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# Paternalism and the Alleviation of Poverty

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Part of the reason for using price subsidies to redistribute income to the poor, rather than the more efficient direct cash transfers, is to produce “happier” taxpayers.

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This paper — a joint product of the Country Operations 2 Division, Country Department II and the Infrastructure and Energy Division, Technical Department, Latin America and the Caribbean Regional Office — is part of a larger effort in the Bank to understand the effectiveness of different poverty alleviation schemes. Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Fresia Betancourt, room 18-126, extension 37703 (40 pages). January 1992.

Typically the tools available for redistribution are price subsidies (in their extreme form, in-kind transfers) and direct cash transfers. Conventional economic theory indicates that the efficiency loss is minimized if cash transfers are used instead of price subsidies. But in almost all economies, including advanced economies, price subsidies are implemented — and cash transfers, the more efficient alternative, are seldom used.

Jesurun-Clements argues that taxpayers enjoy the poorer citizen's specific consumption package (food, housing, education) more than improving the poorer citizens' general economic welfare. Her objective is to identify the conditions under which price subsidies represent a more efficient way of alleviating poverty than cash payments, given taxpayers' paternalistic preferences.

She concludes that when the taxpayers' prevalent behavior is paternalism, and taxpayers

have more weight in society, the option for redistribution would be to target price subsidies to the poor. This brings about a greater improvement in overall social welfare and "happier" taxpayers than any other policy. With this solution, the poor are somewhat better off, even though they would rather receive cash transfers, which would represent the same financial cost to the economy.

When the rich are typically altruistic, there is no distortion in the price system. The preferences of each individual are preserved and the best policy for the economy as a whole and for each individual agency is undoubtedly the use of cash transfers.

Increasing the number of goods, or allowing the rich to enjoy subsidized prices, does not affect the qualitative results. Only the size of the optimum scheme to be used under various circumstances would change.

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## I. OBJECTIVE

Typically, the available tools for redistribution are price subsidies -- which in their extreme form are in-kind transfers-- and direct cash transfers. Conventional economic theory indicates that the efficiency loss of a redistributive policy is minimized if cash transfers are used instead of price subsidies. However, evidence shows that in almost all economies, including the more advanced ones, the implementation of price subsidies as a redistributive policy, is prevalent. That is to say, the seemingly more efficient form of redistribution, cash transfers, is little used.

Using a welfarist approach, this paper claims that the rationale behind the policy choice is the existence of consumption externalities: The taxpayer obtains a certain level of enjoyment out of the consumption package of the poorer fellow citizens, rather than of the general level of the utility of those poorer groups. The objective of this paper is to identify the conditions under which price subsidies represent a more efficient way of alleviating poverty than cash payments, given paternalistic preferences of taxpayers. The development of this topic entails three stages:

1. To establish a conceptual framework for a comparative evaluation of altruistic behavior and paternalistic behavior. The conceptual framework uses a private utility function for the taxpayer that characterizes preferences for redistribution to the poor, under the alternative assumptions of altruistic and paternalistic taxpayer preferences.

2. To find a systematic explanation to justify seemingly inefficient redistributive policies. We seek this explanation by identifying the utility gains that could justify the efficiency cost resulting from the use of poverty alleviation schemes such as price subsidies, rather than lump-sum transfers to the poor.

3. To make policy recommendations by specifying mechanisms for more efficient subsidization of the poor at the existing level of expenditure or on how poverty alleviation schemes could be optimally expanded or contracted.

## II. CONCEPTUAL FOUNDATION

### A. Introduction

An individual has extended preferences when his welfare depends in any way upon the welfare of others. Extended preferences may be expressed in the form of interdependent utilities with either a subset of the community or the entire remainder of the community.

Interdependency of utilities can be either benevolent or malevolent in nature. Within this context, a benevolent (malevolent) individual's welfare increases as a result of increases (decreases) in other people's welfare. Benevolent interdependency of utilities, which is the primary focus of this work, could be utility-related (altruism), or commodity-related (paternalism).

**B. Altruism**

The altruistic individual enjoys any increase in the welfare of others regardless of the source of that increase. An increase in the welfare of others becomes a positive externality to the altruist. Consequently, the altruist is willing to make direct income transfers in order to increase the utility level of others, regardless of the way in which the recipients make their allocation decisions. The altruist does not impose his altruistic consumption preferences; he ignores the tastes of others.

This altruistic behavior means that the utility level of others enters the utility function of the altruist as an argument. This requires that the social preference ordering of the altruistic individual be weakly separable, i.e.:

$$U^\alpha = U^\alpha [u^\alpha (x^\alpha), u^\beta (x^\beta)] \quad \text{where } \frac{\partial U^\alpha}{\partial u^\beta} > 0 \quad (1)$$

and  $\alpha$  and  $\beta$  are two representative individuals;  $\alpha$  is an altruist.

**C. Paternalism**

The paternalistic individual is concerned about the fellow citizens' consumption level of particular goods (e.g., food, housing, education) rather than being concerned about their general economic welfare. Increases in other's consumption of those goods become a positive externality to the paternalistic individual. The paternalistic individual has a preference for certain consumption patterns of others, and not the preference ordering of the recipient. The paternalistic individual's preference ordering function is not weakly separable because at least one element of the individual utility function of one agent appears as an argument in the utility function of at least one other agent:

$$u^{\alpha} = u^{\alpha}(x^{\alpha}, x^{\beta}) \quad \text{where} \quad \frac{\partial u^{\alpha}}{\partial x^{\beta}} > 0 \quad (2)$$

The paternalistic individual, concerned about consumption by others, might be better-off by undertaking unilateral in-kind transfers to ensure that others do in fact consume the goods in question.<sup>1/</sup> These transfers would be Pareto improving because both parties would be better-off as a result. Since the paternalistic agent is placing an additional "value" to that particular good, relative to the value given by others, his willingness to pay for that good is relatively larger than that of others.

The above argument is the foundation of our analysis. For efficiency to prevail under paternalistic interdependence of utilities, the supporting price system would require different prices for different individuals together with a policing system, or direct provision by the state.<sup>2/</sup> This pricing policy prescription is theoretically grounded in the Pigovian tax/subsidy solution: In the presence of external effects, and given the appropriate convexity conditions, a Pareto efficient allocation of resources can be achieved by taxes/subsidies on the commodities generating external effects. If we equate the ratio of each individual's marginal utility for the externality producing good to the price ratio of that good faced by each individual, the result is not necessarily equal to one. For instance, in the case of two consumers, where one consumer creates a positive externality, the price ratio that yields a Pareto efficient solution

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<sup>1/</sup> Or equivalently, the rich may want to subsidize the price of those goods in order to encourage their consumption.

<sup>2/</sup> Most likely, intervention by the state or a control system will be required in order to avoid the "free rider" problem and to ensure that the targeted group is reached as prescribed by the policy.

could be derived as:

$$p^{\beta} / p^{\alpha} = 1 - \frac{\partial u^{\alpha} / \partial x^{\beta}}{\partial u^{\alpha} / \partial x^{\alpha}} \quad (3)$$

The consumer enjoying the externality must face a higher price than the consumer producing the consumption externality for Pareto efficiency. This is the basic argument for subsidizing the price of the pertinent good.

It seems reasonable to think, therefore, that subsidizing the externality-producing good is the right thing to do under paternalism. However, subsidies increase the real income of the recipients and reduce the real income of the taxpayers. Changes in real income would possibly affect the willingness to work. Hence, the effect of subsidies on each agent's labor decision, and the acceptability of this decision to other agents must be considered in order to be able to assess the overall outcome of the policy, an important part of the analysis which will be addressed later.

#### D. The Utility Possibility Function

The potential efficiency improvement stemming from paternalistic through subsidies, can be evaluated by the utility possibility function or utility possibility frontier. This function, which relates the utility levels of two representative individuals, determines the maximum utility an individual can attain given the utility attained by the other individual, in view of the technological constraint imposed by the social transformation function. In the



presence of two-way external effects in consumption,<sup>3/</sup> it can be shown that the slope of this function is defined as:

$$\frac{\partial u^\alpha}{\partial u^\beta} = - \frac{\frac{\partial u^\alpha}{\partial x^\alpha} - \frac{\partial u^\alpha}{\partial x^\beta}}{\frac{\partial u^\beta}{\partial x^\beta} - \frac{\partial u^\beta}{\partial x^\alpha}} \quad (4)$$

The above expression shows that the utility possibility function can slope upwards, i.e. the expression takes a positive sign if the external effects are positive and greater than the internal effect. In all the positions in which the frontier has a positive slope, there is room for welfare enhancement. Welfare gains from redistribution are possible. Everyone can still be made better-off by moving along the upward sloping portion of the utility possibility frontier, until it changes to a downward sloping segment. These movements would be accomplished by increased consumption of the externality-producing good by the individual whose consumption produces the externality.

If we assume that external effects occur only "one-way", the slope of the utility possibility frontier becomes:

$$\frac{\partial u^\alpha}{\partial u^\beta} = - \frac{\frac{\partial u^\alpha}{\partial x^\alpha} - \frac{\partial u^\alpha}{\partial x^\beta}}{\frac{\partial u^\beta}{\partial x^\beta}} \quad (5)$$

This is the case considered in this analysis -- the rich ( $\alpha$ ) benefit from the consumption of one, or more goods by the poor ( $\beta$ ), but not vice-versa.

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<sup>3/</sup> If the only two agents in the economy benefit from each other's consumption.

### III. THE ANALYTICAL MODEL

#### A. Objective

As explained at the beginning of this paper, our objective is to derive the conditions under which it is correct to prescribe price subsidies and cash transfers as redistributive policies. The discussion and analysis will focus on the assumption of extended preferences associated with paternalism. However, for comparative evaluation, the analytical results for both types of benevolent behavior are derived. The treatment of altruism is relatively brief since it is well known that under altruism, direct cash transfer is the best redistributive formula to reach an efficient equilibrium.<sup>4/</sup>

#### B. Description

In our model individuals are divided into two classes, the rich (taxpayers) and the poor (subsidy recipients), differentiated by their human capital as reflected in their wage rates. We take one individual of each class as representative of that class. The postulate of the model is that paternalism exists in the economy. The rich derive utility when the poor consume certain good(s). This consumption by the poor represents a positive externality to the rich. The rich are also self-interested in the sense that they derive utility from their own consumption and leisure.

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<sup>4/</sup> The treatment of altruism is relatively brief since it is well known that under altruism, direct cash transfers is the best redistributive formula to reach to an efficient equilibrium.

In terms of utility function specification, the utility function of the paternalistic rich includes as arguments the consumption and leisure levels of the poor, as well as his own consumption and leisure levels. The poor, on the other hand, only obtain utility from their own consumption of goods and leisure. The preference for own consumption need not differ among classes, so we may assume that both the rich and the poor face the same indifference curve for own consumption.

The presence of paternalistic behavior justifies that the individual producing the consumption externality should pay a lower price for the externality-producing good than the price paid by the individual enjoying the externality. The paternalistic individual is therefore willing to subsidize the price of that good to the recipient. This price differential implies that the set-up of the paternalistic model does not correspond to a competitive equilibrium. The "first-order" conditions are violated in this case. The price ratio is not equal to the marginal rate of substitution of each individual. Hence, prices faced by individuals for the same goods differ from those that would prevail in competition.

Paternalistic behavior of the rich however, does not deter the poor from reducing hours of labor. Reduction in hours of labor by the poor as a response to a redistributive scheme, or work disincentive effect, is seen by the rich as a negative externality because the rich dislike the increase in poor's leisure per se, and because they also dislike the resulting reduction in output in the economy. This disutility effect to the rich is explicitly taken into account in the model by incorporating the poor's labor decision into the rich's preference

ordering. The inclusion of the labor decision of the poor into the rich's utility means that the rich want sure that the poor do not reduce their disposable income for other goods as result of making the externality good more affordable.

In our model there is a government that represents all individuals in the economy and determines the appropriate type and level of redistribution -- subsidies or cash transfers --referring to a Social Welfare Function (SWF) with a capacity for making interpersonal comparisons. This function is a social ordering of the Bergson type, with total welfare measured as a weighted average of the individuals' private utilities. The social weights are chosen by the government according to its value judgement on redistribution. The government chooses the value of social weights based on its concern for each group in the society, as well as on the number of people in each group.

### C. Methodology

We start our analysis with the development of the paternalistic model. For illustrative purposes, we continue our analysis with a simple derivation of the results for the altruistic model in order to enable us to compare the results with the outcome of the different assumptions made under the paternalistic model.

The paternalistic model is developed in ascending order of complexity as we progressively develop more relationships within it:

1. We first refer to a unique bundle of goods for all consumers. We assume a purely redistributive tax system where the rich pay taxes for the

exact amount of the subsidy bill required for the poor to consume an amount of that unique bundle, satisfactory to the rich. The bundle of goods is, therefore, priced differently for the rich (market price) and for the poor (subsidized price).

2. We repeat the exercise using cash transfers rather than subsidies and compare the results.

3. The altruistic model is developed and both redistributive policies are applied to the model.

4. In order to allow for substitution among goods, we drop the one-bundle assumption and consider only two goods in the economy of which both rich and poor consume. One of these goods is the externality-producing good and, hence, it is subsidized. The other good is a market good for both consumer types. Under this case we analyze two possibilities: if only the poor consume the subsidized good at subsidized prices, i.e. if there is perfect targeting; and if also the rich have access to the subsidized prices, i.e. if the subsidy system suffers leakages to other groups different from the poor.

5. Beyond this point we do not add more complexity to the model in order to preserve tractability of results, a point we discuss in more detail later. Instead, we suggest some possible extensions to the analysis, and speculate on the likely results that might arise, based on our own experimentation with the model.

**D. Assumptions of the Model**

The assumptions and specification of our model apply equally to both policy prescriptions- price subsidies and cash transfers:

1. The preferences of the individuals can be written by a strictly quasi-concave utility function increasing in consumption goods and decreasing in own hours of labor.

2. The model is short-term in nature, with fixed prices and wages, fixed technology, and fixed capital stock.

3. In order to maintain analytical tractability, the analysis follows a partial equilibrium approach where many economic relationships and markets are held constant. We choose this tactic rather than a computable general equilibrium (CGE) framework because the additional information we would obtain using a CGE model in this analysis does not justify its complexity. We are however, using a linear production technology with constant returns to scale, which implicitly assumes labor markets clearance, as explained in the next paragraph.

4. Since the capital stock is fixed, changes in output occur through changes in labor supply alone. Labor is supplied by the poor and by the rich. Marginal productivity of the rich is higher than that of the poor, and so is their wage rate. From the point of view of production, goods can be produced by substituting between labor input classes. We assume a simple linear production technology, i.e. constant returns to scale. This

implies<sup>5/</sup>:

$$Y = rK + w^P L^P + w^R L^R = C^P + C^R \quad (6)$$

where  $Y$  = total output

$r$  = rental rate

$K$  = capital stock

$w^P$  = fixed wage rate for the poor

$L^P$  = hours of labor supplied by the poor

$w^R$  = fixed wage rate for the rich.

$L^R$  = hours of labor supplied by the rich

$C^P$  = consumption level of the poor

$C^R$  = consumption level of the rich

5. The marginal utility that the rich derive from their own consumption is larger than the marginal utility they derive from the consumption by the poor. i.e.  $\delta U^R / \delta C^R > \delta U^R / \delta C^P$ . This is to avoid the situation where the rich are willing to let the poor consume so much, that the rich become poor themselves.

6. The marginal utility to the rich of the poor's consumption is decreasing in that good, i.e.  $\delta^2 U^R / \delta^2 C^P < 0$ . This way, as the poor become better-off, the rich reduce their concern for the poor.

7. For equilibrium to exist, the total utility of the rich must be

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<sup>5/</sup> Since with a linear production function the marginal product of the factor always equals its cost, the assumption of linear production technology makes our model closer to a general equilibrium model. Labor market clearance is implicit in the marginal product relationship, in this special case of a linear production function.

greater than the total utility of the poor. That way the rich do not have an incentive to become poor.

8. Preferences are identical within groups or classes in order to avoid pair-wise comparisons between individuals, but there are asymmetric preferences among groups, at least in terms of the preferences for other people's consumption of the externality good in question. Identical preferences for own consumption among groups may be assumed for simplicity, without affecting our results.

9. Our economy consists of three decision makers: the poor, the rich, and the government. Each agent sequentially solves his optimization problem as outlined in the next section.

10. Taxation is purely redistributive. The amount paid by the rich in taxes is the exact amount received in subsidies by the poor.

#### **E. Solution Technique**

Two possible solution techniques could be considered for solving this problem. We call them Benevolent Dictator, and Stackelberg Game solutions respectively.

**Benevolent Dictator:** The problem could be seen as a simultaneous optimization problem where a benevolent dictator type of government maximizes a social welfare function as a weighted average of the private utility functions of each class of individuals, subject to the individual budget constraints. The social weights are chosen by the government according to its value judgement



about redistribution. The solution to the government's problem is the consumption demand function and the labor supply function of each type of individual, as functions of wages, prices, and the social weights.

**Stackelberg Game:** Alternatively, we could consider the problem as an asymmetric Stackelberg game, where we sequentially solve a partial equilibrium problem for each decision maker. First, the poor take the level of subsidy and other parameters as given and decide on their optimum consumption and leisure levels. This decision directly affects the rich's utility level. The paternalistic rich play as a Stackelberg leader, explicitly taking the poor's decisions on consumption and leisure into their optimization problem as reaction functions. Constrained by their own budget and by society's production function, the rich decide on their own optimum consumption and leisure levels. Finally, the government takes the above individual decisions, includes them in its unconstrained<sup>6/</sup> social welfare function, and solves for the optimum level of subsidies or transfers.

The choice between these two solution techniques is based fundamentally in our own belief of the nature of reality. Each of the above techniques gives us different results. The first technique (benevolent dictator) is a simultaneous optimization problem involving the feed-back reactions of each decision maker. The government chooses a level of subsidy and all agents in the economy react to it. The reactions of each individual are already incorporated in the

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<sup>6/</sup> Since the government takes the solution arrived at by each individual, it is optimizing the SWF subject to the constrained optimization of the individual agents.

simultaneous solution. It does not allow for individual adjustments by the rich of their labor and consumption decision once they find out how the poor behaved when facing a certain level of subsidy.

The Stackelberg technique is a sequential optimization problem. Each agent reacts to the solution previously found by another agent. The technique allows for the more powerful decision maker (the rich), to observe those individuals (the poor), whose behavior may affect theirs, and to react accordingly. The poor, on the other hand, respond independently, i.e non-strategically to changes in policy.

Based on our belief about how things work in reality, the Stackelberg technique seems more compelling to be used in the analysis that follows. Generally, it would seem that taxpayers (the rich) want to see their assistance to the poor used for maintaining a minimal level of consumption and labor. If these levels are not reached to the rich's satisfaction, and in particular, if work by the poor is reduced as a response to a poverty alleviation scheme, the rich are powerful enough and sufficiently well organized to reduce the level of assistance to the poor.

**F. Development of the Model: PATERNALISM WITH PRICE SUBSIDIES**

**(ONE-BUNDLE CASE)**

We first develop the case of paternalism. In order to verify the preferability of the policy chosen, the problem is analyzed under both redistributive schemes: price subsidies, and cash transfers. We keep the assumption of paternalism in both cases in order to compare the preferability of

one policy over the other. For illustrative purposes we then proceed with the case of altruism.

1. **The Problem of the Poor:** The poor take the level of subsidy, prices and wages as given, and decide upon their optimal consumption and leisure levels. They supply hours of labor and generate their utility by consumption of goods and leisure. Their budget constraint is determined by their labor income alone. Under this case we are assuming a unique good<sup>7</sup> for the rich and for the poor. The poor face a subsidized price for that good. Normalizing for prices, we take wages and subsidies as real values. The poor's private utility function and budget constraint are:

$$\underset{C^P, L^P}{\text{Max}} U^P = U^P (C^P, L^P) \quad (7)$$

$$\text{s.t. } w^P L^P = (1-s) C^P \quad (8)$$

where:  $C^P$  = consumption level of the poor  
 $L^P$  = hours of labor supplied by the poor  
 $w^P$  = fixed wage rates for the poor  
 $s$  = price subsidy

Assuming that the equilibrium is interior, we can write the solution to the problem as a function of the exogenous components of the optimization problem. The solution of the poor's problem is their consumption demand and labor supply

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<sup>7</sup> We may think of a unique good as a composite good involving a large range of goods.

as functions of wages and subsidies:

$$\begin{array}{ll} C^{P*} = C^P(w^P, s) & \text{Reaction} \\ L^{P*} = L^P(w^P, s) & \text{Functions} \end{array} \quad (9)$$

where \* represents equilibrium values.

2. **The Problem of the Rich:** Facing prices and wages, the rich maximize a utility function that includes their own consumption and leisure levels, and explicitly incorporate their preference towards consumption by the poor, as well as their dislike of the poor's leisure. They explicitly take the optimal decision (reaction functions) on consumption by the poor into their utility function as parameters, not as an additional decision variable<sup>8/</sup>:

$$\text{Max}_{C^r, L^r} U^r = U^r(C^r, L^r, C^{P*}, L^{P*}) \quad (10)$$

where:  $C^r$  = consumption level of the rich

$L^r$  = hours of labor supplied by the rich

The rich's budget constraint is determined by the excess of total output,  $Y$ , after the wages of the poor and subsidy bills are deducted.

$$Y - (w^P L^P + s C^P) = C^r \quad (11)$$

The above constraint implicitly assumes equilibrium in the goods market, i.e., that the output available in the economy is totally consumed.

We can therefore express the rich's budget constraint as:

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<sup>8/</sup> The externality effect that the consumption and labor levels of the poor produce in the economy, shows up in the government problem when the social welfare function is optimized.

$$w^r L^r + rK = C^r + sC^p \quad (12)$$

where  $w^r$  = fixed wage rate for the rich.

The solution of the rich's problem is their consumption demand and labor supply as functions of the exogenous components, wages of the rich and of the poor, and subsidies:

$$C^{r*} = C^r(w^r, w^p, s) \quad (13)$$

$$L^{r*} = L^r(w^r, w^p, s) \quad (14)$$

3. **The Problem of the Government:** The government's role is to maximize social welfare. As an outcome of its maximization behavior, the government improves efficiency in the allocation of resources. The government constructs a Social Welfare Function (SWF) as a weighted sum of the individual utilities. It incorporates the individual preferences of the rich and the poor into its SWF. It then estimates the optimum level of redistribution, and enforces it<sup>9/</sup>. The SWF would have the following form:

$$\underset{s}{\text{Max SWF}} = W = \Omega V^r(C^{r*}, L^{r*}, C^p, L^p) + (1-\Omega) V^p(C^p, L^p) \quad (15)$$

where  $\Omega$ , ( $0 \leq \Omega \leq 1$ ) determines how much weight the government gives to the preferences of the rich.

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<sup>9/</sup> At this point individuals have already solved their private utility problem. Hence, the individual utility levels to be included by the government when setting up its optimization problem is the indirect utility function,  $V$  --the maximum utility level achievable by individuals. The values for consumption and labor to be substituted in equation (15) are those optimum values that each individual chose when the individual problem was solved.

Substituting equations (7) and (10) into (15) and differentiating we get:

$$\frac{dW}{ds} = \frac{d[\Omega V^r(C^r, L^r, C^p) + (1-\Omega)V^p(C^p, L^p)]}{ds} = 0 \quad (16)$$

$$\frac{dW}{ds} = \Omega \left[ \frac{\partial V^r}{\partial C^r} \frac{dC^r}{ds} - \frac{\partial V^r}{\partial L^r} \frac{dL^r}{ds} + \frac{\partial V^r}{\partial C^p} \frac{dC^p}{ds} + \frac{\partial V^r}{\partial L^p} \frac{dL^p}{ds} \right] + (1-\Omega) \left[ \frac{\partial V^p}{\partial C^p} \frac{dC^p}{ds} - \frac{\partial V^p}{\partial L^p} \frac{dL^p}{ds} \right] = 0 \quad (17)$$

The third term in the first bracket on the right-hand-side of equation (17) is the value of the consumption externality. It represents the gain in utility of the rich by increased consumption by the poor that results from a subsidy increase. The next term to the right represents the loss of utility to the rich of reducing hours of labor by the poor as a result of a subsidy increase.

The maximization of the SWF takes into account the decisions that the other two agents have previously made concerning consumption and labor, which are themselves functions of wages and the subsidy level (equations 9, 13, and 14). Applying equation (17) to our model in specific form we can solve for the optimum subsidy value expressed in terms of the exogenous parameters. An application of the model to a standard Cobb-Douglas model is presented immediately after the complete development of the model in its general form.

4. **Comparative Statics:** Once the equilibrium values for consumption and leisure for the poor and for the rich are established, and the equilibrium level of subsidies have been obtained, the full system is totally differentiated, in order to analyze the general equilibrium effect on our solution of any change in the policy parameters.

The variables of the system are:

Endogenous variables:  $W, U^P, U^r, C^P, C^r, L^P, L^r$ .

Exogenous variables:  $w^P, w^r, s, r, K, \Omega$ .

The complete system of equations is:

$$W - \Omega U^r - (1-\Omega) U^P = 0 \quad (18)$$

$$U^r - U^r(C^r, L^r, C^P, L^P) = 0 \quad (19)$$

$$U^P - U^P(C^P, L^P) = 0 \quad (20)$$

$$C^r - C^r(w^r, w^P, s) = 0 \quad (21)$$

$$C^P - C^P(w^P, s) = 0 \quad (22)$$

$$L^r - L^r(w^r, w^P, s, rK) = 0 \quad (23)$$

$$L^P - L^P(w^P, s) = 0 \quad (24)$$

and totally differentiating the complete system:

$$dW - U^r d\Omega - \Omega dU^r - (1-\Omega) dU^P + U^P d\Omega = 0 \quad (18')$$

$$dU^r - U^r_C dC^r - U^r_L dL^r - U^r_C dC^P - U^r_L dL^P = 0 \quad (19')$$

$$dU^P - U^P_C dC^P - U^P_L dL^P = 0 \quad (20')$$

$$dC^r - C^r_w d w^r - C^r_{w^P} d w^P - C^r_s ds - dr - dK = 0 \quad (21')$$

$$dC^P - C^P_w d w^P - C^P_s ds = 0 \quad (22')$$

$$dL^r - L^r_w d w^r - L^r_{w^P} d w^P - L^r_s ds - dr - dK = 0 \quad (23')$$

$$dL^P - L^P_w d w^P - L^P_s ds = 0 \quad (24')$$

Solving for  $dW/ds$ :

$$\begin{aligned} \frac{dW}{ds} = & C^P_s [ (1-\Omega) U^P_C + \Omega U^R_C ] \\ & + \Omega [ U^R_C \cdot C^R_s + U^R_L \cdot L^R_s ] \\ & + L^P_s [ (1-\Omega) U^P_L + \Omega U^R_L ] \end{aligned} \tag{25}$$

The change in welfare due to a change in subsidy may be broken down into three effects:

- i. The change in social welfare due to a change in consumption by the poor after a subsidy change, as it affects both utilities of the rich and of the poor (first row of equation 25.)
- ii. The change in welfare due to a changes in the utility of the rich derived by their own decisions on consumption and on leisure, after a change in subsidies (second row of equation 25.)
- iii. The change in welfare due to a change in labor supply by the poor as a response to a subsidy change, as it affects the utilities of the rich and of the poor (last row of equation 25.)

It is difficult to determine a priori the overall sign of  $dW/ds$ . Several opposing factors are at work in this expression. First, since the total subsidy bill,  $sC^P$ , represents a lump-sum tax to the rich, only a pure income effect occurs to them. Therefore, the rich's labor supply response to a price subsidy is unambiguously positive. Second, to the poor, the price subsidy behaves like an indirect tax in the sense that changes relative prices, hence both income and substitution effects in consumption and leisure, are present. Third, if the net effect on the poor's labor supply is negative, the rich suffer a negative



externality. The total effect on social welfare will depend on the relative intensity of each individual effect.

5. **The Optimal Subsidy:** Equating equation (25) to zero we solve for the optimum level of subsidy. We replace the marginal utilities of consumption and the marginal disutilities of labor with their equilibrium values in terms of prices, subsidies, and wages. The value for the utility of the rich of labor by the poor is more difficult to determine. Let us assume that to the rich, every hour the poor increase leisure as a response to a subsidy scheme, represents a loss in welfare to the rich. This loss to the rich of an additional hour of leisure by the poor is valued by the rich at certain constant  $k > 0$ , embodying the dislike of the resulting loss of output, together with the dislike of the poor's leisure per se. The value of  $k$  would range according to the intensity with which the rich dislike the poor to increase their leisure. It may be assumed to lay between zero and the marginal productivity of the poor's labor as measured by their wage rate,  $w^p$ , i.e.  $0 < k < w^p$ . These boundaries suggest that the paternalistic rich would consider a loss of some positive value but not greater than the value of output that the poor failed to produce by increasing leisure.

The equilibrium values of the marginal utilities of consumption and of leisure for both agents in our model would be:

$$\begin{aligned}U^r_c &= 1 && \text{(competition price)} \\U^r_c &= s && \text{(subsidy level)} \\U^p_c &= (1-s) && \text{(after-subsidy price)}\end{aligned}$$

$$\begin{aligned}
 U^r_L &= w^r && \text{(wage rate of the rich)} \\
 U^p_L &= w^p && \text{(wage rate of the poor)} \\
 U^r_L &= k && \text{(constant dislike value)}
 \end{aligned}$$

and substituting them into equation (25), it becomes:

$$\frac{dW}{ds} = C^p_s [(1-\Omega)(1-s) + \Omega s] + \Omega [C^r_s + w^r L^r_s] + L^p_s [(1-\Omega)w^p + \Omega k] = 0 \quad (26)$$

solving for s\*:

$$s^* = \frac{\Omega [C^r_s + w^r L^r_s + k L^p_s] + (1-\Omega) [C^p_s + w^p L^p_s]}{(1-2\Omega)C^p_s} \quad (27)$$

Equation (27) provides us with the optimum level of subsidy.

The above results help us identify some of the key parameters that deserve empirical estimation. Subsidies will be determined by the elasticities of consumption and leisure of both rich and poor to changes in subsidies, and by the exogenous variables of the model, including  $\Omega$ . It is evident that we need to know more about the relevant elasticities of consumption demand and labor supply, to changes in subsidies (i.e. prices). Using explicit utility functions, we can derive the pertinent parameters to arrive at an expression for the optimum level of subsidy. The application to a specific model --the Cobb-Douglas, will allow us to analyze conditions and results arising from the assumption of paternalism. We develop this application in the next section.

**6. Cobb-Douglas Application:** Applying our model to Cobb-Douglas utility functions the problem to be solved becomes:

The poor:

$$\begin{aligned} \text{Max}_{C^P, L^P} \ln U^P &= a_1 \ln C^P + a_2 \ln(1-L^P) \\ \text{s.t. } w^P L^P &= (1-s)C^P \end{aligned} \quad (28)$$

The rich:

$$\begin{aligned} \text{Max}_{C^R, L^R} \ln U^R &= b_1 \ln C^R + b_2 \ln(1-L^R) + b_3 C^P + b_4 \ln L^P \\ \text{s.t. } w^R L^R + rK - sC^P &= C^R \end{aligned} \quad (29)$$

Using the same solution technique explained above, we obtain the relevant consumption demand, labor supply, and their corresponding elasticities for both the poor and for the rich. They are respectively:

$$C^P = \frac{a_1 w^P}{(1-s)(a_1 + a_2)}; \quad \frac{\partial C^P}{\partial s} = \frac{a_1 w^P}{(1-s)^2(a_1 + a_2)} > 0 \quad (30)$$

$$L^P = \frac{a_1}{a_1 + a_2}; \quad \frac{\partial L^P}{\partial s} = 0 \quad (31)$$

$$C^R = \frac{b_1 w^R}{b_1 + b_2} + \frac{a_1 b_1}{(a_1 + a_2)(b_1 + b_2)} \frac{s w^P}{(1-s)}; \quad \frac{\partial C^R}{\partial s} = - \frac{a_1 b_1 w^P}{(a_1 + a_2)(b_1 + b_2)(1-s)^2} > 0 \quad (32)$$

$$L^R = \frac{b_1}{b_1 + b_2} + \frac{a_1 b_2 w^P s}{(a_1 + a_2)(b_1 + b_2) w^R (1-s)}; \quad \frac{\partial L^R}{\partial s} = \frac{a_1 b_2 w^P}{(a_1 + a_2)(b_1 + b_2) w^R (1-s)^2} > 0 \quad (33)$$

In the Cobb-Douglas case the poor's labor supply is fixed. The intuition behind this result is based on the hypothesis that at low levels of consumption there is little or no substitutability between consumption and leisure. Consequently, subsidies do not affect the labor supply of the poor. They increase consumption as

a response to a subsidy increase. To the rich an increase in the subsidy level represents a direct income reduction because it is a lump-sum tax determined only by the quantity of the subsidized good demanded by the poor. It is a purely redistributive form of tax. The rich give up some of their own consumption as a response to this tax. They also increase their labor supply. The "first-order" conditions of our Cobb-Douglas specification of the problem tell us that the rich are willing to make transfers to subsidize the poor's consumption up to the point where the marginal utility of paying for the poor is equal to the marginal utility of keeping income for their own consumption.

7. **Paternalism with Cash Transfers:** The next step involves the analysis of the model when the policy consists of direct cash transfers,  $T$ , from the rich to the poor, rather than transfers delivered in the form of price subsidies. We retain our assumption of paternalistic behavior. Therefore, we use utility functions, and a social welfare function similar to the functions used under the price subsidy scheme. What differs under this case is the way in which the scheme affects the budget constraint of the poor. This step requires a reformulation of the budget constraints as follows:

**For the poor:**

$$w^P L^P + T = C^P \quad (34)$$

**For the rich:**

$$w^R L^R + rK - T = C^R \quad (35)$$

The "first-order" conditions of the problem in general form would tell us that the rich would make transfers to the poor up to the point where the marginal utility of transferring income to the poor is equal to the marginal utility of keeping it for their own consumption. Following the same solution procedure used in the subsidy

case, we totally differentiate the system and solve for the optimum level of cash transfers which we obtain as:

$$T^* = \frac{\Omega [C^r_T + w^r L^r_T + kL^p_T] + (1-\Omega) [C^p_T + w^p L^p_T]}{\Omega C^p_T} \quad (36)$$

This is the marginal cash transfers that would have to take place in order for the poor to consume the externality good at a level that is satisfactory to the paternalistic rich.

8. **Subsidies vs. Cash Transfers in General Form:** In order to find the conditions under which the subsidy would be less expensive than cash transfers to obtain the same consumption result, we compare the above expression for cash transfers (equation 36) with the optimum subsidy obtained in equation (27). The following condition is derived:

$$s^* C^p < T^* C^p \quad \text{=====>}$$

$$\frac{\Omega [C^r_s + w^r L^r_s + kL^p_s] + (1-\Omega) [C^p_s + w^p L^p_s] C^p_T}{\Omega [C^r_T + w^r L^r_T + kL^p_T] + (1-\Omega) [C^p_T + w^p L^p_T] C^p_s} < \frac{(1-2\Omega)}{\Omega} \quad (37)$$

The change in poor's labor supply as a response to a subsidy change -- the labor elasticity of subsidy-- is zero for the Cobb-Douglas case, and lower than the labor elasticity of cash transfers. This is because in the former, the substitution effect would have an off-setting effect to the income effect, while in the latter, no substitution effect exists. The same argument applies to the elasticity of consumption. Therefore, the numerator of equation (37) is lower than the denominator and we can say that the left-hand side of equation (37) is less than one. This means that the right-hand side must be greater or equal to one in order

for subsidies to be a less expensive policy for the economy:

$$\frac{(1-2\Omega)}{\Omega} \geq 1 \tag{38}$$

$$1 \geq 3\Omega \quad \text{-----} \quad \frac{1}{3} \geq \Omega$$

The above condition tells us what the value of  $\Omega$  should be for subsidies to be less expensive. If for the government the weight of the preferences of the rich is at least 33.3 percent of the total, it is cheaper for the economy to redistribute by using subsidies rather than by using transfers. The more important the rich are to the government (the higher  $\Omega$  is), the more attractive subsidies are relative to cash transfers. As paternalism losses weight, i.e., as  $\Omega$  decreases below 1/3, the more important the preferences of the poor become, and hence, the more important cash transfers become. This particular result takes us into the political economy aspect of the level of  $\Omega$ : Whether or not a high value for  $\Omega$ , which would give a larger social weight to the rich, would penalize the poor even further and a distributive policy does not result in an increase in happiness of the poor as high as it would be potentially feasible with the same cost to the economy, i.e cash transfers.

For the Cobb-Douglas model with cash transfers the model becomes:

**The Poor**

$$\begin{aligned} \text{Max}_{C^P, L^P} U^P &= a_1 \log C^P + a_2 \log(1-L^P) \\ \text{s.t.} \quad w^P L^P + T &= C^P \end{aligned} \tag{39}$$

$$L^P = \frac{a_1 w^P - a_2 T}{(a_1 + a_2) w^P}; \quad \frac{\partial L^P}{\partial T} = -\frac{a_2}{(a_1 + a_2) w^P} < 0 \tag{41}$$

**The Rich**

$$\begin{aligned} \text{Max}_{C^r, L^r} U^r &= b_1 \log C^r + b_2 \log(1-L^r) + b_3 \log(1-C^p) + (b_4) \log L^p \\ \text{s.t. } w^r L^r + rK - T &= C^r \end{aligned} \quad (42)$$

$$C^{r*} = \frac{b_1 w^r - b_1 T}{(b_1 + b_2)}; \quad \frac{\partial C^r}{\partial T} = -\frac{b_1}{b_1 + b_2} < 0 \quad (43)$$

$$L^{r*} = \frac{b_2 T + b_1 w^r}{(b_1 + b_2) w^r}; \quad \frac{\partial L^r}{\partial T} = \frac{b_2}{w^r (b_1 + b_2)} > 0 \quad (44)$$

9. **Subsidies vs Cash Transfers in the Cobb-Douglas Model:** We intend to find conditions under which the underlying redistribution method is output-increasing for the economy as a whole. Considering a partial equilibrium framework, for redistribution to be output-increasing, it is necessary that the increase in output resulting from an increase in the labor supply of the rich, be larger than the decrease in output, if any, resulting from the decrease in the labor supply of the poor, or:

$$w^r \frac{\partial L^r}{\partial S} > w^p \frac{\partial L^p}{\partial S} \quad \text{For subsidies} \quad (45)$$

$$w^r \frac{\partial L^r}{\partial T} > w^p \frac{\partial L^p}{\partial T} \quad \text{For cash transfers} \quad (46)$$

Using the labor elasticity results obtained under subsidies (equation 33) and under cash transfer (equation 44) and substituting them into equations (45) and (46) we get the conditions under which each scheme is output-increasing. For the case of subsidies we get:

$$\frac{a_1 b_2 w^p}{(1-s)^2 (a_1 + a_2) (b_1 + b_2) w^r} > 0 \quad (47)$$

Price subsidies would be output-increasing if the above result holds. Observing the above expression we can establish that the condition is easily met since all values on the left-hand side of equation (48) are positive. This result is not surprising for the Cobb-Douglas case. We already knew that the poor do not reduce their labor supply as a response to a subsidy while the rich increase theirs.

For the case of cash transfers we have:

$$\frac{b_2}{b_1 + b_2} > \frac{a_2}{a_1 + a_2} \quad (48)$$

Standard consumer theory tells us that leisure is a normal good. The share of leisure in the utility function increases with income. Therefore we may assume that  $a_2$  --the share of leisure in the utility of the poor, is lower than  $b_2$  -- the share of leisure in the utility of the rich. We also know that the rich have two more components in their utility function than the poor, namely the levels of consumption and leisure by the poor. Hence we may safely assume that  $(a_1 + a_2)$  is larger than  $(b_1 + b_2)$  and conclude that the condition in equation (48) is satisfied. Consequently, the application of cash transfers as a redistributive scheme under paternalism is undoubtedly output-increasing. Therefore we cannot reject cash transfers as a scheme beneficial to the economy. What we have to find out now is whether the improvement arising from a cash transfers scheme is better than the improvement obtained with subsidies, since we have already found when it is less costly for the economy to use each scheme. We attempt to answer this question by comparing our equations (45) and (46) in Table 1:



TABLE 1

Subsidies vs Cash Transfers in the Cobb-Douglas Model

Subsidies	Cash transfers
$\frac{a_1 b_2 w^p}{(1-s)^2 (a_1 + a_2) (b_1 + b_2) w^r} > 0 \quad (45)$	$\frac{b_2}{b_1 + b_2} > \frac{a_2}{a_1 + a_2} \quad (46)$
$a_1 b_2 w^p > 0 \quad (47)$	$\frac{(a_1 + a_2) b_2 - (b_1 + b_2) a_1}{(a_1 + a_2) (b_1 + b_2)} > 0 \quad (48)$
	$(a_1 + a_2) b_2 - (b_1 + b_2) a_1 > 0 \quad (49)$
	$b_2 a_1 > a_2 b_1 \quad \rightarrow \quad \frac{a_1}{a_2} > \frac{b_1}{b_2} \quad (50)$

Contrasting the labor response of the rich and the poor to changes in subsidies and to changes in cash transfers and simplifying expression (45) we arrive at equations (47) and (50) in the table. We can be certain that equation (47) holds easily. In comparison, equation (50) states that the ratio of the utility coefficient of the poor must be larger than that of the rich. This condition would not hold unambiguously despite the assumptions about the values of the parameters made previously. Thus we may conclude that from the point of view of total output, using subsidies for redistribution is a less limited policy when preferences of the taxpayers are present, i.e. the conditions under which either policy is output

increasing, are more easily met when using subsidies than using cash transfers.

10. **Altruism:** As defined at the introduction of this chapter, the altruistic individual is concerned about the utility of others but not about the precise composition of their preferences. By accepting the utility function of others, their marginal rate of substitution between goods is also being accepted. The conditions for competitive equilibrium are preserved and the solution is Pareto efficient. The preferences of the rich are separable from their altruistic preferences for the poor when the rich take them into account in their utility function. The specification of the problem would be:

**For the Poor:**

$$\underset{C^P, L^P}{\text{Max}} U^P = U^P(C^P, L^P) \quad (51)$$

$$\text{s.t. } w^P L^P = (1-s) C^P \quad (52)$$

**For the Rich:**

$$\underset{C^R, L^R}{\text{Max}} U^R = U^R(C^R, L^R, U^P(C^P, L^P)) \quad (53)$$

$$\text{s.t. } w^R L^R + rK = C^R + s C^P \quad (54)$$

The optimum level of subsidy under altruism is:

$$s^* = \frac{(w^P L^P_s \Omega - C^P_s) \pm \sqrt{(w^P L^P_s \Omega - C^P_s)^2 - 4(C^P_s \Omega)(\Omega C^P_s + \Omega w^R L^R_s + (1-\Omega) w^P L^P_s)}}{2 C^P_s \Omega} \quad (55)$$

The optimum level of cash transfers under altruism is:

$$T^* = \frac{\Omega [C^R_T + w^R L^R_T] + (1-\Omega) [C^P_T + w^P L^P_T]}{\Omega (C^P_T + w^P L^P_T)} \quad (56)$$

Comparing the expression for transfers (equation 56), with the one obtained for paternalism (equation 36) we can see that the required transfers are higher under paternalism than under altruism. The subsidy numerator is smaller and its denominator is larger. The difference is that under paternalism, with unchanged price ratios, the poor would use their transfer for purchasing that level of consumption of the externality good satisfactory to the rich. But it also means, by the income effect, higher consumption of other goods in which the rich have no interest. In order to have the poor obtain a certain level of consumption without changing the price ratio, it is necessary to make a sufficiently high income transfer to ensure that level of consumption, and at the same time allow for consumption of other goods and (possibly) leisure.

On the other hand, under altruism, since what matters is that the poor attain a higher indifference curve, a higher level of utility may be obtained by increasing consumption of all goods available to the poor. The existence for the poor of a wider range of options for increasing their welfare is what makes altruistic behavior more efficient. This is because it is easier to raise the general utility of an individual when he/she is allowed to increase leisure and general consumption for that purpose, than if any negative labor response by the poor, or increased consumption of other goods, is not valued by the rich.

G. PATERNALISM WITH SUBSIDIES (TWO-GOODS CASE)

In order to allow for inter-good substitution, the next stage of our exercise involves the expansion in the number of goods. Instead of considering one good, we will consider two goods, of which both rich and poor may consume. One of the goods is the externality or merit good,  $C_1$ . It is the preference of the rich that the poor should consume at least some of that good. Since it is the externality producing good, it is subsidized. The market price of this good is  $p_1 = 1$  for normalization purposes. The other good is a market good with price  $p_2$  equal for all agents.

Under the two-good case we analyze two possible ways in which the subsidy system is administered: i) if only the poor have access to the subsidized price for that good, while the rich pay the market price i.e. if the administration of the subsidy system is perfectly targeted to the poor; and ii) if there is no specific targeting to the poor of the subsidy system so that the rich have access to that good at the subsidized prices, i.e. if the subsidy delivery system suffers from leakages and subsidized goods are offered to all groups.

We proceed to the development of the application of our model for both redistributive policies, subsidies and cash transfers to the two-good case.

The statement of the two-good case problem would be:

**For the Poor:**

$$\underset{C_1, L^P}{\text{Max}} U^P = U^P (C_1^P, C_2^P, L^P) \quad (57)$$

$$\text{s.t. } w^P L^P = (1-s) C_1^P, p_2 C_2^P \quad (58)$$

**For the Rich:**

$$\underset{C^r, L^r}{\text{Max}} U^r = U^r(C^r_1, C^r_2, L^r, C^P_1, L^P) \quad (59)$$

If subsidies are targeted only to the poor the budget constraint for the rich is:

$$\text{s.t.} \quad w^r L^r + rK = C^r + sC^P_1 \quad (60')$$

If subsidies "leak" to the rich the budget constraint for the rich is:

$$\text{s.t.} \quad w^r L^r + rK = sC^r + sC^P_1 \quad (60'')$$

The resulting subsidy level for perfect targeting and for no-targeting are respectively:

**For perfect targeting:**

$$s^* = \frac{(1-\Omega) [C^P_{1s} + w^P L^P_s + p^2 C^P_{2s}] + \Omega [w^r L^r_s + p^2 C^r_{2s} + C^r_{1s} + kL^P_s]}{(1-2\Omega)C^P_{1s}} \quad (61)$$

**For no targeting (leakages)**

$$s^* = \frac{(1-\Omega) [C^P_{1s} + w^P L^P_s + p^2 C^P_{2s}] + \Omega [w^r L^r_s + p^2 C^r_{2s} + C^r_{1s} + kL^P_s]}{(1-2\Omega)C^P_{1s} - \Omega C^r_{1s}} \quad (62)$$

where:

$C^P_{1s}$  = Marginal consumption by the poor of good 1 , the subsidized good

$C^r_{1s}$  = Marginal consumption by the rich of good 1 , the subsidized good  
(at subsidized prices if no targeting; at market prices otherwise)

$C^P_{2s}$  = Marginal consumption by the poor of good 2 , the market good

$C^r_{2s}$  = Marginal consumption by the rich of good 2 , the market good

$p^2$  = price of the market good.

We cannot tell a priori how the administration of the subsidy system affects the value of the subsidy. Since relative prices for the rich change when their consumption is subsidized, the marginal propensity to consume by the rich and their labor elasticity is certainly different with and without a price subsidy on their own consumption. The marginal propensity to consume would be higher and probably even positive. The response of labor supply would be lower. However, it would be reasonable to assume that the above two effects cancel each other, at least partially. And also that the increased subsidy enjoyed by the rich is exactly matched by increased taxes. Equations (61) and (62) would still differ by the denominator. The difference rests in the presence in the denominator of equation (62) of the marginal consumption by the rich of the externality good as a response to a subsidy change. In the Cobb-Douglas case we saw that this marginal consumption is negative. In that case, when there is perfect targeting, the subsidy level is higher than when there are leakages of the subsidy program to the rich. The higher unit cost of the subsidy represents the cost to the economy of having a price distortion that is not justified or compensated by an externality producing agent, since the poor do not behave paternalistically towards the rich.

The optimum level of cash transfer for the two-goods case is:

$$T^* = \frac{(1-\Omega) [C_{1T}^P + w^P L^P + P_2 C_{1T}^P] + \Omega [w^R L^R + P^2 C_{2T}^R + C_{1T}^R + k L^P]}{\Omega C_{1T}^P}$$

We assume that under this system of cash transfers the only recipients are the poor.

TABLE 2 summarizes the most important general form expressions for each redistributive policy under different assumptions of the behavior of the rich.

TABLE 2  
SUBSIDY vs. CASH TRANSFER UNDER DIFFERENT TAXPAYERS'S BEHAVIORAL ASSUMPTIONS

Subsidy	Cash-Transfer
<b>Altruism - One Bundle</b>	
$s^* = \frac{(w^p L^p_s \Omega - C^p_s) \pm \sqrt{(w^p L^p_s \Omega - C^p_s)^2 - 4(C^p_s \Omega)(\Omega C^p_s + \Omega w^r L^r_s + (1-\Omega) w^p L^p_s)}}{2 C^p_s \Omega}$	$T^* = \frac{\Omega [C^r_T + w^r L^r_T] + (1-\Omega) [C^p_T + w^p L^p_T]}{\Omega (C^p_T + w^p L^p_T)}$
<b>Paternalism - One Bundle</b>	
$s^* = \frac{\Omega [C^r_s + w^r L^r_s + k L^p_s] + (1-\Omega) [C^p_s + w^p L^p_s]}{(1-2\Omega) C^p_s}$	$T^* = \frac{\Omega [C^r_T + w^r L^r_T + k L^p_T] + (1-\Omega) [C^p_T + w^p L^p_T]}{\Omega C^p_T}$
<b>Paternalism - Two Good - Perfect Targeting</b>	
$s^* = \frac{(1-\Omega) [C^p_{1s} + w^p L^p_s + P_2 C^p_{1s}] + \Omega [w^r L^r_s + P^2 C^r_{2s} + C^r_{1s} + k L^p_s]}{(1-2\Omega) C^p_{1s}}$	$T^* = \frac{(1-\Omega) [C^p_{1T} + w^p L^p_T + P_2 C^p_{1T}] + \Omega [w^r L^r_T + P^2 C^r_{2T} + C^r_{1T} + k L^p_T]}{\Omega C^p_{1T}}$
<b>Paternalism - Two Goods - Subsidy Leakages</b>	
$s^* = \frac{(1-\Omega) [C^p_{1s} + w^p L^p_s + P_2 C^p_{1s}] + \Omega [w^r L^r_s + P^2 C^r_{2s} + C^r_{1s} + k L^p_s]}{(1-2\Omega) C^p_{1s} - \Omega C^r_{1s}}$	$T^* = \frac{(1-\Omega) [C^p_{1T} + w^p L^p_T + P_2 C^p_{1T}] + \Omega [w^r L^r_T + P^2 C^r_{2T} + C^r_{1T} + k L^p_T]}{\Omega C^p_{1T}}$
	(same as above)

#### IV. EXTENSIONS TO THE MODEL

In order to obtain additional information from our model, further complexity could be added to it. For instance, because of the partial equilibrium nature of our model, the analysis in this paper does not consider indirect effects that could arise, via effects of poverty alleviation schemes, on conditions in other markets. For instance, our model assumes fixed wages in the short run. If we allow for flexibility of wages of the two labor markets -- the rich's and poor's, the exercise would require a more exact specification of the production function of each type of good (externality and market good), and allow for substitution among the three factors inputs: two kinds of labor and capital.

With the above extension we would keep the assumption of maximization of profits by the firms with respect to the employment of labor holding capital fixed in the short run. However, constant return to scale would not necessarily hold. The respective marginal rates of productivity would determine the factor prices that would directly affect the budget constraints of both the rich and the poor. Demand for labor would be described by the "first-order" conditions of maximization of profits, i.e. firms equate the marginal product of labor to the real wage.

However, only under very special conditions would inclusion of the labor markets result in levels of consumption by the poor opposite to what was intended by the policy (i.e. a reduction in consumption by the poor of the externality producing good is a result of a subsidy increase). That would be the case if the poor are mainly employed in the production of goods with a very high income elasticity of demand, consumed mainly by the rich. An increase in taxes will reduce demand by the rich of the income-elastic good. Demand for labor from that industry



could be curtailed, with a consequent reduction in nominal wages for the poor. If the subsidies or cash transfers fail to compensate for the wage loss of the poor, an increase in the level of redistribution would ultimately reduce total consumption, including consumption of the subsidized good.

Our concern about the validity of our results when wages are endogenous led us to experiment the "supply side" effect of subsidies with our model. We attempted to include specific production functions for each type of good using the three available factors of production. The experiment resulted in loss of tractability as the model became far too complex for a feasible analytical solution. The model grew significantly in the number of variables and results were impossible to interpret. Instead, we will analyze the properties of the more simple model numerically through simulations and empirical estimation in later research.

The other question we would like to consider is the total cost to the economy of each redistribution scheme. That is, how much is the total tax in both cases that needs to be collected in order to bring all agents in the economy to an optimum level of satisfaction. For this particular question more specific information on the value of the parameters would be required in order to assess the full response of each agent to tax/subsidy changes. With some empirical estimation or/and use of existing estimates we would attempt to answer this question at a later stage.

## V. CONCLUSIONS

The results obtained here are quite appealing from the economic policy perspective. Our results provide the economic conditions under which price subsidies and cash transfers may each be considered more effective and efficient

redistributive tools. The conclusions arrived at under each of the problems stated above, could be broadly summarized as follows: When paternalism is the prevalent behavior of the taxpayers, and taxpayers have a higher weight in society, i.e. a higher  $\Omega$ , the option for redistribution should be to target price subsidies to the poor. This option would bring about a greater improvement in overall social welfare and "happier" taxpayers than with any other policy. With this solution the poor are somewhat better-off, but they would rather receive cash transfers at the same financial cost to the economy. When the rich is typically altruistic there is no distortion in the price system. The preferences of each individual are preserved and the best redistributive policy for the economy as a whole, and for each individual agent, is the use of cash transfers. Increasing the number of goods, or allowing the rich to enjoy subsidized prices do not affect our qualitative results. Only the size of the optimum scheme to be used under the various circumstances would change.

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