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# Conditional Demand and Endogeneity? A Case Study of Demand for Juice Products

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The question of endogeneity of conditional expenditures, as well as prices, in conditional demand equations for juices is examined. Both conditional expenditures and prices were found to be uncorrelated with the conditional demand errors, based on Wu–Hausman tests. Conditional demand error variance/covariance estimates and corresponding Slutsky coefficient estimates were approximately proportional, as predicted by the theory of rational random behavior, further supporting independence of conditional expenditures and conditional errors for juice demands.

*Key words:* conditional demand, endogeneity, juices, promotion, rational random behavior.

## Introduction

A popular approach in applied demand analysis in agricultural economics has been to estimate a conditional demand system for a group of commodities of interest, treating quantities of the goods in the group as endogenous, and prices of these goods as well as expenditure on these goods as exogenous or independent of the error term in each conditional demand equation. Recently, LaFrance pointed out that group expenditure may be correlated with the conditional demand error, suggesting that some test, such as the Wu–Hausman specification test (Hausman; Blundell), be conducted to determine whether this is indeed the case. For some specifications, like the Rotterdam model, one might expect group expenditure to be independent of the conditional errors, based on Theil's theory of rational random behavior. Nevertheless, a prudent approach would be to test for exogeneity of group expenditure. If group expenditure is correlated with the disturbances, consistent demand estimates can be obtained by augmenting the conditional demand system with an equation to explain group expenditure in terms of exogenous variables, or using instrumental variables to estimate the conditional demand parameters without explicitly modeling group expenditure, or using an unconditional specification of demand where the expenditure variable is exogenous. Attfield also has shown that imposition of homogeneity restrictions (homogeneity of degree zero in prices and income) allows one to obtain consistent demand estimates when expenditure is endogenous.<sup>1</sup>

In addition to group expenditure, prices also may be endogenous. A general discussion of price endogeneity in demand systems and estimation methods is provided by Theil (1976). More recently, Thurman (1986, 1987), Wahl and Hayes, and Eales and Unnevehr have examined the issue of price versus quantity endogeneity in demand analysis, using the Wu–Hausman test.

In the present study, we examine conditional demands for juices, focusing on the broader endogeneity possibilities suggested above. Retail demands for five types of juices—orange

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juice (OJ), grapefruit juice (GJ), apple juice (AJ), grape juice (GRJ), and remaining juices (RJ)—are examined. The Wu–Hausman test and an informal test suggested by Theil (1980a) are used to examine the possibility of endogeneity of total juice expenditure in conditional demand specifications for the individual juices. Prices are not expected to be endogenous, but the Wu–Hausman test was also used to test this possibility. In the present study, weekly retail grocery sales data were analyzed, and given the relatively short time interval of one week, prices are likely to be exogenous. Grocery stores are also frequently involved in juice promotional programs where consumers are offered cents-off deals which tend to remain unchanged over the period of a week. Juice inventories are usually sufficient for consumers to purchase as much as they want at the going price during a period of a week.

The promotional programs can be expected to affect demand through both discount prices (downward movement along the demand curve) and enhancement of consumer preferences (outward shift in the demand curve) for the products promoted. To allow for the impact of promotional activity on preferences, promotional variables, along with prices and consumer expenditure, were used to explain juice demands. The prices, used as explanatory variables in the juice demand equations, are actual prices paid by consumers and reflect the various price discounts associated with juice promotional programs.

### Model

The demand model chosen for this study is the Rotterdam model (Barten 1964; Theil 1965) which has been shown to be a flexible functional form, comparable to other demand models such as the almost ideal demand system or the translog demand model (see Barnett, Byron, or Mountain for discussion on the flexibility of the Rotterdam model). The basic demand equation for an individual good in the Rotterdam model can be written as

$$(1) \quad w_{it}^u Dq_{it} = \theta_i DQ_t + \sum_j \pi_{ij} Dp_{jt} + \epsilon_{it}$$

where subscripts  $i$  and  $t$  refer to the good in question and time, respectively;  $w_{it}^u = (w_{it} + w_{i,t-1})/2$ , the average unconditional budget share with  $w_{it} = (p_{it}q_{it})/m_t$ , the budget share at time  $t$ , where  $p_{it}$  and  $q_{it}$  are the price and quantity of commodity  $i$  at time  $t$ , and  $m_t$  is total expenditure or income at  $t$ ;  $Dq_{it} = \log(q_{it}/q_{i,t-1})$ ;  $DQ_t = \sum w_{it}^u Dq_{it}$ , the Divisia volume index;  $Dp_{it} = \log(p_{it}/p_{i,t-1})$ ;  $\theta_i = p_{it}/(\partial q_{it}/\partial m_t)$ , the marginal propensity to consume which is treated as a constant to be estimated;  $\pi_{ij} = (p_{it}p_{jt}/m_t)(\partial q_{it}/\partial p_{jt} + q_{jt}(\partial q_{it}/\partial m_t))$ , the Slutsky coefficient which is also treated as a constant to be estimated; and  $\epsilon_{it}$  is an error term. [See Theil (1965, 1975, 1976, 1980a, b) and Barten (1964, 1969) for development and general discussion of the Rotterdam demand equations.]

The basic parameter restrictions for the Rotterdam model are

$$\text{Adding-up:} \quad \sum_i \theta_i = 0 \quad \text{and} \quad \sum_i \pi_{ij} = 0;$$

$$\text{Homogeneity:} \quad \sum_j \pi_{ij} = 0;$$

$$\text{Symmetry:} \quad \pi_{ij} = \pi_{ji}.$$

Promotional variables are included in model (1) using the method of translation. [For general discussion of translation, see, e.g., Pollak and Wales (1980, 1981); for applications to the Rotterdam model and analysis of advertising, see, e.g., Cox, and Brown and Lee (1992a, b).] Translation introduces fixed costs which, in the present analysis, are made functions of the promotional variables. The fixed costs can be viewed as expenditures on psychological needs or requirements. Letting  $\gamma_i$  be the translation term or psychological quantity desired of good  $i$ , the general demand function for good  $i$  is  $q_i = \gamma_i + q_i^*(p_1, \dots, p_m, m - \sum p_j \gamma_j)$  (Pollak and Wales 1981). Note that this general demand relationship includes  $\gamma_i$  as an intercept, as well as  $m - \sum p_j \gamma_j$  or income above the fixed cost amount  $\sum p_j \gamma_j$ . Hence, by making  $\gamma_i$  a function of promotional activity for good  $i$ , the general demand relationship indicates that promotional activities will have direct effects through the intercept terms,  $\gamma_i$ s, and indirect effects through the income term,  $m - \sum p_j \gamma_j$ .

Using the translation approach, promotional effects can be included in Rotterdam model (1) by adding the following term (Cox; Brown and Lee 1992b):

$$(2) \quad \sum_k \beta_{ik} dA_{ikt} - \theta_i \sum_j \sum_k \beta_{jk} dA_{jkt}$$

where  $dA_{ikt} = A_{ikt} - A_{ikt-1}$ , with  $A_{ikt}$  being the level of promotion  $k$  for good  $i$  at time  $t$ ; and  $\beta_{ik} = (p_i/m)(\partial \gamma_i / \partial A_{ik})$  are promotional coefficients, treated as constants for estimation.

For the present study, the commodity subscript is  $i = 1$  for orange juice,  $i = 2$  for grapefruit juice,  $i = 3$  for apple juice,  $i = 4$  for grape juice,  $i = 5$  for other juice, and  $i = 6$  for other food. Assuming the five juices are separable from other goods, a conditional demand system for juices can be developed straightforwardly starting with the unconditional model which, after combining (1) and (2), can be written as

$$(3) \quad w_i^t Dq_i = \sum_k \beta_{ik} dA_{ik} + \theta_i (DQ - \sum_j \sum_k \beta_{jk} dA_{jk}) + \sum_j \pi_{ij} Dp_j + \epsilon_i, \quad \text{or} \\ y_i = a_i + \theta_i(y - a) + \sum_j \pi_{ij} x_j + \epsilon_i,$$

where the time subscript  $t$  has been omitted for convenience;  $y_i = w_i^t Dq_i$ ,  $y = \sum y_i = DQ$ ,  $a_i = \sum_k \beta_{ik} dA_{ik}$ ,  $a = \sum_j a_j = \sum_j \sum_k \beta_{jk} dA_{jk}$ , and  $x_j = Dp_j$ .

Equation (3) can be summed over the goods in the juice group, say group  $A$  ( $i = 1, \dots, 5$ ), to find the conditional real income variable  $y_A$  for the juice group, i.e.,

$$(4) \quad y_A = a_A + \theta_A(y - a) + \sum_j \pi_{Aj} x_j + \epsilon_A,$$

where  $y_A = \sum_{i \in A} y_i$ ,  $a_A = \sum_{i \in A} a_i$ ,  $\theta_A = \sum_{i \in A} \theta_i$ ,  $\pi_{Aj} = \sum_{i \in A} \pi_{ij}$ , and  $\epsilon_A = \sum_{i \in A} \epsilon_i$ .

From (4), we see that  $y - a = (y_A - a_A - \sum_j \pi_{Aj} x_j - \epsilon_A) / \theta_A$ , which can be substituted into (3) to find the conditional demand equation for juice  $i$  in group  $A$ :

$$(5) \quad y_i = a_i + \theta_i^* (y_A - a_A) + \sum_{j \in A} \pi_{ij}^* x_j + \epsilon_i^*,$$

where  $\theta_i^* = \theta_i / \theta_A$ ,  $\pi_{ij}^* = \pi_{ij} - \theta_i^* \pi_{Aj}$ , and  $\epsilon_i^* = \epsilon_i - \theta_i^* \epsilon_A$ .

Note that under the assumption of weak separability,  $\pi_{ij}^*$  in (5) is equal to zero for  $j \notin A$ . For example, for separability involving two groups, say  $A$  and  $B$ , the restriction on the cross-price Slutsky coefficient is  $\pi_{ij} = -\phi_{AB} \theta_i \theta_j$ ,  $i \in A$  and  $j \in B$ , where  $\phi_{AB}$  is a factor of proportionality specific to the groups involved, and hence, for  $i \in A$  and  $j \in B$ ,  $\pi_{ij}^* = \pi_{ij} - (\theta_i / \theta_A) \pi_{Aj} = -\phi_{AB} \theta_i \theta_j + (\theta_i / \theta_A) \phi_{AB} \theta_A \theta_j = 0$ , where we use  $\pi_{Aj} = -\sum_{i \in A} \phi_{AB} \theta_i \theta_j = -\phi_{AB} \theta_A \theta_j$  [see Theil (1976) for development and discussion of weak separability conditions].

An important issue is whether the conditional income variable  $y_A$  is independent of the conditional error term  $\epsilon_i^*$  in equation (5). In general, the error  $\epsilon_A$  in the conditional income variable  $y_A$  can be clearly correlated with the conditional demand error,  $\epsilon_i^* = \epsilon_i - \theta_i^* \epsilon_A$ . However, there is one important case where  $\epsilon_A$  is not correlated with  $\epsilon_i^*$ , and conditional income can be treated as exogenous. In particular, if the covariance between  $\epsilon_i$  and  $\epsilon_j$  is proportional to the Slutsky coefficient  $\pi_{ij}$ , conditional income variable  $y_A$  will be uncorrelated with the conditional error term  $\epsilon_i^*$  in equation (5), shown as follows:

- Let the  $\epsilon_i$ s be contemporaneously correlated normal random variables, with  $E(\epsilon_i) = 0$ , and  $E(\epsilon_i \epsilon_j) = \sigma_{ij}$ .
- Hence,  $E(\epsilon_A \epsilon_i^*) = E((\sum_{j \in A} \epsilon_j)(\epsilon_i - \theta_i^* \sum_{j \in A} \epsilon_j)) = \sum_{j \in A} \sigma_{ij} - \theta_i^* \sum_{i \in A} \sum_{j \in A} \sigma_{ij}$ .
- Let  $\sigma_{ij} = \lambda \pi_{ij}$ , where  $\lambda$  is some factor of proportionality (rational random behavior assumption).
- Homogeneity of demand requires  $\sum_j \pi_{ij} = \sum_{j \in A} \pi_{ij} + \sum_{j \in B} \pi_{ij} = 0$ , or  $\sum_{j \in A} \pi_{ij} = -\sum_{j \in B} \pi_{ij}$ , assuming two groups,  $A$  and  $B$ , for simplicity.
- Given weak separability for  $i \in A$  and  $j \in B$ ,  $\pi_{ij} = -\phi_{AB} \theta_i \theta_j$ , and hence  $\sum_{j \in A} \pi_{ij} = \sum_{j \in B} \phi_{AB} \theta_i \theta_j = \phi_{AB} \theta_i (1 - \theta_A)$ , where we use the adding-up condition,  $\sum_j \theta_j = \sum_{j \in A} \theta_j + \sum_{j \in B} \theta_j = \theta_A + \theta_B = 1$ .
- Combining the previous steps (a) through (e), we see that

$$E(\epsilon_A \epsilon_i^*) = \sum_{j \in A} \sigma_{ij} - \theta_i^* \sum_{i \in A} \sum_{j \in A} \sigma_{ij} \\ = \lambda \sum_{j \in A} \pi_{ij} - \lambda \theta_i^* \sum_{i \in A} \sum_{j \in A} \pi_{ij}$$

$$\begin{aligned}
 &= \lambda \phi_{AB} \theta_i (1 - \theta_A) - \lambda (\theta_i / \theta_A) \sum_{i \in A} \phi_{AB} \theta_i (1 - \theta_A) \\
 &= \lambda \phi_{AB} \theta_i (1 - \theta_A) - \lambda \phi_{AB} (\theta_i / \theta_A) \theta_A (1 - \theta_A) = 0.
 \end{aligned}$$

The above proportionality of the corresponding error covariance and Slutsky terms, and the consequent independence of conditional income and conditional demand error terms are important results of Theil's (e.g., 1975, 1976, 1980a) theory of rational random behavior. In the theory of rational random behavior, errors in quantities demanded are introduced into the general utility maximization problem, and the error covariance matrix is approximated using a Taylor series expansion around the optimal bundle that the consumer plans to purchase (the actual bundle purchased differs from the planned bundle by the errors). To the extent that rational random behavior is a reasonable explanation for the demand errors in the Rotterdam model, independence of conditional income  $y_A$  and the conditional error term  $e_i^*$  in equation (5) actually seems to be a likely possibility. However, the possibility of an endogeneity problem still exists, and the prudent approach is to examine conditional demand estimates for income endogeneity.

In the next section, the Wu-Hausman test and an informal test suggested by Theil (1980a) are considered in examining this potential problem for juice demand. As Theil (1980a) shows, proportionality between unconditional variance/covariance terms ( $\sigma_{ij}$ s) and unconditional Slutsky coefficients ( $\pi_{ij}$ s) also implies proportionality between the conditional variance/covariance terms and conditional Slutsky coefficients ( $\pi_{ij}^*$ s). Hence, the conditional variance/covariance and Slutsky coefficient estimates can be straightforwardly checked for proportionality.

## Application

Weekly data from A. C. Nielsen Company (Nielsen Marketing Research) were used to analyze conditional Rotterdam model (5). The period from week ending 14 November 1987 to 15 May 1993 (228 observations) was studied. Each observation includes dollar and gallon retail sales and measures of promotion, by juice type, in outlets with annual sales of \$4 million or more. Prices were calculated by dividing dollars by gallons, and gallons were divided by the U.S. population to obtain per capita juice sales. Data were provided on two types of promotional activities: (a) A/B ads (printed material in newspapers), and (b) displays accompanied by an ad. The variable for each promotional activity is a measure of the percentage of the market covered by the promotional activity. This measurement of promotion indicates the extent of advertising nationwide; specific data on promotional expenditures or other measures of promotions were not available. Data from the U.S. Department of Commerce on total retail grocery store sales and the consumer price for food also were used. Total retail grocery store sales less Nielsen juice sales divided by the U.S. population and the consumer price index for food was used as a measure of per capita retail grocery store sales other than juice. The consumer price index for food was used as an approximation for the price of retail grocery store goods other than juice. Descriptive statistics for the basic juice data are given in table 1. Mean per capita gallon sales, prices, and conditional budget shares are provided in the table. Orange juice dominates the juice category with a conditional budget share of .64, followed by apple juice, remaining juices, grape juice, and grapefruit juice with conditional budget shares of .16, .09, .05, and .05, respectively.

Past studies (e.g., Tilley; Brown) have found season of the year to affect juice demand and, to allow for seasonality, a fourth-degree polynomial in week of the year was included in the specification of demand. [See Robb for a similar approach to model seasonality using spline functions; an alternative approach suggested by Duffy (1990) would be to 52nd (for the number of weeks in a year) difference the data, as opposed to taking first differences as usually done in defining the Rotterdam model.] The polynomial was restricted so that its value would be continuous from one year to the next. The differences of the polynomial variables (functions of week) were included in the demand specification

**Table 1. Descriptive Statistics for Retail Juice Sales in Grocery Stores with Annual Sales of \$4 Million or More (14 November 1987 through 15 May 1993)**

Juice	Per Capita Sales (ounces/week)	Price (\$/gallon)	Conditional Budget Share <sup>a</sup>
Orange Juice	6.70 (.58) <sup>b</sup>	3.78 (.34)	.645 (.014)
Grapefruit Juice	.49 (.06)	4.17 (.22)	.052 (.005)
Apple Juice	1.97 (.24)	3.16 (.34)	.158 (.010)
Grape Juice	.57 (.06)	3.70 (.21)	.054 (.005)
Remaining Juices	.66 (.07)	5.38 (.21)	.091 (.010)

<sup>a</sup> Figures represent budget shares out of total juice expenditure. The share of total juice expenditure out of total grocery store sales was .012.

<sup>b</sup> Numbers in parentheses are standard errors.

for each juice, with the coefficients (across juices) on each differenced polynomial variable required to sum to zero, based on the adding-up property.

The Wu-Hausman test was used to examine the potential problem of price endogeneity. A separate test was conducted for each unconditional juice demand equation (3)—first-order autocorrelation corrected estimates (prices treated as exogenous) were compared to first-order autocorrelation corrected estimates based on the instrumental variable method. (The instruments were present nonprice explanatory variables, lagged dependent variable, and both lagged price and nonprice explanatory variables; the present value of one of the seasonality variables was omitted due to singularity; and the TSP estimation procedure following Fair was used.) The chi-square test statistics (asymptotically), each with 20 degrees of freedom (the number of explanatory variables), ranged from 2.96 for RJ to 7.02 for GRJ and strongly support (at any reasonable level of significance) independence of prices and equation errors.

The Wu-Hausman test was next used to test for endogeneity of the conditional expenditure variable  $y_A$  in conditional demand equation (5). Again, a separate test was conducted for each conditional juice demand equation and a correction for first-order autocorrelation was made. For each test, the unconditional expenditure variable  $y$  and the log change in the consumer price index for food, along with the other conditional demand explanatory variables except conditional expenditure  $y_A$ , were used as instruments. The chi-square test statistics, now each with 19 degrees of freedom, ranged from 1.50 for RJ to 7.26 for GJ. The results strongly support the hypothesis that conditional income  $y_A$  is independent of the conditional error  $\epsilon_i^*$  in each demand equation. Hence, estimation of conditional juice demand equations (5), treating the group or conditional expenditure variable as exogenous, seems to be appropriate for the present data set.

The theory of rational random behavior further indicates that if conditional expenditure is independent of the conditional errors, proportionality between the corresponding terms of the error covariance matrix and Slutsky matrix should exist. An informal examination of proportionality is considered subsequently.

The five conditional juice demand equations were estimated as a system using the full information maximum likelihood (FIML) method. Homogeneity and symmetry were imposed as part of the maintained hypothesis (e.g., Eales and Unnevehr; Alston and Chalfant). The adding-up conditions were automatically fulfilled, as the juice expenditure data add up by construction—the left-hand-side variables ( $y_i$ ) of equation (5) sum over  $i$  to the conditional expenditure variable ( $\sum_{j \in A} y_j = y_A$ ). Since the data add up, the conditional errors also sum over  $i$  to zero ( $\sum_{i \in A} \epsilon_i^* = \sum_{i \in A} \epsilon_i - \epsilon_A \sum_{i \in A} \theta_i^* = \epsilon_A - \epsilon_A = 0$ ) and the conditional

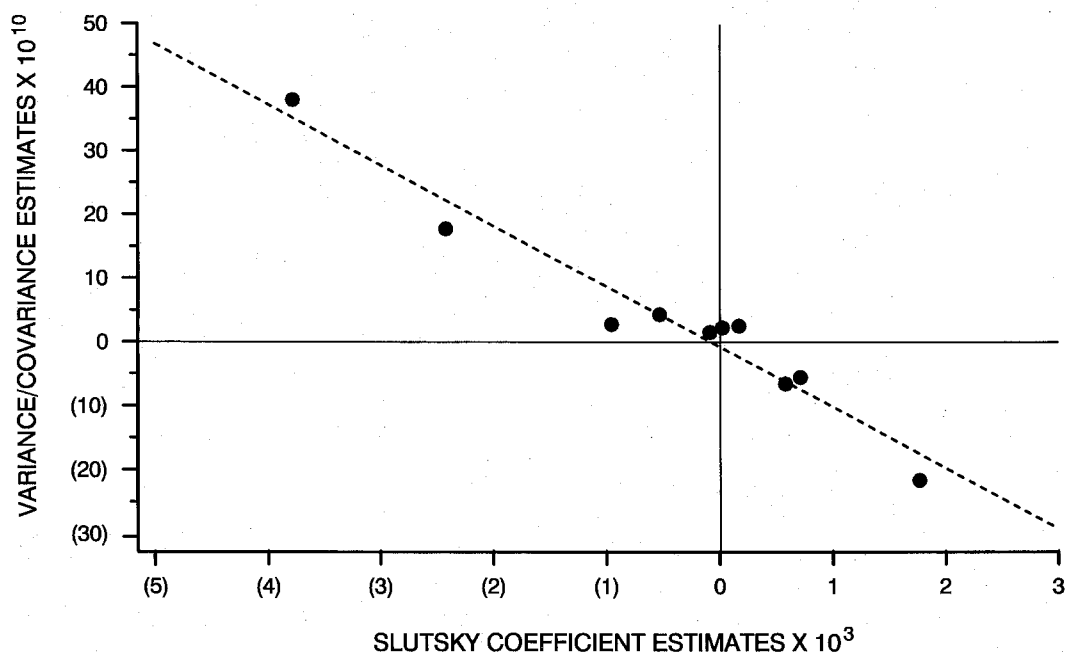


Figure 1. Variance/covariance estimates versus Slutsky coefficient estimates

covariance matrix is singular (Theil 1971; Barten 1969). To overcome this singularity problem, an arbitrary equation (the equation for RJ) was dropped from the system and the FIML method was applied to the system of remaining equations (Barten 1969). This estimation procedure is invariant to the equation dropped; the parameters of the omitted equation can be estimated from the parameter estimates of the included equations and the adding-up conditions. In addition, the system of equations was corrected for first-order autocorrelation by directly estimating an autocorrelation coefficient ( $\rho$ ) for each equation; i.e., the model errors were specified according to the definition of first-order autocorrelation and an autocorrelation coefficient was estimated, along with the other demand parameters, by the FIML method. Since the equations obey adding-up, one autocorrelation coefficient was used for the five equations in the system (Berndt and Savin; Johnson, Hassan, and Green).

Theil (1980a) informally examined the rational random behavior hypothesis by plotting the estimated elements of the conditional error covariance matrix against the corresponding estimated elements of the conditional Slutsky matrix. Proportionality between the covariance matrix terms and the Slutsky coefficients should show up as a straight line through the origin. For the present study, the plot of the estimated conditional variances and covariances against the corresponding estimated Slutsky coefficients followed a line through the origin (fig. 1), supporting the rational random behavior hypothesis and the independence of the conditional expenditure term  $y_A$  and the conditional errors ( $\epsilon_i^*$ s). The ordinary least squares relationship for the plot was

$$\sigma_{ij}^* = -0.820 - 9.522\pi_{ij}^*, \quad R^2 = 0.95,$$

(1.258)      (.791)

where  $\sigma_{ij}^*$  and  $\pi_{ij}^*$  are the conditional covariance term times  $10^{10}$  and conditional Slutsky term times  $10^3$ , respectively, and the numbers in parentheses are estimated standard errors. The insignificance of the intercept and significance of the slope (at any reasonable level of significance) supports the proportionality hypothesis and theory of rational random behavior.

The maximum likelihood estimates for conditional demand model (5) are shown in

table 2. The individual equations fit quite well, with the equation  $R^2$  values ranging from .97 for OJ to .74 for GRJ. The autocorrelation coefficient estimate was  $-.08$  and was more than twice the size of its estimated asymptotic standard error [estimates of model (5), as well as previously discussed Wu-Hausman tests, changed little when the autocorrelation coefficient was restricted to zero].

All estimates of the conditional marginal propensities to consume (MPCs) were positive and twice as large or larger than their corresponding asymptotic standard error estimates, ranging from .67 for OJ to .05 for GJ. All conditional own-Slutsky coefficient estimates were negative, as predicted by theory, and twice as large or larger than their corresponding asymptotic standard error estimates. Seven of the conditional cross-Slutsky coefficient estimates were positive and twice as large or larger than their corresponding asymptotic standard error estimates, indicating substitute relationships; the remaining three cross-Slutsky coefficient estimates had relatively large standard error estimates, indicating neutral cross-price relationships. Seven of the 10 promotional coefficient estimates were twice as large or larger than their corresponding standard error estimates; one was 1.8 times its standard error estimate. Only the two promotional coefficient estimates for OJ were small relative to their corresponding standard error estimates; this result may be due to the large amount of brand promotion and likely brand switching in the OJ market (promotion for a particular brand may expand demand for the brand but at the expense of decreased demand for other OJ brands, and hence demand for the overall OJ category may not significantly change). Lagged promotional effects were found to be insignificant and were excluded from the model. With promotional activity usually including cents-off deals, consumers may have been induced to try a particular juice or buy more, given they were repeat customers, but the effect does not seem to have been lasting. A similar result was found by Brown and Lee (1993).

For each promotional coefficient estimate with a relatively small standard error estimate, the coefficient sign was positive. This result, along with the MPC estimates in the zero-one interval, indicates that the promotional activity in question positively affected demand for the promoted juice; i.e., the direct effect through the translation term was positive and outweighed the negative indirect effect stemming from a reduction in total juice expenditure above fixed expenditures ( $m - \sum_j p_j \gamma_j$ ). Note that the cross-promotional effects, which occur in an indirect manner, are negative, except for one case where the OJ promotional coefficient was negative and insignificant; these results indicate the competitive nature of juice promotions.

The conditional elasticity estimates (at sample mean values) for model (5) are shown in table 3. The elasticity formulas are:

$$\begin{aligned} \text{Expenditure Elasticity:} & \quad e_i = \theta_i^* / (w_i / w_A); \\ \text{Uncompensated Price Elasticity:} & \quad e_{ij} = \pi_{ij}^* / w_i - w_j \theta_j^* / w_i; \quad \text{and} \\ \text{Promotional Elasticity:} & \quad e_{ijk} = \beta_{jk} (\Delta_{ij} - \theta_i^*) A_{jk} / w_i, \end{aligned}$$

where  $w_A = \sum_{i \in A} w_i$ , and  $\Delta_{ij}$  is the Kronecker delta equal to unity if  $i = j$ , and zero otherwise (for conditional elasticity formulas, see, e.g., Theil 1976; Duffy 1987).

The conditional expenditure elasticities ranged from .76 for RJ to 1.17 for GRJ. The conditional own-price elasticities ranged from  $-.89$  for GRJ to  $-1.61$  for GJ. The conditional cross-price elasticities ranged from .50 for a change in the price of OJ on the demand for GJ to  $-.20$  for a change in the price of GRJ (GJ) on the demand for GJ (GRJ). The conditional own-promotional elasticities ranged from .03 for apple juice displays with advertising to zero for the OJ promotions. The conditional cross-promotional elasticities, like the cross-advertising effects previously discussed, were predominately negative and were smaller in magnitude than the own-promotional elasticities. The promotional elasticities, although measures of the general positive own-impacts and negative cross-impacts of promotions, cannot be used to further evaluate the returns of promotions since information on promotional costs was not available.

We complete our empirical analysis by considering the unconditional demand estimates

Table 2. Maximum Likelihood Estimates for Conditional Juice Demands

Juice	Conditional MPC ( $\theta^j$ )			Conditional Slutsky Coefficient <sup>a</sup> ( $\pi_{ij}^*$ )				Conditional Promotional Coefficient <sup>a</sup> ( $\beta_{ik}$ )	
	Orange Juice	Grapefruit Juice	Apple Juice	Grape Juice	Remaining Juices	A/B Ads	Displays w/Ads		
Orange Juice	.6670* (.0110)	.7140* (.0573)	1.7700* (.1460)	.5820* (.0867)	.7000* (.1080)	-.0017 (.0024)	.0087 (.0073)		
Grapefruit Juice	.0550* (.0023)	-.9470* (.0642)	.1770* (.0632)	-.0808 (.0509)	.1370* (.0662)	.0004* (.0002)	.0029* (.0006)		
Apple Juice	.1470* (.0076)		-2.4100* (.1320)	.0307 (.0762)	.4330* (.0954)	.0014 (.0008)	.0051* (.0012)		
Grape Juice	.0630* (.0032)			-.5240* (.0679)	-.0077 (.0879)	.0010* (.0003)	.0032* (.0013)		
Remaining Juices	.0690* (.0042)				-1.2600* (.1230)	.0032* (.0011)	.0153* (.0026)		

Notes: Seasonality coefficient estimates have been omitted for convenience; they are available upon request. An asterisk (\*) indicates coefficient estimate is twice as large or larger than its asymptotic standard error estimate.

<sup>a</sup> Conditional Slutsky and promotional coefficient estimates, and corresponding asymptotic standard errors, are  $10^{-3}$  times table values.

<sup>b</sup> Numbers in parentheses are asymptotic standard errors of coefficient estimates.



Table 3. Conditional Juice Demand Elasticity Estimates

Juice	Expenditure	Juice				
		Orange Juice	Grapefruit Juice	Apple Juice	Grape Juice	Remaining Juices
----- Price Elasticity Estimates -----						
Orange Juice	1.0330* (.0170) <sup>a</sup>	-1.1640* (.0330)	.0410* (.0076)	.0700* (.0200)	.0210 (.0110)	-.0012 (.0140)
Grapefruit Juice	1.0570* (.0440)	.4880* (.0970)	-1.6060* (.1050)	.1220 (.1030)	-.1890* (.0830)	.1280 (.1090)
Apple Juice	.9260* (.0480)	.3540* (.0770)	.0470 (.0350)	-1.4420* (.0730)	-.0330 (.0410)	.1490* (.0510)
Grape Juice	1.1740* (.0590)	.1630 (.1370)	-.1890* (.0800)	-.1370 (.1230)	-.8920* (.1070)	-.1190 (.1400)
Remaining Juices	.7580* (.0460)	.1720 (.0990)	.0900 (.0630)	.2890* (.0910)	-.0480 (.0830)	-1.2600* (.1160)
----- Demand Elasticity Estimates for A/B Ads -----						
Orange Juice		-.0052 (.0072)	-.0007 (.0004)	-.0041 (.0023)	-.0013* (.0004)	-.0021* (.0007)
Grapefruit Juice		.0110 (.0150)	.0130* (.0063)	-.0042 (.0024)	-.0013* (.0004)	-.0021* (.0007)
Apple Juice		.0093 (.0130)	-.0006* (.0003)	.0210 (.0120)	-.0012* (.0003)	-.0019* (.0007)
Grape Juice		.0120 (.0160)	-.0008* (.0004)	-.0047 (.0026)	.0220* (.0060)	-.0024* (.0008)
Remaining Juices		.0077 (.0110)	-.0005* (.0003)	-.0030 (.0017)	-.0010* (.0003)	.0210* (.0072)
----- Demand Elasticity Estimates for Displays with Ads -----						
Orange Juice		.0044 (.0038)	-.0011* (.0002)	-.0064* (.0015)	-.0005* (.0002)	-.0019* (.0003)
Grapefruit Juice		-.0091 (.0078)	.0190* (.0042)	-.0065* (.0015)	-.0005* (.0002)	-.0020* (.0003)
Apple Juice		-.0079 (.0068)	-.0010* (.0002)	.0330* (.0076)	-.0005* (.0002)	-.0017* (.0003)
Grape Juice		-.0100 (.0087)	-.0012* (.0003)	-.0073* (.0017)	.0085* (.0036)	-.0022* (.0004)
Remaining Juices		-.0065 (.0057)	-.0008* (.0002)	-.0047* (.0011)	-.0004* (.0002)	.0190* (.0032)

\* An asterisk indicates elasticity estimate is twice as large or larger than its asymptotic standard error estimate.

<sup>a</sup> Numbers in parentheses are asymptotic standard errors of elasticity estimates.

for conditional juice demands (5), focusing on a result which simplifies the analysis. Estimation of group demand equation (4) for juices suggests that group MPC,  $\theta_A$ , could reasonably be treated as zero—the estimates of  $\theta_A$  and its asymptotic standard error were  $-.0011$  and  $.0010$ , respectively. As shown by Theil (1976), when  $\theta_A = 0$ , the unconditional MPCs,  $\theta_{iS}$ , for the group are also zero.<sup>2</sup> The value of the group MPC,  $\theta_A = 0$ , also implies unconditional juice Slutsky coefficients  $\pi_{ijS}$  are equal to the conditional Slutsky coefficients  $\pi_{ij}^*$ . To see this result, recall  $\pi_{ij}^* = \pi_{ij} - \theta_i^* \pi_{Aj}$ . When  $\theta_A = 0$ , the term  $\pi_{Aj}$  on the right-hand side of this equation is zero, i.e.,

$$(a) \pi_{Aj} = \sum_{i \in A} \pi_{ij};$$

$$(b) \sum_{i \in A} \pi_{ij} + \pi_{6j} = 0, \text{ or } \pi_{Aj} = -\pi_{6j}, \text{ based on adding-up, with } i = 1, \dots, 5 \text{ for group } A, \text{ and } i = 6 \text{ for group } B;$$

$$(c) \pi_{Aj} = \phi_{AB} \theta_6 \theta_j, \text{ based on weak separability; and}$$

$$(d) \pi_{Aj} = 0, \text{ since } \theta_A \text{ and hence } \theta_j \text{ are zero.}$$

Similarly, the cross-group Slutsky coefficients are zero, e.g.,  $\pi_{i6} = -\phi_{AB} \theta_i \theta_6 = 0$ , since  $\theta_i = 0$ ,  $i = 1, \dots, 5$ . In addition, a comparison of unconditional and conditional demand models (3) and (5) indicates that the promotional coefficients are defined the same for the

two models; however, the unconditional and conditional seasonality coefficients generally differ.

The foregoing relationships between conditional and unconditional demand coefficients, for the case when  $\theta_A = 0$ , were checked by comparing estimates of unconditional model (3) with the estimates for conditional model (5). All unconditional juice MPC estimates were relatively small compared to their asymptotic standard error estimates and could reasonably be treated as zero. On the other hand, the unconditional Slutsky and promotional coefficient estimates were only roughly similar to corresponding conditional estimates, and the unconditional cross-group Slutsky coefficient estimates were all twice as large or larger than their asymptotic standard errors, in contrast to expectations. Although all of the unconditional estimates do not conform with expectations based on  $\theta_A = 0$ , the unconditional demand estimates actually seemed quite reasonable—all own-Slutsky coefficient estimates were negative and twice as large or larger than their asymptotic standard error estimates, and all promotion coefficients were positive, with nine out of 10 being twice as large or larger than their asymptotic standard error estimates.

Which set of estimates (conditional or unconditional) better describes juice demands? As with any empirical analysis, the answer depends, in part, on judgment, and for this purpose, we take a closer look at the data. The Nielsen juice data were weekly and are quite accurate, as sales are measured at check-out scanners at grocery stores. Since our conditional demand model estimates are based entirely on the Nielsen data, we feel somewhat confident these estimates reflect juice demands. On the other hand, the unconditional demand estimates are based on both the Nielsen data and U.S. Department of Commerce data for total grocery store sales. We need to admit that aggregating food other than juice into a single category ( $i = 6$ ) is only a rough approximation. Moreover, the raw U.S. Department of Commerce data for food were on a monthly basis and interpolated to obtain weekly data, consistent with the weekly juice data<sup>3</sup> (see, e.g., Thurman 1987, for use of similar interpolated data). Both aggregation and interpolation are possible sources of error, giving us somewhat less confidence in the unconditional results.

The results of the present study are not directly comparable to other studies of juice demands due to differences in juice categories studied, models used, and data analyzed. Nevertheless, the results of the present study generally are consistent with results found by Brown and Lee (1992a). Both studies found conditional juice expenditure elasticities similarly varying around unity and conditional own-price elasticities in, or close to, the interval between  $-1$  to  $-2$ . The insignificant unconditional expenditure responses in the present study are similar to those found by Brown, except that the latter study found a significant positive unconditional expenditure response for orange juice. The latter study also found that the own-price response for apple juice, although negative, was insignificant, while the present study found this response to be negative and significant to the extent the estimate was relatively large compared to its asymptotic standard error; the price response for grapefruit juice was also stronger in the present study, while the price responses for orange juice and grape juice were similar in the two studies.

### Concluding Comments

Conditional expenditures, as well as prices, may be endogenous in conditional demand systems, as recently discussed by LaFrance. Tests such as the Wu-Hausman specification test can be used to determine whether endogeneity exists, and if so, corrective measures can be taken, including instrumental variable estimation, use of homogeneity restrictions as discussed by Attfield, extension of the model by explicitly specifying equations to explain conditional expenditures and/or prices, and estimation of an unconditional model where explanatory variables can be treated as exogenous.

In the present study of conditional demands for juices, application of the Wu-Hausman test indicated conditional expenditures and prices can be treated as exogenous. An informal test suggested by Theil (1980a) also indicated independence of conditional expenditures

and conditional demand errors. As predicted by the theory of rational random behavior, the conditional variance/covariance estimates were approximately proportional to the correspondent conditional Slutsky coefficient estimates, implying independence of conditional expenditures and conditional errors. For conditional juice demands, Theil's plot of variance/covariance estimates against Slutsky coefficient estimates clearly revealed the predicted proportionality of rational random behavior. For studies where good instrumental variables are not available to apply the Wu-Hausman test, such plots may prove to be useful diagnostic tools for examining the possibility of expenditure endogeneity in conditional demand specifications.

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## Notes

<sup>1</sup> For  $n$  goods, each demand equation has a total of  $n + 1$  responses ( $n$  price responses and one income response). Imposition of homogeneity reduces the number of independent responses (that need to be directly estimated) per equation to  $n$ . With treatment of income as endogenous and prices as exogenous, the  $n$  independent responses in a homogeneity-restricted demand equation can be estimated consistently by the instrumental variable method, using the  $n$  prices as instruments.

<sup>2</sup> When  $\theta_A$  is zero, the conditional MPC,  $\theta_i^*$ , is undefined by  $\theta_i/\theta_A$ ; however, this does not mean that  $\theta_i^*$  does not exist or that  $\theta_A$  must be nonzero for weak separability. Conditional  $\theta_i^*$  is well defined by  $\theta_i/\theta_A$  for strong separability, which requires  $\theta_A$  to be positive (e.g., Theil 1976). For weak separability,  $\theta_A$  can be positive, zero, or negative. When  $\theta_A = 0$ ,  $\theta_i^*$  requires an alternative definition in terms of decomposed demand effects (see Theil 1976). The alternative definition of  $\theta_i^*$  continues to satisfy  $\sum_{i \in A} \theta_i^* = 1$  and  $\theta_i = \theta_i^* \theta_A$ , and the definition of conditional demand specification (5) is otherwise unchanged.

<sup>3</sup> Instead of interpolating monthly data to obtain weekly data, weekly data might be aggregated to obtain monthly data. However, for the present study, aggregation of the Nielsen weekly data to monthly levels for all variables was not possible. Although weekly sales, in both dollars and gallons, could straightforwardly be aggregated to monthly sales, there was insufficient information to aggregate weekly promotional variables, measuring market coverage, to a monthly basis.

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