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The Long-Term Behavior of Commodity Prices

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and
Brian Wright

The long-term net barter terms of trade between primary commodities and manufactures has been declining 0.6 percent a year.

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This paper — a product of the International Commodity Markets Division, International Economics Department — is part of a larger effort in PRE to understand the short- and long-run behavior of primary commodity prices and the implications of movements in these prices for the developing countries. Copies are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Audrey Kitson-Walters, room S7-053, extension 33712 (54 pages with figures and tables).

Most earlier studies of the long-term trend in the net barter terms of trade between primary commodities and manufactures have suffered from statistical shortcomings, argue Ardeni and Wright.

In their analysis, they use a fairly new statistical approach called structural time series, which, they claim, overcomes those shortcomings.

The tests they ran indicate that the deflated commodity price index is a stationary series with a unit root. The derived structural model outperforms ARIMA models in terms of fit and forecasting. They argue against the idea of "structural breaks" in the data which Sapsford, and Cuddington and Urzua found.

Their results support the conclusion that the net barter terms of trade has declined an estimated 0.6 percent a year — a result consistent with that of Grilli and Yang.

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FOREWORD

The debate over whether there is a downtrend in the long-term net barter terms of trade between primary commodities and manufactures is long-standing. There has been a recent upsurge of interest in this topic with modern time-series techniques being applied to the data. The work reported here uses a fairly new statistical approach called Structural Time Series which, it is claimed, overcomes the shortcomings of techniques used in earlier studies.

The International Commodity Markets Division has an ongoing interest in this topic, because of its important policy implications, and therefore supports research in this area by its own staff or, as in this case, by consultants.

AN ANALYSIS OF THE AGGREGATE LONG-TERM BEHAVIOR OF COMMODITY PRICES

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1. INTRODUCTION

The statistical debate on the net barter terms of trade between primary commodities and manufactures has received a great deal of attention in the recent years. Starting from the Prebisch-Singer hypothesis, the empirical research has focused on the long term movements in commodity prices as well as their cyclical variations. Many studies (Spraos (1980), Sapsford (1985), Thirlwall and Bergevin (1985), Grilli and Yang (1988)) have concluded that there has actually been a deterioration in the net barter terms of trade, although to a lesser extent than predicted by Prebisch (1950) and Singer (1950). Works of Cuddington and Urzúa (1987, 1989), on the other hand, have given no support to the deterioration hypothesis, by emphasizing the cyclical variation of secular movements in commodity prices around a steady level.

Notwithstanding this recent flourishing of empirical evidence on long term movements in commodity prices, the debate is still unsettled, as the statistical methodologies that have been used have several shortcomings, some of which make the results unreliable. Spraos (1980) fitted a simple log-linear time trend variable to the data in a regression estimated via OLS. Sapsford (1985), interpreted the results of Spraos (who found no negative trend in the postwar period) in the light of a possible "omitted" structural break in 1950. By introducing a dummy variable and correcting for serial correlation through the Cochrane-Orcutt technique, Sapsford has been able to recover a negative trend in the net barter terms of trade on post-war data, too. Thirlwall and Bergevin (1985), using quarterly data for disaggregated commodity price indices on the postwar period, have also fitted exponential time trend models, finding evidence of either constant or deteriorating terms of trade. Grilli and Yang (1988), using a series of newly constructed price indices, have estimated a simple time trend model (correcting for serial correlation) finding significant downward trends in the net barter terms of trade.

As Cuddington and Urzúa (1987, 1989) noted, all these studies (if we exclude Grilli's and Yang's) appear to have overlooked the importance of the serial correlation reflected in the price series. In the absence of any inspection of the statistical properties of the univariate representations of the series, all inferences that have been drawn are potentially subject to spurious regression problems. In a regression of a variable against a time trend and a constant, the distribution of the OLS estimator does not have finite moments and is not consistent if the error process is nonstationary (Plosser and Schwert (1978)), and tests of a time trend are biased towards finding one when none is present, if the disturbance is nonstationary (Nelson and Kang (1986)).

The problem appears thus to be the appropriate description of the error process and, therefore, of the series at hand. Cuddington and Urzúa (1987, 1989), following the identification approach suggested by Box and Jenkins (1976), find that most of the series they analyze appear to be nonstationary in the mean. For their study of the Grilli and Yang indices they reject the deterministic trend model in favor of a stochastic trend one by testing the null hypothesis of non-stationarity in the price series using the tests proposed by Dickey and Fuller (1979) and Perron (1988). Excluding a one-time jump that they assume occurred in 1920, they conclude that no deterioration has occurred in the net barter terms of trade from 1900 to 1983. Unfortunately, the limits of their approach tend to weaken the force of their conclusions. In the first place, the simple analysis of the correlograms of the series is not, per se, sufficient evidence in favor of a certain model, since there can be several other models that are consistent with a given set of data. In the second place, tests of unit roots like Dickey and Fuller's (and their corrected versions as suggested by Perron) have been proven to have low power against close alternatives.

In summary, the evidence on the net barter terms of trade between primary commodities and manufactures appears, quite mixed. While the evidence of a deterioration seems incomplete and incorrect on several statistical grounds, the hypothesis of a trendless evolution does not look robust either. The purpose of the present study is to analyze commodity prices by using a fairly new statistical approach that overcomes the shortcomings of the recent literature suggested above. This approach to time series modeling goes under the name of Structural Time Series and tries to model explicitly what we can call the "structural" components of a time series, i.e. the trend, the cycle, and the residual (irregular) components.

The Structural Time Series approach has been proposed by Harvey and others in a number of papers (Harvey and Todd (1983), Franzini and Harvey (1983), Harvey (1985), Harvey and Durbin (1986)). The idea is to formulate a time series model directly in terms of trend, cyclical and irregular components. Since it is often difficult to understand which properties different ARIMA specifications will have in terms of potential decompositions into "secular", "cyclical" and "irregular" components, the alternative is thus to express unobserved components models that have these components explicitly built into their structure.

This approach requires no preliminary assumptions about the properties of the series, e.g. stationarity of the first differences which underlies the decomposition method proposed by Beveridge and Nelson (1981) and used by Cuddington and Urzúa (1987, 1989). Moreover, a structural time series model can be transformed into an ARIMA model which can thus be interpreted as the reduced (restricted) form of the structural model. All the "components" of the series are assumed to follow an individual pattern and are independent and statistically uncorrelated[1]. Technically, structural time series model can be cast in a state-space framework, and estimated through the Kalman filter. Estimates of the individual components can be obtained through a Kalman Smoothing algorithm.

In this study we analyze the aggregate commodity price index (CPI) constructed by Grilli and Yang (1988). They proposed also two alternative deflators: the U.S. Manufacturing Price Index (USMPI) and the United Nations Manufacturing Unit Value (UNMUV) based on internationally traded manufacturing prices.

The paper is organized as follows. In the next section the Structural Time Series approach is briefly outlined. The remainder is then devoted to the statistical analysis of the aggregate deflated commodity price index (CPI). We show that the autocorrelation function of the log of the deflated index is consistent with a stationary ARMA representation and that the evidence of non-stationarity in the data is not as clear-cut as previously claimed. We then perform unit root tests in some univariate representations. Whereas for the Dickey-Fuller tests we can reject the hypothesis of a root of unity, for the Augmented Dickey-Fuller tests (with 4 lags) we are unable to reject the same hypothesis. Several structural and ARIMA models are then estimated and compared. The former appear to have better fit and forecasting performance than the latter. Moreover, the deterministic trend model proves superior in terms of fitting to the stochastic trend model, although in both cases the trend appears to be significantly negative over the entire time series.

2. THE STRUCTURAL TIME SERIES APPROACH

The approac troduced by Box and Jenkins (1976) is based on the idea that a "parsimonious" model from the class of autoregressive integrated moving average

[1]In the Beveridge and Nelson decomposition method, the trend and the cyclical components have the same variance, i.e. they are perfectly correlated.

(ARIMA) process can be identified on the basis of the correlogram and the sample partial autocorrelation function of the observed series. The data are thus used to identify a suitable model, although this can have properties that are difficult to interpret in terms of underlying components.

The structural approach, on the other hand, is based on the idea that a model containing unobserved components can be fitted to the data. As Harvey states it, "the structural times series model is not intended to represent the underlying data generation process. Rather, it aims to present the [stylized] "facts" about the series in terms of a decomposition into trend, cycle, seasonal, and irregular components" (1985, p. 225).

Let yt be the observed variable (in logs). The ***basic structural model*** can be written as 1/

(1) $$y_t = \mu_t + \psi_t + \varepsilon_t \qquad t = 1,\ldots,T$$

where μ_t is a trend component, ψ_t is a cycle component and ε_t is an irregular component. We assume that ψ_t is a stationary linear process, ε_t is a white noise disturbance with variance σ^2, and all the components are uncorrelated with each other. The linear trend can be written as

(2) $$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \quad t = 1,\ldots,T$$

(3) $$\beta_t = \beta_{t-1} + \xi_t \qquad t = 1,\ldots,T$$

where η_t and ξ_t are independent white noise processes with variances σ^2_η and σ^2_ξ respectively. The cyclical component can be modeled as

(4) $$\begin{bmatrix}\psi_t\\ \psi^*_t\end{bmatrix} = \rho\begin{bmatrix}\cos\lambda & \sin\lambda\\ -\sin\lambda & \cos\lambda\end{bmatrix}\begin{bmatrix}\psi_{t-1}\\ \psi^*_{t-1}\end{bmatrix} + \begin{bmatrix}\omega_t\\ \omega^*_t\end{bmatrix} \qquad 0 \le \lambda \le \pi, 0 \le \rho \le 1$$

1/ Here we follow Harvey (1985).

where ω_t and ω_t^* are uncorrelated white noise processes with variances σ^2_ω and $\sigma^2_{\omega^*}$ respectively (ψ_t^* appears by construction). Here λ can be thought of as the frequency of the cycle and ρ as the damping factor of the amplitude. Although this formulation appears rather peculiar, it allows a great variety of processes. The cycle can be rewritten as

$$(5)\qquad \left(1-2\rho\cos\lambda\,L+\rho^2L^2\right)\psi_t=(1-\rho\cos\lambda\,L)\omega_t+(\rho\sin\lambda)\omega_t^*$$

which is an ARMA (2,1) (L is the lag operator). If $\sigma^2_\omega=0$, it reduces to an AR (2) with complex roots, whereas if either $\lambda=0$ or $\lambda=\pi$, then $\psi_t \sim \mathrm{AR}(1)$. Also we assume that $\sigma^2_\omega=\sigma^2_{\omega^*}$.

The basic structural model (1), (2), (3), (5) can be written in *state space* form. The *state* or *transition* equation is:

$$(6)\qquad \begin{bmatrix}\mu_t\\ \beta_t\\ \psi_t\\ \psi_t^*\end{bmatrix}=\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & \rho\cos\lambda & \rho\sin\lambda\\ 0 & 0 & -\rho\sin\lambda & \rho\cos\lambda\end{bmatrix}\begin{bmatrix}\mu_{t-1}\\ \beta_{t-1}\\ \psi_{t-1}\\ \psi_{t-1}^*\end{bmatrix}+\begin{bmatrix}\eta_t\\ \xi_t\\ \omega_t\\ \omega_t^*\end{bmatrix}$$

or, more compactly

$$(7)\qquad \alpha_t = S\alpha_{t-1}+\tau_t\,.$$

where $\alpha_t=\left[\mu_t,\beta_t,\psi_t,\psi_t^*\right]'$, and so on. The *measurement* equation is

$$(8)\qquad y_t=\begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\mu_t\\ \beta_t\\ \psi_t\\ \psi_t^*\end{bmatrix}+\varepsilon_t=z_t'\alpha_t+\varepsilon_t\,.$$

The parameters $\lambda,\rho,\sigma^2_\eta,\sigma^2_\xi,\sigma^2_\omega,\sigma^2$ can be estimated by maximizing the likelihood of the observed sample with respect to these parameters, through the Kalman filter. Maximum likelihood estimators can be obtained either in the time domain or in the frequency domain. The time domain procedure is based on the state space representation above (see Harvey (1981)).

A comparison of different non-nested models can be made on the basis either of the maximized likelihood function or of the prediction error variance (PEV) $\tilde{\sigma}_\rho^2$, which is the steady-state variance of the one-step-ahead prediction error. The R_D^2, defined as (Harvey (1984)):

(9) $$R_D^2 = 1 - T^*\tilde{\sigma}_\rho^2 / \sum_{t=2}^{T}(\Delta y_t - \Delta\bar{y})^2$$

where T* is the number of residuals and T is the total number of observations, is a standardized measure of the goodness of fit reflected in $\tilde{\sigma}_\rho^2$.

Once the estimates of the unknown parameters are obtained, the Kalman filter gives the minimum mean squared estimates of the state vector at time T, i.e. the level, the slope and the cyclical components. The estimate of β_T will be the final estimate of the long run growth rate of y_t, while the estimate of μ_T will be the final estimate of the level of the trend. Estimates of the unobserved components can then be obtained for the whole sample period by Kalman smoothing.

One of the essential characteristics of this model is that it is a local approximation to a linear trend. The level and the slope change slowly over time according to a random walk process. Also, the disturbances in the cyclical component make the cycle stochastic, so that its pattern too varies over time. Equations (6), (7), and (8) also imply an ARIMA representation for y_t:

(10) $$y_t = \frac{\xi_{t-1}}{\Delta^2} + \frac{\eta_t}{\Delta} + \frac{(1-\theta_1 L)\omega + \theta_2\omega^2}{(1-\Phi_1 L - \Phi_2 L^2)} + \varepsilon_t \quad t = 1,\ldots,T$$

(10a) $$y_t = \frac{1}{\Delta^2}[\xi_{t-1} + \Delta\eta_t] + \frac{(1-\theta_1 L)\omega + \theta_2\omega^*}{(1-\Phi_1 L - \Phi_2 L^2)} + \varepsilon_t$$

(10b) $$y_t = \frac{\xi_t}{\Delta^2} + \frac{\theta(L)\omega}{\phi(L)} + \varepsilon_t$$

where $\Delta = (1-L)$ is the first difference operator, $\phi(L)$ is a 2-nd order polynomial in L, $\theta(L)\omega = (1-\theta_1 L)\,\omega + \theta_2\,\omega^*$ and ξ_t is an IMA(1,1) process, since $\xi_t = \Delta\eta_t + \xi_{t-1}$. The first component of the right hand side of (10b) is the trend, whereas the second is the cyclical component. It must be noticed that in order for the minimum mean square estimates of the two components to have finite variances the two polynomials Δ^2 and $\phi(L)$ should not have any root in common. This means that changes in the cyclical component occur independently from the changes in the trend component . Expression (10b) shows that the structural time series model can be thought of as an unobserved component ARIMA model (UCARIMA) as discussed by Engle (1978) and Nerlove, Grether and Carvalho (1979).

The general ARIMA representation of the trend plus cycle model is therefore an ARIMA (2, 2, 4) with no constant term. The two unit roots come from the fact that both the level and the slope of the trend follow a random walk. Thus, if $\sigma_\xi = 0$, i.e. the slope is constant, y_t will be an ARIMA (2, 1, 3). Provided that $\sigma_\eta^2 > 0$, y_t will then be stationary in the first differences. A model where $\sigma_\xi^2 = \sigma_\omega^2 = \sigma^2 = 0$ will correspond to

(11) $$\Delta y_t = \beta + \eta_t$$

which Nelson and Plosser (1982) dubbed as *difference stationary*[1/] In this case, all the variance of the process is attributed to the (stochastic) trend level. Conversely, if $\sigma_\eta^2 = \sigma_\xi^2 = \sigma_\omega^2 = 0$, i.e. all the variance is attributed to the irregular component, the model reduces to

(12) $$y_t = \mu + \beta t + \varepsilon_t$$

[1]This is also the model that Cuddington and Urzúa (1987, 1989) select.

which Nelson and Plosser (1982) called *trend stationary*. Here, μ_t is a deterministic linear trend plus a constant drift. If $\sigma^2_\omega > 0$, then y_t is an ARMA (2, 2).

If the cycle ψ_t is just an AR(2), and $\sigma^2_\xi = 0$ (constant slope), then y_t is an ARIMA (2, 1, 2). However in this case as well as in the previous ones, it is an ARIMA model with restrictions on the parameters. An ARIMA (2, 1, 2) has 5 parameter to be estimated while the basic structural model, with $\sigma^2_\xi = \sigma^2_\omega = 0$ has only 4.

3. TRENDS AND CYCLES IN THE AGGREGATE COMMODITY PRICE INDEX

3.1 ANALYSIS OF THE CORRELOGRAMS.

Both the logarithm of the commodity price index (CPI) deflated by the US Manufacturing price index (USMPI) and the log of CPI deflated by the UN Manufacturing unit value index (UNMUV) visibly show some decline over the whole period 1980-1986, particularly the latter. However, they show a great deal of randomness, too. There are peaks in the 1910's, in the '20's, the '50's, and the '70's (Figures 1A).

In what follows, we will focus on log (CPI/UNMUV), hereafter LPV, on the ground that it appears as a better candidate for a real world commodity price index than log (CPI/USMPI). Although the United States has certainly played a central role in the international trade in commodities during the whole century, a world-trade-weighted price index seems more representative. Moreover, since Cuddington and Urzúa made the same choice, we would like to have a reference for an appropriate comparison of the results. 1/

The correlogram of LPV decays rather slowly, and at lag 14 it is not significantly different from zero. However, for longer lags, there is a substantial negative autocorrelation (see Figures 2A and 2B). Individual values are not

1/ The possibility of greater bias in the manufactured goods index than in the primary commodities index due to quality changes has to be acknowledged, which means a downwards bias in the deflated series.

significant, but there seems to be a long "wave" with a trough at lag 25. Just as the visual inspection of the series suggests that LPV is not mean-stationary (Figure 1A), the rather slow decay of the correlogram of LPV indicates the possibility that first differencing may be needed in order to achieve stationarity. However, although LPV may not be mean-stationary, it does not seem to be a simple random walk.

The correlogram of the first differences of LPV (D1LPV) shows peaks or troughs at lags 2, 10, 16, 25, 36 significant at lags 2, 10, and 16 (Figure 2C). This seems to rule out the possibility that the process generating the LPV series is actually an ARIMA (0,1,1), i.e. a random walk with an MA(1) error component, since for such a process the autocorrelation function is zero at lags greater than 1. Moreover, the sample autocorrelation function from D1LPV is not positive at lag 1 and is not zero at all higher lags, as we would expect if LPV was following a simple random walk process. (For this case, however, the plot of the first differences (Figure 1B) strongly implies mean-stationarity, in line with the assessment of Cuddington and Urzúa (1989)). In sum, neither LPV nor its first differences appear to follow a simple random walk, and their time series processes seems to be considerably more complex.

Denote the autocorrelation at lag τ from the d-th difference of a stochastic process as $\rho_d(\tau)$. For the basic structural model (1), a restriction like $\rho_1(\tau)=0$ for $\tau \geq 2$ implies that $\psi_t = 0$, that is, the process has no cyclical behavior. If we do not impose such a restriction, then we can have a number of stochastic processes that are stationary but whose first difference are consistent with the actual correlogram of the first differences of LPV. Moreover, having a negative value of $\rho_1(1)$ is perfectly compatible with the structural model in (1). In fact, since by construction ε_t and η_t are uncorrelated, $\rho_1(1)$ has to be less than or equal to zero (see Appendix 1).

Consider again the basic structural model (1) and suppose, for simplicity, that $\sigma_\eta^2 = \sigma_\xi^2 = \sigma^2 = 0$ so that (1) can be written as

(13) $$y_t = \mu + \beta t + \psi_t ,$$

that is, the model reduces to the trend-stationary model in (12), since ψ_t is stationary by construction. In its most general form, ψ_t can be, as we have seen above, an ARMA(2,1). Since any ARMA(p,q) model can be approximated by an AR(m), with m large, then we can approximate ψ_t with a higher order AR(p) model. For a stationary AR(p) process we know that (Box and Jenkins, (1976, p. 54)):

(14a) $$\rho_o(\tau) = \phi_1 \rho_0(\tau-1) + \phi_2 \rho_o(\tau-2) + \ldots \phi_p \rho_o(\tau-p)$$

and that, upon solving the Yule-Walker p equations:

(14b) $$\begin{cases} \rho_o(1) = \phi_1 + \phi_2 \rho_o(1) + \ldots + \phi_p \rho_o(p-1) \\ \rho_o(2) = \phi_1 \rho_o(1) + \phi_2 + \ldots \phi_p \rho_0(p-2) \\ \ldots \\ \rho_o(p) = \phi_1 \rho_o(p-1) + \phi_2 \rho_o(p-2) + \ldots + \phi_p \end{cases}$$

we can get the autocorrelation coefficients in terms of the autoregressive parameters. In matrix form

(14c) $$\boldsymbol{\rho} = \mathbf{P}^{-1} \boldsymbol{\phi}$$

where $\boldsymbol{\rho}$ is a vector of p autocorrelation coefficients, $\boldsymbol{\phi}$ is a vector of p autoregressive parameters and $\mathbf{P}$ is the pxp matrix:

$$\begin{bmatrix} (1-\phi_2) & -\phi_3 & \cdots & -\phi_p & 0 \\ -(\phi_1+\phi_3) & (1-\phi_4) & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\phi_{p-1} & -\phi_{p-2} & \cdots & -\phi_1 & 1 \end{bmatrix}$$

If ψ_t is an AR(p), so is y_t. The autocorrelation function of Δy_t will thus satisfy the same difference equation as that of ψ_t, but starting at $\tau = 1$. It turns out that we can express the autocorrelations of the first differences in terms of the autocorrelations of the levels (Harvey (1985, p. 219), see also Appendix 2)

$$\rho_1(\tau) = \frac{2\rho_0(\tau) - \rho_0(\tau-1) - \rho_0(\tau+1)}{2(1-\rho_0(1))} \,. \tag{15}$$

The correlograms of LPV and D1LPV are shown in Table 1. As we can see, the sample autocorrelation function of D1LPV shows significant negative values at lags 2 and 16 and a positive one at lag 10. We have chosen an AR(16) process as a possible candidate able to pick those features. An AR(16) is a process of sufficiently high order (p is almost T/5) that can capture the significant correlation at lag 16. Thus, we have estimated eq. (13) with $\psi_t \sim$ AR(16), getting the following values for the $\hat{\phi}_i$:

$$\begin{aligned}
&\hat{\phi}_1 = .716; \hat{\phi}_2 = -.164; \hat{\phi}_3 = .089; \hat{\phi}_4 = -.019; \hat{\phi}_5 = -.014; \\
&\hat{\phi}_6 = -.014; \hat{\phi}_7 = .085; \hat{\phi}_8 = -.030; \hat{\phi}_9 = .018; \hat{\phi}_{10} = .174; \\
&\hat{\phi}_{11} = -.077; \hat{\phi}_{12} = -.138; \hat{\phi}_{13} = .121; \hat{\phi}_{14} = .016; \hat{\phi}_{15} = -.234; \\
&\hat{\phi}_{16} = -.039.
\end{aligned}$$

From the estimated coefficients we have computed the theoretical autocorrelations and, from these, the theoretical autocorrelation function $\rho_1(\tau)$ for the first differences of y_t. Values are listed in Table 1. Interestingly, the three peaks at lags 2, 10, and 16 are picked rather well by the theoretical autocorrelation function of the first differences of y_t with a deterministic trend and cyclical AR(16) disturbances (Figure 2D), and it can be seen that the pattern is not dissimilar to that of the observed correlogram in Figure 2C.

In sum, the conclusion is that first differencing is not necessarily needed and that a difference-stationary model of the ARIMA type is probably not the best description of the actual process since it would require a positive autocorrelation at lag 1 and zero autocorrelations at higher lags for the first differences. On the contrary, the autocorrelation at lag 1 is negative (although small), whereas some of the autocorrelations at higher lags are nonnegligible, and the overall pattern is not inconsistent with the structural model in (1) with a deterministic trend and cyclical disturbances. Moreover, what the example above shows is that, although the correlogram of LPV may indicate the need for differencing, the correlogram of D1LPV is quite at odds with that. Since the correlogram of D1LPV seems to fit different processes, we may conclude that the mere inspection of the correlogram of LPV is not enough to justify first differencing. Obviously, the mean-stationarity of D1LPV does not imply that LPV is nonstationary, as the first differences of any stationary process are stationary in any case.

3.2 TESTING FOR NONSTATIONARITY

The issue of whether the trend plus cycle model (with a stationary cyclical component and a linear trend) is appropriate depends primarily on the stationarity of the error process. If the disturbances in eq. (13) are to have a single unit root, in fact, then the difference stationary model (11) would be more appropriate, the latter being just a nested model of a more general specification including a linear trend variable

$$y_t = \mu + \beta t + \psi_t \qquad (16)$$

where $\Delta\psi_t$ is a stationary ARMA process. This is the way Cuddington and Urzúa actually specify their research hypothesis (1989, p. 433), although they do not actually test their specification against eq. (16) as the null. Instead, they test it against the null of a unit root.

In the approach introduced by Dickey and Fuller (1979, 1981) unit root tests are performed under the null hypothesis that one root is unity against the alternative that is not. For an AR(1) representation like $y_t = \rho y_{t-1} + \varepsilon_t$, the distribution of the OLS estimator of ρ is not standard under the null hypothesis of $\rho = 1$ and the "t - statistics" do not follow a Student t distribution (Fuller (1976)). Dickey and Fuller

(1979) have computed the limiting distributions for the "t-statistics" of the $\hat{\rho}$'s in the following three models:

(17) $$\Delta y_t = \rho y_{t-1} + \gamma \Delta y_{t-1} + \varepsilon_t$$

(18) $$\Delta y_t = \mu + \rho y_{t-1} + \gamma \Delta y_{t-1} + \varepsilon_t$$

(19) $$\Delta y_t = \mu + \beta t + \rho y_{t-1} + \gamma \Delta y_{t-1} + \varepsilon_t$$

where ε_t is white noise, under the null hypothesis of $\rho = 0$. Here μ is a constant drift, while t is a linear trend. The alternative hypothesis, in the three cases, is that $\rho \neq 0$. Similarly, augmented Dickey-Fuller tests are tests on the "t-statistics" of the $\hat{\rho}$'s in the following:

(20) $$\Delta y_t = \rho y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \varepsilon_t$$

(21) $$\Delta y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \varepsilon_t$$

(22) $$\Delta y_t = \mu + \beta t + \rho y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \varepsilon_t$$

where more lagged differenced terms are included to capture the dynamics (and to insure the $\hat{\varepsilon}$'s are white noise). The limiting distributions, under the null of $\rho = 0$, are the same as above.[1/]

[1/] Several unit-root tests have been introduced in the recent literature, e.g. Sargan and Bhargawa (1983), Said and Dickey (1984), Phillips (1987), Phillips and Perron (1988), Perron (1989). Most of them are based on Dickey's and Fuller's tabulated distributions.

We have performed the Dickey-Fuller unit-root tests on LPV under the three representations in eq. (17) - (19), getting the following results ($\hat{\tau}$ indicates the estimated "t-statistic" of $\hat{\rho}$):

				Critical Values	
Eq.(17)	$H_o: \rho = 0$	$H_A: \rho \neq 0$	$\hat{\tau} = -2.13$	−2.60(1%)	−1.95(5%)
Eq.(18)	$H_o: \rho = 0$	$H_A: \rho \neq 0$	$\hat{\tau} = -2.15$	−3.51(1%)	−2.89(5%)
Eq.(19)	$H_o: \rho = 0$	$H_A: \rho \neq 0$	$\hat{\tau} = -3.79$	−4.04(1%)	3.45(5%)

We are able to reject H_o in the estimation of (17) and (19) at the 5% but we fail to reject it in (18). Also, $\hat{\mu}$ is significant in (19) but not in (18), while $\hat{\gamma}$ is significant in (19). Therefore nonstationarity, if there is any, is only borderline and, overall, we may confidently reject the hypothesis that LPV has a unit root on the basis of the Dickey-Fuller tests.

Nevertheless, the Augmented Dickey-Fuller (ADF) tests give quite a different result. The ADF tests performed on LPV under the three representations in eq. (20) - (22) with p = 4, in fact, give the following results (the null and alternative hypotheses the same are as above):

Eq.(20)	$\hat{\tau} = -1.71$
Eq.(21)	$\hat{\tau} = -1.43$
Eq.(22)	$\hat{\tau} = -2.78$

In all three cases we fail to reject the null hypothesis of a unit root. Therefore, although the latter tests seem to indicate quite uniformly that LPV is nonstationary, these results are overall quite unsatisfactory. It is not simply that the two sets of tests are at odds[1/] but that unit root tests are difficult to reject (and to interpret).

[1/] On the other hand, white-noise Box-Ljung Q tests on the residual of *each of the* six equations could not reject the null that these were indeed white-noise, at the 5% significant level.

Although the case for stationarity is actually quite strong since we *are* able to reject the null of nonstationarity in two cases, these tests lack power. 1/

In summary, the preliminary investigation of the data, based both on the correlograms and on the statistical tests, shows that the evidence of non-stationarity in the aggregate commodity price index is mixed. Although first differencing may be advocated in order to avoid the risks of incorrect inferences due to the presence of unit roots, the dangers of overdifferencing are as grave, (as shown by Plosser and Schwert (1978)), particularly in such a borderline case. It is therefore desirable to have a procedure that would bypass the trade-off between differencing and not differencing. One such a procedure is the structural time series approach we will explore next.

3.3 ESTIMATION, TESTING AND MODEL EVALUATION OF THE BASIC STRUCTURAL MODEL

One of the attractive features of the structural time series approach is that estimation and testing of the basic structural model outlined in eq. (1) - (4) require no preliminary assumptions about the characteristics of the underlying data generating process (e.g. stationarity). Moreover, it seems desirable to have a model that allows, at least in theory, the explicit modeling of the cyclical movements displayed by the series (which showed up, for instance, in the fitting of a deterministic trend model with autoregressive disturbances in the previous section).

The basic structural model, as cast in state-space form in eq. (6) - (8), can be estimated in the time-domain through the Kalman Filter, which gives maximum likelihood estimates of the structural parameters $\sigma^2, \sigma^2_\eta, \sigma^2_\xi, \sigma^2_\omega, \lambda$ and ρ (for details see Harvey and Todd (1983)). A comparison of various models that are non-nested

1/The unit root test performed by Cuddington and Urzúa based on Perron (1988) is conditional on the presence of a one-time jump in the drift in 1921. If the series is non-stationary in the mean, it is actually quite difficult to distinguish a one-time jump from the continuously wandering pattern of a stochastic non-mean-reverting process.

can be made on the basis either of the maximized likelihood function, or of the prediction error variances, or of the R_D^2.

We have estimated several different versions of the following models: (a) the stochastic trend model without the cyclical component, i.e. with $\psi_t = 0$;

(b) the trend plus cycle model, that is, with stochastic trend, stochastic slope and stationary cycle;

(c) the cyclical trend model $y_t = \mu_t + \varepsilon_t$, where $\mu_t = \mu_{t-1} + \beta_{t-1} + \psi_{t-1} + \eta_t$, and β_t and ψ_t are the same as before;

(d) the trend plus cycle model as in (b) with the imposition of an AR(1) cycle, i.e. $\lambda = 0$ or $\lambda = \pi$;

(e) the trend plus cycle model with deterministic linear trend and constant slope, i.e. $\sigma_\eta = \sigma_\xi = 0$;

(f) the trend plus cycle model with constant average growth rate (constant slope), i.e. $\sigma_\xi = 0$;

(g) the stochastic trend model with constant slope and no cyclical component.

In summary, the seven models can be looked at as restricted versions of the basic structural model (1) - (4), if we exclude model (c) where the cycle is "built into" the trend component.

Results for LPV over the sample period (1900-1986) are shown in Table 2. The period of the cycle corresponding to a frequency of λ radians is given by $2\pi/\lambda$ years. The white-noise test is given by the Box and Ljung Q statistic:

$$(23) \qquad Q = T^*(T^* + 2)\sum_{\tau=1}^{P}(T^* - \tau)^{-1} r^2(\tau)$$

where T* is the number of residuals and $r(\tau)$ is the rth autocorrelation in the residuals and T is the total number of observations. In a model with n parameters, Q has χ^2

distribution with P-n+1 degrees of freedom under the null hypothesis. In Table 2A, we chose a value of p = 18.[1/] The heteroscedasticity test is given by

(24) $$H=\left[\sum_{t=T-m+1}^{T}\left(v_t^2/f_t\right)\right]/\sum_{t=k+1}^{m+k}\left(v_t^2/f_t\right)$$

where k=T–T* and m=T*/3 or the nearest integer to it (see Harvery (1985)). Here, v_t is the one-step-ahead prediction error and f_t is the estimate of its variance (both v_t and f_t are obtained from the Kalman filter). The H statistic is approximately distributed as an F with (m, m) degrees of freedom. Harvey (1985) recommends a choice of m=T/3. In our case m=28.

Several interesting features emerge from the estimation results. Whenever the cyclical component is not explicitly set to zero (models (a) and (g)), it has substantial variance. The period of the cycle, however, is very variable.

All the models are satisfactory with respect to the diagnostic Q (serial correlation) and H (heteroscedasticity) tests, despite their different goodness of fit. The maximized log of the likelihood function is not too different across the models, while the prediction error variance and the R_D^2 are quite variable. The variance parameter of the slope, σ_ξ^2, is always found to be zero. The variance of the irregular component σ^2, also, is always found to be basically zero (if rounded off at the fourth decimal), although it must be noticed that a positive value is consistent with the negative (but insignificant) value of $r_1(1)$, the sample autocorrelation at lag 1 of the first differences of LPV.

Models (d) and (e), i.e. the stochastic trend-stochastic slope-AR(1) cycle model and the constant trend-constant slope-cycle model respectively, are the ones to be preferred in terms of goodness of fit (they have reasonable R_D^2 and lowest PEV). Interestingly, the variances of the level $\left(\sigma_\eta^2\right)$ and the slope $\left(\sigma_\xi^2\right)$ are found to be

[1/] Wherever a parameter has been estimated as zero, we did not count it.

MODEL	FEATURES	RESTRICTIONS WITH RESPECT TO B.S.M. (1) - (4)
(b)	Stochastic trend Stochastic slope Cycle	None
(a)	Stochastic trend Stochastic slope No cycle	$\psi_t = 0 \Rightarrow \sigma_\omega = 0$
(g)	Stochastic trend Constant slope No cycle	$\psi_t = 0 \Rightarrow \sigma_\omega = 0$ $\sigma_\xi = 0$
(d)	Stochastic trend Stochastic slope Cycle-AR(1)	$\lambda = 0$
(e)	Constant trend Constant slope Cycle	$\sigma_\eta = \sigma_\xi = 0$
(f)	Stochastic trend Constant slope Cycle	$\sigma_\xi = 0$

zero in the former model, and the cycle basically follows an AR(1) process in the latter model, so that the two estimated models are fundamentally the same. The fact that these two models are preferred is, in itself, evidence in favor of the trend stationary model (12). In both cases, in fact, the trend is estimated as a linear deterministic one, with a non-zero constant drift.

The superiority of the deterministic trend models is confirmed by their better goodness of fit. The stochastic trend models with no cycle (a) and (g) reduce to a random walk plus drift model, since $\sigma_\xi^2 = 0$ in both cases. The gain over the random walk plus drift model, however, is very little (as measured by the R_D^2). The stochastic trend plus cycle model (b) and (f) indicate that a better goodness of fit can be obtained by including a cyclical component in the model, but still the gain over the random walk plus drift is not that satisfactory (up to a 9% better fit). A far better goodness of fit is obtained by the cyclical trend model (c), although it is still inferior to deterministic trend models (d) and (e).

The fact that the cyclical trend model fares better than the trend plus cycle model implies that the trend and the cycle components (if stochastic) cannot be separated (see Harvey (1985, p. 223)). The rate of change of the trend is decomposed into a long-run component, a transitory cyclical component and a random component. However, since σ_ξ^2 is zero and σ_η^2 is basically zero also, the rate of change of the trend is thus equal to a constant term plus the cyclical component. Rewrite the cyclical trend model:

$$\begin{aligned}
y_t &= \mu_t + \varepsilon_t \\
\mu_t &= \mu_{t-1} + \beta_{t-1} + \psi_{t-1} + \eta_t \\
\beta_t &= \beta_{t-1} + \xi_t \\
\psi_t &\sim \text{ARMA}(2,1)
\end{aligned}$$

if $\sigma_\eta^2 = \sigma_\xi^2 = \sigma^2 = 0$, then

$$\mu_t = \mu_{t-1} + \beta + \psi_{t-1} = \mu_0 + \beta t + \sum_{i=0}^{\infty} \psi_{t-1-i} = \mu_0 + \beta t + \frac{1}{1-L}\psi_{t-1}$$

so that

(25) $$y_t = \mu_0 + \beta t + \frac{1}{1-L}\psi_{t-1}$$

that is, y_t is an ARIMA (2,1,2) with a deterministic linear trend and a drift.

The difference between this model and the deterministic trend model implied by the estimated models (d) and (e) is subtle but dramatic. Both models (d) and (e), in fact, reduce to

(26) $$y_t = \mu_0 + \beta t + \psi_t$$

that is y_t is an AR(1) with a deterministic linear trend and a drift. Therefore, both model (c) and model (d) or (e) imply a model with a linear deterministic trend and a drift but two different error processes. In the first case, the error process is stationary in the first differences, while in the second it is stationary in levels. Thus, the two models are nested. However, if one had to choose on the basis of the unit-root tests alone, the choice would have probably been for the former, whereas on the grounds of the better fit and the smaller PEV we would certainly prefer the latter.

In any case, the estimation results show that a cyclical pattern in the LPV series is clearly present, either built within the trend, or in addition to a deterministic trend. The level and the slope of the trend together with the cyclical pattern are estimated by a smoothing algorithm. The final estimates at the end of the sample period are given by the Kalman filter (see Table 2B). In our case, in all the models the slope of the trend remains constant over time. At 1986, the components of the trend have been estimated as follows:

LEVEL $(\hat{\mu}_T)$	SLOPE $(\tilde{\beta}_T)$
–.1427	–.0060
[.0851]	[.0017]

The figures in brackets are RMSE's. Since the observations are in logarithms, the estimated level of the trend at 1986 was exp (–.1427) = .867, while the growth rate of the trend was –0.6% per year.

These end-of-period state coefficients are those resulting from model (d).[1/] They imply a negative value of the trend at 1986 with a negative slope of –0.6% a year. Such values are confirmed by the estimated coefficients of the constant and the slope for model (e): starting from a value of exp (.3768) = 1.46, the trend has decreased along the 87 years of the sample at a rate of –0.6% per year, to end in 1986 at the level of exp (–.1415) = .86.

The estimated components for models (d), (e), and (b) are shown in Figure 3, 4, and 5 respectively.[2/] Comparing the estimated trends from the various models, it is noteworthy that all specifications indicate evidence of a secular deterioration in the real commodity price index all over the sample and no one-time drop in any year. This seems to confirm the thrust of the Prebisch - Singer hypothesis of a persistent steady worsening of the net barter terms of trade (apparently modified by cyclical movements) as opposed to some one-time shift (perhaps due to the resetting of the international trade conditions after the world wars).

Interestingly, these findings are completely at odds with Cuddington and Urzúa's (1989), which give no support to the deterioration hypothesis. In their work, neither the trend-stationary model nor the difference-stationary model show any evidence of secular deterioration in commodity prices. There are two possible explanations for such differences. The first is that Cuddington and Urzúa use a dummy variable to account for an apparent one-time downward shift in the mean after 1920.

[1/] The state coefficients resulting from model (c), as well as those from model (a), are not very informative since they are given by the *actual* values of the level and the slope at the end of the period.

[2/] Again, in the case of model (c), the plot of individual components is not very informative. As a matter of fact, we don't have "individual" components, since the cycle is built within the trend and the two cannot be separated. The plot of the trend coincides with the actual values of the series.

However, although a visual inspection of the plot of the commodity price index series might suggest such a shift, one could argue that other one-time shifts may have occurred, e.g., in 1930, 1950, 1973 or even 1982. Also, a shift may have occurred in the years pre-1900, and the 1982 shift may have been just the beginning of a new twenty-year downward slump. In other words, it seems arbitrary to isolate a single one-time jump in a series that over the long-run varies widely.

The second difference arises from the different approach to the analysis of economic time series. Cuddington and Urzúa, following the Box-Jenkins identification approach, are led to the conclusion that first differencing is needed. The difference-stationary model seems superior to the trend-stationary one (although the actual significance of the latter is obscured by the inclusion of a dummy variable). However, the DS model itself appears to be unable to capture all the characteristics of the series, and this is mainly because of the narrowness of the OLS fitting of ARIMA models. If, in fact, one has to choose on the basis of the significance of the coefficients alone (and keeping in mind the "parsimony" criterion), then the price for simplicity will necessarily be paid in terms of richness of the model. Moreover, the assumption that differencing leads to stationarity is not one to be taken for granted. Although the correlogram of the first differences may die out in the classical fashion, there may be other features of the series that are not captured in a parsimonious ARIMA model.

The structural time series approach tries to explain the characteristics of the observed correlograms with unobserved components which have some desired properties, namely the trend (long-run component), the cycle (transitory cyclical component), and the irregular component. Since it requires no preliminary assumptions on the characteristics of the series (e.g. stationarity of the first differences) it avoids the dangers of incorrect inferences arising from assuming a unit root when none is present. The fact that we were able to find such components both in a stochastic-trend model and in a deterministic-trend one confirms that a similar decomposition is actually a reasonable one, being supported by the data. Also, the evidence of a significant downward sloping trend in either specifications is certainly in favor of the secular deterioration hypothesis and confirms that the finding of Cuddington and Urzúa (1989) of no such deterioration is due to the incorrect treatment of the characteristics of the series and to the decomposition arising from that treatment.

3.4 UNRESTRICTED ARIMA MODEL ESTIMATION

As a matter of comparison, we have estimated several ARIMA models to check whether the standard ARIMA-model selection methodology could have led to the selection of models displaying the same characteristics as the one estimated through the structural time series approach. As we have seen above, the structural model corresponds to an ARIMA model in which the AR and MA parameters are subject to binding restrictions. Given these restrictions, from any structural model it is thus possible to recover an ARIMA model. The question is whether these restrictions would lead to any improvement over the correspondent unrestricted model. If they do, the structural model will prove superior in displaying the desired characteristics, which could have not been uncovered in the unrestricted estimation.

A natural way to compare unrestricted and restricted ARIMA models is the estimated prediction error variance. In the restricted model, this is the one-step-ahead prediction error variance estimated by the Kalman filter. In the unrestricted ARIMA model, the prediction error variance is simply the variance of the disturbances (the squared SSE).

The estimation results for a number of ARIMA models are presented in Tables 3A and 3B. Models in Table 3A correspond to the following forms. For d = 0,

$$\left(1-\phi_1 L-\phi_2 L^2\right)y_t=\mu+\beta t+\left(1-\theta_1 L-\theta_2 L^2\right)\varepsilon_t\,,$$

i.e. an ARMA (p, q) with constant μ and linear trend t, with $p \leq 2$ and $q \leq 2$. For d=1,

$$\left(1-\phi_1 L-\phi_2 L^2\right)\Delta y_t=\mu+\beta t+\left(1-\theta_1 L-\theta_2 L^2\right)\varepsilon_t$$

i.e. an ARIMA (p, 1, q) with constant μ and linear t, and $p \leq 2$ and $q \leq 2$. Models in Table 3B correspond to the same forms, with no linear trend included.

On the basis of the prediction error variance, one would choose an ARIMA (2, 1, 2) from Table 3B $\left(\hat{\sigma}_p^2=0.0109\right)$ and either an ARIMA (1,1,2) with trend $\left(\hat{\sigma}_p^2=0.0107\right)$ or an ARIMA (2, 1, 2) with trend $\left(\hat{\sigma}_p^2=0.0108\right)$ from Table 3A. However, since the "t-statistics" associated with the coefficients are too low, we would drop some of the coefficients, and choose models where all the estimated parameters are significant. In the ARIMA (1, 1, 2) with trend in Table 3A, the estimated $\hat{\beta}$ is not significant. By dropping it, we get the ARIMA (1, 1, 2) in Table 3B with $\hat{\sigma}_p^2=0.116$. Conversely, if we drop the second-order lag MA parameter, we get an ARIMA (1,1,1) with trend, but this turns out to have a non-invertible MA polynomial (the same happens if we exclude the linear trend).

A slightly better fit could be obtained by dropping the time trend from the ARIMA (1,1,2) and estimating it in levels, i.e. with an ARMA (2,2). This gives a $\hat{\sigma}_p^2=0.115$. No other ARMA model without trend fares better. Conversely, a better fit can be obtained with an ARMA (2, 1) with trend $\left(\hat{\sigma}_p^2=0.112\right)$ but, still, the MA coefficient is barely significant. Thus, if we stick to the significance of all the estimated parameters the best we can get is, for the models with no trend, an ARMA (0, 2) (but $\hat{\sigma}_p^2=0.0162$) or an ARMA (1,0) (with $\hat{\sigma}_p^2=0.0123$, but $\hat{\mu}$ is not significant.). An ARMA (2, 1) with trend gives $\hat{\sigma}_p^2=0.0112$ (but θ_1 is barely significant) whereas an ARMA (1, 0) with trend yields $\hat{\sigma}_p^2=0.0114$, with all the coefficients highly significant.

In conclusion, the apparent trade-off between overall fitting and significance of the individual coefficients tends to make the choice of a good model rather arbitrary. The problem with ARIMA-model selection is exactly that, in looking for an adequate parsimonious representation, many different processes may actually yield similar fits, even though they can have very different properties. ARMA models obviously imply very different processes from ARIMA models, just as models with linear trends are very different from models with no trends. Since many of the estimated models seem actually to pass the white-noise test on the residuals, when the estimated prediction error variances are the same, then the choice really boils down to some "a-priori" beliefs regarding the true nature of the process. But this is one reason why we can claim the ARIMA-model selection as being unsatisfactory.

Consider, for instance, the estimated ARMA (1, 2) model in Table 3B:

$$(1-.94L)y_t = \text{constant} + (1+.07L+.3L^2)\varepsilon_t .$$

By multiplying both sides by a common factor (1-L) (if, in fact, both y_t and ε_t have this factor in common, it cancels out) we have

$$(1-.94L)(1-L)y_t = \text{constant} = + (1-L)(1+.07L+.3L^2)\varepsilon_t .$$

Now consider the estimated ARIMA (1, 1, 2) model in Table 3B

$$(1-.68L)(1-L)y_t = \text{constant} + (1+.82L+.22L^2)\varepsilon_t .$$

By multiplying both sides by the common factor (1-.94L) we get

$$(1-.68L)(1-.94L)(1-L)y_t = \text{constant} + (1-.94L)(1+.82L+.22L^2)\varepsilon_t$$

that is

(*) $$(1-1.62L =.6392L^2)(1-L)y_t = \text{constant} + (1+.82L+22L^2)(1-.94L)\varepsilon_t .$$

Now, take the estimated ARMA (2,2) model in Table 3B and multiply it through by (1-L):

(**) $$(1-1.72L+.71L^2)(1-L)y_t = \text{constant} + (1+.86L+.24L^2)(1-L)\varepsilon_t .$$

Although the ARMA (2, 2) shows a slightly better fit, (*) and (**) are approximately the same since

$$\left(1-1.62L+.64L^2\right) \approx (1-.7L)(1-.95L) = \left(1-1.72L+.71L^2\right).$$

The actual difference between the two polynomials is really small and, if the MA root of .94 is approximated to 1, then the ARIMA (1, 1, 2) and the ARMA (2, 2) are almost undistinguishable. Thus, if one is to choose on the parsimony criterion alone (and maybe *a priori* is strongly in favor of non-stationarity), then he would maybe choose the ARIMA (1, 1, 2) model as a good approximation of the underlying process. However, the two models indicate two different views of the world: the ARMA model outlines a persistent cycle around a smooth trend, while the ARIMA one indicates a random walk with a small cyclical variance, where all deviations are persistent.

To conclude, it is interesting to compare the results from unrestricted ARIMA models with the one coming from the restricted ARIMA representations deriving from the structural estimations. As we have seen above, the deterministic linear trend model with AR(1) disturbances was the one with better overall fit $\left(\hat{\sigma}_p^2 = 0.0103\right)$. The unrestricted estimation of such a model, however, gives a prediction error variance 11% larger $\left(\hat{\sigma}_p^2 = 0.0114\right)$. The random walk with drift gives a $\hat{\sigma}_p^2 = 0.0123$, the same as the one given by the structural trend model (as it should be). The cyclical trend model had a $\hat{\sigma}_p^2 = 0.0109$, as does the unrestricted ARIMA (2,1,2). Thus, not only does the structural model yield estimates of the "components" that have a meaningful interpretation, but also it proves superior in terms of actual fit to the data.

3.5 MODEL RELIABILITY AND FORECASTING ACCURACY

With the estimated parameters obtained through the Kalman filter we can make predictions of future values, together with their conditional mean square errors, from the state-space form. These predictions can be made either within the sample or in a post-sample period. The sum of squares of the one-step-ahead prediction errors will give a measure of forecasting accuracy and this measure, too, can be used in order to compare alternative non-nested models. Also, prediction errors in the post-sample period can be compared with the prediction errors within the sample. A statistic to test whether the prediction errors in the post-sample period are significantly greater than the prediction errors within the sample period is given by a Chow-like test, which is distributed as an F with (ℓ, T^*) degrees of freedom, where ℓ is the number of post-sample observations considered, and T* is the number of residuals (as before).

We have reestimated the three structural models (b), (c), and (d) over various subsample periods. One reason to do this was to check the stability of the models over different time intervals: a dramatic change in the estimated parameters would have certainly implied substantial unreliability of the models under investigation. Over the whole sample 1900-1986, model (b), the stochastic trend with stationary cycle model, was the one with least satisfactory fit $(R_D^2 = .03, \hat{\sigma}_p^2 = .0125)$. Model (c), the cyclical trend model, had quite a good fit $(R_D^2 = .15, \hat{\sigma}_p^2 = .0109)$, whereas model (d), the trend plus AR(1) cycle, had the best fit $(R_D^2 = .20, \hat{\sigma}_p^2 = .0103)$.

Over different subsample periods, things change a little (results are shown in Table 4, the first ten columns from left). Starting from the 1900-1985 down to the 1900-1967 sample, the estimated parameters for model (b) change substantially. In the smallest sample, the estimated variance of the cycle is about half the size it had in the original (largest) sample, while the variance of the trend is ten times bigger. Model (b) is clearly not robust. Over the 1900-1985 sample (just one less observation than the original one), the R_D^2 increases to .11 and the PEV decreases to .0111. Breakdowns for model (b) seem to have occurred not only in 1986, but also after 1973 (over the 1900-1973 sample, the PEV was .0104).

Model (c) shows greater stability over the different sub-periods. The estimated parameters change very little (although the variance of the cycle is larger in the 1900-1967 sample than in the original). However, the overall fit of the model worsens considerably. Over the 1900-1985 sample, the R_D^2 falls to .05 and $\hat{\sigma}_p^2$ rises to .0119. Under this criterion, the 1986 year seems to be the only real breakdown for this model, as the fit does not change very much over the other sub-samples.

Model (d) is by far the most stable. The estimated parameters change very little, and the overall fit remains more than acceptable over the various sub-periods. This means that having estimated the model up to 1985, or 1983, or even 1982 would have not made very much difference. The fit is actually worse for the 1900-1980 sample $\left(R_D^2 = .08, \hat{\sigma}_p^2 = .0115\right)$, but it is amazingly better for the 1900-1967 sample $\left(R_D^2 = .22, \hat{\sigma}_p^2 = .0092\right)$.

In conclusion, the trend plus cycle model with deterministic trend, constant slope, and AR(1) cycle, seems the more reliable among the structural models. It has stable parameters over various sub-samples and better fit over the 1900-1985, 1900-1983, 1900-1982, and 1900-1967 periods than both model (b) and (c). Over two samples, the 1900-1980 and the 1900-1973 ones, the stochastic trend model seems to fare better in terms of fit (but only slightly) although it appears to be rather unstable in the estimated parameters.

To obtain a better feel for the overall performance of the three structural models (b), (c), and (d) (which somewhat represent three different views of the world), we have tried to verify their forecasting accuracy from the various sub-samples over to 1986, the final observation year in the original sample period. Two forecasts measures are presented in Table 4. The first ones are based on the conditional predictions given by the one-step-ahead forecasts, whereas the second ones are based on the unconditional predictions, given by the forecasts over long horizons based on the original sample.[1/] The unconditional forecasts are the forecasts made for the period from T+1 to T+ℓ using the observations (and the estimated parameters) up

1/ The results for the long horizons may be the more relevant; the one-year-ahead forecasts may not be very meaningful because of sharp year-to-year variations.

to time T only. The conditional forecasts are made by updating the sample at each step.

Over all sub-sample periods (excluding 1900-1980, and 1900-1973), model (d) shows the best (lowest) final MSE based on the conditional one-step-ahead prediction errors. The post-sample conditional prediction error sum of squares (SS) are lower for model (c), which, also, does not fail the predictive - F test for any of the samples. Model (b) and (d) both fail the latter test for the 1900-1985 sample only. Table 4 shows also results for the CUSUM tests. CUSUMs of standardized generalized recursive residuals are used for detecting structural changes over time (Harvey and Durbin (1986)). However, since the CUSUM is more a diagnostic rather than a formal test we have just indicated if the CUSUM values, for the various models, were within the significance lines. As we can see, all models seem to have passed this test.

Conditional state coefficients at 1986 are also shown in Table 4 (again, the ones for model (c) are not informative). The ones from model (b) change quite a bit, although they tend to decrease, while the ones from model (d) are quite stable. Since these are the would-be estimates of the level and the slope (the cycle is irrelevant) at 1986 obtained from the various sub-periods, they show that in either models the estimated trend at the fixed year is negative as well as the estimated slope. In particular, having estimated model (d) over the 1900-1982 period would have given almost the same responses as over the 1900-1986 period: the estimated level is in fact about exp (–.14) = .87, with a slope of –.06% per year.

Over longer horizons, model (d) appears to be the most satisfactory as it was for the one-step-ahead forecasting. The final MSE at 1986 is always lower, except for the 1900-1980 period, for which model (b) final MSE is the smaller. Unconditional post-sample prediction error sum of squares vary greatly across models and sub-sample periods. An interesting comparison can be made between the unconditional predicted state coefficient at 1986 from the various sub-samples and the ones estimated from the original 1900-1986 sample.

For model (b), the estimated trend level at 1986, given the whole sample, was exp (–.1378) = .87 (Table 2) with a slope of –1.0% per year. The unconditional predicted levels (slopes) at 1986 were: .92 (–0.6%) for 1900-1985; .96 (–.4%) for 1900-1983; .92 (–.5%) for 1900-1982; 1.06 (–.3%) for 1900-1980; 1.01 (–.3%) for 1900-1973; .86 (–.6%) for 1900-1967. Interestingly, although the level varies greatly, the predicted slope is always negative!

For model (d), the estimated level at 1986, given the whole sample, was exp (–.1427) = .87, with a slope of –.6% a year. The unconditional predicted levels (slopes) at 1986 were: .90 (–.5%) for 1900-1985; .91 (–.5%) for 1900-1983; .90 (–.5%) for 1900-1982; 1.00 (–.2%) for 1900-1980; .91 (–.5%) for 1900-1973; .83 (–.7%) for 1900-1967). The trend is almost always predicted as declining, with a negative slope stable around –.6%!

In conclusion, these results suggest that structural models, by allowing a richer representation of complex observed time series than ARIMA models, are able to capture unobserved characteristics of the series that would otherwise have been lost. The trend plus AR(1) cycle, which we find more satisfactory than other structural models, although actually fairly simple, seems to have a very good performance over different sub-samples, both in terms of fit and of forecasting accuracy. The evidence of a secular deterioration in the permanent component of commodity prices (which turns out to be deterministic and linear) is confirmed over all samples, whereas the ability of the model to predict large spikes like the 1973 one appears rather poor (although it is still better than a random walk with drift).

One last issue that should be addressed with this respect is the one of the "structural breaks" as dubbed by Cuddington and Urzúa. As mentioned above, they allow for such structural breaks by adding a dummy variable to an ARIMA model. In this way, the residuals are obviously less irregular (and so the SSE is lower) and the estimated time coefficient, whenever is present, appears to be insignificant. The treatment of structural breaks in such a matter with dummy variables is, however, rather specious.

Nevertheless, we wanted to check whether the addition of a dummy variable to our structural model, in the spirit of *intervention models* of Box and Tiao (1975), would ever change the results we have obtained above. We used two dummy variables, as suggested by Cuddington and Urzúa (1989): the first, called DUMMY, defined as 1 up 1920, and 0 thereafter; the second, called DUM21, defined as 1 for 1921 only, and 0 otherwise. The estimation results for the trend plus AR(1) cycle show little change in the estimated parameters, and a better fit $\left(R_D^2 = .28,\ \hat{\sigma}_p^2 = .0094\right)$. With the addition of DUMMY the state parameters at the end of period change to exp (–.1367) = .87 for the level (the same as before) and –.99% for the slope (it gets steeper than before). The smoothed components are shown in Fig. 6. Amazingly, the trend shows an (obviously artificial) increase in the first part, but then it turns negative anyway! Thus, the addition of that dummy variable really seems to have no implication for the secular movements in the trend and the cycle components.

The addition of DUM21 to the trend plus AR(1) cycle has just a small effect (the damping factor changes to .79), although the fits improve $\left(R_D^2 = .34,\ \hat{\sigma}_p^2 = .0086\right)$. The estimated smoothed components as well as the state coefficients do not change *at all*. As before, we get a level of .87 decreasing at –.6% a year. Finally, the inclusion of both dummies gives basically the same picture as before: a slight change in $\hat{\rho}$ (now .79), and a better fit $\left(R_D^2 = .34,\ \hat{\sigma}_p^2 = .0086\right)$. Interestingly, the effect of DUMMY is now irrelevant. The smoothed components as well as the state coefficients are unchanged. In conclusion, the addition of the dummies appears to add indeed little new information, contributing only to a better fit, and it seems to us more difficult to justify their presence than to give them up.

As a curiosity, one might ask how would have Prebisch's original opinion been formulated, if he had this model? We have estimated the trend plus AR(1) cycle model up to 1938, to see how different the results would have been. The estimated parameters were the following:

$$\hat{\sigma}_{\eta}^{2} = .0;\ \hat{\sigma}_{\zeta}^{2} = .139\text{x}10^{-4};\ \hat{\sigma}_{\omega}^{2} = 138\text{x}10^{-4};\ \hat{\sigma}^{2} = .0;\ \hat{\rho} = .75;\ R_{D}^{2} = .03;\ \hat{\sigma}_{p}^{2} = .0158\,.$$

The state coefficients at 1938 were –.0141 for the level and –.0158 for the slope. Thus, a level of the trend of .99 with a slope of –1.6% a year are indeed much steeper than the actual estimated ones to 1986! In fact, the unconditional prediction for the state coefficients at 1986 were of exp (–.7733) = .46 for the level, with a slope of –1.6%. This means that if we had estimated this model up to 1938 we would have gotten a prediction of falling commodity prices to 50 years later much worse than what they have actually been. But this, nevertheless, shows that Prebisch and Singer were certainly not completely wrong in trying to draw the attention of the world to the deteriorating net barter terms of trade of developing countries. If this is not true now to the same extent, it does not imply that it does not hold, as we have seen above, and that deteriorating tendencies have persisted throughout the last 50 years also.

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Appendix 1.

$$\rho_1(1) = E\left[(y_t - y_{t-1})(y_{t-1} - y_{t-2})\right] / E\left[(y_t - y_{t-1})^2\right]$$

where $y_t = \frac{\xi_{t-1}}{\Delta^2} + \frac{\eta_t}{\Delta} + \frac{(1-\theta_1 L)\omega_t + \theta_2 \omega_t^*}{\phi(L)} + \varepsilon_t$.

Call $\theta(L)\omega_t = (1-\theta_1 L)\omega_t + \theta_t\, \theta_2\, \omega_t^*$

Then:

$$E\left[(y_t - y_{t-1})(y_{t-1} - y_{t-2})\right] =$$

$$= E\left[(\Delta y_t)(\Delta y_{t-1})\right] =$$

$$= E\left[\left(\frac{\xi_{t-1}}{\Delta} + \eta_t + \frac{\Delta\theta(L)\omega_t}{\phi(L)} + \varepsilon_t - \varepsilon_{t-1}\right) x\right.$$

$$\left. x\left(\frac{\xi_{t-2}}{\Delta} + \eta_{t-1} + \frac{\Delta\theta(L)\omega_{t-1}}{\phi(L)} + \varepsilon_{t-1} - \varepsilon_{t-2}\right)\right]$$

$$= E\left[\frac{\xi_{t-1}\xi_{t-2}}{\Delta^2} + \frac{\xi_{t-1}\eta_{t-1}}{\Delta} + \frac{\theta(L)}{\phi(L)}\xi_{t-1}\omega_{t-1} + \frac{\xi_{t-1}\varepsilon_{t-1}}{\Delta} - \frac{\xi_{t-1}\varepsilon_{t-2}}{\Delta}\right.$$

$$+\frac{\xi_{t-2}}{\Delta}\eta_t + \eta_t\eta_{t-1} + \frac{\Delta\theta(L)}{\phi(L)}\eta_t\omega_{t-1} + \eta_t\varepsilon_{t-1} - \eta_t\varepsilon_{t-2}$$

$$+\frac{\theta(L)}{\phi(L)}\omega_t\xi_{t-2} + \frac{\Delta\theta(L)}{\phi(L)}\omega_t\eta_{t-1} + \frac{\Delta^2(\theta(L))^2}{(\phi(L))^2}\omega_t\,\omega_{t-1}$$

$$+\frac{\Delta\theta(L)}{\phi(L)}\omega_t\varepsilon_{t-1} - \frac{\Delta\theta(L)}{\phi(L)}\omega_t\varepsilon_{t-2} + \frac{\varepsilon_t\,\xi_{t-2}}{\Delta} +$$

$$+\varepsilon_t\eta_{t-1} + \frac{\Delta\theta(L)}{\phi(L)}\varepsilon_t\omega_{t-1} + \varepsilon_t\varepsilon_{t-1} - \varepsilon_t\varepsilon_{t-2}$$

$$\left.-\varepsilon_{t-1}\frac{\xi_{t-2}}{\Delta} - \varepsilon_{t-1}\eta_{t-1} - \frac{\Delta\theta(L)}{\phi(L)}\varepsilon_{t-1}\omega_{t-1} - \varepsilon_{t-1}^2 + \varepsilon_{t-1}\varepsilon_{t-2}\right]$$

$$= E\left[\frac{\xi_{t-1}\xi_{t-2}}{\Delta^2}\right] + E\left[\eta_t\eta_{t-1}\right] + E\left[\left(\frac{\theta(L)}{\phi(L)}\right)^2\omega_t\omega_{t-1}\right] -$$

$$-E\left[\left(\frac{\theta(L)}{\phi(L)}\right)^2\omega_{t-1}^2\right] + E\left[\varepsilon_t\varepsilon_{t-1}\right] - E\left[\varepsilon_t\varepsilon_{t-2}\right]$$

$$-E\left[\varepsilon_{t-1}^2\right] + E\left[\varepsilon_{t-1}\varepsilon_{t-2}\right]$$

Since η_t, ξ_t, ω_t, and ε_t are white noises

$$\operatorname{cov}\left[\varepsilon_t \varepsilon_{t-j}\right] = E\left[\varepsilon_t \varepsilon_{t-j}\right] - E\left[\varepsilon_t\right] t\left[\varepsilon_{t-j}\right]$$
$$= E\left[\varepsilon_t \varepsilon_{t-j}\right] = 0$$

Therefore

$$E\left[(y_t - y_{t-1})^2 (y_{t-1} - y_{t-2})\right] = -E\left[\left(\frac{\theta(L)}{\phi(L)}\right)^2 \omega_{t-1}^2\right] - E\left[\varepsilon_{t-1}^2\right]$$
$$= -\left(\frac{\theta(L)}{\phi(L)}\right)^2 \sigma_\omega^2 - \sigma^2$$

Also

$$E\left[(y_t - y_{t-1})^2\right] = \frac{\sigma_\xi^2}{\Delta} + \sigma_\eta^2 + \left[\frac{\theta(L)}{\phi(L)}\right]^2 \sigma_\xi^2 + 2\sigma^2$$

Therefore

$$\rho_1(1) = \frac{-\left[\left(\frac{\theta(L)}{\phi(L)}\right)^2 \sigma_\omega^2 + \sigma^2\right]}{\frac{\sigma_\xi^2}{\Delta} + \sigma_\eta^2 + \left(\frac{\theta(L)}{\phi(L)}\right)^2 \sigma_\omega^2 + 2\sigma^2} < 0$$

Appendix 2.

Claim:

$$\rho_1(\tau)=\frac{2\rho_0(\tau)-\rho_0(\tau-1)-\rho_0(\tau+1)}{2(1-\rho_0(1))}$$

Proof

$$2\rho_0(\tau)-\rho_0(\tau-1)-\rho_0(\tau+1)=$$

$$=\frac{1}{VAR(y_t)}\cdot\left\{2E\left[y_t y_{t-\tau}\right]-E\left[y_t y_{t-\tau-1}\right]-E\left[y_t y_{t-\tau+1}\right]\right\}$$

$$=\frac{1}{VAR(y_t)}\cdot\left\{E\left[2y_t y_{t-\tau}-y_t y_{t-\tau-1}-y_t y_{t-\tau+1}\right]\right\}$$

$$=\frac{1}{VAR(y_t)}\left\{E\left[y_t y_{t-\tau}-y_t y_{t-\tau-1}-y_{t-1}y_{t-\tau}+y_{t-1}y_{t-\tau-1}\right.\right.$$

$$\left.\left.+y_t y_{t-\tau}-y_t y_{t-\tau+1}+y_{t-1}y_{t-\tau}-y_{t-1}y_{t-\tau-1}\right]\right\}$$

$$=\frac{1}{VAR(y_t)}\left\{E\left[(y_t-y_{t-1})(y_{t-\tau}-y_{t-\tau-1})\right]+\right.$$

$$\left.+E\left[y_t y_{t-\tau}-y_t y_{t-\tau+1}+y_{t-1}y_{t-\tau}-y_{t-1}y_{t-\tau-1}\right]\right\}$$

$$=\frac{E\left[(y_t y_{t-1})(y_{t-\tau}-y_{t-\tau-1})\right]}{VAR\left[y_t\right]}$$

$$=\frac{\rho_1(\tau)\cdot E\left[(y_t-y_{t-1})^2\right]}{E\left[y_t^2\right]}=\rho_1(\tau)\cdot\frac{E\left[y_t^2\right]-2E\left[y_t y_{t-1}\right]+E\left[y_{t-1}^2\right]}{E\left[y_t^2\right]}$$

$$=\rho_1(\tau)\frac{2E\left[y_t^2\right]-2E\left[y_t y_{t-1}\right]}{E\left[y_t^2\right]}=\rho_1(\tau)\cdot 2\left(1-\frac{E\left[y_t y_{t-1}\right]}{E\left[y_t^2\right]}\right)$$

$$=\rho_1(\tau)\cdot 2(1-\rho_0(1))$$

TABLE 1. OBSERVED CORRELOGRAM OF THE SERIES (LPV) AND OF ITS FIRST DIFFERENCES (D1LPV) AND THEORETICAL AUTOCORRELATION FUNCTION FOR THE FIRST DIFFERENCES OF Y_t WITH DETERMINISTIC LINEAR TREND AND AR(16) DISTURBANCES (EQ. 13).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
LPV	.805*	.644*	.571*	.506*	.435*	.405*	.413*	.402*	.397*	.403*	.341*	.264*	.251*	.201	.105	.045
D1LPV	–.015	–227*	–.051	–040	–.064	–.066	.002	–.038	.015	.203*	.060	–.075	.040	.038	–.155	–.230*
Δy_t	–.060	–280*	.007	.006	–.162	–.087	.080	–.062	–.041	.259*	.076	–.199	.081	.175	–.216*	–.212*

Note: $y_t = \mu + \beta t + \psi_t$, with $\psi_t \sim AR(16)$. The theoretical autocorrelation function $\rho_0(\tau)$ for y_t was computed using eq. (14c), given the estimated coefficients $\hat{\phi}_i$ from eq. (13). The autocorrelation function for Δy_t was computed using the formula given in (15). The standard error is 1.96 x $T^{-1/2}$ = .210. A star (*) indicates a significant value.

TABLE 2A

STRUCTURAL TIME SERIES MODELS   SAMPLE PERIOD: 1900-1986 (OBS)   SERIES: LPV

MODEL	ESTIMATES x 10^{-4} σ^2_η (LEVEL)	σ^2_ξ (SLOPE)	σ^2_ω (CYCLE)	σ^2 (IRREG.)	ρ	λ (FREQ.)	$2\pi/\lambda$ PERIOD	Log L	$\tilde{\sigma}^2_p \times 10^{-1}$ PEV	R^2_D	Q* Box-Ljung	H**.
(a)	129.0	0	—	0.003	—	—	—	140.13	.123	.05	21.31(17)	.8149
(b)	2.753	0	99.153	0	.90	.30	20.9	139.13	.125	.03	11.25(14)	.8813
(c)	0.007	0	99.122	0	.48	1.58	4.0	142.32	.109	.15	9.703(15)	.6580
(d)	0	0	112.00	0	.75	.0	—	142.87	.103	.20	16.52(17)	.8652
(e)	—	—	112.00	0.112	.75	0.36	175.8	142.86	.104	.20	11.19(15)	.8540
(f)	25.016	-	61.821	0.079	.68	.62	10.1	143.44	.117	.09	6.22(14)	.8221
(g)	128.00	—	—	0.534	—	—	—	140.20	.129	.00	21.78(17)	.8142

0

*Box-Ljung Q-statistic with (18-r+1) degrees of freedom (in parenthesis).
**Hsteroscedasticity statistic with (28, 28) degrees of freedom.
Values in the square brackets are RMSE's. for the state coefficients at the end of period.

TABLE 2 B

STRUCTURAL TIME SERIES MODELS SAMPLE PERIOD: 1900-1986 (OBS)
SERIES: LPV

	STATE COEFFICIENTS			
	LEVEL	SLOPE	FIXED CONSTANT	FIXED SLOPE
(a)	–.3884	–.0113	—	—
	[.0000]	[.0168]		
(b)	–.1378	–.0062	—	—
	[.0916]	[.0027]		
(c)	–.3884	–.0099	—	—
	[.0220]	[.0124]		
(d)	–.1427	–.0060	—	—
	[.0851]	[.0017]		
(e)	—	—	.3708	–.0059
			[.0651]	[.0017]
(f)	.3380	—	—	–.0065
	[.4747]			[.0055]
(g)	.2868	—		–.0078
	[1.061]			[.0122]

TABLE 3A

ARIMA MODELS WITH A LINEAR TREND   SAMPLE PERIOD: 1900-1986 (OBS) — SERIES: LPV

		$\hat{\mu}$	$\hat{\beta}$	$\hat{\phi}_1$	$\hat{\phi}_2$	d	$\hat{\theta}_1$	$\hat{\theta}_2$	R^2	$\hat{\sigma}^2_p \times 10^{-1}$	DW	$\phi(P)$.
ARMA												
1.	(1,0)	.11	−.002	.72	—	0	—	—	.73	.114	1.76	22.38(26)
		(2.91)	(−2.87)	(8.99)								
2.	(2,0)	.13	−.002	.82	−.15	0	—	—	.74	.114	1.93	20.07(25)
		(3.18)	(−3.11)	(7.26)	(−1.29)							
3.	(1,1)	.16	−.002	.59	—	0	.25	—	.74	.112	1.97	19.22(25)
		(2.85)	(−2.80)	(4.73)			(1.72)					
4.	(1,2)	.12	−.002	.69	—	0	.14	−.11	.74	.114	1.96	17.87(24)
		(1.60)	(−1.67)	(3.66)			(.66)	(−.59)				
5.	(2,1)	.18	−.002	.34	.19	0	.50	—	.74	.115	1.97	18.36(24)
		(2.49)	(−2.42)	(.69)	(.48)		(1.05)					
6.	(2,2)	.13	−.002	.64	−.02	0	.18	−.10	.76	.116	1.96	17.87(23)
		(.57)	(−.58)	(.46)	(−.03)		(.13)	(−.27)				
7.	(0,1)	.37	−.006	—	—	0	.67	—	.69	.131	1.50	45.03(26)
		(9.12)	(−7.03)				(7.97)					
8.	(0,2)	.38	−.006	—	—	0	.81	.30	.72	.120	1.85	26.50(25)
		(7.59)	(−5.90)				(7.52)	(2.83)				
ARMA												
9.	(1,1,0)	.006	−.0003	−.018	—	1	—	—	.00	.121	.198	28.67(26)
		(.23)	(−.60)	(−.16)								
	(1,1,1)=>NON INVERTIBLE											
10.	(2,1,0)	.005	−.0003	−.028	−.24	1	—	—	.06	.129	2.00	23.12(25)
		(.17)	(−.59)	(−.25)	(−2.17)							
11.	(1,1,2)	.001	0.0	.72	—	1	−.92	−.21	.22	.107	2.07	20.66(24)
		(.25)	(−.40)	(5.87)			(−5.16)	(−1.33)				
	(2,1,1)=>NON INVERTIBLE											
12.	(2,1,2)	.002	0.0	.42	.26	1	−.61	−.55	.23	.108	2.09	20.04(23)
		(.34)	(−.43)	(.72)	(.50)		(−1.08)	(−.89)				
13.	(0,1,1)	.006	0.0	—	—	1	−.03	—	0.0	.133	1.95	28.80(26)
		(.23)	(−.61)				(−.31)					
14.	(0,1,2)	0.0	0.0	—	—	1	−.13	−.36	.09	.125	1.86	23.14(25)
		(−.04)	(−.51)				(−1.26)	(−3.33)				

TABLE 3B

ARIMA MODELS WITH NO LINEAR TREND SAMPLE PERIOD: 1900-1986 (OBS) — SERIES: LPV

		$\hat{\mu}$	$\hat{\phi}_1$	$\hat{\phi}_2$	d	$\hat{\theta}_1$	$\hat{\theta}_2$	R^2	$\hat{\sigma}_p^2 \times 10^{-1}$	DW	$\phi(P)$.
ARMA											
1.	(1,0)	.009	.87	—	0	—	—	.71	.123	1.85	23.03(26)
		(.64)	(14.21)								
2.	(2,0)	.01	.92	–.06	0	—	—	.71.	.126	1.92	22.09(25)
		(.72)	(8.10)	(–.51)							
3.	(1,1)	.01	.86	—	0	.11	—	.71	.125	1.98	21.42(25)
		(.79)	(10.78)			(.86)					
4.	(1,2)	.0	.94	—	0	–.07	–.30	.72	.121	1.89	20.66(24)
		(.01)	(17.37)			(–.56)	(–2.42)				
	(2,1)=>NON INVERTIBLE										
5.	(2,2)	.0	1.72	–.71	0	–.86	–.24	.76	.115	2.01	18.60(23)
		(–.48)	(10.16)	(–4.72)		(–3.59)	(–1.50)				
6.	(0,1)	.13	—	—	0	.75	—	.52	.201	1.15	131.88(26)
		(4.80)				(10.18)					
7.	(0,2)	.13	—	—	0	.96	.43	.62	.162	1.63	66.91(25)
		(4.04)				(9.44)	(4.35)				
ARIMA											
8.	(1,1,0)	–.008	–.02	—	1	—	—	.00	.132	1.97	28.54(26)
		(–.63)	(–.14)								
9.	(2,1,0)	–.009	–.02	–.24	1	—	—	.06	.128	2.00	23.14(25)
		(–.75)	(–.23)	(–2.17)							
	(1,1,1)=>NON INVERTIBLE										
10.	(1,1,2)	.0	.68	—	1	–.82	–.22	.14	.116	1.98	19.00(24)
		(–.31)	(5.55)			(–5.39)	(–1.48)				
	(2,1,1)=>NON INVERTIBLE										
11.	(2,1,2)	.0	.53	.18	1	–69	–.45	.22	.109	2.09	20.46(23)
		(.23)	(.86)	(.33)		(–1.12)	(–.67)				
12.	(0,1,1)	–.007	—	—	1	–.03	—	.00	.130	1.95	28.90(26)
		(–.64)				(–.28)					
13.	(0,1,2)	–.006	—	—	1	–.13	–.36	.08	.121	1.86	23.44(25)
		(–1.02)				(–1.21)	(–3.34)				
					1						

TABLE 4

CONDITIONAL AND UNCONDITIONAL FORECASTS FOR STRUCTURAL MODELS

MODEL	σ^2_η (x 10^{-4})	σ^2_ξ (x 10^{-4})	σ^2_ω (x 10^{-4})	σ^2 (x 10^{-4})	ρ	λ	$2\pi/\lambda$	$\tilde{\sigma}^2_p$ (x 10^{-1})	R^2_D	SS
(b)										
1900-1985	5.738	.0	90.596	.0	.80	.31	20.0	.111	.11	1.0335
1900-1983	9.870	.012	80.539	.0	.66	.45	13.9	.115	.09	.9760
1900-1982	19.370	.0	64.423	.0	.69	.60	10.4	.113	.11	.9596
1900-1980	20.150	.0	60.990	.0	.70	.62	10.1	.111	.12	.9175
1900-1973	22.322	.0	54.852	.0	.72	.60	10.6	.104	.10	.7908
1900-1967	23.232	.0	55.540	.0	.72	.60	10.6	.106	.10	.7400
(c)										
1900-1985	.0	.0	96.786	.0	.48	1.63	3.9	.119	.05	1.0372
1900-1983	.0	.0	99.046	.0	.46	1.62	3.9	.121	.04	1.0265
1900-1982	.0	.0	100.000	.0	.45	1.60	3.9	.121	.04	1.0192
1900-1980	.0	.0	102.000	.0	.43	1.60	3.9	.121	.03	.9889
1900-1973	.0	.0	101.000	.0	.36	1.57	4.0	.114	.02	.8523
1900-1967	.0	.0	105.000	.0	.33	1.59	3.9	.117	.01	.8034
(d)										
1900-1985	.0	.0	108.000	.0	.76	—	—	.099	.21	1.0089
1900-1983	.0	.0	109.000	.0	.74	—	—	.100	.21	.9930
1900-1982	.0	.0	109.000	.0	.75	—	—	.100	.21	.9855
1900-1980	.0	.011	107.000	.0	.74	—	—	.115	.08	.9540
1900-1973	.0	.001	102.000	.0	.78	—	—	.105	.09	.8236
1900-1967	.0	.0	103.000	.0	.77	—	—	.092	.22	.7687

TABLE 4 continued-1

CONDITIONAL AND UNCONDITIONAL FORECASTS FOR STRUCTURAL MODELS

MODEL	FINAL MSE	CONDITIONAL FORECASTS AT 1986 ERROR SS	CHOW (ℓ, T^*)	CUSUM	CONDITIONAL FORECASTS STATE COEFFICIENTS AT 1986 Level	Slope
(b)						
1900-1985	.0117	.0458	3.92(1,84)*	OK	–.1451	–.0065
1900-1983	.0116	.0776	2.23(3,82)	OK	–.1709	–.0081
1900-1982	.0115	.0794	1.72(4,81)	OK	–.2108	–.0076
1900-1980	.0113	.1216	1.79(6,79)	OK	–.2154	–.0077
1900-1973	.0106	.2514	1.81(13,72)	OK	–.2262	–.0078
1900-1967	.0108	.3024	1.47(19,66)	OK	–.2281	–.0080
(c)						
1900-1985	.0121	.0387	3.20(1,84)	OK	–.3884	–.0099
1900-1983	.0123	.0496	1.35(3,82)	OK	–.3884	–.0099
1900-1982	.0123	.0572	1.16(4,81)	OK	–.3884	–.0099
1900-1980	.0123	.0895	1.21(6,79)	OK	–.3884	–.0099
1900-1973	.0116	.2355	1.56(13,72)	OK	–.3884	–.0099
1900-1967	.0119	.2883	1.28(19,66)	OK	–.3884	–.0099
(d)						
1900-1985	.0113	.0491	4.36(1,84)*	OK	–.1405	–.0060
1900-1983	.0113	.0651	1.91(3,82)	OK	–.1406	–.0060
1900-1982	.0114	.0726	1.59(4,81)	OK	–.1420	–.0060
1900-1980	.0116	.1119	1.61(6,79)	OK	–.1394	–.0068
1900-1973	.0107	.2370	1.71(13,72)	OK	–.1497	–.0062
1900-1967	.0108	.2907	1.42(19,66)	OK	–.1457	–.0061

*Reject at 5%.

TABLE 4 continued-2

CONDITIONAL AND UNCONDITIONAL FORECASTS FOR STRUCTURAL MODELS

UNCONDITIONAL FORECASTS (EXTRAPOLATIONS): 1986

MODEL	FINAL MSE	ERROR SS	STATE COEFFICIENTS AT 1986 Level	Slope
(b)				
1900-1985	.0117	.0458	–.0880	–.0065
1900-1983	.0255	.0512	–.0437	–.0044
1900-1982	.0293	.0305	–.0842	–.0051
1900-1980	.0332	.0623	.0563	–.0025
1900-1973	.0570	.0241	.0078	–.0033
1900-1967	.0837	.0246	–.1466	–.0058
(c)				
1900-1985	.0121	.0387	–.1916	–.0065
1900-1983	.0323	.0344	–.0856	–.0047
1900-1982	.0421	.0173	–.1592	–.0062
1900-1980	.0635	.0615	.0687	–.0019
1900-1973	.1536	.0517	.1298	–.0008
1900-1967	.2551	.0315	–.1826	–.0059
(d)				
1900-1985	.0113	.0491	–.1055	–.0054
1900-1983	.0229	.0334	–.0906	–.0061
1900-1982	.0264	.0209	–.1057	–.0053
1900-1980	.0339	.0481	.0018	–.0015
1900-1973	.0397	.0156	–.0935	–.0050
1900-1967	.0433	.0300	–.1795	–.0065

FIG 1A: AGGREGATE PRICE INDEX - LPV

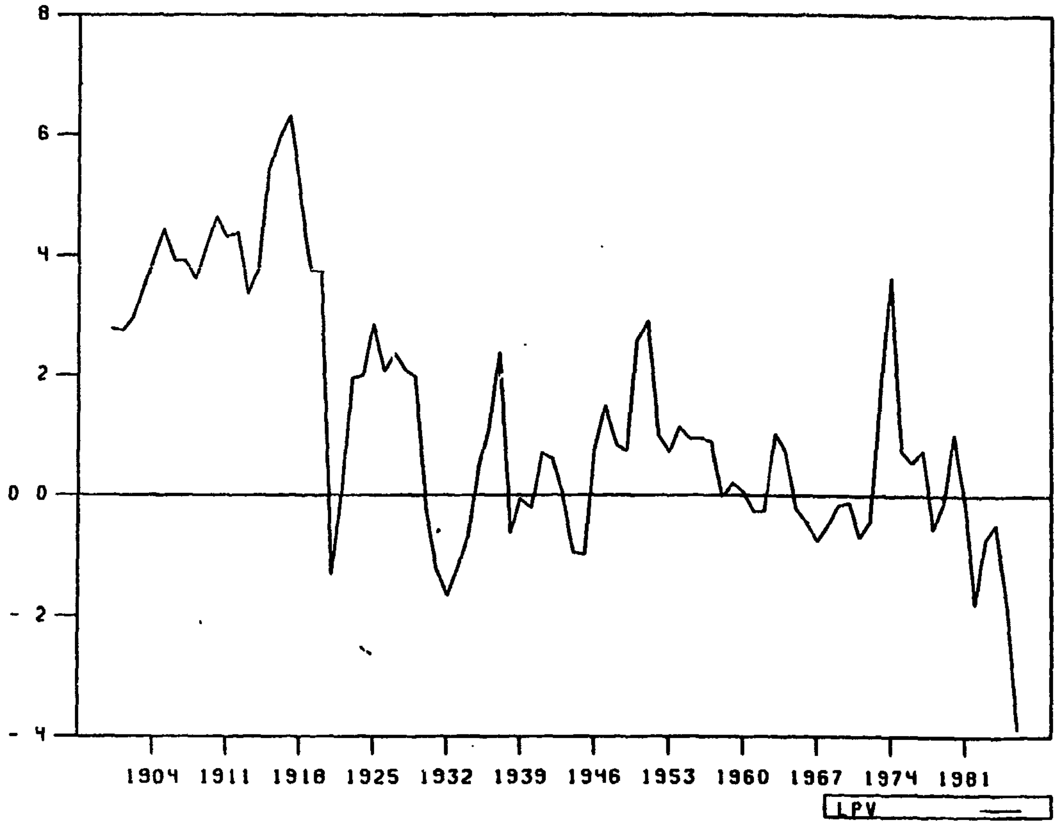


FIG 1B: FIRST DIFFERENCE OF LPV - D1LPV

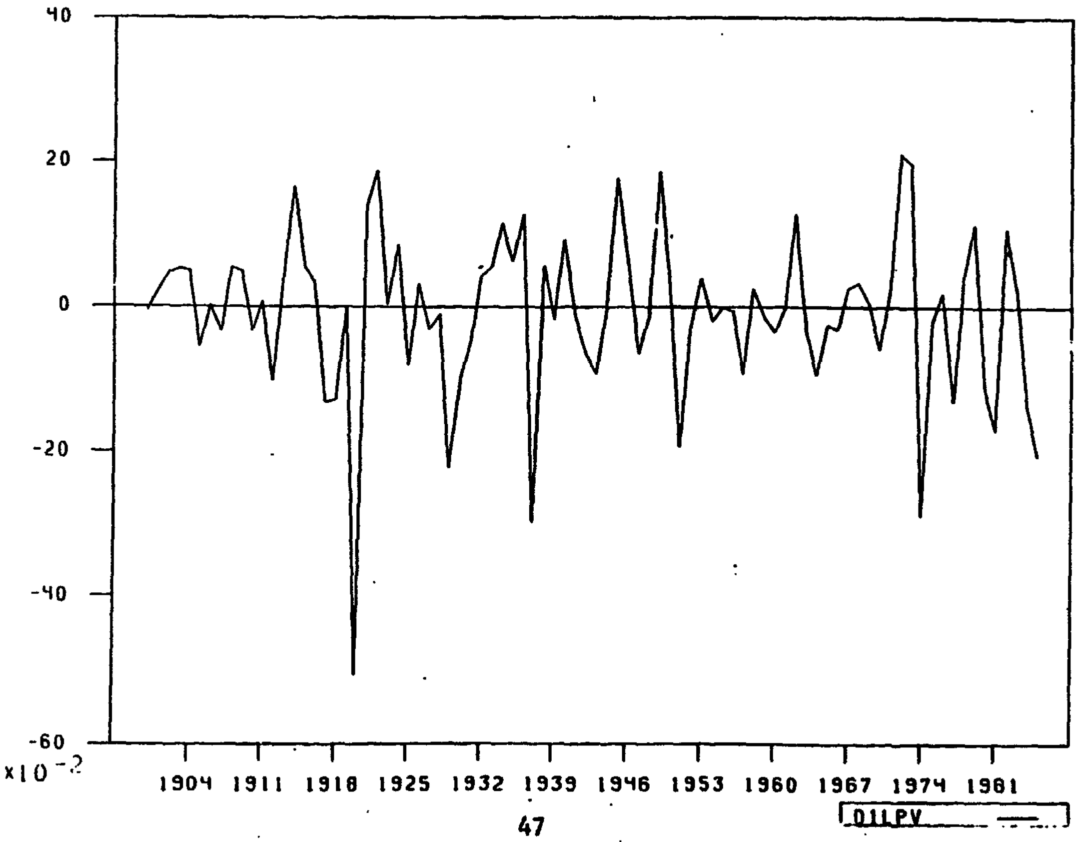


FIG 2A: AUTOCORRELATION FUNCTION OF LPV

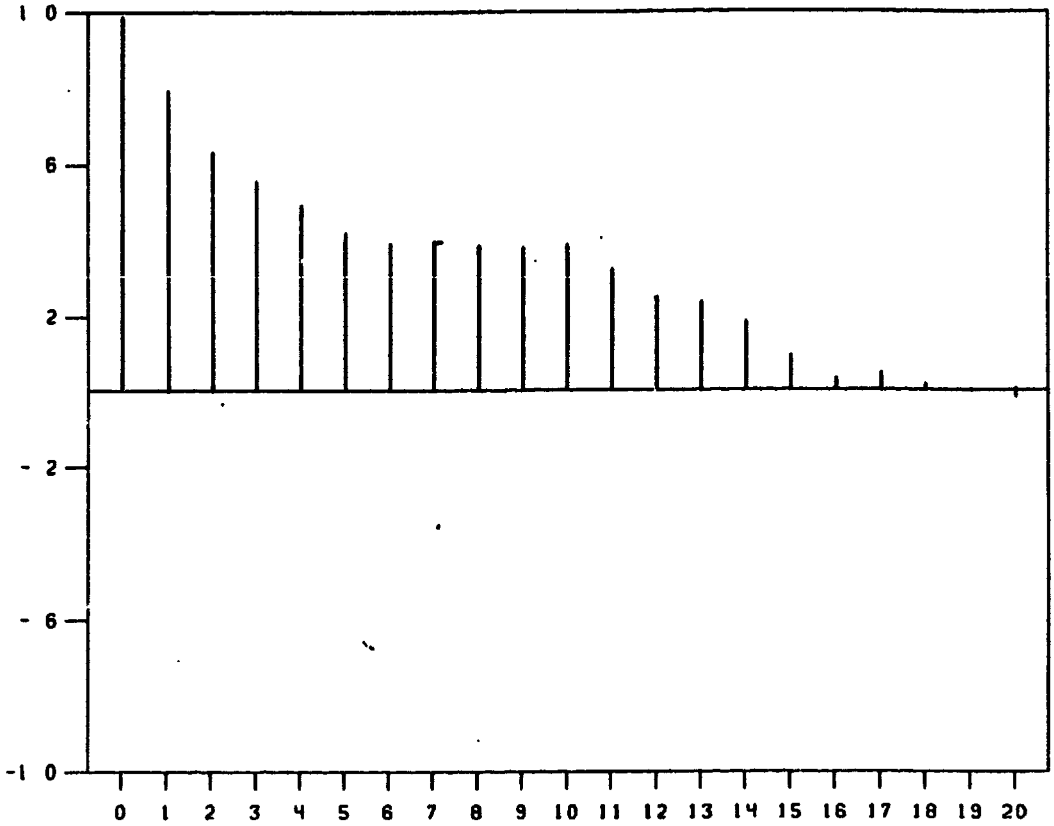


FIG 2B: AUTOCORRELATION FUNCTION OF LPV

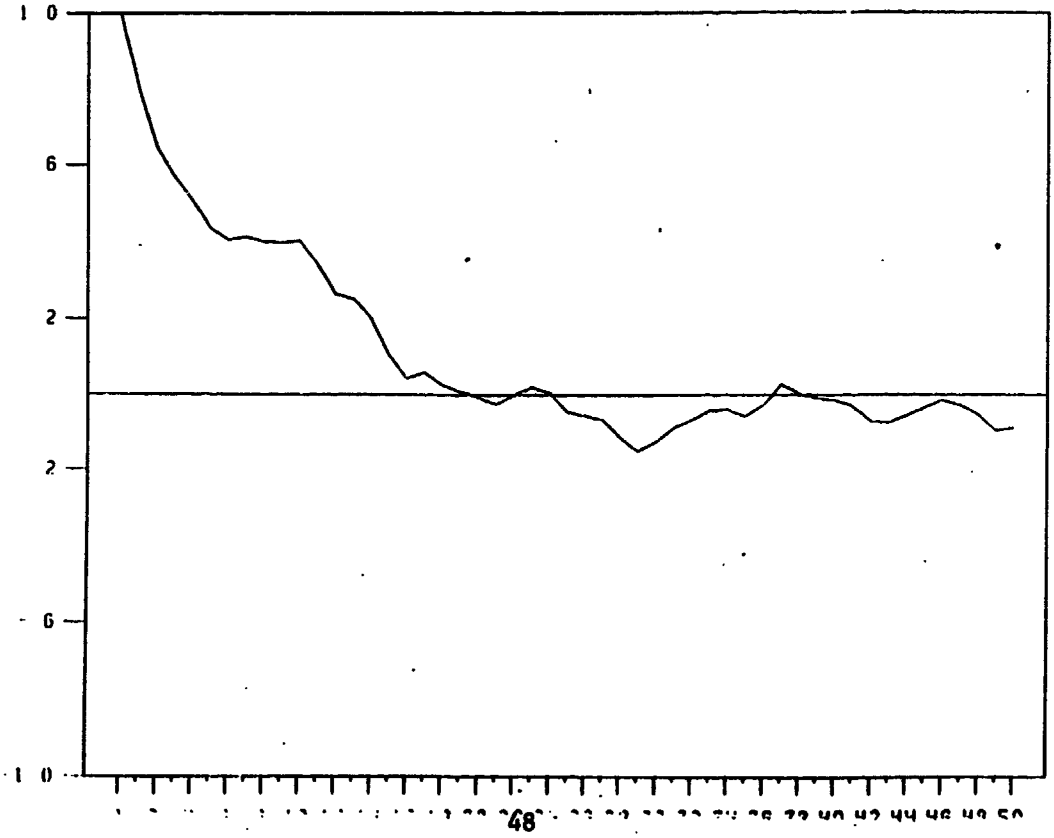


FIG 2C: AUTOCORRELATION FUNCTION OF DILPV

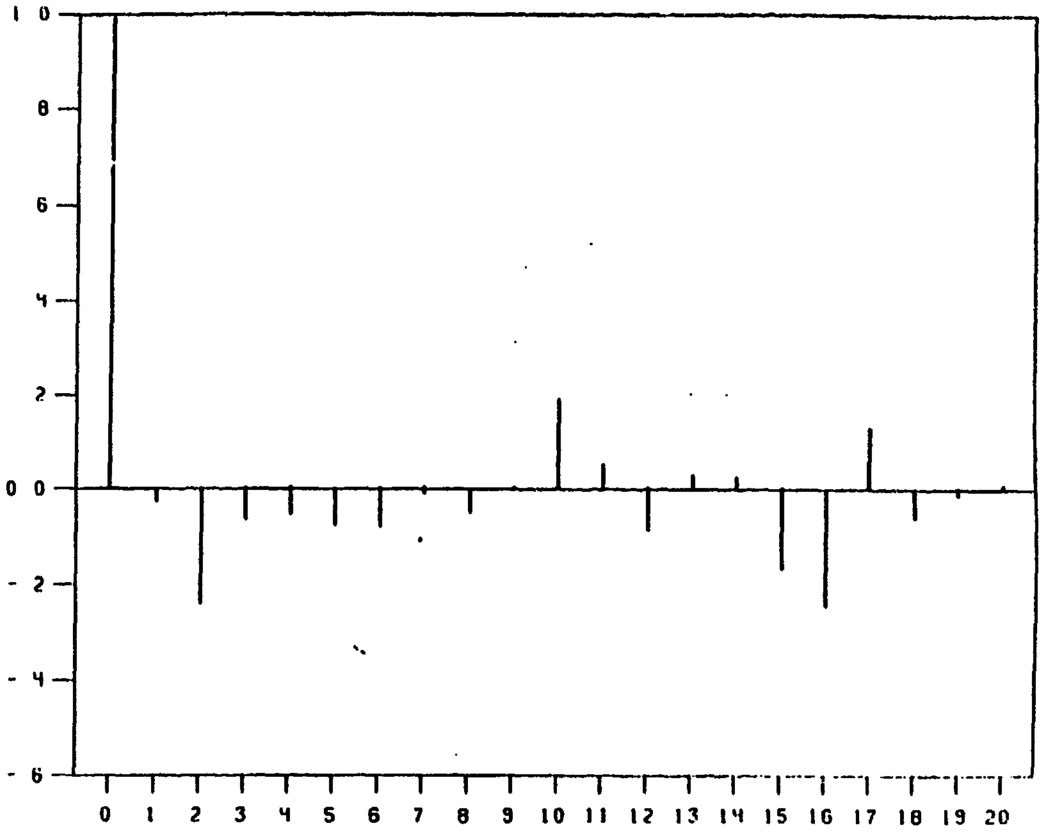


FIG 2D: THEOR AUTOCORR OF FDIFF OF Y, AR(16) RES

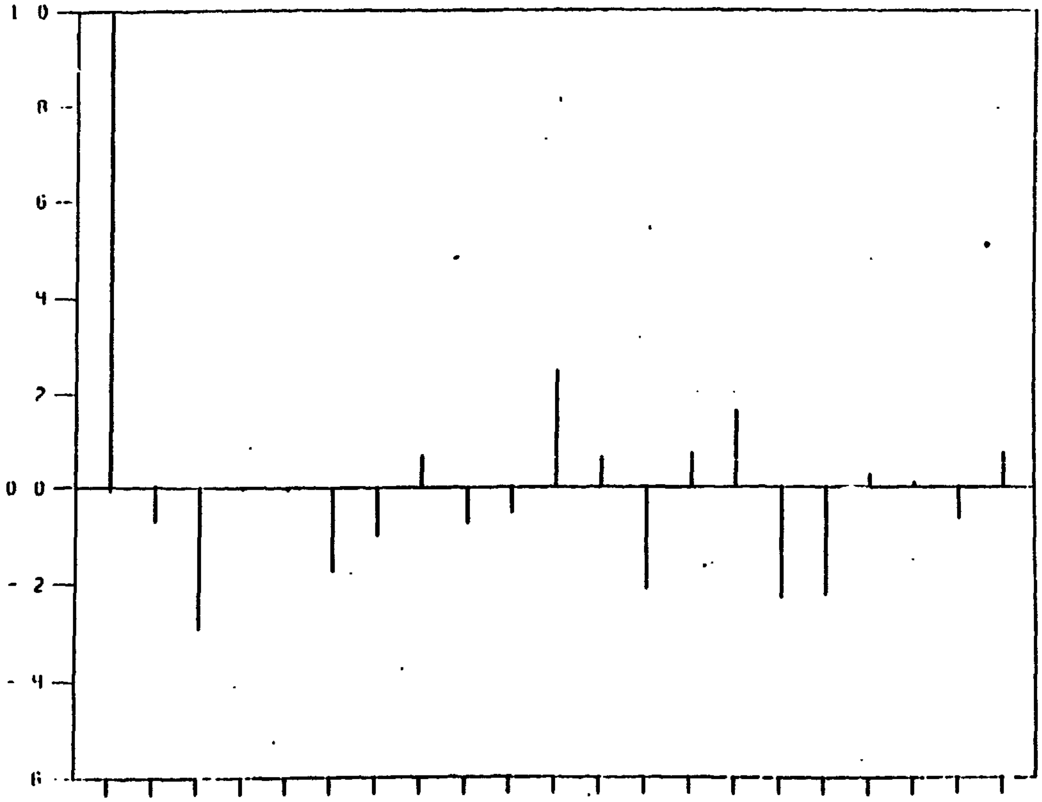


FIG 3A - MODEL STOCHASTIC TREND PLUS CYCLE

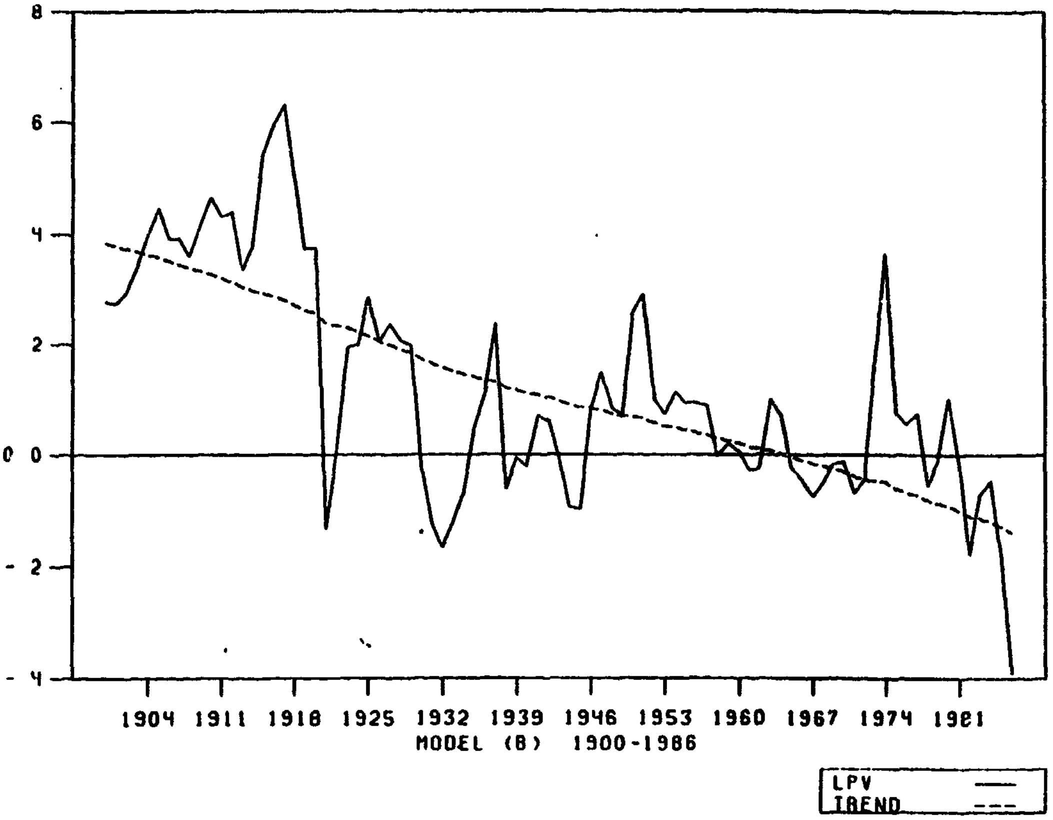


FIG 3B - MODEL: STOCHASTIC TREND PLUS CYCLE

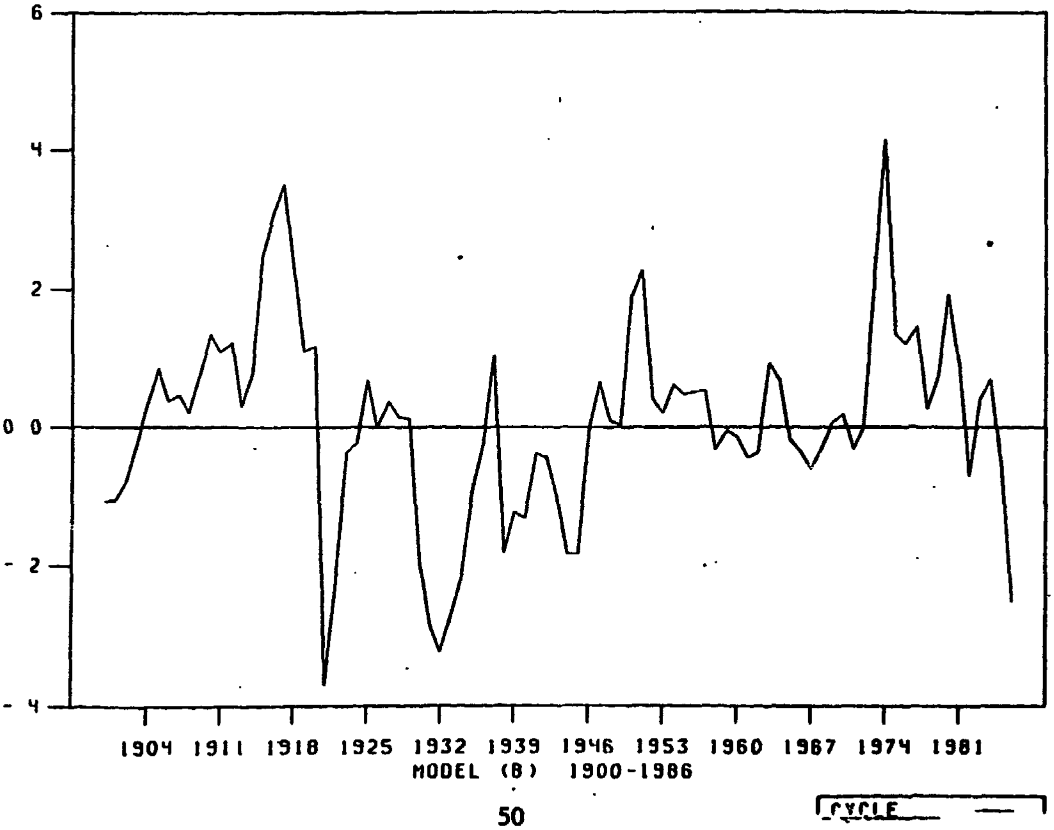


FIG 4A - MODEL: STOCHASTIC TREND PLUS AR(1) CYCLE

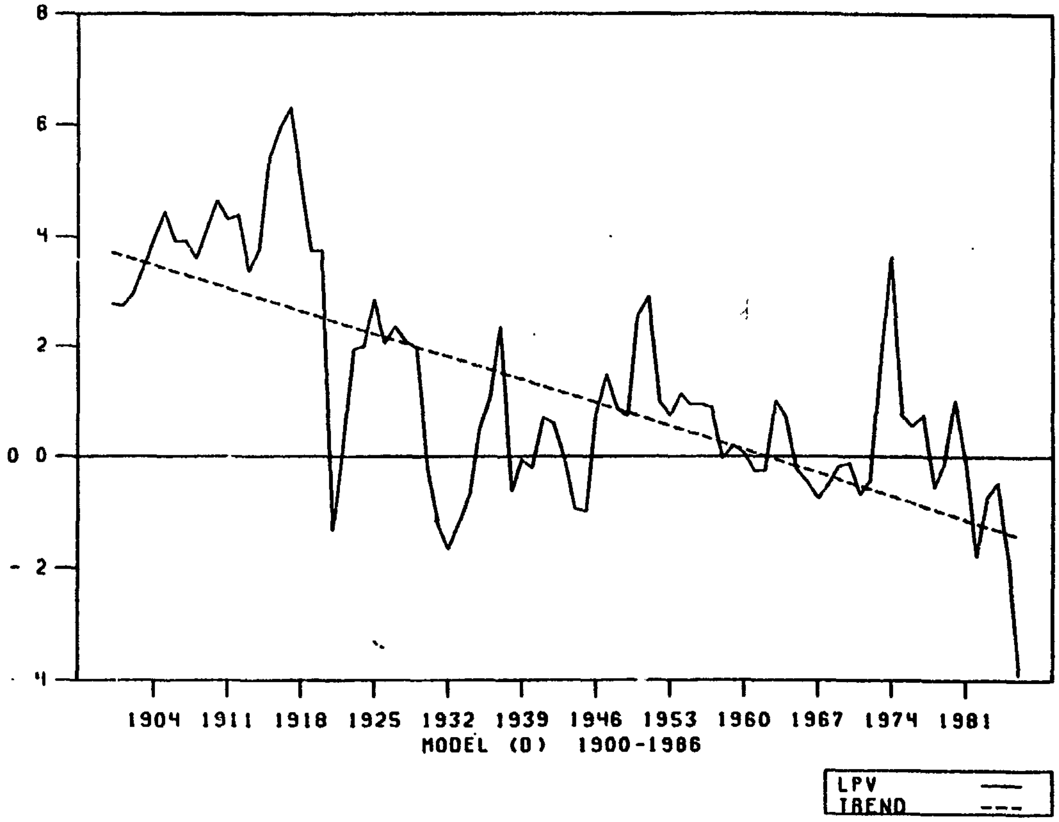


FIG 4B - MODEL STOCHASTIC TREND PLUS AR(1) CYCLE

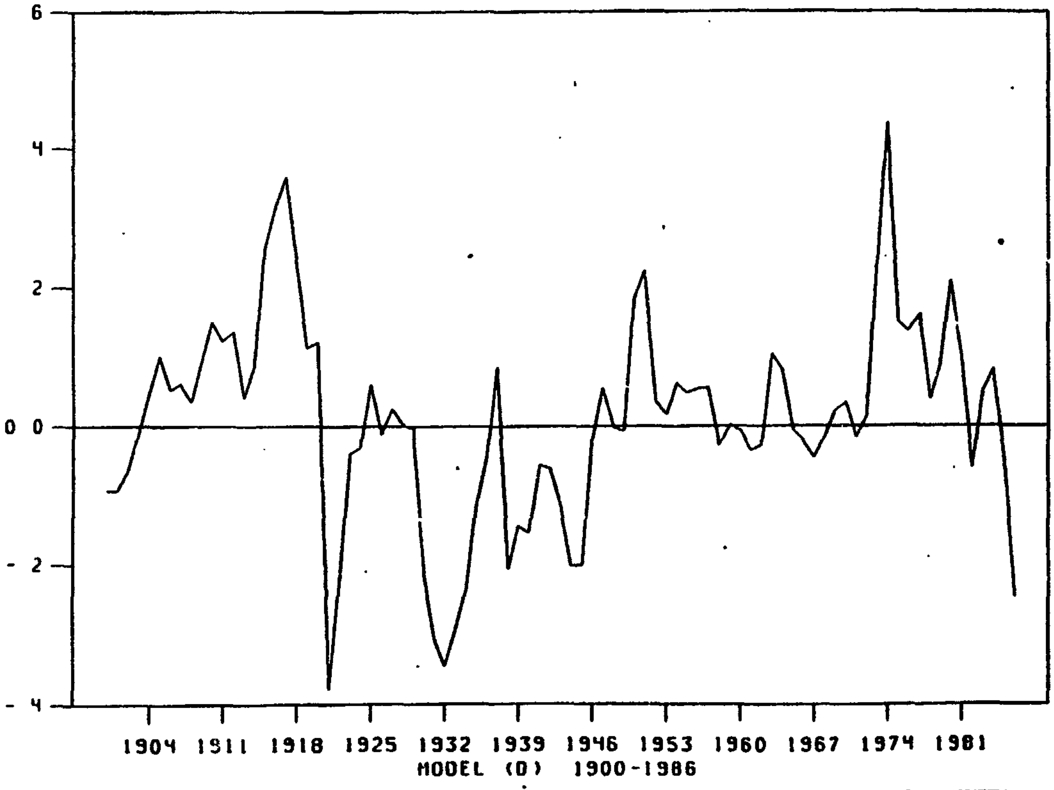


FIG 5A - MODEL: DETERMINISTIC TREND PLUS CYCLE

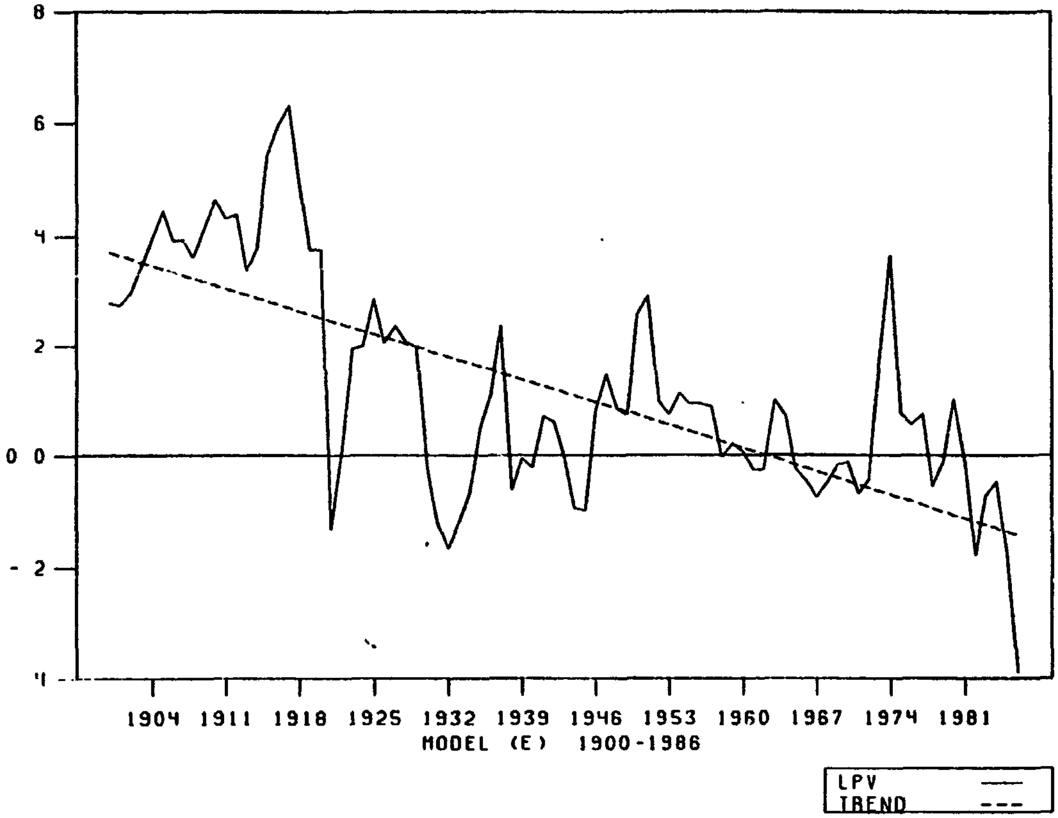


FIG 5B - MODEL. DETERMINISTIC TREND PLUS CYCLE

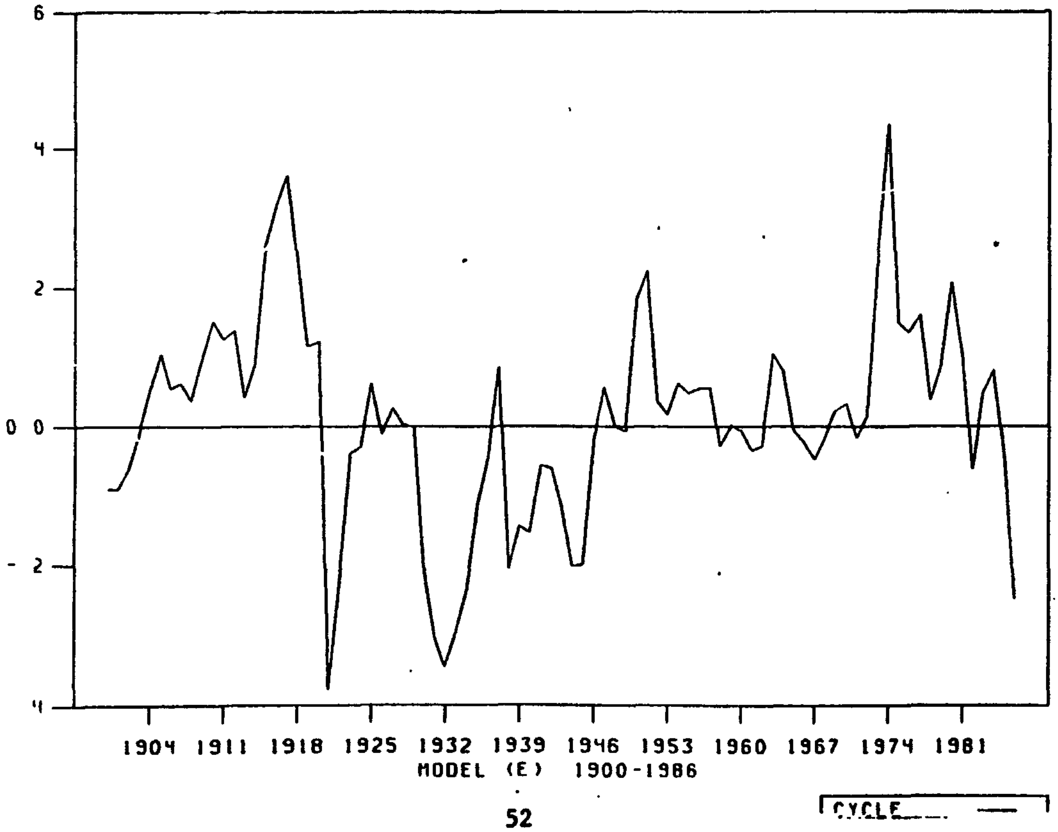


FIG 6A - MODEL. STOCHASTIC TREND PLUS AR(1) CYCLE

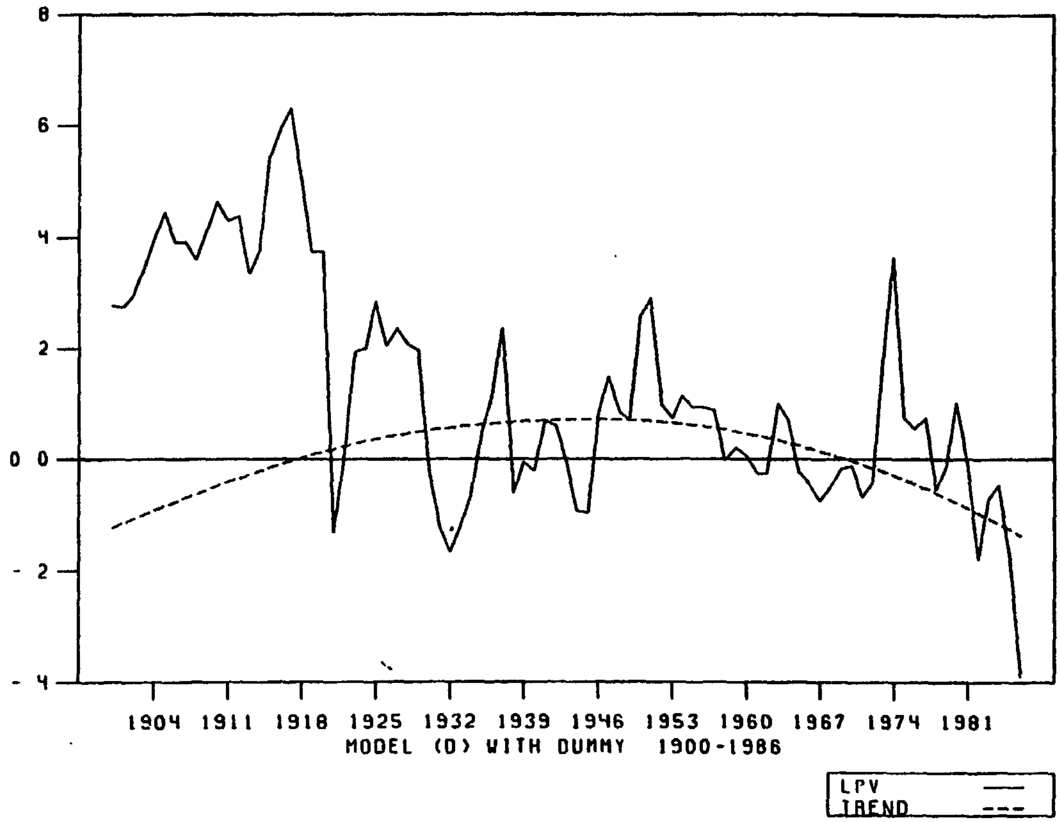


FIG 6B - MODEL: STOCHASTIC TREND PLUS AR(1) CYCLE

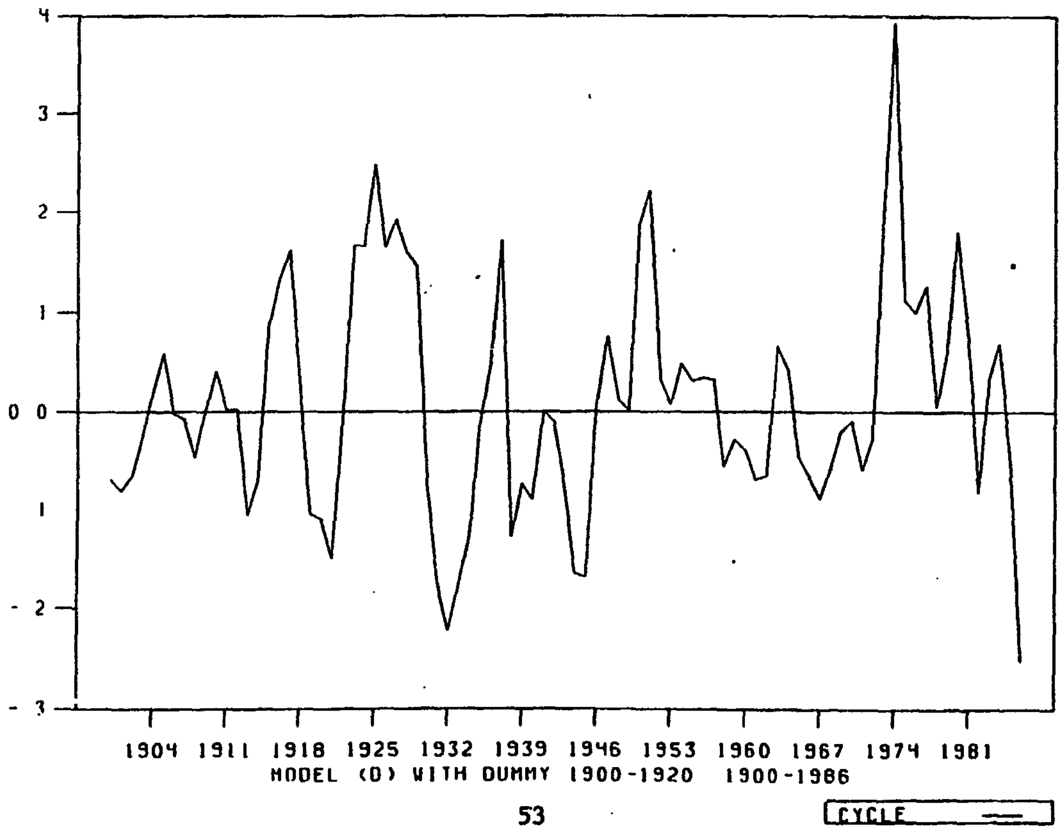
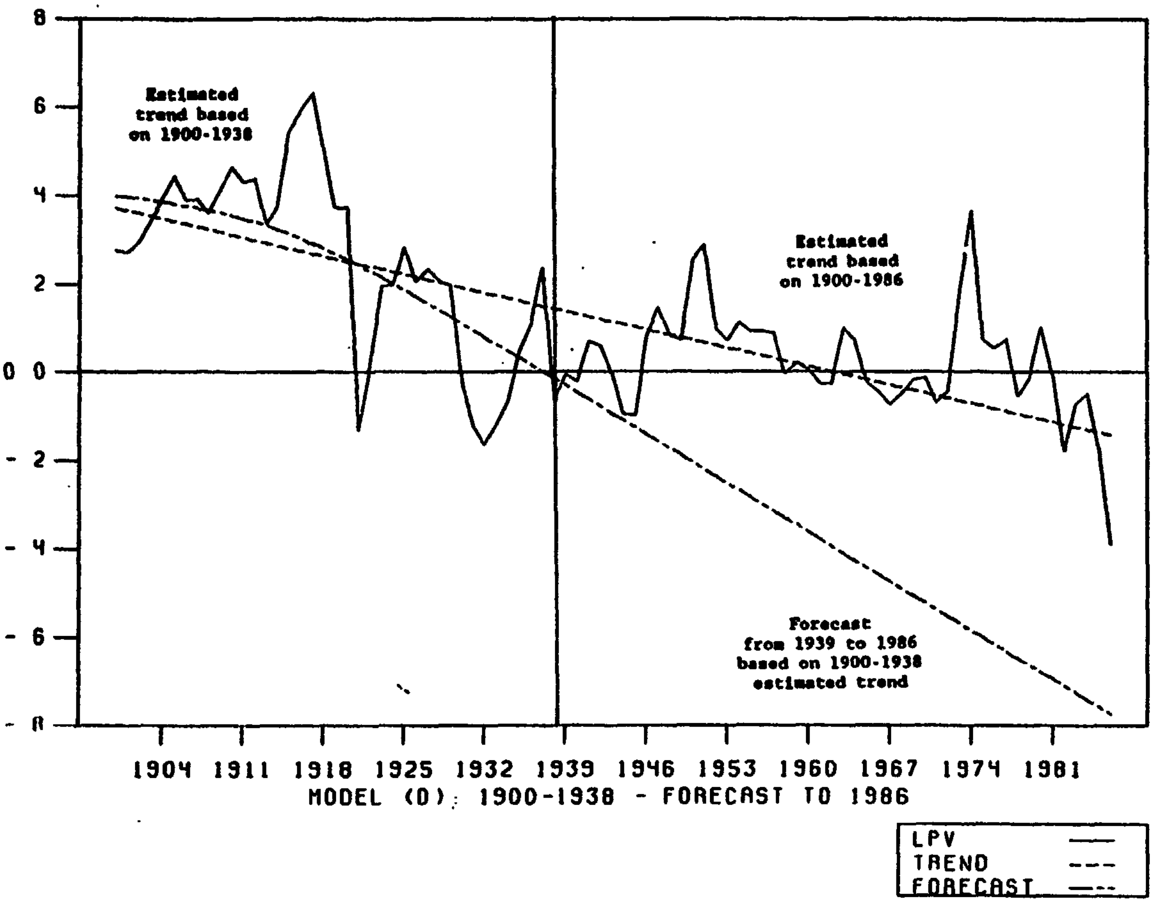


FIG. 7 - MODEL: STOCHASTIC TREND PLUS AR(1) CYCLE



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