Open Space Allocation and Travel Costs

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Abstract

Open space allocation in a city has largely been addressed by simultaneously allocating land for open space and residential housing in a general equilibrium model. Open space competes with lot sizes of residential housing to determine the optimal density and allocation of open space in a city. Residents derive enjoyment from the open space, but they never actually visit it. In other words, these models ignore that people must transport themselves to the open space to enjoy it.

Open space is often defined as any pristine natural area, but the focus in this paper is land for parks. Rather than addressing whether parks take too much land away from residential housing, this paper focuses on how parks should be allocated to maximize the net benefits from visitation to the parks. The net benefit from a visit to a park is the value derived from the park less the travel costs to reach the park. The planning authority is modeled to make two decisions when allocating a fixed amount of open space in the city. The planner decides how many parks to have and where the parks should be located in the city. When there are more parks, the travel costs from visitation are reduced; however, when there are more parks, every park is smaller than before, and the value of the open space is reduced. A condition is derived to inform the planner how to optimally allocate the open space when both the number and the location of the parks are variable.

The model originally has travel costs constant throughout the city. However, later in the paper, travel costs are allowed to vary over space. In particular, travel costs are modeled to be more concentrated over the city center because there are demographic differences. The first order conditions for the optimal open space allocation are determined analytically, but the optimal placements are computed numerically. The

results indicate that a fewer number of parks are relatively more optimal when travel costs are concentrated at the city center.

Among the drawbacks to the model in this paper is that the planner allocates a fixed amount of land. More realistically, the planner would simultaneously determine how much land there should be for open space, how to divide the land and where to place the land. Such an approach has been used in past papers except that those papers did not consider the travel costs incurred by residents to reach the open space. Another drawback is that the planner allocates all the open space at once. However, parks are usually created in a city when there is public demand or money in the budget for them.

There are many possible extensions to this research. For instance, some travel costs are roadblocks so that travel costs spike at those areas in the city. Further, the value derived from a park is probably dependent on its nearness to other amenities like an ocean or mountain. Clearly, the number of parks, their location, and their size has a great influence on the net benefits residents derive from them. These characteristics of open space require consideration before a meaningful model of competing land use between open space and residential housing is developed.

Introduction

Open space in urban areas, such as parks, parkways, greenbelts and public squares provide numerous services to city residents. Clean air, scenic vistas and recreational opportunities are among the benefits available to open space visitors, and city planners utilize open space to shape and contain urban areas (Fujita, 1997). Large proportions of some cities are occupied by open space, but there has been relatively little research done from an economic perspective to determine how this open space is best distributed throughout a city.

There are many considerations that go into determining how to allocate parks in a city. When the question is what allocation of parks provides the greatest surplus to city residents, the economic content is the ingredients people use to decide if and what parks to visit. In particular, what are the travel costs to visit a park and how do the benefits of a park depend on its characteristics. A city planner's point of view is taken to conduct the analysis of the open space allocation. The city planner is a benevolent ruler that aims to find the open space allocation that provides the city residents with the maximum surplus, i.e. the value of the visit less the travel costs, from the open space.

The city planner has various options at its discretion to increase the surplus residents derive from open space. A subset of these options is that the city planner can manipulate the spatial characteristics of the open space. For example, the city planner can mold the open space into any shape it wants. Another option the city planner has is to divide into pieces a fixed amount of land for parks. In other words, rather than being forced to put a single park someplace in a city, the city planner has the freedom to split the park into numerous pieces and distribute those pieces anywhere it likes in the city. The city planner's use of this option to allocate parks in a city is the focus of this paper.

Before going into further detail about the model, an examination into how this analysis relates to the past literature identifies the contribution made here. Open space has received attention recently because economists have taken an interest in urban sprawl. The research on urban sprawl examines the factors that cause open space outside a city to be developed into residential housing or farmland. In Wu (2001), leapfrog development is explained by a communities' interest in locating near a natural amenity such as a river or a scenic hill. While Wu investigates how an urban area expands to fill around natural amenities, this paper examines how natural amenities are optimally placed inside an already existing city.

Yang (1990) looked at the provision of a central park, and Fujita and Lee (1997) examined the efficient configuration of a greenbelt amenity. A more general approach by Fujita and Yang (1983) allowed the amenity density distribution to vary across the city. In particular, the finding in that paper was that if households have a log-linear or Cobb-Douglas utility function, the efficient density of the amenity is uniform across the city.

Unfortunately, the form that the uniformly dense amenities have throughout the city is left a mystery. It could be that there are a few medium sized parks located about the city in uniform intervals, or there may be numerous small parks distributed about the city. In this paper, the model addresses how much the open space should be split apart. The purpose here is to emphasize that there are details to the allocation of open space never explicitly considered in more complex models that have an important impact on the total surplus derived by city residents.

Multiple parks

The investigation of the optimal allocation of city parks is simplified by considering a city that is represented by a line. While this is not representative of most cities, small towns where a main street is the center of activity are well represented by a line. If the small town explanation does not appeal to the reader, then consider that a city planner is trying to decide how to place a park along a single street in a major city. The generality of the results reached are not affected by using a line, but there is likely more richness in the two dimension results that is lost.

Consider a city planner that is required to place open space of length, l, in a city. The city planner is free to divide the park into however many pieces it desires, and those pieces can be placed wherever it wishes throughout the city. The length of the city not including the park is normalized to one. People in this city pay a constant cost of k per unit of length traveled.

The city planner is posed with the dilemma of whether to create one park or multiple equally sized smaller parks that have a total length equal to the single park. Evaluating these alternatives requires knowledge about the value people derive from a park, and the travel costs people incur to obtain that value. Suppose that people in a city visit a park only once, and the only value people derive from the park is from the visit. There is no value derived from the travel to the park. All people derive an identical value from a park visit, but the value derived is dependent on the size of the park. The only cost incurred from a park visit is the travel cost. In other words, there is no fixed fee for entrance to the parks.

In order to determine what allocation of the open space is best, the surplus (i.e. the total value derived from the park less the total costs to society from visiting the park) for an abstract number of parks is computed. The total cost of visiting the parks is determined in two steps. First, the optimal placement of the parks to minimize travel costs is found. Second, the total cost to society associated with that optimal park placement is found. The benefit from the park system depends on the number of parks the city planner chooses to create. The benefit from having r parks is $U(s, t, z)$ where $s = (A / r)$ is park size, $t = b\, r^2$ is the number of trips to the park, and z is a vector of other park characteristics. $U(\bullet)$ is quasiconcave and monotonically increasing in its arguments. Since A is the fixed amount of land for parks the planner has to distribute, the parks get smaller, i.e. s falls, when their number is increased. Although it seems natural to assume diminishing returns to park size, the extent of the increased value from combining two parks into one is an empirical question. The number of trips, t, to the parks increases with the square of the number of parks because people visit parks more frequently when they are nearby. The choice to the make trips increasing with the square of the number of parks is ad hoc; a look at estimated travel cost demand curves for city parks should allow a more realistic derivation of the relationship between trips to the park and the number of parks.

The diagram below shows the appearance of a city when the city planner decides to create r parks. The parks are the darkened portions of the line. The areas where the resident live and commute to the parks are $x_1, x_2, \ldots, x_n$. These areas sum to one so that parks are in fact only points on the line.

Since travel costs are a constant k per mile, the societal cost of visiting a park for people that live along a line of length a is: $\int_0^a kx\,dx = k\left[\frac{1}{2}x^2\right]_0^a = \frac{1}{2}ka^2$. Since the total cost of the park is only the travel costs, the cost minimization problem where there are n areas that the residents live and $r = (n-1)$ parks is:

$$\min_{x_1,\ldots,x_n} \frac{1}{2}k(x_1^2 + \frac{1}{2}\sum_{i=2}^{n-1} x_i^2 + x_n^2) \quad s.t. \quad \sum_{i=1}^{n} x_i = 1$$

The first order conditions are that $x_1 = x_n$ and that $2x_1 = x_i \ \forall\, i = 2, \ldots, n-1$. In other words, the parks are placed symmetrically along the line so that no resident travels further than $x_1 = x_n$ distance to reach a park. Solving these first order conditions, it is found that $x_1 = x_n = (1/2r)$ and that $x_i = (1/r) \ \forall\, i = 2, \ldots, n-1$. When these areas are plugged into the objection function, the total travel costs when $r = (n-1)$ parks are optimally placed in a city is found to be $(k/4r)$.

There is no indication yet what the optimal number of parks is since the benefits from creating parks have not been introduced. Below net benefits are maximized to yield a first order condition for the number of parks, and there are some comparative statics for changes in the parameters.

$$\max_{r} \ NB(r) = U(s, t, z) - \frac{k}{4r}$$

The first order condition is:

$$\frac{dNB(r)}{dr} = G(r) = -\frac{\partial U}{\partial s}\frac{A}{r^2} + \frac{\partial U}{\partial t}2br + \frac{k}{4r^2}$$

The comparative statics are:

$$\frac{\partial r}{\partial k} = -\frac{\partial G / \partial k}{\partial G / \partial r} = \frac{-(1/4r^2)}{(-\frac{\partial U}{\partial r \partial s}\frac{A}{r^2} - 2\frac{\partial U}{\partial s}\frac{A}{r^3} + \frac{\partial U}{\partial r \partial t} 2b\,r + \frac{\partial U}{\partial t} 2b - \frac{k}{4r^3})} = \frac{(-)}{(-/+)}$$

$$\frac{\partial r}{\partial b} = -\frac{\partial G / \partial b}{\partial G / \partial r} = \frac{-2r(\partial U / \partial t)}{\partial G / \partial r} = \frac{(-)}{(+/-)}$$

$$\frac{\partial r}{\partial A} = -\frac{\partial G / \partial A}{\partial G / \partial r} = \frac{\left(\frac{\partial U}{\partial s}\right)\left(\frac{1}{r^2}\right)}{\partial G / \partial r} = \frac{(+)}{(+/-)}$$

The argument in the benefits function, that looks like a utility function, for park trips gives the comparative statics their ambiguous signs. If the park trips argument is dropped and only park size is considered, then the comparative statics have expected signs. In particular, $\partial r / \partial k$ is positive, and this is expected since a planner would want to create more parks when each mile to a park is more costly to travel. Also, $\partial r / \partial A$ is negative. Here it is difficult to say what the expected sign is since more land might mean you want to combine it with an original park or create a new park in the city.

The single first order condition result in this model is due to the simplicity of the model, but there is more happening here than the choice about what park configuration yields the highest surplus. There are clearly equity issues raised from this analysis. In particular, suppose that the higher surplus park configuration is the one where there is a single central park. The majority of the surplus goes to the people living close to the large park while the people on the fringe of the city receive far less surplus than if the park were split into two. When the park is split into pieces, the travel costs incurred by the city residents are much more similar than when there is a single central park. Equivalently, the surplus from the park amenity is much more equitably distributed if a park is split into pieces. Accordingly, if equity is an issue, the city planner might consider foregoing a higher surplus central park option for a more equitable two parks option.

Additionally, besides using the first order condition to illuminate the analysis about how to optimally distribute parks in a city, this condition can be used to infer the beliefs city planners have about the values of different sized parks to visitors. The first order condition says to equate the change in benefits from an additional park with the change in travel costs. An empirically determined change in total travel costs allows for an estimate on the change in value the planner has for that configuration. In particular, GIS data allows a researcher to determine the total travel costs associated with an alternative numbers of parks. Next, by noting what park configuration the city planner actually chose, the researcher retrieves some information about the difference in values the city planner believes the residents have for the different sized parks.

For instance, if a central park is chosen for a city and the difference in total travel costs from the two parks option is some amount x, the researcher would conclude that the city planner believes the value from a visit to a large park less the value from a visit to a park half its size is greater than x. Hedonic studies on housing prices have established the values people place on nearby open space. If the hedonic studies have established values for parks of different sizes, the difference in values in those studies can be compared to the inferred beliefs that the city planner has about peoples' values for different sized parks. Noting whether those value differences deviate significantly from each other provides a check on whether city planners are appropriately allocating open space in a city.

The next section examines more rigorously how a city planner should best distribute parks when travel costs are bunched up over a particular area. The opportunity costs of time incurred by park visitors are considered.

A closer look at travel costs

Suppose that there is uniform difficulty traveling throughout the city, i.e. no roadblocks, but people in the city have varying opportunity costs of time. On the outskirts of the city the opportunity cost of time is low while at the city center the opportunity cost of time is the highest. This representation of the varying opportunity costs of time is based upon the assumption that the high paying jobs are located at the city center, and people visit the park during the day. At each spot in the city there is a different opportunity cost of time, i.e. wage, and for the person emanating from that spot that cost is constant per mile amount wherever the person goes in the city.

All people still pay the constant k per mile related to the costs associated with their vehicle. The curve $cx(x-1)$ describes the varying opportunity cost of time where x is the location along the line and c controls the magnitude of this opportunity cost. Note, that at $x=0$ and $x=1$, the opportunity cost of time is zero no matter how large c is made. At the fringes of the city, no people work, and their opportunity cost of time is zero. The curve reaches its maximum where $x=0.5$ at the city center. Presumably, the CEOs working downtown have the highest opportunity costs of time. These two costs combined describe the travel costs faced by the city residents.

Since travel costs have been made more complex, this analysis simply compares the one versus two parks alternatives to keep the math from obscuring the information provided by the new economic content. The surplus from each option is sought to determine the best alternative. The benefits side remains simple since the values derived from visits to different sized parks are given. However, the park placements to minimize travel costs have become more involved. The minimization problem for the one park scenario is:

$$\min_{a} \quad \phi(a) = \int_0^a (k + cx(1-x)(a-x)\ dx + \int_a^1 (k + cx(1-x)(x-a)\ dx$$

Leibniz's Rule is applied to obtain the necessary condition to this problem:

$$\phi_a = \int_0^a (k + cx(1-x))\ dx - \int_a^1 (k + cx(1-x))\ dx = 0$$

$$= \left[kx - \frac{c}{3}x^3 + \frac{c}{2}x^2 \right]_0^a - \left[kx - \frac{c}{3}x^3 + \frac{c}{2}x^2 \right]_a^1 = 2ka - \frac{2}{3}ca^3 + ca^2 - k - \frac{c}{6} = 0$$

The only solution to the necessary condition that lies within the interval [0,1] is $a = 0.5$. In other words, for all k and c, the optimal placement of the park is at the center of the city. Recall that in the first model where k is the only travel cost component, the optimal placement for the single park was also at the center of the city. Since the opportunity costs of time component to travel costs reaches its maximum at the center of the city, the conclusion is not surprising that travel costs are minimized when the park is placed at the center of the city. At that placement, the CEOs located downtown, those with the highest opportunity cost of time, travel the least.

Since the park is always placed at the center of the city, the total travel costs are easily determined in terms of the parameters k and c. In particular, $\phi\left(\frac{1}{2}\right) = \frac{8k + c}{32}$, and the surplus from this alternative is $v_1 - \frac{8k + c}{32}$.

The minimization problem for the two parks scenario is:

$$\min_{a,b} \ \phi(a,b) = \int_0^a (k + cx(1-x)(a-x)\ dx + \int_a^{\frac{(b+a)}{2}} (k + cx(1-x)(x-a)\ dx$$
$$+\int_{\frac{(b+a)}{2}}^{b} (k + cx(1-x)(b-x)\ dx + \int_b^1 (k + cx(1-x)(x-b)\ dx$$

Again, Leibniz's Rule is applied to get the necessary conditions:

$$\phi_a = \int_0^a (k + cx(1-x))\ dx - \int_a^{\frac{(b+a)}{2}} (k + cx(1-x))\ dx = 0$$
$$= \frac{1}{24}\left(((a+b)^3 - 3(a+b)^2)c - 12(a+b)k + 8a((3-2a)ac + 6k)\right) = 0$$

$$\phi_b = \int_{\frac{(b+a)}{2}}^{b} (k + cx(1-x))\ dx - \int_b^1 (k + cx(1-x))\ dx = 0$$
$$= \frac{1}{24}\left((a^3 + 3a^2(b-1) + 3ab(b-2) + 3b^2(7-5b) - 4)c - 12k(a+b+2) + 48bk\right) = 0$$

Now the optimal a and b vary based upon the values that k and c have. Accordingly, explicit solutions for a and b in terms of k and c would be useful. However, the first order conditions are too intractable to find a and b other than numerically. Below the table shows the optimal park placement(s) for both the one park and two park scenarios for different values of k and c. Further, the table shows the total travel costs for both scenarios and the ratio of those total travel costs.

One park vs. Two parks Alternatives for Different k and c

k/c	Single Park		Two Parks			Ratio of Total Travel Costs: TC2/TC1
	Park Location	Total Travel Cost: TC1	Park 1 Location	Park 2 Location	Total Travel Cost: TC2	
500/1	0.5	125	0.25	0.75	62	0.496
50/1	0.5	12.5	0.25	0.75	6.3	0.504
1	0.5	0.281	0.26	0.74	0.144	0.512
1/50	0.5	1.81	0.32	0.67	0.99	0.547
1/500	0.5	15.8	0.33	0.68	8.7	0.551

When the problem is where to place a single park, the conclusion is to put it at the center of the city for all k and c. However, when the problem is where to place two parks, the optimal placements vary based upon k and c; in particular, the ratio k/c rather than the absolute values of k and c determine the optimal placements. When k is large relative to c, the constant across space cost per mile component to total travel costs dominates, and the optimal placements are equivalent to those found when k is the only travel cost component. When c is large relative to k, the optimal placements are squeezed closer to the center of the city. Since the opportunity cost of time component to travel costs is the most significant, travel costs are concentrated over space at the city center. The best way to alleviate the high travel costs at the city center is to bring the parks closer to it. In this way, those people that have the highest opportunity costs of time are those that travel the least.

Further, the table shows how the total travel costs of each alternative vary with k and c. Note that for every k/c ratio, the total travel costs are lower in the two park alternative since the total distance people have to travel is reduced. Also, if k or c increase, the cost of every mile traveled has risen so that naturally the total travel costs in both alternatives increase. Accordingly, the appropriate way to compare the total travel costs is to calculate the ratio between them. In this way, the gain observed from the reduced travel costs in the two parks scenario does not depend on the levels that k or c have. Observe that the total travels costs ratio rises as the k/c ratio falls. When the opportunity cost of time is the dominant travel cost component, the gain from choosing the two parks alternative is less.

The ratio, TC2/TC1=0.5, when k is the only travel cost is clear because the two parks alternative means the distance traveled by people is cut in two. That is, the two parks alternative exactly halves the total travel costs. However, the conclusion that the two parks alternative is even less beneficial when the opportunity cost of time component is the most significant is less clear. When the time cost component dominates, the distance traveled by people near the city center is cut by more than two, but the total travel cost is cut by less than two.

When costs are the same for all people in a city, the optimal placements for the two parks option are those that cut the distance traveled by people in half. When travel costs are greater for those at the city center, the intuitive response is to bring the parks closer to those with the high travel costs. Those with the highest travel costs are made to travel less since each mile less they travel contributes more to cost savings than a mile less for someone with small travel costs. However, the optimal placements moved closer to the city center mean that the distance traveled is cut by less than two. Since residents are traveling further when the optimal placements are closer to the city center, the total travel costs of the two parks option is cut by less than two. When travel costs are heterogeneous because of demographic differences in some city regions, the two parks option is less effective at reducing total travel costs. A policy prescription is that a homogeneous mix of people with different opportunity costs of time should be encouraged in a city.

The extension here with the opportunity costs of time concentrated over an area in space is a way to consider varying demographics of all kinds over space. For instance, the chief demographic feature of a city center might not be that it is where people with high opportunity costs of time reside during the day. Rather, a city center might be the place where the poor live who have malfunctioning automobiles or rely on public transportation. At the outskirts of town people have expensive automobiles to take them easily anywhere throughout the city. In this instance, the stereotypical demographics of a city center complement each other to suggest that travel costs increase for people around a city center so that the two parks option is the most beneficial. Information about the true demographics in a city and a clear understanding about what those demographics mean for travel costs for those people is crucial to determining how travel costs are concentrated over space.

Conclusion

The placement and number of parks throughout a city has a strong influence on the surplus residents derive from those parks. In particular, splitting open space into pieces is a powerful option the city planner has to increase surplus for residents. By increasing the number of parks, travel costs are always reduced, but the size of the reduction depends on the kinds of travel costs in the city. If travel costs are concentrated in regions of the city, creating two parks is relatively more costly for residents although it is more equitable.

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