribution of current and past fluctuations in the separation rate and the job finding rate, β^0 is the contribution of initial deviation from flow steady-state, where the contributions are defined recursively by:

$$\Psi_t^s = \lambda_{t-1}[(1 - \tilde{u}_{t-1})\Delta \ln s_t + \frac{1 - \lambda_{t-2}}{\lambda_{t-2}}\Psi_{t-1}^s], \Psi_0^s = 0 \quad (35)$$

$$\Psi_t^f = \lambda_{t-1}[-(1 - \tilde{u}_{t-1})\Delta \ln f_t + \frac{1 - \lambda_{t-2}}{\lambda_{t-2}}\Psi_{t-1}^f], \Psi_0^f = 0 \quad (36)$$

$$\Psi_t^0 = \frac{\lambda_{t-1}(1 - \lambda_{t-2})}{\lambda_{t-2}}\Psi_{t-1}^0, \Psi_0^0 = \Delta \ln u_0. \quad (37)$$