AJAE Appendix:

Grading, Minimum Quality Standards, and the Labeling of Genetically Modified Products

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Proof of Proposition 5 – Comparative Statics Results

With a uniform distribution of types $H(\beta)=\beta$, and thus in the uncovered market when both GM and non-GM products are produced, market demands are, respectively, $D_g=\hat{\beta}$ and $D_n=\tilde{\beta}-\hat{\beta}$, so that total demand is $D_T \equiv D_n + D_g = \tilde{\beta}$. Upon recalling the arbitrage relations of competitive equilibrium, that is,

$$p_n^0 = p_g^0 + \delta$$

$$p_g^1 = p_g^0 + \eta$$

$$\left(p_n^1 - p_g^1\right)F(R) = \delta + \sigma$$

we have

$$\tilde{\beta}(R) \equiv \frac{u - p_g^0 - \eta - \dfrac{\delta+\sigma}{F(R)}}{a\overline{s}(R)}$$

$$\hat{\beta}(R) = \frac{\delta+\sigma}{aF(R)\left[1-\overline{s}(R)\right]}.$$

In what follows we simplify notation and omit the functional dependence on R by writing $F(R)=F$, $f(R)=f$ and $\overline{s}(R)=\overline{s}$. Also, we define $k\equiv\delta+\sigma$, $A\equiv u-p_g^0-\eta$, and $P\equiv p_g^0$, so that

$$\hat{\beta} = \frac{k}{Fa(1-\overline{s})}$$

$$\tilde{\beta} = \frac{AF-k}{Fa\overline{s}}.$$

Aggregate consumer surplus here is $CS=\frac{1}{2}a\left[\tilde{\beta}^2\overline{s}+\hat{\beta}^2(1-\overline{s})\right]$. Substituting and simplifying obtains

$$CS = \frac{1}{2a}\left[\frac{\left(A-\dfrac{k}{F}\right)^2}{\overline{s}} + \frac{\left(\dfrac{k}{F}\right)^2}{(1-\overline{s})}\right].$$

Hence, the welfare function is

$$W = \Pi(P) + \frac{1}{2a}\left[\frac{\left(A - \frac{k}{F}\right)^2}{\bar{s}} + \frac{\left(\frac{k}{F}\right)^2}{(1-\bar{s})}\right]$$

where $\Pi(P)$ is producer surplus. The optimality conditions for welfare maximization (yielding the optimal standard purity R^* and the competitive farm-level equilibrium price P^*) are

(1) $$W_P = \Pi'(P) - \frac{1}{a}\frac{\left(A - \frac{k}{F}\right)}{\bar{s}} = 0$$

(2) $$W_R = \frac{1}{2a}\left[\frac{2\left(A - \frac{k}{F}\right)\frac{k}{F^2}f}{\bar{s}} - \frac{\left(A - \frac{k}{F}\right)^2}{\bar{s}^2}\frac{f}{F}(R-\bar{s}) - \frac{2\left(\frac{k}{F}\right)^2\frac{f}{F}}{(1-\bar{s})} + \frac{\left(\frac{k}{F}\right)^2}{(1-\bar{s})^2}\frac{f}{F}(R-\bar{s})\right] = 0.$$

Upon substitution and simplification we obtain

(3) $$W_R = \left(\frac{af}{2F}\right)\left(\tilde{\beta} - \hat{\beta}\right)\left[\hat{\beta}\left((1-\bar{s}) + (1-R)\right) - \tilde{\beta}\left(R-\bar{s}\right)\right] = 0 \rightarrow \left[\hat{\beta}\left((1-\bar{s}) + (1-R)\right) - \tilde{\beta}\left(R-\bar{s}\right)\right] = 0.$$

Consider now the comparative statics effect of the parameter $k \equiv \sigma + \delta$. Differentiating the optimality conditions in (1) and (2) and expressing the results in matrix form yields

$$\begin{bmatrix} W_{RR} & W_{RP} \\ W_{PR} & W_{PP} \end{bmatrix}\begin{bmatrix} R_k \\ P_k \end{bmatrix} = \begin{bmatrix} -W_{Rk} \\ -W_{Pk} \end{bmatrix}.$$

Solving by Cramer's rule obtains

$$R_k = \frac{1}{\Delta}\begin{vmatrix} -W_{Rk} & W_{RP} \\ -W_{Pk} & W_{PP} \end{vmatrix} = \frac{-W_{Rk}W_{PP} + W_{Pk}W_{RP}}{\Delta}$$

$$P_k = \frac{1}{\Delta}\begin{vmatrix} W_{RR} & -W_{Rk} \\ W_{PR} & -W_{Pk} \end{vmatrix} = \frac{-W_{RR}W_{Pk} + W_{PR}W_{Rk}}{\Delta}$$

where $\Delta = \begin{vmatrix} W_{RR} & W_{RP} \\ W_{PR} & W_{PP} \end{vmatrix} < 0$, $W_{RR} < 0$ and $W_{PP} > 0$ by the second-order conditions of the welfare optimization problem (saddle point).

We now compute the partial effects that enter these comparative statics expressions. Differentiating the optimality conditions in (1) and (2) yields

$$W_{PR} = -\frac{1}{a}\frac{k}{F^2}\frac{f}{\bar{s}} + \frac{1}{a}\frac{\left(A - \frac{k}{F}\right)}{\bar{s}^2}\frac{f}{F}(R - \bar{s})$$

which can be simplified to

$$W_{PR} = \frac{f}{F\bar{s}}\left[-\hat{\beta}(1-\bar{s}) + \tilde{\beta}(R-\bar{s})\right].$$

Evaluating this partial effect at the optimality conditions, such that (3) holds, we obtain

$$W_{PR} = \frac{f}{F\bar{s}}\hat{\beta}(1-R) \quad > 0 .$$

Next, differentiating (1) we find

$$W_{Pk} = \frac{1}{aF\bar{s}} > 0 .$$

And differentiating (2) we obtain

$$W_{Rk} = \frac{1}{a}\left[-\frac{kf}{F^3\bar{s}} + \left(A - \frac{k}{F}\right)\frac{1}{\left(F\bar{s}\right)^2}\left(f\bar{s} + f(R-\bar{s}\right) - \frac{k}{F^3}\frac{f}{(1-\bar{s})} + \frac{k}{F^2}\frac{1}{(1-\bar{s})^2}\frac{f}{F}(R-\bar{s})\right]$$

which simplifies to

$$W_{Rk} = \frac{f}{F^2}\frac{1}{\bar{s}(1-\bar{s})}\left[\tilde{\beta}R(1-\bar{s}) - \hat{\beta}\left((1-\bar{s})^2 + \bar{s}(1-R)\right)\right].$$

Evaluating this partial effect at the optimality conditions, such that (3) holds, we obtain

$$W_{kR} = \frac{f}{F^2}\frac{1}{\bar{s}(1-\bar{s})}\hat{\beta}\left[\frac{(1-R)\left[R(1-\bar{s}) - \bar{s}(R-\bar{s})\right] + \bar{s}(1-\bar{s})^2}{(R-\bar{s})}\right].$$

Thus, a sufficient condition for $W_{kR} > 0$ is $R(1-\bar{s}) \geq \bar{s}(R-\bar{s})$, which does hold because $R \geq \bar{s}$ and $R \leq 1$. Hence, we conclude that $W_{kR} > 0$.

The foregoing partial effects allow us to sign the comparative statics effect on farm price:

$$P_k = \frac{W_{RR}W_{Pk} - W_{PR}W_{Rk}}{-\Delta} < 0 .$$

But $sign(R_k) = sign(W_{Rk}W_{PP} - W_{Pk}W_{RP})$. Note that

$$W_{PP} = \Pi''(p_g^0) + \frac{1}{a\bar{s}}.$$

Because $\Pi''(p_g^0) = S'(p_g^0) > 0$ (the profit function is convex) and $W_{Rk} > 0$, to conclude that $R_k > 0$ it suffices to show that $Z \equiv W_{Rk}\frac{1}{a\bar{s}} - W_{Pk}W_{RP} \geq 0$. From earlier derivations,

$$Z \equiv \frac{f}{F^2}\frac{1}{\bar{s}(1-\bar{s})}\hat{\beta}\left[\frac{(1-R)\left[R(1-\bar{s}) - \bar{s}(R-\bar{s})\right] + \bar{s}(1-\bar{s})^2}{(R-\bar{s})}\right]\frac{1}{a\bar{s}} - \frac{1}{aF\bar{s}}\frac{f}{F\bar{s}}\hat{\beta}(1-R)$$

which can be simplified to yield

$$Z = \frac{1}{a}\frac{f}{F^2}\frac{1}{\bar{s}^2(1-\bar{s})}\hat{\beta}\left[\frac{\bar{s}(1-R)^2 + \bar{s}(1-\bar{s})^2}{(R-\bar{s})}\right] > 0$$

and so we can conclude that $R_k > 0$. Recalling that $k \equiv \delta + \sigma$, we have therefore established parts (i) and (ii) of Proposition 4.

The comparative statics analysis for the parameter k is readily adapted to the comparative statics of the "GM aversion" parameter a. Specifically,

$$R_a = \frac{W_{Ra}W_{PP} - W_{Pa}W_{RP}}{-\Delta}$$

$$P_a = \frac{W_{RR}W_{Pa} - W_{PR}W_{Ra}}{-\Delta}.$$

The partial effects of interest here are

$$W_{Pa} = \frac{1}{a^2F\bar{s}}(AF - k) = \frac{1}{a}\tilde{\beta} > 0$$

$$W_{Ra} = -\frac{1}{a}W_R = 0$$

and so we find

$$P_a = \frac{W_{RR}W_{Pa}}{-\Delta} < 0$$

$$R_a = \frac{-W_{Pa}W_{RP}}{-\Delta} < 0$$

which establishes part (iii) of Proposition 4.

Finally, concerning the parameters u and η, we note that they enter the problem only through the term $A \equiv u - p_g^0 - \eta$. For the comparative statics of this term we have

$$R_A = \frac{W_{RA}W_{PP} - W_{PA}W_{RP}}{-\Delta}$$

$$P_A = \frac{W_{RR}W_{PA} - W_{PR}W_{RA}}{-\Delta}.$$

The partial effects of interest here are

$$W_{PA} = -\frac{1}{a\overline{s}} < 0$$

and $W_{RA} < 0$ because $W_{RA} = -W_{PR}$ and we showed earlier that $W_{PR} > 0$. Thus we can immediately conclude that $P_A > 0$. The sign of R_A is the sign of $Z \equiv \left(W_{RA}W_{PP} - W_{PA}W_{RP}\right)$. By using $W_{RA} = -W_{PR}$ we find $Z \equiv W_{RA}\left(W_{PP} + W_{PA}\right)$, and by noting that $W_{PP} = \Pi''(p_g^0) - W_{PA}$ we get $Z \equiv W_{RA}\Pi''(p_g^0) < 0$, and so we conclude that $R_A < 0$. Recalling again that $A \equiv u - p_g^0 - \eta$, this concludes the comparative statics of parameters u and η (part (iv) of Proposition 5). ■