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A Valuation Formula
for LDC Debt

Daniel Cohen

Looking at the true cost of debt buy-back on the secondary
market requires a valuation formula for LDC debt.

This paper — a product of the Debt and International Finance Division, International Economics Department — is part of a larger effort in PRE to understand the determination of secondary market prices for developing country debt and the impact of debt buy-backs on developing countries. Copies are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Sheilah King-Watson, room S8-040, extension 31047 (20 pages).

A large gap may lie between the amount of debt relief that is nominally granted to a debtor and that which is actually given up by the creditors. To help put that gap in perspective, Cohen proposes a valuation formula that provides:

• The price at which a buy-back of the debt, on the secondary market, is advantageous to the country.

• The value to creditors of having the flows of payment guaranteed against factors that hinder a country in servicing its debt.

• The degree of tradeoff between growth of payments and levels of payments.

It is not good business for a country to announce its intention to buy back debt. Accord-ing to Cohen's calculation, doing so immediately raises the price — by as much as 45 percent when half the debt is repurchased. A favorable (small) buy-back price is shown to be (on average) about half the observed market price.

The value of guarantees, Cohen argues, cannot exceed 25 percent of the market price of the debt. Typically they're worth only about 10 percent.

As for the degree of tradeoff, Cohen's formula finds that a 1 percent additional growth rate is worth a 15 percent increase in the flows of payments.

Cohen also offers an assessment of the Mexican debt-relief agreement reached in 1990.

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by
Daniel Cohen*
(Université de Nancy, CEPREMAP, and CEPR)

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I - INTRODUCTION

The Brady plan has triggered a number of proposals aimed at reducing the face value of LDC debt. As now well understood in the literature on this topic, there is a gap which may be very large between the debt relief which is nominally granted to (or purchased by) a debtor and the relief which is actually given up by the creditors (when measured in terms of the net transfers which the debtor is expected to pay).

As indeed pointed out by Dooley (1988), a country that announces or that is expected to repurchase a significant of its debt on the secondary market immediately raises the price at which the transaction must be undertaken (at a level that corresponds to its ex-post value). One direct consequence of this observation is that it is certainly counter-productive to set up an institution or to create a facility that openly repurchases a given quantity of LDC debt on the secondary market. Furthermore, as pointed out by Bulow and Rogoff (198?), even if the country repurchases a small amount (or "ride-up" the supply curve by repurchasing its debt one dollar after the other), the price at which the transaction is performed on the secondary market corresponds to the _average_ value of the debt which is (perhaps well) above the _marginal_ price that properly measures the actual reduction of the burden of the debt that is obtained by the debtor.

In this paper, I attempt to give some empirical flesh to these crucial (qualitative) remarks by giving an exact valuation formula for LDC debt which mimicks, it is hoped, the pricing that is observed on the secondary market. Using some reasonable numbers, I will show that, following Doley's point, a country that announces in advance that it will seek to repurchase half of the face value of its debt may end-up over-paying the market value of debt by about 45 %. Following Bulow and Rogoff's argument, I then show that a country (that is solvent in "average") may over-pay the buy-back of its debt by a ratio of one to two. I also offer to estimate other transactions such as the value of guaranteeing against exogeneous stock the payments that are

made by the country : I will show that such a guarantee is not likely to exceed 25 % of the market value of the debt.

Section II offers a motivation and a theoretical background for this paper. Section III gives the valuation formula and gives the discount associated to various parameters of the country. Section IV shows the difference between average and marginal prices and estimate the value for the country of making a "take it or leave it" offer to its creditors ; section V gives an estimate of the cost of guaranteeing the debt. A conclusion gives some perspectives on the Mexican deal that has been negotiated in July 1989.

II - MOTIVATION AND THEORETICAL BACKGROUND

The literature on sovereign risk can be compactly summarized as in the following model. Take a two-period model and assume that a country has to repay in period 2 a debt whose contractual value is D. Furthermore, assume that the country always has the option to repudiate its debt and also assume that the banks can (credibly) impose -in retaliation- a sanction that amounts to a fraction λQ of the country's income, Q. Finally, assume that the banks can always get the country to pay that fraction λQ that the country would forego by defaulting. Call $dF(Q)$ the density of the (random) distribution of the country's income. Let r be the riskless rate and take the banks to be risk-neutral. One can write the market value of the debt D as :

$$(1) \quad V(D) = \frac{1}{1+r} \left[\int_{0}^{D/\lambda} \lambda Q \, dF(Q) + \int_{D/\lambda}^{\infty} D. \, dF(Q) \right]$$

The first term in the bracket represents how much the banks can get when the income of the country satisfies $\lambda Q \leq D$. In that case indeed the country would rather default than paying the face value of the debt and we assumed that the banks can get the country to pay the fraction λQ that the country would lose by doing so.

The second term in the bracket measures the expected payments that accrue to the banks when the country does honor the contractual value of the debt. This happens with a probability $1-F(D/\lambda)$.

There are obviously many questions that such a model does not address. Why is it that the debt is not a contract that is contingent on the realization of Q ? Where does the debt comes from and why is it that there is not a clause that raises the face value D when –in expected terms– the country is insolvent ? What would happen if bargaining considerations were introduced in the second period ? How to handle the incentive questions when Q is endogeneously determined by capital accumulation ? These questions are not all specific to the debt literature (in particular the uncontingent dimension of the debt). Some are, such as the bargaining questions. We can only, here, refer to earlier works such as the surveys in Eaton, Gersovitz and Stiglitz (1986), or the papers contained in Dealing With The Debt Crisis (1990) or to our own work (Cohen, 1991) for an attempt to answer some of these questions.

The issue that we want to address here is one that has been raised by Bulow and Rogoff (1988) and Dooley (1988) in a similar set up and is related to the valuation of the debt and its implication for debt buy-backs. Consider equation (1). The market price of the debt (such as observed on the secondary market) can simply be written as :

$$(2) \quad q(D) = \frac{V(D)}{D} = \frac{1}{1+r} \left[\int_0^{D/\lambda} \frac{\lambda Q}{D} \, dF(Q) + 1-F(D/\lambda) \right]$$

If a country were to repurchase one dollar of its debt on the secondary market, this is the price that it would have to pay.

Now Dooley (1988) has made the following remark. Assume that the country is known in advance to want to repurchase a share B of its debt. At which price will the transaction take place ? If lenders act competitively and if they are all aware of the transaction B that the country wants to undertake, they will only accept to sell at the price

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that will prevail ex-post, after the transaction is completed. This price is simply :

$$q(D-B) = \frac{1}{1+r} \left\{ \int_{0}^{(D-B)/\lambda} \frac{\lambda Q}{D-B} \, dF(Q) + 1 - F\,[(D-B)/\lambda] \right\}$$

and is obviously larger (perhaps much larger) than the initial price. This argument is aimed at showing that it is not a good business to buy-back a large amount of debt and letting the creditors know it.

The second key remark is the one made by Bulow and Rogoff (1988). Even when it only performs a small transaction, the country has to pay a price such as (2) that does not properly measure the benefit accruing to it when it repurchases its debt. Indeed, one dollar repurchased by the country reduces the burden of the repayment by an amount which is measured by the marginal (rather than the average) price of the debt. Here, this marginal price is :

$$\rho(D) \equiv V'(D) = 1 - F(D/\lambda)$$

which is strictly (perhaps much) lower than q(D).

So, even if the country could "ride-up" the price-quantity schedule and repurchase, one dollar after the other, the quantity B, it would end up paying the reduction of the market value of the debt by an amount $\int_{D-B}^{D} q(D) \, dD$ which is above the actual reduction of the debt's market value $\Delta V \equiv V(D) - V(D-B) = \int_{D-B}^{D} \rho(D) dD.$

All these questions are obviously extremely important in assessing the key question of determining which strategy the country should undertake in order to alleviate -through the secondary market- the burden of its debt.

The model above, however, is very qualitative and does not help very much assessing the empirical magnitude of the effects that it points out to. It is the purpose of the following sections to address this empirical dimension.

III - A VALUATION FORMULA FOR LDC DEBT

We want to generalize a formula such (1) so as to give it an empirical content. In order to do that, we shall make the following assumptions. We consider a continuous time model of an infinitely lived economy. We assume that the resources Q_t of the country follow a Brownian process and we let $P_t \equiv \lambda Q_t$ represent how much the banks can, at most, oblige the country to pay. We write the law of motion of P_t as follows :

$$(2) \quad \frac{dP_t}{P_t} = \mu \, dt + \sigma \, dz_t \quad ; \; z_t \text{ a Wiener process.}$$

We also assume that the debt is short-term and continuously rescheduled at the riskless rate of interest by the creditors. With that hypothesis, the banks can always capture $P_t (=\lambda Q_t)$ as long as the debt is not entirely repaid.

The hypothesis that the debt is short-term is somehow farfetched, but not too much. When they reschedule the debt of an insolvent country, the banks like to keep a short-leash approach so as to make sure that they do not lose an opportunity to monitor the country's choices.

The assumption that the debt is rescheduled at the riskless rate may appear more debatable. Analyzing a fixed spread over libor would not be very interesting ,however, since the spread should decline as the country becomes solvent. Our assumption is instead litterally true for the public debt that is negotiated at the Paris Club. For commercial banks,the spread over libor is actually quite small and has been steadily declining. In table IV.4 of the World Debt Tables (1989), for instance, one sees that the spread over libor (for the debt restructuring agreements) was (in percentage points): 1.8 in 1982-85,

1.3 in 1986, 1.0 in 1987 and 0.8 in 1988. As these data show, the banks may have responded to the early stage of the debt crisis by a large spread that has helped them raise the face value of their debt and then -perhaps for incentive reason- they left the spread go down (see Cohen (1989) for a potential explanation of this behavior).

With these assumptions, one can write the law of motion of the face value of the debt as :

$$d\ D_t = \left[\ r\ D_t - P_t\ \right]\ dt \qquad \text{as long as} \qquad D_t \geq 0$$

Call V_t the market value of the debt. Assuming risk-neutral lenders, it is a solution to

(3) $\quad E_t\ dV_t + P_t\ dt = r\ V_t\ dt$

whenever $D_t > 0$. (E_t is the expectation at time t).

Call q_t the market price of the debt so that $V_t = q_t\ D_t$ and call $x_t = \dfrac{P_t}{D_t}$ the apparent yield on the country's debt. We shall seek a function $q(x)$ which is a solution to (3). Making use of Ito's lemma, one can show that $q(x)$ is a solution to the following differential equation:

(4) $\quad \dfrac{1}{2}\ q''(x)\ x^2\ \sigma^2 + x\ q'(x)\ [\mu - r + x] - q(x)\ x + x = 0$

There are two boundary conditions. When x approaches 0 (the face value of the debt becomes infinite), then the banks simply get the present discounted value of all P_t. Therefore $\lim\limits_{x \to 0} \dfrac{q(x)}{x} = \dfrac{1}{r-\mu}$.

The other boundary condition is the following. When $x = \infty$ (the face value of the debt goes to zero) the country is solvent, so that :
$\lim\limits_{x \to \infty} q(x) = 1$ (the debt is quoted at par).

One can then check (see appendix for details) that the price $q(x)$ of the debt can then be written :

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$$(5) \begin{cases} \text{(a)} \ q(x) = \dfrac{x}{x_0} - C(1 - \dfrac{x}{x_0}) \displaystyle\int_0^{x/x_0} \varphi(t)dt \qquad \text{when } x < x_0 \\[20pt] \text{(b)} \ q(x) = 1 - C(\dfrac{x}{x_0} - 1) \displaystyle\int_{x/x_0}^{\infty} \varphi(t)dt \qquad \text{when } x > x_0 \\[20pt] \text{(c)} \ q(x_0) = 1 - C\, e^{-\beta} \end{cases}$$

in which

$$(6) \begin{cases} x_0 = r - \mu \\[14pt] \varphi(t) = \dfrac{1}{(t-1)^2} \, e^{-\beta t} \, t^{\beta} \qquad ; \qquad \beta = 2 \dfrac{r - \mu}{\sigma^2} \\[20pt] C = \dfrac{1}{\beta \displaystyle\int_0^{\infty} e^{-\beta t} \, t^{\beta} \, dt} \end{cases}$$

C is obtained by imposing that q(x) is a differentiable function. It can be depicted as in diagram 1[1].

THE PRICE q(x) OF THE DEBT

1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

0   0.25   0.5   0.75   1   1.25   1.5   1.75   2 x/x_0

$q_0(x)$
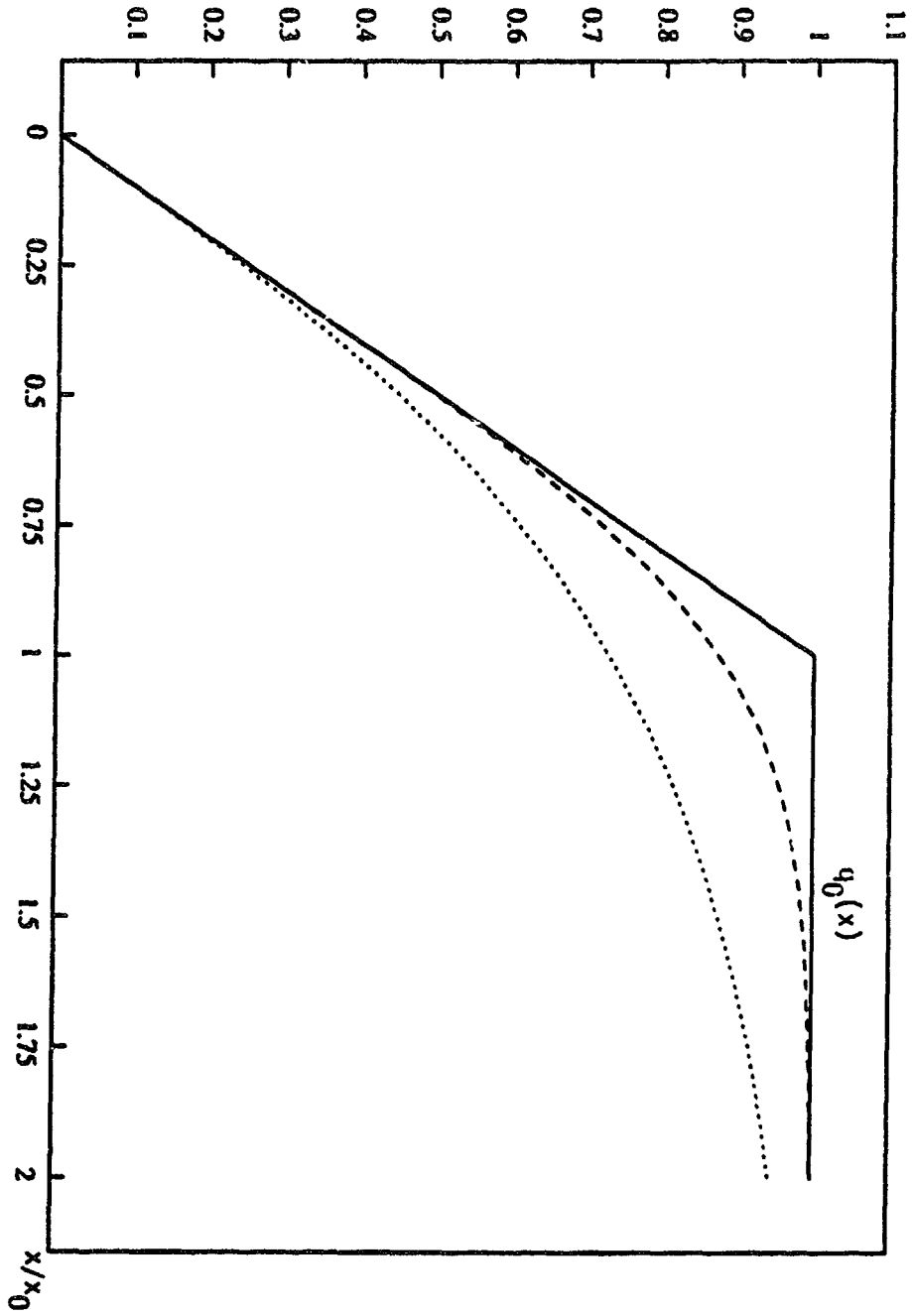
- - - beta = 10   ...... beta=2   ——— no risk

DIAGRAM 1

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On Diagram 1 the solid live $q_0(x)$ represents the market price of the debt when there is no risk ($\sigma=0$). It is worth $q_0(x) = \dfrac{x}{x_0}$ when $x < x_0$ and 1 otherwise. In the zero-risk case, the country's debt is quoted below par whenever the present discounted value of the transfers paid by the debtor is below the face value of the debt. When the present discounted value of these transfers exceeds the face value of the debt, then there will necessarily come a point when the country will have repaid all its debt, so that the price is necessarily one all the time.

When the transfers are risky ($\sigma > 0$), the expected present discounted value of all P_t is again $\dfrac{P_0}{r-\mu}$ and it does not depend on σ. The difference between the risky and the non risky case is due to the fact that a _good_ fortune may help the country repay all its debt so that the value of the claim held by the debtor is _less_ than the present discounted value of _all_ future transers P_t. (A similar picture emerges in Genotte, Kharas and Sadeq 1987).

In the case when $x = x_0$, for instance, present discounted value calculation (based on the assumption that creditors would receive P_t for ever) would lead to the misleading implication that the debt should be quoted at par. In table 1, we have shown the discount which instead appears when the risk component is taken into account, in that case (when $x = x_0$) and in the case when $x = \dfrac{x_0}{2}$.

TABLE 1 - THE MARKET PRICE OF THE DEBT

β	Case $x = x_0$	Case $x = x_0/2$
1	0.63	0.390
2	0.73	0.444
5	0.825	0.487
10	0.875	0.498

$\beta = 2 \frac{r-\mu}{\sigma^2}$ so that a standard deviation $\sigma = 0.2$ (which is that which is observed on Wall Street) and a difference between interest rate and growth of 4 percentage points lead to $\beta = 2$ which we shall take as our benchmark case in the sequel.

As a simple application of table 1, one may ask : what is the equivalence, from the creditors' view point, between an increase in the growth rate of payments and a reduction of current payments?

When the value of the debt is infinite, the equivalence is straightforward to calculate. The value of the debt is simply $V_0 = \frac{P_0}{r-\mu}$ so that $\frac{dP}{P} = -\frac{d\mu}{r-\mu}$. If growth is increased by 1 percentage point then, say when $r-\mu = 4 \%$, lenders can accept a 25 % reduction of the transfers made by the country. This 25 % reduction is obviously the maximum amount of debt service reduction that lenders can accept when the debt is finite. When the debt is very small, for example, the trade-off becomes a negligible one as the lenders care less about the future growth of the country's ability to pay in the future (since they do not expect to cash it in full).

As an intermediate case, it is possible to use formula (1) to see how this trade-off operates when $x = x_0$, $\beta = 2$ and $r-\mu = 0.04$. One can calculate in that case that an additionnal 1 percentage point of the country's growth can "buy" a 15 % reduction of the current payments that the country makes to its creditors.

IV - THE VALUE OF A DEBT WRITE-OFF

1 - Let us first assume that the country wants to repurchase its debt on the secondary market. As we argued in section II, if it makes an offer to the banks, and if we assume that the banks are a group of competitive investors, the only price at which the transaction can take place is the ex-post market value of the debt which will prevail after the transaction has taken place. As an application of the numbers displayed in table 1, consider the case when the country's debt is initially such that $x = \frac{1}{2}x_0$ and assume that the country wants to repurchase half of its debt. Consider for instance the case when $\beta = 2$;

we see from table 1 that the debt must be repurchased at 73 cents on the dollar, despite an initial price of 45 cents on the dollar before the transaction was announced. This indicates that repurchasing half the face value of the debt costs 62 % of its initial market value.

2 - Assume now instead that banks can rationnally coordinate their collective behavior and that the country can make the banks a credible "take it or leave it" offer such as : "repurchase the debt at such price or I use the money in an alternative way which yields no benefits to you (say I consume it)". What is now the cost -under this hypothesis- of repurchasing half the debt. When x goes from $\frac{x_0}{2}$ to x_0, the value of the banks' claims is reduced by a number which is

$$\Delta V = q(\frac{x_0}{2}) \ D - q(x_0) \ D/2$$

This implies that the banks must be compensated for reducing the debt by half by a fraction θ of their initial claim which is equal to :

$$\theta = \frac{\Delta V}{q(x/2).D} = \left[1 - \frac{1}{2} \frac{q(x)}{q(x/2)} \right]$$

When $\beta = 2$, we find that $\theta = 18$ %. This number can therefore be advantageously compared to the 62 % which was found in the previous paragraph. Under the hypotheses which have adopted, the capability of making a credible offer to the banks therefore represents as much as 44% of the market value of the debt. This shows that the point made by Dooley can indeed be potentially very important.

3 - As pointed out by Bulow and Rogoff (1988), we have seen that, when small transactions are involved, the relevant statistic to analyze the value of a debt reduction is the marginal value of the debt. Call $\rho = \frac{dV}{dD}$ this marginal price. ρ measures the market value which the banks as a whole are actually giving up when they reduce the face value of the debt by one dollar. Mathematically one finds $\rho = q(x) - xq'(x)$. As x approaches zero (the debt becomes very large), it is easy to check that the ratio of the marginal to the average price of the debt is zero (the lenders -as a whole- do not care at all about one more or one less dollar). Conversely, as the debt goes to zero, the marginal and the

average prices do converge one towards the other. In table 2, I have calculated the marginal price of the debt in the two corresponding cases which were desplayed in table 1. We see from tables 1 and 2 that the difference between the marginal and the average price remains very substantial, even in the case when $x = x_0$. Indeed, in such cases, the marginal price always appears to be less than half the value of the average price. For instance, a relatively solvent country such as one obtained when $x = x_0$ and $\beta = 10$, whose debt only shows a 12.5% discount, exhibits a marginal price of its debt of only 42 cents on the dollar. For a country with the same characteristic ($\beta = 10$) and twice bigger a debt, the marginal price is virtually zero, while the discount which is observed on the secondary market is about 50%.

These results confirm the econometric evidences which are shown in Cohen (1989) where it was indeed found -out of a direct analysis of the data on the secondary market- that the hypothesis that the marginal price was zero could not be statistically rejected when the debt was quoted at a 50 % discount.

(The evidence brought by Bulow and Rogoff (1988) for the Bolivian debt also strongly pointed to the fact that its marginal price was zero).

TABLE 2 - MARGINAL PRICE OF THE DEBT

	(1) $x = x_0$	(2) $x = x_0/2$
$\beta = 1$	0.266	0.148
$\beta = 2$	0.336	0.110
$\beta = 5$	0.388	0.017
$\beta = 10$	0.420	0.007

V - THE VALUE OF GUARANTEEING THE DEBT

Offering a guarantee on the payment of the debt usually involves two distinct mechanisms. One is to guarantee that, say the interest will always be paid. The other one amounts to protect the lenders from the stochastic disturbances which afflict the ability of the country to service its debt. The first mechanism is generally not a pure guarantee and may actually enhance the average ability of the country to service its debt. To that extent, it involves a partial bail out of the banks as well as a pure insurance mechanism. In this section we limit our analysis to the second mechanism and ask : what is the value of protecting the banks against the fluctuations of the countries' ability to pay. Mathematically, this simply amounts to substitute to the stochastic streams of repayment P_t a deterministic pattern $\hat{P}_t = P_0 \, e^{\mu t}$ which has the same expected mean, and offer the banks $\text{Min}\left[D_0, \dfrac{P_0}{r-\mu} \right]$.

When the face value of the debt is infinite, the market value of such an insurance scheme is simply zero. Indeed, the bank are assumed to be risk-neutral and they already expect -in present value terms- to receive $\dfrac{P_0}{r-\mu}$. When the debt is not infinite, the value of the guarantee is simply given by the difference between the market price of the debt and the $q_0(x)$ line displayed in diagram 1. From this diagram, it is apparent that the maximum value of such a guarantee is obtained at the point when $x = x_0$. At this point the country would be solvent if the banks could make sure to get $P_0 \, e^{\mu t}$ rather than P_t. One also sees that the maximum value of the guarantee is nothing else but the market discount which the debt would exhibit at this point. Table 3 also shows the value of guaranteeing the debt when $x = \dfrac{x_0}{2}$.

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TABLE 3 - THE VALUE OF GUARANTEING THE DEBT
AGAINST DEBTORS' RISK
(in % of the market value of the debt)

	Maximum value of such guarantee $(x=x_0)$	Value of the guarantee when $x=x_0/2$
$\beta = 1$	37 %	22 %
$\beta = 2$	27 %	10 %
$\beta = 5$	17.5 %	2.6 %
$\beta = 10$	12.5 %	1.4 %

We therefore see, for instance, that the value (to the banks) of offsetting the stochastic disturbances of the debtor's transfers represents (only) approximately 10 % of the market value of the debt in the case when $\beta = 2$ and when the debtor is "half-solvent".

VI - SOME PERSPECTIVES ON THE MEXICAN DEAL

In July 1989, Mexico and its creditors agreed on a debt relief plan offering the banks three options : 1) Reduce the face value of the debt by 35 % ; 2) Reduce the interest rate down to 6.25 % ; 3) Capitalize about 2/3 of the interest due. The negotiated settlement (such as eventually signed early 1990) involves the following combination :
1) 42 % of the debt is swapped against a bond whose face value is written down by 35 % ; 2) 46 % of the debt is swapped against a bond with a reduced nominal interest rate ; 3) 12 % of the debt will be rescheduled along the line of the third option. One may estimate that a nominal write-off of about 15 bls emerged from the deal (see Claessens and Van Winjbergen, 1990). The market value of this nominal

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write-off must be estimated at the marginal price shown in table 2. For $\beta = 2$, we get the following :

TABLE 4 - MARKET VALUE OF THE DEBT WRITE-OFF

$x = x_0$	$x = x_0/2$
5.0	1.6

In exchange, 4 bls of new money (2 from the World Bank and the IMF, 2 from the Japanese Government) were put on the table to "encourage" the deal. Let us interpret those 4 bls as a loan which is understood by the parties to be _junior_ to the newly issued bonds. The value to the senior creditors of a _junior_ claim is simply measured by its face value _minus_ its marginal price (which corresponds to the case when the junior money is indeed cashed in back by the junior creditors). It is reported in table 5. Depending upon which interpretation we have of the Mexican situation we can come up with the two following numbers (taking the case $\beta=2$ as a benchmark).

TABLE 5 - MARKET VALUE TO SENIOR CREDITORS
OF 4 bls OF "NEW" JUNIOR MONEY

$x = x_0$	$x = x_0/2$
2.7	3.6

Compared to table 4, one sees that it is a bad or a good deal for the banks depending upon whether $x = x_0$ or $x = x_0/2$. If one takes the observed market price (of about 45 cents) to be an accurate measure of Mexico's solvency, one should conclude (from table 1) that the hypothesis that $x = x_2/2$ is the correct one. I^r that case, the banks

gave up 1.6 bls (in market terms), while the sponsors put 3.6 bls on the table.

Let us show how to get the solution to equation (4). Call $z(x) = q(x)-1$. Equation (4) can then be written :

(A1) $\frac{1}{2} \sigma^2 z''(x) x + z'(x) [\mu - r + x] - z(x) = 0$

When $x \neq x_0$, define the auxilliary function $F(x)$ as the solution to:

(A2) $z(x) = (x + \mu - r) F(x)$

$F(x)$ is a solution to the following differential equation :

(A3) $\frac{1}{2} x(x + \mu - r) \sigma^2 F''(x) + F'(x) \left[x \sigma^2 + (x + \mu - r)^2 \right] = 0$

That is :

$$\frac{F''(x)}{F'(x)} = - \left\{ \frac{2}{x + \mu - r} + \frac{2}{\sigma^2} \frac{x + \mu - r}{x} \right\}$$

Integrating (A4) yields :

$$\text{Log } F'(x) = - 2 \text{ Log } (x + \mu - r) - \frac{2}{\sigma^2} x + 2 \frac{r - \mu}{\sigma^2} \text{ Log } x + C$$

So that one can write

(A5) $F'(x) = \dfrac{C}{\left(\dfrac{x}{x_0} - 1\right)^2} \left(\dfrac{x}{x_0} \right)^{\beta} e^{-\beta \frac{x}{x_0}}$

in which β and x_0 are defined as in equation (6).

Using the two boundary conditions, one can then show that :

(A6) $x < x_0 \Rightarrow q(x) = \dfrac{x}{x_0} - C_- \left(1 - \dfrac{x}{x_0}\right) \displaystyle\int_0^{x/x_0} \varphi(t)\, dt$

(A7) $x > x_0 \Rightarrow q(x) = 1 - C_+ \left(\dfrac{x}{x_0} - 1\right) \displaystyle\int_{x/x_0}^{\infty} \varphi(t)\, dt$

Integrating φ by part shows that

(A8) $x < x_0 \Rightarrow \displaystyle\int_0^{x/x_0} \varphi(t)\,dt = -\dfrac{1}{x/x_0 - 1}\, e^{-\beta\, x/x_0}\left(\dfrac{x}{x_0}\right)^\beta - \beta \displaystyle\int_0^{x/x_0} t^{\beta-1} e^{-\beta t}\, dt$

and similarly that

(A9) $x > x_0 \Rightarrow \displaystyle\int_{x/x_0}^{\infty} \varphi(t)\,dt = \dfrac{1}{x/x_0 - 1}\, e^{-\beta\, x/x_0}\left(\dfrac{x}{x_0}\right)^\beta + \beta \displaystyle\int_{x/x_0}^{\infty} t^{\beta-1} e^{-\beta t}\, dt.$

One can readily see that the continuity of $q(x)$ (imposed by the no-arbitrage condition) yields $C_- = C_+ \equiv C$ and that $q(x_0)$ is as in equation (5.c). In order to find C, we impose that $q(x)$ is differentiable (hence twice differentiable since we want q to be a solution to a differential equation of order 2).

From (A6), one can write :

$x < x_0 \Rightarrow q'(x) = \dfrac{1}{x_0} + \dfrac{C}{x_0} \displaystyle\int_0^{x/x_0} \varphi(t)\,dt - \dfrac{C}{x_0}\left(1 - \dfrac{x}{x_0}\right)\varphi(x/x_0)$

and, out of equation (A8), we can then write :

$x < x_0 \Rightarrow q'(x) = \dfrac{1}{x_0} - \dfrac{C}{x_0}\,\beta \displaystyle\int_0^{x/x_0} t^{\beta-1}\, e^{-\beta t}\, dt.$

Similarly, one would get from equation (A7) :

$$x > x_0 \Rightarrow q'(x) = \frac{C}{x_0} \beta \int_{x/x_0}^{\infty} t^{\beta-1} e^{-\beta t} \, dt.$$

Writing that $q'(x_0-) = q'(x_0+)$ consequently imposes that

$$C = \frac{1}{\beta \int_0^{\infty} t^{\beta-1} e^{-\beta t} \, dt}$$

Integration $t^{\beta-1} e^{\beta t}$ by part shows that C is indeed as in equation (6).

BIBLIOGRAPHY

Bulow, J. and Rogoff, K. (1988) - "The Buy-Back Boondoggle", Brookings Papers on Economic Activity, n°2.

Claessens, S. and S. Van Winjbergen (1990) - "Secondary Market Prices under Alternative Debt Reduction Strategies : An Option Pricing Approach with an Application to Mexico", Working Paper, World Bank.

Cohen, D. (1989) - "How to Cope with a Debt Overhang : Cut Flows Rather than Stocks", in Dealing with the Debt crisis, Husain and Diwan eds, the World Bank.

Cohen, D. (1991) - "Private Lending to Sovereign States : A Theoretical Autopsy", Forthcoming MIT Press.

Dooley, M. (1988) - "Buy-Backs and Market Valuation of External Debt", IMF Staff Papers, pp. 215-229.

Genotte, G., Kharas H. and Sadeq S. (1987) - "A Valuation Model for Developing Country Debt with Endogeneous Rescheduling", World Bank Economic Review, Vol.1,n°2.

Husain, H. and I.Diwan (eds) (1990) - Dealing with the Debt Crisis, World Bank: Washington D.C.

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