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Industrial Structure, Appropriate Technology and Economic Growth in Less Developed Countries

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Abstract

The authors develop an endogenous growth model that combines structural change with repeated product improvement. That is, the technologies in one sector of the model become not only increasingly capital-intensive, but also progressively productive over time. Application of the basic model to less developed economies shows that the (optimal) industrial structure and the (most) appropriate technologies in less developed economies are endogenously determined by their factor endowments. A firm in a less developed country that enters a capital-intensive, advanced industry in a developed country would be nonviable owing to the relative scarcity of capital in the factor endowments of less developed countries.

This paper—a product of the Development Economics Vice Presidency—is part of a larger effort in the World Bank to contribute to a better understanding of the economic and social development process. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The author may be contacted at research@worldbank.org.

Industrial Structure, Appropriate Technology and Economic Growth in Less Developed Countries*

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1. Introduction

Since the Industrial Revolution in the eighteenth century, the world's countries have evolved into two groups. The first group includes rich, industrialized, developed countries (DCs), while the second group includes poor, agrarian, less developed countries (LDCs) (Lin, 2003). Nevertheless, prior to World War II, only a few governments (most notably the Soviet Union) regarded economic growth as their direct responsibility and adopted policies for which economic growth was the primary stated objective, and development economics was not a separate field of study (Krueger, 1995). In the great revival of interest in economic development that has marked the past decade, attention has centered on two main questions: First, what determines the overall rate of economic advance? Second, what is the optimal allocation of given resources to promote growth (Chenery, 1961)? There are two different and occasionally controversial approaches to tackle the questions above, respectively. Analysis of the determinants of the growth rate is the main purpose of modern growth theory, i.e., neoclassical growth theory and recently endogenous growth theory. Efforts to provide solutions to the second question have relied mainly on the principles, e.g., comparative advantage, from trade theory.[1]

According to neoclassical growth theory (e.g., Ramsey, 1928; Solow, 1956; Swan, 1956; Cass, 1965; Koopmans, 1965), which focuses on the process of capital formation with the assumption of the same given technology between LDCs and DCs, LDCs would grow faster than DCs and the gap in per capita income between LDCs and DCs would narrow because of diminishing returns to capital. Furthermore, if the marginal returns to capital continue to fall, the economy will enter a steady state with an unchanging standard of living. These unsatisfying conclusions of neoclassical growth theory have led the current generation of new growth theorists to formulate models in which per capita income grows indefinitely (e.g., Arrow, 1962; Shell, 1967; Romer, 1986, 1990; Lucas, 1988; Jones and Manuelli, 1990; King and Rebelo, 1990; Segerstrom, et al. 1990; Grossman and Helpman 1991a; Rebelo, 1991; Aghion and Howitt, 1992).[2] Regardless of the great contribution that modern growth theory and trade theory have made, neither of them can successfully explain the following economic phenomenon: after World War II, although the governments of many LDCs adopted various policy measures to industrialize their economies, only a small number of economies in East Asia have actually succeeded in raising their level of per capita income to the level in DCs.

Lin (2003, 2007) addressed this problem by providing a reasonable explanation with intrinsic logical consistency. The argument is that the tremendous differences in economic performance among LDCs can be explained largely by their governments' strategies for development. Motivated by the dream of nation building, most LDC governments, both socialist and non-socialist alike, pursued a catch-up type of comparative-advantage-defying (CAD) strategy to accelerate the development of the then advanced capital-intensive industries after World War II (Lin, 2003). The firms in the government's priority industries are not viable in an open, competitive market because these industries do not match the comparative advantage of their particular economy (Lin and Tan, 1999; Lin, 2003). As such, it is imperative for the government to introduce a series of regulations and interventions in international trade, the financial sector, the

[1] The chief criticism is that comparative advantage is essentially a static concept which ignores a variety of dynamic elements (Chenery, 1961).

[2] Please refer to Grossman and Helpman (1994) as well as Barro and Sala-i-Martin (2003) for the details of three approaches to formulate models in which per capita income grows indefinitely.

labor market, and so on so as to mobilize resources for setting up and supporting the continuous operation of non-viable firms (Lin, 2007; Lin and Zhang, 2007a; Lin, et al. 2003). This kind of development mode might be good for mobilizing scarce resources and concentrating on a few clear, well-defined priority sector (Ericson, 1991); but an economy of this type becomes very inefficient as the result of misallocation of resources, rampant rent seeking, macro instability, and so forth (Lin, 2003, 2007). By contrast, an LDC governments, e.g. the newly industrialized economies in Asia and recently China,[3] may pursue a comparative-advantage-following (CAF) strategy. In CAF, the government attempts to induce a firm's entry in a industry according to the economy's existing comparative advantage, and to facilitate the firm's adoption of appropriate technology by borrowing at low cost from the more advanced countries. With this strategy, the economy may enjoy rapid growth that could be greater than that in the DCs owing to the advantage of the latter-comers and the faster upgrades in factor endowments in the LDC (Lin, 2003; Lin and Zhang, 2006; and Zhang, 2006). Thus, the convergence of LDCs with DCs could be realized.

At the Marshall Lectures at Cambridge University on October 31-November 1, 2007, Lin used per capita income as a proxy for the relative abundance of capital and labor in an economy and argued:

> When Japan initiated its automobile-production in the mid-1960s, its per capita income was more than 40 percent of that in the United States. The automobile industry was not the most advanced, capital-intensive industry at that time nor was Japan a capital-scarce economy. Thus, Japan has achieved great success in automotive industry since the mid-1960s. When South Korea instituted an industrial policy for automotive production in the mid-1970s, its per capita income was about only 20 percent of that of the United States and about 30 percent of that of Japan. This could explain why South Korea has achieved a limited degree of success in automotive industry after the mid-1970s. The automotive industries in China and India were started in the 1950s when their per capita income was less than 10 percent of that of the United States. The automobile firms in both countries were not viable; therefore their survival required continuous government protection 30 years after their establishment. (cited in Lin, 2007)

Table 1. Level of Per Capita Income (1990 Geary-Khamis dollars)

	United States	Japan	South Korea	India	China
1955	10,970	2,695	1,197	665	818
1965	14,017	5,771	1.578	785	945
1975	16,060	10,973	3,475	900	1,250

Source: Maddison Angus, Monitoring the World Economy, 1820-1992. Washington, DC: Organization for Economic Cooperation and Development, 1995, pp. 196-205.

The main purpose of the present paper is to develop an endogenous growth model that combines structural change with repeated product improvements to discuss the issues of optimal industrial structure, the (most) appropriate technology, and economic growth in an LDC in a

[3] Hsieh and Klenow (2007) argues that both China and India would get big TFP (Total Factor Productivity) gains from rationalizing allocation of capital and labor in both countries (TFP would double), while China appears to have benefited from recent reform efforts, but India shows little gain.

dynamic general-equilibrium framework. Our paper pertains to work on structural change, i.e., the systematic change in the relative importance of various sectors (e.g., Kuznets, 1957, 1973; Chenery, 1960; Baumol, 1967; Laitner, 2000; Kongsamut, et al. 2001; and Ngai and Pissarides, 2007). In Kongsamut, et al. (2001), the production function of the different sectors, i.e., agriculture, services, and manufacturing, are proportional, while in Ngai and Pissarides (2007), who focuses on exogenous total factor productivity differences across different sectors, all sectors have identical Cobb-Douglas production functions. More closely related to our paper are Acemoglu and Guerrieri (2006, 2008), Zhang (2006), as well as Zuleta and Young (2007). Acemoglu and Guerrieri (2006) first illustrate, when the elasticity of substitution of different products with different capital intensities in the aggregate production function of the final good is not equal to unity, the inevitable outcome of directed technical change is non-balanced growth between different sectors. Zuleta and Young (2007) developed a two-sector model of non-balanced economic growth with induced innovation, in which one sector ("goods" production) has technology differentiated by the elasticity of output with respect to capital and becomes increasingly capital-intensive over time.[4] Zuleta and Young (2007) further assume that although every technology is available at any instant, the adoption of a technology (i.e., innovation) is costly and the cost of innovation is increasing in its capital, thus creating a tradeoff between investment in capital and capital-intensity. In Zuleta and Young (2007), however, both the investment in capital deepening, and the investment to adopt a more capital intensive production function are the results of optimal decisions by an identical firm at any instant, and there is no creative destruction, i.e., more advanced products render previous ones obsolete (Schumpeter, 1942, Segerstrom, et al. 1990, Grossman and Helpman, 1991a, and Aghion and Howitt, 1992).[5]

There are two sectors in the present model, the traditional sector, and the modern sector. Technological change in the traditional sector takes the form of horizontal innovation based on expanding variety (Romer, 1990), while technological progress in the modern sector is accompanied by incessantly creating advanced capital-intensive industry to replace backward labor-intensive industry, which is the distinctive characteristic of the present model. Our paper is the first attempt, to our knowledge, to address structural change with creative destruction, and to simultaneously address the fact that some products in the modern economy – such as the personal computer (PC) – become not only increasingly capita-intensive, but also progressively productive over time.

The results of our model show that the optimal industrial structure in LDCs should not be the same as that in DCs, the (most) appropriate technology adopted in the modern sector in LDCs ought to be inside the technology frontier of the DCs, and the firm in the LDCs that enters capital-intensive, advanced industry in the DCs would be nonviable owing to the relative scarcity of capital in the LDCs' factor endowments. Appropriate technology was first introduced by Atkinson and Stiglitz (1969), and has been revived recently by Diwan and Rodrik (1991) as well as Basu and Weil (1998). Basu and Weil (1998) is the first paper that provides the formal model to discuss appropriate technology in the economic growth framework. The authors argue that technology is specific to particular combinations of inputs, i.e., the capital-labor ratio in their paper. Nevertheless, technological progress in Basu and Weil (1998) is the by-product of

[4] Seater (2005) developed a one sector exogenous growth model with the similar technical change as in Zuleta and Young (2007).
[5] There is no change of capital-intensity in Segerstrom, et al. (1990), Grossman and Helpman (1991a), or Aghion and Howitt (1992).

"localized learning by doing", as introduced by Atkinson and Stiglitz (1969). In the present paper, technological progress in the modern sector accompanied by increased capital intensity in the generation of products requires an intentional investment of resources by profit-seeking firms or entrepreneurs, which is emphasized in Grossman and Helpman (1994). Based on the endogenous growth model with expanding variety, Acemoglu and Zilibotti (2001) argue that many technologies used by LDCs are developed in the OECD economies and are designed to make optimal use of the skills of the work force in the richer countries. Thus, the necessary outcome is low productivity in the LDCs owing to the scarcity of skills in these countries.[6]

The rest of the paper is organized as follows. Section 2 constructs a specific model of endogenous economic growth that combines industrial structural upgrading with creative destruction. In section 3, we investigate the issues of endogeneity of industrial structure and appropriate technology as well as the firm's viability in the LDC based on the dynamic trajectory of the present economy. Section 4 contains some brief concluding remarks. Finally, some details of the model that do not appear in the text are provided in the appendix.

2. The Basic Model

We consider a theoretical world consisting of a DC and an LDC that share the identical demographics but have distinct factor endowment structures,[7] i.e., the relative abundance of capital in the DC at time t_0, denoted by $\bar{K}(t_0)$, and the relative scarcity of capital in the LDC at time t_0, denoted by $\underline{K}(t_0)$. To simplify the analysis, we assume there is no international trade and no capital mobility in the present theoretical world.

2.1 Consumer Behavior

In both the DC and the LDC, there are $L(t)$ workers at time t, supplying their labor without any disutility.[8] The population has a constant exponential growth rate $\tilde{g}$. We also assume that all households share identical constant relative risk aversion (CRRA) preferences over total household consumption index $C(t, j)$, and all population growth takes place in existing households, which implies that the economy admits a representative agent with CRRA preferences:

$$U_\tau = \int_\tau^\infty \frac{C(t, j)^{1-\theta} - 1}{1-\theta} \exp[-\rho(t-\tau)]dt \tag{1}$$

where ρ is a subjective discount rate, $\theta \geq 0$ is the coefficient of relative risk aversion, and

6 There is never a problem of countries using technologies that do not match their level of development or endowment structure in Basu and Weil (1998), while an LDC in Acemoglu and Zilibotti (2001) never has an opportunity to choose his appropriate technology.

7 In the present paper, the upper bar is used as a superscript to indicate the variables in the DC, while the lower bar is an index of the LDC.

8 We suppress time and country indexes when this causes no confusion.

$C(t, j)$ represents an index of consumption (sub-utility function) of j th generation goods at time t. To reflect the household's tastes for diversity in consumption, we adopt for $C(t, j)$ a specification that imposes constant elasticity of substitution (CES) between consumption of traditional goods, denoted by C_1, and consumption of modern goods of j th generation, denoted by $C_2(t, j)$. Specifically, we have

$$C(t, j) = \left[\gamma C_1^{\varepsilon} + (1-\gamma)\left(C_2(j)\right)^{\varepsilon}\right]^{1/\varepsilon}, \quad 0 < \varepsilon < 1 \tag{2}$$

where $\gamma \in (0,1)$ is the share parameter of the two goods above, and $\varepsilon \in (0,1)$ determines the elasticity of substitution between consumption goods in the traditional sector and in the modern sector. It is convenient for us to choose traditional goods as numeraire and denote the price of modern goods of j th generation to be p_j.

The representative consumer maximizes (1) subject to an inter-temporal budget constraint. The consumption optimization problem can be solved in two stages. First, the representative consumer takes price p_j as given and chooses C_1 and $C_2(j)$ to maximize static utility in (2) for a given level of expenditure at time t, denoted by $E(t)$.

$$\max_{C_1, C_2(j)} \left[\gamma C_1^{\varepsilon} + (1-\gamma)\left(C_2(j)\right)^{\varepsilon}\right]^{1/\varepsilon}$$

subject to static budget constraint:

$$C_1 + p_j C_2(j) = E \tag{3}$$

The first-order conditions of the above maximization problem yield the following demand functions for C_1 and $C_2(j)$:

$$C_1 = \frac{E}{1 + p_j \left[\frac{(1-\gamma)}{\gamma p_j}\right]^{\frac{-1}{\varepsilon-1}}} \tag{4}$$

and

$$C_2(j) = \frac{E}{\left[\frac{(1-\gamma)}{\gamma p_j}\right]^{\frac{1}{\varepsilon-1}} + p_j} \tag{5}$$

Substituting (4) and (5) into (2) yields

$$C(j) = E\aleph\left(p_j\right)$$

where $\aleph\left(p_j\right)$ amounts to

$$\left\{\gamma\left[1+p_j\left(\frac{(1-\gamma)}{\gamma p_j}\right)^{\frac{-1}{\varepsilon-1}}\right]^{-\varepsilon}+(1-\gamma)\left[\left(\frac{(1-\gamma)}{\gamma p_j}\right)^{\frac{1}{\varepsilon-1}}+p_j\right]^{-\varepsilon}\right\}^{1/\varepsilon}$$

Substituting $C(j)=E\aleph\left(p_j\right)$ into (1), the representative agent's utility function becomes

$$U_\tau=\int_\tau^\infty\frac{\left[E(t)\aleph\left(p_j\right)\right]^{1-\theta}-1}{1-\theta}\exp[-\rho(t-\tau)]dt \tag{6}$$

The second-stage consumption optimization problem involves choosing the time pattern of expenditures E to maximize (6) subject to the representative consumer's inter-temporal budget constraint:

$$\int_t^\infty\exp[R(t)-R(x)]E(x)dx=B(t) \tag{7}$$

where $R(t)$ is the cumulative nominal interest factor from time 0 to time t, i.e., $R(t)\equiv\int_0^t\exp[r(x)]dx$ with $R(0)\equiv 1$, and $B(t)$ is the representative agent's present value of the stream of factor incomes plus the value of initial asset holding at time t.

The inter-temporal optimization problem of the above representative agent implies the following Euler equation

$$\frac{\dot{E}}{E}=\frac{r(t)-\rho}{\theta} \tag{8}$$

and the transversality condition

$$\lim_{t\to\infty}\exp(-\rho t)\vartheta(t)B(t)=0$$

where $r(t)$ is the nominal interest rate at time t.

Before leaving the consumption side of the economy, it will be useful for our later analysis to consider the relationship of the representative consumer's spending allocated to traditional goods with respect to modern goods. Differentiating (3) with respect to time t yields

$$\frac{\dot{C}_1}{C_1}\frac{C_1}{E}+\frac{p_jC_2(j)}{E}\frac{\dot{C}_2(j)}{C_2(j)}+\frac{p_jC_2(j)}{E}\frac{\dot{p}_j}{p_j}=\frac{\dot{E}}{E}$$

Denote the share of the representative consumer's spending allocated to traditional goods by $s_1\equiv\frac{C_1}{E}$, and it is obvious that we have $1-s_1=\frac{p_jC_2(j)}{E}$, which is the share of the representative consumer's spending allocated to modern goods. Then we have

$$s_1\frac{\dot{C}_1}{C_1}+(1-s_1)\frac{\dot{C}_2(j)}{C_2(j)}+(1-s_1)\frac{\dot{p}_j}{p_j}=\frac{\dot{E}}{E} \tag{9}$$

2.2 Producer Behavior and Static Equilibrium

Turning to the production side, there are only two primary factors of production, capital K and labor L, and two sectors in both the DC and the LDC. One is the traditional sector and the other is the modern sector. We assume the product in the traditional sector, denoted by Y_1, can be used as consumption goods only, while the product in the modern sector, denoted by $Y_2(j)$ which is the product of j th generation, can be consumed by households, installed by firms as capital, or invested by entrepreneurs as R&D expenditures.

2.2.1 Production in the Traditional Sector

We assume that the production of the homogenous goods Y_1 in the competitive traditional sector only requires an index of intermediates D, and the production function of traditional goods is

$$Y_1 = D$$

which is a standard assumption in the economic growth literature.

Following Grossman and Helpman (1991b), the index of intermediates D is represented by

$$D = \left[\int_0^N z(i)^\alpha \, di \right]^{1/\alpha}, \quad 0 < \alpha < 1 \tag{10}$$

where $z(i)$ denotes the input of intermediate good i; N is the number (measure) of available intermediate goods, i.e., the technology in the traditional sector; and α is the elasticity of substitution between different intermediate inputs. At every moment in time, the existing producers of intermediate goods engage in oligopolistic price competition, and intermediate good $z(i)$ is produced with the following Cobb-Douglas production function:

$$z(i) = l(i)^\beta k(i)^{1-\beta}, \quad 0 < \beta < 1 \tag{11}$$

where $l(i)$ and $k(i)$ are labor and capital employed in the production of the existing intermediate good $z(i)$.

Facing the given price of the existing intermediate good $z(i)$, which is denoted by $q(i)$, and the price for the product of the traditional sector, which is normalized to be 1, the inverse demand function for the existing intermediate good $z(i)$ by the competitive firm in the traditional sector is given by:

$$q(i) = (Y_1)^{1-\alpha} z(i)^{\alpha-1} \tag{12}$$

And the profit maximization problem of the existing intermediate firm i can be equivalently written as

$$\underset{l(i),k(i)}{Max}\ (Y_1)^{1-\alpha}.l(i)^{\alpha\beta}k(i)^{\alpha(1-\beta)}-w.l(i)-r.k(i) \tag{13}$$

The first-order conditions in (13) are

$$\alpha\beta(Y_1)^{1-\alpha}.l(i)^{\alpha\beta-1}k(i)^{\alpha(1-\beta)}=w \tag{14}$$

$$\alpha(1-\beta)(Y_1)^{1-\alpha}.l(i)^{\alpha\beta}k(i)^{\alpha(1-\beta)-1}=r \tag{15}$$

Combining (14) with (15) yields the existing intermediate firm i's factor demand functions

$$l(i)=\left[(1-\beta)^{\alpha(1-\beta)}\beta^{(1-\alpha+\alpha\beta)}\ \alpha w^{-(1-\alpha+\alpha\beta)}r^{-(\alpha-\alpha\beta)}(Y_1)^{(1-\alpha)}\right]^{\frac{1}{1-\alpha}} \tag{16}$$

$$k(i)=\frac{w(1-\beta)}{r\beta}\left[(1-\beta)^{\alpha(1-\beta)}\beta^{(1-\alpha+\alpha\beta)}\ \alpha w^{-(1-\alpha+\alpha\beta)}r^{-(\alpha-\alpha\beta)}(Y_1)^{(1-\alpha)}\right]^{\frac{1}{1-\alpha}} \tag{17}$$

Substituting existing intermediate firm i's factor demand functions in (16) and (17) into (12), then $q(i)$, i.e., the price of the existing intermediate good $z(i)$, satisfies

$$q(i)=\alpha^{-1}(1-\beta)^{-(1-\beta)}\beta^{-\beta}\ w^{\beta}r^{1-\beta} \tag{18}$$

Thus, in a symmetric equilibrium, all the existing intermediate firms in the traditional sector would charge the same price and share identical factor demand functions, which implies

$$l(i)=\frac{L_1}{N}\ \text{ and }\ k(i)=\frac{K_1}{N} \tag{19}$$

where L_1 and K_1 are the total amount of labor and capital used in the traditional sector, respectively.

$$L_1=N\left[(1-\beta)^{\alpha(1-\beta)}\beta^{(1-\alpha+\alpha\beta)}\ \alpha w^{-(1-\alpha+\alpha\beta)}r^{-(\alpha-\alpha\beta)}\right]^{\frac{1}{1-\alpha}}Y_1 \tag{20}$$

$$K_1=N\frac{w(1-\beta)}{r\beta}\left[(1-\beta)^{\alpha(1-\beta)}\beta^{(1-\alpha+\alpha\beta)}\ \alpha w^{-(1-\alpha+\alpha\beta)}r^{-(\alpha-\alpha\beta)}\right]^{\frac{1}{1-\alpha}}Y_1 \tag{21}$$

Now the production function of the existing intermediate good $z(i)$ in (11) becomes

$$z(i)=\frac{1}{N}\left(L_1\right)^{\beta}\left(K_1\right)^{1-\beta} \tag{22}$$

and the production function of the traditional sector in (10) could be rewritten as

$$Y_1=N^{\frac{1-\alpha}{\alpha}}\left(L_1\right)^{\beta}\left(K_1\right)^{(1-\beta)} \tag{23}$$

Combining (19) and (23) with (14) and (15) implies the wage rate and interest rate satisfy

$$w=\alpha\beta N^{\frac{1-\alpha}{\alpha}}\left(K_1/L_1\right)^{1-\beta} \tag{24}$$

$$r=\alpha(1-\beta)N^{\frac{1-\alpha}{\alpha}}\left(K_1/L_1\right)^{-\beta} \tag{25}$$

Substituting (24) and (25) into (13), the profit function of the existing intermediate firm i in the traditional sector can be obtained by

$$\pi_1(i) = (1-\alpha)N^{\frac{1-2\alpha}{\alpha}}\left(L_1\right)^{\beta}\left(K_1\right)^{1-\beta} \tag{26}$$

As in Judd (1985) and Romer (1990), we also assume that production of a new intermediate good requires R&D expenditures X_1 in terms of the modern goods devoted to the invention of a new blueprint. Moreover, we also assume that process innovation outlays are made by private, profit-making entrepreneurs, who receive indefinite patent protection and will appropriate some of the benefits from a new process innovation in the form of oligopoly profits. The oligopolistic entrepreneur of intermediate firm i in the traditional sector's present value of future operating profits from producing $z(i)$ discounted to time t is given by

$$V_1(i,t) = \int_t^{\infty} \exp[R(t)-R(x)]\pi_1(i,x)dx$$

where $\pi_1(i,x)$ is the flow profits of firm i from producing intermediate good $z(i)$ in the traditional sector, which is expressed by (26) at time x.

Differentiating $V_1(i,t)$ with respect to time t yields

$$\frac{\dot{V}_1(i,t)}{V_1(i,t)} = r - \frac{\pi_1(i,t)}{V_1(i,t)} \tag{27}$$

With the spillover effect from the current stock of knowledge in the traditional sector to future process innovations emphasized in Romer (1990) in mind, we assume that if X_1 units of modern goods engage in research in the traditional sector, they generate a flow of new products $\dot{N}$ given by

$$\dot{N} = b_1 N^{-\varphi_1} X_1$$

where b_1 is a strictly positive constant measuring the technical difficulty of creating new blueprints in the traditional sector, and $\varphi_1 \in (-1,\infty)$ measures the degree of spillovers in technology creation.

Then, with free entry by the intermediate firm i, if there are positive but finite resources devoted to R&D in the traditional sector at time t, we must have the zero-profit condition for firm i as

$$V_1(i,t) = p_j \frac{N^{\varphi_1}}{b_1} \tag{28}$$

2.2.2 Production in the Modern Sector

Producing the product in the modern sector also requires variable inputs capital K and labor L, but not intermediates. The production function of the product of j th generation in the modern sector is given by

$$Y_2(j) = \mathrm{F}_2[A_2(j), K, L] = A_2(j) K_2^{\delta_j} L_2^{1-\delta_j}, \quad 1-\beta < \delta_j \le \tilde{\delta}$$

where $\tilde{\delta}$ is an exogenously given parameter that satisfies $\tilde{\delta} < 1$, K_2, L_2 and δ_j is capital and labor used, as well as the capital intensity in the modern sector for the product of j th generation, respectively, and $A_2(j)$ is the productivity of the $j = 1, 2, \ldots$ th generation product. The parameters in the present paper that satisfy $1-\beta < \delta_j < \tilde{\delta}$ imply that the modern sector is more capital-intensive than the traditional sector at any moment.

Following the literature on horizontal innovation or creative destruction (e.g., Segerstrom, et al., 1990; Grossman and Helpman, 1991a; as well as Aghion and Howitt, 1992), we assume the productivity of the $j = 1, 2, \ldots$ th generation in the modern sector, denoted by $A_2(j)$, is exactly λ times that of the generation before it. That is, we have

$$A_2(j) = \lambda A_2(j-1)$$

where λ is an exogenously given constant that satisfies $\lambda > 1$. We choose units so that the productivity of the lowest generation with $j = 0$, i.e., the one available at time $t = 0$ (the starting point of the analysis), is equal to unity; that is we assume $A_2(0) = 1$.[9]

In contrast with the literature on horizontal innovation or creative destruction mentioned above – which focus on productivity (or product quality) rising only – the present paper embodies product innovation in technological progress that is incessantly capital intensive and progressively productive over time. We assume that the relationship between capital intensity δ_j of the generation $j = 1, 2, \ldots$ with that of generation $j-1$ satisfies the following condition for simplicity

$$\delta_j = \delta_{j-1} + b_2(\tilde{\delta} - \delta_{j-1})^{\varphi_3} \tag{29}$$

where $b_2 > 0$ and $\varphi_3 > 0$.[10]

Now the production function in the modern sector could be rewritten as

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[9] As the starting point of the analysis, we assume that the modern sector begins at time $t = 0$ with one firm which has access to a universally known backstop technology in the perfectly competitive output market and factor market until the first generation product is invented.

[10] Equation (29) implies in infinite horizon we have $\lim_{t\to\infty} \delta_{j(t)} = \lim_{j\to\infty} \delta_j = \tilde{\delta}$.

$$Y_2(j) = \mathrm{F}_2[A_2(j), K, L] = \lambda^j K_2^{\delta_j} L_2^{1-\delta_j} \tag{30}$$

Of course, more advanced products, i.e., products with higher productivity and rising capital intensity, could not be produced until they have been invented. We follow the approaches taken in Aghion and Howitt (1992) as well as in chapter 4 of Grossman and Helpman (1991b), and assume that research in the modern sector produces a random sequence of product innovations. Any firm in the modern sector that carries out R&D at intensity ι for a time interval of length dt will succeed in its attempt to develop the product of generation $j = 1, 2, \ldots$ based on the existing generation before it with probability ιdt which follows a Poisson distribution. R&D expenditures per unit of time in this activity are $X_2 \equiv (\lambda^j)^{\varphi_2} \iota$ in terms of modern goods when the entrepreneur attempts to develop the product, where $\varphi_2 > 1/(1-\tilde{\delta}) > 1$ reflects the fact that the more advanced the technology in modern sector, the more R&D expenditures are needed for further product innovation in this sector. Furthermore, we assume the parameters of the present model satisfy $\alpha\beta(1+\varphi_1) = \varphi_2(1-\tilde{\delta})(1-\alpha)$ to guarantee balanced growth between the traditional sector and the modern sector on the infinite horizon.[11]

Once the product of generation $j = 1, 2, \ldots$ has been invented in the research lab, the successful innovator obtains a patent that is assumed to last forever on condition that no new generation has been invented; otherwise, the present generation product in the modern sector will be replaced by the next generation/vintage. And the producers with the requisite know-how and patent rights can manufacture the product of j th generation in the modern sector according to the production function in (30). We assume that all firms in the modern sector engage in price competition in the output market and are price-takers in the factor market, and we also assume that only one leader-firm, e.g., firm j, in the modern sector has access to the state-of-the-art technology. Another firm, a follower-firm, i.e., firm $j-1$, masters the technology that is one step behind it.

For the moment, we assume innovations are always drastic, which means the successful innovator is unconstrained by potential competition from the previous patent.[12]

From (4) and (5), the inverse demand function faced by a monopolistic firm j in the modern sector charging price p_j can be solved as follows:

$$p_j = \frac{(1-\gamma)\left(C_2(j)\right)^{\varepsilon-1}}{\gamma C_1^{\varepsilon-1}}$$

Let us denote the fraction of modern goods of the j th generation consumed by households

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[11] When $\alpha\beta(1+\varphi_1) \neq \varphi_2(1-\tilde{\delta})(1-\alpha)$, non-balanced growth between the traditional sector and the modern sector implies that either the traditional sector or the modern sector will be trivial compared to the other sector in infinite horizon.

[12] Please see Lin and Zhang (2007b) for the details of the case of nondrastic innovations.

by $\mu_j \equiv \dfrac{C_2(j)}{Y_2(j)}$,[13] then we have

$$p_j = \frac{(1-\gamma)\left(\mu_j Y_2(j)\right)^{\varepsilon-1}}{\gamma C_1^{\varepsilon-1}} \tag{31}$$

Facing the given inverse demand function in (31), given factor prices w_j and r_j, the firm j in the modern sector will choose K_2 and L_2 to maximize profit, given by:

$$\pi_2(j) = \frac{(1-\gamma)\left(\mu_j\right)^{\varepsilon-1}}{\gamma C_1^{\varepsilon-1}}\left(\lambda^j K_2^{\delta_j} L_2^{1-\delta_j}\right)^{\varepsilon} - r_j K_2 - w_j L_2 \tag{32}$$

The first-order conditions in (32) are

$$\frac{(1-\gamma)\left(\mu_j\right)^{\varepsilon-1}\lambda^{j\varepsilon}}{\gamma C_1^{\varepsilon-1}}\varepsilon\delta_j K_2^{\varepsilon\delta_j-1} L_2^{\varepsilon(1-\delta_j)} = r_j \tag{33}$$

$$\frac{(1-\gamma)\left(\mu_j\right)^{\varepsilon-1}\lambda^{j\varepsilon}}{\gamma C_1^{\varepsilon-1}}\varepsilon(1-\delta_j) K_2^{\varepsilon\delta_j} L_2^{\varepsilon(1-\delta_j)-1} = w_j \tag{34}$$

Combining (33) with (34) implies that the factor demand functions of firm j in the modern sector should satisfy

$$L_2 = \left\{\frac{(1-\gamma)\left(\mu_j\right)^{\varepsilon-1}\lambda^{j\varepsilon}}{\gamma C_1^{\varepsilon-1}}\varepsilon(1-\delta_j)^{(1-\varepsilon\delta_j)}(\delta_j)^{\varepsilon\delta_j}(w_j)^{-(1-\varepsilon\delta_j)}(r_j)^{-\varepsilon\delta_j}\right\}^{\frac{1}{1-\varepsilon}} \tag{35}$$

$$K_2 = \frac{\delta_j w_j}{(1-\delta_j) r_j} L_2 \tag{36}$$

Substituting (35) and (36) into (32), we can solve the profit function of firm j in the modern sector as follows

$$\pi_2(j) = (1-\varepsilon)\frac{(1-\gamma)\left(\mu_j\right)^{\varepsilon-1}\lambda^{j\varepsilon}}{\gamma C_1^{\varepsilon-1}}\left(K_2^{\delta_j} L_2^{1-\delta_j}\right)^{\varepsilon} \tag{37}$$

And the price of the product of the j th generation in the modern sector is determined by

$$p_j = \lambda^{-j}\varepsilon^{-1}\left(\delta_j\right)^{-\delta_j}(1-\delta_j)^{-(1-\delta_j)}(w_j)^{(1-\delta_j)}(r_j)^{\delta_j} \tag{38}$$

At time t, the value to an outside research firm j that aims to develop a product whose productivity is λ times as great as the state of the art and carries out R&D at intensity ι when this firm is successful in the j th product innovation, which is denoted by $V_2(j,t)$, is the expected present value of the flow of monopoly profits $\pi_2(j,x)$ discounted to time t, where

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[13] It is obvious that $1-\mu_j$ denotes the savings rate in the present model.

the duration of $\pi_2(j,x)$ follows the exponential distribution with parameter ιx:

$$V_2(j,t)=\int_t^{\infty}\exp[R(t)-R(x)]\pi_2(j,x)\prod(j,x)dx$$

where $\prod(j,x)$ equals the probability that there will be exactly j innovations from the starting point to time x, thus, we have

$$\prod(j,x)=\frac{(\iota x)^j e^{-\iota x}}{j!}$$

Newcomer firm j in the modern sector would choose research intensity ι for a time interval of length dt to maximize

$$\max_{\iota} V_2(j,t)\iota dt-p_j(t)(\lambda^j)^{\varphi_2}\iota dt \tag{39}$$

The maximization problem in (39) implies

$$p_j(t)(\lambda^j)^{\varphi_2}\geq V_2(j,t),\ \ \iota\geq 0 \text{ and } \left[p_j(t)(\lambda^j)^{\varphi_2}-V_2(j,t)\right]\iota=0$$

Thus, as long as the R&D operates at a positive but finite scale, we must have $\iota>0$, and $p_j(t)(\lambda^j)^{\varphi_2}=V_2(j,t)$. And the variation of the value to an outside research firm j discounted to time t, denoted by $\dot{V}_2(j,t)$, can be expressed as

$$\frac{\dot{V}_2(j,t)}{V_2(j,t)}=r+\iota-\frac{\pi_2(j,t)}{V_2(j,t)} \tag{40}$$

2.3 Market Clearing Conditions

We close the model by describing market clearing conditions. The output market clearing condition in the traditional sector implies

$$C_1=Y_1$$

If we neglect capital depreciation in our model for simplicity, then the output market clearing condition in the modern sector is:

$$C_2(j)+\dot{K}+X_1+X_2=Y_2(j)$$

According to the analysis above, the factor market clearing conditions can be expressed as:

$$L_1+L_2=L$$

$$K_1+K_2=K$$

where L_1 (K_1) and L_2 (K_2) denotes the levels of labor (capital) used in the traditional and modern sectors, respectively. It is convenient for the analysis below to denote the fraction of labor and capital used in the traditional sector by $\kappa_L = \dfrac{L_1}{L_1 + L_2}$ and $\kappa_K = \dfrac{K_1}{K_1 + K_2}$. From (20), (21), (35), and (36), we find

$$\kappa_L = \frac{1}{1 + \dfrac{\left[\dfrac{1}{\gamma}(1-\gamma)\left(\mu_j\right)^{\varepsilon-1} \lambda^{j\varepsilon} \varepsilon (1-\delta_j)^{(1-\varepsilon\delta_j)} (\delta_j)^{\varepsilon\delta_j} (w_j)^{-(1-\varepsilon\delta_j)} (r_j)^{-\varepsilon\delta_j} \right]^{\frac{1}{1-\varepsilon}}}{N\left[(1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \ \alpha w^{-(1-\alpha+\alpha\beta)} r^{-(\alpha-\alpha\beta)}\right]^{\frac{1}{1-\alpha}}}} \tag{41}$$

and

$$\kappa_K = \frac{1}{1 + \dfrac{\left[\dfrac{1}{\gamma}(1-\gamma)\left(\mu_j\right)^{\varepsilon-1} \lambda^{j\varepsilon} \varepsilon (1-\delta_j)^{(1-\varepsilon\delta_j)} (\delta_j)^{\varepsilon\delta_j} (w_j)^{-(1-\varepsilon\delta_j)} (r_j)^{-\varepsilon\delta_j} \right]^{\frac{1}{1-\varepsilon}}}{\dfrac{w(1-\beta)}{r\beta} \dfrac{(1-\delta_j) r_j}{\delta_j w_j} N\left[(1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \ \alpha w^{-(1-\alpha+\alpha\beta)} r^{-(\alpha-\alpha\beta)}\right]^{\frac{1}{1-\alpha}}}}$$

Since the traditional sector and the modern sector share the identical factor prices in equilibrium, we have

$$\kappa_K = \frac{\kappa_L}{\kappa_L + \xi_j (1-\kappa_L)} \tag{42}$$

where $\xi_j \equiv \dfrac{\delta_j \beta}{(1-\delta_j)(1-\beta)}$. Thus, κ_L could characterize the industrial structure in the present model.

2.4 Dynamic Equilibrium with an Infinite Horizon

The dynamic equilibrium in this economy is given by paths for prices of factors, intermediates and modern goods w, r, $[q(i)]_{i=1}^{N}$, p, allocations of factors L_1, L_2, K_1, K_2, as well as R&D expenditures X_1, X_2 such that producers maximize profits, and the representative consumer chooses consumption and savings C_1, C_2 and E to maximize his utility under the market clearing conditions. It is convenient for us to study equilibrium with an infinite horizon first, and then characterize the dynamic trajectory of the present economy. As regards equilibrium with an infinite horizon, we have the following prosposition (see the Appendix for the proof).

Proposition 1: There exists a constant growth equilibrium (CGE) in the present economy with an infinite horizon, which means consumer expenditures E grow at a constant rate g_E^*

with an infinite horizon, i.e., $\lim_{t\to\infty}\frac{\dot{E}}{E}=g_E^*$, and the interest rate in CGE is also a constant, i.e., $\lim_{t\to\infty} r\equiv r^*=\theta g_E^*+\rho$. Furthermore, $\alpha\beta(1+\varphi_1)=\varphi_2(1-\tilde{\delta})(1-\alpha)$ implies that the CGE is also a unique balanced growth equilibrium (BGE), such that the modern sector and the traditional sector grow at the same constant rate with an infinite horizon, where $\lim_{t\to\infty}\frac{\dot{Y}_1}{Y_1}\equiv g_{Y_1}^*=\frac{\alpha\beta(1+\varphi_1)}{\alpha\beta(1+\varphi_1)-(1-\alpha)}\tilde{g}$, $\lim_{t\to\infty}\frac{\dot{Y}_2}{Y_2}\equiv g_{Y_2}^*=\frac{\varphi_2(1-\tilde{\delta})}{\varphi_2(1-\tilde{\delta})-1}\tilde{g}$, and $g_E^*=g_{Y_1}^*=g_{Y_2}^*$.

3. Industrial Structure, Appropriate Technology, and Firm Viability in LDCs

In this section, we will explore the issues of endogenous industrial structure, appropriate technology and viability of a firm in the LDCs based on the dynamic trajectory of the above basic model.

3.1 Structural Change and Technological Progress in the Dynamic Trajectory

Now we begin with the dynamic trajectories of the economy described in the present paper. The dynamic trajectories of this economy can be characterized by an autonomous system of nonlinear differential equations which contains three control variables, $e\equiv\frac{E}{LN^{\frac{1-\alpha}{\alpha\beta}}}$, X_1, and X_2 as well as seven state variables, δ, κ_L, p, μ, $k\equiv\frac{K}{LN^{\frac{1-\alpha}{\alpha\beta}}}$, $n\equiv\frac{N^{\frac{(\varphi_1+1)\alpha\beta-(1-\alpha)}{\alpha\beta}}}{L}$, and $\mathrm{I}\equiv\frac{\lambda^{(\varepsilon-\varphi_2)\int_0^t\iota(x)dx}L}{N^{\frac{(1-\alpha)(\varepsilon-\delta\varepsilon-1)}{\alpha\beta}}}$.

First and foremost, we need to solve the equilibrium interest rate r in the dynamic trajectories. From (25), we know that the equilibrium interest rate in the dynamic trajectories is determined by:

$$r=\alpha(1-\beta)\left[\frac{\xi+(1-\xi)\kappa_L}{k}\right]^{\beta}$$

where $\xi\equiv\frac{\delta\beta}{(1-\delta)(1-\beta)}>1$.

Second, we need to calculate the dynamics of capital intensity in the modern sector. We could invoke the property of a Poisson distribution to argue that the expected time of a firm in the modern sector that carries out R&D at intensity $\iota(t)$ to develop the product of generation

$j = 1, 2, ...$ based on the existing generation before it is $dt = 1/\imath(t)$ (Feller, 1968). Therefore, from (29), the dynamics of capital intensity in the modern sector, denoted by $\dot{\delta}$, is given by

$$\dot{\delta}(t) \equiv \lim_{dt \to 0} \frac{\delta(t+dt) - \delta(t)}{dt} = b_2[\tilde{\delta} - \delta(t)]^{\varphi_3} \imath(t) \quad (43)$$

Third, we should discover the evolution of the optimal R&D intensity in the dynamic paths. Differentiating $p\lambda^{\varphi_2 \int_0^t \imath(x)dx} = V_2(t)$ with respect to time t implies

$$\frac{\dot{p}}{p} + \varphi_2 \ln \lambda \imath(t) = \frac{\dot{V}_2(t)}{V_2(t)} \quad (44)$$

Combining (40) with (44), we obtain

$$(\varphi_2 \ln \lambda - 1)\imath = r - \frac{\dot{p}}{p} - \frac{\pi_2(t)}{V_2(t)}$$

where $\frac{\pi_2(t)}{V_2(t)} = (1-\varepsilon)\frac{(1-\gamma)\mu^{\varepsilon-1}}{\gamma p} \frac{(1-\kappa_K)^{\delta\varepsilon}(1-\kappa_L)^{(1-\delta)\varepsilon}}{(\kappa_L)^{\beta(\varepsilon-1)}(\kappa_K)^{(1-\beta)(\varepsilon-1)}} \mathrm{I} k^{\delta\varepsilon-(1-\beta)(\varepsilon-1)}$.

Thus, the evolution of normalized accumulated R&D intensity, denoted by I, should satisfy

$$\imath = \frac{r - \frac{\dot{p}}{p} - (1-\varepsilon)\frac{(1-\gamma)\mu^{\varepsilon-1}}{\gamma p} \frac{(1-\kappa_K)^{\delta\varepsilon}(1-\kappa_L)^{(1-\delta)\varepsilon}}{(\kappa_L)^{\beta(\varepsilon-1)}(\kappa_K)^{(1-\beta)(\varepsilon-1)}} \mathrm{I} k^{\delta\varepsilon-(1-\beta)(\varepsilon-1)}}{(\varphi_2 \ln \lambda - 1)} \quad (45)$$

where $\kappa_K = \frac{\kappa_L}{\kappa_L + \xi(1-\kappa_L)}$.

Fourth, we need to characterize the evolution of technology in the traditional sector. Differentiating the zero-profit condition for firm i in the traditional sector, which is expressed by (28) with respect to time t, yields

$$\frac{\dot{V}_1(i,t)}{V_1(i,t)} = \frac{\dot{p}}{p} + \varphi_1 \frac{\dot{N}}{N} \quad (46)$$

Combining (27) with (46) yields

$$\frac{\dot{p}}{p} + \varphi_1 \frac{\dot{N}}{N} = r - \frac{\pi_1(i,t)}{V_1(i,t)} \quad (47)$$

Substituting $\frac{\pi_1(i,t)}{V_1(i,t)} = b_1 \frac{(1-\alpha)(\kappa_L)^{\beta}(\kappa_K)^{1-\beta}}{p} \frac{k^{1-\beta}}{n}$ and $\frac{\dot{N}}{N} = \frac{\alpha\beta\left(\frac{\dot{n}}{n} + \tilde{g}\right)}{(\varphi_1 + 1)\alpha\beta - (1-\alpha)}$

into (47) implies the dynamics of normalized technology in the traditional sector, denoted by $\frac{\dot{n}}{n}$, is determined by

$$\frac{\dot{p}}{p}+\varphi_1\frac{\alpha\beta\left(\frac{\dot{n}}{n}+\tilde{g}\right)}{(\varphi_1+1)\alpha\beta-(1-\alpha)}=r-b_1\frac{(1-\alpha)\left(\kappa_L\right)^{\beta}\left(\kappa_K\right)^{1-\beta}}{p}\frac{k^{1-\beta}}{n} \tag{48}$$

Fifth, from (41), we obtain the fraction of labor used in the traditional sector κ_L as

$$\frac{1}{\kappa_L}=1+\frac{\left[\frac{(1-\gamma)}{\gamma}\varepsilon\right]^{\frac{1}{1-\varepsilon}}\mu^{-1}(1-\delta)^{\frac{1-\varepsilon\delta}{1-\varepsilon}}\delta^{\frac{\varepsilon\delta}{1-\varepsilon}}\left(\frac{k}{\xi+(1-\xi)\kappa_L}\right)^{\frac{\varepsilon(\delta-1+\beta)}{1-\varepsilon}}(n\mathrm{I})^{\frac{\varepsilon}{(\varepsilon-\varphi_2)(1-\varepsilon)}}}{\left[(1-\beta)^{\alpha(1-\beta)}\beta^{(1-\alpha+\alpha\beta)}\ \alpha\right]^{\frac{1}{1-\alpha}}(\alpha\beta)^{\frac{\varepsilon-\alpha}{(1-\varepsilon)(1-\alpha)}}\left(\frac{1-\beta}{\beta}\right)^{\frac{\varepsilon\delta}{1-\varepsilon}+\frac{\alpha(\beta-1)}{1-\alpha}}} \tag{49}$$

Sixth, it is time for us to understand the law of price change in the present model. From (31), the price of the product in the modern sector can be rewritten as

$$p=\frac{(1-\gamma)}{\gamma}\left(\frac{\mu\lambda^{\psi(t)}K_2^{\delta}L_2^{1-\delta}}{N^{\frac{1-\alpha}{\alpha}}L_1^{\beta}K_1^{1-\beta}}\right)^{\varepsilon-1} \tag{50}$$

Differentiating (50) with respect to time t, we obtain the law of the price of modern goods, denoted by $\frac{\dot{p}}{p}$, as

$$\frac{\dot{p}}{p}=(\varepsilon-1)\left[\begin{array}{c}\frac{\dot{\mu}}{\mu}+\iota\ln\lambda+\left(\delta+\beta-1\right)\frac{\dot{k}}{k}+\frac{(1-\alpha)(\delta-1)\alpha\beta}{\alpha\beta(\varphi_1+1)\alpha\beta-(1-\alpha)}\left(\frac{\dot{n}}{n}+\tilde{g}\right)\\ -\delta\frac{\dot{\kappa}_K}{1-\kappa_K}-(1-\delta)\frac{\dot{\kappa}_L}{1-\kappa_L}-\beta\frac{\dot{\kappa}_L}{\kappa_L}-(1-\beta)\frac{\dot{\kappa}_K}{\kappa_K}\end{array}\right] \tag{51}$$

where $\dot{\delta}\ln\left(\frac{(1-\kappa_K)K}{(1-\kappa_L)L}\right)$ is infinitesimal and we neglect it in (51).

Seventh, the Euler equation of the representative agent requires that the optimal path for the normalized consumption expenditure e must satisfy

$$\frac{\dot{e}}{e}=\frac{r-\rho}{\theta}-\tilde{g}-\frac{(1-\alpha)\left(\frac{\dot{n}}{n}+\tilde{g}\right)}{(\varphi_1+1)\alpha\beta-(1-\alpha)} \tag{52}$$

Eighth, from (9), the dynamics of the fraction of modern goods consumed by households μ could be determined by

$$\frac{r-\rho}{\theta} = s_1\left[\frac{2(1-\alpha)\beta\left(\frac{\dot{n}}{n}+\tilde{g}\right)}{(\varphi_1+1)\alpha\beta-(1-\alpha)} + \beta\left(\frac{\dot{k}}{k}+\frac{\dot{\kappa}_K}{\kappa_K}\right) + \tilde{g} + (1-\beta)\frac{\dot{\kappa}_L}{\kappa_L}\right] + (1-s_1)\frac{\dot{p}}{p}$$

$$+(1-s_1)\left[\frac{\dot{\mu}}{\mu} + \iota\ln\lambda + \frac{\delta\dot{k}}{k} + \frac{\delta(1-\alpha)\left(\frac{\dot{n}}{n}+\tilde{g}\right)}{(\varphi_1+1)\alpha\beta-(1-\alpha)} - \frac{\delta\dot{\kappa}_K}{1-\kappa_K} + \tilde{g} - \frac{(1-\delta)\dot{\kappa}_L}{1-\kappa_L}\right] \quad (53)$$

where $s_1 = \frac{(\kappa_L)^{\beta}(\kappa_K)^{1-\beta}}{e}k^{1-\beta}$ and again we neglect $b_2[\tilde{\delta}-\delta]^{\varphi_3}\iota\ln\frac{(1-\kappa_K)K}{(1-\kappa_L)L}$, which is infinitesimal, in (53).

Ninth, the market clearing condition in the modern sector implies the dynamics of normalized capital, denoted by $\frac{\dot{k}}{k}$, follows

$$\frac{\dot{k}}{k} = (1-\mu)(1-\kappa_K)^{\delta}(1-\kappa_L)^{1-\delta}(\mathrm{I}n)^{\frac{1}{\varepsilon-\varphi_2}}k^{\delta-1}$$

$$-\frac{\mathrm{I}^{\frac{\varphi_2}{\varepsilon-\varphi_2}}n^{\frac{\varepsilon}{\varepsilon-\varphi_2}}\iota}{k} - \tilde{g} - \frac{\left(\frac{\alpha\beta n}{b_1 k}+1-\alpha\right)\left(\frac{\dot{n}}{n}+\tilde{g}\right)}{(\varphi_1+1)\alpha\beta-(1-\alpha)} \quad (54)$$

Finally, the other two control variables X_1 and X_2 can be computed as $X_1 = \frac{1}{b_1}\dot{N}N^{\varphi_1}$ and $X_2 = \lambda^{\varphi_2\int_0^t \iota(x)dx}\iota$ with the initial value of technology in the traditional sector at the starting point of the analysis, i.e., the exact value of $N(t)$ at time $t=0$, denoted by $N(0)$, as well as $A_2(0)=1$ assumed above.

Summarizing the results above, we can characterize the dynamics of the present economy as well as industrial structural change and technological progress in the following proposition.

Proposition 2: The dynamic equilibrium of the present economy could be characterized by an autonomous system of nonlinear differential equations which contains eight variables, δ, I, n, κ_L, p, e, μ, and k, given by eight equations (43), (45), (48), (49), (51), (52), (53), (54). Furthermore, in both LDCs and DCs, the evolution of industrial structure $\kappa_L(t_0)$ and appropriate technology $N(t_0)$, $j(t_0)$, $\delta(t_0)$ at time $t=t_0$ is endogenously determined by the above autonomous system of nonlinear differential equations, as well as the capital stock $K(t_0)$ at time $t=t_0$ and initial conditions in labor and technology in the traditional sector and the modern sector, i.e., $L(0)$, $N(0)$, and δ_0 at the starting point of the analysis $t=0$.

3.2 Catch-up and Firm Viability in LDCs

It is obvious from proposition 2 that we have $\underline{\kappa}_L(t_0) \neq \overline{\kappa}_L(t_0)$, $\underline{\delta}(t_0) < \overline{\delta}(t_0)$, $\underline{j}(t_0) < \overline{j}(t_0)$, and $\underline{n}(t_0) \leq \overline{n}(t_0)$ when $\overline{K}(t_0)/\underline{K}(t_0) > 1$ at time $t=t_0$. The ratio of capital in the DC to that in the LDC, denoted by $\overline{K}/\underline{K}$, could be interpreted broadly, as a metaphor for the LDC's current stage of development. The larger $\overline{K}/\underline{K}$ is, the more backward the economy in this LDC, and $\overline{K}/\underline{K}$ will decrease and approach unity eventually as the LDC converges to the DC. Moreover, the ratio of technology in the DC to that used in the LDC denotes the distance to the technology frontier in the LDC, where $\overline{N}/\underline{N}$ denotes the distance to the technology frontier of the traditional sector, and $\overline{j}/\underline{j}$ denotes the distance to the technology frontier of the modern sector. When these terms are large, the LDC is far from the world technology frontier.

As pointed out in section 1, motivated by the dream of nation building, most of the LDC governments pursued a catch-up type strategy to accelerate the development of the then advanced capital-intensive industries after World War II. Thus, in reality, the actual industrial structure and technology adopted in LDCs may deviate from that endogenously determined in proposition 2. If we define the industrial structure and technology in an LDC endogenously determined in proposition 2 as the optimal industrial structure and (most) appropriate technology in that country, by summarizing the analysis above, we obtain the following result.

Corollary 1: Owing to the relative scarcity of capital in LDCs, endogenous industrial structural change and technological progress on the dynamic trajectory imply that the (optimal) industrial structure in LDCs should not be the same as that in DCs; the (most) appropriate technology adopted in the modern sector in LDCs ought to be inside the technology frontier of the DCs.

On the viability of the firm in the government's priority industries when this government pursued a catch-up type strategy to accelerate development of the then advanced capital-intensive industries, we obtain the following proposition.

Proposition 3: When an LDC government pursues a catch-up type strategy, the firm in this LDC that enters a capital-intensive, advanced industry of the modern sector in the DCs would be nonviable owing to the relative scarcity of capital in the factor endowments of the LDCs.

Proof: We will prove proposition 3 based on an extreme assumption first, then extend it to the general case. The extreme assumption is that we model the catch-up type strategy in the LDC whose capital stock equals to $\underline{K}(t_0)$ at time $t=t_0$ by assuming that the actual industrial structure and technology chosen by the government in the LDC coincides with those in the DC whose capital stock equals $\overline{K}(t_0)$ at time $t=t_0$ for tractability, i.e., $\underline{N}(t_0)=\overline{N}(t_0)$,

$\underline{j}(t_0)=\overline{j}(t_0)$, $\underline{\delta}(t_0)=\overline{\delta}(t_0)$, and $\underline{\kappa}_L(t_0)=\overline{\kappa}_L(t_0)$ at time $t=t_0$, when $\underline{K}(t_0)<\overline{K}(t_0)$.

From the analysis in subsection 2.2, we know that, without external subsidies, as long as R&D operates at a positive but finite scale, the present value of firm j in any country discounted to time t_0, denoted by $V_2(j,t_0)$, must satisfy the free-entry condition, i.e., $p_j(t_0)(\lambda^j)^{\varphi_2}=V_2(j,t_0)$. Naturally, the present value of the firm in the DC, whose capital stock equals $\overline{K}(t_0)$, which enters industry $\overline{j}$ discounted to time t_0, denoted by $\overline{V}_2(\overline{j},t_0)$, meets the above free-entry condition precisely, because there is positive and bounded R&D intensity in the DC, which implies that we have

$$p_{\overline{j}}(t_0)(\lambda^{\overline{j}})^{\varphi_2}=\overline{V}_2(\overline{j},t_0) \tag{55}$$

Equation (40) could be reformulated as

$$\frac{V_2(j,t)}{p_j}=\frac{\dot{V}_2(j,t)}{(r+\iota)p_j}+\frac{\pi_2(j,t)}{(r+\iota)p_j} \tag{56}$$

Substituting $\frac{\pi_2(j)}{p_j}=(1-\varepsilon)Y_2(j)$ into (56) yields

$$\frac{V_2(j,t)}{p_j}=\frac{\dot{V}_2(j,t)}{(r+\iota)p_j}+\frac{(1-\varepsilon)Y_2(j,t)}{(r+\iota)} \tag{57}$$

where $p_j=\frac{(1-\gamma)}{\gamma}\left(\frac{\lambda^{\psi(t)}[(1-\kappa_K)K]^{\delta}[(1-\kappa_L)L]^{1-\delta}-\left(X_1+X_2\right)}{N^{\frac{1-\alpha}{\alpha}}(\kappa_L L)^{\beta}(\kappa_K K)^{1-\beta}}\right)^{\varepsilon-1}$ and satisfies $\frac{\partial p_j}{\partial K}<0$.

Differentiating (57) with respect to K obtains

$$\frac{\partial}{\partial K}\left[\frac{V_2(j,t)}{p_j}\right]=\frac{\frac{\partial}{\partial K}\dot{V}_2(j,t)}{(r+\iota)p_j}-\frac{\dot{V}_2(j,t)}{(r+\iota)^2p_j}\frac{\partial r}{\partial K}-\frac{\dot{V}_2(j,t)}{(r+\iota)(p_j)^2}\frac{\partial p_j}{\partial K}+\frac{(1-\varepsilon)}{(r+\iota)}\frac{\partial Y_2(j)}{\partial K}-\frac{(1-\varepsilon)Y_2(j)}{(r+\iota)^2}\frac{\partial r}{\partial K} \tag{58}$$

It is obvious that $\dot{V}_2(j,t)>0$, $\frac{\partial Y_2(j)}{\partial K}>0$, $\frac{\partial r}{\partial K}<0$, which imply that we have $\frac{\partial}{\partial K}\left[\frac{V_2(j,t)}{p_j}\right]>0$, owing to $\frac{\partial}{\partial K}\dot{V}_2(j,t)$ is a higher-order infinitesimal term which could be neglected in (58). Therefore, given $\underline{N}(t_0)=\overline{N}(t_0)$, $\underline{j}(t_0)=\overline{j}(t_0)$, $\underline{\delta}(t_0)=\overline{\delta}(t_0)$, and

$\underline{\kappa}_L(t_0) = \overline{\kappa}_L(t_0)$ at time $t = t_0$, $p_{\underline{j}}(t_0)(\lambda^{\underline{j}})^{\varphi_2} > \underline{V}_2(\underline{j}, t_0)$ is a direct conclusion from (55) when $\underline{K}(t_0) < \overline{K}(t_0)$, which means that the firm in the modern sector of the LDC would be nonviable if this LDC imitates the industrial structure and copies the most advanced technologies used in the DC exactly.

Furthermore, as a matter of fact, $\dfrac{V_2(j,t)}{p_j}$ is a continuous function of all its arguments, thus, when there is a severe scarcity of capital in the LDC, i.e., $\overline{K}(t_0) \big/ \underline{K}(t_0)$ is much greater than 1, the conditions that $\underline{N}(t_0) = \overline{N}(t_0) - \Delta_N$, $\underline{j}(t_0) = \overline{j}(t_0) - \Delta_j$, and $\underline{\kappa}_L(t_0) = \overline{\kappa}_L(t_0) + \Delta_\kappa$ when $\Delta_N > 0$, $\Delta_j > 0$, and Δ_κ are all small enough, could still suffice for $p_{\underline{j}}(t_0)(\lambda^{\underline{j}})^{\varphi_2} > \underline{V}_2(\underline{j}, t_0)$, thereby we would obtain proposition 3. *Q.E.D.*

Therefore, it is imperative for the government to introduce a series of regulations and interventions to mobilize resources for setting up and supporting the continuous operation of the non-viable firms. This type of economy might become very inefficient as the result of misallocation of resources (Lin, 2007, Lin and Zhang, 2007a).

4. Concluding Remarks

In the present paper, we have developed an endogenous growth model that combines structural change with repeated product improvements. The distinctive characteristic of our model originates from the technology in the modern sector, which becomes not only increasingly capital-intensive, but also progressively productive over time as the result of innovation by the profit-seeking firms. Each technology in the modern sector is appropriate for one and only one capital-labor ratio, i.e., the technologies in the modern sector are specific to particular factor endowment structures. Therefore, we could draw the conclusion that an LDC's optimal industrial structure and the (most) appropriate technology are endogenously determined by that economy's endowment structure, and the optimal industrial structure in LDCs should not be the same as that in DCs. That is, the (most) appropriate technology adopted in the modern sector in the LDCs ought to be inside the technology frontier of the DCs, and a firm in an LDC that enters capital-intensive, advanced industry in a DC would be nonviable owing to the relative scarcity of capital in the factor endowments of LDCs. We hope the framework developed in the present paper provides a new line of thought for analyzing the root cause of the differences in economic performance in LDCs. Our argument is that whether the industrial structure and technology adopted in LDCs match the factor endowment structure is the fundamental reason for the diversity in economic performance among LDCs.

References

Acemoglu, Daron and Veronica Guerrieri. "Capital Deepening and Non-Balanced Economic Growth", (August, 2006). NBER Working Paper No. W12475, Available at SSRN: http://ssrn.com/abstract=926054.

Acemoglu, Daron and Veronica Guerrieri. "Capital Deepening and Non-Balanced Economic Growth", Journal of Political Economy，Vol.116, No. 3. (June 2008), pp. 467-498.

Acemoglu, Daron, and Fabrizio Zilibotti, "Productivity Differences", Quarterly Journal of Economics, Vol. 116, No. 2. (May, 2001), pp. 563-606.

Aghion, Philippe and Peter Howitt. "A Model of Growth through Creative Destruction", Econometrica, Vol. 60, No. 2. (Mar., 1992), pp. 323-351.

Arrow, Kenneth J. "The Economic Implications of Learning by Doing", Review of Economic Studies, Vol. 29, No. 3. (Jun., 1962), pp. 155-173.

Atkinson, Anthony B and Joseph E. Stiglitz. "A New View of Technological Change", Economic Journal, Vol. 79, No. 315. (Sep., 1969), pp. 573-578.

Barro, Robert J. and Xavier Sala-i-Martin, Economic Growth, 2nd Edition. Cambridge: MIT Press. (2003).

Basu, Susanto. and David N. Weil. "Appropriate Technology and Growth", Quarterly Journal of Economics, Vol. 113, No. 4. (Nov., 1998), pp. 1025-1054.

Baumol, William J. "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis", American Economic Review, Vol. 57, No. 3. (Jun., 1967), pp. 415-426.

Caselli, Francesco and Jaume Ventura. "A Representative Consumer Theory of Distribution", American Economic Review, Vol. 90, No. 4. (Sep., 2000), pp. 909-926.

Cass, David. "Optimum Growth in an Aggregative Model of Capital Accumulation", Review of Economic Studies, Vol. 32, No. 3. (Jul., 1965), pp. 233-240.

Chenery, Hollis B. "Patterns of Industrial Growth", American Economic Review, Vol. 50, No. 4. (Sep., 1960), pp. 624-654.

Diwan, Ishac and Dani Rodrik. "Patents, Appropriate Technology, and North-South Trade", Journal of International Economics, Vol. 30, Issue, 1-2, (February, 1991), pp. 27-47.

Ericson, Richard E. "The Classical Soviet-Type Economy: Nature of the System and Implications for Reform", Journal of Economic Perspectives, Vol. 5, No. 4. (Autumn., 1991), pp. 11-27.

Feller, William. "An introduction to probability theory and its applications", 3rd Edition, New York: John Wiley, 1968.

Grossman, Gene M. and Elhanan Helpman. "Quality Ladders in the Theory of Growth", Review of Economic Studies, Vol. 58, No. 1. (Jan., 1991a), pp. 43-61.

Grossman, Gene M. and Elhanan Helpman. Innovation and Growth in the Global Economy, Cambridge: MIT Press. (1991b).

Grossman, Gene M. and Elhanan Helpman. "Endogenous Innovation in the Theory of Growth", Journal of Economic Perspectives, Vol. 8, No. 1. (Winter, 1994), pp. 23-44.

Hsieh, Chang Tai and Peter Klenow, "Misallocation and Manufacturing TFP in China and India", (May, 2007). Available at: http://www.klenow.com/MMTFP.pdf.

Jones, Larry E. and Rodolfo E. Manuelli. "A Convex Model of Equilibrium Growth: Theory and Policy Implications", Journal of Political Economy, Vol. 98, No. 5, Part 1. (Oct., 1990), pp. 1008-1038.

Judd, Kenneth L. "On the Performance of Patents", Econometrica, Vol. 53, No. 3. (May, 1985), pp.

567-586.
King, Robert G. and Sergio Rebelo. “Public Policy and Economic Growth: Developing Neoclassical Implications”, Journal of Political Economy, Vol. 98, No. 5, Part 2: The Problem of Development: A Conference of the Institute for the Study of Free Enterprise Systems. (Oct., 1990), pp. S126-S150.
Kongsamut, Piyabha. Sergio Rebelo, and Danyang Xie. “Beyond Balanced Growth”, Review of Economic Studies, Vol. 68, No. 4. (Oct., 2001), pp. 869-882.
Koopmans, Tjalling C. “On the Concept of Optimal Economic Growth”, in The Econometric Approach to Development Planning, Pontificiae Academiae Scientiarum Scripta Varia, 28, (1965), pp. 225-300; reissued North-Holland (1966).
Krueger, Anne O. “Policy Lessons from Development Experience Since the Second World War”, in Jere Richard Behrman and T.N. Srinivasan eds., Handbook of Development Economics, vol. 3B, Amsterdam: Elsevier Science Publishers (North-Holland), (1995), pp 2497-2550.
Kuznets, Simon. “Quantitative Aspects of the Economic Growth of Nations: II. Industrial Distribution of National Product and Labor Force”, Economic Development and Cultural Change, Vol. 5, No. 4, Supplement. (Jul., 1957), pp. 1-111.
Kuznets, Simon. “Modern Economic Growth: Findings and Reflections”, American Economic Review, Vol. 63, No. 3. (Jun., 1973), pp. 247-258.
Laitner, John. “Structural Change and Economic Growth”, Review of Economic Studies, Vol. 67, No. 3. (Jul., 2000), pp. 545-561.
Lin, Justin Yifu. “Development Strategy, Viability and Economic Convergence,” Economic Development and Cultural Change, Vol. 53, No. 2 (January 2003), pp. 277-308.
Lin, Justin Yifu. “Viability, Economic Transition and Reflection on Neoclassical Economics”, Kyklos, Vol. 8, Issue. 2. (May 2005), pp 239-264.
Lin, Justin Yifu. “Development and Transition: Idea, Strategy, and Viability”, Preliminary draft prepared for the Marshall Lectures of Cambridge University on October 31-November 1, 2007. Available at Peking University: http://www.ccer.edu.cn/download/8300-1.pdf.
Lin, Justin Yifu, Guofu Tan. “The Soft Budget Constraint, Policy Burdens, Accountability, and the Soft Budget Constraint”, American Economic Review, Vol. 89, No. 2, Papers and Proceedings of the One Hundred Eleventh Annual Meeting of the American Economic Association. (May, 1999), pp. 426-431.
Lin, Justin Yifu, Pengfei Zhang. “Appropriate Technology, Technology Choice and Economic Growth in Developing Countries”, Chinese Economic Quarterly, (in Chinese), Vol. 5, No. 4, (Jul, 2006), pp. 985-1006.
Lin, Justin Yifu, Pengfei Zhang. “Development Strategy and Economic Institutions in Less Developed Countries”, CID Graduate Student and Postdoctoral Fellow Working Paper No. 17, Harvard University, (March., 2007a). Available at CID Harvard University: http://www.cid.harvard.edu/cidwp/grad/017.htm.
Lin, Justin Yifu, Pengfei Zhang. “Development Strategy, Optimal Industrial Structure and Economic Growth in Less Developed Countries”, CID Graduate Student and Postdoctoral Fellow Working Paper No. 19, Harvard University, (July., 2007b). Available at CID Harvard University: http://www.cid.harvard.edu/cidwp/grad/019.htm.
Lin, Justin Yifu, Fang Cai, and Zhou Li. China’s Miracle: Development Strategy and Economic Reform, the Chinese University of Hong Kong Press, 2003 (Revised English edition).

Lucas, Robert Jr. “On the mechanics of economic development”, Journal of Monetary Economics, vol. 22, issue1, (July., 1988), pp. 3-42.

Maddison Angus, Monitoring the World Economy, 1820-1992. Washington, DC: Organization for Economic Cooperation and Development, 1995.

Matsuyama, Kiminori. “Structural Change”, forthcoming in L. Blume and S. Durlauf, eds., the New Palgrave Dictionary of Economics, 2nd Edition, Macmillan. (May, 2005), Available at: http://www.faculty.econ.northwestern.edu/faculty/matsuyama/Structural%20Change.pdf.

Ngai, L. Rachel and Christopher A. Pissarides, “Structural Change in a Multisector Model of Growth”, American Economic Review. Vol. 97, No. 1, (March., 2007), pp. 429-443.

Ramsey F. P. “A Mathematical Theory of Saving”, Economic Journal, Vol. 38, No. 152. (Dec., 1928), pp. 543-559.

Rebelo, Sergio. “Long-Run Policy Analysis and Long-Run Growth”, Journal of Political Economy, Vol. 99, No. 3. (Jun., 1991), pp. 500-521.

Romer, Paul M. “Increasing Returns and Long-Run Growth”, Journal of Political Economy, Vol. 94, No. 5. (Oct., 1986), pp. 1002-1037.

Romer, Paul M. “Endogenous Technological Change”, Journal of Political Economy, Vol. 98, No. 5, Part 2: The Problem of Development: A Conference of the Institute for the Study of Free Enterprise Systems. (Oct., 1990), pp. S71-S102.

Schumpeter, Joseph Alois. Capitalism, Socialism, and Democracy, New York and London, Harper & Brothers, (1942).

Seater, John J. “Share-Altering Technical Progress”, in L.A. Finley eds., Focus on economic growth and productivity, New York: Nova Science Publishers, Inc., (2005).

Segerstrom, Paul S., T. C. A. Anant. and Elias Dinopoulos. “A Schumpeterian Model of the Product Life Cycle”, American Economic Review, Vol. 80, No. 5. (Dec., 1990), pp. 1077-1091.

Shell, Karl. “A Model of Incentive Activity and Capital Accumulation”, in Karl Shell eds., Essays on the theory of optimal economic growth. Cambridge, Mass., M.I.T. Press, (1967).

Solow, Robert M. “A Contribution to the Theory of Economic Growth”, Quarterly Journal of Economics, Vol. 70, No. 1. (Feb., 1956), pp. 65-94.

Swan, Trevor W. “Economic growth and Capital Accumulation”, Economic Record, Vol. 32, Issue. 63. (Nov., 1956), pp. 334-61.

Zhang, Pengfei. “Factor Endowments, Industrial Structure, Technology Choices and Economic Development”, Ph.D. Dissertation (in Chinese), Peking University, (May, 2006).

Zuleta, Hernando and Andrew T. Young. “Labor’s Shares-Aggregate and Industry: Accounting for Both in a Model of Unbalanced Growth With Induced Innovation”, (January 17, 2007). Available at SSRN: http://ssrn.com/abstract=880062.

Appendix: Proof of Proposition 1

We guess-and-verify the existence of a unique constant growth equilibrium (CGE) in infinite horizon, i.e., $t \to \infty$, such that consumer expenditures E grow at a constant rate g_E^*

$$\lim_{t\to\infty} \frac{\dot{E}}{E} = g_E^* \tag{59}$$

Substituting (8) into (59) implies

$$\lim_{t\to\infty} r \equiv r^* = \theta g_E^* + \rho$$

which means the interest rate in CGE is also a constant.

We focus here the special case of CGE, i.e., a balanced growth equilibrium (BGE), such that modern sector and traditional sector grow at the same constant rate in infinite horizon for simplicity. The conditions that guarantee the existence of a BGE in the present model is

$$\alpha\beta(1+\varphi_1) = \varphi_2(1-\tilde{\delta})(1-\alpha)$$

which will be proved in the analysis below.

In BGE, the fraction of modern goods of the j th generation consumed by the households is constant, i.e., $\lim_{t\to\infty} \mu_{j(t)} = \lim_{j\to\infty} \mu_j \equiv \mu^*$, thus, we have:

$$\lim_{t\to\infty} \frac{\dot{C}_2(j)}{C_2(j)} \equiv g_{C_2}^* = \lim_{t\to\infty} \frac{\dot{Y}_2}{Y_2} \equiv g_{Y_2}^*$$

Differentiating $p_j = \frac{(1-\gamma)\left(\mu_j Y_2(j)\right)^{\varepsilon-1}}{\gamma Y_1^{\varepsilon-1}}$ with respect to time t implies in BGE

$$g_p^* \equiv \lim_{t\to\infty} \frac{\dot{p}}{p} = 0$$

Equation (9) and $g_p^* = 0$ imply that in BGE the share of the representative consumer's spending allocated to traditional sector, denoted by s_1^*, is a constant and

$$g_{C_1}^* = g_{Y_1}^* = g_{C_2}^* = g_{Y_2}^* = g_E^*$$

where $\lim_{t\to\infty} s_1 \equiv s_1^*$, $\lim_{t\to\infty} \frac{\dot{C}_1}{C_1} \equiv g_{C_1}^*$, and $\lim_{t\to\infty} \frac{\dot{Y}_1}{Y_1} \equiv g_{Y_1}^*$.

Let us first derive the growth rates of the key objects in traditional sector in BGE. Differentiating (23) with respect to time t yields

$$\frac{\dot{Y}_1}{Y_1} = \frac{1-\alpha}{\alpha}\frac{\dot{N}}{N} + \beta\frac{\dot{L}_1}{L_1} + (1-\beta)\frac{\dot{K}_1}{K_1}$$

Thus, the growth rate of traditional sector in BGE is given by

$$g_{Y_1}^* = \frac{1-\alpha}{\alpha} g_N^* + \beta g_{L_1}^* + (1-\beta) g_{K_1}^* \tag{60}$$

where $\lim_{t\to\infty} \frac{\dot{L}_1}{L_1} = g_{L_1}^*$ and $\lim_{t\to\infty} \frac{\dot{K}_1}{K_1} \equiv g_{K_1}^*$.

Differentiating the interest rate in (25) with respect to time t implies that we have

$$g_{K_1}^* = \frac{1-\alpha}{\alpha\beta} g_N^* + g_{L_1}^* \tag{61}$$

where $\lim_{t\to\infty} \frac{\dot{N}}{N} \equiv g_N^*$.

Combining (60) with (61), we find

$$g_{Y_1}^* = \frac{1-\alpha}{\alpha\beta} g_N^* + g_{L_1}^* \tag{62}$$

Differentiating the zero-profit condition for firm i in traditional sector which is expressed by (28) with respect to time t yields

$$\frac{\dot{V}_1(i,t)}{V_1(i,t)} = \frac{\dot{p}}{p} + \varphi_1 \frac{\dot{N}}{N} \tag{63}$$

Combining (27) with (63) yields

$$\frac{\dot{p}}{p} + \varphi_1 \frac{\dot{N}}{N} = r - \frac{\pi_1(i,t)}{V_1(i,t)} \tag{64}$$

In BGE, we have show that $\lim_{t\to\infty} \frac{\dot{N}}{N} \equiv g_N^*$ is a constant, thus we have

$$\lim_{t\to\infty} \frac{\dot{V}_1(i,t)}{V_1(i,t)} = \lim_{t\to\infty} \frac{\dot{\pi}_1(i,t)}{\pi_1(i,t)} = \varphi_1 g_N^*$$

Differentiating the profit function of the existing immediate firm i in traditional sector which is expressed by (26) with respect to time t, we obtain

$$\lim_{t\to\infty} \frac{\dot{\pi}_1(i,t)}{\pi_1(i,t)} = \varphi_1 g_N^* = \frac{1-2\alpha}{\alpha} g_N^* + \beta g_{L_1}^* + (1-\beta) g_{K_1}^* \tag{65}$$

Combining (61) and (62) with (65) yields

$$g_{Y_1}^* = \frac{\alpha\beta(1+\varphi_1)}{\alpha\beta(1+\varphi_1) - (1-\alpha)} g_{L_1}^* \tag{66}$$

Now we turn to the growth rate of the key objects in modern sector in BGE. The properties of the Poisson distribution imply that in infinite horizon, the expected number of product innovations in a time interval of length t is $i^* t$ (Feller, 1968). Thus, in infinite horizon, the production function in modern sector becomes

$$Y_2 = \lim_{t\to\infty, j\to\infty} \mathrm{F}_2[A_2(j), K, L] = \lambda^{\iota^* t} K_2^{\tilde{\delta}} L_2^{1-\tilde{\delta}} \tag{67}$$

where ι^* is the optimal rate of innovations in the long-run.

Differentiating (67) with respect to time t yields

$$\frac{\dot{Y}_2}{Y_2} = \iota^* \ln\lambda + \tilde{\delta}\frac{\dot{K}_2}{K_2} + (1-\tilde{\delta})\frac{\dot{L}_2}{L_2}$$

Therefore, the growth rate of modern sector in BGE is given by

$$g_{Y_2}^* = \iota^* \ln\lambda + \tilde{\delta} g_{K_2}^* + (1-\tilde{\delta}) g_{L_2}^* \tag{68}$$

where $g_{K_2}^* \equiv \lim_{t\to\infty}\frac{\dot{K}_2}{K_2}$, and $g_{L_2}^* \equiv \lim_{t\to\infty}\frac{\dot{L}_2}{L_2}$.

From (33), the interest rate in BGE can be expressed by

$$r^* = \frac{(1-\gamma)\left(\mu^*\right)^{\varepsilon-1} \lambda^{\varepsilon\iota^* t}}{\gamma Y_1^{\varepsilon-1}} \varepsilon\tilde{\delta} K_2^{\varepsilon\tilde{\delta}-1} L_2^{\varepsilon(1-\tilde{\delta})} \tag{69}$$

Differentiating (69) with respect to time t yields

$$(\varepsilon-1) g_{Y_1}^* = \varepsilon\iota^* \ln\lambda + (\varepsilon\tilde{\delta}-1) g_{K_2}^* + \varepsilon(1-\tilde{\delta}) g_{L_2}^* \tag{70}$$

Combining (68) with (70) yields

$$g_{Y_1}^* = g_{K_1}^* = g_{Y_2}^* = g_{K_2}^*$$

The fact that $g_{K_1}^* = g_{K_2}^*$ implies the fraction of capital used in traditional sector is a constant in BGE, i.e., κ_K^* is a constant, where $\kappa_K^* = \lim_{t\to\infty}\kappa_K(t)$. Thus, from (42) and $\xi^* \equiv \lim_{j\to\infty}\xi_j = \frac{\tilde{\delta}\beta}{(1-\tilde{\delta})(1-\beta)}$, we know the fraction of labor used in traditional sector is also a constant in BGE, i.e., κ_L^* is a constant, where $\kappa_L^* \equiv \lim_{t\to\infty}\kappa_L(t)$, which implies that

$$g_{L_1}^* = g_{L_2}^* = \tilde{g}$$

Substituting $g_{L_1}^* = \tilde{g}$ into (66) yields

$$g_{Y_1}^* = g_{K_1}^* = g_{Y_2}^* = g_{K_2}^* = \frac{\alpha\beta(1+\varphi_1)}{\alpha\beta(1+\varphi_1)-(1-\alpha)}\tilde{g}$$

Because the product innovations occur in the modern sector according to a time-varying Poisson process with instantaneous arrival rate $\iota(x)$ and the expected number of success before time t equals to $\psi(t) \equiv \int_0^t \iota(x)dx$, thus, the properties of the Poisson distribution imply that λ^j amounts to $\lambda^{\psi(t)}$ at time t (Feller, 1968). From the analysis above, we must have $p_j(t)\lambda^{\varphi_2 \int_0^t \iota(x)dx} = V_2(j,t)$ as long as there is positive and bounded growth in modern sector, which implies

$$\varphi_2 \iota^* \ln \lambda = \lim_{t\to\infty} \frac{\dot{V}_2(t)}{V_2(t)}$$

Thus, substituting ι^* into (40) implies

$$r^* + \iota^*(1-\varphi_2 \ln \lambda) = \lim_{t\to\infty} \frac{\pi_2(t)}{V_2(t)}$$

and

$$\lim_{t\to\infty} \frac{\dot{\pi}_2(t)}{\pi_2(t)} = \varphi_2 \iota^* \ln \lambda$$

In BGE, the profit function of the firm j in modern sector which is expressed by (37) reduces to

$$\lim_{t\to\infty} \pi_2(t) = (1-\varepsilon)\frac{(1-\gamma)\left(\mu^*\right)^{\varepsilon-1}\lambda^{\varepsilon\iota^* t}}{\gamma Y_1^{\varepsilon-1}}\left(K_2^{\tilde{\delta}} L_2^{1-\tilde{\delta}}\right)^{\varepsilon} \tag{71}$$

Differentiating (71) with respect to time t yields

$$\varphi_2 \iota^* \ln \lambda + (\varepsilon-1) g_{Y_1}^* = \varepsilon\iota^* \ln \lambda^* + \varepsilon\tilde{\delta} g_{K_2}^* + \varepsilon(1-\tilde{\delta}) g_{L_2}^* \tag{72}$$

Combining (70) with (72) yields

$$g_{K_2}^* = \varphi_2 \iota^* \ln \lambda \tag{73}$$

Substituting (73) into (68), we obtain

$$g_{Y_2}^* = \frac{\varphi_2(1-\tilde{\delta})}{\varphi_2(1-\tilde{\delta})-1}\tilde{g}$$

And the optimal intensity of innovations in the long-run, denoted by ι^*, is determined by

$$\iota^* = \frac{(1-\tilde{\delta})}{\ln \lambda[\varphi_2(1-\tilde{\delta})-1]}\tilde{g}$$

From the analysis above and comparing the growth rate in modern sector with that in traditional sector in BGE, we know the parameters of our model which satisfy

$$\alpha\beta(1+\varphi_1) = \varphi_2(1-\tilde{\delta})(1-\alpha)$$

could indeed guarantee the existence and uniqueness of the BGE in the present paper.

From (20) and (35), the fraction of labor used in traditional sector in BGE can be expressed as $\kappa_L^* = \dfrac{1}{1+\ell^*}$, where

$$\ell^* = \lim_{t\to\infty} \frac{\left[\dfrac{1-\gamma}{\gamma}\varepsilon(1-\tilde{\delta})^{(1-\varepsilon\tilde{\delta})}\tilde{\delta}^{\varepsilon\tilde{\delta}}\right]^{\frac{1}{1-\varepsilon}}(r^*)^{\frac{(\alpha-\alpha\beta)}{1-\alpha}-\frac{\varepsilon\tilde{\delta}}{1-\varepsilon}}}{\mu^*\left[(1-\beta)^{\alpha(1-\beta)}\beta^{(1-\alpha+\alpha\beta)}\;\alpha\right]^{\frac{1}{1-\alpha}}}\lambda^{\frac{\varepsilon\iota^* t}{1-\varepsilon}} w^{\frac{(1-\alpha+\alpha\beta)}{1-\alpha}-\frac{(1-\varepsilon\tilde{\delta})}{1-\varepsilon}}\frac{1}{N}$$

and

$$\lim_{t\to\infty}\frac{\dot{w}}{w} = \frac{(1-\alpha)}{\alpha\beta(1+\varphi_1)-(1-\alpha)}\tilde{g}$$

Finally, the fraction of modern goods consumed by households in BGE, denoted by μ^*, can be solved by

$$\dot{K} + \frac{\dot{N}N^{\varphi_1}}{b_1} + \lambda^{\varphi_2\iota^* t}\iota^* = (1-\mu^*)\lambda^{\iota^* t}\left[\frac{\xi^*(1-\kappa_L^*)K}{\kappa_L^* + \xi^*(1-\kappa_L^*)}\right]^{\tilde{\delta}}[(1-\kappa_L^*)L]^{1-\tilde{\delta}}$$

as well as the capital stock $K(t_0)$ at time $t = t_0$ and initial conditions of labor and technologies in traditional sector and modern sector, i.e., $L(0)$, $N(0)$, δ_0 and $j(0)$ at the starting point of the analysis, i.e., at time $t = 0$.