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Option Value of Harvesting: Theory and Evidence

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Abstract *Real options analysis shows that fishery harvesting becomes more conservative, and catch efficiency is raised, when exploitation is subject to irreversibility, uncertainty, and delay for a cooperative profit-maximizing fleet. With Rock Lobster catch and effort data from Fisheries Victoria of the Department of Natural Resources and Environment in Australia, the options-augmented surplus-yield model is tested and compared with the performance of its conventional profit-maximizing and biological maximum-sustainable-yield counterparts.*

Key words Catch per unit effort, delay, irreversibility, options, uncertainty.

Introduction

Commercial aggregate catch per unit effort (CPUE) is affected by numerous factors: individual fishing effort, stock abundance and concentration, capture success probability, licenses and quotas, and seasonal/spatial variations (Seber 1982; Burnham *et al.* 1987; Hilborn and Walters 1992; Wallace, Lindner, and Dole 1997). In this paper, a theory of harvesting is developed to show that *stock-size uncertainty* can affect optimal fishing effort, harvesting catch and the resulting CPUE.

We begin with the institutional framework of cooperative profit-maximizing license-restricted fishery. Suppose the fishermen's co-op targets a biologically sustainable harvest and a mutually agreed and legally binding division of economics returns that allows also for differing management goals, such as those arising from different rates of discount or supply conditions among fishermen. This can be achieved in practice through internal transfer payments with the economic returns from the fishery to any one fisherman not dependent solely upon that fisherman's harvest (Munro 1979, 1996).

We further assume that the fishermen face a competitive output market, are given deterministic market conditions about price and cost, enjoy the absence of illegal poaching or encroachment from outsiders, harvest sustainably with a stable technology, and have identical, rational expectations about uncertain future fish stock dynamics, whose movements are spanned by existing financial instruments or their combinations in well-functioning capital markets, where all risks are efficiently priced (Dixit and Pindyck 1994, ch. 9). The following decision problem is then posed for the fishermen: for logistical reasons the fleet could only be deployed once in each harvesting period, and the fishermen were to determine their collective profit-maximizing effort.

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Preliminaries

First let us examine three preconditions: future stock uncertainty, irreversible harvesting, and delay. Before periodic harvesting occurs, fishermen face an uncertain future dynamic profile of the stock size under surplus-yield harvesting. For example, habitat can change, regenerated stock can migrate, new recruitment can come from outside spawning stock, and there may be unforeseen environmental changes. Supposing that the present stock or its initial *state* is known with certainty, we model the stochastic behavior of fish stock (X) by the following *geometric Brownian motion*:

$$dX = \alpha X dt + \sigma X dz \tag{1}$$

where dz is the increment of a standard Wiener process (Dixit 1991); α is the intracycle stock movement within a harvest period and is distinct from intercycle stock regeneration that gives rise to the usual biological stock-recruitment process, viz. metered age-structured models (Clark 1990); σ is the instantaneous standard deviation on fish stock capturing the uncertainty surrounding future regeneration. Since regeneration would be offset by a sustainable harvest in each harvesting cycle and all harvesting cycles are identical, the resulting stock X, as specified in equation (1), will represent the sustained stock resulting from identical harvest and regeneration in the previous cycle. To harvest from such a stock, we postulate the following production function:

$$H(E, X) \equiv \text{ASY} = qEX \tag{2}$$

where ASY is the average sustainable surplus yield; E the harvesting effort (control variable); and q is the *fixed* catchability coefficient.

The second precondition is *irreversible* harvesting. In general, an investment is irreversible if capital invested cannot be recouped should adverse market conditions render it unworthy after investing (Dixit and Pindyck 1994). Harvesting can be irreversible since the harvested stock cannot be returned to the habitat. As a result, a harvest decision based on deterministic market conditions and uncertain future stock size cannot be unwound after it has been exercised.

Finally, the harvesting decision can be *delayed*. If the fish stock is not harvested now it will still be available for harvesting at any future time within the harvesting cycle. Therefore, harvesting is not a now-or-never proposition. Without illegal poaching or encroachment from outsiders, this would be the case as if a single decision-maker were dominating the management and controlling the fishery (Munro 1979, 1996). Using the real options analogy, the fleet has the *right*, but not the *obligation* to exercise the harvesting *option*, depending on how *deep in the money* that option is.

The Model

Using equation (2), the profit $\pi(E, X)$ in a harvesting period by exerting effort E under stock size X is:

$$\pi(E, X) = pqEX - cE \tag{3}$$

where p and c are per unit catch price and effort cost, respectively. Quantity $\pi(E, X)$ is realized only if harvesting takes place. Let $F(X)$ be the value of the harvesting opportunity, which includes equation (3) and the option value of harvesting. This is the value of the right or the option to exercise the *irreversible* harvesting decision. $F(X)$

is obtained by maximizing equation (3) over the effort level E subject to a constraint on the stochastic dynamics of the stock in equation (1). Assuming a no-arbitrage-opportunities equilibrium discount rate ρ (Brealey and Myers 1992) and applying Ito's lemma, we derive the behavior of $F(X)$ while harvesting is *not* taking place as:

$$\frac{1}{2}\sigma^2 X^2 F_{XX} + \alpha X F_X - \rho F = 0. \tag{4}$$

When harvesting takes place, the irreversible harvesting option is exercised and $F(X)$ becomes $\pi(E, X)$. Using equations (3) and (4), our objective is to find the optimal effort solution. Label it E_{op}. We design a decision rule which requires that whenever the stock level reaches a certain threshold (X_{op}), fishermen harvest according to E_{op}. Since $\pi(E, X)$ is linear in X, the regime we describe will consist of a "bang-bang" controlled harvesting at E_{op} whenever $X \geq X_{op}$, and not harvesting otherwise (Clark 1990). Consequently, the harvesting decision can be *delayed*.

The differential equation (4) has two auxiliary quadratic roots: the larger-than-positive-one root β_1, and the negative root β_2. Assuming $\alpha < \rho$, then by the method of undetermined coefficients and preventing F from becoming infinitely large, F can be written as:

$$F = \begin{cases} \pi(E, X) & \text{if } X \geq X_{op} \\ AX^{\beta_1} & \text{otherwise} \end{cases} \tag{5}$$

where A is an undetermined coefficient and X_{op} is the threshold above which optimal harvesting occurs. When $X < X_{op}$, F consists entirely of the option value (AX^{β_1}). To solve for A and X_{op} we need two conditions:

$$A(X_{op})^{\beta_1} = \pi(E, X_{op}) \quad \text{(value-matching)} \tag{6}$$

$$A\beta_1 (X_{op})^{\beta_1 - 1} = \frac{\partial \pi(E, X_{op})}{\partial X_{op}} \quad \text{(smooth-pasting)} \tag{7}$$

Condition (6) requires that, at the boundary between harvesting and not harvesting, the fleet be indifferent and, therefore, attach the same F to the value of the harvesting opportunity and profit. Condition (7) guarantees continuity of the slope of F at the point of indifference so that F remains the same whether X_{op} is approached from the left or the right (Dixit 1993). Solving equations (3), (6), and (7) gives the level of profit required before harvest is initiated:

$$\pi_{op} = \frac{p \cdot qEX_{op}}{\beta_1} \tag{8}$$

Therefore, the harvesting decision will only be profitable if harvesting revenue $p \cdot qEX_{op}$ covers the harvesting cost cE and equation (8):

$$\Pi(E, X_{op}) \equiv p \cdot qEX_{op} - cE - \frac{p \cdot qEX_{op}}{\beta_1} \tag{9}$$

$$= p\left(\frac{\beta_1 - 1}{\beta_1}\right) qEX_{op} - cE$$

where $\Pi(E, X_{op})$ is the harvesting profit function that also accounts for the option value. Since the fleet has power to exert its effort at E_{op} in equation (9), it maximizes its group harvesting profit at the optimal level represented by $\Pi_{op}(E_{op}, X_{op})$. Comparing with traditional profit maximization vis-à-vis equation (3), the profit in equation (9) is a result of exercising the fishing option whenever the stock size is favorable at $X \geq X_{op}$. This profit is diminished by the value of such an option that is killed as a consequence of fishing. The diminution is measured by the term $(\beta_1 - 1)/\beta_1$. Since equation (9) does not involve any ad hoc fisheries stock-recruitment parameters, it becomes significant in its generality. For example, although different fisheries are better explained by species-specific models, they are invariably expressed in a relationship linking harvesting effort and the sustained stock. We demonstrate this generality by the Gordon-Schaefer average sustainable yield model:

$$ASY \equiv qEX = qkE\left(1 - \frac{qE}{r}\right) \tag{10}$$

(Clark 1990, ch.2) where k is the ceiling stock size and r is the intrinsic natural growth rate. After inserting E_{op} and X_{op}, solving for X_{op}, and substituting it out in equation (9), we can maximize Π_{op} for the option-value effort solution:

$$E_{op} = \frac{r}{2q}\left(1 - \frac{c\left(\frac{\beta_1}{\beta_1 - 1}\right)}{pqk}\right) \tag{11}$$

where q is the fixed catchability coefficient. Since β_1 is larger than one, $\beta_1/(\beta_1 - 1)$ must also be larger than one and is known as the *option-value multiple* (hence the subscript *op*). This multiple has the effect of slowing down harvest rate and making the benefit-cost decision more conservative because benefits have to overcome both costs as well as the value of keeping the harvesting options open and alive (Dixit and Pindyck 1994). Furthermore, the option-value multiple will become larger and the fishing effort correspondingly more conservative as the uncertainty element becomes more pervasive.[1] In the effort equation (11) with fixed q and k, fish price p will not only have to overcome unit effort cost c, but also the option-value multiple $\beta_1/(\beta_1 - 1)$ before effort is justified. The resulting option-value CPUE solution is then:

$$CPUE_{op} = \frac{qk}{2}\left(1 + \frac{c\left(\frac{\beta_1}{\beta_1 - 1}\right)}{pqk}\right). \tag{12}$$

Therefore, conservative effort leads to more subdued harvesting and increases CPUE in equation (12).

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[1] It can be shown that an increase in σ decreases β_1, increasing the option-value multiple.

Estimation Results

We statistically test equation (12) using catch and effort data for the Victorian rock lobster fishery, which is license-controlled with individual transferable quotas under a co-management system involving fishermen, managers, and recreationists. To facilitate empirical testing, equation (12) is rewritten as:

$$\left(CPUE_{op} - \frac{qk}{2}\right) = \mu \frac{c}{2p}\left(\frac{\beta_1}{\beta_1 - 1}\right) + \varepsilon. \tag{13}$$

Equation (13) is estimated using monthly time-series data for $CPUE_{op}$. Each harvesting cycle is sufficiently small to assume that the intracycle stock movement α is practically zero. Quantities q and k are estimated from Schnute's procedure (Schnute 1977). Quantity p is proxied by gross value of production divided by total catch. The Victorian unit fishing effort cost, c, is proxied by Melbourne all-group consumer price index which covers a fisherman's basic fishing and vessel expenses. Quantity μ is a scale parameter whose size depends on the units of measurement for c and p.

The hypothesis $H_0{:}\mu > 0$ versus $H_1{:}\mu \leq 0$ was tested on the Eastern and Western Zones of Port Phillip Bay, Australia, separately. Appendix A contained a description of the data. Schnute's procedure produced the surplus-yield model estimates in appendix table B1. Using the estimated q and k, a linear regression was then fitted to equation (13) with a suppressed constant. The estimation results were reported in appendix tables B2 and B3.

The regression results suggest that, at a 5% level of significance, the hypothesis of a positive μ—a positive relationship between CPUE and the option-value multiple—cannot be rejected by evidence. We thus conclude that the model given in section 3 for Victorian rock lobster harvesting under uncertainty can be accepted as a competing hypothesis.

We have also provided the corresponding estimates for the conventional profit-maximizing Gordon-Schaefer model and the biological maximum sustainable yield (MSY) model in appendix B. The Gordon-Schaefer model restricts the option-value multiple to unity. The MSY model further restricts the cost/price ratio to be constant. Under our cooperative framework comparable with the incentive-compatible sole-owner controlled fishery, both model adequacy and parameter estimate precision decrease with the level of reality in the order of the three models, with the option-value model having the best performance.

Conclusion

In addition to harvesting profit, there is also an intrinsic value of keeping the harvesting options alive when stock exploitation is subject to irreversibility, uncertainty, and delay. This renders harvesting more conservative. Using the Victorian rock lobster catch and effort data in Australia, the uncertainty-augmented maximum-profit surplus-yield model was tested and found to have outperformed its conventional and maximum-sustainable-yield counterparts. There is one caveat: we assumed *a priori* that the uncertain component of the future stock evolution was described by Brownian motion in the form of a standard Wiener process; *i.e.*, uncertainty was bias-neutral and could be positive or negative with equal likelihood. If uncertainty is biased against the downside—more likely for stock to turn out higher than lower, then the option value of harvesting will be increased and the conservatism will be reinforced. On the other hand, if uncertainty is biased against the upside, then the conservatism will be muted and option value will be lowered. In the latter case, we cannot say definitively that fishing effort will still be lower than when there exists no uncertainty.

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Appendix A

Data

Variable	Description	Sample Period	Source
Catch (*H*)	Monthly lobster catch in kg	June 1977 to December 1996 excluding out-of-season months	Victorian Fisheries Catch and Effort Information Bulletin (DNRE 1996)
Effort (*E*)	Monthly number of potlifts	ibid.	ibid.
$CPUE_{op}$	Average fisherman daily CPUEs for the month (kg per potlift)	ibid.	ibid.
Standard deviation of $CPUE_{op}$ (σ)	Standard deviation of fisherman daily CPUEs for the month (kg per potlift)	ibid.	ibid.
Lobster price (*p*)	Monthly lobster catch price per kg	December 1978 to December 1996	ibid.
Unit effort cost (*c*)	Monthly Melbourne all-purpose consumer price index (base = 100 at 1979/80)	April 1985 to December 1996	Victorian Monthly Summary of Statistics (Australian Bureau of Statistics, Cat. No. 5229.2)
Discount rate (ρ)	All capitals monthly average indicative interest rate (%)	July 1983 to December 1996	ibid.

Appendix B

Estimation Results

Table B1
Schnute's Estimates for Victorian Rock Lobster Catch and Effort

Surplus Yield Model	Eastern Zone	Western Zone
Intrinsic growth rate *r*	0.31	0.83
Catchability coefficient *q*	1.8×10^{-6}	4.3×10^{-6}
Ceiling stock size *k*	360,000	250,000
Model adequacy	$F(2,197) = 10.79$ ($p = 0.00$)	$F(2,202) = 9.17$ ($p = 0.00$)

Table B2
Explaining Victorian Rock Lobster CPUE (Western Zone) by: (1) Option-value Model, (2) Conventional Profit-maximization Model, and (3) Biological MSY Model

Western Zone	(1) Option Value Model	(2) Profit-maximization Model	(3) Biological MSY Model *
μ (scale parameter)	3.4×10^{-3}	1.7×10^{-3}	19.6×10^{-3}
Standard error	1.5×10^{-3}	3.7×10^{-3}	17.5×10^{-3}
p-value	0.02	0.65	0.27
Model adequacy	$F(1,111) = 4.13$ ($p = 0.04$)	$F(1,111) = 1.03$ ($p = 0.31$)	$F(1,111) = 0.00$ ($p = 1.00$)

Table B3
Explaining Victorian Rock Lobster CPUE (Eastern Zone) by: (1) Option-value Model, (2) Conventional Profit-maximization Model, and (3) Biological MSY Model

Eastern Zone	(1) Option Value Model	(2) Profit-maximization Model	(3) Biological MSY Model *
μ	9.6×10^{-3}	18.0×10^{-3}	72.4×10^{-3}
Standard error	9.2×10^{-4}	2.5×10^{-3}	12.7×10^{-3}
p-value	0.00	0.00	0.00
Model adequacy	$F(1,112) = 58.94$ ($p = 0.00$)	$F(1,112) = 15.99$ ($p = 0.00$)	$F(1,112) = 0.00$ ($p = 1.00$)

* Assuming a constant unit effort cost-catch price ratio