**Is It "Economics and Psychology"?: The Case of Hyperbolic Discounting**

Ariel Rubinstein*

School of Economics

Tel Aviv University

Tel Aviv, Israel

and

Department of Economics

Princeton University

Princeton, New Jersey

rariel@post.tau.ac.il

http://www.princeton.edu/~ariel/

* My thanks to Michal Ilan, my research assistant in this project, to Yoram Hamo who helped me with the experiments and to Leeat Yariv for her comments. Thanks to David Laibson who helped me find my way through the bibliography. Special thanks to Rani Spiegeler for the many conversations we had on the subject and for his encouragement in writing the paper.

**Abstract**

The paper questions the methodology of "economics and psychology". It focuses on the case of hyperbolic discounting. Using some experimental results, I argue that the same sort of evidence which rejects the standard constant discount utility functions can just as easily reject hyperbolic discounting as well. Furthermore, a decision-making procedure based on similarity relations better explains the observations and is more intuitive. The paper concludes that combining "economics and psychology" requires opening the black box of decision-makers rather than modifying functional forms.

Key words: Hyperbolic discounting, choice over time, similarity, procedure, decision making.

## 1. Introduction

My interest in this paper is how we, theoretical economists, interpret experimental evidence to justify our assumptions. Rather than presenting an abstract discussion, I will focus on one currently fashionable topic in the application of economic theory: "hyperbolic discounting". A recent spate of papers have replaced the standard "constant discount utility function" with a particular form of hyperbolic discount utility function, $v(x_0)+\sum_{t=1,.2,.}(\prod_{s=1,..,t}\delta_s)v(x_t)$, where $(\delta_s)$ is an increasing sequence. Most of the literature uses a special case of this functional form: $u(x_0,x_1\ldots,x_t,\ldots)=v(x_0)+\beta\sum_{t=1,.2,.}\delta^t v(x_t)$. Rewards obtained in period 0,1,2,3,… are discounted by 1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$…. respectively. In this functional form the rate of substitution between today and tomorrow is smaller than that between any other pair of successive periods. The use of this functional form was introduced by Phelps and Pollak (1968) and gained prominence in the wake of the very influential work of David Laibson originated in Laibson (1997).

This functional form triggered a revival of the discussion of time inconsistency in the tradition of Strotz (1956). The literature assumed that at each period t*, the decision-maker uses the utility function $u(x_{t^*},x_{t^*+1},\ldots)$ to evaluate the stream of payments from period t* onward. This implies time inconsistency since $\delta$, the marginal rate of substitution between t* and t*+1 from the point of view of any previous period, is replaced by $\beta\delta$ at t*. Time inconsistency complicates the modeling of the decision-maker since assumptions must be added which specify the decision-maker's analysis of his future behavior.

I leave it to other survey papers, presently being written, to list and classify the dozens of applications of this functional form in economic contexts. Following Laibson (1997) it has been applied to a wide range of issues: growth, self-regulation, information acquisition, job search, choice of retirement age, procrastination, addiction, investment in human capital, etc. Phenomena which cannot be explained by standard discounting utility functions appear as equilibrium outcomes once the decision maker is assumed to use hyperbolic discounting. Policy questions were freely discussed in these models even though welfare assessment is particular tricky in the presence of time inconsistency. In the context of the simplified hyperbolic utility functions the literature often assumed, though with some hesitation, that the welfare criterion is the utility function with stationary discounting rate $\delta$ (which is independent of $\beta$).

It is interesting how the economic literature has justified the abandonment of the constant discounting utility and the adoption of hyperbolic discount functions. The justification has a common structure as demonstrated in Laibson (1996). The justification goes as follows: (1) Evidence: "Research on animal and human behavior has led psychologists to conclude (see Ainslie (1992) and Loewenstein and Prelec (1992)) that discount functions are generalized hyperbolas…"; (2) Hyperbolic discount functions are then introduced: "Hyperbolic discount functions generate a preference structure which is a special case of the general class of dynamically inconsistent preferences…." and finally (3) Reference to the response of the economic profession is mentioned: "Despite the availability of this analytical framework, and the substantial body of evidence supporting hyperbolic discounting, few economists have studied the implications of hyperbolic discount functions."

Over time, the justification became even more sweeping: Harris and Laibson (1999) stated (the numbers in the text were added to emphasize the structure): "(1) Laboratory and field studies of time preferences find that discount rates are much greater in the short run than in the long run. (2) To model this phenomena, psychologists have adopted discount functions from the class of generalized hyperbolas, and (3) economists have used the discrete-time quasi hyperbolic discount function: $1, \beta\delta, \beta\delta^2, \ldots, \beta\delta^t, \ldots$ ."

Within a few months the "facts" were "established" with even more certainty. Brocas and Carrilo (1999), state in a footnote: "There is well documented literature both in psychology and more recent in economics showing that individuals' discount rates are best approximated by hyperbolic rather than the traditional exponential functions. We refer the reader to Ainslie (1975), Thaler (1981) and Benzion et al. (1989) for empirical support of this theory both in animals and humans…"

We see that the literature refers not only to evidence from human beings but also from animals. As Laibson (1996) says in the abstract: "Studies of animal and human behavior suggest that discount functions are approximately hyperbolic". My own reading of several papers on animal subjects (such as Ainslie and Herrenstein (1981)) makes me wonder, whether the evidence from "nature" strengthens the case of hyperbolic discounting. Needless to say, the connection between findings on pigeons or even monkeys and behavior of human seems rather tenuous. We believe that an animal does not understand the choice it is facing in the way a human being does. What we observe are the results of a learning process. However, even if we ignore this "minor" problem, the interpretation of the results to support hyperbolic discounting involves a leap of faith. In the experiments, the "subject" is presented with a binary choice between a small amount of food, x, following a short delay t and a larger amount of food, y, following a longer delay, s. The results seem to show that the ratio of frequencies with which amount y with delay s is chosen to that of amount x with delay t, is roughly (y/s):(x/t). How do those results support the thesis that animals maximize hyperbolic discounting utility functions? The hyperbolic discount utility function "predicts" a clear deterministic choice of the alternative with the higher utility as opposed to a random choice. Thus, even if pigeons and human beings behaved similarly in their selection of saving plans and pension funds, I cannot see how these results support the hyperbolic discounting method.

In any case, the main justification for the adoption of the hyperbolic discounting utility function is empirical evidence in the cognitive psychology literature which contradicts the predictions of utility functions with stationary fixed discount rates. The results of two types of experiments were introduced to support the hyperbolic discounting case:

The first type is discussed first by Thaler (1981): Some people prefer "one apple today" to "two apples tomorrow" but at the same time they prefer "two apples in one year plus one day" to "one apple in one year". Ainslie and Haslam (1992) reports that "a majority of subjects say they would prefer to have a prize of $100 certified check available immediately over a $200 certified check that could not be cashed before 2 years; the same people do not prefer a $100 certified check that could be cashed in 6 years to a $200 certified check that could be cashed in 8 years".

Experiments of this type have been replicated with choices involving a wide range of goods (e.g., real cash, hypothetical cash, food, and access to video games) and a wide range of subject populations. Most importantly, the results seem to be confirmed by our intuition.

The second class of experiments is discussed in Thaler (1981) and Benzion, Rapoport and Yagil (1989). Subjects were asked to imagine that they had won a sum of money in a lottery and that they could either take the money now or wait for an increased amount later. They were presented with several variations of the amount and date. For each pair they had to specify the minimal amount of money they would settle for in return for the delay. If a subject was indifferent between the amount $x at time t and $y immediately, then we say that the subject's choice is consistent with the discount rate $\delta(x,t)$ defined by the equation $y=\delta(x,t)^t x$. The results show that the average discount rate is decreasing in t. However, it was also found that $\delta(x,t)$ is not constant but is an increasing function of x. The larger the sum of money at stakes, the higher (closer to 1) the discount factor.

The experiments cited above are quite persuasive. The hyperbolic discounting functional form is only marginally different from the standard utility function and seems to provide an "explanation" of the evidence. The economic paradigm of optimizing a simple functional form is "safe" and we are tempted to call the new set of models "psychology and economics".

Recall that there is an infinite number of functional forms consistent with the psychological findings. Therefore, it is sensible to pause and examine the experimental justification for hyperbolic discounting. How exactly does the experimental evidence support hyperbolic discounting? Is the choice of this form just a matter of convenience or does it capture certain psychological processes? How does this functional form stand up in other tests?

## 3. An Alternative Psychological Explanation of the Evidence

My own reading of the experimental results relies on ideas presented in Rubinstein (1988) within the context of decision making under uncertainty. This approach holds that the decision-maker uses a procedure which attempts to simplify the choice by applying similarity relations. (The important role of similarity in decision making was emphasized by Amos Tversky: see for example Tversky (1977). The first formalization of a notion of similarity is due to Luce (1956)).

The objects of choice are of the form (x,p) which is interpreted as a lottery yielding $x with probability p and $0 with probability 1-p. I believe that when comparing the lottery ($3,000, 0.25) with the lottery ($4000, 0.2), many subjects consider the two probability numbers 0.2 and 0.25 as similar; this is not the case for the dollar amounts $3000 and $4000. Thus, in the choice between ($3000, 0.25) and ($4000, 0.2), the money dimension is the decisive factor. On the other hand, when comparing the degenerate lottery ($3,000, 1) and the lottery ($4000, 0.8) the typical decision maker does not consider the dollar amounts or the probability numbers to be similar and applies a different criterion (such as maximizing expected payoff or risk aversion).

In our present discussion, the objects of choice are of the form (x,t) whereby $x is received with a delay of t units of time. I think that when comparing two pairs (x,t) and (y,s), many decision makers go through a three-stage procedure using two similarity relations, one in the money dimension and one in the time dimension:

Stage I) The decision maker looks for dominance: If x>y and t<s then the pair (x,t) is preferred over (y,s).

Stage II) The decision maker looks for similarities between x and y and between t and s. If he finds similarity in one dimension only, he determines his preference between the two pairs using the dimension in which there is no similarity. For example, if t is similar to s but x is not similar to y, and x>y , then (x,t) is preferred over (y,s).

Stage III) If the first two stages were not decisive the choice is made using a different criterion.

Much of the analysis in Rubinstein (1988) can be applied here. However, note that the role of probability 0 is replaced here with time equal to infinity and the role of probability 1 is replaced with time equal to 0.

The experimental findings of time inconsistency described in the previous section are compatible with the application of this procedure. Consider, for example, a decision-maker who is applying the above procedure and determines that "today" and "a year from now" are not similar while "10 years" and "11 years" are. Then, if the decision maker is indifferent between $x today and $y in a year from now, it must be that x<y and that x and y are not similar (if they were similar then the subject would prefer $x today over $y in a year from now). On the other hand, if a subject is indifferent between $x in 10 years and $z in 11 years, then it must be that x<z and that x and z are similar (if x and z were not similar, then since 10 and 11 years are, the above procedure would find $z in 11 years to be preferred over $x in 10 years). If y is similar to x and z is not and both are greater than x, then one would expect z to be smaller than y.

Both the hyperbolic discounting approach and the procedural approach are consistent with the evidence. However, in the next section I will try to persuade the reader that the procedural approach is in fact superior to hyperbolic discounting in explaining the experimental results.

## 3. Experiments

The following two pairs of experiments demonstrate that while the behavior of a significant number of subjects is incompatible with the hyperbolic discounting hypothesis, it is consistent with a plausible application of the above procedural approach.

Following some pilot experiments done primarily at Tel-Aviv University, a two-stage experiment was conducted. The subjects were students in a Political Science class at

Princeton University (39% freshmen, 27% sophomores, 19% juniors and 15% seniors). The teacher estimated that 10-15% of the subjects were majoring in economics. The students were approached twice with an interval of 14 days. In each round, the students were asked to enter a web site designed for the experiment and to respond on-line to three questions. (See http://www.princeton.edu/~ariel/discounting1 and http://www.princeton.edu/~ariel/discounting2 for the original forms.) A prize of $100 was randomly awarded to one of the participants in each round.

The technique was quite successful with 165 students responding to the first message and 145 to the second. Of the 145 students, 45% did not participate in the first round making it possible to check whether the fact that students participated or not in the first round made a difference (which it did not).

**Experiment 1-2**

1) You can receive the amounts of money indicated according to one of the two following schedules:

| | | | | |
|---|---|---|---|---|
| A | April 1<br>$ 1000 | July 1<br>$1000 | Oct 1<br>$1000 | Dec 1<br>$1000 |
| B | March 1<br>$ 997 | June 1<br>$997 | Sept 1<br>$997 | Nov 1<br>$997 |

Which do you prefer?

2) You have to choose between:

A Receiving $ 1000 on Dec 1st.

B Receiving $ 997 on Nov 1st.

Your choice is:

The hyperbolic discounting approach predicts that every subject choosing B in Q2 will choose B in Q1. If a subject chooses B in Q2, then he is ready to sacrifice $3 in order to advance the payment due in December by one month. The hyperbolic discounting theory predicts that he would find the three dollar sacrifice worthwhile in order to advance any of the other three scheduled payments by one month.

The results contradict this prediction: 54% of the subjects chose B in Question 2, while only a smaller number of subjects, 34%, chose schedule B in question 1. In particular, 22% of the subjects who answered both questions chose 1A and 2B whereas only 6% chose 1B and 2A.

| | | **Q2** | | | | |
|---|---|---|---|---|---|---|
| | | 1x1000 | 1x997 | No answer | | |
| | 4x1000 | 33 | 19 | 56 | **108** | 66% |
| **Q1** | 4x997 | 6 | 23 | 27 | **56** | 34% |
| | No answer | 28 | 36 | | 64 | |
| | | **67** | **78** | 83 | 228 | |
| | | 46% | 54% | | | |

My explanation of the results in terms of the above procedural approach is as follows: In Q1 many subjects viewed the alternative as a pair, a sequence of dates and a sequence of $ amounts. The sequence of dates (April 1, July 1, Oct 1, Dec 1) was considered similar to the sequence (March 1, June 1, Sept 1, Nov 1) while the sequence of payments ($1000, $1000, $1000, $1000) was considered less similar to ($997, $997, $997, $997) than $1000 was to $997. These subjects chose A over B in Q1; however, responding to Q2, they found both the dates and the amounts similar and activated stage III of the decision procedure.

**Experiment 3-4**

| |
|---|
| 3) In 60 days you are supposed to receive a new stereo system to replace your current one. Upon receipt of the system, you will have to pay $960. Are you willing to delay the transaction by one day for a discount of $2? |

| |
|---|
| 4) Tomorrow you are supposed to receive a new stereo system to replace your current one. Upon receipt of the system, you will have to pay $1,080. Are you ready to delay the delivery and the payment by 60 days for a discount of $120? |

According to the hyperbolic discount approach whoever is not willing to accept $2 for a one-day delay in delivery 60 days from now should not agree to a postponement of 60 days from the present in exchange for $120. The results don't confirm this prediction: 43% of the subjects rejected the delay in Q3 while only 31% rejected the delay in Q4. Of the 84 participants who answered both questions, almost a quarter made a switch which contradicted the direction predicted by the hyperbolic discounting approach.

| | | Q4 ($120) | | | | |
|---|---|---|---|---|---|---|
| | | delay | immediate | No answer | | |
| | Delay | 37 | 10 | 46 | **93** | 57% |
| **Q3 ($2)** | Immediate | 20 | 14 | 37 | **71** | 43% |
| | No answer | 43 | 21 | | 64 | |
| | | **100** | **45** | 83 | 228 | |
| | | 69% | 31% | | | |

The manipulation of subjects' behavior in this experiment, as in Experiment 1-2, was accomplished by triggering the similarity relation with regard to the <u>money</u> dimension. In Q3, some subjects stopped in the second phase of the procedure after judging that a two dollar difference leaves the payments in the two alternatives quite similar whereas a delay in payment of 60 days is much less similar to one of 61 days. In Q4, I think that subjects found both the payments and the delays not to be similar.

## 4. Experiment 5-6

The last pair of experiments introduced an additional element of the tradeoff between time and money. It demonstrates that there are other psychological phenomena which are not exhibited by hyperbolic discount utility functions.

| | |
|---|---|
| 5) | Choose between:<br>A) Receiving $ 1000 on 1/1/2002.<br>B) Receiving $ 500 on 1/1/2001 and $500 on 1/1/2003. |

| | |
|---|---|
| 6) | You have just bought a computer and you have to<br>Choose between two payment schemes:<br><br>A) Pay $1500 on April 15th .<br>B) Pay $500 on each of these dates: March 15th , April 15th and May 15th . |

The results are presented in the following table:

| | | Q6 | | | | |
|---|---|---|---|---|---|---|
| | | Pay $1500 | Pay 3x$500 | No answer | | |
| | Get 2x$500 | 19 | 27 | 38 | **84** | 58% |
| **Q5** | Get $1000 | 20 | 15 | 26 | **61** | 42% |
| | No answer | 32 | 51 | | 83 | |
| | | **71** | **93** | 64 | 228 | |
| | | 43% | 57% | | | |

In Q5, present value calculations should have lead subjects to prefer the option with two installments, however 42% of the subjects preferred one payment, presumably activated a sort of preference of the "middle". In Q6, time considerations should have lead subjects to prefer one payment, however, 57% preferred 3 payments. Since subjects had to choose between options which involved "losses", they presumably adopted "middle" aversion. This interpretation of the results, namely the existence of "middle preference" regarding gains, and "middle aversion" regarding losses, is compatible with Kahneman and Tversky's notion of loss aversion (see, for example, Tversky and Kahneman (1991)).

## 6. Discussion

The main justification for the use of hyperbolic discount utility functions is based on experimental observations which reject constant discount utility functions. In this paper I have argued that the same sort of evidence can just as easily reject hyperbolic discounting as well. Furthermore, the procedure based on similarity better explains the observations and is more intuitive.

In defense of hyperbolic discounting one might mention its analytical convenience. The hyperbolic discount utility functions leads to richer results than the standard constant discount function, yet possesses a functional form close enough to the standard one to allow the employment of standard techniques. However, analytical convenience should be used with caution especially when the model is motivated by real life evidence in the first place. In my opinion, the hyperbolic discounting approach misses the core of the psychological process and thus constitutes only a minor modification of the standard discounting approach while suffering from some normative disadvantages.

I fully agree that the hyperbolic discounting approach does capture psychological phenomena in which the present is given special treatment. However, the fact that it uses a very specific functional form means that much more than this psychological phenomena is being implicitly assumed.

It is interesting to contrast the warmth of the "profession" towards the hyperbolic discount literature with the more hostile response to non-expected utility theories. Observations such as the Allais paradox motivated many economists and decision theorists in the Fifties and Sixties to search for alternative functional forms to expected utility which would be consistent with the psychological evidence regarding decision making under uncertainty. The functional forms which were suggested as substitutions for expected utility were not less empirically motivated than the hyperbolic discount utility functions and were certainly more established axiomatically. Despite this, we have seen only a few (recent) applications of non-expected utility theories to economic problems whereas the hyperbolic discounting approach immediately entered the mainstream of economics. I find it puzzling.

Adopting the similarity-based procedural approach may require revolutionary changes in our theories. As discussed in Rubinstein (1988), the application of similarity-based procedures may result in conflicts with transitivity since the transitive closure of the partial relation as determined in the first two stages of the procedure is not likely to be consistent with the third stage. Doing "economics and psychology" requires much more than citing experimental results and marginally modifying our models. We need to open the black box of decision-makers, investigate the procedures used and come out with completely new and fresh modeling devices.

## References

Ainslie, G. (1992) Picoeconomics, Cambridge: Cambridge University Press.

Ainsley, G. and N. Haslam (1992) "Hyperboloc Disocunting", in Choice over Time, Loewenstein, G. and J. Elster (editors), Russell Sage Foundation.

Ainslie, G. and R. Herrenstein (1981) "Preference Reversal and Delayed Reinforcement", Animal Learning and Behavior, 9, 476-482.

Benzion, U., A. Rapoport and J.Yagil (1989) "Discount Rates Inferred from Decisions: An Experimental Study", Management Science, 35, 270-284.

Brocas, I. and J.Carrilo (1999), “Entry Mistakes, Entrepreneurial Boldness and Optimism”, mimeo.

Fishburn, P. and A. Rubinstein (1982) "Time Preference", International Economic Review, 23, 677-694.

Harris, C. and D. Laibson (1999) "Dynamic Choices of Hyperbolic Consumers", mimeo.

Laibson, D. (1996) "Hyperbolic Discount Functions, Undersaving, and Savings Plans", NBER Working Paper 5635.

Laibson, D. (1997) "Golden Eggs and Hyperbolic Discounting", Quarterly Journal of Economics, 112, 443-477.

Lowenstein, G. and R.H. Thaler (1989) "Intertemporal Choice", Journal of Economic Perspectives, Volume 3, Number 4, 181-193.

Luce, L. (1956) "Semiorders and a Theory of Utility Discimination", Econometrica, 24, 178-191.

Phelps, E.S. and R.A.Pollak (1968) "On Second-Best National Saving and Game-Equilibrium Growth", Review of Economic Studies, 35, 201-208.

Rubinstein, A. (1988) " Similarity and Decision-Making Under Risk", Journal of Economic Theory, 46 (1988), 145-153.

Strotz, R.H. (1956) "Myopia and Inconsistency in Dynamic Utility Maximization", Review of Economic Studies, 23, 165-180.

Thaler, R. (1981) "Some Empirical Evidence on Dynamic Inconsistency", Economic Letters, 8, 201-207.

Tversky, A. (1977) "Features of Similarity", Psychological Review, 84, 327-352.

Tversky, A. and D.Kahneman (1991) "Loss Aversion in Riskless Choice: A Reference-Dependent Model", Quarterly Journal of Economics, 106, 1039-1061.