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CAPTURING ASYMMETRY IN REAL EXCHANGE RATE WITH QUANTILE AUTOREGRESSION[*]

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SUMÁRIO

RESUMO

Autorregressão quantílica é empregada para explorar possíveis assimetrias no processo de ajustamento da taxa real de câmbio entre a lira italiana, o franco francês, o marco alemão e a libra esterlina. Baseando-se na melhor especificação para cada percentil foram construídas funções de densidade condicional. A partir dessas funções identificamos duas fontes de assimetria: 1) a dispersão depende do valor condicional da taxa real de câmbio, ou seja, existe heterocedasticidade condicional; 2) a probabilidade de elevação e queda também depende do valor condicionado da taxa real de câmbio: a probabilidade de apreciação (depreciação) é maior quando a moeda já se encontra depreciada (apreciada).

Forte heterocedasticidade foi apenas verificada nas relações envolvendo a lira, o franco e o marco. Esse problema foi resolvido estimando-se um modelo quadrático autorregressivo. As relações envolvendo a libra apresentaram-se mais homocedásticas, porém a maior dispersão sugere elevada probabilidade de grandes oscilações.

ABSTRACT

Quantile autoregression is used to explore asymmetries in the adjustment process of pair wise real exchange rate between the Italian lire, French franc, Deutsch mark, and the British pound.

Based on the best specification for each quantile we construct predicted conditional density functions which guided us to identify two sources of asymmetry: 1) dispersion depends on the conditioned value of the real exchange rate, i.e., "*conditional*" heterokedasticity; 2) the probability of increases and falls also changes according to the conditioned value, i.e., there is higher probability for the real exchange rate to appreciate (depreciate) given the currency is depreciated (appreciated).

We only verified strong heterokedasticity in relations among the lire, franc, and mark, which was resolved by estimating quadratic autoregressive model for some quantiles. Relations involving the pound presented stable but higher dispersion indicating larger probability of wider oscillation.

Keywords: Exchange Rate, Quantile Autoregression, Unit Root, Asymmetry

JEL Classification: C14, C22, F31

I. INTRODUCTION

Several recent works have investigated the possibility of asymmetry in real exchange rate (RER) time series. Relying on threshold autoregression (TAR) models, Michael, Nobay, and Peel (1997), Obstfeld and Taylor (1997), Bec, Carrasco and Salem (2004), Leon and Najarian (2005) found evidences of asymmetry. According to these works, the RER in period *t* follows a random walk if it assumes central values in $t-1$, and behaves as a convergent AR process it takes extreme values in $t-1$. These findings favor the notion that transaction cost is important for determining the behavior of RER.

In this paper we innovate and explore asymmetry in RER using *quantile autoregression* (QAR), as developed by Koenker and Xiao (2002, 2004). Differing from other methods, QAR permits the characterization of the entire distribution a time series, allowing for a better understanding of its stochastic process. The flexibility of the QAR makes it possible carrying unit root test at each quantile which also allows for the assessment of local and global persistent process. Another gain when relying on QAR is the possibility of having alternative specifications for modeling different quantiles.

Our analysis is conducted for pair wise real exchange rate between the Italian lira, French franc, Deutsch mark, and the British pound for a sample ranging from January of 1973 to December of 1998.

The main contribution of this paper, besides the use of QAR to analyze real exchange rate data, is the identification and measurement of two sources of asymmetry in the adjustment process of RER. The first is the "conditioned" heterokedasticity: dispersion, measured in terms of standard deviation and range, varies with the conditioned value of the real exchange rate. The second refers to tail behavior: the probability of increases and falls of a RER also changes according to the conditioned value, i.e., there is higher probability for the real exchange rate to appreciate (depreciate) given the currency is depreciated (appreciated). This last finding goes in the direction of the results obtained with the use of TAR family of models but it does not necessarily validate the use of symmetric thresholds so commonly employed in previous studies. Our analysis actually gives more support to non symmetric TAR models as implemented by Leon and Najarian (2005).

Important to emphasize that our notion of "conditional" heterokedasticity should not be interpreted with that used in ARCH models, which refers to a situation where the variance at time *t* depends on past variances.

Looking specifically at each currency, the heterokedasticity was only identified for relations between the lira, franc, and mark. Dispersion for RER involving the pound are larger but stable, meaning that estimated standard deviation and range are invariant to the conditioned value used to predict the real exchange rate.

The findings of the current work were based on the information contained in predicted conditional density functions constructed after estimating the best specification for each quantile.

The rest of this paper proceeds as follows. In the next section we review recent empirical literature on the behavior of the real exchange rate. In section 3 we briefly discuss quantile autoregression developed by Koenker and Xiao (2002, 2004). Section 4 brings the results and analysis, while 5 concludes. We plotted some predicted conditional density functions to help us visualize the asymmetric behavior followed by each RER time series.

II. EVIDENCES ON THE BEHAVIOR OF THE REAL EXCHANGE RATE

The purchasing power parity (PPP) theory is based on the validity of the following equation: $P_t = E_t P_t^*$, where P_t and P_t^* refer to domestic and foreign price levels, respectively, and E_t is the nominal exchange rate between the two currencies (or the home price of the foreign currency). This relation says that a devaluation in the home currency (increase of E_t) will be reflected in similar increase of the domestic price level, P_t, and/or in a reduction in the foreigner's, P_t^*. If this is indeed the case, one should expect a constant real exchange rate; $q_t = E_t P_t^* / P_t$.

It is well known, however, that due to price stickiness, at least in the short run the real exchange rate is influenced almost entirely by variations in the nominal exchange rate, which implies in oscillation of q_t over time and failure of the PPP in the short run. Over a longer horizon deviations from equilibrium should disappear as prices start to adjust. This is the same as saying that a real exchange rate series should not feature a unit root. Based on this economic rationale, several works tried to verify if q_t behaves like a stationary time series.

The first tests for the validity of the PPP were based on the Augmented Dickey Fuller (ADF) type of equation which features the following specification:

$$q_t = \alpha_0 + \alpha_1 q_{t-1} + \sum_{j=1}^{p} \alpha_{j+1} \Delta q_{t-j} + u_t \tag{1}$$

where $u_t \sim iid(0, \sigma^2)$ is the disturbance term. The validity of the PPP in the long run implies $|\alpha_1| < 1$.

Rejection of the null hypothesis of a unit root was hardly obtained, implying a random walk behavior for *RER* and therefore failure of the PPP theory[1]. It is known, however, that the low power of ADF and Phillips-Perron (PP) unit root tests makes it hard to distinguish between $\alpha_1 = 1$ from α_1 just close to 1. And indeed the estimated values for α_1 were all very close to 1[2].

The use of more powerful unit root tests using panel regression models delivered ambiguous results, sometimes favoring the stationarity of the *RER*[3]and some others favoring a random walk[4] behavior.

More recently, the literature has considered another two types of tests. One is related to the close to unity behavior of the *RER* and has been addressed by Kim and Lima (2004). They argued that the *RER* may be better described by a local persistent process, which postulates a great similarity with

[1] The constant non rejection of the unit root has been considered one of the six major puzzles in the international finance literature (Obstfeld and Rogoff, 2000).

[2] . See, for example, Darby (1980), Enders (1988) and Mark (1990).

[3] Froot and Rogoff (1995); Frankel and Rose (1996); Wu and Wu (2001); Papell (2002).

[4] O 'Connell (1996); Engel (1996); Canzoneri, Cumby, and Diba (1996).

the unit root in the short run, but that would present a convergent behavior in longer horizons. They applied the Lima-Xiao (2002) test to capture this local persistent process and did not reject the hypothesis that the *RER* of the G7 countries has a root near to unit, but not exactly 1.

The other direction pursued by this literature is the incorporation of a non-linear adjustment process for the RER, which has been analyzed with threshold autoregressive (TAR) models. The economic intuition for the use of this model is that transaction costs may create a region (called *band of inaction*) where market arbitrage is non-profitable, justifying the random walk behavior for the RER. However, if the RER is smaller than a lower threshold or greater than an upper one, international trade would be profitable causing the RER to behave like a stationary autoregressive process.

The work of Michael, Nobay and Peel (1997), Peel, Sarno and Taylor (2001), Bec, Carrasco and Salem (2004), and Leon and Najarian (2005) indicated that a three regime TAR better describes the stochastic process followed by several RER, corroborating the transaction cost theory. An important difference between the work of Leon and Najarian resides on the fact that they do not impose symmetric thresholds, which they found to be an important restriction.

Among the previous TAR work, only Bec, Carrasco and Salem (2004) used the same data we do in the current article. Based on their findings, the pair wise RER between the French Franc, Italian Lira, and the Deutsch Mark are better characterized by a three regime TAR process, since they rejected the null of unit root in favor of the TAR process. Quantile autoregression has not yet been considered to analyze the dynamics of a RER process. By filling this gap we obtain results not explored under previous econometric techniques.

III. QUANTILE AUTOREGRESSION

In this section we briefly discuss the quantile unit root test developed by Koenker and Xiao (2002, 2004).

Let the autoregressive process of any time series y_t be represented by

$$y_t = \alpha_1 y_{t-1} + u_t,\ t = 1,\dots,n. \tag{2}$$

and denote the τth quantile of u as $Q_u(\tau)$. Let $Q_{y_t}(\tau \mid \mathrm{y}_{\mathrm{t-1}})$ be the τth conditional quantile of y_t conditional on y_{t-1} which can be represented as

$$Q_{y_t}(\tau \mid \mathrm{y}_{\mathrm{t-1}}) = Q_u(\tau) + \alpha y_{t-1}. \tag{3}$$

Let $\alpha_0(\tau) = Q_u(\tau)$, $\alpha_1(\tau) = \alpha$ and define $\alpha(\tau) = (\alpha_0(\tau), \alpha_1(\tau))^T$ and $x_t = (1, y_{t-1})^T$. The previous equation can be rewritten as

$$Q_{y_t}(\tau \mid \mathrm{y}_{\mathrm{t-1}}) = x_t^T \alpha(\tau). \tag{4}$$

The quantile autoregressive parameter $\alpha(\tau)$ is estimated according to the linear programming problem suggested by Koenker and Basset (1978). Each solution $\hat{\alpha}(\tau)$ is the τth autoregressive quantile coefficient. Given $\hat{\alpha}(\tau)$, the τth quantile function of y_t, conditional on the past information, can be estimated by

$$\hat{Q}_{y_t}(\tau \mid x_{t\text{-}1}) = x_t^T \hat{\alpha}(\tau), \tag{5}$$

while the conditional density of y_t can be estimated by the following difference quotients

$$\hat{f}_{y_t}(\tau \mid x_t) = \frac{(\tau_i - \tau_{i-1})}{\hat{Q}_{y_t}(\tau_i \mid x_t) - \hat{Q}_{y_t}(\tau_{i-1} \mid x_t)} \tag{6}$$

The previous model can be used to test for unit root in each of the estimated quantiles. In particular, it can be extended to include higher order lagged difference terms resulting in the ADF type of equation:

$$y_t = \alpha_1 y_{t-1} + \sum_{j=1}^{p} \alpha_{j+1} \Delta y_{t-j} + u_t \,. \tag{7}$$

By letting $\alpha_j(\tau) = \alpha_j,\, j = 1, \ldots, p+1$, we can define $\alpha(\tau) = (\alpha_0(\tau), \alpha_1, \ldots, \alpha_{p+1})$ and $x_t = (1, y_{t-1}, \Delta y_{t-1}, \ldots, \Delta y_{t-q})^T$, leading to

$$Q_{y_t}(\tau \mid \Im_{t\text{-}1}) = x_t^T \alpha(\tau), \tag{8}$$

where $\Im_t$ is the σ-field generated by $\{u_s, s \le t\}$, and $Q_{y_t}(\tau \mid \Im_{t\text{-}1})$ is the τth conditional quantile of y_t, conditional on $\Im_{t-1}$.

Just as in the case of the ADF test, Koenker and Xiao showed that the asymptotic distribution of α_1, under the null hypothesis of a unit root, is the same irrespective of the value assumed by α_j, $j = 0, 1, \ldots, p+1$. They derived the following *t-ratio statistics* to test for the presence of a unit root at each desired quantile τ :

$$t_n(\tau) = \frac{f\left(\hat{F}^{-1}(\tau)\right)}{\sqrt{\tau(1-\tau)}} \left(\mathbf{Y}_{-1}^T \mathbf{P}_X \mathbf{Y}_{-1}\right)^{1/2} \left(\hat{\alpha}_1(\tau) - 1\right), \tag{9}$$

where $f\left(\hat{F}^{-1}(\tau)\right)$ is a consistent estimator of $f\left(F^{-1}(\tau)\right)$, $\mathbf{Y}_{-1}$ is the vector of lagged dependent variables y_{t-1}, and $\mathbf{P}_X$ is the projection matrix onto the space orthogonal to $\mathbf{X} = (1, \Delta y_{t-1}, \ldots, \Delta y_{t-p})$.

The limiting distribution of t_n is the same of the covariate-augmented Dickey-Fuller test of Hansen (1995). The critical values provided by Hansen (1995, page 1155) depend on a nuisance parameter δ^2 which is the square of the long-run correlation coefficient δ between $\{w_t\}$ and $\{\psi_\tau(u_{t\tau})\}$, where $w_t = \Delta y_t$ and $\psi_\tau(u_{t\tau}) = \tau - I(u_{t\tau} < 0)$. This correlation is expressed as

$$\delta = \delta(\tau) = \frac{\sigma_{w\psi}(\tau)}{\sigma_w \sigma_\psi(\tau)} = \frac{\sigma_{w\psi}(\tau)}{\sigma_w \sqrt{\tau(1-\tau)}} \qquad (10)$$

where $\sigma_{w\psi}(\tau)$ is the long run covariance between $\{w_t\}$ and $\{\psi_\tau(u_{t\tau})\}$, and σ_w^2 is the long run variance of $\{w_t\}$.

Once $t_n(\tau)$ is computed, and δ^2 is estimated according to $\hat{\delta}^2(\tau) = \dfrac{\hat{\sigma}_{w\psi}^2(\tau)}{\tau(1-\tau)\hat{\sigma}_w^2}$, one simply needs to compare $t_n(\tau)$ with the appropriate critical values.

Hansen constructed critical values for δ^2 in steps of 0.1. Instead of using these ranges, we followed the suggestion of Koenker and Xiao (2004) and fitted a quadratic equation associating δ^2 with the 5% critical value, resulting in the following estimated equation[5]:

$$CV5\% = -\underset{(0.0202)}{1.98667} - \underset{(0.0929)}{1.56576}\left(\delta^2\right) + \underset{(0.0906)}{0.742424}\left(\delta^2\right)^2$$

$$R^2 = 0.9952, \text{ and } F = 836.9242\ (pvalue = 0.000)$$

After computing δ^2 we can plug it in the previous equation to obtain the predicted 5% critical value which can be compared to the test statistic $t_n(\tau)$.

The procedure just described captures the local behavior of a time series y_t, but it does not provide a long run perspective. For instance, although a series can have local persistence and explosive behavior depending on the magnitude of the shock, it can still be stationary in the long run. This is guaranteed by corollary 2.1 in Koenker and Xiao (2002). According to this corollary, if

1. $\{u_t\}$ are *iid* random variables with mean 0 and variance $\sigma^2 < \infty$, and the distribution function of u_t, F, has a continuous density f with $f(u) > 0$ on $U = \{u : 0 < F(u) < 1\}$;
2. y_t is determined by equation 8; and
3. $E(\alpha)^2 < 1$,

then y_t is covariance stationary. In this case we could compute $E(\alpha)^2$ by $\int_\varepsilon^{1-\varepsilon} \alpha(\tau)^2 d\tau$ to check whether this expectation is smaller than 1[6].

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[5] Standard deviation is in parenthesis below its respective coefficient.

[6] Notice that a formal test has not yet been developed, but by computing this integral we can at least have an idea about the global behavior of the series considered.

Nuisance Parameters

In order to compute the test statistic $t(n)$ we need to estimate $f(F^{-1}(\tau))$, σ_w^2 and $\sigma_{w\psi}(\tau)$. Following Koenker and Xiao[7] (2004), the quantile density function is estimated according to

$$f_n\left(F_n^{-1}(t)\right) = \frac{2h_n}{F_n^{-1}(t+h_n) - F_n^{-1}(t-h_n)} \tag{11}$$

where $F_n^{-1}(s)$ is an estimate of $F^{-1}(s)$ and h_n is a bandwidth[8]. $F^{-1}(s)$ is obtained by using the following empirical quantile function for the linear model proposed by Bassett and Koenker (1982),

$$\hat{Q}(\tau \mid \bar{x}) = \bar{x}^T \hat{\alpha}(\tau). \tag{14}$$

The density $f\left(F^{-1}(t)\right)$ is estimated according to

$$f_n\left(F_n^{-1}(t)\right) = \frac{2h_n}{x^T\left(\hat{\alpha}(t+h_n) - \hat{\alpha}(t-h_n)\right)} \tag{15}$$

For the long-run variance and covariance parameters, Koenker and Xiao suggest, respectively, the following Kernel estimators:

$$\sigma_w^2 = \sum_{h=-M}^{M} k\left(\frac{h}{M}\right) C_{ww}(h) \text{ and } \sigma_{w\psi} = \sum_{h=-M}^{M} k\left(\frac{h}{M}\right) C_{w\psi}(h),$$

where $k(\bullet)$ is the lag window defined on $[-1,1]$ with $k(0)=1$, and M is the bandwidth (truncation) parameter. $C_{ww}(h)$ and $C_{w\psi}(h)$ are sample covariances defined by $C_{ww}(h) = n^{-1}\sum_{\circ} w_t w_{t+h}$ and $C_{w\psi}(h) = n^{-1}\sum_{\circ} w_t \psi_{t+h}(\hat{u}_{(t+h)\tau})$, where $\sum_{\circ}$ stands for summation over $1 \le t, t+h \le n$.

We chose to work with the Bartlett kernel, $k\left(\frac{h}{M}\right) = 1 - \left|\frac{h}{M}\right|$, for computing the consistent correlation coefficient δ, with the bandwidth truncation parameter (M) being chosen according to Andrews (1991).

[7] The authors follow Siddiqui (1960) when deciding on the estimation of $f_n\left(F_n^{-1}(t)\right)$.

[8] The R function "bandwith.rq", which is available in the library "quantreg", was used to obtain the values for h_n.

IV. ESTIMATION AND RESULTS

We worked with monthly data from January of 1973 to December of 1998 obtained from the IMF International Financial Statistics CD-ROM. The nominal exchange rate is the end of period, and the price deflator is the CPI. The logarithm of each RER series can be visualized in Figure 1.

FIGURE 1

Logarithm of the real exchange rate between Italian lire, French franc, Deutsch mark, and the British pound from Jan/1973 to Dec/1998

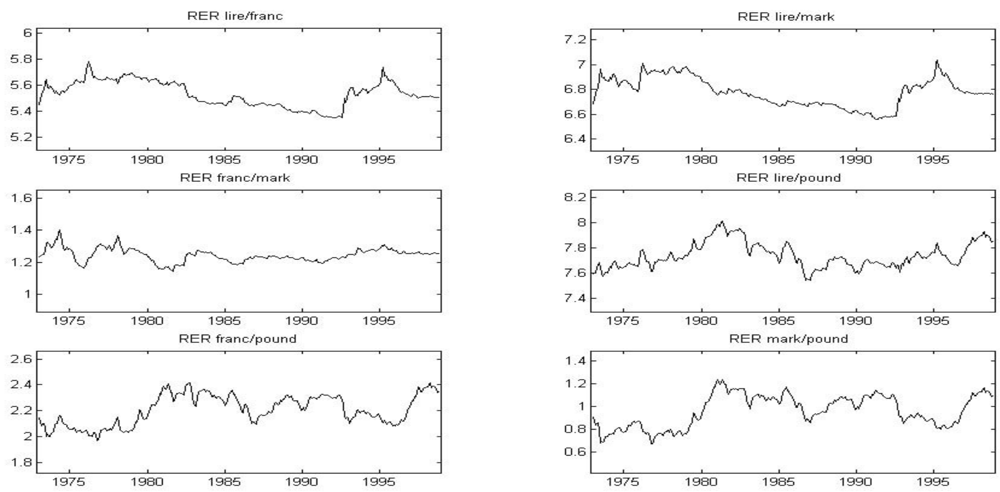


TABLE 1

Test statistics of the ADF and PP unit root tests and estimated value of $E(\alpha)^2$

Test	$RER^{it/fr}$	$RER^{it/gr}$	$RER^{fr/gr}$	$RER^{it/uk}$	$RER^{fr/uk}$	$RER^{uk/gr}$
ADF	-1.95	-1.98	-3.28*	-2.76	-2.35	-2.07
PP	-1.9	-1.86	-2.90*	-2.45	-1.98	-1.87
$E(\alpha)^2$	0.991	0.981	0.934	0.938	0.953	0.965

* stands for rejection at 5% level of significance.

Unit Root Tests

Two standards unit root tests (Augmented Dickey-Fuller and Phillips-Perron) were initially applied to our series. The lag order of the ADF equation was chosen according to the BIC criteria and used in the OLS and also in the quantile regressions.

Results reported in table 1 are coherent with previous works: unit root was only rejected for real exchange rate between the French franc and the Deutsch mark. The tests failed to reject the null of a unit root in the other series.

We also estimated $E(\alpha)^2$ by $\int_{\varepsilon}^{1-\varepsilon} \alpha(\tau)^2 d\tau$ using $\tau \in [0.05, 0.95]$. The results are in the last row of table 1. Consistent with the previous two tests, the lowest value was obtained for $RER^{fr/gr}$ (0.934). The value for $RER^{it/uk}$ was very similar (0.9384). A value closer to 1 was estimated for $RER^{it/fr}$ and $RER^{it/gr}$: 0.9912 and 0.9807, respectively. Despite the absence of critical values to compare these estimates, the statistics suggest that $RER^{fr/gr}$ and $RE^{it/uk}$ behave as a stationary series in the long run. Very likely we would conclude that $RER^{it/fr}$ and $RER^{it/gr}$ feature a unit root given the proximity of $\int_{\varepsilon}^{1-\varepsilon} \alpha(\tau)^2 d\tau$ to 1. It is harder to draw conclusions about $RER^{fr/uk}$ and $RER^{uk/gr}$ given their intermediate values.

TABLE 2

Estimated values of $\alpha_1(\tau)$, test statistic t_n, and δ^2 for $RER^{it/fr}$ $RER^{it/gr}$ and $RER^{fr/gr}$ at selected quantiles

Quantile	$RER^{it/fr}$			$RER^{it/gr}$			$RER^{fr/gr}$		
	$\alpha_1(\tau)$	t_n^A	δ^2	$\alpha_1(\tau)$	t_n^A	δ^2	$\alpha_1(\tau)$	t_n^A	δ^2
0.05	0.893*	-5.347	0.091	0.909*	-5.225	0.076	0.872*	-3.666	0.283
0.10	0.938*	-2.601	0.194	0.942*	-3.346	0.051	0.929	-2.291	0.355
0.15	0.965	-2.082	0.219	0.955*	-4.151	0.147	0.930*	-2.933	0.477
0.20	0.967*	-3.664	0.254	0.966*	-3.194	0.226	0.933*	-4.753	0.447
0.25	0.973*	-3.426	0.289	0.974*	-2.997	0.275	0.936*	-4.954	0.45
0.30	0.983	-2.123	0.316	0.976*	-2.959	0.267	0.947*	-4.435	0.509
0.35	0.99	-1.193	0.313	0.985	-2.269	0.23	0.958*	-3.519	0.507
0.40	0.992	-1.022	0.293	0.987	-1.929	0.341	0.965*	-2.729	0.444
0.45	0.994	-0.832	0.316	0.992	-1.064	0.373	0.965*	-2.967	0.45
0.50	0.994	-0.933	0.349	0.998	-0.327	0.437	0.978	-1.934	0.466
0.55	0.995	-0.668	0.347	1.003	0.455	0.416	0.982	-1.468	0.47
0.60	1.002	0.353	0.396	1.002	0.267	0.464	0.992	-0.601	0.406
0.65	1.005	0.664	0.402	1.004	0.656	0.558	0.989	-0.832	0.473
0.70	1.016	2.06	0.427	1.007	0.902	0.568	0.976	-1.618	0.452
0.75	1.024	2.773	0.481	1.01	1.027	0.526	0.979	-1.038	0.439
0.80	1.032	2.842	0.444	1.012	1.017	0.489	0.984	-0.674	0.42
0.85	1.036	2.222	0.486	1.009	0.592	0.482	0.99	-0.365	0.519
0.90	1.035	1.681	0.562	1.011	0.567	0.516	1.012	0.303	0.572
0.95	1.044	0.848	0.613	1.039	0.757	0.669	1.075	0.933	0.36

* stands for rejection at 5% significance level.

A. The test statistic tn is compared to the critical value that is found by plugging δ^2 in the following equation: $CV5\% = -1.98667 - 1.56576(\delta^2) + 0.742424(\delta^2)^2$

Quantile estimation of $\alpha_1(\tau)$ and their unit root tests are reported in tables 2 and 3. The results show different patterns among the series analyzed. In table 2 we verify that a local random walk behavior is rejected in lower quantiles of $RER^{it/fr}$, $RER^{it/gr}$, and $RER^{fr/gr}$, with increasing coefficients in τ. The pattern is different for relations involving the British pound (table 3) where unit root was only rejected at the 15th and the 25th quantiles of $RER^{it/uk}$.

TABLE 3

Estimated values of $\alpha_1(\tau)$, test statistic t_n, and δ^2 for $RER^{it/uk}$ $RER^{fr/uk}$ and $RER^{uk/gr}$ at selected quantiles

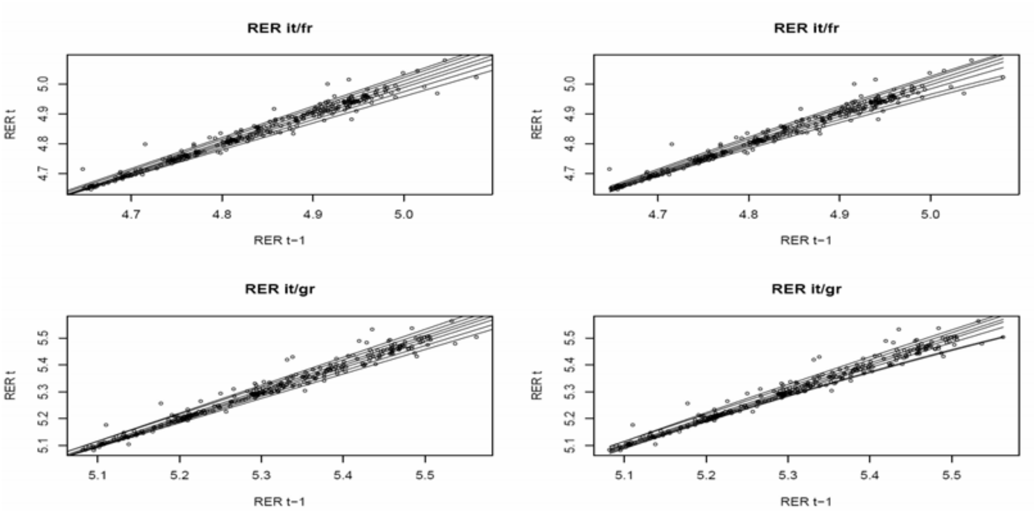
Quantile	$RER^{it/uk}$			$RER^{fr/uk}$			$RER^{uk/gr}$		
	$\alpha_1(\tau)$	t_n^A	δ^2	$\alpha_1(\tau)$	t_n^A	δ^2	$\alpha_1(\tau)$	t_n^A	δ^2
0.05	0.935	-1.25	0.199	0.958	-1.726	0.315	0.979	-0.757	0.192
0.10	0.959	-1.359	0.261	0.962	-1.877	0.346	0.991	-0.622	0.325
0.15	0.953*	-2.632	0.421	0.966	-1.822	0.451	0.993	-0.467	0.391
0.20	0.956	-2.447	0.509	0.981	-1.123	0.507	0.986	-0.918	0.484
0.25	0.961*	-2.591	0.469	0.972	-1.812	0.478	0.977	-1.72	0.577
0.30	0.976	-1.645	0.525	0.981	-1.18	0.541	0.98	-1.576	0.533
0.35	0.971	-2.302	0.529	0.984	-1.194	0.502	0.981	-1.578	0.57
0.40	0.973	-2.473	0.547	0.978	-1.87	0.533	0.98	-1.888	0.518
0.45	0.976	-1.902	0.615	0.977	-2.018	0.564	0.983	-1.509	0.604
0.50	0.973	-2.212	0.68	0.978	-2.183	0.633	0.987	-1.297	0.586
0.55	0.98	-1.695	0.645	0.977	-2.213	0.599	0.982	-1.914	0.56
0.60	0.978	-1.697	0.669	0.985	-1.375	0.679	0.98	-2.243	0.657
0.65	0.977	-1.638	0.628	0.987	-1.01	0.661	0.984	-1.585	0.731
0.70	0.974	-1.835	0.708	0.983	-1.237	0.599	0.978	-1.964	0.635
0.75	0.98	-1.215	0.651	0.983	-1.155	0.54	0.976	-1.785	0.631
0.80	0.969	-1.622	0.547	0.967	-2.291	0.421	0.979	-1.438	0.564
0.85	0.966	-1.468	0.516	0.974	-1.533	0.443	0.992	-0.649	0.433
0.90	0.971	-0.944	0.378	0.969	-1.813	0.298	0.978	-2.143	0.357
0.95	0.97	-0.628	0.207	0.971	-0.833	0.255	0.989	-0.507	0.235

* stands for rejection at 5% significance level.

A. The test statistic tn is compared to the critical value that is found by plugging δ^2 in the following equation: $CV\,5\% = -1.98667 - 1.56576\left(\delta^2\right) + 0.742424\left(\delta^2\right)^2$

FIGURE 2

Fitted lines for linear (left) and quadratic (right) models of some quantile autoregression: $RER^{it/fr}$ and $RER^{it/gr}$



The results obtained for $RER^{it/fr}$, $RER^{it/gr}$, and $RER^{fr/gr}$ indicate asymmetric adjustment, which is responsible for a heterokedasticity not yet explored in previous works on real exchange rate. Such a heterokedasticity can be dealt with by a quadratic autoregressive specification[9] that is similar to the

[9] The heterokedasticity obtained in our work is similar to that of the Sydney temperature analyzed by Koenker (2005). He suggested a quadratic model to deal with this problem.

previous QAR (equation 8), but with y_{t-1}^2 included as a covariate and β introduced in the vector of parameters, which results in: $x_t = (1, y_{t-1}, y_{t-1}^2, \Delta y_{t-1}, \ldots, \Delta y_{t-q})^T$ and $\alpha(\tau) = (\alpha_0(\tau), \alpha_1, \beta, \alpha_2, \ldots, \alpha_{p+1})$.

FIGURE 3

Fitted lines for linear (left) and quadratic (right) models of some quantile autoregression: $RER^{it/uk}$ and $RER^{fr/gr}$

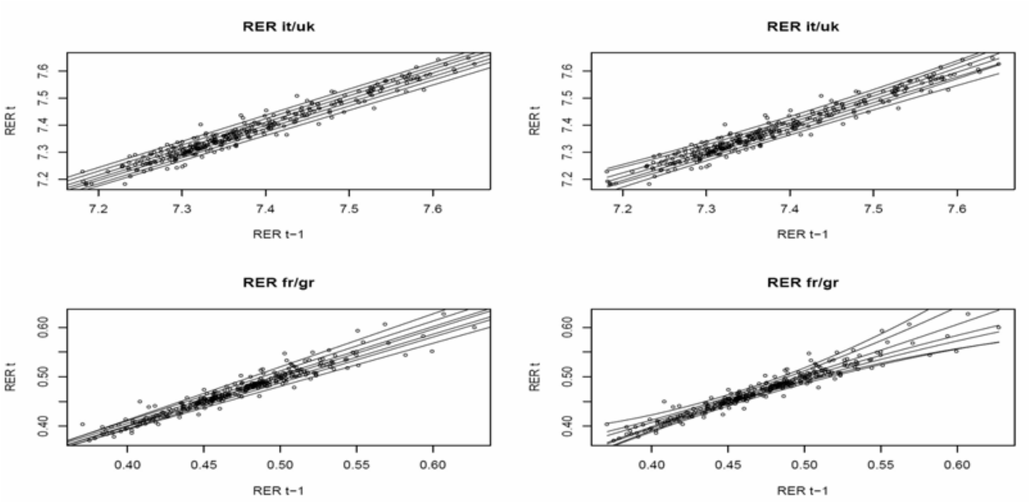
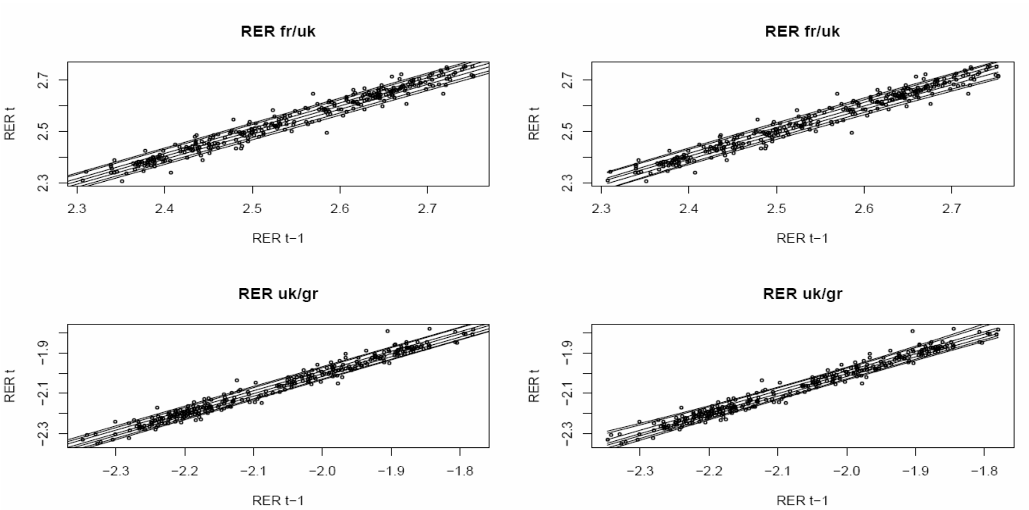


FIGURE 4

Fitted lines for linear (left) and quadratic (right) models of some quantile autoregression: $RER^{fr/uk}$ and $RER^{uk/gr}$.

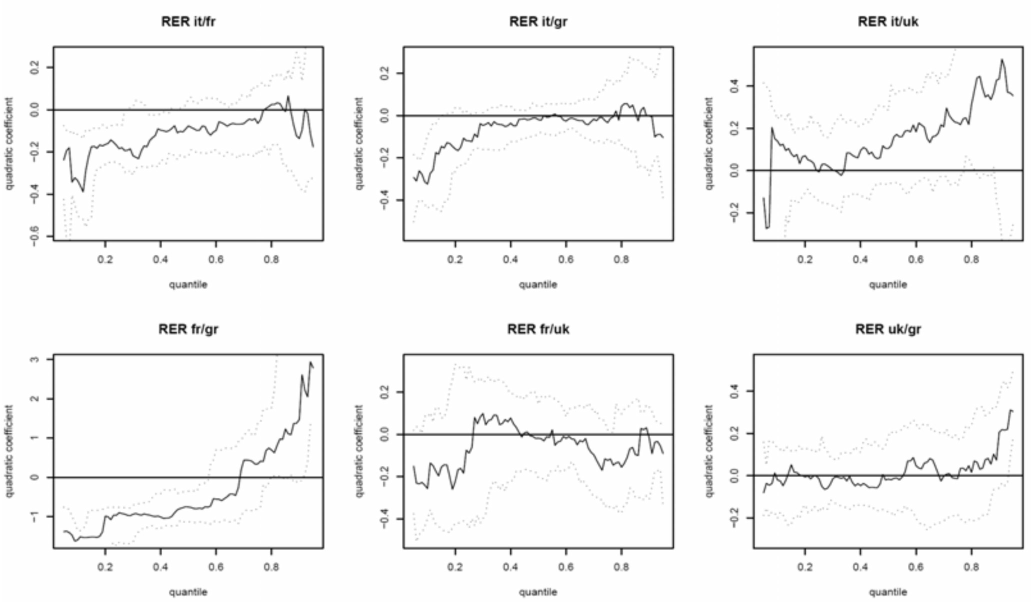


In figures 2, 3, and 4 we show the fit of linear (left side) and quadratic specifications (right side). The fit of the linear model for relations involving the pound look parallel, suggesting a more symmetric local adjustment. In these cases we notice the quadratic specification not changing the result, which evidences the lack of significance of the quadratic term as also witnessed by figure 5.

The situation is different for $RER^{it/fr}$, $RER^{it/gr}$, and $RER^{fr/gr}$. The fitted lines of the linear quantile model are not parallel. Steeper slopes are observed in upper quantiles forming a fan shaped graph. This reflects the real exchange rate can assume wider range in period t if higher values are observed in $t-1$. The quadratic specification seems appropriate to capture this heterokedasticity, as it can be observed in the right plots of figures 2 and 3, and also by observing the behavior of all estimated quadratic coefficients and their respective 90% confidence interval plotted in figure 5.

FIGURE 5

Estimated quadratic coefficients and their respective 90% confidence interval



We gain better understanding about the stochastic processes followed by each RER after constructing conditional predicted density functions based on the most appropriate specification for each quantile. For example, in the case of $RER^{it/uk}$ we verify a significant quadratic coefficient only for $\tau = 0.90$; for all remaining τ the linear model seems more appropriate. In this case q_t was predicted (conditioned on values of q_{t-1}) using the linear model for every quantile, except for $\tau = 0.90$ that was estimated with the quadratic model. The same procedure was employed to the other time series.

TABLE 4

Quantifying the asymmetry in the *RER*[it/fr]

$Q_{q_{t-1}}(\tau)$	$\hat{\sigma}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\Pr\_L(z=1)$	$\Pr\_U(z=1)$	$\Pr\_L(z=1.5)$	$\Pr\_U(z=1.5)$	$\Pr\_L(z=2)$	$\Pr\_U(z=2)$
$4.66_{\tau=0.05}$	0.005	0.023	0.253	0.077	0.110	0.066	0.000	0.055
$4.69_{\tau=0.1}$	0.005	0.025	0.165	0.110	0.066	0.066	0.000	0.055
$4.76_{\tau=0.25}$	0.006	0.032	0.143	0.132	0.033	0.077	0.022	0.055
$4.81_{\tau=0.4}$	0.008	0.039	0.132	0.154	0.044	0.077	0.033	0.044
$4.83_{\tau=0.5}$	0.008	0.042	0.154	0.154	0.055	0.077	0.033	0.044
$4.86_{\tau=0.6}$	0.010	0.047	0.143	0.154	0.066	0.077	0.033	0.022
$4.92_{\tau=0.75}$	0.012	0.057	0.154	0.132	0.088	0.055	0.044	0.022
$4.95_{\tau=0.9}$	0.014	0.065	0.165	0.110	0.088	0.033	0.044	0.000
$4.97_{\tau=0.95}$	0.015	0.069	0.165	0.110	0.099	0.022	0.055	0.000

$\Pr\_L(z) = \Pr(\hat{q}_t < Q_{y_{t-1}}(\tau) - z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$, $\Pr\_U(z) = \Pr(\hat{q}_t > Q_{y_{t-1}}(\tau) + z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$

Each conditional predicted density function allowed us to construct measures of dispersion and tail behavior. This information is condensed in tables 4 to 9 and also in the appendix, where we plotted some predicted density functions. Dispersion was analyzed in terms of conditional standard deviation, $\hat{\sigma}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$, and conditional range, $\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$. The tail behavior was analyzed after computing probabilities or the real exchange rate $\hat{q}_t$ to be above and below z standard deviations away from the conditioned value q_{t-1}: $\Pr(\hat{q}_t >\mid Q_{y_{t-1}}(\tau) \pm z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)) \mid)$.

TABLE 5

Quantifying the asymmetry in the *RER* [it/gr]

$Q_{q_{t-1}}(\tau)$	$\hat{\sigma}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\Pr\_L(z=1)$	$\Pr\_U(z=1)$	$\Pr\_L(z=1.5)$	$\Pr\_U(z=1.5)$	$\Pr\_L(z=2)$	$\Pr\_U(z=2)$
$5.11_{\tau=0.05}$	0.006	0.027	0.121	0.143	0.044	0.099	0.000	0.066
$5.14_{\tau=0.10}$	0.006	0.027	0.099	0.154	0.022	0.099	0.000	0.066
$5.21_{\tau=0.25}$	0.007	0.031	0.165	0.143	0.011	0.088	0.000	0.066
$5.29_{\tau=0.40}$	0.009	0.039	0.165	0.143	0.055	0.077	0.000	0.044
$5.30_{\tau=0.50}$	0.009	0.040	0.165	0.143	0.066	0.077	0.000	0.033
$5.33_{\tau=0.60}$	0.010	0.044	0.165	0.132	0.099	0.077	0.000	0.033
$5.39_{\tau=0.75}$	0.012	0.055	0.187	0.099	0.099	0.055	0.044	0.022
$5.47_{\tau=0.90}$	0.015	0.070	0.198	0.088	0.088	0.033	0.088	0.000
$5.49_{\tau=0.95}$	0.017	0.075	0.198	0.077	0.088	0.033	0.088	0.000

$\Pr\_L(z) = \Pr(\hat{q}_t < Q_{y_{t-1}}(\tau) - z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$, $\Pr\_U(z) = \Pr(\hat{q}_t > Q_{y_{t-1}}(\tau) + z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$

TABLE 6

Quantifying the asymmetry in the RER [fr/gr]

$Q_{q_{t-1}}(\tau)$	$\hat{\sigma}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\Pr\_L(z=1)$	$\Pr\_U(z=1)$	$\Pr\_L(z=1.5)$	$\Pr\_U(z=1.5)$	$\Pr\_L(z=2)$	$\Pr\_U(z=2)$
$0.40_{\tau=0.05}$	0.007	0.033	0.165	0.154	0.022	0.055	0.000	0.055
$0.41_{\tau=0.10}$	0.006	0.029	0.143	0.187	0.022	0.110	0.000	0.055
$0.45_{\tau=0.25}$	0.005	0.023	0.099	0.198	0.022	0.132	0.000	0.077
$0.46_{=0.40}$	0.005	0.023	0.110	0.187	0.022	0.132	0.000	0.066
$0.47_{\tau=0.50}$	0.005	0.025	0.154	0.165	0.033	0.110	0.000	0.066
$0.48_{\tau=0.60}$	0.006	0.027	0.187	0.154	0.033	0.088	0.000	0.044
$0.50_{\tau=0.75}$	0.007	0.031	0.209	0.132	0.044	0.088	0.011	0.044
$0.52_{\tau=0.90}$	0.009	0.044	0.253	0.088	0.077	0.055	0.000	0.055
$0.54_{\tau=0.95}$	0.012	0.054	0.275	0.077	0.110	0.055	0.000	0.055

$\Pr\_L(z) = \Pr(\hat{q}_t < Q_{y_{t-1}}(\tau) - z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$, $\Pr\_U(z) = \Pr(\hat{q}_t > Q_{y_{t-1}}(\tau) + z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$

Dispersion

$\hat{\sigma}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$ and $\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$ are larger for relations involving the British pound. The estimated standard deviation situates around 0.015, 0.016 and 0.017 for RER$^{it/uk}$, RER$^{fr/uk}$, and RER$^{gr/uk}$, respectively, regardless to where we condition q_{t-1}. Conditioning q_{t-1} at central quartiles delivers the following estimates for RER$^{it/gr}$, RER$^{it/fr}$, and RER$^{fr/gr}$: 0.008, 0.009, and 0.005, respectively.

In the case of the last three relations we observe higher variability in the values of $\hat{\sigma}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$. In order to illustrate this fact, notice that $\hat{\sigma}^{it/fr}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$ is equal to 0.005 for $Q_{q_{t-1}}(\tau = \{0.05, 0.1\})$, and it becomes 0.14 and 0.15 for $Q_{q_{t-1}}(\tau = \{0.90, 0.95\})$, respectively. Similarly, $\hat{\sigma}^{it/gr}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$ equals 0.006 at $\tau = \{0.05, 0.1\}$ and 0.15 and 0.17 when $\tau = \{0.90, 0.95\}$, respectively. The estimates for RER$^{fr/gr}$ are more stable, except for high percentiles of q_{t-1}.

TABLE 7
Quantifying the asymmetry in the *RER* $^{it/uk}$

$Q_{q_{t-1}}(\tau)$	$\hat{\sigma}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\Pr\_L(z=1)$	$\Pr\_U(z=1)$	$\Pr\_L(z=1.5)$	$\Pr\_U(z=1.5)$	$\Pr\_L(z=2)$	$\Pr\_U(z=2)$
$7.24_{\tau=0.05}$	0.016	0.072	0.088	0.198	0.022	0.154	0.000	0.088
$7.27_{\tau=0.25}$	0.016	0.072	0.099	0.198	0.033	0.143	0.000	0.077
$7.31_{\tau=0.25}$	0.015	0.072	0.110	0.198	0.044	0.088	0.011	0.033
$7.33_{\tau=0.40}$	0.015	0.073	0.132	0.176	0.055	0.077	0.022	0.033
$7.36_{\tau=0.50}$	0.015	0.074	0.154	0.154	0.066	0.055	0.022	0.033
$7.38_{\tau=0.60}$	0.015	0.074	0.165	0.132	0.077	0.044	0.033	0.033
$7.44_{\tau=0.75}$	0.015	0.077	0.187	0.121	0.088	0.044	0.033	0.011
$7.53_{\tau=0.90}$	0.017	0.080	0.231	0.121	0.121	0.033	0.033	0.000
$7.57_{\tau=0.95}$	0.018	0.084	0.231	0.132	0.121	0.033	0.033	0.000

$\Pr\_L(z) = \Pr(\hat{q}_t < Q_{y_{t-1}}(\tau) - z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$, $\Pr\_U(z) = \Pr(\hat{q}_t > Q_{y_{t-1}}(\tau) + z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$

TABLE 8
Quantifying the asymmetry in the *RER* $^{fr/uk}$

$Q_{q_{t-1}}(\tau)$	$\hat{\sigma}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\Pr\_L(z=1)$	$\Pr\_U(z=1)$	$\Pr\_L(z=1.5)$	$\Pr\_U(z=1.5)$	$\Pr\_L(z=2)$	$\Pr\_U(z=2)$
$2.37_{\tau=0.05}$	0.016	0.063	0.099	0.231	0.033	0.176	0.000	0.066
$2.38_{\tau=0.10}$	0.016	0.063	0.099	0.220	0.044	0.176	0.000	0.055
$2.44_{\tau=0.25}$	0.016	0.064	0.132	0.209	0.044	0.121	0.000	0.044
$2.50_{\tau=0.40}$	0.016	0.065	0.165	0.198	0.055	0.110	0.000	0.011
$2.54_{\tau=0.50}$	0.016	0.065	0.176	0.187	0.066	0.077	0.022	0.011
$2.60_{\tau=0.60}$	0.016	0.066	0.187	0.176	0.099	0.066	0.022	0.000
$2.65_{\tau=0.75}$	0.016	0.067	0.209	0.143	0.110	0.055	0.022	0.000
$2.69_{\tau=0.90}$	0.016	0.067	0.220	0.121	0.121	0.044	0.044	0.000
$2.72_{\tau=0.95}$	0.016	0.068	0.231	0.110	0.121	0.044	0.055	0.000

$\Pr\_L(z) = \Pr(\hat{q}_t < Q_{y_{t-1}}(\tau) - z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$, $\Pr\_U(z) = \Pr(\hat{q}_t > Q_{y_{t-1}}(\tau) + z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$

Similar pattern was also verified for *estimated range*: relations involving the pound has higher but stable $\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$; for the remaining series $\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$ is smaller but less stable as it increases with τ. This last characteristic is better illustrated once we notice that the estimated range of RER$^{it/fr}$ and RER$^{it/gr}$ at low values of q_{t-1} is about three times smaller than those computed for high q_{t-1}.

TABLE 9

Quantifying the asymmetry in the *RER*$^{uk/gr}$

$Q_{q_{t-1}}(\tau)$	$\hat{\sigma}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\hat{R}(\hat{q}_t \mid Q_{q_{t-1}}(\tau))$	$\Pr\_L(z=1)$	$\Pr\_U(z=1)$	$\Pr\_L(z=1.5)$	$\Pr\_U(z=1.5)$	$\Pr\_L(z=2)$	$\Pr\_U(z=2)$
$-2.26_{\tau=0.05}$	0.017	0.075	0.165	0.198	0.044	0.099	0.000	0.022
$-2.24_{\tau=0.10}$	0.017	0.072	0.165	0.198	0.044	0.099	0.000	0.022
$-2.20_{\tau=0.25}$	0.017	0.067	0.165	0.187	0.066	0.099	0.000	0.022
$-2.14_{\tau=0.40}$	0.016	0.063	0.176	0.176	0.077	0.099	0.000	0.000
$-2.08_{\tau=0.50}$	0.016	0.062	0.198	0.165	0.099	0.088	0.000	0.000
$-2.02_{\tau=0.60}$	0.016	0.062	0.231	0.154	0.110	0.077	0.011	0.000
$-1.95_{\tau=0.75}$	0.017	0.063	0.242	0.143	0.121	0.077	0.011	0.000
$-1.88_{\tau=0.90}$	0.017	0.069	0.253	0.132	0.121	0.044	0.011	0.000
$-1.86_{\tau=0.95}$	0.017	0.070	0.253	0.132	0.132	0.044	0.011	0.022

$\Pr\_L(z) = \Pr(\hat{q}_t < Q_{y_{t-1}}(\tau) - z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$, $\Pr\_U(z) = \Pr(\hat{q}_t > Q_{y_{t-1}}(\tau) + z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)))$

Tail Behavior

Columns 3 to 8 of tables 4-9 display $\Pr(\hat{q}_t >\mid Q_{y_{t-1}}(\tau) \pm z\sigma(\hat{q}_t \mid Q_{y_{t-1}}(\tau)) \mid)$, for $z = \{1, 1.5, 2\}$. These measures inform not only the asymmetric behavior in the adjustment of each real exchange rate, but also quantify the probabilities of being in the tails. Despite the difference in magnitude, similar pattern was observed for the six RER studied regardless the value of z.

Relations involving the pound (tables 7, 8 and 9) have higher probability of depreciation given a very appreciated q_{t-1}, and vice versa. A more equal probability for moving in either direction is obtained for q_{t-1} at central quantiles. These patterns are also observed for the other relations, with some local exception.

For RER$^{fr/gr}$, $z = 1$ and q_{t-1} at $\tau = 0.05$ we verified almost the same probabilities for moving in either direction: 16.5% for falling more than 1 standard deviation and 15.4% for increasing.

For RER$^{it/fr}$, $z = \{1, 1.5\}$ and q_{t-1} conditioned at $\tau = 0.05$ (i.e. very appreciated lira) we observed a much higher probability for appreciation (25.3% for $z = 1$ and 11% for $z = 1.5$) than depreciation (7.7% for $z = 1$ and 6.6% for $z = 1.5$).

The analysis when $z = 2$ is particularly interesting because it informs the probabilities of being in very extreme tails. Except for RER$^{fr/gr}$, the general pattern already mentioned remains: higher probability of appreciation (depreciation) given an already depreciated (appreciated) currency. Exception was verified for RER$^{fr/gr}$, since the probability for the franc to depreciate was around 5%

regardless of the conditioned value at $t-1$. Positive probability of appreciation (1.1%) was only obtained when predictions were computed for q_{t-1} at $\tau = 0.75$.

Another issue to notice is the low probability for the $RER^{uk/gr}$ to assume very extreme values, which contrasts with the estimates for other relations.

Transaction Cost

Transaction cost theory says that the RER more likely depicts a random walk behavior at central values, while a convergent autoregressive pattern would be observed if it assumes extreme values.

The results reported in tables 4-9 showing higher probability for appreciation (depreciation) if RER is depreciated (appreciated) favor the intuition behind the transaction cost theory. The only exception was $\Pr(\hat{q}_t^{fr/gr} > |Q_{q_{t-1}^{fr/gr}}(\tau) \pm 2\sigma(\hat{q}_t^{fr/gr} \mid Q_{q_{t-1}^{fr/gr}}(\tau))|)$, since we estimated higher probability for devaluation of the franc independent of τ.

Similar to the study of Leon and Najarian, our results also indicate that is dangerous relying on symmetric TAR models for modeling the stochastic behavior of the real exchange rate. This happens because the probability of appreciating conditioned on depreciated RER is not necessarily the same as the probability of depreciating given an appreciated currency.

V. CONCLUSIONS

Quantile autoregression was used to analyze the behavior of the pair wise real exchange rate between the Italian lire, French franc, Deutsch mark, and the British pound, using data from January of 1973 to December of 1998.

The main contribution of this paper, besides the use of QAR to analyze real exchange rate data, is the identification and measurement of two sources of asymmetry in the adjustment process of RER. The first is the "conditioned" heterokedasticity: dispersion, measured in terms of standard deviation and range, varies with the conditioned value of the real exchange rate. The second refers to tail behavior: the probability of increases and falls of a RER also changes according to the conditioned value, i.e., there is higher probability for the real exchange rate to appreciate (depreciate) given the currency is depreciated (appreciated). These probabilities were not symmetric.

This last finding goes in the direction of the results obtained with the use of TAR family of models but it does not necessarily validate the use of symmetric thresholds so commonly employed in previous studies. Our analysis actually gives more support to non symmetric TAR models as implemented by Leon and Najarian (2005).

Specific analysis of each currency showed heterokedasticity for $RER^{it/gr}$, $RER^{it/fr}$, and $RER^{fr/gr}$, which was dealt with by estimating a quadratic autoregressive model. The quadratic specification revealed to be a better model for several quantiles. The linear specification was on general superior for relations against the pound, for which we also observed higher dispersion (measured in terms of standard deviation and range).

It would still be interesting to include covariates to verify if there exists any relation between them and the asymmetry of the real exchange rate. This is left for future research.

APPENDIX: CONDITIONAL PREDICTED DENSITY FUNCTION

We show in figures 6-10 conditional density functions for predicted real exchange rate at some selected q_{t-1}. Above each density plot we have the value of q_{t-1} and its respective quantile. At the bottom we report the probability of $\hat{q}_t$ being smaller and greater than q_{t-1}. The vertical line in each plot corresponds to the value of q_{t-1}.

FIGURE 6

Predicted density of $RER^{fr/gr}$ using the best specification at each τ

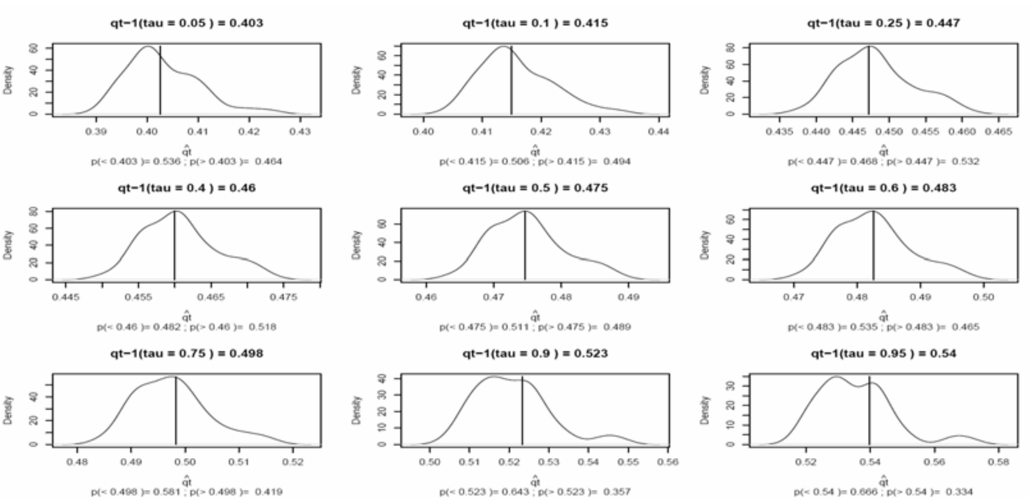


FIGURE 7

Predicted density of $RER^{it/fr}$ using the best specification at each τ . (dens_final_if.pdf)

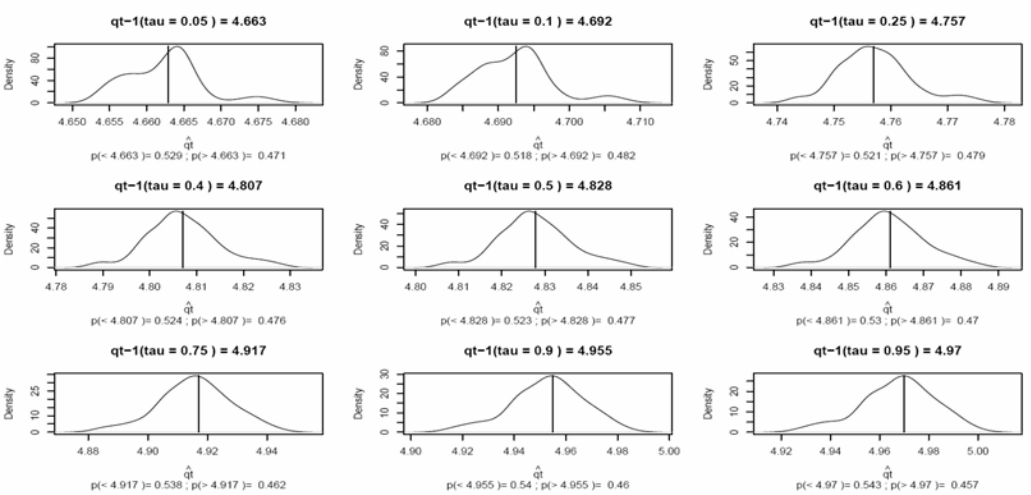


FIGURE 8

Predicted density of $RER^{it/gr}$ using the best specification at each τ

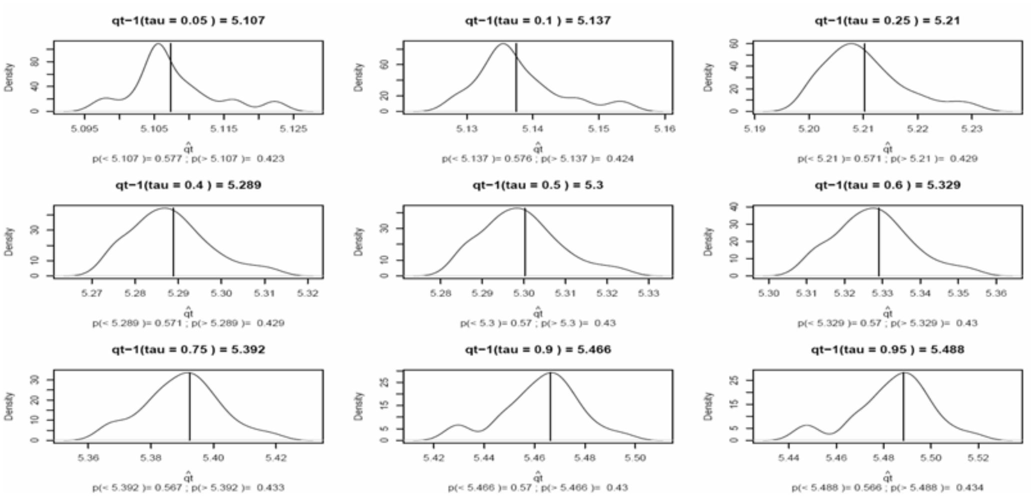


FIGURE 9

Predicted density of $RER^{it/uk}$ using the best specification at each τ . (dens_final_iuk.pdf)

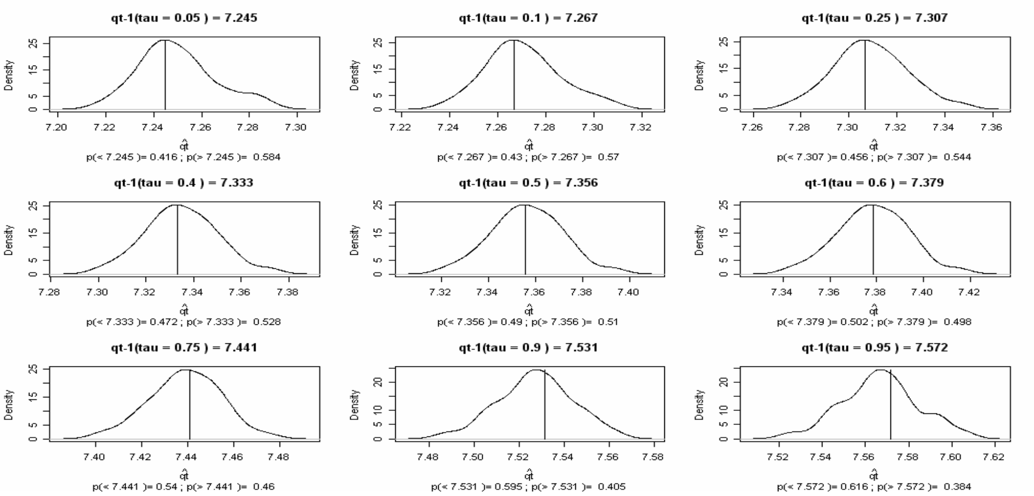
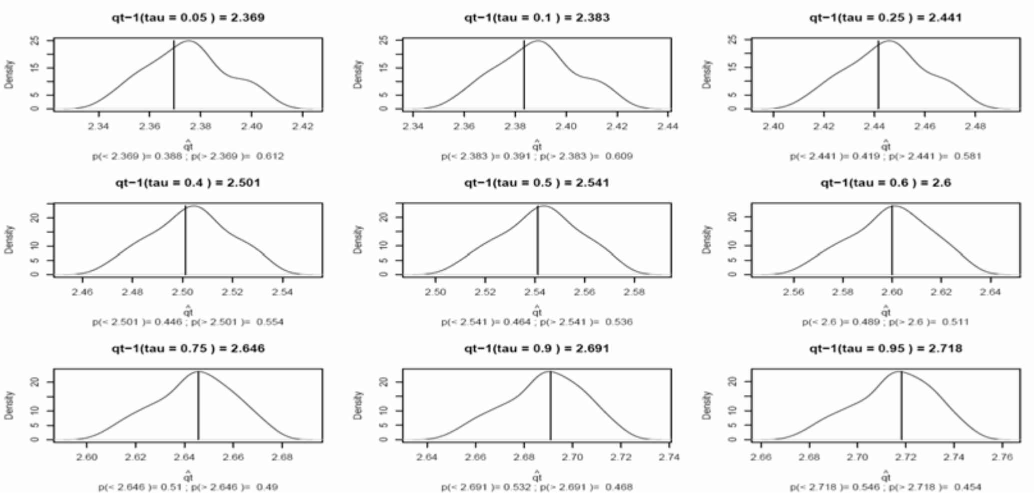


FIGURE 10

Predicted density of $RER^{fr/uk}$ using the best specification at each τ



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