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An Option-Pricing Approach to Secondary Market Debt

(Applied to Mexico)

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Stijn Claessens
and
Sweder van Wijnbergen

This pricing model for secondary market debt is designed to assess the impact of debt reduction on the value of remaining claims and the market value of different types of guarantees.

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Claessens and van Wijnbergen present a pricing model for secondary market debt designed to assess the market value of various forms of guarantees and the impact of debt reduction on the value of remaining claims.

Their model is more flexible and realistic than other models. The technique used — option pricing — accounts explicitly for the sources and nature of risks on sovereign debt. By so doing it is possible to assess the market value of various forms of guarantees as well as the impact of debt reduction on secondary market pricing.

The model is extremely flexible in handling different maturity schedules, differences in seniority, and expectations about the availability of foreign exchange and willingness to pay.

Claessens and van Wijnbergen apply the model to Mexico. They first price the value of a general obligation claim. They then price claims with fixed and rolling interest guarantees. They derive specific market values for general obligation debt and for collateralized exit bonds and show the impact of different debt reduction shemes on the secondary market price. They conclude that the terms of the new bonds are in accord with recent secondary market prices of the existing debt.

The authors show that the three debt restructuring options offered to individual banks are not equivalent if the newly created exit bonds are senior to new-money claims. The new-money option is worth considerably less.

This paper is a product of the Debt and International Finance Division, International Economics Department and the Country Operations Division, Latin America and the Caribbean Country Department II. Copies are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Sheilah King-Watson, room S7-033, extension 33730 (21 pages with figures and tables).

An Option-Pricing Approach to Secondary Market Debt (Applied to Mexico)

by

Stijn Claessens
World Bank

and

Sweder van Wijnbergen*
World Bank
and CEPR

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1 INTRODUCTION

The Brady plan, with its emphasis on negotiated debt reduction with possible support from official sources, has increased the importance of a good understanding of the pricing of external debt in secondary markets. It is clearly difficult to assess the feasibility of different debt restructuring schemes without a better understanding of the pricing of existing debt in the secondary market, and the likely effects of different debt reduction strategies on secondary market prices. After all, these prices represent the opportunity cost to holders of the claims being restructured and define the limits within which the bargaining process can produce an outcome. Three major issues are at stake. First the impact of debt reduction per se on the valuation of the claims that remain. Second the extent to which credit enhancements, through collateralization and forms of official guarantees for newly created claims, increase the market value of the instruments to which such enhancements are attached. Third, the impact of changes in seniority structure due to the newly created claims. These problems are interrelated in that debt reduction itself may affect the valuation of credit enhancements on remaining or newly created instruments.

Existing models fall short of providing the necessary consistent approach to secondary market pricing of existing debt and newly created, partly enhanced claims. We will argue below that most existing models are insufficiently equipped to discuss the dynamics of secondary market prices under alternative debt reduction strategies and few are able to provide insights on the value of credit enhancement.

Most models used for pricing secondary market debt can be classified in one of two classes:

1) the all-or-nothing approach: in each year, the debtor pays its debt service obligation in full with a certain probability and pays nothing with one minus that probability. Special cases are a constant probability p over time, in which case the secondary market price equals p (for instance, Martin and van Wijnbergen (1989)); or a geometrically declining probability, which would warrant use of a constant risk adjusted discount rate to calculate the present value of contractual repayments (for instance, Lamdany (1988)).

2) the certainty approach: here the secondary market price is the present value of future trade balances (with some corrections), taken as exogenous and deterministic, and divided by the face value of debt (for instance, Dooley and Symansky (1989), and Rodriguez (1988)).

Both approaches have their weaknesses. The all-or-nothing approach does not allow one to discuss the effects of a debt reduction on the secondary market price: this price remains the exogenous probability of (willingness of) repayment and is not affected by any amount of debt reduction. Marginal and average price of debt are equal <u>by assumption</u> under this approach. Neither does it allow for a full discussion of guarantees that are tied to debt reduction schemes, as the value of a guarantee is independent of any debt reduction taking place.

The certainty approach ignores the impact of debt reduction on the present value of expected repayments completely and can by construction not be used to evaluate guarantees or the impact of changes in the seniority structure, since these issues are inherently related to existing uncertainty about likelihood and magnitude of repayment. The approach can however account for different price paths in response to debt reduction.

The weaknesses of these models point to the importance of modelling explicitly the sources of uncertainty driving secondary market prices. The improved understanding of secondary market prices that would result is, in turn, important for an assessment of debt reduction schemes; an evaluation of the market value of new instruments is essential for an assessment of the feasibility of any given proposal. Finally, explicit modeling of the sources of uncertainty allows for an assessment of the impact of different repayment schedules and seniority structure on secondary market prices. This explicit modeling is the more important as some debt reduction schemes involve enhancements through guarantees and collaterals whose values are state-contingent since they depend on the stochastic pattern of amounts available for repayments.

This paper presents a model for pricing and evaluating existing and new (possibly credit enhanced) claims using option pricing techniques which explicitly models sources and natures of risks on sovereign debt. The paper is structured as follows. Section 2 sets out the basic approach; we present a pricing model of secondary market debt using option pricing tools. In Section 3 we apply the approach to pricing of claims with fixed and/or rolling interest and principal guarantees. We also show how to price recapture clauses that can be associated with newly created debt claims. Section 4 presents an application to Mexico and discusses the valuation and likely impact on secondary markets of the recent agreement between Mexico and its commercial creditors. Section 5 concludes.

II. PRICING SECONDARY MARKET DEBT USING OPTION PRICING

II.1 A Secondary Market Model

We develop a more complete model for pricing a country's secondary market commercial debt using option pricing techniques [1]/. The setup is the following. Due to uncertainty in the country's export earnings, import requirements and net scheduled capital in-or outflows, the net amount of financing available each period to service foreign commercial debt is uncertain. The uncertainty in the amount of resources available to service foreign obligations can be due to ability to pay as well as willingness to pay factors. For convenience, we lump these factors together and assume that the creditors have appropriability of any resources falling short of contractual debt service, or, alternatively and equivalently, that the country is a perfectly willing, but sometimes unable payer. Thus, each period the country will pay as much as its financial resources allow to the commercial banks, but

[1] Option pricing has been used before in the pricing of LDC debt by Kharas et alii (1987); Cohen (1989) gives an analytical solution to the pricing problem they solve numerically. These papers focus on the option a creditor has to call a default, whereas we focus on the option the country has not to service its debt in periods of low foreign exchange availability.

never more than its contractual obligations in the period. Consequently, repayments may fall short of commercial debt service obligations due.

We can represent this repayment behavior by the following:

(1a) $R^*(t) = \min(R_t, FX_t)$

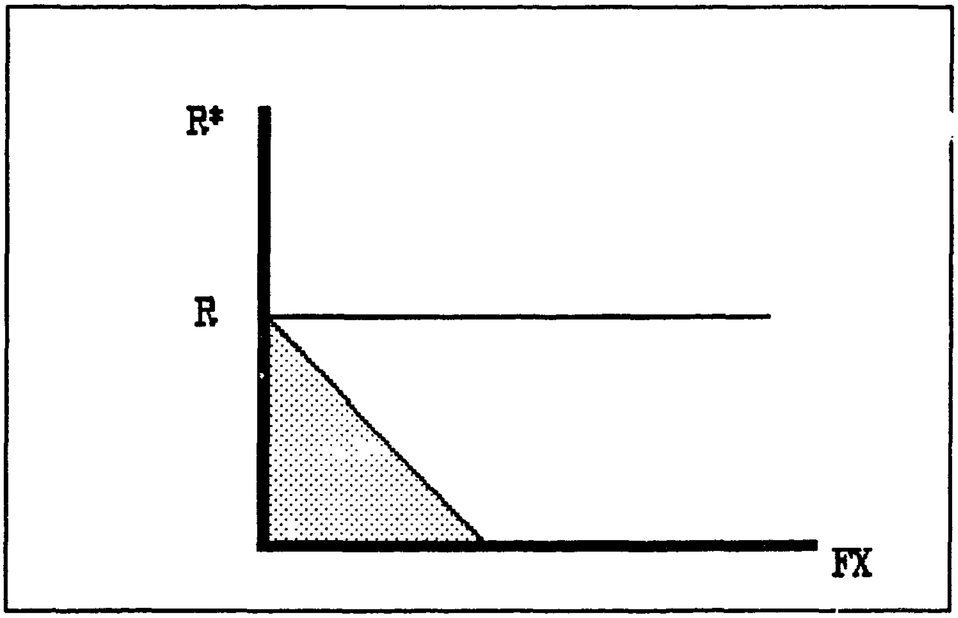
with $R^*(t)$ equal to the repayment in period t; R_t equals the contractual debt service in period t and FX_t the resources available to service commercially held debt, also in period t. R_t is assumed known, although it is straightforward to extend the methodology to stochastic contractual debt service, such as in the case of floating interest rate debt (see for instance, Fischer (1978) and Margrabe (1978)).

(1a) can be rearranged to yield:

(1b) $R^*(t) = R_t - \max[0, R_t - FX_t]$

But $\max[0, R_t - FX_t]$ equals the value of a put, with a strike price of R_t, which is written on the value of the foreign exchange available, FX_t. [2]/ Thus equation 1b shows that the uncertain repayment can be represented by a certain repayment R_t minus a put, with a strike price of R_t, which is written on the value of the foreign exchange available, FX_t.

FIG. 1a

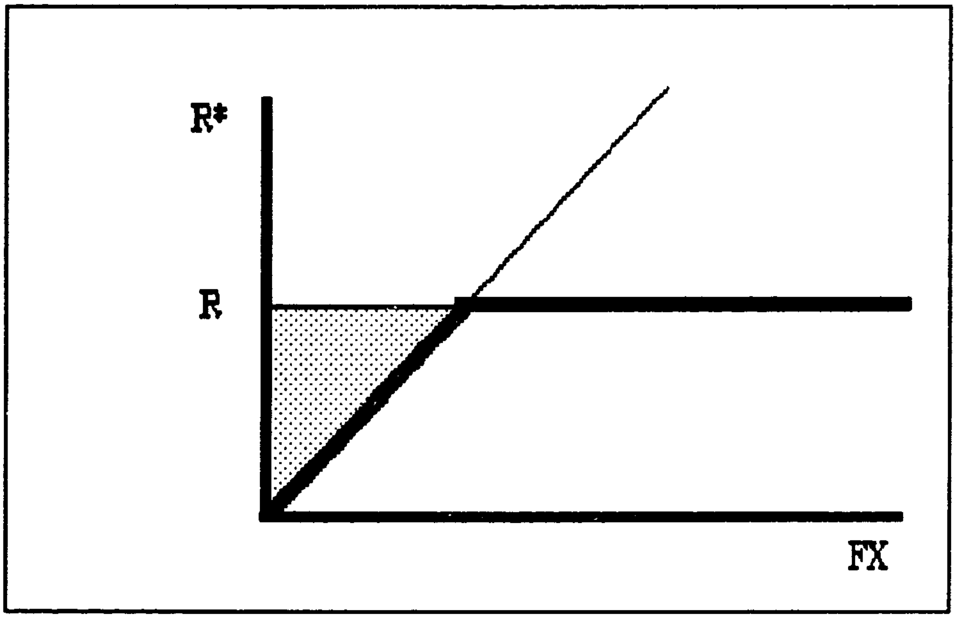


[2] The state variable FX is a non-traded asset and not as such priced in the market. But if the state variable is spanned by other traded instruments, one can price the non-traded asset and all results go through identically as in the case of traded assets. See also section III.

This is shown graphically in Figures 1a and 1b. In Figure 1a, the shaded area represents the value of a put written on FX_t with exercise price R_t. The put pays $\max[0, R_t - FX_t]$: whenever FX_t falls below R_t, the put is in the money and its value is equal to $R_t - FX_t$; and whenever FX_t is above R_t, the put is out of the money and thus worthless.

Figure 1b below shows first of all the payment obligation, R_t, which is independent of FX_t and thus represented by a horizontal line (FX is on the horizontal axis). Subtracting the put (shaded area) from the fixed payment R_t, yields the desired payoff function, $R^*_t = R_t - \max[0, R_t - FX_t]$. This is represented by the heavy line in Fig.1b, the line that goes from the origin out at a 45-degree angle until it cuts R_t and then moves horizontally to the right. For any outcome of FX_t above R_t, full repayment results and thus $R^*_t = R_t$. For a value of FX_t below R_t, only FX_t is paid and hence $R^*_t = FX_t$. Thus R^*_t clearly also equals $\min(R_t, FX_t)$.

FIG.1b



Now that we have replicated the payoff stream at maturity, it is easy to calculate the current value of the uncertain payoff stream as the current value of the certain future obligation R_t minus the current value of the put. This equals the discounted value of R_t, $\exp(-rt)*R_t$ (where r is the (continously compounding) interest rate), minus the current value P of a put with an exercise price of R_t, written on FX_t. [3]/ If $V(R_t)$ is the present value of the claim, we can represent this as:

[3] The formula assumes a constant interest rate r for notational convenience only. The empirical application presented below allows for different maturity structures of interest rates.

(2) $V(R_t) = \exp(-rt)*R_t - P(FX_t,R_t,r,t,\sigma)$.

where $P(FX_t,R_t,r,t,\sigma)$ is the current value of a put written on FX_t with exercise price R_t, interest rate r, maturity t and standard deviation σ. If one furthermore assumes that FX behaves lognormally, the pricing of the put can be done using the Black and Scholes option pricing formula (see Black and Scholes (1973)). [4]/ P is then equal to the following expression:

(3) $P(FX_t,R_t,r,t,\sigma) = - FX_0*\exp((\mu-r)t)*N(d1) + \exp(-rt)R*N(d2)$

where

$$d1 = [-\log(FX_0*\exp(\mu t)/R_t) - (\sigma^2/2)*t]/(\sigma\sqrt{t})$$

$$d2 = d1 + \sigma\sqrt{t}$$

μ = the drift in FX_t over the period 0..t [5]/

The current value of a loan with the series R_t falling due over time is imply the sum of the current values of a series of these claims over the maturity of the contract. The present value V_L of a series of contractual obligations R_t, for a maturity T, is thus equal to:

(4) $V_L = \Sigma_t R_t \exp(-rt) - \Sigma_t P(FX_t,R_t,r,t,\sigma)$

where R_t can be different for each period depending on the terms on the loan and the summation is over t=1,..,T. Note that this implies that we can study the implications of different maturity structures on the price of debt, something which in most other pricing models by assumption does not affect the price of debt.

II.2 Pricing Exit Bonds with Fixed Guarantees

The methodology explained above can also be used to price guarantees that are provided by a third party for a specific payment falling due at a specific maturity date. Assume that the third party provides a guarantee for full payment of K at maturity date τ. Following a similar line of reasoning, one can represent the guarantee as a put option with an exercise price of K, a maturity date τ and written on an underlying asset FX. Such a put can again be priced using the Black and Scholes formula:

(5) $V_{FG} = P(FX_\tau,K_\tau,r,\tau,\sigma)$

---

[4] Other density functions can easily be incoporated using numerical integration techniques.

[5] The formula assumes a constant drift μ for notational convenience only. The empirical application presented below allows for time varying drift parameter μ. The valuation formula differs from the Black-Scholes equation in that we do not assume μ=r.

Define the set of years τ over which guarantees are provided as $\{\tau'\}$; furthermore, assume for simplicity that $K_\tau = R_\tau$ for all $\tau\epsilon\{\tau'\}$. Then the value of such a set of guarantees equals:

$$(6) \quad V_{FG} = \Sigma_{\{\tau'\}}P(FX_\tau,R_\tau,r,\tau,\sigma)$$

and the value of the loan with this set of guarantees attached becomes:

$$(7) \quad V_{L,FG} = \Sigma_t R_t \exp(-rt) - \Sigma_t P(FX_t,R_t,r,t,\sigma) + \Sigma_{\{\tau'\}}P(FX_\tau,R_\tau,r,\tau,\sigma)$$

Any type of fixed guarantee, whether of principal or interest and whether single or multiple years, can be priced using this methodology.

II.3 Pricing Exit Bonds with Rolling Interest Guarantees

A bond with rolling guarantees can also be priced using the same option pricing methodology. Assume the following rules. The guarantee is at time zero extended for coverage of one year of interest. If the country remains current on the guaranteed obligation, the guarantee will be extended for another year, and so on. [6]/ In terms of our model, the guarantee will cover next year's debt service obligation provided the foreign exchange available in each of the previous periods was above the debt service obligation of the corresponding year. As before, it is assumed that in case of partial repayment the claimholders acquire all the foreign exchange available in this period if it falls below the debt service obligation and can at most retain their debt service obligation if the state of nature is better this period.

In period one the repayment of R_1 is assured through the guarantee, implying that the current value of the debt service obligation is $\exp(-rt_1)*R_1$. In period two the repayment is assured provided the country did not default in period one on its obligation, in which case the guarantee would have been called. If however the guarantee was called, the repayment in period two will be $\min[FX_2,R_2]$ as under the regular claim without any guarantee. This implies that the current value of the second period obligation will be equal to $\exp(-rt_2)*R_2$ minus the current value of a put on FX_2 with exercise price R_2, plus a put which is conditional on the guarantee not being called the first period PC:

$$(8) \quad V(R_2)_{RG} = \exp(-rt_2)*R_2 - P(FX_2,R_2,r,t,\sigma) + PC(FX_2,R_2,r,t,\sigma)$$

The first two terms are equal to the standard expression for a claim on a country, the contractual obligation discounted minus the value of a put. The third term represents the value of the guarantee, which is the value of a put conditional on no prior calls so that the guarantee is indeed effective. If FX_t is serially independent over time, an assumption we make, the pricing of this last conditional put is particularly simple and yields:

$$(9) \quad PC(FX_2,R_2,r,t,\sigma) = \Omega(2,R_1,FX_1,\sigma)*P(FX_2,R_2,r,t,\sigma)$$

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[6] The pricing is done for a guarantee. Identical results obtain for an escrow account as long as the interest earnings on the escrow account are not retained in the account.

where $\Omega(2,R_1,FX_1,\sigma)$ is the probability that the guarantee is not called prior to time 2. The value of the put which is conditional on no prior call simplifies to the value of an unconditional put multiplied by the probability of no prior call in any previous periods. Similar expressions follow for later periods.

Multiperiod Guarantees

More general expressions for N-period rolling guarantees can easily be derived using similar methodology. For an N-year rolling guarantee, the first N repayments are fully guaranteed and thus valued without any credit risk discount. The claim value for period N+1 is the discounted contractual value minus the value of an unconditional put, plus the value of the guarantee. The value of the guarantee in that period equals the value of a put which is conditional on less than N calls in the preceding N periods. This last put option can similarly be priced as the conditional put derived for the one-year rolling guarantee. The only difference is that, for a N-period rolling guarantee, Ω now refers to the cumulative probability of at most N-1 prior calls.

It is convenient to index Ω by the number of years covered by the rolling guarantee: Ω_N. Define, furthermore, $\zeta_N(t)$ as the amount left in the guarantee fund at the start of year t, expressed in number of years of interest covered, for a fund that originally covered N years. Thus the following holds by definition:

(10) $\zeta_N(t) > 0$ for $t \leq N$

Ω_N at time t depends on all R and FX of the periods preceding t. Call {t'} the set of t' preceding t. From the definition of Ω_N and ζ_N it is clear that:

(11) $\Omega_N(t,R_{\{t'\}},FX_{\{t'\}},\sigma) = \text{Prob}(\zeta_N(t) > 0)$

(10) and (11) together imply:

(12) $\Omega_N(t,R_{\{t'\}},FX_{\{t'\}},\sigma) = 1$ for $t \leq N$

< 1 for $t > N$ and $\sigma > 0$

Martin and van Wijnbergen (1989) show that the value of $\Omega_N(t,R_{\{t'\}},FX_{\{t'\}},\sigma)$ can be derived using a simple recursion formula in conjunction with the initial conditions in (12). This recursion formula greatly simplifies the numerical analysis and is incorporated in the computer programs used for the empirical analysis presented below.

With all this machinery developed, one can express the increment of the value of rolling guarantees with N years coverage over the value of a N-year fixed guarantee:

(13) $V_{L,RG-N} - V_{L,FG-N} = \Sigma_{t>N}\ \Omega_N(t,R_{\{t'\}},FX_{\{t'\}},\sigma)*P(FX_t,R_t,r,t,\sigma)$

≥ 0

with obvious definitions of $V_{L,RG-N}$ and $V_{L,FG-N}$. Also, R_t in equ. (13) should be understood to only include interest payments. The inequality in (13) shows that, for the creditors, rolling guarantees are at worst of equal value to a corresponding fixed interest guarantee; and if there is any positive $\Omega_N(t,R_{\{t'\}},FX_{\{t'\}},\sigma)$ for t>N, even if only one, rolling guarantees are strictly preferable from the creditors' point of view over fixed length guarantees with similar coverage.

Equ.(13) suggests that the incremental value of switching from fixed to rolling guarantees is influenced by the initial level of debt D_0, through the impact of D_0 on R_t:

$$(14) \quad \delta(V_{L,RG-N} - V_{L,FG-N})/\delta D_0 =$$

$$\Sigma_{t>N}\ \delta\Omega_N(t,R_{\{t'\}},FX_{\{t'\}},\sigma)/\delta D_0 * P(FX_t,R_t,r,t,\sigma) \quad < 0$$

$$+ \Sigma_{t>N}\ \Omega_N(t,R_{\{t'\}},FX_{\{t'\}},\sigma) * \delta P(FX_t,R_t,r,t,\sigma)/\delta D_0 \quad > 0$$

The first set of terms is negative since higher debt and thus higher R_t implies greater credit risk and thus smaller Ω_N; the fund is more likely to be exhausted at any given time beyond period N. However the second term is positive, since the value of the put increases with an increase in the striking price R_t. The net effect is a priori ambiguous and thus needs to be addressed empirically (cf. Section III).

II.4 Pricing Bonds with Recapture Clauses

The methodology used above is also easily extended to account for the possibility of recapture clauses, where future payments obligations depend in some fashion on the amount of foreign exchange available in each individual period. Assume, for instance, that in exchange for a certain amount of debt reduction at time zero, the creditors receive a recapture clause which entitles them, whenever foreign exchange exceeds a certain level L, to a share α of the excess foreign exchange over L in every period after time τ. Assume further that the maximum amount that creditors can receive per period under this sharing rule is limited by an amount M. [7] The Mexican debt package negotiated in the summer of 1989 contains a similar sharing rule.

Such a sharing rule can easily be represented in terms of option terminology: the creditors hold, in addition to their regular claim, a fraction α of a series of calls that are written on FX with exercise prices L, maturity dates $\tau+1$, $\tau+2,..,T$, and are short a fraction α of a series of calls that are written on FX with excercise price $U=L+M/\alpha$ and maturity date $\tau+1$, $\tau+2,..,T$.

To see the equivalence between the sharing rule and the portfolio of options just described, consider the payoff structure for the recapture clause, which we call I.

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[7] L, M and α can be made time dependent. In addition, L, M and α can be made dependent on other stochastic variables, such as world inflation rates in case of indexed clauses; in that case, one needs to use the stochastic option pricing formula of Fischer (1978) and Margrabe (1978).

$$(15)\quad I = \Sigma_{\tau'>\tau} \max[\alpha\max[FX_{(\tau')}-L,0],M]$$

$$= \Sigma_{\tau'>\tau}\ \alpha*(\max[FX_{(\tau')}-L,0] - \max[FX_{(\tau')}-U,0]);\ U=L+M/\alpha$$

The expressions in the two brackets in the last equation are the two calls mentioned above, with exercise prices L and U=L+M/α. The value of the calls can once again be evaluated using the Black-Scholes formula. Alternative recapture clauses, which may depend in a more complicated manner on FX, can be handled similarly.

III SECONDARY MARKET PRICING AND THE VALUE OF GUARANTEES: MEXICO 1989

In this Section, we first assess the characteristics of the stochastic process governing foreign exchange availability in Mexico. The results are used in an analysis of the determinants of secondary market prices. We then assess the valuation of different forms of interest guarantees (fixed versus "rolling" guarantees). This is done within the context of Mexico's situation mid-1989. Section IV analyses the Mexican debt package negotiated over the summer of 1989.

III.1 Behavior of Foreign Exchange Available

The availability of foreign exchange to service Mexico's commercial bank debt depends predominantly on the behavior of Mexico's non-interest current account, which in turn depends to a large extent on the behavior of oil export earnings. Thus the variability of the financial resources available to service external debt is in the case of Mexico predominantly a result of the uncertainty of the price of oil. Even though the foreign exchange earnings of Mexico are non-traded assets, and as such not priced directly in the market, they are likely spanned by assets which are traded and whose current values are known. For example, Mexico's oil earnings can easily be spanned through forward or futures contracts traded on over-the-counter and exchange markets Consequently, the pricing methodology underlying the option valuation, which assumed traded assets, can be used.

The behavior of Mexico's future oil earnings will depend on price behavior and expected quantity. It is projected that the quantity of oil produced will remain at its current level over the near future (1.2 million barrels per day) and will decrease in the late 1990s (to 0.8 million barrels a day). The standard deviation of the average price of Mexican oil over the last 8 years has been 23%. Similar standard deviations are observed for prices that are close substitutes of Mexican oil, such as Borneo light (25% over 87-89), and for the average OPEC oil price (40% over 87-89, 21 percent over 85-89). The standard deviation of the annual changes in most (nominal) oil prices over the period 1975-1988 has been at least 20% annually. Correcting for any trend in oil prices does not change these estimates significantly.

Another way to get an estimate of expected standard deviation is to use market information, such as actual prices of oil options. Given a pricing model, observed option prices can be used to back out volatilities that are consistent with those prices. Doing that one finds that the historical estimates of the standard deviation of oil prices are in fact consistent with those implied by the prices of options on oil traded on exchanges. Using the

Black and Scholes formula on recent option prices implies volatilities of around 20%. Thus historical values for the volatility of oil prices closely approximate the market's assessment of future volatility. We therefore use the historical volatility in our pricing exercise. [8]/

Commercial banks claims are de facto junior to many other claims on Mexico, e.g. official sector claims and bonds. Thus, the resources available to service the commercial debt have to be determined after these other creditors are serviced. This implies that amount of foreign exchange available for commercial bank debt servicing contains a component which is dependent on oil revenues and another, more deterministic part. The following procedure is therefore used. First, the non-oil, non-interest current account is projected in a deterministic fashion. The projections are based on the model reported in van Wijnbergen (1989) and van Wijnbergen and Pena (1989). Second, the non-oil, non-interest current account is adjusted for debt service to more senior claim holders, for foreign direct investment flows and for capital account transactions such as reserve accumulation (see van Wijnbergen and Pena (1989) for details). Third, oil earnings are added to the flow, thus introducing the stochastic element in FX_t.

III.2 Secondary Market Pricing

Using the option pricing model outlined above, we calculated the value of existing commercial bank claims on Mexico. At the "base case" values for the distribution of FX_t and σ^2, the model predicts that the secondary market price of the part of Mexico's commercial debt under negotiation in 1989 ($52.7 bUS), is 37 cents before any debt reduction. The secondary market price in February, 1989, just before the Brady plan was proposed, was in fact around that value.

The matrix below presents prices for alternative combinations of foreign exchange expected to be available to service commercial bank held debt and different degrees of uncertainty regarding these expected values. The bold numbers in the matrix represent combinations of the expected present value of foreign exchange available (PV(FX)) and oil price variance σ^2 that yield a valuation close to the pre-Brady price, between 35 and 40 cents on the dollar.

[8] Consistent with our assumption of no serially dependence of FX_t, we modelled in the application not the uncertainty in the change in the price of oil but instead the uncertainty in the level of the price of oil. The standard deviation of annual changes in the price of oil is therefore converted into the standard deviation of (the logarithm of) the price of oil.

Table 1: Secondary Market Price as a function of PV(FX) and σ^2

FX \ σ^2	0.25	0.36	0.49
0.75	0.37	0.31	0.26
1.00	0.43	0.37	0.31
1.25	0.49	0.41	0.35

Note: PV(FX) is presented as a share of the base case value and variances are expressed relative to the logarithm of the oil price.

The Table demonstrates the sensitivity of the secondary market price to the expected value and the variance of resources to service commercial debt. Consider the impact of the variance first. The value of any put increases with the degree of uncertainty:

(16) $\delta P/\delta\sigma^2 = FX_t * \sqrt{t} * N' > 0,$

Thus, since the secondary market value equals the discounted face value minus the value of a put, the value of the claim decreases as uncertainty increases, something Table 1 confirms. The Table also demonstrates that the value of the put falls with the initial value of the underlying asset; therefore, and not surprizingly, the secondary market evaluation in fact rises with a higher expected ability to pay.

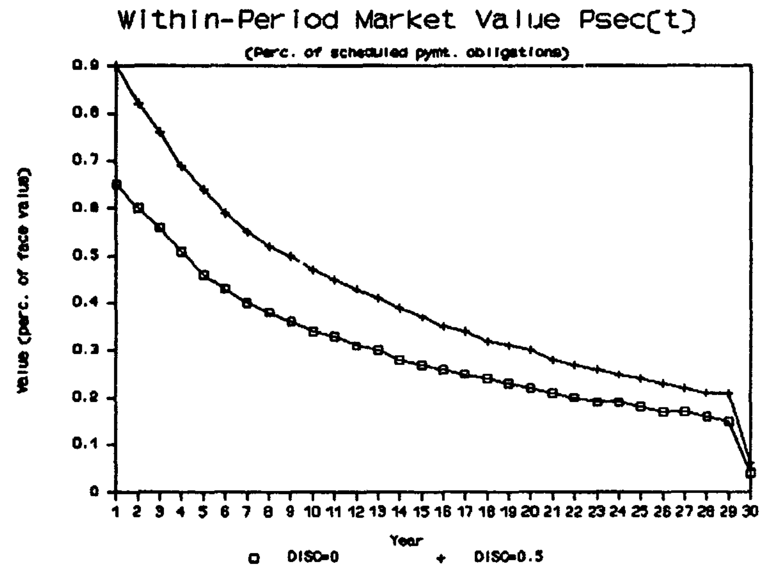
Figure 2 illustrates the impact of the structure of payments on secondary market evaluation, something that cannot be assessed with most existing models of debt pricing. The figure shows the within period price of a claim falling due in that period; this within period price can be derived from formula (2) by dividing through the discounted face value of the claim falling due in year t:

(17) $P_{sec}(t) = (\exp(-rt)*R_t - P(FX_t,R_t,r,t,\sigma))/(\exp(-rt)*R_t)$

$= 1 - P(FX_t,R_t,r,t,\sigma)*\exp(rt)/R_t$

The figure shows the $P_{sec}(t)$ associated with the original amount of debt under negotiation in 1989, $52.7 bUS, but with the amortization assumed due as a bullet payment at the end of thirty years. $P_{sec}(t)$ declines gradually over time because uncertainty increases as time progresses; this increases the value of the put constituting the discount and thus depresses the price. This gradual decline would seem to lend some respectability to the practice of using a risk-adjusted discount rate. That respectability is lost, however, once we look at the last period, the period in which the bullet payment comes due. Using a risk-adjusted discount rate implies a declining probability to repay, but one that is independent of the amount due in any given period. Thus there is no difference in valuation, using that method, between a period in which scheduled debt service just consists of $5 bUS in interest, and a period in which, in addition, over $50 bUS principal comes due. But Figure 2 shows what common sense would also suggest, that such a large difference in scheduled debt service has a major impact on relative valuation. $P_{sec}(t)$, which was falling gradually at less than one point per period, suddenly drops by 11 points in the period in which the bullet payment is due.

FIG.2



The impact of the size of an obligation on its relative value explains something that one misses using risk adjusted rates, the importance of full collateralization of principal in any exchange offer. With a repayment probability independent of the size of the obligation, such collateralization seems inefficient; it ties up funds for an event that takes place so far in the future so as to be of little importance. Fig. 2 shows that, while collateralization guarantees a payment far in the future, it could still be valuable and an efficient use of enhancement resources because the credit risk in the bullet period is so much larger than in other periods. The impact of the size of an obligation on its relative value is also clear from the line in Figure 2 which depicts the $P_{sec}(t)$ in case the contractual interest payments are halved. The relative values $P_{sec}(t)$ are considerably higher.

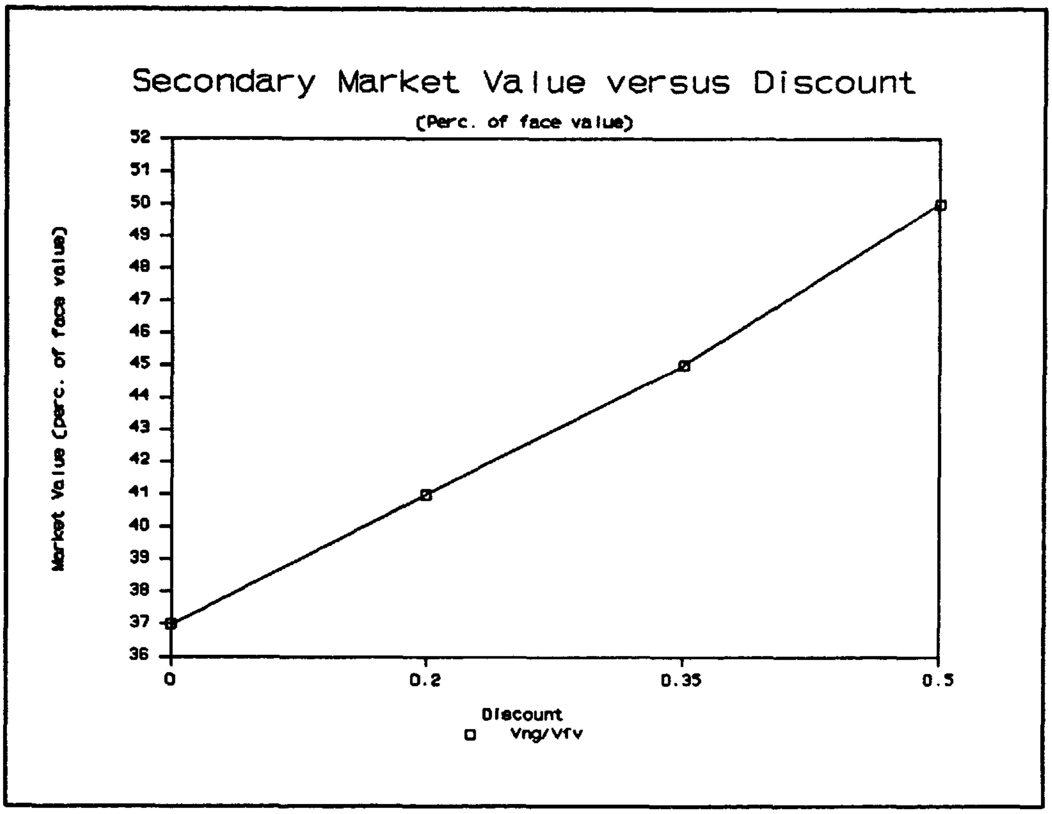
One can derive the secondary market price of a bond from the series ot within-period valuations $P_{sec}(t)$ as follows:

(18) $P_{sec,L} = \Sigma_t\ P_{sec}(t) * R_t e^{-rt} / (\Sigma_t\ R_t e^{-rt})$

Figure 3 shows the sensitivity of this secondary market price with respect to the amount of debt. This figure is especially illustrative since it can show the effects of debt reduction on the secondary market price, something other models did not account for. A debt service reduction of 50 percent for instance, increases the price from 37 cts to 50 cts. As a consequence, the 50% debt reduction does reduce the market value, from $19.3 billion to $13.2 billion, but by 32% only, much less than 50%. The difference between the 50%

face value reduction and the 32% reduction in market value is caused by the increase in unit value from 37 cts to 50 cts.

FIG.3



The next exercise demonstrates the impact of seniority. Consider a package not unlike the one Mexico negotiated mid 1989 (cf. Section IV for details): creditors can choose between an exchange offer for an exit bond at 35% discount, or a new money commitment of 25% over the original claims, spread out over four years (7%,6%,6%,6%). Assume, moreover, that the debt with the new money calls attached is officially recognized as junior to the exit bond. Of course, the respective valuation of the two options depends on how much of the original debt is brought under each of the two options. Take as a benchmark 80% exchanged at a 35% discount and 20% new money.

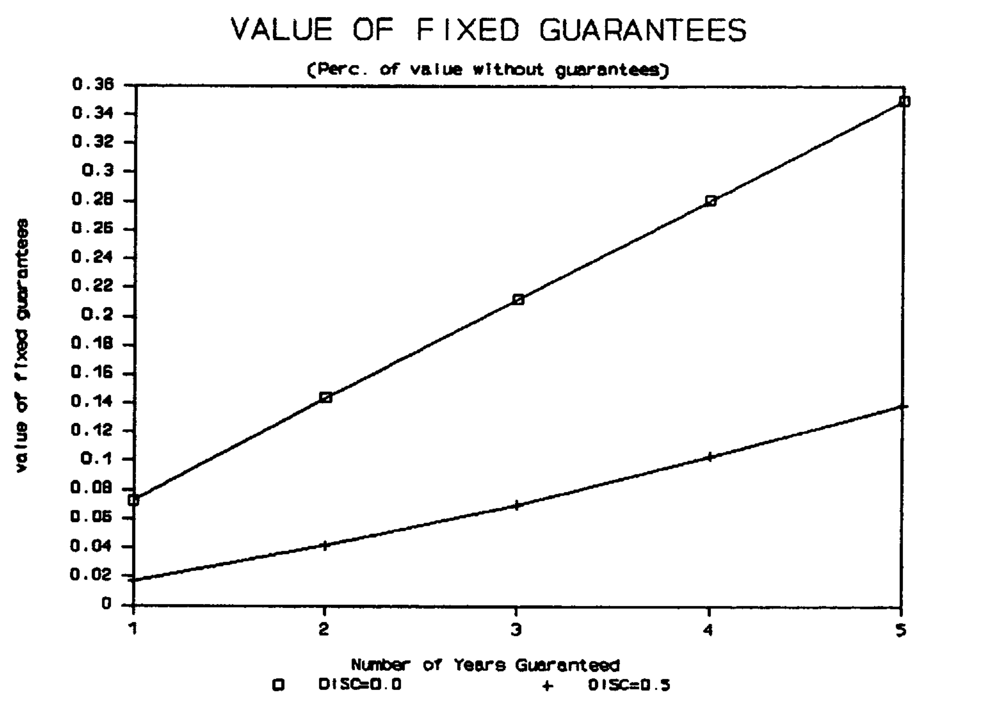
The model predicts a secondary market price of the new exit bond of 49 cents per dollar of new face value, up from the old price of 37 cts/$. However, this ignores the 35% discount granted at the time of the exchange of the old instrument for the new. Once the discount is taken into account (by expressing the value of the new claims as a percentage of the old, pre-discount face value), the unit value drops by 35% to 32 cts/$. This is however still in excess of the projected value of the new money claims. These claims are valued at no more than 26 cts/$ in the configuration used in this exercise, although they carry exactly the same interest rate as the new bonds. The 6 cts/$ difference in price is exclusively due to the lack of seniority of the new money claims.

In the next section we discuss what happens if the bonds derive their value in excess of a regular claim on the country not only from seniority but in addition from the guarantees provided on (parts) of interest payments and principal.

III.3 Valuation of Fixed Interest Guarantees

The value of a fixed guarantee on any payment for a specific year was shown above to be equal to the value of a put with an exercise price equal to the payment. In other words, a fixed guarantee on a year's interest payments just cancels the within period secondary market discount ($1-P_{sec}(t)$) in the year the guarantee applies. Fig. 4 indicates the value of a fixed interest guarantee for a range of years. The figure is based on a 30-year bond with bullet payment due in the last year and lists the difference in market value of a bond with and without a given number of years in which interest payments are up-front guaranteed, as a function of that number of years and as a percentage of the market value without interest guarantees: $(V_{LF}-V_L)/V_L$.

FIG.4



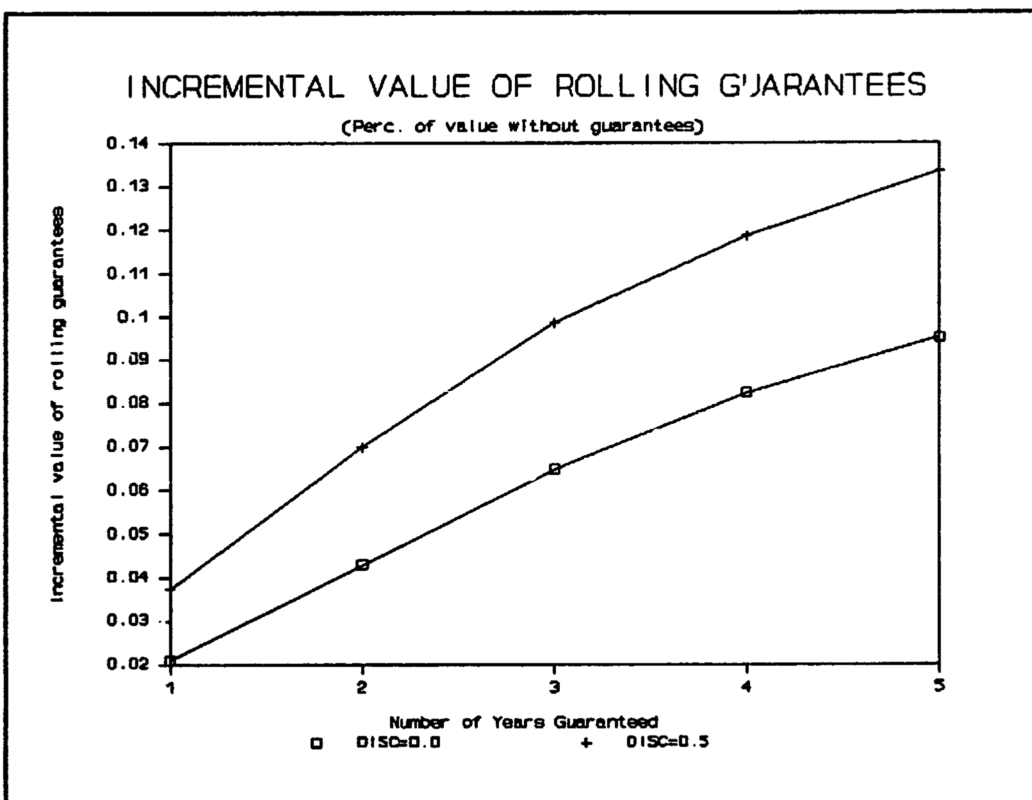
The value of a fixed guarantee depends on the likelihood of non-performance in a given year and hence on the amount of debt service due in that year. Thus, any reduction in contractual values will lower the value of a fixed guarantee. This can be seen in Fig.4. The higher line indicates the value of guarantees as a function of the number of years guaranteed for a bond with $52.7 bUS principal; the lower line gives the value of guarantees as a function of the number of years covered for a similarly structured bond, but after a 50% discount on principal. The value of the guarantees, as a percentage of the value of the now much smaller bond with interest guarantees, drops by almost two thirds as the discounted claim is much more assured to begin with.

III.4 The Value of Rolling Interest Guarantees

To assess the incremental value of rolling over fixed length guarantees, compare a fixed length and a rolling guarantee, both with N years of interest coverage. Equation (13) shows that a rolling guarantee is always worth more to creditors as long as there is positive probability, however small, that the guarantee will be called by less than N times in the first N years.

The superiority of rolling guarantees over fixed guarantees is demonstrated in Figure 5; this figure lists the incremental value of rolling over fixed guarantees as a percentage of the market value of the unguaranteed instrument, $(V_{LR}-V_{LF})/V_L$. The instrument guaranteed is a 30-year bond with a bullet payment of principal at the end. The lower line assumes the face value of the bond equals the entire debt under negotiation, \$52.7 bUS; the higher line is based on a 50% discount for a bond with face value of \$26.4 bUS. The diagram shows that the incremental value of the rolling guarantee increases with the number of years covered, but at an increasingly slower rate. This is because extra years provide benefits that are increasingly further in the future and hence discounted more heavily.

FIG.5



The diagram also sheds light on the impact of debt relief on the incremental value of rolling guarantees, an impact that is theoretically ambiguous (cf. Equ.(14) in Section II.3). In the case and numbers under consideration here, the net impact is positive: more debt relief increases the incremental value of rolling guarantees. Evidently the effect through the derivative of Ω_N dominates the effect due to the reduction in face value. With more debt relief, Mexico would become a better credit risk, which diminishes the incremental value of rolling guarantees. But at the same time, the chance that there is anything left in the fund in any given year after N is greater and this increases the incremental value of rolling guarantees. Figure 5 shows that the latter effect dominates. Debt relief itself increases the incremental value of rolling over fixed guarantees.

IV MEXICO'S 1989 EXTERNAL DEBT AGREEMENT

In this Section we demonstrate the power of the methods developed through an assessment of the debt package concluded between Mexico and its creditors in the summer of 1989.

IV.1 Outline of the July 1989 Debt Agreement

On July 23 Mexico and its commercial creditors reached a tentative agreement on a restructuring of part of Mexico's external debt. The package tentatively agreed upon covers the debt under the Restructuring agreements, the 1983-84 Credit Agreements (New Money Loans), and the 1987 Multi-Facility agreement. This amounts to $52.7 billion. Creditor banks holding credits under these facilities are presented with a menu of options including two debt and debt service reduction facilities, and a new money facility. Banks can choose to participate through any combination of the new money and debt relief options. The three options are:

A Discount bonds, to be exchanged against existing debt at 35% discount;
B Par bonds, exchanged at par but carrying a 6.25% fixed interest rate;
C New money, at LIBOR plus 13/16, with a 7 year grace period and tenor of 15 years; the commitment should equal 25% of the amount brought under this option. Disbursement will be 7% at effectiveness and 6% each in 1990, 1991 and 1992.

On the discount and par bonds, principal will be fully secured, in addition to at least 18 months of interest coverage through an escrow account (rolling guarantee). If sufficient funds are available, interest coverage will be increased up to 24 months. Banks holding loans contracted in the 1983-88 period will reschedule them with 7 years grace and 15 years maturity to the extent they are not swapped for par or discount bonds.

Banks choosing the debt relief options A and B are eligible for recovering some of the money given up through a "recapture clause". Under this clause, beginning July 1996, 30% of the extra oil revenues Mexico gets if the price of oil rises above $14 per barrel (to be adjusted for US inflation), will accrue to the banks that have granted debt service relief. This amount is in no year to exceed 3% of the nominal value of the debt exchanged for these bonds at the time of the exchange (i.e. there is no indexation of this cap).

The amount available under this clause will be scaled back by the percentage of the total debt brought under the two debt relief options.

IV.2 Debt Relief

Table 1 summarizes the element of debt relief embedded in each of the three options the commercial banks can choose between. We also list debt relief if a combination of 54% interest reduction, 20% principal reduction and 26% new money is chosen.

Table 2: DEBT RELIEF IMPLIED BY THE THREE OPTIONS (percentage of face value)

	Debt Relief Percentage: Without Recapture	With Recapture
New Money	0	0
Interest reduction	28	25
Principal reduction	35	32
54% IR/20% PR/26% NM	22	20

Note: Debt relief is defined as the reduction in the discounted value of debt service as a percentage of the face value of the outstanding debt.

Of course, new money implies no debt relief, but offers the highest immediate reduction in net transfers. The principal reduction option involves 35% debt relief, since the mark-up on LIBOR will not be changed under this scenario. Evaluated at current interest rate projections, the low interest rate option implies 28% debt relief. This number is sensitive to the projections used for international interest rates. To calculate debt relief, we use a LIBOR of 8.5% for the remainder of 1989 and for 1990, and 8% for the remaining 28 years. The two debt relief options are equivalent in terms of implied debt relief if LIBOR would stay at 9.1% for the next thirty years.

However, the interest reduction option provides more than debt service relief. Because the interest rate on this exit instrument will be fixed, it also provides insurance against interest rate fluctuations.

Furthermore, comparison of the new money option with the debt relief options is influenced by the fact that the latter qualify for the recapture clause and the new money option does not. The value of these provisions depends both on expected future oil prices and on the variability of these prices. Thus any evaluation needs explicitly to incorporate the impact of uncertainty on the expected cost of this clause; evaluating the impact of the recapture clause on debt service obligations at some point estimate of future oil prices is not enough. Estimates using the methodology of Section 2 and market information on the pricing of oil options suggest that the recapture clause is worth about 3% of the amount brought under the debt relief options. Thus, the debt relief would be reduced as indicated in Table 2. In absolute amounts, this would imply $1.6 billion less debt relief if the 54/20/26 division is chosen.

IV.3 Attractiveness of the Various Options for the Creditors

The final impact of the package on debt relief and creditworthiness indicators depends on the particular mix chosen by the creditors. It is thus of interest to assess the factors likely to influence that choice. Four factors are likely to dominate. First, Mexican credit risk and the extent to which different instruments are affected differently. Second, for given credit risk, the amount of debt relief embedded in each of the three options. Third, the extent to which different instruments are "enhanced" throught the use of official moneys in the form of collateralization, direct guarantees or arms' length guarantees through escrow accounts. Fourth, the tax and regulatory treatment of the income and balance sheet consequences of any option.

The impact of tax and regulatory treatment does not allow for a general discussion. This is country-specific, and may even depend on the particular profit-and-loss and balance sheet situation of an individual creditor. However, the impact of the first three factors, credit risk, debt relief and credit enhancement, on the value of the new instruments can be assessed without entering details specific to particular creditor countries or even to individual creditors. The results of such an evaluation are presented in Table 3. This table summarizes the projected secondary market valuation of the different instruments, assuming that they are chosen in a 54/20/26 mix between interest reduction, principal reduction and new money. The Table lists the expected value, as a percentage of face value, with and without enhancements, based on the model presented in the preceding Sections.

Table 3 Projected Secondary Market Valuation of the New Instruments

	Percentage chosen	Without enhancements; Perc. of new face value	With scheduled enhancements; Perc. of new face value	Perc. of old face value
IR	0.54	0.34	0.44	0.44
PR	0.20	0.50	0.61	0.40
NM	0.26	0.25	0.25	0.25

IR : Interest Rate Reduction
PR : Principal Reduction
NM : New Money

Enhancement for IR,PR options: 18 months rolling interest guarantee, full collateralization principal

The Table suggests that as a percentage of the new face value and without any enhancement, the principal reduction exit bond would be quoted at the highest price, because it receives market interest rates as opposed to 6.25% fixed. With interest coverage as stipulated in the tentative agreement, the value would increase further. The low-interest instrument would trade for less, simply because it carries a lower interest rate.

However, the unit value of the new claims is not the only factor entering the decision on which option to choose. After all, with principal reduction, old claims are exchanged at a 35% discount for new claims, while the low interest bond would be exchanged at par. To incorporate that discount, the secondary market valuation needs to be compared to the old face value; in this way any discount at the time of the exchange of the old debt for the new instrument is taken into account. This reverses the outcome of the comparison: the low interest option would remain the same with enhancements, since old and new face value are the same; but the value of the discount bond, inclusive of enhancements, would fall by 35%. Thus, unless tax and regulatory matters affect the two options differentially, one would expect commercial creditors to prefer interest reduction over principal reduction.

The second striking result in the table is the low valuation of the new money option. This option has clearly been presented as junior debt by the Mexican authorities, junior to the exit instruments. This has a major impact on valuation. Without subordination, the new money option would have traded at close to the unenhanced, new face value quotation of the principal reduction deal, since it carries market interest rates too but no guarantees. Its junior status reduces the valuation to 25 cts, however. This 25 cts for the most "junior" claim holders is thus the marginal price of Mexico's debt, almost half the average price of 44 cts, the price the model predicts if the majority indeed goes for interest rate reduction. 44 cts was in fact the value immediately after the negotiations ended in August 1989, lending some credence to these results.

V CONCLUSIONS

Existing models of secondary market pricing of sovereign debt can either not address the impact of debt reduction on the secondary market price, or they cannot address valuation of credit enhancements. Both issues are not only of intrinsic interest, but also of great practical importance in the voluntary debt reduction exercises that form the core of the current Brady initiative. The voluntary nature requires that the enhanced new debt instruments should have a market value at least as high as the market value of the old instruments. But in that set up, answers to both questions raised at the beginning of this paragraph can assist in assessing how much credit enhancement is necessary to make a certain amount of debt relief acceptable to creditors. Or, if the sequence of events is such that the moneys available for credit enhancement are known before the amount of debt relief granted, the answers to these questions can assist in finding out how much debt relief should be expected for given enhancement moneys.

The model developed in this paper was designed to shed light on the determinants of secondary market prices and the likely impact on valuation of different debt reduction strategies and forms of enhancements. The technique used is option pricing. The model was used to demonstrate that debt reduction has a substantial impact on the value of those claims that remain. We also assess the value of fixed and rolling guarantees. In particular, we prove theoretically and demonstrate empirically that from the creditors' point of view rolling guarantees dominate fixed guarantees as a technique of credit enhancement. We furthermore explore the impact of debt relief on the incremental value of rolling guarantees over fixed guarantees. Also, the model was used to assess empirically the value of seniority.

The final section provides a preliminary assessment of the debt restructuring agreement recently reached between Mexico and its commercial creditors. The three options are not equivalent if the newly created bonds are senior to the new money option. We show that the terms of the two exit bonds are such that their market value is likely to be close to the pre-Brady plan pricing of claims on Mexico, although they imply substantial debt relief for Mexico. It thus seems a fair agreement and, since the rolling guarantee was shown to be more valuable to the creditors than a fixed guarantee, with efficient use of official resources.

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