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EFFICIENT SEARCH BY COMMITTEE

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Abstract

This note constructs an efficient mechanism for finding the best candidate for a committee from a sequence of potential candidates. Committee members have independent private values information about the quality of the candidate. The mechanism selects the best candidate according to the standard utilitarian welfare criterion. Furthermore, the mechanism can be modified to have a balanced budget.

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1 Introduction

A recent paper by Albrecht, Anderson & Vroman (2010) investigated equilibria in models of search by committees. As is natural for many committees, that paper does not allow side payments to be made during the search process. Nevertheless the committee members have often other trade-offs even if outright monetary transfers are not allowed. Consider the example of an academic job applicant. Different research groups may derive different payoffs from hiring the candidate. Often there are other decisions within the department that can be adjusted to compensate a given group. At the very least, the department is also likely to hire in coming years. It is possible to provide some incentives for the committee members by linking current decisions to future ones.

Motivated by considerations of this type, we make the opposite assumption in this note. We assume that utility is perfectly transferable between the committee members. In other words, we assume that the preferences of the committee members are quasilinear in transfers, and we adopt a mechanism design approach to the committee decisions. It seems natural to require that the budget be balanced in the committee. In other words, the members cannot get outside financial help to resolve their conflict and they do not burn money.

The dynamic pivot mechanism defined in Bergemann & Välimäki (2010) can be used in the current setting to support the efficient choice of a candidate. The transfers required to implement this mechanism are easily computed. In fact, these computations are much easier than the equilibrium computations in the model without transfers. The committee decision problem has similarities to a public goods provision problem. The chosen candidate will have an effect on the welfare of all committee members. The opportunity cost of not continuing the search can be thought of as the cost of the chosen candidate. Nevertheless, this cost is just an opportunity cost and therefore not reflected when balancing the budget. Hence in contrast to the problem of providing a public good with a real production cost, we show that the dynamic pivot mechanism generates a surplus. By usual arguments, this surplus can be rebated to the committee members in a manner that balances the budget.

The insights in this note suggest that even if the committees cannot use transfers, search can be conducted more effectively than by using simple voting rules. In more finely tuned deliberations, trade-offs between committee members' preferences are possible. This can be done e.g. by putting less weight in future deliberations on the opinions of the committee members that are blocking the selection of the current candidate.

In Section 2, we describe the model. In Section 3, we define the dynamic pivot mechanism for this setting and show that an efficient mechanism with a balanced budget exists. In Section 4, we discuss some extensions of the result.

2 Model

2.1 Uncertainty and Payoffs

A committee consisting of N symmetric members $i \in \{1, ...N\}$ must choose one alternative from a countable sequence of candidates indexed by their time of appearance $t = 0, 1, ...$ All committee members discount future with the same discount factor δ. A candidate t yields a payoff $x_t^i \in \mathbb{R}$ to committee member i if t is selected. Following Albrecht, Anderson & Vroman (2010) , we assume that the x_t^i are i.i.d. across the i and t and that they are distributed according to a distribution function F. Denote the sum of payoffs amongst the committee members by

$$z = \sum_{i=1}^{N} x_t^i.$$

By independence, z has distribution $G^N(\cdot)$ that can be readily computed by the convolution formula. The sum of the payoffs of all committee members except i is denoted by z^{-i} and again by independence, this random variable has distribution $G^{N-1}(\cdot)$ for all values of x_t^i. We denote the realization of of the vector of payoffs in period t by x_t and the sequence of such vectors by $x = (x_0, x_1, ...)$.

The candidate selection is denoted by $d = (d_0, d_1, ...)$ where $d_t = 1$ indicates that candidate t was selected. Since only one candidate can be chosen, we have

$$d_t = 1 \Rightarrow d_{t'} = 0 \text{ for all } t \neq t'.$$

Given the linear structure of payoffs, it is without loss of generality to consider only deterministic allocations.

The committee members can also be asked to make transfers in each period t. Let $p = (p_t^1, ..., p_t^N)$ denote the vector of transfers in period t and let $p = (p_0, p_1, ...)$ be the sequence of transfer vectors

The expected payoff for player i from an allocation (d, p) is given by

$$v^i(x, d) - p_t^i := \mathbb{E} \sum_{t=0}^{\infty} \delta^t [x_t^i d_t - p_t^i],$$

where the expectation is taken over the realizations x_t^i.

Social welfare in the model is defined by the utilitarian welfare criterion on the committee members ignoring the transfers for the moment.

$$W(x, d) = \sum_{i=1}^{N} v^i(x, d).$$

2.2 Direct Mechanism

Each x_t^i is observed only by committee member i. We construct a direct revelation mechanism where each committee member reports in each period her payoff x_t^i from this candidate. We denote these reports by r_t^i. The hiring decisions $d_t \in \{0, 1\}$ must be decided based on the vector of reports r_t rather than on the true payoffs x_t.

In order to give the committee members incentives to report truthfully, i.e. to choose $r_t^i = x_t^i$, we assume that they can be asked to make or receive transfers p_t^i based on the vector of realized reports. We denote the profile of reports in period t by r_t and we let $r = (r_0, r_1, ...)$.

A public history in period t is a sequence of decisions and reports in the past periods, $h_t := (r_0, d_0, r_1, d_1, ..., r_{t-1}, d_{t-1})$. Each player i also knows her own past realized types $h_t^i := (x_0^i, r_0, d_0, x_1^i, r_1, d_1, ..., x_{t-1}^i, r_{t-1}, d_{t-1})$. A reporting strategy of player i is a sequence of functions

$$\rho_t^i : H_t^i \times \mathbb{R} \to \mathbb{R}.$$

Following (Bergemann & Välimäki 2010), a direct dynamic mechanism is a pair of functions $(d_t(r_t; t_t), p_t(r_t; h_t))$. The mechanism is *ex-post incentive compatible* if for all i, all t, and all x_t^i

$$\begin{aligned} x_t^i \in \arg\max_{r_t^i} & \left[x_t^i d_t\left(r_t^i, x_t^{-i}; h_t\right) - p_t^i\left(r_t^i, x_t^{-i}; h_t\right)\right] \\ & + \mathbb{E}_{x_{s>t}}\left[\sum_{s=t+1}^{\infty} \delta^{s-t} x_s^i d_s\left(x_s^i, x_s^{-i}; h_s\right) - p_s^i\left(x_s^i, x_s^{-i}; h_s\right) \mid h_t, x_t\right]. \end{aligned}$$

Notice that here we are assuming that committee members other than i report truthfully. By the one-shot deviation principle, it is sufficient to check the incentives in period t only for player i. Notice also that in this ex post criterion, all the values of the x_t^i are assumed to be known up to t, but expectations are taken over these variables for $s > t$.

3 The Dynamic Pivot Mechanism

In order to compute the dynamic pivot mechanism we need to solve two social welfare maximization problems for the case of complete information. Let d^* solve

$$\max_d \mathbb{E}\left[\sum_{i=1}^{N}\sum_{s=t}^{\infty} \delta^{s-t}[x_s^i d_s\left(x_s^i, x_s^{-i}\right) | h_t, x_t\right],$$

and let d^{*-i} solve

$$\max_d \mathbb{E}\left[\sum_{j\neq i}\sum_{s=t}^{\infty}\delta^{s-t}[x_s^i d_s\left(x_s^i, x_s^{-i}\right)|h_t, x_t\right].$$

It is a standard exercise in search theory to show that the optimal decision rule for each of these cases is described by a cutoff policy that requires hiring the first candidate for whom $z_t \geq z^{*k}$, where k is the number of players, and z^{*k} solves

$$(1-\delta)\, z^{*^k} = \int_{z-z^{*k}}^{\infty}\left(z - z^{*k}\right) dG^k(z).$$

Denote the optimal values by obtained using the optimal cutoff strategies after history h_t and at current type profile x_t by $W(x_t; h_t)$ and $W^{-i}\left(x_t^{-i}; h_t\right)$.[1] The marginal contribution of player i is

$$M^i(x_t; h_t) := W(x_t; h_t) - W^{-i}\left(x_t^{-i}; h_t\right).$$

The transfers are flow variables in the sense that they are paid each period and the quantities on the right hand side are stock variables reflecting the players' payoffs in the dynamic game. Hence it is clear that the dynamic pivot mechanism must account for this. For this reason, we define the dynamic marginal contributions

$$m_i(x_t; h_t) := M^i(x_t) - \delta \mathbb{E} M^i(x_{t+1}; h_{t+1}).$$

Suppose that the transfers in the model are defined as follows:

$$v^i\left(x_t^i, d_t^*\right) - p_t^{*i}\left(r_t^i, r_t^{-i}; h_t\right) = m_i(r_t; h_t) \tag{1}$$

Bergemann & Välimäki (2010) show that the mechanism defined by $\left(d_t^*(r_t, h_t), p_t^*(r_t, h_t)\right)$ is ex post incentive compatible and satisfies ex post

[1]Even though the optimal decision rule does not depend on histories, we are keeping it in the notation to cover also the case discussed in the extensions where the committee members learn about their own distribution of candidate types.

individual rationality. The payoff from not participating in the mechanism is understood to be the payoff to i resulting from the choice rule d^{*-i}. Therefore we may use the realized values x_t^i for the reported values in the formulas that follow.

The next step is to calculate the payments in the dynamic pivot mechanism. As in any pivot mechanism, $p_t^i = 0$ if $d_t^* = d_t^{*-i}$. Hence we have to cover only cases where $d_t^* < d_t^{*-i}$ or $d_t^* > d_t^{*-i}$. In the first, we have by definition

$$z_t < z^{*N} \text{ and } z_t^{-i} > z^{*N-1}.$$

Therefore

$$W\left(x_t; h_t\right) = z^{*N} \text{ and } W^{-i}\left(x_t; h_t\right) = z_t^{-i}.$$

Since $d_t^* = 0$, we have

$$v^i\left(x_t^i, d_{*t}\right) = 0, \quad \delta \mathbb{E} W\left(x_{t+1}; h_{t+1}\right) = z^{*N},$$

and

$$\delta \mathbb{E} W^{-i}\left(x_{t+1}; h_{t+1}\right) = z^{*N-1}.$$

Substituting into equation (1), one can solve

$$p_t^{*i}\left(x_t^i, z_t^{-i}; h_t\right) = z_t^{-i} - z^{*N} + z^{*N} - z^{*N} > 0.$$

The last inequality follows from the assumption that

$$z_t^{-i} > z^{*N-1}.$$

Consider next the case where

$$z_t > z^{*N} \text{ and } z_t^{-i} < z^{*N-1}.$$

Since $d_t^* = 1$, it is easy to see that

$$W\left(x_t; h_t\right) = z_t \text{ and } W^{-i}\left(x_t; h_t\right) = z^{*N-1}.$$

For the next period,

$$\delta \mathbb{E} W\left(x_{t+1}; h_{t+1}\right) = \delta \mathbb{E} W^{-i}\left(x_{t+1}; h_{t+1}\right) = 0.$$

Therefore

$$p_t^{*i}\left(x_t^i, z_t^{-i}; h_t\right) = z_t - z^{*N-1} > 0.$$

Since the payments are non-negative for all players at all realization of the x_t^i, this calculation shows that the dynamic pivot mechanism generates a surplus.

4 Discussion

Budget Balance The dynamic pivot mechanism constructed is ex post incentive compatible, individually rational and results in an ex post budget surplus. In order to balance the budget, the transfers could be modified to bring the transfers to the level of the associated expected externality mechanism. This would nevertheless come at a cost. In the dynamic pivot mechanism, the committee members get as their payoff their marginal contribution. In the modified balanced budget mechanism, this would no longer be true. Hence there could be a potential trade-off between budget balance and correct incentives to pay private participation costs in the committee.

Correlated Committee Members Since the dynamic pivot mechanism satisfies ex post incentive constraints, the same construction remains valid if the x_t^i are correlated across i. The only adjustment needed to cover this case is to note that the distribution $G^N(z^n)$ is no longer a convolution of the marginal distributions.

Learning the Payoff Distributions The dynamic pivot mechanism can be modified to cover situations where the x_t^i are correlated across t.. An example of this would be models where each committee member draws payoffs from a conditionally i.i.d. draws from independent distributions $F\left(\cdot : \theta^i\right)$ and where the they learn in a Bayesian fashion about the θ^i. The main difference in this setting would be that the optimal decisions now depend in a non-trivial manner on the histories.

Interdependent Values Sometimes it is natural to think that the committee members have independent payoff relevant signals y_t^i on the candidates. To satisfy the appropriate monotonicity requirements, it must be assumed that the signal of member i has a larger impact on i's payoff than on the payoffs of the other committee members. For example, we could have:

$$x_t^i = y_t^i + \gamma \sum_{j \neq i} y_t^j \text{ for some } \gamma \leq 1.$$

In this case, we could construct a dynamic version of the generalized VCG mechanism to implement the efficient decision rule based on the reported signals y_t^i. The main difference compared to the dynamic pivot mechanism would be in the calculation of the transfers. In the generalized VCG mechanism, the transfer of a pivotal player i that blocks an otherwise acceptable candidate is calculated based on the societal payoffs from the highest y_t^i that overturns the decision of the other members. Similarly the transfer of the pivotal player that forces the acceptance of an otherwise rejected candidate would be computed based on the lowest signal overturning the decision.

It should perhaps be noted that even in the pure common values setting where $\gamma = 1$, voting games perform quite badly whereas the generalized VCG mechanism implements the efficient allocation without requiring transfers.

More Complicated Committee Decisions The dynamic pivot mechanism can be computed for committee decisions when multiple candidates must be selected. The insights gained from the simplest model give some suggestions for further work along this dimension even in cases where transfers are ruled out. If the committee must select two candidates from a sequence of potential candidates, an optimal decision rule can discipline those committee members that block the appointment of the current candidate by putting less weight on their reports when deciding on future candidates. With two candidates, the member that insists on getting her favorite candidate chosen for the first slot can be made to pay an implicit price by discounting her opinions when filling the second slot.

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