Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Providence, Rhode Island, July 24-27, 2005.

TIME VARYING COEFFICIENT: AN APPLICATION OF FLEXIBLE LEAST SQUARES TO CATTLE CAPTIVE SUPPLY

By

Andrew C. Lee
Economist
Environmental Protection Agency
Office of Pesticide Program
Washington, DC 20460
Email: lee.andrew@epa.gov
Phone: 703-308-7226

Man-Keun Kim
Postdoctoral Fellow
Joint Global Change Research Institute
University of Maryland
College Park, MD 20740
Email: man-keun.kim@pnl.gov
Phone: 301-314-6743

May 15, 2005

Corresponding author:
Andrew C. Lee
Environmental Protection Agency
Office of Pesticide Program
1200 Pennsylvania Ave., NW
Washington, DC 20460
Mail Code: 7503C
Email: lee.andrew@epa.gov
Phone: 703-308-7226

This paper reflects work done by the authors at their prior jobs and does not represent the views of the U.S. Environmental Protection Agency nor the Joint Global Change Research Institute of University of Maryland.

Abstract

Although conventional linear regression techniques assume time constancy of parameters time varying coefficient or the problem of structural instability in econometric relationships has been recognized by econometricians. In this study, time varying impact of captive supply on fed cattle cash market price is investigated via flexible least squares approach. Time path of flexible least squares coefficient estimate indicates an approximately four fold increase in price impact of captive supply over the sample period, but even this multiplied price impact is small compared to the effect of boxed beef price which shows negligible time variation. The time path also aids in identification of structural breaks in the price impact of captive supply.

Key words: time varying coefficient, flexible least squares, structural break, captive supply

TIME VARYING COEFFICIENT: AN APPLICATION OF FLEXIBLE LEAST SQUARES TO CATTLE CAPTIVE SUPPLY

Key words: time varying coefficient, flexible least squares, structural break, captive supply

Structural changes in the U.S. fed cattle industry include increasing firm size, concentration, vertical integration through contracts, and regulations (e.g., the Livestock Mandatory Reporting Act of 1999 (LMR)). In particular, captive supply, a form of backward integration by packers, is becoming an increasingly controversial issue. The Grain Inspection Packers and Stockyard Administration (USDA_GIPSA, p. vi) defines captive supply as cattle owned or fed by a packer, procured through forward contracts and marketing agreements, and cattle that are otherwise committed to a packer more than 14 days prior to slaughter.

Arguments in favor of captive supply include reduced transaction costs, reduced market risk, efficiency, quality enhancement, and global competitiveness (Feuz et al.). Opponents argue that it has adverse impact on fed cattle cash market prices, reduces competition and market access by small cattle producers, and increases market power of packers (Conner et al.). In particular, with fed cattle input cost accounting for the most of packer's production costs, combined with the projected increase in captive supply use, the potential impact of captive supply on fed cattle cash market price is becoming an increasingly contentious issue for market participants and policymakers. In this study, time varying nature of the price effect of captive supply is investigated using the flexible least squares (FLS) approach.

Conventional statistical techniques such as OLS impose time constancy of parameters in an economic model. Time varying coefficient or the problem of structural instability in econometric relationships has been recognized by econometricians (Dusenberry and Klein). Cooley and Prescott argue that it would often be more reasonable to assume that the parameters vary over time. In many instances economic theory suggests that econometric relationships vary over time (Lucas).

The current econometric literature on the relationship between use of captive supply and fed cattle cash market price (e.g., Parcell, Schroeder and Dhuyvetter; Elam; Ward, Koontz, and Schroeder; and Hayenga and O'Brien) assumes time constancy of coefficients. This study was motivated by a conspicuous break in monthly captive supply data (January 1988 to December 2001) around January 1999 and its potential impact on fed cattle cash market price. Captive supply volume remains at about 20% of the total procurement (spot market plus captive supply) from January 1988 to January 1999 and starting around January 1999 it increases to more than 40% in December 2002. Dummy variable analysis and the Chow test confirm a statistically significant structural break in the model at this point. The flexible least squares approach, by explicitly tracing out time paths of coefficient estimates, may provide a useful complement to traditional statistical techniques in investigation of effects of structural changes in a system.

In the next section, time varying linear regression and flexible least squares approach of Kalaba and Testfatsion (1989) is briefly presented. Then, a simple linear model of fed cattle market is specified and structural break tests are performed. The model is then estimated via flexible least squares and the results are discussed. Finally, implications and limitations of the study are noted in the concluding section.

Method

Time Varying Linear Regression Problem

Kalaba and Testfatsion (1989) formulates time varying linear regression problem as follows. Suppose noisy observations $y_1, \cdots, y_T$ over a time-span 1, . . ., T have been generated by a linear regression model with coefficients that evolve only slowly over time, if at all. More precisely, these prior theoretical beliefs are stated as follows:

Measurement specification [linear measurement]:

(1a) $y_t - x_t' b_t \sim 0$, $t = 1, \ldots, T$

Dynamic specification [coefficient stability]:

(1b) $b_{t+1} - b_t \sim 0$, $t = 1, \ldots, T-1$

where $x_t' = (x_{t1}, \cdots, x_{tK})$, 1 × K row vector of known exogenous regressors and

$b_t = (b_{t1}, \cdots, b_{tK})'$, $K \times 1$ column vector of unknown coefficients.

The measurement and dynamic specifications in (1) reflect the prior beliefs of linear measurement and coefficient stability in a simple direct way, without any distributional assumptions about the error term that are required of conventional OLS estimation.

A basic problem is then to determine whether this theory is compatible with the data. That is, can one find a coefficient estimate time path $(b_1, \cdots, b_T)$ so that the theoretical specifications (1) satisfy the realized sequence of observations $(y_1, \cdots, y_T)$ in an acceptable approximate sense? The flexible least squares approach provides a means for finding such coefficient estimate sequence.

The Flexible Least Squares Approach

In many linear regression applications in the natural and social sciences the coefficients evolve slowly over time (Kalaba and Testfatsion, 1989). In such cases one can think of model specification error arising from two sources for each choice of an estimate time sequence vector $b = (b_1, \cdots, b_s)$: residual measurement error given by the discrepancy between the observed dependent variable y_t and the estimated linear regression model $x_t' b_t$ at each time *t* and residual dynamic error given by the discrepancy $[b_{t+1} - b_t]$ between the coefficient vector estimates for each successive pair of times *t* and *t* + 1.

Kalaba and Testfatsion (1989) defines the flexible least squares solution as the collection of all coefficient sequence estimates **b** which yield vector-minimal sums of squared measurement and dynamic errors for the given observations -- that is, which attain the "residual efficiency frontier". This is analogous to the usual Pareto-efficiency frontiers which characterize the efficient attainable trade-offs between two quantities. In the flexible least squares context, the frontier reveals the cost in terms of measurement error that must be paid to reduce the dynamic error.

In model (1), a coefficient estimate sequence **b** could fail to satisfy the measurement specification (1a) and/or the dynamic specification (1b). Let the cost from the first type of error be measured by the sum of squared residual measurement errors

$$\text{(2a)} \qquad r_M^2(b;T) = \sum_{t=1}^{T} [y_t - x_t' b_t]^2$$

and the cost from for the second type of error be measured by the sum of squared residual dynamic errors

$$\text{(2b)} \qquad r_D^2(b;T) = \sum_{t=1}^{T-1} [b_{t+1} - b_t]'[b_{t+1} - b_t]$$

Then the (time *T) residual possibility set* is defined as the collection

$$(3) \qquad P(T) = \{r_D^2(b;T), r_M^2(b;T) \mid b \in E^{TK}\}$$

of all possible combinations of (2a) and (2b) attainable at time *T*, conditional on the given observations $y_1, \cdots, y_T$. The residual possibility set is depicted in figure la.

The lower envelope of the residual possibility set represents the locus of vector-minimal sums of squared residual dynamic and measurement errors attainable at time *T*, conditional on the given observations (figure 1b). This lower envelope, denoted by $P_F(T)$, will be referred to as the (time *T*) *residual efficiency frontier* and reveals the cost in terms of residual measurement error that must be paid in order to achieve the *zero* residual dynamic error (time-constant coefficients) required by OLS estimation. The coefficient sequence estimates **b** which attain this frontier are referred to as *FLS estimates.* For the given observations, the FLS estimates are the coefficient sequence estimates which are *minimally incompatible* with the linear measurement and coefficient stability specifications of (1). Thus, formally, the flexible least squares estimation is finding the coefficient sequence estimates b which minimizes the following "incompatibility cost function."

$$(4) \qquad C(b;\mu,T) = \delta\sum_{t=1}^{T-1}[b_{t+1} - b_t]'[b_{t+1} - b_t] + \sum_{t=1}^{T}[y_t - x_t'b_t]^2$$

where δ is the weight factor that assigns a relative priority to the two priors in the model specification (1). The OLS can be viewed as a limiting case of FLS in which absolute priority is given to the dynamic prior (1b) over the measurement prior (1a) (Kalaba and Testfatsion, 1989, *Theorem 6.1*). The OLS solution can also be interpreted as a particular way of aggregating the information embodied in the FLS estimates $(b_1, \cdots, b_T)$. Therefore, a key difference between FLS and OLS is that the FLS approach seeks to understand which coefficient vector actually obtained at each time *t* and the OLS approach seeks to understand which coefficient vector obtained on average over time (Kalaba and Testfatsion, 1989, *Theorem 6.2*).

Data

The monthly data from January 1988 to December 2001 for the following variables were used. Since Nebraska tended to be the center for price discovery for the major cattle feeding region including Texas/Oklahoma, Kansas, Nebraska, Colorado, and Iowa/Minnesota (Ward), the Nebraska steer prices (Slaughter Steer Price, Choice 2-4, Nebraska Direct, 1100-1300 lb, USDA_AMS) were used as the fed cattle cash market prices. Captive supply data are from the USDA_GIPSA. Captive supply is the sum of the cattle fed by a packer, procured through forward contracts and marketing agreements, and the cattle that are otherwise committed to a packer more than 14 days prior to slaughter as a percentage of the total slaughter for the four largest packing firms. Boxed beef prices are the Wholesale Boxed Beef Cut-Out Value, Choice 1-3, Central U.S., 600-750 lb. from USDA_AMS. Fed cattle futures prices were obtained from the Knight-Ridder.

Model

Model 1is a base model representing the fed cattle cash market. Model 2 is a model with an intercept dummy, Model 3 is a slope dummy model, and Model 4 contains both dummies. Presence of a structural break in the model is confirmed by the dummy variable analysis and the Chow test.

Model 1 - Base model:

$$(5) \qquad Log(FEDP_t) = \beta_{11} + \beta_{21} Log(CS_t) + \beta_{31} Log(BOXP_t) + \beta_{41} Log(FP_t) + \varepsilon_t$$

where, *FEDP* = deflated monthly fed cattle cash market price (\$/cwt), *CS* = monthly captive supply (%), *BOXP* = deflated monthly boxed beef price (\$/cwt), *FP* = deflated monthly fed cattle futures price (\$/cwt), and ε_t = disturbance term.

The sample is divided into two parts at January 1999. This point was selected because of a distinct break in the captive supply time series at this point. Also, this point approximately coincides with the enactment of the Livestock Mandatory Reporting Act of 1999).

$$\text{(6)} \qquad \mathrm{D_t} = \begin{cases} 0 & \text{if t = Jan,88} \sim \text{Dec,98} \\ 1 & \text{if t = Jan,99} \sim \text{Dec,01} \end{cases}$$

Model 2 - Intercept dummy:

$$\text{(7)} \qquad \begin{aligned} Log(FEDP_t) = \beta_{12} + \delta_{12}D_t + \beta_{22}Log(CS_t) \\ + \beta_{32}Log(BOXP_t) + \beta_{42}Log(FP_t) + \varepsilon_t \end{aligned}$$

The dummy variable $\delta_{12}D_t$ allows for potentially different intercept terms in the two sample partitions.

Model 3 - Slope dummy for captive supply:

$$\text{(8)} \qquad \begin{aligned} Log(FEDP_t) = \beta_{13} + \beta_{23}Log(CS_t) + \delta_{23}Log(CS_t)D_t \\ + \beta_{33}Log(BOXP_t) + \beta_{43}Log(FP_t) + \varepsilon_t \end{aligned}$$

The dummy variable $\delta_{23}D_t$ allows for potentially different slope parameters in the two sample partitions

Model 4 - Both dummy variables:

$$\text{(9)} \qquad \begin{aligned} Log(FEDP_t) = \beta_{14} + \delta_{14}D_t + \beta_{24}Log(CS_t) + \delta_{24}Log(CS_t)D_t \\ + \beta_{34}Log(BOXP_t) + \beta_{44}Log(FP_t) + \varepsilon_t \end{aligned}$$

The Durbin Watson and other tests revealed a second order autocorrelation in the disturbance term for all models and the models are estimated using Cochrane-Orcutt procedure to correct the autocorrelation problem.

As reported in table 1, Model 3 with a slope dummy was the best fit according to the log (AIC). Estimated captive supply coefficient is -0.0118 during the first sub-period (January 1988 to December 1998), but -0.0251 (= -0.0134-0.0118) for the second sub-period (January 1999 to December 2001). The Chow tests performed on the base model (Model 1) and the two sub-models resulting from a break at January 1999 further confirm a presence of a structural break. The base model (Model 1) is estimated using the FLS procedure in SHAZAM and results are discussed below.

Results and Discussion

The residual efficiency frontier is graphed in figure 2. The shape of the residual efficiency frontier can provide a qualitative indication of whether or not the OLS solution provides a good description of the observations. If the true model generating the observations has time-constant coefficients, then, the frontier should be rather flat in a neighborhood of the OLS extreme point in the $r_D^2 - r_M^2$ plane. That is, the cost that must be paid in terms of measurement error is small for even large decreases in dynamic error in this neighborhood. On the other hand, if the true model generating the observations has time-varying coefficients, the frontier should be fairly steeply sloped in a neighborhood of the OLS extreme point because large increases in measurement error are required for small decreases in dynamic error. In this case the OLS solution is unlikely to provide a good description of the given observations (Tesfatsion and Veitch). In figure 2, the efficiency frontier for the fed cattle market is quite steeply sloped in a neighborhood of the OLS extreme point indicating that the OLS estimate would not be compatible with the given data.

The FLS estimation results for the alternative values of δ, along with summary statistics are shown in table 2. The standard deviation of the FLS kth coefficient estimates provides a summary measure of the extent to which these estimates deviate from constancy. For example, for δ = 0.5, the standard deviation of the FLS captive supply coefficient sequence is 0.00941 or the coefficient of variation (CV) is -47.75% as shown in table 2. A large CV implies large time variation in the coefficient and in this case, time constancy assumption of OLS may not be appropriate. The coefficients of variation for the FLS coefficient estimates of boxed beef price and futures price are 0.70% and 5.04%, respectively, indicating much less time variation than captive supply coefficient. Thus, the summary statistics of the FLS estimates can be used to assess the extent to which the OLS solution is representative of the typical FLS estimates along the frontier (Tesfatsion and Veitch).

Time paths of coefficient estimates are shown in figures 3a to 3c. The captive supply estimate sequence (figure 3a) indicates that the price impact of captive supply starts to accelerate starting around the mid-1993. Since our model is specified as log-log, the coefficients are directly interpreted as elasticities. Thus, a 1% increase in captive supply is associated with approximately 0.01% and 0.04% decrease in price in the beginning and end of the sample period, respectively (400% increase). Price effect of boxed beef price shows a slight (2%) decline over the sample period (figure 3b) and that of fed cattle futures price shows a modest (10%) increase over the sample period (figure 3c).

In this study, price impact of a 1% increase in captive supply ranges from \$0.01/cwt to \$0.02/cwt over the sample period. Ward, Koontz, and Schroeder estimated that a 1% increase in captive supply cattle was associated with less than 1% decrease in spot market prices. Elam found that price reductions range from \$0.15/cwt to \$0.37/cwt. Parcell, Schroeder and Dhuyvetter estimated that a 1% increase in captive supply shipment was associated with a \$0.02/cwt and \$0.03/cwt reduction in basis (cash price minus futures price) in Colorado and Texas. In this study, the basis decreases on average by \$0.01/cwt for the national fed cattle market. Thus, the results of this study are in general agreement with the literature. Even with the four fold increase over the sample period, the magnitude of impact of captive supply on fed cattle cash market price is very small compared to that of boxed beef price, for example.

Dramatic structural changes in cattle market in recent years might have contributed to the time variations in coefficients. However, the mechanisms that might explain the time paths of variables may be difficult because of the complexities of the operation of fed cattle market. Although the FLS estimate sequence does not explain *how* it happens, it traces out the net *effects* of structural changes in the fed cattle market. The time path of captive supply coefficient sheds light on the time varying price impact of captive supply since the time of the structural break.

The structural shift may have been caused by the Livestock Mandatory Reporting Act of 1999 and/or other forces in and out of the fed cattle market. For example, packers might increase the use of captive supply in anticipation of the potential increase in feeder bargaining power that might result from the increased market information made available by LMR. Whatever the cause(s) of the structural break might be, as shown in this study, even the four fold increase in the price impact of captive supply is relatively small compared to the effects of other variables.

Conclusion

The essence of the FLS approach can be viewed as follows. Economists (Dusenberry and Klein; Cooley and Prescott; Lucas) find it reasonable for parameters in an econometric model to change over time. For example, in a consumption function $y = x'\beta + \varepsilon$, where y is consumption and x' is a vector of variables that influence consumption such as disposable income, and β is the coefficient vector. The marginal propensity to consume, the coefficient for the disposable income, for instance, may change over time due to structural changes in the economy such as policy shifts or changes in consumer preference. The information on net effects of these structural changes on the system is captured in time series data on the variables of the system. The FLS approach makes use of such information in tracing out time paths of estimated coefficients of a system. Thus, the approach is particularly useful where the potential impacts of the structural changes are not well understood.

The flexible least squares approach formulates the estimation problem as the prior beliefs of linear measurement and coefficient stability in a simple direct way without any distributional assumptions about the error term that are required by conventional OLS estimation. In addition, the FLS approach does not require any assumptions about the motion of the coefficients as required by the Kalman filtering techniques (Kalaba and Tesfatsion, 1990). A visual inspection of the FLS estimate sequence of captive supply coefficient indicates a break around the mid-1993 and a major structural break is confirmed at this point by dummy variable analysis. Based on the time path of captive supply coefficient estimate this break appears to be more significant than the one at January 1999 although the dummy variable analysis indicates both breaks are of the similar magnitude. This is another use of the FLS approach as an exploratory tool in identification of a structural shift.

In the FLS approach, the choice of a value for the weight factor δ is arbitrary without the prior knowledge of the relative importance of measurement and dynamic errors. The value of 0.5 used in this study was selected for the lack of a better choice. Functional forms other than specified in this study with a more complete set of relevant variables (e.g., packing plant utilization rate, etc) may provide more accurate time paths. These limitations require that the results of this study should be interpreted with caution.

References

Conner, J., P. Carstensen, R. McEowen, and N. Harl. *The Ban on Packer Ownership and Feeding of Livestock: Legal and Economic Implications*. Available at www.econ.iastate.edu/research/abstracts/NDN0126.pdf.

Cooley, T., and E. Prescott. "Estimation in the presence of stochastic parameter variation." *Econometrica* 44(1976):167-184.

Dusenberry and Klein. *The Brookings Quarterly Econometric Model of the United States*. Chicago: Rand McNally, 1965.

Elam, E. "Cash Forward Contracting vs. Hedging of Fed Cattle, and the Impact of Cash Contracting on Cash Prices." *Journal of Agricultural and Resource Economics* 17(1992):205-217.

Feuz, D., G. Grimes, M.L. Hayenga, S. Koontz, J.D. Lawrence, W.D. Purcell, T.C. Schroeder and C.E. Ward. *Comments on Economic Impacts of Proposed Legislation to Prohibit Beef and Pork Packer Ownership, Feeding, or Control of Livestock*, January 14, 2002. Available at www.econ.istate.edu/outreach/agriculture/reports/johnsonamendment.pdf.

Hayenga, M.L., and D. O'Brien. "Packer Competition, Forward Contracting Price Impacts, and the Relevant Market for Fed Cattle." In W.D. Purcell, Ed. *Pricing and Coordination in Consolidated Livestock Markets, Captive Supplies, Market Power, IRS Hedging Policy*, Blacksburg, VA: Virginia Tech University, Research Institute on Livestock Pricing, 1992.

Kalaba, R., and L. Tesfatsion. “Time-Varying Linear Regression via Flexible Least Squares.” *Computer and Mathematics with Applications* 17(1989):1215-1245.

Kalaba, R., and L. Tesfatsion. “A Further Note on Flexible Least Squares and Kalman Filtering.” *Journal of Economic Dynamics and Control* 14(1990):183-185.

Lucas, R. "Econometric policy Evaluation: A Critique." *Carnegie-Rochester Conference Series on Public Policy* 1(1976):19-46.

Parcell, J., T. Schroeder, and K. Dhuyvetter. “The Effect of Captive Supply Cattle on a Live Cattle Basis.” Paper presented at Western Agricultural Economics Association Meeting, Reno NV, 1997.

Tesfatsion, L., and J. Veitch. “U.S. Money Demand Instability: A Flexible Least Squares Approach.” *Journal of Economic Dynamics and Control* 14(1990):151-173.

Unites States Department of Agriculture, Grain Inspection, Packers and Stockyards Administration (USDA_GIPSA). *Captive Supply of Cattle and GIPSA’s Reporting of Captive Supply*. January 18, 2002.

Ward, C.E., S.R. Koontz, and T.C. Schroeder. “Impacts from Captive Supplies on Fed Cattle Transaction Prices.” *Journal of Agricultural and Resource Economics* 23(1998):494-514.

Table 1. OLS and Dummy Estimation Results

Dependent variable = *Log*(*FEDP*)

Variable	**Model 1**	**Model 2**	**Model 3**	**Model 4**
Name	**Base**	**Incercept dummy**	**Slope dummy**	**Both**
Constant	-0.4707	-0.5714	-0.5697	-0.5735
	(-1.134)*	(-3.440)*	(-3.441)*	(-3.441)*
d1		**-0.0466**		0.0981
		(-4.551)*		(1.034)
d2			**-0.0134**	-0.0405
			(-4.703)*	(-1.535)
Log(CS)	-0.0185	-0.0126	-0.0118	-0.0107
	(-2.196)*	(-1.172)*	(-2.015)*	(-1.789)
Log(BOXP)	0.7252	0.7769	0.7764	0.7728
	(14.22)*	(16.29)*	(16.36)*	(16.21)*
Log(FP)	0.3215	0.2868	0.2863	0.2905
	(5.480)*	(5.276)*	(5.298)*	(5.346)*
R-Squared	0.9798	0.9816	0.9818	0.9819
Durbin-Watson	1.9254	1.9914	1.9910	1.9908
Loglik value	434.724	442.727	442.386	443.938
Log(AIC)	-7.9749	-8.0567	-8.0645	-8.0592

Values in parentheses are t-values and * indicates the estimate is significant under 5% significance level.

Table 2. FLS estimate summary statistics

δ		Constant	Log(CS)	Log(BOXP)	Log(FP)
0.1	Mean	**0.05903**	**-0.01979**	**0.79513**	**0.11326**
	St.Dev	0.00109	0.00951	0.00570	0.00562
	CV	1.85%	-48.07%	0.72%	4.96%
0.3	Mean	**0.05548**	**-0.01976**	**0.79716**	**0.11185**
	St.Dev	0.00108	0.00948	0.00567	0.00558
	CV	1.94%	-47.95%	0.71%	4.99%
0.5	Mean	**0.04937**	**-0.01971**	**0.80068**	**0.10939**
	St.Dev	0.00106	0.00941	0.00561	0.00551
	CV	2.1%	-47.75%	0.70%	5.04%
0.7	Mean	**0.03637**	**-0.01961**	**0.80834**	**0.10399**
	St.Dev	0.00101	0.00928	0.00547	0.00535
	CV	2.77%	-47.29%	0.68%	5.14%
0.9	Mean	**-0.01232**	**-0.01914**	**0.83806**	**0.08249**
	St.Dev	0.00083	0.00869	0.00499	0.00469
	CV	-6.74%	-45.41%	0.60%	5.68%
0.95	Mean	**-0.06155**	**-0.01845**	**0.86738**	**0.06141**
	St.Dev	0.00068	0.00802	0.00457	0.00398
	CV	-1.11%	-43.48%	0.53%	6.49%
0.99	Mean	**-0.25394**	**-0.01391**	**0.94755**	**0.01604**
	St.Dev	0.00045	0.00554	0.00359	0.00204
	CV	-0.18%	-39.81%	0.38%	12.74%
0.999	Mean	**-0.59449**	**-0.00768**	**1.00650**	**0.02992**
	St.Dev	0.00050	0.00287	0.00256	0.00177
	CV	-0.08%	-37.32%	0.25%	5.91%
0.9999	Mean	**-0.58343**	**-0.02572**	**0.97730**	**0.07389**
	St.Dev	0.00035	0.00157	0.00164	0.00136
	CV	-0.06%	-6.11%	0.17%	1.84%
1.00	Mean	**-0.74397**	**-0.06246**	**0.98442**	**0.13466**
OLS	St.Dev	0	0	0	0
	CV	0	0	0	0

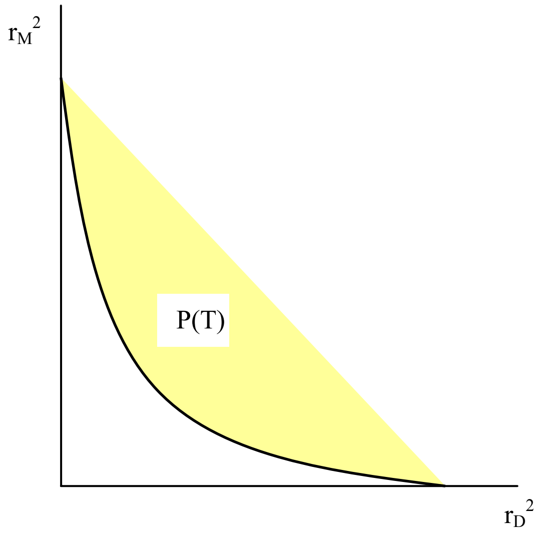

Figure 1a. Residual possibility set *P*(*T*)

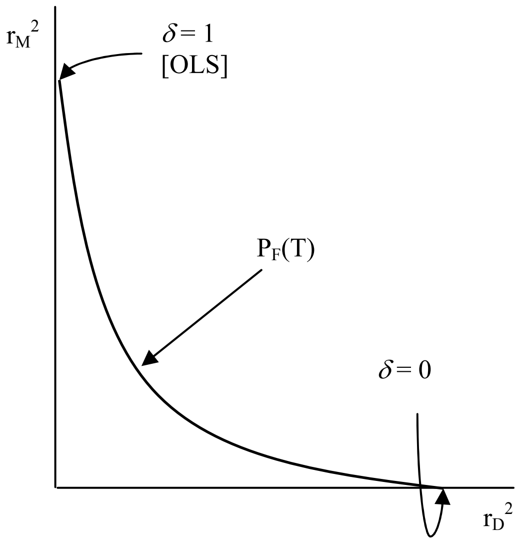

Figure 1b. Residual Efficiency Frontier $P_F(T)$

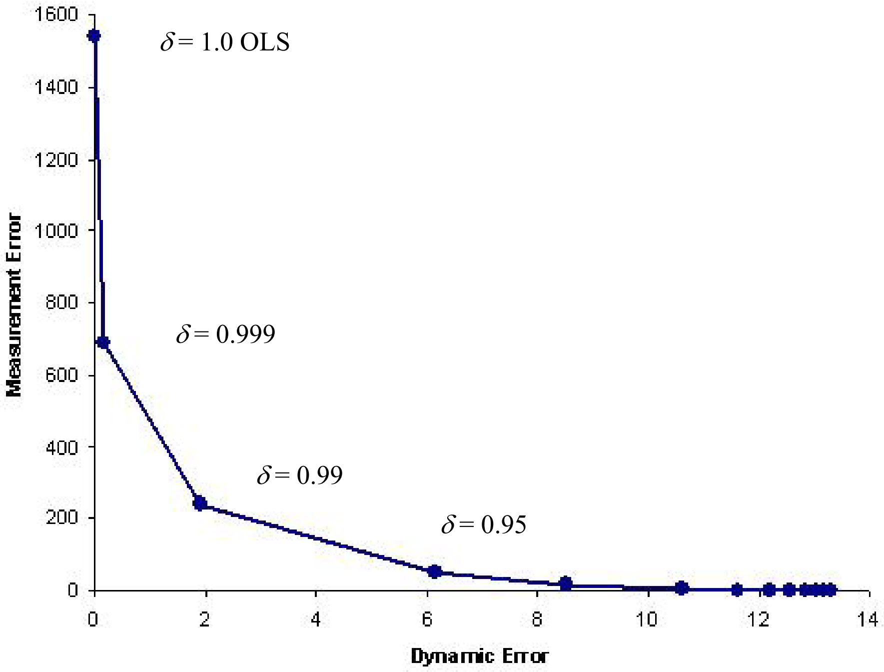

Figure 2. Residual Efficiency Frontier for Captive Supply Model

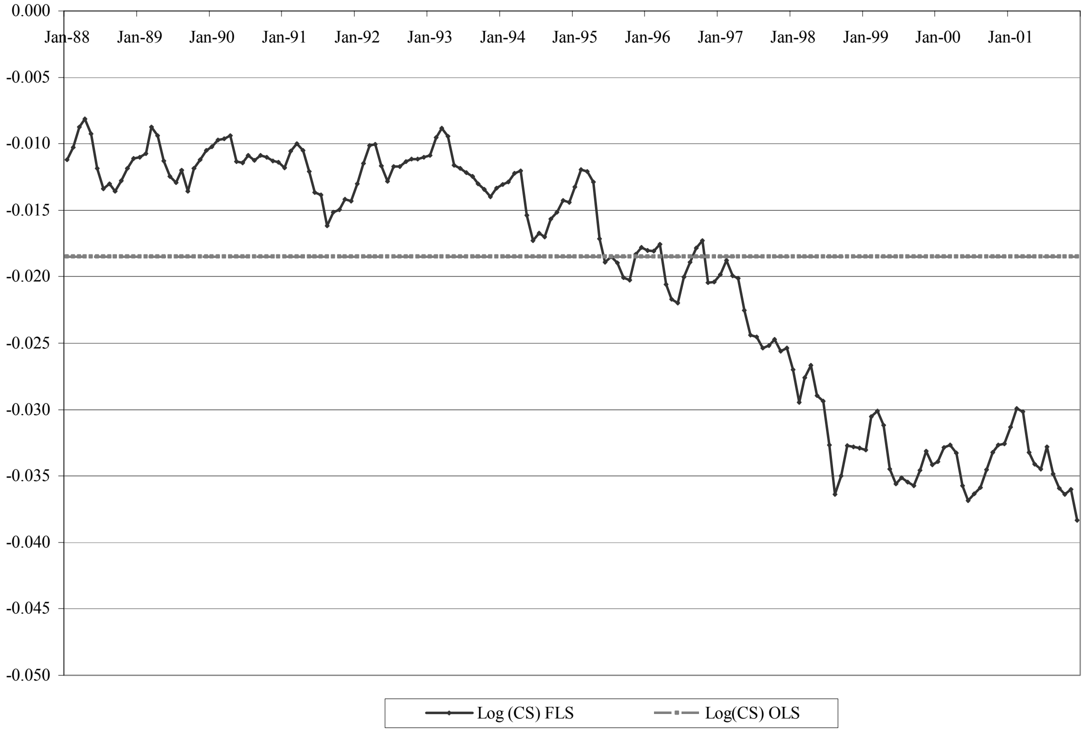

Figure 3a. FLS Time Path and OLS Estimate of Captive Supply Coefficient

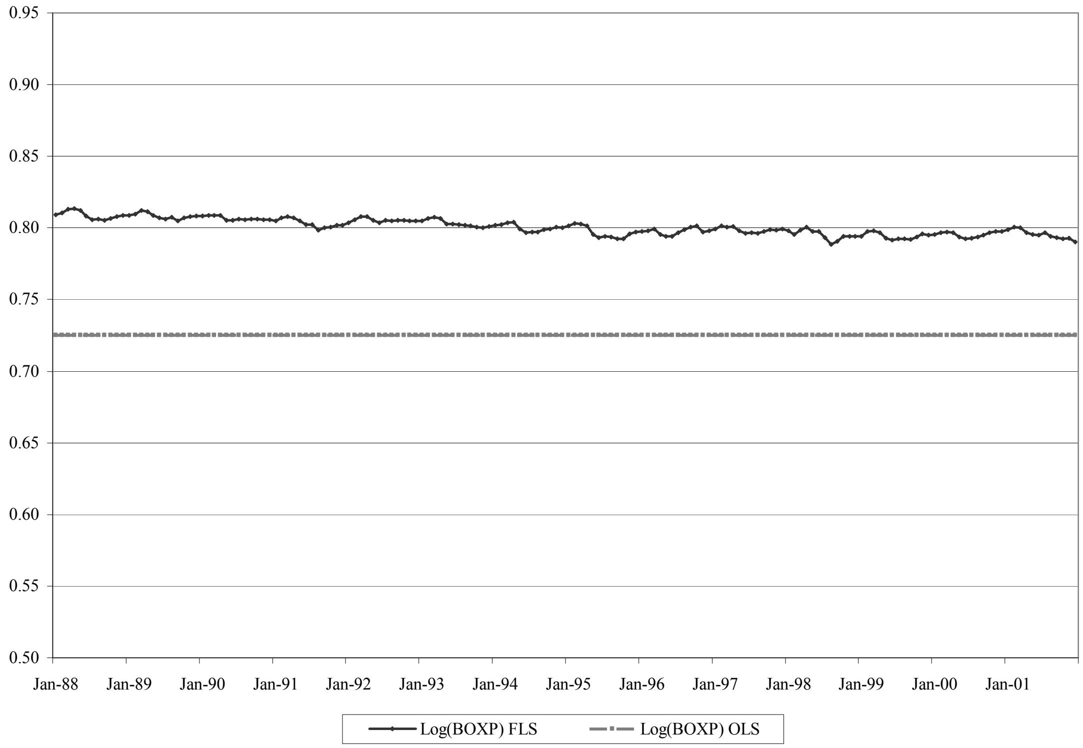

Figure 3b. FLS Time Path and OLS Estimate of Boxed Beef Price Coefficient

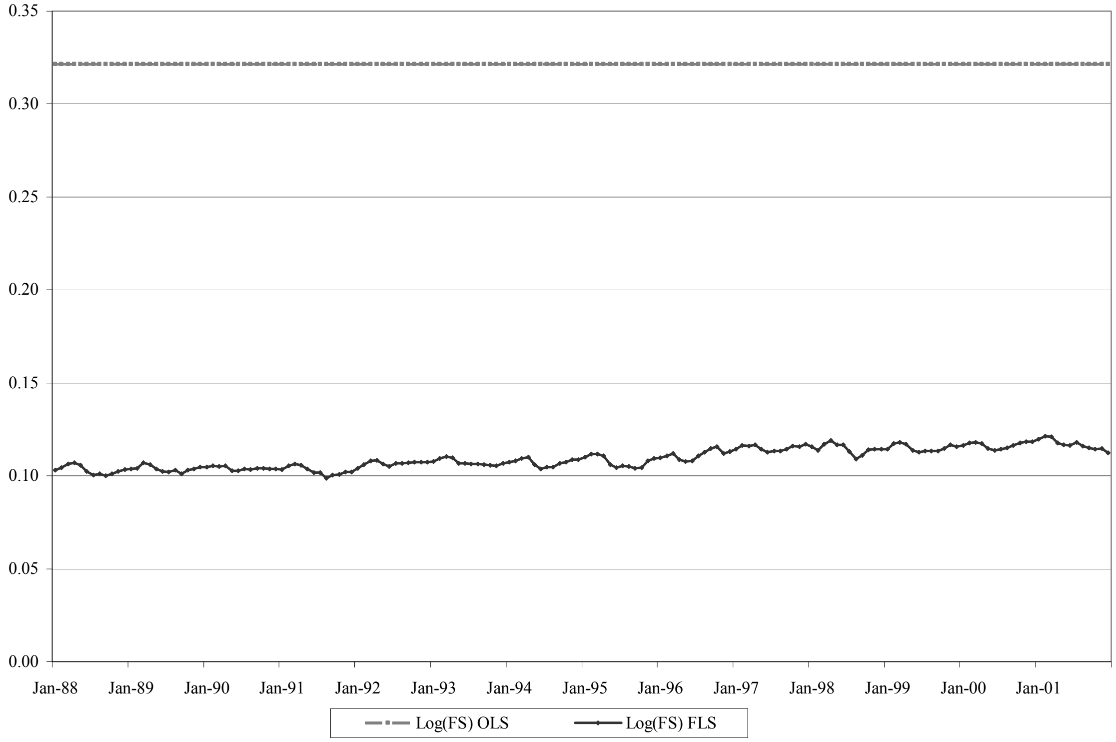

Figure 3c. FLS Time Path and OLS Estimate of Fed Cattle Futures Price Coefficient