The Dynamics of an Open-access Fishery: Baltic Sea Cod

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Abstract *This paper sets up a dynamic open-access model of a single industry exploiting a single resource stock. The model is applied empirically to describe the dynamics of the eastern Baltic Sea cod fishery. The theoretical model is based on the benchmark papers by Smith (1968, 1969). Types of steady state are discussed theoretically and the theory is applied to the eastern Baltic Sea cod fishery. The empirical path the fishery has been following since 1982 is determined and how it relates to the optimal path to steady state is discussed. Comparisons are made to other empirical studies, and the stability of the steady state is evaluated. The paper concludes that the Baltic Sea cod stock likely is on a path to a stable steady state, and it might not be a problem that the stock is below safe biological limits.*

Key words Baltic sea cod, bio-economics, dynamic entry/exit, fisheries, open-access, stability of steady state.

JEL Classification Codes Q21, Q22.

Introduction

Since Warming (1911), Gordon (1954), and Scott (1955), there has been an increasing amount of literature describing fisheries economics. Warming (1911) and Gordon (1954) sought to explain why an open-access fishery resulted in little or no profit. It was pointed out that entry of the mobile factors proceeds beyond the numbers sufficient for economically efficient use of the fish stock. Hence, in the absence of property rights, too much effort and too little biomass characterize the open-access fishery. Scott examined the difference in the intensity of fishing between a common property resource and a resource owned by a sole owner.

These early models of fisheries characterized different equilibria without discussing the dynamics behind the equilibrium. Smith (1968, 1969) addressed this issue by describing the dynamics of the use of an open-access resource and is thereby a pioneer in applying phase diagrams in fishery economics. He characterized open-access equilibrium, including some hypotheses about the dynamics of an open-access resource overexploitation. Even though stock equilibrium under open-access might be positive, the stock may be driven close to extinction along the path of adjustment due to overshooting. Wilen (1976) applied the dynamic theory of Smith to the Pacific fur seal. He showed that the sealing industry followed a pattern

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quite close to that predicted by Smith. The industry followed a convergent 'boom-and-bust' process characterized in the later periods by low, falling profits and industry exit. Bjørndal and Conrad (1987) also applied the theory of Smith (1968), with special reference to the question of stock extinction under open access. A non-linear deterministic model for the North Sea herring fishery showed that the industry overshoots, but the increase in the stock, and hence the first loop of a convergent spiral, was not completed because the fishery was closed in 1977. Therefore, one can only guess that extinction might have occurred if the fishery did not close. These empirical models applying difference equations have still left the theoretical evaluation of the stability of the steady state unaddressed.

This paper contributes to the literature on the dynamics of an open-access fishery in two ways. It provides an empirical evaluation of the eastern Baltic Sea cod fishery. Further, it extends the previous empirical work of Bjørndal and Conrad (1987) by examining the stability properties of an empirically evaluated steady state.

This paper sets up a discrete-time, deterministic model of a single industry, which is used to describe the dynamics of recovery of a renewable resource. The model is based on Smith's theories of the dynamics of resource recovery with a biological constraint and a technological constraint (Smith 1968, 1969). The setup in Smith (1969) applies general functional forms, and he discusses the scenario with multiple steady states. The theoretical part of this paper discusses the steady state and also applies general functional forms. The theoretical part of our model contributes to the literature since we also analyse the steady states theoretically. The setup we apply differs slightly from Smith (1969). The biological function is divided into a function describing growth in the biomass not considering recruits and a function describing recruits entering the biomass, a so-called delay-difference model. The theoretical model is applied to the eastern Baltic Sea cod fishery, which has been subject to *de facto* open-access since records began in 1966. The resource is currently considered to be below safe biological limits and the scientists at the ICES recommend a total closure of the fishery (ICES 2000).[1,2] The applied model shows that the eastern Baltic Sea cod fishery most likely has a stable steady state and that the open-access path to this steady state is a spiral, which is why it involves overshooting of effort and hence an extremely low level of biomass. If the resource truly is on a path to steady state, then regulations, as a closure of the fishery, might not be necessary.

The Dynamics of an Open-access Fishery

Consider a renewable resource, such as a fishery, that is exploited by an industry. The industry is characterized by open-access and hence faces a free entry-exit procedure. The dynamics of the system have three behavioral restrictions; namely, the interactions of the resource stock, individual firms, and the industry. These are the main ideas in the theory of Smith (1968, 1969). This section sets up a model describing the dynamics of an open-access fishery based on Smith's theory.

The industry exploits the resource according to a production function that depends on the size of the resource stock at time t and the number of vessels in the industry at time t,[3] which can be written as:

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[1] ICES is an abbreviation for International Council for the Exploration of the Sea (www.ices.dk).

[2] There was a total closure of the cod fishery in the spring of 2003, just before the regular summer ban.

[3] Smith (1969) also allows for mesh size considerations (or any other technological measure) in his general setup. This does, however, complicate the analysis significantly, and he omits it in what he refers to as a simple illustration. Likewise, it is omitted in our model description.

$$Y_t = H(K_t, S_t), \tag{1}$$

where Y_t is the production of the industry as a whole at time t; $H(S_t, K_t)$ is the harvest from the biomass, where S_t is the exploited biomass at time t; and K_t is the amount of effort employed in the industry at time t. It is assumed that the first derivatives of the production function with respect to stock and effort, respectively, are positive: $H_S \geq 0$ and $H_K \geq 0$.

The industry as a whole faces an instantaneous profit function according to following equation:

$$\pi_t = p_t H(K_t, S_t) - c_t K_t, \tag{2}$$

where p_t is the unit price the industry receives for its production at time t, and c_t is the unit cost of employing effort faced by the industry at time t. It is implicitly assumed that the industry is one of several sources to the market in question, otherwise the price would depend on yield and cost per effort unit. It is further assumed that the industry faces constant marginal costs and that unit prices and unit costs are constant through time.[4]

The entry and exit process depends on the profit level in the industry; positive profit attracts effort to the industry, while negative profit makes effort withdraw.[5] By assumption, if there is a positive profit, there are an unlimited number of potential vessels attracted to the industry. The entry/exit process, therefore, occurs according to the following equation:

$$\Delta K_t = K_{t+1} - K_t = \mu \pi_t = \mu \big[pH(S_t, K_t) - cK_t \big], \tag{3}$$

where $\mu > 0$ is an adjustment parameter or a behavioral constant; the larger the μ, the faster the industry reacts to changes in profit.[6] This behavioral restriction contains both the industry and the individual firm's behavioral restrictions. When deciding whether an additional vessel is to enter the industry, the number of vessels employed by the industry and stock size are exogenous. Since we are assuming open-access, the behavioral restriction for the single fleet is determined by the difference between the profit and zero.[7] Vessels enter the industry when there is a positive profit and leave the industry when there is a negative profit. For simplicity, we assume constant average costs of production.[8] The open-access nature of the resource uniquely influences the cost structure of the recovery process. As a

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[4] This assumption is a simplification compared to the original model by Smith (1968, 1969). Smith (1969) allows price to vary with the harvest. However, a constant price is more appropriate in this model, since the Baltic Sea is a comparatively small supplier of cod to a global whitefish market in which there are many substitute species.

[5] One could assume without loss of generality that a certain level of profit greater than zero is required in order to enter and remain in the industry.

[6] Smith (1968) suggests a difference in the speed of reaction depending on whether it is entry or exit from the industry that is being considered. By assuming that vessels can easily be used in other fisheries, we can argue for an equivalent speed of entry and exit. One could also apply a response function to make the model more descriptive of real world situations. Such a response function might include the alternative profit from other fisheries. This does, however, complicate the computations and is therefore omitted.

[7] One could assume a minimum profit required to enter the industry because there are sunk costs connected with entering the fishery. Assuming the minimum profit is zero does not change the general conclusion and the minimum profit is, therefore, omitted in this analysis.

[8] General cost hypotheses are: An increase in biomass increases the density, and the species are easier to catch, thereby decreasing the cost. On the other hand, if increases in the biomass do not change the density, then it may be that costs are unchanged, hence $\partial C/\partial X \leq 0$. This prevents extinction of the stock. If the inequality is strict, then recovery costs show stock externalities. The efficiency of each unit of effort employed in the industry may be lowered by congestion, hence $\partial C/\partial X \geq 0$. If the inequality is strict, then recovery cost is said to suffer from crowding externalities.

consequence, there might be direct and significant diseconomies of production with divergence of private and social optima.

The population is assumed to follow a biological delay-difference model. We have chosen the delay-difference model with the Baltic Sea cod fishery in mind. This fishery is estimated by ICES (2000) applying an extensive cohort model with seven cohorts and a stock-recruitment function, but such an age-structured model for growth would complicate the analysis. We, therefore, approximate the cohort model estimated by ICES with an aggregate cohort model including only two cohorts, namely the recruits and the rest. The reason for this is that the recruits are not entering the biomass immediately after spawning. The primary concern of the delay-difference model is to identify the harvestable biomass in weight. Since we assume the harvestable biomass is identical to the spawning stock biomass for the Baltic Sea cod fishery, the model will be formulated in terms of this variable. The stock at time t, S_t, thus refers to spawning stock biomass at time t. Other examples of delay-difference models applied to fisheries are given by Bjørndal (1990). The delay-difference model is described in equation (4):

$$\Delta S_t = S_{t+1} - S_t = F(S_t) + G(S_{t-j}) - H(S_t, K_t). \qquad (4)$$

The delay-difference model is divided into three parts on the right side. The first part, $F(S_t)$, determines the stock excess after growth and the natural mortality of the biomass. This is evaluated on the biomass before it is harvested and recruits are entering the biomass. We believe it is realistic to let growth and mortality be density dependent, because there will be relatively more food available for a small stock than a large one. Therefore, growth and mortality is described by the $F(S_t)$ function. The second part, $G(S_{t-j})$, represents the addition to the stock due to recruitment, which is assumed to occur at discrete time intervals, measured in weight. Moreover, recruits will not join the parent stock until several years, j, after spawning, this is referred to as a delay. The delay can be explained by the recruits not entering the biomass until they have reached a certain size or reached maturity, *etc*. If $j = 0$, there is no lag and the recruits are entering the biomass at the end of the year. The third part, $H(S_t, K_t)$, represents the deduction from the spawning stock due to harvesting. The fishery is assumed to take place at the end of the period. In the delay-difference function, it is assumed that there is no interaction between the three parts of equation (4) — growth and mortality of the biomass, recruitment to the biomass, and harvest from the biomass.

With given initial values of the system, it is possible to iterate forward in time, and trajectories (S_t, K_t) can be plotted in a phase-space. The stationary point(s) (or steady states) are defined where $\Delta S_t = 0$ and $\Delta K_t = 0$; *e.g.*, where there is no change in either stock or the capital stock over time. Assume we are in steady state, therefore $S_{t+1} = S_t$ and $K_{t+1} = K_t$. A possible plot of the isoclines is illustrated in figure 1. The intersection of isoclines determines the steady state.

The slopes of the isoclines are likely similar to those illustrated. To explain this, consider that we are located on the K-isocline. If the stock increases, it is realistic to believe that a higher capital stock-level is required to retain the zero-profit assumption (*e.g.*, retain our location on the K-isocline). Likewise, assume that we are located on the S-isocline and that the capital stock is increased. Then it is realistic to believe that the stock-level must also decrease to keep our location on the S-isocline. It should be emphasized that the slope of isoclines may differ and that the steady state does not need to be unique.

Since our system of difference equations is non-linear, we linearize the system around steady state. By evaluating the stability of the linear system, we are able to
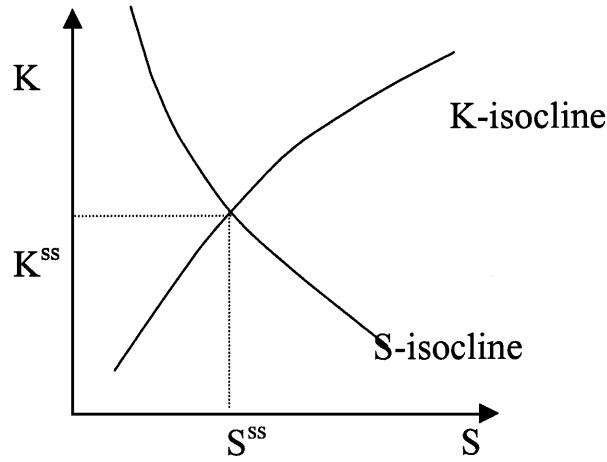
Figure 1. Possible Plot of the Stock and the Capital Stock Isoclines

determine the local stability of our steady state. Thus, we evaluate the behavior of trajectories in the neighbourhood of the steady state. Since our system is of higher order, we cannot directly apply the Jacobian matrix. We linearize the system around the steady state and then deduce the system into a single unknown and determine the characteristic function related to this higher order difference equation. The roots of the characteristic function are determining the general solution. A Taylor approximation around steady state looks like:

$$S_{t+1} = \frac{\partial S_{t+1}}{\partial S_t}\bigg|_{SS} S_t + \frac{\partial S_{t+1}}{\partial K_t}\bigg|_{SS} K_t + \frac{\partial S_{t+1}}{\partial S_{t-j}}\bigg|_{SS} S_{t-j} + \text{constant} \tag{5a}$$

$$K_{t+1} = \frac{\partial K_{t+1}}{\partial S_t}\bigg|_{SS} S_t + \frac{\partial K_{t+1}}{\partial K_t}\bigg|_{SS} K_t + \text{constant}. \tag{5b}$$

Applying the associated homogeneous case, we can eliminate K_t and K_{t+1} in equation (5b) by isolating K_t in equation (5a) and by replacing t with $t + 1$.[9] The difference system has been deducted to depend on a single unknown, as follows:

$$S_{t+2+j} + (-S_S - K_K)S_{t+1+j} + (K_K S_S - S_K K_S)S_{t+j} + S_{S_{-j}} S_{t+1} + S_{S_{-j}} K_K S_t = 0, \tag{6}$$

where S_S, S_K, and $S_{S_{-j}}$ refer to the partial derivative of S_{t+1} wrt. S_t, K_t, and S_{t-j}, respectively, evaluated at steady state, and K_K and K_S refer to the partial derivative of K_{t+1} wrt. K_t, and S_t evaluated at steady state. They are all numbers. The correspond-

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[9] The stability properties of the steady state are not affected by the constants in equations (5) and (6).

ing characteristic equation is:

$$\lambda^{2+j} + (-S_S - K_K)\lambda^{1+j} + (K_K S_S - S_K K_S)\lambda^j + S_{S_{-j}}\lambda + S_{S_{-j}}K_K = 0, \qquad (7)$$

where λ represents the eigenvalues of the polynomial. The solution to the characteristic polynomial yields $(2 + j)$ eigenvalues.[10] For our system of difference equations to be locally stable, we need two conditions to be satisfied: the initial point (and the rest of the points in the sequence) must be close to the steady state and the eigenvalues have all moduli less than one.[11] If eigenvalues are not less than one, then our solution to the system explodes and diverges from steady state. If our initial point is not close to the steady state, then we are not sure whether the path follows the revealed properties, since our solution is only an approximation to the original system. Our empirical section determines the eigenvalues of the system in the example based on the Baltic Sea cod fishery.

The Baltic Sea Cod Fishery

The Baltic Sea Fishery is a resource stock shared among members of the European Union (EU) (Denmark, Finland, Germany, and Sweden) and Estonia, Latvia, Lithuania, Poland, and the Russian Federation. The Baltic Sea consists of the central Baltic Sea, the Gulf of Bothnia, the Gulf of Finland, the Sound and the Danish Straits. The Baltic Sea is a shallow sea with an average depth of approximately 60 meters. The only connection with the Skagerak and the North Sea is the Sound and the Danish Straits, and total replacement of the water in the sea takes some 35 years. Figure 2 depicts the subdivisions in the Baltic Sea.

The sea consists of a two-layered water mass with brackish water characteristics to the north and east (salinity of 0.5 per mill) and higher salinity to the south and west (salinity of 18 per mill).

Due to the diversity in salinity, two distinct types of cod are present: the Eastern Baltic cod, often referred to as Baltic cod (*Gadus morhua callarias L.*), and western Baltic cod, which is the Atlantic cod (*Gadus morhua L.*) (ICES 2000; IBSFC 2000; Christensen and Jørgensen 1989). The two types of cod differ in population genetics. The eastern cod occurs in the central and the northern part of the Baltic (subdivisions 25-32 in figure 2), the western cod inhabits the areas west of Bornholm Island and the Danish Straits (subdivisions 22-24 in figure 2). The species overlap in the area near Bornholm Island, but mixing is assumed to be minimal (Christensen and Jørgensen 1989). The Eastern cod population is the largest, accounting for approximately 90% of the cod stock in the Baltic Sea (IBSFC 2000). The main difference between cod in the Baltic Sea and that in the North Sea is the ability of the Baltic cod to spawn in lower salinity levels. The eggs of the Baltic cod have increased diameter, which allows them to float when lower salinity levels are present. The data applied in the empirical study are data for the eastern Baltic Sea cod.

The scientists at the ICES regard the stock of the eastern Baltic cod to be below safe biological limits (ICES 2000). The spawning stock declined from historically low levels in 1980–84 to the lowest level on record in 1992. The spawning stock has

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[10] The two values may coincide and appear as single eigenvalue.
[11] Sydsæter (1981) p. 415.

Figure 2. Subdivisions in the Baltic Sea

increased in recent years, but is still below the long-term average.[12] In almost all years, landings have been far above levels recommended by the ICES. The fleet capital stock and fishing effort have not been reduced accordingly, and fishing mortality has increased during the stock decline. The fishery is not sustainable under the current environmental conditions. ICES (2000) estimates of the spawning stock biomass (SSB) indicate that the current level is the second lowest recorded since 1994 and is below the safe biological limit of the SSB (B_{lim}) estimated to be 160,000 tonnes. The fishing mortality is presently 0.82, which is below the limit F_{lim} at 0.96 but above the precautionary approach estimated to be 0.6. Because of the very low SSB, the number of recruitments is considered to be dependent on the SSB. Considering the precautionary approaches for stock and fishing mortality set by the ICES in 1999, the stock was within safe biological limits only in 1978–79. For approximately half of the years on record, the stock exceeded the precautionary approach for fishing mortality and is below the precautionary approach for the SSB and below safe biological limits.

For successful spawning, the cod eggs need minimum values of salinity (11 per mill) and oxygen concentration (2 ml/l). These conditions only occur in the deepest areas, such as Bornholm Deep, Gdansk Deep, and Gotland Deep, and are variable with inflows from the North Sea. The size of the reproductive volume is defined as

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[12] Due to favorable hydrographic conditions, there were unusually strong year classes in the late 1970s and early 1980s, and these formed the basis for additional exploitation in this period. It attracted vessels normally operating outside the Baltic Sea, and catch levels more than doubled, which probably explains the historically low levels in 1984–92. There were no TACs established by the IBSFC from 1982–88.
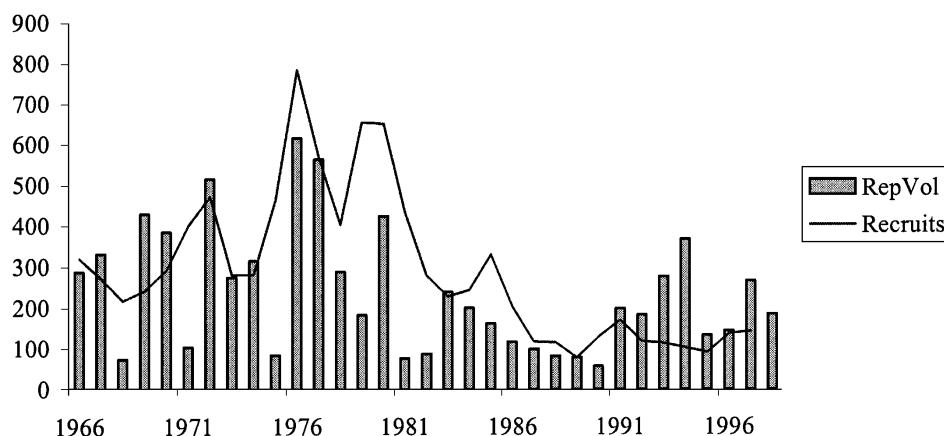
Figure 3. Reproduction Volume in km³ and Recruits in Millions
Source: Aro (2000).

the volume of water providing suitable conditions for successful cod spawning. The reproduction volume plotted against the recruits is shown in figure 3.

Figure 3 shows that since 1966 the cod fishery in the eastern Baltic has gone through an interesting development. Until 1981, the Baltic Sea experienced regular inflows of saline water from the North Sea, which yielded a relatively high reproduction volume. During the same period the recruitment, measured at age 2 lagged 2 periods, was relatively high. Post 1981, the reproduction volume was lower and recruitment was settled on a reduced level. The stock recruitment model in the most recent assessment (ICES 2001) was based on a Ricker curve only for the years from 1982.

The high level of reproductive volume until 1982 combined with a total allowable catch (TAC) measure rebuilds the biomass to a high level. From 1982 to 1989 the biomass was sustained on a higher, but decreasing, level, and the fishery experienced open access, which resulted in huge effort, since the number of trawls increased and gillnets were introduced. In addition, the Baltic Sea experienced a lack of regular inflows from the North Sea. This resulted in an undermining of the biomass until 1989, when an extremely low level was reached and a new TAC was introduced. Since 1989, the biomass has fluctuated on a critically low level.

International Management of Baltic Sea Cod

The International Baltic Sea Fisheries Committee (IBSFC) controls the fishing activity in the Baltic Sea. It was established pursuant to Article V of the Convention on Fishing and Conservation of the Living Resources in the Baltic Sea and the Belts (the Gdansk Convention) which was signed on the 13th September 1973.

The main tool for management is TACs, which the IBSFC sets annually based on recommendations from the ICES. After the IBSFC has allocated TACs to the participating agents, it is up to the authorities in the participating states to regulate and reinforce them.

Until 1977, when the IBSFC introduced the first TAC on cod in the Baltic Sea, the fishery was subject to access based on bilateral agreements. In 1977 the exclusive economic zones were increased to 200 nautical miles, dividing the sea according to the centre line, which created some disputes around the islands of Bornholm and Gotland. In particular, the dispute between Sweden and the Soviet Union around Gotland Island gave rise to an area frequently called the 'white zone,' where there existed open-access until 1987, when Sweden and the Soviet Union came to an agreement. From 1977–81 (both years included), TACs were set in other areas in the range between 174 to 235 thousand tonnes, but for all these years, catch exceeded the TAC. From 1982 to 1988 the fishery was not subject to any TAC and worked as an open-access fishery.[13] In 1989 a TAC of 220 thousand tonnes was reintroduced. The historically low levels of both the eastern and the western stocks resulted in extremely low TAC levels in 1992. In 1993 and 1994, TACs were set at 40 and 60 thousand tonnes, respectively. Comparing the TAC with actual harvests indicates that the TAC was often exceeded. There were no effective constraints on or regulation of effort or activity of the existing fleet; therefore, the fishery is considered to be *de facto* open access.

The Empirical Model

This section establishes an empirical model based on the theoretical setup. The model is divided into a biological part and an economic part, each estimating specific functional forms.

Population Dynamics

Following the setup, we assume a delay-difference model. It is assumed that the increase in biomass results from natural growth and recruits, while the decrease in biomass is described by natural mortality and fishery. The relationship is shown in figure 4.

Data on the eastern Baltic Sea cod fishery for harvests, biomass, recruits, and other biological parameters are available from 1966–99 (ICES 2000). In the data set, the recruits are measured at age two, but are not assumed to enter the spawning stock biomass before age three, since they do not reach maturity until they are at least three.[14] We, therefore, find it relevant to apply the delay-difference model with $j = 3$. The spawning stock biomass and the fished biomass are assumed to be equivalent, consisting of year class three and older.

The population dynamics are estimated as two separate functions: the delay stock-recruitment function and the growth function including natural mortality. The stock-recruitment function describes the relationship between the spawning stock biomass and the recruits measured at age two. The growth function determines the natural growth and the natural mortality within the fished biomass. Mortality from fishing is finally subtracted.

The specific functional form of the stock recruitment function is determined in the empirical section. We can conclude, however, that since the dataset only supplies

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[13] There might have been some technical measures regulating mesh size, etc., and the EEZ existed, but still the fishery was subject to *de facto* open-access.
[14] On average, app. 48% of year class three, over the period 1966–99, are mature at the start of the year (ICES 2000 and own calculations).
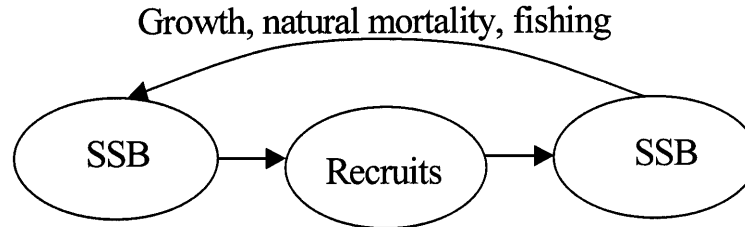
Dynamics of the Baltic Sea Cod Fishery

Figure 4. Assumed Relationship between SSB and Recruits

recruits measured at two years, there must be a stock-recruitment relationship between stock and recruits age two, $R_t(S_{t-2})$. However, since we are not assuming recruits entering the fished biomass until age three, only a fraction, d, of recruits $R_t(S_{t-2})$ survive to the next period and enter the SSB.

$$G(S_{t-3}) = dR_{t-1}(S_{t-3}). \tag{8}$$

Various recruitment functions are tested that include different environmental variables to test which one gives the best fit.[15] The recruits are tested to follow a Ricker function, a Beverton-Holt function, a logistic growth function, or a quadratic relationship with constant recruits for low levels of biomass. Two environmental variables have been included in these different tests. First, it is tested whether a dummy for years with high salinity inflows is significant. This is not the case in any of the functions when they are corrected for first-order autocorrelation. Second, a variable measuring reproduction volume is included. This variable turns out to be significant in a Ricker recruitment function, but since there is no reliable way of predicting the size of the reproduction volume, we have omitted it here. We have instead decided to apply the approach suggested in ICES (2001). Here, according to figure 3, the absence of regular inflows from 1982 to current time reduces reproduction volume and recruitment experience, a downward shift reflecting a reduced level of recruitment since 1982. Therefore, the recruitment function is estimated as a pre-1981, a post-1981, and a conventional Ricker function (covering the whole period). The estimates are summarized in table 1.

Data from industry applied later are newer data, and the pre-1981 Ricker curve has a low adjusted R-square. Therefore, the post-1981 Ricker curve is most relevant, but might be considered as a pessimistic view of the recruits, since it only reflects the period with low recruitment. A more optimistic view is the conventional Ricker curve, which also includes years with salinity inflows and a higher recruitment level. We assume the recruitment curve is located somewhere in the span between the two curves and apply both curves for further analysis.

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[15] A referee pointed out that data mining can be associated with some problems (Charemza and Deadman 1992). We are dealing with this by applying only the same explanatory variables for functions compared.

Table 1
Ricker Stock-Recruitment Function Estimations

Conventional 1966–99	Pre-1981 1966–81	Post-1981 1982–99
Significant at the 9% Level	Significant at the 12% Level	Significant at the 5% Level
$Log\ (R_{t-1}/S_{t-3})$	$Log\ (R_{t-1}/S_{t-3})$	$Log\ (R_{t-1}/S_{t-3})$
$= -1.17 - 0.0018S_{t-3}$	$= -0.98 - 0.000898S_{t-3}$	$= -1.53 - 0.0016S_{t-3}$
(−2.91) (−1.79)	(−5.51) (−1.70)	(−7.17) (−2.89)
	$\bar{R}^2 = 0.12$	$\bar{R}^2 = 0.31$
$R_{t-1} = S_{t-3}e^{(-1.17-0.0018S_{t-3})}$	$R_{t-1} = S_{t-3}e^{(-0.98-0.000898S_{t-3})}$	$R_{t-1} = S_{t-3}e^{(-1.53-0.0016S_{t-3})}$

The first part of the delay-difference equation (equation [4]) determines how the existing biomass is subject to growth (in their weight) and to natural mortality in the absence of recruits. We believe it is realistic to assume this is a density-dependent function. This stock-dependent function is assumed to follow a quadratic function similar to the logistic law of growth:

$$F(S_t) = aS_t - bS_t^2, \tag{9}$$

where a and b are parameters describing the growth and natural mortality in the biomass. We have that for small stock, the biomass grows at a higher rate compared to a large stock, which can be related to the feeding foundations. The function is assumed not to experience any shifts in the period. When considering the stock, recruits are subtracted since we are interested in estimating the pure excess growth in the stock. The measure of the stock available is the exploited stock, so harvest is subtracted. The logistic growth curve can be estimated by rewriting equation (4) in discrete-time and applying data for recruits and harvests. It is assumed that growth occurs before the stock is harvested and recruits have entered the SSB. This can be written as:

$$S_{t+1} - G(S_{t-3}) = 1.43S_t - 0.00021S_t^2 - H_t \qquad \bar{R}^2 = 0.96. \tag{10}$$
$$(31.50) \qquad (-2.51)$$

Thus, the assumption that the logistic function can describe excess growth is strengthened by the empirical results. The fraction of cod surviving from year class two to year class three, d, is assumed to be 0.76.[16] Hence, the estimated biological dynamic is as follows:

$$\tag{11}$$

where parameter estimates are collected in table 2.

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[16] It is only the relation coefficient between year classes two and three that is of interest here; therefore, R-square and t-statistics are not reported. In addition, an anonymous reviewer pointed out that R-squared would give a false measure, since it most likely reflects how the sizes of the year classes are estimated.

Table 2
Parameter Estimates for Biological Dynamics

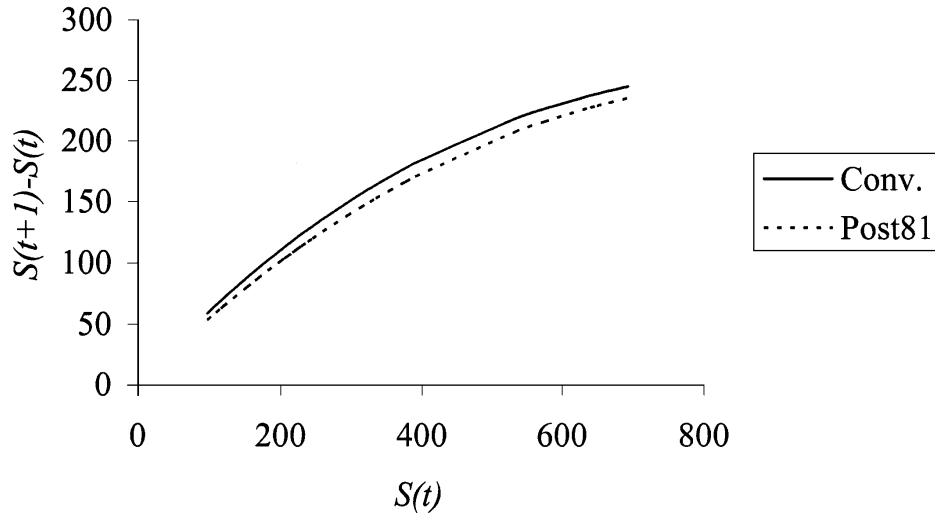|	a	b	c_1	c_2	d
Post-1981 Ricker	0.43	0.00021	−1.529	−0.001568	0.76
Conv. Ricker	0.43	0.00021	−1.167	−0.001771	0.76

Figure 5. Post-1981 and Conventional Biological
Note: Dynamics, harvest are ignored.

The combined unexploited growth curves with the post-1981 and the conventional views are plotted in figure 5.

Productivity Dynamics

Problems with availability of data forced us to change the model slightly for the productivity dynamics. The productivity dynamics are measured in effort (in our case, days at sea) instead of numbers of vessels.[17] We found it appropriate to apply days at sea, since the effort employed in the cod fishery varied significantly during the period in question. Applying an effort measure allows the inclusion of effort also employed in other fisheries. There might be a problem in applying days at sea as a homogeneous measure of effort over a longer period. This problem is, however, ignored here, since there has not been any significant technological development during the period considered. By significant, we mean that there have not been any

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[17] Wilen (1976) and Bjørndal and Conrad (1987) apply number of vessels in their models.

new boats entering the fishery. The data for the production dynamics are based on a sample period 1987–99, and the data source is the Danish Research Institute of Food Economics (2001). A single vessel is representative for the fleet.

A typical vessel for the Danish cod fishery in the Eastern Baltic Sea is selected in order to describe the fishery. The selected vessel has a tonnage of 49.35 GRT, which is a medium to large vessel in the Baltic Sea. It is a trawler, which is the most common type of vessel catching cod in the Baltic Sea (Frost and Andersen 2001). The vessel harvested cod in the Baltic Sea during the whole sample period. At the beginning of the sample period, the harvest consisted of almost 100% cod from the Baltic Sea. Later in the sample period, the stock decreased and the composition of the harvest changed. The cod ratio from cod harvested in the Baltic Sea decreased to as little as 2.6%. This is a typical picture of a fleet in the Baltic Sea. At the end of the period, the cod ratio increased to almost 50%. Data for the selected vessel are shown in Kronbak (2002).

Effort is measured in days at sea in subdivisions 25–32, which refers to the eastern Baltic Sea, which is the area east of Bornholm Island.[18] The data for the selected vessel is denoted by the superscript *i*. Effort for the selected vessel, *i*, in the cod fishery, E_t^i, is measured as days in area 25–32 times the share of the value of the total catch, which is cod compared to other species harvested in the area:

$$E_t^i = days_t^i \frac{Value\ of\ cod\ in\ 25 - 32^i}{Total\ value\ of\ catch^i}.$$

The total effort employed in the Danish industry is determined. The Danish industry is denoted by the superscript *DK*. Dividing effort by the amount of cod caught by the selected vessel gives an inverse catch per unit of effort (CPUE) measure. By multiplying by the total amount of cod caught in the Danish industry in the Baltic Sea, the days at sea are measured for the respective year for the Danish industry:

$$E_t^{DK} = \frac{E_t^i}{Amount\ of\ cod^i}\ Amount\ of\ cod^{DK}.$$

The effort employed in the rest of the industry is calculated by assuming it is proportional to the effort employed in Denmark, where the proportion is determined by the harvest ratio.

In the eastern Baltic Sea, 77% of the catch value of cod landed in Denmark in 1997 was caught by vessels from Bornholm (Frost and Andersen 2001). Therefore, yearly costs per vessel applied in the data set are the yearly costs per vessel belonging to the region of Bornholm. The data on total yearly costs are found in Ministeriet for Fødevarer (2000). For calculation of the variable costs in the cod fishery see Kronbak (2002). The data for the eastern Baltic cod fishery are summarized in table 3, where numbers are rounded.

Production functions, based on the Danish industry, are estimated in order to determine the steady state. To find the best fit, four different types of production functions are estimated for the Danish trawler fleet harvesting cod in the eastern Baltic Sea. These production functions are: (I) a production function only dependent

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[18] Measuring effort as days at sea is only a somewhat simplistic method, since in real-world fisheries effort is a vector of various inputs. With various inputs effort might change without changing days at sea. This aspect is ignored in our model.

Table 3
Data for the Eastern Baltic Sea Cod Fishery

Year	SSB 1,000 Tonnes	DK Harvest 1,000 Tonnes	DK Effort Days at Sea	Yearly Var. Cost/Trawler 1,000 DKK	Price DKK/kg	Daily Cost 1,000 DKK
1987	311.52	60.55	44,346		7.08	
1988	292.91	56.50	40,997		6.80	
1989	237.78	48.45	32,631		6.96	
1990	216.01	41.96	39,885		9.31	
1991	152.10	35.10	28,559		10.19	
1992	96.96	14.92	17,359		9.74	
1993	119.27	4.56	7,843		7.08	
1994	199.45	12.23	8,825		6.85	
1995	244.13	20.10	8,559	175.91	6.48	5.80
1996	163.55	29.66	10,451	259.92	5.99	8.40
1997	133.00	19.33	13,145	277.41	7.44	5.37
1998	109.10	15.50	12,942	653.70	10.05	6.28
1999	116.04	20.14	16,333.54	686.33	10.41	6.61

Source: Ministeriet for Fødevarer, Landbrug og Fiskeri (1995, 1996, 1997, 1998, and 1999) and own calculations.

on days at sea, (II) a Cobb-Douglas production function, (III) a Cobb-Douglas production function where the exponent of days equals one, and (IV) an exponential production function, ensuring no more than the size of the stock is harvested. Estimation results for the different production functions can bee seen in Kronbak (2002).

The Cobb-Douglas function has the most significant estimates and is used in further analysis. The estimation is significant at a 7% level and has a high adjusted R^2. The regression is OLS with t-statistics in brackets.

The Cobb-Douglas production function is:

$$Ln(H_t) = -7.472 + 0.748 \ln(E_t) + 0.644 \ln(S_t) \qquad \overline{R}^2 = 0.75 \qquad (12a)$$

$$(-4.358) \quad (3.969) \qquad (2.086)$$

$$H_t = 0.00057 E_t^{0.748} S_t^{0.644}. \qquad (12b)$$

Hence, the assumption that the total industry follows the estimated Cobb-Douglas production function[19] and vessel dynamics occur according to the normalized profit per day at sea, yields the following entry-exit procedure:

$$E_{t+1} - E_t = \mu \left(\frac{p \cdot q \cdot E_t^\alpha \cdot S_t^\beta - c \cdot E_t}{p \cdot E_t} \right), \qquad (13)$$

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[19] Since data for the whole industry not are available, the procedure used for estimating the production function follows the idea from Bjørndal and Conrad (1987); namely it is assumed that the industry follows the production function estimated for a single, highly representative country, in our case Denmark.

where μ is the response parameter determining how fast the number of days at sea is changed according to changes in profit. Estimates indicate that $\mu = 4.07$ is highly significant with an adjusted $R^2 = 0.92$.

Total effort in the industry is calculated by assuming that the total effort is proportional to the Danish effort employed, where the proportion is assumed to depend on the harvest ratio,

$$E_t = E_t^{DK} \frac{Amount\ of\ cod}{Amount\ of\ cod^{DK}} = \frac{E_t^i}{Amount\ of\ cod^i} Amount\ of\ cod.$$

The calculated effort for the industry is illustrated in table 4, where numbers are subject to rounding.

Since no estimates of effort are available before 1987, effort is assumed to depend on the stock size and the fishing mortality. It is tested whether the effort is linear in fishing mortality, linear in the product of the fishing mortality and the biomass, and whether it is log-linear in the fishing mortality and the biomass level. The most significant estimation indicates that effort is linear in the product of the biomass and the fishing mortality.[20] Assuming effort follows this relation allows for estimates of effort level from 1966–99. The estimates of effort level for the total industry from 1966–99 are presented in Kronbak (2002) and in figure 6 as a state-space diagram that shows the combinations of the stock size and the days at sea for the period 1987–99 for the total industry.

Table 4
Stock, Danish and Total Harvest/Danish and
Total Effort for the Eastern Baltic Sea Cod Fishery

Year	Stock 1,000 Tonnes	DK Harvest 1,000 Tonnes	Total Harvest 1,000 Tonnes	DK Effort Days at Sea	Total Effort Days at Sea
1987	311.515	60.55	207.081	44,346	151,660
1988	292.91	56.50	194.787	40,997	141,340
1989	237.777	48.45	179.178	32,631	120,680
1990	216.007	41.96	153.546	39,885	145,969
1991	152.103	35.10	122.517	28,559	99,672
1992	96.963	14.92	54.882	17,359	63,855
1993	119.265	4.56	45.183	7,843	77,758
1994	199.454	12.23	93.354	8,825	67,373
1995	244.13	20.10	107.718	8,559	45,878
1996	163.546	29.66	121.889	10,451	42,946
1997	133.003	19.33	88.6	13,145	60,260
1998	109.1	15.50	67.429	12,942	56,312
1999	116.037	20.14	72.989	16,334	59,197

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[20] The estimation is significant at the 1% level with an adjusted $R^2 = 0.60$.
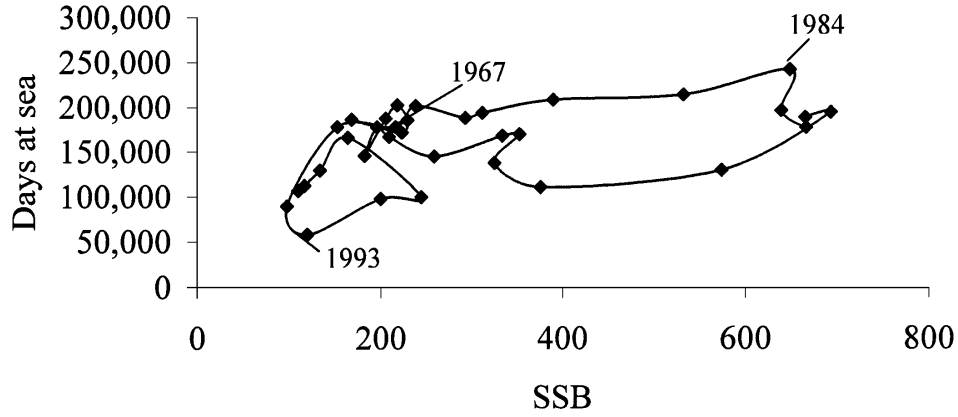
Figure 6. State-space Diagram showing Combinations of Stock Size and Days at Sea for the Total Fleet (1966–99).

Analysis

Assuming the dynamics follow the estimates for stock-recruitment, growth, and harvest, then the system of equations looks like the following:

$$E_{t+1} - E_t = \mu\left(\frac{p \cdot q \cdot E_t^\alpha \cdot S_t^\beta - c \cdot E_t}{p \cdot E_t}\right). \tag{14}$$

Equation (14) describes the dynamics of the harvesting activity. This activity is increased between two periods if there is a positive profit to be gained from the fishery. The speed of increase in the harvesting activity is determined by μ.

   A change in the exploited biomass from one period to another depends on the growth in the stock, the recruitment to the stock, and also the harvesting activity employed in the past period. The following equation describes the change in the exploited biomass over time:

$$S_{t+1} - S_t = aS_t - bS_t^2 + d(S_{t-3}e^{c_1 + c_2 S_{t-3}}) - q \cdot E_t^\alpha \cdot S_t^\beta. \tag{15}$$

The estimated parameter values are collected in table 5.

Table 5
Estimated Parameter Values for the Set of Equations describing the Dynamic System

a	α	b	β	q	c_1	c_2	d	μ
0.43	0.748	0.00021	0.644	0.00057	−1.529	−0.001568	0.76	4.07
0.43	0.748	0.00021	0.644	0.00057	−1.167	−0.001771	0.76	4.07

Solving Numerically for Steady State

The steady state is solved numerically for different values of price and cost. This is done by an initial guess for days at sea inserted in the first equation for steady state. It provides a value for the stock, which is substituted into the second equation for the steady state. The difference between the initial guess and the calculated value of days at sea is evaluated to see whether it is within an arbitrarily small value, ε; if not, then the guess is readjusted to the mean of the former guess and the calculated value. The process will converge to the steady state either from above or below.

Table 6 illustrates the steady state for different cost and price values. Plotting the steady-states for the years 1995–99 yields figure 7.

We conclude that 1996 is an extreme because it has a very low price level and a very high level of costs per day, which leads to an extremely (unrealistic) high level of steady state, since the steady state is located far above the actual range of the stock and far above the actual range of effort employed. This is seen from comparing the actual state-space diagram plotted in figure 6 with the steady states in figure 7.

When the price-cost ratio is high, as in 1998 and 1999, the biomass in steady

Table 6

Numerical Simulations of the Steady-state Values for the Eastern Baltic Sea Cod Fishery

| Year	Daily Cost DKK	Price DKK/kg	Price/Cost Ratio	SS Stock Conv. 1,000 Tonnes	SS Days Conv.	SS Stock Post-1981 1,000 Tonnes	SS Days Post-1981
1995	5,795.59	6.56	0.001131	529.17	248,154	514.57	231,025
1996	8,396.18	5.98	0.000712	1383.50	460,160	1380.38	457,514
1997	5,371.55	6.97	0.001298	408.62	221,060	393.84	201,196
1998	6,277.47	10.25	0.001633	267.03	185,361	253.94	163,025
1999	6,605.13	11.26	0.001705	246.46	179,432	233.85	156,899
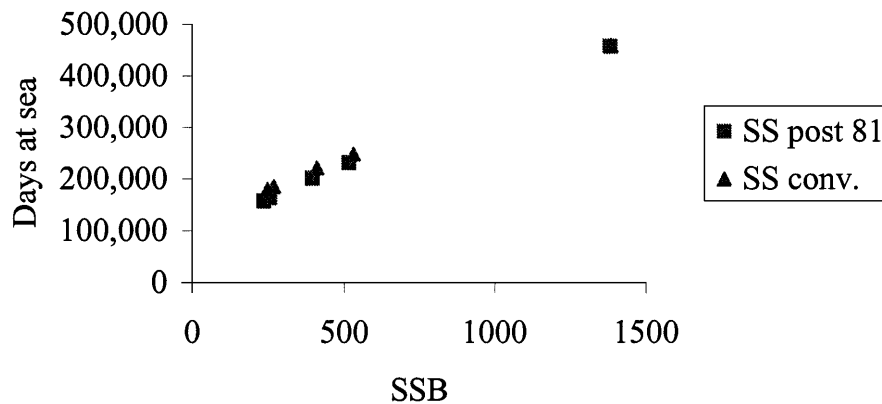
Figure 7. Conventional and Post-1981 Steady States (1995–99)

state is low, since it becomes more profitable to harvest down the stock. When the price-cost ratio is low, as in 1996, the biomass in steady state is high and it is relatively more profitable to invest in the stock.

Assuming that the 1998 and 1999 prices and costs are fixed gives an area within which the steady state is believed to be located. Plotting the steady state and the effort level in the industry from 1982–99 yields a diagram indicating a movement to the steady state almost following a stable spiral where overshooting occurs in late 1970s and early 1990s (see figure 8). Hence the cod fishery in the Baltic Sea seemingly follows the same pattern as the North Pacific fur seal fishery described in Wilen (1976) and the North Sea herring fishery described in Bjørndal and Conrad (1987) (see figure 8).

To evaluate the type of steady state, we determine the characteristic polynomial defined in equation (7). With our specific functional forms and the steady state in question, we can evaluate the eigenvalues. These are summarised in table 7.
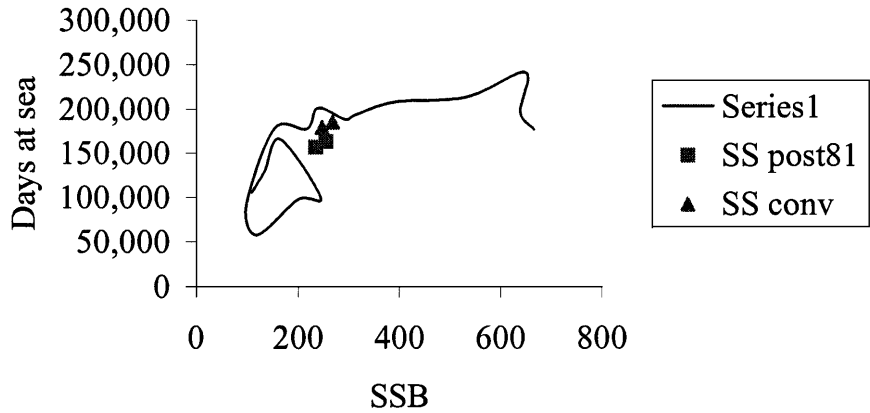

Figure 8. Stock Size and Days at Sea (1982–99), Steady States
at Fixed Price-Cost Ratio 1998–99 Level

Table 7
Evaluation of the Stability of Steady State

	1995	1996	1997	1998	1999
Conv. Ricker	0.19	1.06	−0.26	−0.30	−0.31
Eigenvalues	0.07±0.22i	−0.2±0.21i	0.06±0.34i	0.04±0.42i	0.04±0.42i
	1.09±0.12i	0.36±0.34i	0.99±0.22i	1.05±0.29i	1.06±0.30i
Stability of Steady State	Unstable	Unstable	Unstable	Unstable	Unstable
Post 1981 Ricker	−0.21	1.06	−0.26	0.31	0.19
Eigenvalues	0.06±0.27i	−0.2±0.21i	0.06±0.35i	0.05±0.42i	0.07±0.22i
	0.94±0.15i	0.36±0.34i	1.0±0.23i	1.07±0.30i	1.07±0.30i
Stability of Steady State	Stable	Unstable	Unstable	Unstable	Unstable

Note: The eigenvalues with the different recruitment functions are identical in 1996 only due to rounding.

The eigenvalues have modulo some above and some below 1. If at least one eigenvalue has modulo greater than 1, then the fixed point is unstable. An unstable steady state would result in severe depletion or possible extinction of the resource. Only with the conventional Ricker curve and the 1995 prices do we have a stable steady state. Since we have a lag in our population dynamics, the requirements for a stable steady state become larger. For comparison, we have also evaluated the stability of the steady states assuming there is no lag in the recruitment ($j = 0$). It yields a stable steady state for all years in question.[21] This shows how sensitive the stability of the steady state is to changes in functional forms and parameter values. Even with ambiguity in our results, the fishery is likely to be on its path to steady state. We have, however, only evaluated the stability of the steady state close to the steady state and are not able to conclude what happens in the whole diagram.

Conclusion

The results presented are not the first in the area, but are more advanced then the first presentations of Wilen (1976) and Bjørndal & Conrad (1987) who tested the dynamic model presented by Smith (1968) applying basic functional forms. Our model uses a biological delay-difference function, separating recruits and growth including natural mortality in the biomass, and a Cobb-Douglas production function. We contribute to the literature by also deriving the stability of the determined steady state. The empirical model shows that, assuming a fixed price-cost ratio, the eastern Baltic Sea fishery is likely to have a stable steady state. If the optimal path to the steady state is a convergent spiral, then it involves overshooting of effort and hence a (extremely) low level of biomass as is also the case in the earlier applications of the theory. The model indicates that a low stock level is not necessarily a problem if the resource is on a path to a stable steady state.

These results of the empirical model for the Baltic Sea cod fishery show that even though the biomass was critically low in 1992, the decrease in the pressure on the fishery led to a stock increase, avoiding extinction and starting an oscillation. The overshooting of effort only occurs in the first loop of the spiral, and the biomass does not increase enough for overshooting of effort to occur again. The biomass does not increase enough to end the convergent spiral path. This might be explained by two different effects; either by the ambiguity in the stability results of the steady state or by a change in the steady state resulting from a change in the price/cost ratio over time. Extinction of the stock is another possible steady state, but is not considered for further discussion in our analysis. The Baltic Sea cod fishery is thus shown likely to follow a convergent spiral to steady state, where overshooting occurs in 1991, resulting in an historically low level of biomass in 1992. The steady state level is, however, very sensitive to changes in price and cost levels. This is a critical limitation of the simulation model, since the price-cost ratio is assumed to be fixed after determining the steady state. In general, it should be noted that the steady state changes over time as parameter values or functional forms are changing; we have only exploited the model for changes in recruitment curve and for different prices and costs. The steady state is also changing as the system is subject to stochastic jolts; these are assumed away in our model.

Further limitations are that the data for effort level are only based on a single, typical Danish vessel; other country-specific data was not available. It could, however, be interesting to examine the effect of other production functions estimated on

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[21] The two eigenvalues for each year are less than 1.

the basis of data for the whole area. The measurement of effort as a function of only days at sea is another limitation that might overlook important aspects of the fishery, since effort in real-world settings is a vector of different parameters. Furthermore, changes in the steady state level as price and cost ratios change are not taken into account and, price and cost are, again, based solely on the Danish data. Here, further research would be needed to examine the effect of changing price-cost ratios and the effect of a moving steady state.

Although this model is slightly more advanced in functional setup compared to earlier empirical work in the area, more research is still needed. Among other things, the limitation of having constant average costs in the fishery and the effect of replacing the response parameter with a response function including profits in alternative fisheries need to be examined.

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